Improved Rotor-Position Estimation by Signal Injection in Brushless AC Motors, Accounting for Cross-Coupling Magnetic Saturation

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Abstract—This paper presents an improved signal-injection-based sensorless-control method for permanent-magnet brushless ac (BLAC) motors, accounting for the influence of cross-coupling magnetic saturation between the d- and q-axes. The d- and q-axis incremental self-inductances, the incremental mutual inductance between the d-axis and q-axis, and the cross-coupling factor are determined by finite-element analysis. An experimental method is proposed for measuring the cross-coupling factor which can be used directly in the sensorless-control scheme. Both measurements and predictions show that a significant improvement in the accuracy of the rotor-position estimation can be achieved under both dynamic and steady-state operation compared with that which is obtained with the conventional signal-injection method.

Index Terms—Brushless ac (BLAC) motor, cross-coupling effect, sensorless, signal injection.

I. INTRODUCTION

Accurate rotor-position information is an essential requirement for permanent-magnet (PM) brushless ac (BLAC) drives. Thus, either an encoder or a resolver is usually employed. However, this increases the cost and complexity and may compromise the reliability. Hence, there has been a significant development effort on sensorless techniques, which estimate the rotor position indirectly from the phase voltages and currents.

Various back-EMF-based rotor-position-estimation methods have been developed for PM BLAC motors [1]–[3], which all estimate the rotor position from the governing voltage equation, their difference being in the way that the rotor position and speed estimation are deduced. However, they require accurate stator-winding inductances and resistances, may be problematic at low speed, and cannot be used for starting.

Hence, the most popular sensorless-control method for starting and low-speed operation is based on injecting a high-frequency voltage signal into the phase windings and measuring the resultant high-frequency current. Such a method was originally developed for induction motors and was subsequently extended to PM BLAC motors with saliency [4]–[6]. The identification of the initial rotor polarity, which was not reported in [4]–[6], was achieved in [7] and [8] by comparing the sine and cosine terms of the second harmonic component in the d-axis high-frequency current. In all the foregoing papers, the signal-injection technique was applied to BLAC motors, which have geometric rotor saliency. In [9], it was extended to a BLAC motor with a surface-mounted PM rotor by utilizing the saliency which resulted from magnetic saturation. In [5]–[9], the high-frequency signal was injected into the d-axis, which was aligned with the rotor. Alternatively, the high-frequency signals can also be injected into an α/β-axis [10], [11], which are fixed with the stator winding. Since the signals are injected into the α/β-axis rather than the estimated d-axis, the control strategy in [10] and [11] may be more stable than the other methods.

However, it was found experimentally [5] that the error in the estimated rotor position increased with the load current, although the problem was not specifically addressed. More recently, it has been shown in [12] and [13] that the rotor-position-estimation error was caused by dq-axis cross-coupling, i.e., \( L_{dqh} \neq 0 \) and was dependent on the machine design, although no measures were taken to reduce the error. In [10] and [11], the equivalent machine model was used as in [5], except that it was transformed from the dq-axis to the α/β-axis reference frame. When \( L_{dqh} \neq 0 \), it was described as the ‘saliency offset’ in \( L_{rh}, L_{qh}, \) and \( L_{αβh} \). It is well known that the mutual inductance between the d- and q-axes \( (L_{dqh}) \) of a BLAC motor results from cross-coupling due to magnetic saturation, as shown in [14], by both measurement and finite-element analysis. However, for simplicity, the influence of cross-coupling magnetic saturation is usually neglected in the electromagnetic modeling [15], [16], vector control [17], [18], and sensorless position control [5]–[9] of BLAC motors.

This paper improves the accuracy of the rotor-position estimation by accounting for the influence of cross-coupling magnetic saturation in the signal-injection-based sensorless method. Section II presents the underlying theory and analyzes the rotor-position-estimation error which results when the influence of cross-coupling is neglected. Section III proposes a simple way of accounting for the cross coupling based on data which can be obtained either by finite-element analysis or from measurements. Finally, predicted and measured results are compared in


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Section IV to validate the proposed improved signal-injection sensorless-control method.

