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# Measurement uncertainty assessment of Coordinate Measuring Machines by simulation and planned experimentation

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ABSTRACT

Current standard procedures for the evaluation of measurement uncertainty of Coordinate Measuring Machines (CMM) being not fully satisfactory, other methods are considered. Conformity to geometric and dimensional tolerances, specified for an industrial workpiece previously involved in a comprehensive round robin test, was checked with CMM. Measurement uncertainty was also assessed exploiting a simulation method, developed at Politecnico di Torino, offering several advantages in terms of CMM operability and substantial cost savings. Estimates of measurement variability due to single and combined effects of factors considered, obtained from testing in a representative verification, are discussed in the light of results of simulation.



## 1. Introduction

The challenges of globalization, product life-cycle time reduction and matching customer demands mandate reliable control of production systems. Enhancement of product quality standards implies delving into control and inspection methods. In this scenario quality control demands require tight standards of product inspection.

A current industrial problem is addressed in this paper, namely how to develop a reliable approach to measurement of industrial pieces on a Coordinate Measuring Machine in order to obtain dependable measurements.

CMMs are versatile instruments used for precision inspection in industry, and their unique properties more than justify substantial investment; nevertheless uncertainties associated with them are to be reckoned with [1–3]. A correct statement of measurement uncertainty is nowadays a must for companies wishing to comply with ISO 9000 standards [4–6], which requires an effective measurement management and measurement process control [7]. Only a dependable, cost effective inspection system can ensure quality products, reducing to an acceptable level the risk of accepting substandard parts and/or discarding conforming ones.

Such considerations have pushed companies to invest resources on these issues. Estimation of measurement uncertainty entails taking into account the multipurpose characteristics of coordinate measuring systems. A substantial number of sources of uncertainty is to be reckoned with, including machine components, fixtures and probes, strategy for data acquisition and/or sampling, data processing and measurement environment [8].

In the light of these considerations, an assessment of measurement variability was undertaken, exploiting a comparison between a simulative method, developed at Politecnico di Torino, and results of a recent national round robin test. Evaluation of measurement uncertainty using a simulative method was shown to yield substantial advantages in terms of cost savings, lead time reduction and management integration.

## 2. Background on CMM measurement uncertainty estimation

Uncertainty evaluation of CMM measurements is made awkward by the sheer complexity of such machines. Substantial efforts were carried out by academia, national and international institutes and organizations, leading to a number of different approaches. These methods cover e.g. empirically approximated methodologies, counseling from measurement experts, and numerical simulations, the latter made possible by the dramatic

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increase in computing power currently experienced. A general rigorous application of GUM guidelines is often difficult; standardized and simulative methods, objects of this study, are briefly described.

### 2.1. Standardized methods

Standardized methods to evaluate CMMs measurement uncertainty owe much to the work of ISO TC213 Committee, especially WG 10. According to GPS standard framework, the well known series ISO 15530 [9–11] was produced, covering such items as:

- Part 2: dealing with the multiple measurement strategies approach to uncertainty estimation [9];
- Part 3: pointing out a method based on comparative measurements to approximate empirically evaluation of measurement uncertainty [10];
- Part 4: drawing guidelines to evaluate measurement uncertainty by means of computer simulations [11].

At the present time Part 2, based on measurement results originating from an EU project called EasyTrac [12], has not been published. This project collected a large amount of experimental data; the scientific community however decided to broaden investigation and organized several additional data collecting projects. Multiple measurement strategy combines several orientations of the workpiece and point distribution pattern on surfaces considered, in order to get a better estimation for the conventional "true" value of the measurand. The workpiece must be measured in at least four different orientations, corresponding to positions ensuring good measurement conditions. In each orientation, the workpiece must be measured with at least five different point distribution patterns. Whenever distance/size measurements are required, subsidiary measurements on an artifact of similar size must be performed along the three CMM coordinate axes, and repeated three times. Calibration value and related calibration uncertainty are determined by calculations based upon the database generated by all measurement results obtained.

