CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Archivio istituzionale della ricerca - Università di Trieste

ScienceDirect
IFAC PapersOnline CONFERENCE PAPER ARCHIVE
IFAC-PapersOnLine 48-21 (2015) 066–071

Distributed Adaptive Fault-Tolerant Control of Uncertain Multi-Agent Systems

Mohsen Khalili * **Xiaodong Zhang** ** **Marios Polycarpou** *** **Thomas Parisini** **** **Yongcan Cao** †

* *Wright State University, Dayton, OH 45435, USA (e-mail: khalili.4@wright.edu).*
** *Wright State University, Dayton, OH 45435, USA (e-mail: xiaodong.zhang@wright.edu).*
*** *Department of Electrical and Computer Engineering, University of Cyprus, Nicosia, Cyprus (e-mail: mpolycar@ucy.ac.cy).*
**** *Imperial College London and University of Trieste (e-mail: t.parisini@gmail.com).*
† *Air Force Research Laboratory, Wright-Patterson AFB, OH 45435, USA (e-mail: yongcan.cao@gmail.com).*

Abstract: This paper presents an adaptive fault-tolerant control (FTC) scheme for a class of nonlinear uncertain multi-agent systems. A local FTC scheme is designed for each agent using local measurements and suitable information exchanged between neighboring agents. Each local FTC scheme consists of a fault diagnosis module and a reconfigurable controller module comprised of a baseline controller and two adaptive fault-tolerant controllers activated after fault detection and after fault isolation, respectively. Under certain assumptions, the closed-loop system's stability and leader-follower consensus properties are rigorously established under different modes of the FTC system, including the time-period before possible fault detection, between fault detection and possible isolation, and after fault isolation.

© 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Fault-tolerant control, Multi-agent systems, Consensus

1. INTRODUCTION

Several modern technical systems can be characterized by distributed multi-agent systems, that is, systems comprised of various distributed and interconnected autonomous agents/subsystems. Examples of such systems include cooperative unmanned vehicles, smart grids, air traffic control system, etc. In recent years, cooperative control using distributed consensus algorithms has received significant attention (see, e.g., Ren and Beard (2008)). Since the overall distributed multi-agent systems are required to operate reliably at all times, despite the possible occurrence of faulty behaviors in some agents, the development of fault diagnosis and accommodation schemes is a crucial step in achieving reliable and safe operations.

In the last two decades, significant research progress has been made in the design and analysis of fault diagnosis and accommodation schemes (see, for instance, Blanke et al. (2006)). Most of these methods utilize a centralized architecture, where the diagnostic module is designed based on a global mathematical model of the overall system and is required to have real-time access to all sensor measurements. Because of limitations of computational resource and communication overhead, such centralized methods may not be suitable for large-scale distributed interconnected systems. As a result, in recent years, there has been a significantly increasing research interest in distributed fault diagnosis schemes for multi-agent systems (see, for instance, Yan and Edwards (2008); Ferrari et al. (2012); Shames et al. (2011)).

This paper presents a distributed adaptive FTC methodology for accommodating faults in a class of nonlinear uncertain multi-agent systems. A FTC scheme is designed for each agent in the distributed system by utilizing local measurements and suitable information exchanged between neighboring agents. Each local FTC scheme consists of two main modules: 1) the online health monitoring (fault diagnosis) module consists of a bank of nonlinear adaptive estimators. One of them is the fault detection estimator, while the others are fault isolation estimators; and 2) the reconfigurable controller (fault accommodation) module consists of a baseline controller and two adaptive fault-tolerant controllers used after fault detection and after fault isolation, respectively. Under certain assumptions, the closed-loop system's stability and leader-following consensus properties are established for the baseline controller and adaptive fault-tolerant controllers. This paper significantly extends the results of (Zhang et al. (2004)) by generalizing the centralized FTC method to the case of leader-follower formation of distributed multi-agent systems.

2. GRAPH THEORY NOTATION

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \cdots, v_P\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and P is the number of nodes. An edge is an ordered pair of distinct nodes (v_j, v_i) meaning that the ith node can receive information from jth node. For an edge (v_j, v_i), node v_j is called the parent node, node v_i the child node, and v_j is a neighbor of v_i. An undirected graph can be considered as a special case of a directed graph where $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$ for any $v_i, v_j \in \mathcal{V}$. An undirected graph is connected if there is a path between any pair of nodes. A directed graph contains a directed spanning tree if there exists a node called the root such

2405-8963 © 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.506

that the node has directed paths to all other nodes in the graph.

The set of neighbors of node υ_i is denoted by $N_i = \{j : (\upsilon_j, \upsilon_i) \in \mathcal{E}\}$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \Re^{P\times P}$ associated with the directed graph $\mathcal{G}$ is defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(\upsilon_j, \upsilon_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The topology of an intercommunication graph $\mathcal{G}$ is said to be fixed, if each node has a fixed neighbor set and a_{ij} is fixed. It is clear that for undirected graphs $a_{ij} = a_{ji}$. The Laplacian matrix $L = [l_{ij}] \in \Re^{P\times P}$ is defined as $l_{ii} = \sum_{j\in N_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. Both $\mathcal{A}$ and L are symmetric for undirected graphs and L is positive semidefinite.