II. ANALYSIS OF ROTOR-POSITION-ESTIMATION ERROR CAUSED BY CROSS COUPLING

When cross coupling between the d- and q-axes is considered, the voltage equations of a BLAC motor, in the rotor dq-axis reference frame, are given by [14]

\[
\begin{bmatrix}
    v_d \\
    v_q
\end{bmatrix}
= \begin{bmatrix}
    R_s + L_{dh}p & -\omega_r L_q + L_{dqh}p \\
    \omega_r L_d + L_{qdh}p & R_s + L_{qh}p
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    \omega_r \psi_m
\end{bmatrix}
\]  

(1)

where \( R_s \) is the stator-winding phase resistance and \( \psi_m \) is the flux linkage per phase due to the PMs, \( \omega_r \) is the angular rotational speed, and \( p = d/dt \). \( v_d, v_q \) and \( i_d, i_q \) are the \( d- \)and \( q- \)axis voltages and currents, \( L_d \) and \( L_q \) are the apparent \( d-q \)-axis self-inductances, respectively, and \( L_{dh}, L_{qh}, L_{dqh}, \) and \( L_{qdh} \) are the \( d- \) and \( q- \)axis incremental self- and mutual inductances, defined by

\[
\begin{align*}
L_{dh} &= \frac{\psi_d(i_d + \Delta i_d, i_q, \Phi_m) - \psi_d(i_d, i_q, \Phi_m)}{\Delta i_d} \\
L_{qh} &= \frac{\psi_q(i_d, i_q + \Delta i_q, \Phi_m) - \psi_q(i_d, i_q, \Phi_m)}{\Delta i_q} \\
L_{dqh} &= \frac{\psi_q(i_d, i_q + \Delta i_q, \Phi_m) - \psi_q(i_d, i_q, \Phi_m)}{\Delta i_d} \\
L_{qdh} &= \frac{\psi_q(i_d + \Delta i_d, i_q, \Phi_m) - \psi_q(i_d, i_q, \Phi_m)}{\Delta i_q}
\end{align*}
\]  

(2)

where \( \Phi_m \) is the PM flux.

Due to magnetic saturation, \( L_{dh}, L_{qh}, L_{dqh}, \) and \( L_{qdh} \) vary with both \( i_d \) and \( i_q \). By way of example, Fig. 1 shows the finite-element calculated incremental-winding inductances for the interior-magnet BLAC motor whose parameters are given in Table I. As can be seen, \( L_{dh} \) and \( L_{qh} \) reduce as \( i_d \) and \( i_q \) are increased.

When only the high-frequency signal-injection components are considered, (1) can be approximated by

\[
\begin{bmatrix}
    v_{dh} \\
    v_{qh}
\end{bmatrix}
= \begin{bmatrix}
    L_{dh} & L_{dqh} \\
    L_{qdh} & L_{qh}
\end{bmatrix}
\begin{bmatrix}
    i_{dh} \\
    i_{qh}
\end{bmatrix}
\]  

(3)

Equation (3) can be transformed from the rotor-position reference frame \((\theta_c)\) to the estimated rotor-position reference frame \((\theta_e)\) by the following transformation matrix \( T(\Delta \theta) \), viz.,

\[
T(\Delta \theta) = \begin{bmatrix}
    \cos(\Delta \theta) & \sin(\Delta \theta) \\
    -\sin(\Delta \theta) & \cos(\Delta \theta)
\end{bmatrix}
\]  

(4)

where \( \Delta \theta \) is the error in the estimated rotor position, i.e., \( \Delta \theta = \theta_e - \theta_c \), is the difference between the estimated rotor position and the actual rotor position. Hence, we have (5), shown at the bottom of the next page, where

\[
\begin{align*}
L_{avg} &= \frac{L_{gh} + L_{dh}}{2} \\
L_{dif} &= \frac{L_{gh} - L_{dh}}{2}
\end{align*}
\]  