### 2.2. Computer simulations

Computer simulation represents the next frontier for uncertainty assessment, for CMM as well as for a large number of measurement instruments and methods for which a direct application of the GUM is difficult if at all possible [13,14]. In the case of CMM several methods were developed, e.g. "Expert CMM" [15], "Virtual CMM" [16], "Simulation by constraints" [17] and "Virtual Instrument" [18]. Some applications of the general theory of Monte Carlo simulations were also carried out, for a comprehensive review see e.g. [8] and [19]. The above mentioned methods are called parametric, since the CMM is modeled using parametric equations; the measurement process is modeled as well. Simulation allows propagating uncertainty from the coordinates of the measured points to measurement results. While these methods exhibit good performances with respect to multiple measurement strategies [9], a major issue is preservation of CMM versatility. Some methods may be run off-line to simulate a measurement process and its uncertainty before making any real measurement. However some remarks are in order whenever measurements are not performed in metrological laboratories. While variability due to environmental factors may be kept under control when measurements are performed in controlled rooms, and continuous and careful checks of CMM are carried out, such conditions seldom apply in industrial environment. Major problems originate therefore from strong statistical assumptions concerning input quantities, quality and quantity of the information, required to characterize the CMM, and from requirements on environmental conditions potentially affecting measurements.

### 2.3. The proposed method for approximated uncertainty evaluation (PoliTo)

An approximated method was developed and patented at Politecnico di Torino (PoliTo), providing dependable, cost effective uncertainty estimation in industrial applications [20,21]. It exploits Monte Carlo technique to simulate errors in the coordinates of each measured point, using CMM part program to propagate measurement uncertainty from point coordinates to measurands. The main departure from parametric methods (implying mathematical models according to a rigid body error assumption), consists in the exploitation of a probabilistic approach based upon *bootstrap* method [22], a numerical approach to evaluate variability of statistical estimators, such as least square estimators of geometrical parameters. Given an original experimental sample of size $n$, a single *bootstrap sample* is obtained by $n$ extractions with replacement from the original experimental sample. A proper number of bootstrap samples enables estimation of relevant population parameters, provided of course that the original experimental sample is representative of the population under investigation.

The proposed approximated method provides straightforward, reasonable uncertainty evaluation, proven by extensive application to be adequate for industrial application.

#### 2.3.1. Two steps approach to CMM measurement tasks

This method is based on the assumption that every CMM measurement task may be divided into two elementary steps, both affected from uncertainty contributors:

- step 1 covers measurement of the coordinates of a given number of points on the workpiece, taking into account most uncertainty contributors;
- in step 2 the set of coordinates of measured points is exploited according to part program, in order to obtain the values of the measurands (intrinsic or relational parameters of associated features [23], dimensional – geometrical tolerances). Uncertainty pertaining to fitting algorithms is also taken into account.

Thus in step 1 uncertainty contributors influence the coordinates of each sampled point, while step 2 propagates uncertainty from the coordinates of measured points to the measurands.

It must be underlined that in case of non-comparative measurements [9], such as the proposed method, CMM accuracy should be taken into consideration as one of the uncertainty contributors affecting step 1. Another remark concerns uncertainty due to interaction between form error of the workpiece and discrete sampling of its surface. This term, usually provided as an input quantity for simulative methods, may prove awkward to be accounted for whenever detailed, specific information on the workpiece at hand is lacking. The proposed method does not require any input regarding this contributor, already present in measurement data, and taken into account by the algorithm generating perturbed data.

The approximated method evaluating measurement uncertainty acts in between the two stages, by first extracting information related to measurement uncertainty from the coordinates of the sampled points using bootstrap technique, and then exploiting step 2 of the measurement process to propagate uncertainty to measurands.

According to the block diagram of Fig. 1, in a normal CMM measurement operation, the sequence is straightforward: the