3. PROBLEM FORMULATION

Consider a set of M agents with the dynamics of the ith agent, $i = 1, \cdots, M$, being described by

$$\begin{aligned} \dot{x}_i &= \phi_i(x_i) + u_i(x_i, x_J) + \eta_i(x_i, t) \\ &\quad + \beta_i(t - T_i) f_i(x_i, u_i(x_i, x_J)), \end{aligned} \tag{1}$$

where $x_i \in \Re^n$ and $u_i \in \Re^n$ are the state vector and input vector of the ith agent, respectively. Additionally, x_J contains the state variables of neighboring agents that directly communicate with agent i, including the time-varying leader to be tracked (i.e., x^r) as agent number $M + 1$, i.e., $J = \{j : j \in N_i\}$, $\phi_i : \Re^n \mapsto \Re^n$, $\eta_i : \Re^n \times \Re^+ \mapsto \Re^n$ and $f_i : \Re^n \times \Re^n \mapsto \Re^n$ are smooth vector fields. Specifically, ϕ_i and η_i represent the known nonlinearity and modeling uncertainty, respectively. The term $\beta_i(t - T_i) f_i(x_i, u_i)$ denotes the changes in the dynamics of ith agent due to the occurrence of a fault. Specifically, $\beta_i(t - T_i)$ represents the time profile of a fault which occurs at some unknown time T_i, and $f_i(x_i, u_i)$ is a nonlinear fault function. In this paper, $\beta_i(\cdot)$ is assumed to be a step function (i.e., $\beta_i(t - T_i) = 0$ if $t < T_i$, and $\beta_i(t - T_i) = 1$ if $t \geq T_i$). It is assumed that in each agent only one fault possibly occurs at any time.

For isolation purposes, we assume that there are r_i types of possible nonlinear fault functions in the fault class associated with the ith agent; specifically, $f_i(x_i, u_i)$ belongs to a finite set of functions given by

$$\mathcal{F}_i \triangleq \{f_i^1(x_i, u_i), \cdots, f_i^{r_i}(x_i, u_i)\}. \tag{2}$$

Each fault function f_i^s, $s = 1, \cdots, r_i$, is described by

$$f_i^s(x_i, u_i) \triangleq [(\theta_{i1}^s)^T g_{i1}^s(x_i, u_i), \cdots, (\theta_{in}^s)^T g_{in}^s(x_i, u_i)]^T, \tag{3}$$

where θ_{ip}^s, for $i = 1, \cdots, M$, and $p = 1, \cdots, n$, is an unknown parameter assumed to belong to a known compact set Θ_{ip}^s (i.e., $\theta_{ip}^s \in \Theta_{ip}^s \subseteq \Re^{q_{ip}^s}$), and $g_{ip}^s : \Re^n \times \Re^n \mapsto \Re^{q_{ip}^s}$ is a known smooth vector field. As described in Zhang et al. (2004), the fault model described by (2) and (3) characterizes a general class of nonlinear faults where the vector field g_{ip}^s represents the functional structure of the sth fault affecting the pth state equation, while the unknown parameter vector θ_{ip}^s characterizes the fault magnitude.

The objective of this paper is to develop a robust distributed fault-tolerant leader-following consensus control scheme, using diagnostic information, for the class of distributed multi-agent systems described by (1). The following assumptions are made throughout the paper:

Assumption 1. Each component of the modeling uncertainty, represented by $\eta_i(x_i, t)$ in (1), has a known upper bound, i.e., $\forall p = 1, \cdots, n$, $\forall x_i \in \Re^n$ and $\forall u_i \in \Re^n$,

$$|\eta_{ip}(x_i, t)| \leq \bar{\eta}_{ip}(x_i, t), \tag{4}$$

where the bounding function $\bar{\eta}_{ip}$ is known and uniformly bounded.

Assumption 2. The intercommunication topology of the distributed system described by (1) is a fixed connected undirected graph.

Assumption 1 characterizes the class of modeling uncertainty under consideration. The bound on the modeling uncertainty is needed in order to distinguish between the effects of faults and modeling uncertainty during the fault diagnosis process. Assumption 2 is needed to ensure that the information exchange among agents is sufficient for the team to achieve the desired team goal.

Let us define three important time–instants: T_i is the fault occurrence time; $T_d > T_i$ is the time–instant when a fault is detected; $T_{\text{isol}} > T_d$ is the time–instant when the monitoring system (possibly) provides a fault isolation decision, that is, which fault in the class $\mathcal{F}_i$ has actually occurred. The structure of the fault-tolerant controller for the ith agent takes on the following general form (Zhang et al. (2004)):

$$\begin{aligned} \dot{\omega}_i &= \begin{cases} g_0(\omega_i, x_i, x_J, t), & \text{for} \quad t < T_d \\ g_D(\omega_i, x_i, x_J, t), & \text{for} \quad T_d \leq t < T_{\text{isol}} \\ g_I(\omega_i, x_i, x_J, t), & \text{for} \quad t \geq T_{\text{isol}} \end{cases} \\ u_i &= \begin{cases} h_0(\omega_i, x_i, x_J, t), & \text{for} \quad t < T_d \\ h_D(\omega_i, x_i, x_J, t), & \text{for} \quad T_d \leq t < T_{\text{isol}} \\ h_I(\omega_i, x_i, x_J, t), & \text{for} \quad t \geq T_{\text{isol}} \end{cases} \end{aligned} \tag{5}$$

where ω_i is the state vector of the distributed controller; g_0, g_D, g_I and h_0, h_D, h_I are nonlinear functions to be designed according to the following qualitative objectives:

(1) In a fault free mode of operation, a baseline controller guarantees the state of ith agent $x_i(t)$ should track the leader's time-varying state x^r, even in the possible presence of plant modeling uncertainty.
(2) If a fault is detected by diagnostic scheme, the baseline controller is reconfigured to compensate for the effect of the (yet unknown) fault, that is, the fault-tolerant controller is designed in such a way as to exploit the information that a fault has occurred, so that the controller may recover some control performance. This new controller should guarantee the boundedness of system signals and some leader-following consensus performance, even in the presence of the fault.
(3) If the fault is isolated by diagnostic scheme, then the controller is reconfigured again. The second fault-tolerant controller is designed using the information about the type of fault that has actually occurred so as to improve the control performance.