(6)

\[
\begin{align*}
\theta_m &= \arctan(L_{dqh}/L_{dif}) = \arctan[2L_{dqh}/(L_{gh} - L_{dh})] \\
\Delta \theta &= \sqrt{L_{dif}^2 + L_{dqh}^2}
\end{align*}
\]  

(7)

Since the high-frequency voltage signal, \( v_{sig} = V_{inj} \sin(2\pi f_H t) \), is applied to the \( d- \) axis, (5) becomes (8), as shown at the bottom of the next page.
The high-frequency $d$- and $q$-axis currents in the estimated rotor-position reference frame are obtained as

$$
\begin{align*}
\vec{i}_{dh}^e &= \frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} \left[ L_{avg} + \tilde{L}_{dif} \cos(2\Delta \theta + \theta_m) \right] \\
\vec{i}_{qh}^e &= -\frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} \tilde{L}_{dif} \sin(2\Delta \theta + \theta_m). 
\end{align*}
$$

(9)

In the conventional signal-injection-based sensorless method, cross coupling between the $d$- and $q$-axes is neglected, and the high-frequency component in the $q$-axis current is assumed to be proportional to the estimation error in rotor position [9]. Therefore, $\vec{i}_{qh}^e$ is forced to zero in order to make the estimation error zero in the conventional method, i.e.,

$$
\vec{i}_{qh}^e = \frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} \tilde{L}_{dif} \sin(2\Delta \theta + \theta_m) = 0. 
$$

(10)

In this case, an error exists in the estimated rotor position as a result of the incremental mutual inductance between the $d$- and $q$-axes $L_{dqh}$. The error is given by

$$
\Delta \theta = \theta_r^e - \theta_r = -\theta_m/2 \approx \frac{1}{2} \arctan \left( \frac{2L_{dqh}}{L_{dh} - L_{qh}} \right) 
$$

(11)

where $\theta_r^e$ and $\theta_r$ are the estimated and actual rotor positions, respectively.

Clearly, the rotor-position error will be zero only when $L_{dqh} = 0$, while the stronger the cross coupling between the $d$- and $q$-axes, the larger will be the error, as will be shown later.

III. IMPROVED SIGNAL-INJECTION SENSORESS METHOD

In [19], the error in the estimated rotor position due to the neglect of the incremental mutual inductance between the $d$- and $q$-axes $L_{dqh}$ was analyzed by finite-element analysis and experimentally, and a method was proposed to directly compensate for the error by employing (11) based on either the finite-element predicted incremental inductances, $L_{dh}$, $L_{qh}$, $L_{dqh}$, which are shown in Fig. 1, or using premeasured errors. However, this improved method was still essentially based on the conventional sensorless method which forces the high-frequency component in the $q$-axis current to be zero, together with the error compensation.

In this section, an alternative method of improving the signal-injection-based sensorless method is proposed to eliminate the potentially large error in the estimated rotor position caused by the influence of $d$- and $q$-axis cross coupling, i.e., $L_{dqh} \neq 0$. Rather than forcing the high-frequency component in the $q$-axis current to be zero, as in the conventional sensorless method, an optimal relationship between the high-frequency components of the $d$- and $q$-axis currents is established from the analysis presented in Section II.

When the rotor-position-estimation error $\Delta \theta$ is sufficiently small, (9) can be approximated by

$$
\vec{i}_{dh}^e \approx \frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} (L_{qh} - 2L_{dqh} \cdot \Delta \theta) 
$$

(12a)

$$
\vec{i}_{qh}^e \approx \frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} (-2L_{dqh} - 2L_{dih} \cdot \Delta \theta). 
$$

(12b)

Both sides of (12a) are multiplied by $L_{dqh}/L_{qh}$ and added to (12b) to give

$$
\vec{i}_{qh}^e + \lambda \vec{i}_{dh}^e = \frac{v_{sig}}{p(L_{avg}^2 - L_{dif}^2)} (-2L_{dih} - 2L_{dqh}) \cdot \Delta \theta 
$$