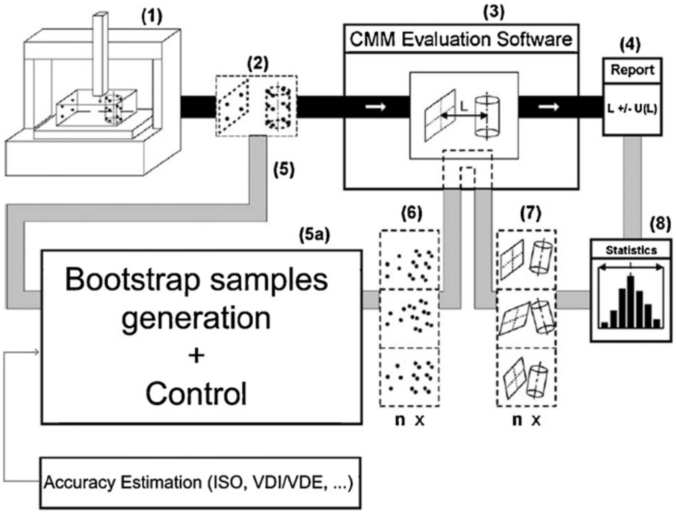


**Fig. 1.** Traditional CMM operation data flow (black straight line); uncertainty evaluation with approximated method (gray line).Modified from [6].

process starts with the collection of points for each feature of interest in the measurement (1), followed by computation of associated features performed by CMM evaluation software (2). Geometric characteristics of associated features are therefore used to compute geometrical tolerances or dimensional tolerances as specified in technical drawings (3). Measurands are listed in a measurement report to arrive at an estimate of conformance (4) for each measured feature.

With the proposed method, the initial input consists in the set of points (5) to be sampled on a particular feature, specified with respect to the ideal design geometry. For each set of sampled points pertaining to a particular feature, the bootstrap algorithm (5a) generates a perturbed dataset [20]. To perform this operation an estimate of CMM accuracy is required, information based on acceptance and reverification test [24,25] being usually adequate. A control on the adequacy of perturbed dataset is performed, and aberrant bootstrap samples if any are rejected. The control procedure, currently covered by the patent, is set up considering the accuracy of the CMM as per ISO 10360-2 [24] (updated in 2009). The method may also be considered task specific, since the uncertainty contributor related to CMM accuracy may be related to $MPE_E$ or $MPE_P$ according to the particular measurement task being performed.

Thereafter, for any particular feature, a set of simulated results is generated (6) and fed into typical CMM evaluation software, to produce a set of associated features (7). By evaluating each associated feature, a probability density function of the reported parameter may be computed to characterize its uncertainty. Statistics (8) associated with such a probability density function may then be exploited to express the uncertainty of the measurement following the notation of the GUM.

## 3. A case study: industrial workpiece tolerance verification

In order to compare results obtained with the approximated method with those of a standardized one, measurement data belonging to verification of dimensional and geometrical tolerances were used. Data set exploited originates from a measurement exercise planned in 2002–2004 as part of the National Interest Research Project (PRIN) on ‘‘Quality in Manufacturing Measurements by means of Coordinate Measuring Machines’’ funded by the Ministry for University in Italy [26].