4. BASELINE CONTROLLER DESIGN

In this section, we design the baseline controller and investigate the closed-loop system stability and performance before fault occurrence. Without loss of generality, let the leader be agent number $M + 1$ with a time-varying reference state (i.e., $x_{M+1} = x^r$). The baseline controller for the ith agent can be designed as:

$$u_{ip} = -\sum_{j\in N_i} k_{ij}\tilde{x}_{ij} - \phi_{ip}(x_{ip}) - \bar{\kappa}_{ip}\, sgn\Big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\Big), \tag{6}$$

where u_{ip} and x_{ip} are the pth component of the input and state vectors of the ith agent, respectively, $p = 1, \cdots, n$, $i = 1, \cdots, M$, $\tilde{x}_{ij} \triangleq x_{ip} - x_{jp}$, $\bar{\kappa}_{ip} \triangleq \bar{\eta}_{ip} + \kappa_p$, κ_p is a positive bound on $|\dot{x}_p^r|$ (i.e., $\kappa_p \geq |\dot{x}_p^r|$), $sgn(\cdot)$ is the sign function, N_i is the set of neighboring agents that directly communicate with the ith agent including the leader, and

k_{ij}, for $j \in N_i$, are positive constants. Notice that $k_{im} = 0$, for $m \notin N_i$.

Note that, by considering the leader as agent $M+1$, the topology graph for the $M+1$ agents has a spanning tree with the leader as its root. First, we need the following Lemmas:

Lemma 1. (Ren and Beard (2008)) The Laplacian matrix $L \in \Re^{P\times P}$ of a directed graph $\mathcal{G}$ has at least one 0 eigenvalue with $\mathbf{1}_P$ as its right eigenvector, where $\mathbf{1}_P$ is a $P \times 1$ column vector of ones, and all nonzero eigenvalues of L have positive real parts. 0 is a simple eigenvalue of L if and only if the directed graph $\mathcal{G}$ has a spanning tree.

Lemma 2. Consider a connected graph $\mathcal{G}$ with the leader as the $(M+1)$th node. The matrix

$$\bar{\mathcal{L}} \triangleq \Psi\mathcal{L} + \mathcal{L}^T\Psi \tag{7}$$

is positive semidefinite and has a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector, where $\Psi \in \Re^{(M+1)\times(M+1)}$ is the Laplacian matrix of the graph with an undirected leader, and $\mathcal{L} \in \Re^{(M+1)\times(M+1)}$ is the Laplacian matrix of the graph with a directed leader.

Proof. Due to space limitation, the proof is omitted in this paper and can be found in (Khalili et al. (2015)). □

The following result characterizes the stability and leader-following performance properties of the controlled system before fault occurrence.

Theorem 1. In the absence of faults in the ith agent, the baseline controller described by (6) guarantees that the leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e. $x_i - x^r \to 0$ as $t \to \infty$.

Proof. Based on (6) and before occurrence of the fault, the closed-loop system dynamics are given by

$$\dot{x}_{ip} = -\sum_{j\in N_i} k_{ij}\tilde{x}_{ij} + \eta_{ip} - (\bar{\eta}_{ip}+\kappa_p)sgn\big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\big)\,, \tag{8}$$

We can represent the collective state dynamics as

$$\dot{x}^p = -\mathcal{L}x^p + \zeta^p - \bar{\zeta}^p\,, \tag{9}$$

where $x^p \in \Re^{M+1}$ is comprised of the pth state component of the $M+1$ agents, including the leader as the $(M+1)$th agent, i.e., $x^p = [x_{1p},\, x_{2p},\, \cdots,\, x_{Mp},\, x_p^r]^T$, the terms $\zeta^p \in \Re^{M+1}$ and $\bar{\zeta}^p \in \Re^{M+1}$ are defined as

$$\zeta^p \triangleq [\eta_{1p},\, \cdots,\, \eta_{Mp},\, 0]^T,\ \bar{\zeta}^p \triangleq \big[\bar{\zeta}_{1p},\, \cdots,\, \bar{\zeta}_{Mp},\, 0\big]^T, \tag{10}$$

where $\bar{\zeta}_{ip} \triangleq (\bar{\eta}_{ip}+\kappa_p)sgn\big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\big)$, $i = 1,\cdots,M$. We consider the following Lyapunov function candidate:

$$V_p = x^{pT}\Psi x^p = \frac{1}{2}\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{M} k_{i(M+1)}\tilde{x}_{i(M+1)}^2, \tag{11}$$

where Ψ is defined in Lemma 2, $\tilde{x}_{i(M+1)} \triangleq x_{ip} - x_{(M+1)p}$, and $x_{(M+1)p}$ is the pth component of the leader's time-varying state x^r. Then, the time derivative of the Lyapunov function (11) along the solution of (9) is given by

$$\begin{aligned}\dot{V}_p = &-x^{pT}\bar{\mathcal{L}}\,x^p + 2\dot{x}_p^r\sum_{i=1}^{M} k_{i(M+1)}(x_p^r - x_{ip})\\ &+2x^{pT}\Psi(\zeta^p - \bar{\zeta}^p)\,,\end{aligned} \tag{12}$$

where $\bar{\mathcal{L}}$ is defined in (7). Based on (10), we have

$$x^{pT}\Psi\zeta^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}(x_{ip}-x_{jp})\eta_{ip}\,, \tag{13}$$

$$x^{pT}\Psi\bar{\zeta}^p = \sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}(\bar{\eta}_{ip}+\kappa_p)sgn\big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\big). \tag{14}$$