(13)

where $\lambda = L_{dqh}/L_{qh}$ is defined as the coupling factor, which can either be calculated from a knowledge of the machine parameters or deduced experimentally, as shown next. Thus, when the effect of cross coupling is considered, the rotor-position-estimation error $\Delta \theta$ can be calculated as

$$
\Delta \theta = \frac{L_{avg}^2 - L_{dif}^2}{-2v_{sig}(L_{dif} + \lambda L_{dqh})} \cdot \Delta \theta = \tilde{k}_{HF} \cdot (\vec{i}_{qh}^e + \lambda \vec{i}_{dh}^e) 
$$

(14)

where $\tilde{k}_{HF} = (L_{avg}^2 - L_{dif}^2)/(-2v_{sig}(L_{dif} + \lambda L_{dqh}))$ is a coefficient which enables the rotor-position-estimation error to be determined from $(\vec{i}_{qh}^e + \lambda \vec{i}_{dh}^e)$. In (13) and (14), it is noted that the rotor-position-estimation error $\Delta \theta$ is proportional to

$$
\begin{align*}
\begin{bmatrix}
\vec{i}_{dh}^e \\
\vec{i}_{qh}^e
\end{bmatrix} &= T(\Delta \theta) \begin{bmatrix}
\vec{i}_{dh} \\
\vec{i}_{qh}
\end{bmatrix} = T(\Delta \theta) \begin{bmatrix}
L_{dh} & L_{dqh} \\
L_{dqh} & L_{qh}
\end{bmatrix} T^{-1}(\Delta \theta) \cdot p \begin{bmatrix}
\vec{i}_{dh}^e \\
\vec{i}_{qh}^e
\end{bmatrix} \\
&= \begin{bmatrix}
L_{avg} - \tilde{L}_{dif} \cos(2\Delta \theta + \theta_m) & \tilde{L}_{dif} \sin(2\Delta \theta + \theta_m) \\
\tilde{L}_{dif} \sin(2\Delta \theta + \theta_m) & L_{avg} + \tilde{L}_{dif} \cos(2\Delta \theta + \theta_m)
\end{bmatrix} \begin{bmatrix}
\vec{i}_{dh}^e \\
\vec{i}_{qh}^e
\end{bmatrix} 
\end{align*}
$$

(5)

$$
\begin{align*}
\begin{bmatrix}
v_{sig} \\
0
\end{bmatrix} &= \begin{bmatrix}
L_{avg} - \tilde{L}_{dif} \cos(2\Delta \theta + \theta_m) & \tilde{L}_{dif} \sin(2\Delta \theta + \theta_m) \\
\tilde{L}_{dif} \sin(2\Delta \theta + \theta_m) & L_{avg} + \tilde{L}_{dif} \cos(2\Delta \theta + \theta_m)
\end{bmatrix} \begin{bmatrix}
\vec{i}_{dh}^e \\
\vec{i}_{qh}^e
\end{bmatrix} 
\end{align*}
$$

(8)
can be controlled to be zero. Hence, the determination of the coupling factor is simplified by representing the data in Fig. 2 by the following simple function, viz.:

\[
\lambda = \begin{cases} 
-k_1 \cdot \lambda^e, & \lambda^e \geq 0 \\
-(k_1 + k_2 \cdot \lambda^e) \cdot \lambda^e, & \lambda^e < 0 
\end{cases}
\]  

(18)

where \(k_1\) and \(k_2\) are coefficients. For the BLAC motor under consideration, \(k_1 = 0.05 \text{ A}^{-1}\) and \(k_2 = 0.011 \text{ A}^{-2}\) from the finite-element calculated value of \(\lambda\) [Fig. 2(a)], and \(k_1 = 0.06 \text{ A}^{-1}\), \(k_2 = 0.011 \text{ A}^{-2}\) from the measured values of \(\lambda\) [Fig. 2(b)].