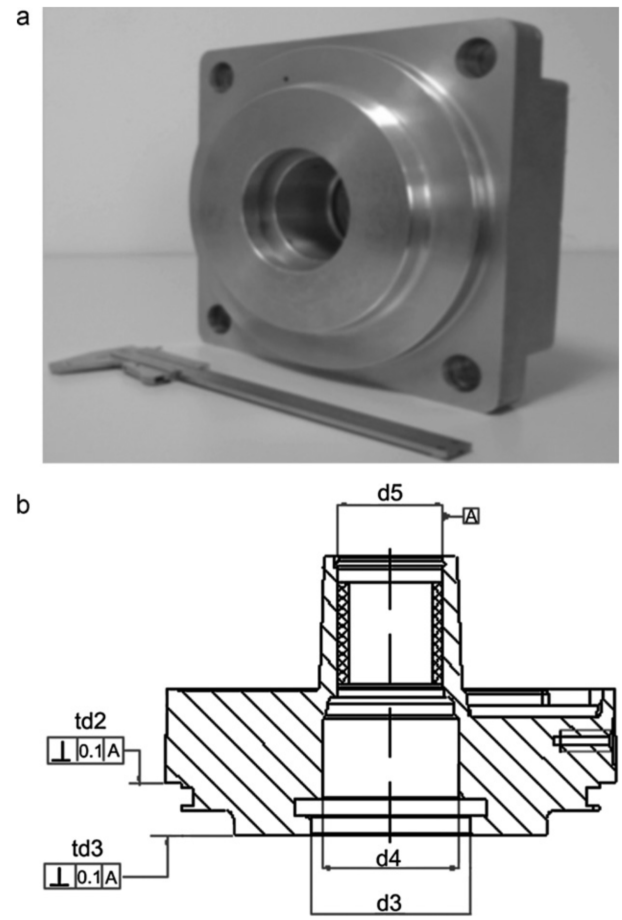


**Fig. 2.** Measured workpiece (a), and cross section showing dimensions and tolerances considered (b).

Three pieces were then examined, namely a sleeve, a spindle and a platform with a bushing (Fig. 2), the latter being considered in the present work. The round robin measurement exercise involved three machine tool component manufacturers (A1, A2, A3) and one CMM manufacturer (C1), all exploiting certified CMMs with qualified operators fully familiar with the parts under consideration.

The round robin exercise was planned as a five factor, full factorial nested design ABCDE with $3 \times 3 \times 2 \times 4 \times 4 = 288$ treatment combinations, replicated five times [26]. Factors and levels are listed in Table 1. Analysis of results, performed according to a general linear model, yielded detailed information on average and scatter of single and combined effects.

Robust estimates of central tendency and spread, namely median and reduced range, were preferred in the analysis to average and standard deviation, definitely more vulnerable to the influence of outliers. Instead of resorting to robust statistics, outliers may be dealt with well-known methods [27].

Factor A, day of the week, did not exhibit a significant effect, nor does it appear in significant interactions; this may not however be read as proof that there’s no such thing as a ‘‘Monday morning effect’’. In fact evidence to the contrary may be found in some individual data files; being limited to a few instances, and barely exceeding noise level, it hardly surfaced in the overall picture.

Factor B, part measured, is the largest single factor affecting median, and the third largest concerning range. This contribution is best read into the context of the sizable BC, BCE interactions, pointing to the combined effect of differences in the approaches of

**Table 1**
Factors and levels.

| Factors | Code | Levels | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| Day of week | A | Monday | Wednesday | Friday | – |
| Part measured | B | Sleeve | Spindle | Platform | – |
| Measurements | C | Size | Tolerance | – | – |
| Zone measured | D | 1 | 2 | 3 | 4 |
| Participating firm | E | A1 | A2 | A3 | C1 |

different CMM operators, concerning measurements on remarkably different parts.

Factor C, measurement, does not appear among significant single effects with median, yet it does by far exceed all other ones when range (especially log transformed) is considered. Tolerance appears to be affected by a substantially larger uncertainty than size, as might be expected on account of metrological considerations.

Factor D, zone measured (nested in B) – a major contributor to total sum of squares – was found to explain a good deal of what would be otherwise (wrongly) attributed to factor B, together with a whole slew of fictitious interactions, and exhibits a standard error not far from that due to pure error.

Factor E, participating firm, takes the second place among significant main effects in the three ANOVA tables, and appears also in the largest triple interaction, BCE. It highlights both single and combined effects of differences in measurement practices among firms, covering effects such as (minor) thermal effects, and CMM operator's habits. Non significant effects, and error, account for something in the order of one third to one half in terms of sum of squares, and almost four fifths in terms of degrees of freedom, a rather conservative split, enabling detection of fairly small effects.

### *3.1. Uncertainty statement of reference values*

Measurement uncertainties of reference values were obtained approximating the multiple measurements strategy technique [9]. Uncertainty estimation may be deemed approximated, since ISO standard rules were not exactly adhered to. Each participant to the round robin exercise carried out as many as 15 replications. Measurements were performed using different strategies by each participant; the workpiece was set up on, and taken out from CMM's measuring table several times according to a specified sequence. Measurements were spread over several days, in order to broaden adequately the basis for estimation of measurement uncertainty. However, different measurement strategies (i.e. probing in different points from a repetition to another) were not performed, as opposed to ISO procedure; therefore uncertainty belonging to part form error was not fully taken into account. Moreover CMM accuracy contribution is not taken into account. These factors may be observed as a contributor to the spread of all the reference values.

## 4. Uncertainty evaluation by PoliTo method

An additional, independent set of measurements was performed with another CMM (DEA Global), belonging to a private company, well maintained and used by skilled operators, with the following main characteristics [24]:

- $MPE_E = (2.9 + L/250)\ \mu m$, where $L$ is given in mm.
- $MPE_p = 2.8\ \mu m$.