Using the property that $k_{ij} = k_{ji}$ (based on Assumption 2), we know that $\sum_{i=1}^{M}\sum_{j\in N_i, j\neq M+1} k_{ij}(x_{ip}-x_{jp}) = 0$. Therefore, we have

$$2\dot{x}_p^r\sum_{i=1}^{M} k_{i(M+1)}(x_p^r - x_{ip}) = -2\dot{x}_p^r\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}. \tag{15}$$

By substituting (13), (14) and (15) into (12), we have

$$\begin{aligned}\dot{V}_p = &-x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}(\eta_{ip} - \dot{x}_p^r)\\ &-2\sum_{i=1}^{M}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}(\bar{\eta}_{ip}+\kappa_p)sgn\Big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\Big).\end{aligned} \tag{16}$$

Based on (16) and Assumption 1, we have

$$\dot{V}_p \leq -x^{pT}\bar{\mathcal{L}}\,x^p\,.$$

Therefore, using Lemma 2, we know that $\dot{V}_p$ is negative definite with respect to $x_{ip} - x_{jp}$, because the only x^p that makes $-x^{pT}\bar{\mathcal{L}}\,x^p$ zero is $x^p = \mathbf{1}_{M+1}c$, where c is a constant. Therefore, consensus is reached asymptotically, i.e., $x_{ip} - x_{jp} \to 0$ as $t \to \infty$. More specifically, $x_{ip} - x_p^r \to 0$ as $t \to \infty$. □

5. DISTRIBUTED FAULT DIAGNOSIS

The distributed fault detection and isolation (FDI) architecture is comprised of M local FDI components, with one FDI component designed for each of the M agents. The objective of each local FDI component is to detect and isolate faults in the corresponding agent. Specifically, each local FDI component consists of a fault detection estimator (FDE) and a bank of r_i nonlinear adaptive fault isolation estimators (FIEs), where r_i is the number of different nonlinear fault types in the fault set $\mathcal{F}_i$ (2) associated with the corresponding agent. If a fault is detected in a particular agent i, then the corresponding r_i local FIEs are activated for the purpose of determining the particular type of fault that has occurred in the agent. Due to space limitation, the details are omitted in this paper and can be found in (Khalili et al. (2015)).

6. FAULT-TOLERANT CONTROLLER MODULE

In this section, the design and analysis of the FTC schemes are rigorously investigated for two different operating modes of the closed-loop system: 1) during the period after fault detection and before isolation, and 2) after fault isolation. To facilitate the analysis of the distributed adaptive FTC systems, from now on we assume that the general fault function $f_i^s(x_i, u_i)$ given in (3) takes on the following specific forms:

(1) Process faults represented by

$$f_i^s(x_i) \triangleq [(\theta_{i1}^s)^T g_{i1}^s(x_i), \cdots, (\theta_{in}^s)^T g_{in}^s(x_i)]^T\,, \tag{17}$$

(2) Actuator fault represented by partial loss of effectiveness of the actuators. Specifically,

$$f_i^s(u_i) \triangleq [\theta_{i1}^s u_{i1}, \cdots, \theta_{in}^s u_{in}]^T\,, \tag{18}$$

where the parameter $\theta_{ip}^s \in (-1, 0]$, $p = 1, \cdots, n$, characterizes the magnitude of the actuator fault.

6.1 Accommodation before Fault Isolation

After the fault is detected at time $t = T_d$, the isolation estimators are activated to determine the particular type of fault that has occurred. Before the fault is isolated, no information about the fault type and fault function is available. Adaptive approximators such as neural-network models can be used to estimate the unknown process fault function $\beta_i f_i$. The term "adaptive approximator" (Farrell and Polycarpou (2006)) is used to represent nonlinear multivariable approximation models with adjustable parameters, such as neural networks, fuzzy logic networks, polynomials, spline functions, etc. Specifically, we consider linearly parametrized network (e.g., radial-basis-function networks with fixed centers and variances) described as follows: for $p = 1, \cdots, n$,

$$\hat{f}_{ip}(x_i, \hat{\vartheta}_{ip}) = \sum_{j=1}^{\varrho} c_{pj}\varphi_j(x_i)\,, \tag{19}$$

where $\varphi_j(\cdot)$ represents the fixed basis functions, and $\hat{\vartheta}_{ip} \triangleq col(c_{pj} : j = 1, \cdots, \varrho)$ is the adjustable weights of the nonlinear approximator. In the presence of a process fault, $\hat{f}_{ip}$ provides the adaptive structure for online approximating the unknown fault function $f_{ip}(x_i)$. This is achieved by adapting the weight vector $\hat{\vartheta}_{ip}(t)$. Therefore, the system dynamics described by (1) can be rewritten as, for $p = 1, \cdots, n$,