In practice, multipliers and low-pass filters are used to obtain the high-frequency \(d-\) and \(q-\)axis current components, \(i_{dh}^e\) and \(i_{qh}^e\) [9], for use in (14) to calculate the error \(\Delta \theta\) in the estimated rotor position. \(\Delta \theta\) is then input to a proportional–integral regulator to correct the estimated rotor position \(\theta^e\) and speed \(\omega^e\). Hence, since the cross coupling is now accounted for in the rotor-position estimation, the accuracy is improved significantly.

Clearly, when the cross coupling is sufficiently small compared with \(L_{qh}\), i.e., \(L_{dqh} \approx 0\) and, hence, \(\lambda \approx 0\), the proposed sensorless-control strategy results in \(i_{dqh} = 0\) and is identical to the conventional signal-injection-based sensorless method.

**IV. SIMULATED AND MEASURED RESULTS**

In order to demonstrate the utility of the improved signal-injection method, both simulated results and measurements have been obtained for the BLAC motor, whose parameters are given in Table I.

The sensorless controller was implemented on a TMS320C31 DSP, while a 1024 pulse/r encoder was used to measure the actual rotor position \(\theta_r\). The A/D sampling and pulsewidth-modulation switching frequency were both 5 kHz, while the injected sinusoidal signal was 35 V, 330 Hz. The control-loop cycle was 200 \(\mu\)s, and the measured coupling factor \(\lambda\), shown in Fig. 2(b) and represented by (18), was used (Fig. 3), for which a flowchart is shown in Fig. 4.

Fig. 5 shows the estimated and actual rotor position when the demanded rotor speed is changed from \(-10\) to \(+10\) Hz, i.e., \(-200\) to \(+200\) t/min. As will be evident from Fig. 5(a), when the conventional signal-injection-based sensorless method is employed, good agreement is only achieved when the current is relatively small, the error in the estimated rotor position...
increasing significantly with the current, e.g., the error is $25^\circ$ electrical when $i_q = 4$ A. However, when the improved signal-injection method is employed, this error reduces to...
Fig. 6. Simulated step-speed responses (speed command: ±10 Hz). (a) Conventional signal-injection method. (b) Improved signal-injection method.

5° (electrical) Fig. 5(b). This improvement is also confirmed by the simulated results shown in Fig. 6.

Fig. 7 shows the measured steady-state error in the estimated rotor position for various d- and q-axis currents, $i_d$ and $i_q$, when the estimated rotor position is used for position feedback. With the conventional signal-injection method Fig. 7(a), the error increases as both the absolute value of $i_q$ and the positive value of $i_d$ are increased, due to increased saturation and the consequent increase in the influence of cross coupling. For example, the maximum error in full dq-axis current ranges, as shown in Fig. 7(a), to 45.0° electrical. From Fig. 7(b), when the improved signal-injection method is employed, the maximum error is reduced to 6.1° electrical. This improvement is confirmed by the simulated results shown in Fig. 8, for which the corresponding maximum errors in the estimated rotor position are 37.4° electrical and 2.7° electrical, respectively.

V. CONCLUSION

Cross-coupling magnetic saturation between the d- and q-axes of a PM BLAC motor can significantly influence the accuracy of rotor-position estimation by signal-injection-based methods. This paper has presented an improved signal-injection method which accounts for the cross-coupling effect. The d- and q-axis incremental self-inductances and the incremental mutual inductance between the d- and q-axes have been calculated by finite-element analysis and a cross-coupling factor determined. An experimental method has been proposed to

Fig. 7. Measured rotor-position-estimation error. (a) Conventional method, MAX($\theta_{e_r} - \theta_r$) = 45.0°. (b) Improved method, MAX($\theta_{e_r} - \theta_r$) = 6.1°.

Fig. 8. Simulated rotor-position-estimation error. (a) Conventional method, MAX($\theta_{e_r} - \theta_r$) = 37.4°. (b) Improved method, MAX($\theta_{e_r} - \theta_r$) = 2.7°.
determine the cross-coupling factor directly for subsequent use in the proposed sensorless-control scheme. Both measurements and simulations show that a significant improvement in the accuracy of the rotor-position estimation can be achieved, in both dynamic and steady-state modes of operation, compared with the conventional signal-injection sensorless method.

References


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