**Table 2**
Dimensions and tolerances considered.

| Code | Measurement | Nominal value/mm |
|---|---|---|
| d3 | Diameter | 76.00 |
| d4 | Diameter | 65.00 |
| td2 | Perpendicularity | 0.10 |
| td3 | Perpendicularity | 0.10 |

**Table 3**
Number of points for defining features involved.

| Code | Feature's points | Datum's points |
|---|---|---|
| d3 | 8 | – |
| d4 | 8 | – |
| td2 | 12 | 16 |
| td3 | 12 | 16 |

All measurands specified in technical drawings (5 basic dimensions and 6 geometrical tolerances) were considered in the comparative study. Results discussed here pertain to two basic dimensions and two geometrical tolerances as indicated in Table 2, similar considerations applying to other results.

Measurement strategy regarding repeated measurements was according to the factorial plan defined in PRIN project [21,26]; the number of points measured for each feature is shown in Table 3. These points were used in simulation for uncertainty estimation, sample size being crucial for the bootstrap process.

## 5. Results and discussion

Measurement results and related uncertainties (at 95% confidence level) pertaining to references A1, A2, A3, C1, and an added set obtained at Politecnico di Torino with the approximated method referred to above, are shown in Figs. 3–6. CMM accuracy contributor for the case at hand is 3 μm; over 1000 bootstrap samples were taken.

Some considerations are in order. First of all, expanded uncertainties attached to reference values show that although measurement were performed in different days, with different operators, mounting on and removing from CMM measuring table repeatedly the workpiece, uncertainty due to form error of the measurand was not taken into account. For most companies involved, measurements were taken always probing the same points; the effect of form error of the measurand was therefore only visible as a contribution to the spread of the results obtained from different companies. In fact each of them developed its own particular part program, so each one probed different points with respect to the others. As a consequence, the feature of approxi-