$$\dot{x}_{ip} = \phi_{ip}(x_{ip}) + (1 + \theta_{ip})u_{ip} + \eta_{ip} + \hat{f}_{ip}(x_i, \vartheta_{ip}) + \delta_{ip}(x_i), \tag{20}$$

where the parameter θ_{ip} is defined in (18), $\delta_{ip} \triangleq f_{ip}(x_i) - \hat{f}_{ip}(x_i, \vartheta_{ip})$ is the network approximation error for the pth state of the ith agent, and ϑ_{ip} is the optimal weight vector given by

$$\vartheta_{ip} \triangleq \arg \inf_{\hat{\vartheta}_{ip} \in \Theta_{ip}} \left\{ \sup_{x_i \in \mathcal{X}_i} |f_{ip}(x_i) - \hat{f}_{ip}(x_i, \hat{\vartheta}_{ip})| \right\},$$

where $\mathcal{X}_i \subseteq \Re^n$ denotes the set to which the variables x_i belongs for all possible modes of behavior of the controlled system. To simplify the subsequent analysis, in the following we assume that the bounding conditions on the network approximation error are global, so we set $\mathcal{X}_i = \Re^n$. For each network, we make the following assumption on the network approximation error:

Assumption 3. for each $i = 1, \cdots, M$, and $p = 1, \cdots, n$,

$$|\delta_{ip}| \le \alpha_{ip}\bar{\delta}_{ip}(x_i)\,, \tag{21}$$

where $\bar{\delta}_{ip}$ is a known positive bounding function, and α_{ip} is an unknown constant.

Based on the system model (20), the neural network model (19), and Assumption 3, an adaptive neural controller can be designed using adaptive approximation and bounding control techniques (Farrell and Polycarpou (2006)). Specifically, we consider the following controller algorithm:

$$u_{ip} = \frac{1}{1 + \hat{\theta}_{ip}}\bar{u}_{ip}\,, \tag{22}$$

$$\begin{aligned}\bar{u}_{ip} = &-\phi_{ip}(x_{ip}) - \sum_{j \in N_i}\left(k_{ij}\tilde{x}_{ij}\right) - \hat{f}_{ip}(x_i, \hat{\vartheta}_{ip}(t)) - \psi_{ip} \\ &-(\bar{\eta}_{ip} + \kappa_p) sgn\left(\sum_{j \in N_i} k_{ij}\tilde{x}_{ij}\right),\end{aligned} \tag{23}$$

$$\dot{\hat{\vartheta}}_{ip} = \Gamma_{ip}\sum_{j \in N_i} k_{ij}\tilde{x}_{ij}\varphi_{ip}(x_i)\,, \tag{24}$$

$$\psi_{ip} = \hat{\alpha}_{ip}\bar{\delta}_{ip}(x_i) sgn\left(\sum_{j \in N_i} k_{ij}\tilde{x}_{ij}\right), \tag{25}$$

$$\dot{\hat{\alpha}}_{ip} = \Upsilon_{ip}\left|\sum_{j \in N_i} k_{ij}\tilde{x}_{ij}\right|\bar{\delta}_{ip}(x_i)\,, \tag{26}$$

$$\dot{\hat{\theta}}_{ip} = \mathcal{P}_{\bar{\theta}_{ip}}\left\{\bar{\Gamma}_{ip}\sum_{j \in N_i} k_{ij}\tilde{x}_{ij}u_{ip}\right\}, \tag{27}$$

where k_{ij} are positive constants, $\hat{\theta}_{ip}$ is an estimation of the actuator fault magnitude parameter θ_{ip} with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_{ip}$ to the corresponding set $[\bar{\theta}_{ip}, 0]$ for $\bar{\theta}_{ip} \in (-1, 0)$, $\hat{\vartheta}_{ip}$ is an estimation of the neural network parameter vector ϑ_{ip}, $\varphi_{ip} \triangleq col(\varphi_j : j = 1, \cdots, \varrho)$ is the collective vector of fixed basis functions, $\hat{\alpha}_{ip}$ is an estimation of the unknown constants α_{ip}, and Γ_{ip} and Υ_{ip} are symmetric positive definite learning rate matrices.

Using some algebraic manipulations, we can rewrite (22) as $u_{ip} = \bar{u}_{ip} - \hat{\theta}_{ip}u_{ip}$. Therefore, using (20) and (22), we can represent the collective closed-loop state dynamics as

$$\dot{x}^p = -\mathcal{L}x^p + \zeta^p - \bar{\zeta}^p + \tilde{f}^p + \delta^p - \psi^p + \xi^p\,, \tag{28}$$

where $x^p \in \Re^{M+1}$, $p = 1, \cdots, n$, is comprised of the pth component of the M agents and the leader as the $(M+1)$th agent, i.e., $x^p = [x_{1p},\, x_{2p},\, \cdots,\, x_{Mp},\, x_p^r]^T$, the terms $\zeta^p \in \Re^{M+1}$ and $\bar{\zeta}^p \in \Re^{M+1}$ are defined in (10), and the terms $\tilde{f}^p \in \Re^{M+1}$, $\delta^p \in \Re^{M+1}$, $\psi^p \in \Re^{M+1}$ and $\xi^p \in \Re^{M+1}$ are defined as

$$\tilde{f}^p \triangleq \left[(\tilde{\vartheta}_{1p})^T\varphi_{1p}\ \cdots\ (\tilde{\vartheta}_{Mp})^T\varphi_{Mp}\ 0\right]^T, \tag{29}$$

$$\delta^p \triangleq [\delta_{1p}\ \cdots\ \delta_{Mp}\ 0]^T\,, \tag{30}$$

$$\psi^p \triangleq [\psi_{1p}\ \cdots\ \psi_{Mp}\ 0]^T\,, \tag{31}$$

$$\xi^p \triangleq \left[\tilde{\theta}_{1p}u_{1p}\ \cdots\ \tilde{\theta}_{Mp}u_{Mp}\ 0\right]^T, \tag{32}$$

where $\tilde{\theta}_{ip} = \theta_{ip} - \hat{\theta}_{ip}$ is the actuator fault magnitude estimation error, and $\tilde{\vartheta}_{ip} = \vartheta_{ip} - \hat{\vartheta}_{ip}$ and φ_{ip} are the parameter estimation errors and basis functions corresponding to the neural network model associated with the pth state component of the ith agent, respectively. To derive the adaptive algorithm and to investigate analytically the stability properties of the feedback system, we consider the following Lyapunov function candidate:

$$\begin{aligned}V_p = &\, x^{pT}\Psi\, x^p + (\tilde{\vartheta}^p)^T(\Gamma^p)^{-1}\tilde{\vartheta}^p + (\tilde{\alpha}^p)^T(\Upsilon^p)^{-1}\tilde{\alpha}^p \\ &+ (\tilde{\theta}^p)^T(\bar{\Gamma}^p)^{-1}\tilde{\theta}^p\,,\end{aligned} \tag{33}$$

where Ψ is defined in Lemma 2, $\tilde{\vartheta}^p = \left[\tilde{\vartheta}_{1p}^T,\, \cdots,\, \tilde{\vartheta}_{Mp}^T\right]^T$ is the collective parameter estimation errors, $\tilde{\alpha}^p = [\tilde{\alpha}_{1p},\, \cdots,\, \tilde{\alpha}_{Mp}]^T$ is the collective bounding parameter estimation errors defined as $\tilde{\alpha}_{ip} = \alpha_{ip} - \hat{\alpha}_{ip}$, $\tilde{\theta}^p = \left[\tilde{\theta}_{1p},\, \cdots,\, \tilde{\theta}_{Mp}\right]^T$ is the collective actuator fault magnitude parameter estimation errors, and $\Gamma^p = \mathrm{diag}\{\Gamma_{1p}, \cdots, \Gamma_{Mp}\}$, $\Upsilon^p = \mathrm{diag}\{\Upsilon_{1p}, \cdots, \Upsilon_{Mp}\}$ and $\bar{\Gamma}^p = \mathrm{diag}\{\bar{\Gamma}_{1p}, \cdots, \bar{\Gamma}_{Mp}\}$ are adaptive learning rate matrices.

Following the same procedure as given in the proof of Theorem 1, using (29), (30), (31) and (32), and selecting the adaptive algorithm for $\hat{\vartheta}_{ip}$ and $\hat{\theta}_{ip}$ as (24) and (27), respectively, it can be shown that the time derivative of the Lyapunov function (33) along the solution of (28) satisfies

$$\dot{V}_p \leq -x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\Bigg(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}(\delta_{ip}-\psi_{ip}) - \tilde{\alpha}_{ip}(\Upsilon_{ip})^{-1}\dot{\hat{\alpha}}_{ip}\Bigg).$$

It is worth noting that since the projection modification can only make the Lyapunov function derivative more negative, the stability properties derived for the standard algorithm still hold (Farrell and Polycarpou (2006)). By using (25) and based on Assumption 3, we have

$$\begin{aligned}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}(\delta_{ip}-\psi_{ip}) &= \sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\big(\delta_{ip}-\hat{\alpha}_{ip}\bar{\delta}_{ip}sgn(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij})\big)\\ &\leq |\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}|\tilde{\alpha}_{ip}\bar{\delta}_{ip}\,. \end{aligned} \tag{34}$$

By using (34), we have

$$\dot{V}_p \leq -x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\Bigg(|\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}|\tilde{\alpha}_{ip}\bar{\delta}_{ip} - \tilde{\alpha}_{ip}(\Upsilon_{ip})^{-1}\dot{\hat{\alpha}}_{ip}\Bigg).$$

Therefore, by using (26) and after some algebraic manipulations, we have

$$\dot{V}_p \leq -x^{pT}\bar{\mathcal{L}}\,x^p = -2\sum_{i=1}^{M}\Bigg(\sum_{j\in N_i} k_{ij}(x_{ip}-x_{jp})\Bigg)^2. \tag{35}$$

Based on the same reasoning logic as reported in the proof of Theorem 1, we conclude that $\dot{V}_p$ is negative semidefinite, and $x_{ip}-x_{jp}$, $\hat{\vartheta}_{ip}$ and $\hat{\alpha}_{ip}$ are uniformly bounded. Integrating both sides of (35), we know that $(x_{ip}-x_{jp}) \in L_2$. Since $(x_{ip}-x_{jp}) \in L_\infty \cap L_2$ and $\dot{x}_{ip}-\dot{x}_{jp} \in L_\infty$, based on Barbalat's Lemma, we can conclude that consensus is reached asymptotically, i.e., $x_{ip}-x_{jp} \to 0$ as $t\to\infty$. More specifically, $x_{ip}-x_p^r \to 0$ as $t\to\infty$ and therefore, the leader-follower consensus is reached asymptotically.