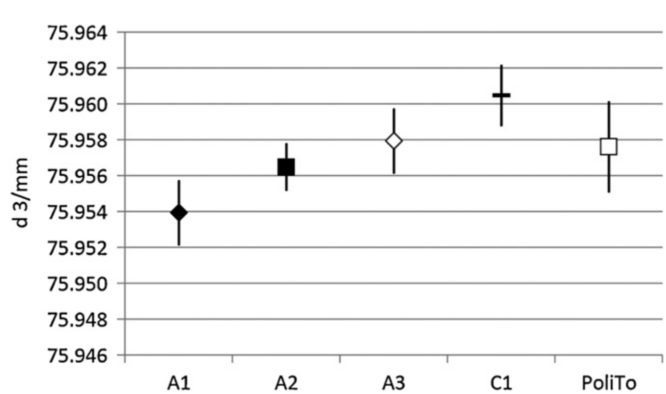


**Fig. 3.** Diameter of bore d3; results for reference values A1, A2, A3, C1 and for PoliTo.

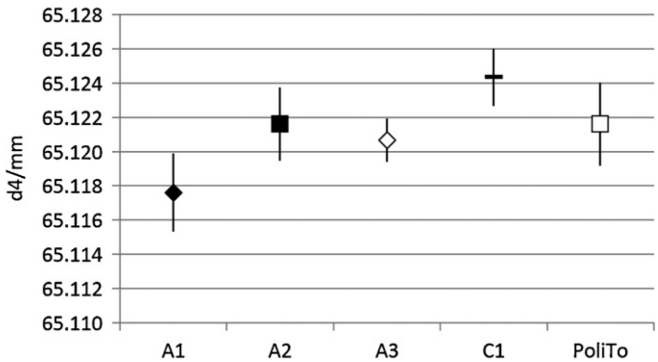


**Fig. 4.** Diameter of bore d4; results for reference values A1, A2, A3, C1 and for PoliTo.

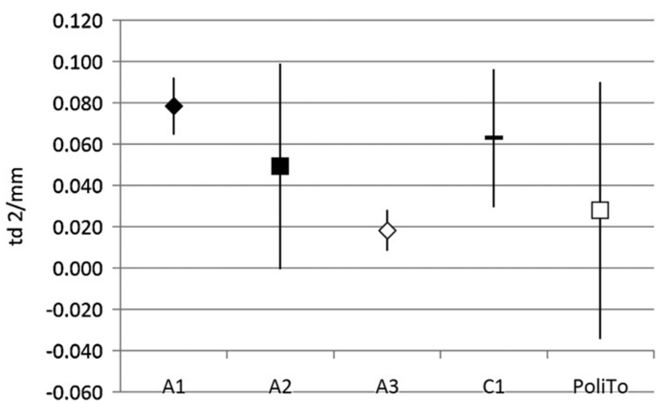


**Fig. 5.** Perpendicularity td2; results for reference values A1, A2, A3, C1 and for PoliTo. Negative values are devoid of physical significance.

mated method, leading to overestimate somehow uncertainty with respect to a single reference value, should be considered as a definite asset, since thanks to the particular method adopted of perturbing data, uncertainty due to form error is properly accounted for. Results for dimensional tolerances appear reasonable.

Figs. 3 and 4 show agreement between measured value pertaining to PoliTo with respect to the other four; measurement uncertainty obtained with the approximated method appears reasonable, perhaps slightly overestimated. A strong systematic effect appears to be due to the different participants, the pattern of measured values being strikingly similar in either figure.

As expected, geometrical tolerances present a different situation, see Figs. 5 and 6; wider spread between results can be observed, differences being about an order of magnitude larger than those pertaining to dimensional tolerances. Relative agreement may be observed regarding perpendicularity tolerances, most measures lying within the tolerance band obtained by the approximated method. In fact for the case of td2 the approximated method estimated an expanded uncertainty of 62 μm, definitely comparable with the stated value of 50 μm pertaining to CMM belonging to machine tool manufacturer A2. Pretty much the same results are obtained for td3 tolerance, where again the approximated method yielded results close to uncertainty evaluation obtained by participant A2, those pertaining to others appearing somehow underestimated.

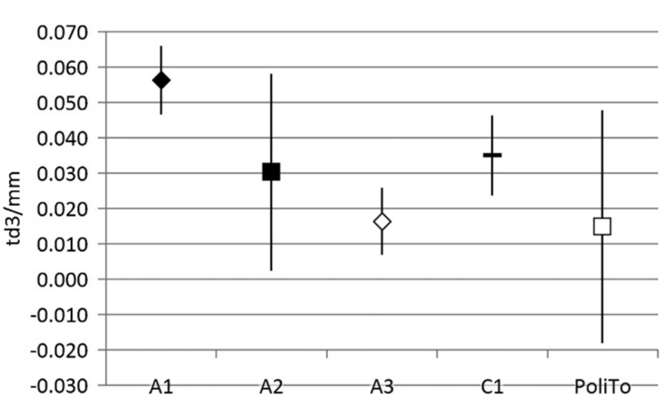


**Fig. 6.** Perpendicularity td3; results for reference values A1, A2, A3, C1 and for PoliTo. Negative values are devoid of physical significance.

## 6. Conclusion

A comparison is presented, based upon an actual case, between a method for CMM measurement uncertainty estimation based upon simulation, and a standard method. Major problems of current simulative methods were related to strong statistical assumptions in input quantities, quality and quantity of the information required for CMM characterization, and in terms of stringent environmental requirements concerning the laboratory where measurements are performed. The approximated method for uncertainty estimation pertaining to CMM measurements, developed and patented at Politecnico di Torino [20,21], presented and applied in an experimental case, was compared to a standardized procedure for uncertainty estimation [9]. Measurement task involved tolerance verification of an industrial workpiece, reference values and related uncertainties being provided by results obtained in the course of a comprehensive round robin exercise [26]. The approximated method exhibited good performances: substantial agreement with different measurements of both dimensional and geometrical tolerances was obtained, with no undue overestimation of measurement uncertainty. Form error estimation, detectable only by analyzing variability between groups (i.e. participants), was correctly taken into account by the proposed method.

In the light of these considerations, advantages related to application of a simulative method in order to evaluate measurement uncertainty were highlighted, namely potential lead time reduction, and substantial savings.

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