The aforementioned design and analysis procedure is summarized as follows:

Theorem 2. Suppose that the bounding Assumption 3 holds. Then, if a fault is detected, the adaptive fault-tolerant law (22), the weight parameter adaptive law (24), the bounding parameter adaptive laws (25) and (26), and the actuator fault parameter adaptive law (27) guarantee

(1) all the signals and parameter estimates are uniformly bounded, i.e., x_{ip}, $\hat{\vartheta}_{ip}$ and $\hat{\alpha}_{ip}$ are bounded for all $t\in(T_i, T_{isol})$;
(2) leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e. $x_i - x^r \to 0$ as $t\to\infty$.

6.2 Accommodation after Fault Isolation

Let us now assume that the isolation procedure described in Section 5 provides the information that fault s has been isolated at time T_{isol}. Based on the FTC architecture described by (5), the controller is reconfigured again to further improve control performance based on the diagnostic information of isolated fault type. Below, we describe the cases of process fault described by (17) and actuator fault given by (18), respectively. Without loss of generality, let the leader be the agent number $M+1$ with a set of neighborhoods N_{M+1}.

Adaptive Fault-Tolerant Controller for Process Fault

After the isolation of the fault type s, i.e., $t \geq T_{isol}$, the dynamics of the system takes on the following form: for $p = 1,\cdots,n$,

$$\dot{x}_{ip} = \phi_{ip}(x_{ip}) + u_{ip} + \eta_{ip}(x_i,t) + \theta_{ip}^{s}g_{ip}^{s}(x_i)\,. \tag{36}$$

The following adaptive fault-tolerant controller is adopted:

$$u_{ip} = -\phi_{ip}(x_{ip}) - \sum_{j\in N_i}\big(k_{ij}\tilde{x}_{ij}\big) - \hat{\theta}_{ip}^T g_{ip}^s(x_i) - (\bar{\eta}_{ip}+\kappa_p)sgn\Bigg(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\Bigg), \tag{37}$$

$$\dot{\hat{\theta}}_{ip} = \Gamma_{ip}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}g_{ip}^s(x_i)\,, \tag{38}$$

where $\hat{\theta}_{ip}$ is an estimation of the unknown fault parameter vector, and Γ_{ip} is a symmetric positive definite learning rate matrix. Then, we have the following:

Theorem 3. Assume that process fault s occurs at time T_i and that it is isolated at time T_{isol}. Then, the fault-tolerant controller (40) and fault parameter adaptive law (41) guarantee that the leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e., $x_i - x^r \to 0$ as $t\to\infty$.

Proof. Due to space limitation, The proof is omitted in this paper and can be found in (Khalili et al. (2015)). □

Adaptive Fault-Tolerant Controller for Actuator Fault

In the case of an actuator fault, i.e., $t \geq T_{isol}$, the dynamics of the system takes on the following form: for $p = 1,\cdots,n$,

$$\dot{x}_{ip} = \phi_{ip}(x_{ip}) + (1+\theta_{ip})u_{ip} + \eta_{ip}(x_i,t)\,. \tag{39}$$

The following adaptive fault-tolerant controller is adopted:

$$u_{ip} = \frac{1}{1+\hat{\theta}_{ip}}\bar{u}_{ip}\,, \tag{40}$$

$$\dot{\hat{\theta}}_{ip} = \mathcal{P}_{\bar{\theta}_{ip}}\Bigg\{\bar{\Gamma}_{ip}\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}u_{ip}\Bigg\}, \tag{41}$$

where $\bar{u}_{ip} \triangleq -\phi_{ip}(x_{ip}) - \sum_{j\in N_i} k_{ij}\tilde{x}_{ij} - \bar{\kappa}_{ip}sgn\big(\sum_{j\in N_i} k_{ij}\tilde{x}_{ij}\big)$, $\hat{\theta}_{ip}$ is an estimation of the unknown actuator fault magnitude parameter θ_{ip} with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_{ip}$ to the corresponding set $[\bar{\theta}_{ip}, 0]$ for $\bar{\theta}_{ip} \in (-1, 0)$, and $\bar{\Gamma}_{ip}$ is a symmetric positive definite learning rate matrix. Then, we have the following:

Theorem 4. Assume that an actuator fault occurs at time T_i and that it is isolated at time T_{isol}. Then, the fault-tolerant controller (40) and fault parameter adaptive law (41) guarantee that the leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e., $x_i - x^r \to 0$ as $t\to\infty$;

Proof. Due to space limitation, The proof is omitted in this paper and can be found in (Khalili et al. (2015)). □

7. SIMULATION RESULTS

In this section, a simulation example of a networked multi-agent system consisting of 5 agents is considered to illustrate the effectiveness of the distributed fault-tolerant control method. The dynamics of each agent is given by

$$\dot{x}_i = u_i + \eta_i + \beta_i(t - T_i) f_i(x_i, u_i)\,, \tag{42}$$

where, for $i = 1, \cdots, 5$, $x_i = [x_{i1}, x_{i2}]^T$ and $u_i = [\nu_i cos(\bar{\psi}_i), \nu_i sin(\bar{\psi}_i)]^T$ are the state and input vector of ith agent, respectively, $\bar{\psi}_i$ and ν_i in the input vector u_i are the orientation and the linear velocity of each agent representing a ground vehicle.

The ground vehicle given model in (42) is a standard unicycle-like model that can be controlled with the orientation $\bar{\psi}_i$ and vehicle linear velocity ν_i. Using the developed algorithms, the desired orientation and linear velocity of the ground vehicle robot can be obtained uniquely. Then, a low level controller can be designed to track the desired orientation and linear velocity for driving the ground vehicles to desired positions.

The unknown modeling uncertainty in the local dynamics of the agents are assumed to be sinusoidal signals $\eta_i = [0.5sin(t), 0.5sin(t)]^T$ bounded by $\bar{\eta}_i = [0.6, 0.6]^T$. The objective is for each agent to follow the leader's position described by $x^r = [x_1^r, x_2^r]^T = [5 + sin(t), 5 + cos(t)]^T$.

The Laplacian matrix of the intercommunication graph of agents plus leader is given as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & -1 & -1 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The fault class under consideration is defined as follows

(1) A process fault described by $f_i^1 = \theta_i^1 g_i^1$, where $g_i^1 = x_i^2 cos(x_i)$ and the fault magnitude $\theta_i^1 \in [0, 1]$.
(2) An actuator fault described by $f_i^2 = \theta_i^2 g_i^2$, where $g_i^2 = u_i$ and the fault magnitude $\theta_i^2 \in [-0.8, 0]$.

The details of the distributed FDI algorithm is omitted here due to space limitation.

A radial basis function (RBF) neural network is used for approximation of the process fault function after its detection and before its isolation. The RBF network considered in this paper consists of 21 neurons with 21 adjustable gain parameters. The center of radial basis functions are equally distributed on interval $[-10, 10]$ with a variance of 0.5. The initial parameter vector of the neural network is set to zero. We set the learning rate as $\Gamma_i = 5$ and consider a constant bound on the network approximation error, i.e., $\bar{\delta}_i = 1$. The adaptive gains in (26) and (27) are chosen as $\Upsilon_i = 2$ and $\bar{\Gamma}_i = 1$, respectively.

Figures 1 and 2 show the case of a process fault in the form of $f_1^1 = \theta_1^1 g_1^1$ with a magnitude of 0.8 occurs to agent 1 at $T_i = 5$ second. Regarding the performance of the adaptive fault-tolerant controllers, as can be seen from Figure 1, the leader-following consensus is achieved using the proposed adaptive FTCs, while the agents cannot follow the leader without the FTC controllers after fault occurrence (see Figure 2). Thus the benefits of the FTC method can be clearly seen. The tracking errors for the second states have the same behavior and therefore is omitted due to space limitation.

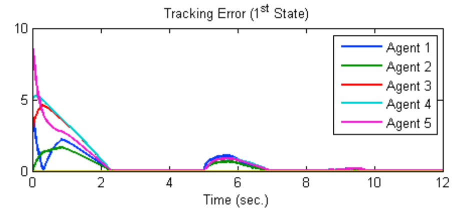

Fig. 1. The tracking errors in the case of a process fault in agent 1: with adaptive fault-tolerant controllers

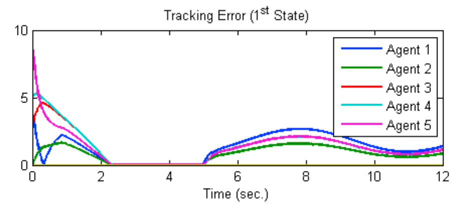

Fig. 2. The tracking errors in the case of a process fault in agent 1: without adaptive fault-tolerant controllers

8. CONCLUSION

In this paper, we investigate the problem of a distributed FTC for a class of multi-agent uncertain systems. Under certain assumptions, adaptive FTC controllers are developed to achieve the leader-following consensus in the presence of faults. The extensions to systems with more general structure is an interesting topic for future researches.

REFERENCES

Blanke, M., Kinnaert, M., Lunze, J., and Staroswiecki, M. (2006). *Diagnosis and Fault-Tolerant Control*. Springer, Berlin.

Farrell, J. and Polycarpou, M.M. (2006). *Adaptive Approximation Based Control*. J. Wiley, Hoboken, NJ.

Ferrari, R., Parisini, T., and Polycarpou, M.M. (2012). Distributed fault detection and isolation of large-scale discrete-time nonlinear systems: An adaptive approximation approach. *IEEE Transactions on Automatic Control*, 57(2), 275–790.

Khalili, M., Zhang, X., Polycarpou, M.M., Parisini, T., and Cao, Y. (2015). Distributed adaptive fault-tolerant control of uncertain multi-agent systems. *Technical Report, Department of Electrical Engineering, Wright State University*. http://cecs.wright.edu/~frank.zhang/SAFEPROCESS2015_full.pdf.

Ren, W. and Beard, R. (2008). *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*. Springer-Verlag, London, U.K.

Shames, I., Teixeira, A.M., Sandberg, H., and Johansson, K.H. (2011). Distributed fault detection for interconnected second-order systems. *Automatica*, 47, 2757–2764.

Yan, X. and Edwards, C. (2008). Robust decentralized actuator fault detection and estimation for large-scale systems using a sliding-mode observer. *International Journal of Control*, 81(4), 591–606.

Zhang, X., Parisini, T., and Polycarpou, M.M. (2004). Adaptive fault-tolerant control of nonlinear systems: a diagnostic information-based approach. *IEEE Transactions on Automatic Control*, 49(8), 1259–1274.