Synchronization of chaotic systems using time-delayed fuzzy state-feedback controller

H.K. Lam[1], *Member, IEEE,* W.K. Ling[1], H.H.C. Iu[2], *Member, IEEE* and S.H. Ling[2], *Member, IEEE*
[1]Department of Electronic Engineering, Division of Engineering, The King's College London, WC2R 2LS, United Kingdom, (email: hak-keung.lam@kcl.ac.uk)
[2]School of Electrical, Electronic and Computer Engineering, The University of Western Australia

Abstract: This paper presents the fuzzy-model-based control approach to synchronize two chaotic systems subject to parameter uncertainties. A fuzzy state-feedback controller using the system state of response chaotic system and the time-delayed system state of drive chaotic system is employed to realize the synchronization. The time delay which complicates the system dynamics makes the analysis difficult. To investigate the system stability and facilitate the design of fuzzy controller, T-S fuzzy models are employed to represent the system dynamics of the chaotic systems. Furthermore, the membership grades of the T-S fuzzy models become uncertain due to the existence of parameter uncertainties which further complicates the system analysis. To ease the stability analysis and produce less conservative analysis result, the membership functions of both T-S fuzzy models and fuzzy controller are considered. Stability conditions are derived using Lyapunov-based approach to aid the design of fuzzy state-feedback controller to synchronize the chaotic systems. A simulation example is presented to illustrate the merits of the proposed approach.

INTRODUCTION

Fuzzy-model-based control approach is a promising approach to deal with complex nonlinear systems. It has been successfully applied in various applications. Recently, fuzzy-model-based control approach has been employed to synchronize chaotic systems, which is a useful application in communication system to ensure a secure communication.

In fuzzy-model-based control approach, generally, T-S fuzzy model [1] is employed to describe the dynamical behaviors of the response and drive chaotic systems. It was shown in [2] that most common chaotic systems can be represented by T-S fuzzy models with simple rules. Based on the T-S fuzzy model, a fuzzy state-feedback controller [3] is then designed to realize the synchronization. Under a design criterion that the grades of membership of both response and drive chaotic system are known, LMI-based exact linearization conditions were given to design a fuzzy state-feedback controller to synchronize two identical chaotic systems. In [2], this design criterion was alleviated by using the H_∞ tracking control approach. Under the approach in [2], the grades of membership of the drive chaotic system are not necessarily known and the tracking performance is guaranteed by an H_∞ tracking performance index. The fuzzy-model-based control approach has combined with adaptive ability [3]-[4] to deal with chaotic systems subject to parameter uncertainties. With the outstanding approximation ability of the fuzzy system, the uncertain parameter values of the chaotic systems can be estimated in an online manner according to some update rules. A fuzzy controller can generate an appropriate control action based on the estimated parameters. The adaptive fuzzy approach offers a superior robustness property, however, computational demand and structural complexity of the controller are increased. In some operating environment, the system state information of the drive chaotic system reaches the responses system with time delay owing to the long-distance transmission. Under such a situation, the current state information of the drive chaotic system cannot be obtained to realize the synchronization. Synchronization using time-delayed feedback control was also investigated. Linear controller using constant time-delayed system state information of both drive and response chaotic system, and the current system state information of response chaotic system was proposed to realize the synchronization. Both time-delay independent and dependent stability conditions were derived using the Lyapunov-Krasovksii function. This delayed-feedback control approach was extended to adaptive fuzzy framework [5].

NOVELTY AND METHOD

In this paper, a fuzzy controller is proposed to synchronize two chaotic systems. The fuzzy controller makes use of current system state information of the response chaotic system and the time-delayed system state information of the drive chaotic system to realize the synchronization. The time delay to be considered is time varying and uncertain in value. It is due to this reason, the proposed fuzzy state-feedback controller cannot use the time-delayed system state information of the response chaotic system compared with the linear control and the adaptive fuzzy control [5] approaches of which constant time delay was considered. To cope with the time-varying delay, the boundedness property of the system states of the drive chaotic system is taken advantage to investigate system stability. Furthermore, the parameter uncertainties of the chaotic systems eliminate the favourable properties of the fuzzy-model-based control approach to facilitate the stability analysis and produce relaxed stability conditions [3], [5]. To alleviate the difficulties introduced by parameter uncertainties, membership functions of both fuzzy model and fuzzy controller are considered. Consequently, some free matrices are allowed to be introduced to the stability conditions to ease the stability analysis and produce less conservative stability conditions. LMI-based stability conditions are derived to aid the design of a fuzzy controller to realize the synchronization.

LMI-based stability conditions governing the system stability of the fuzzy-model-based chaotic systems with time-delay fuzzy controller have been obtained. The system performance of the chaotic synchronization of two chaotic systems is guaranteed by an H_∞ performance function. The following figure 1 shows the system state responses of the response (dotted lines) and drive (solid lines) Rössler systems with $u(t) = 0$ for $0 \le t < 50$s and the proposed fuzzy controller applied for $t \ge 50$s with the time delay of $\tau_d(t) = 0.01\left(1 + \frac{(1+\sin(t))}{4}\right)$. Both the chaotic system are subject to parameter uncertainties. Fig. 2 shows the tracking error. It can be seen that the proposed fuzzy controller is able to synchronize both the response and drive chaotic systems subject to parameter uncertainties and time-varying delay.

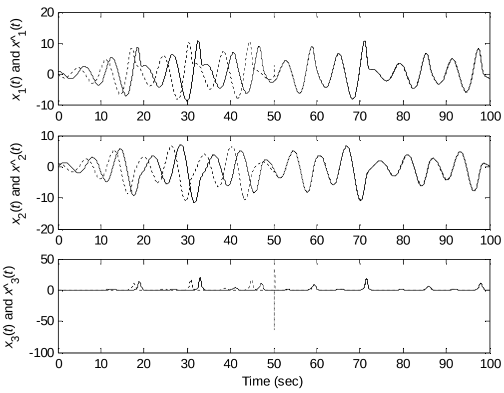

Fig. 1

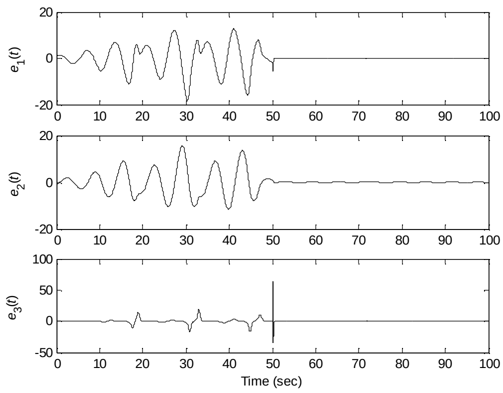

Fig. 2

The synchronization of chaotic systems subject to parameter uncertainties using timed-delayed fuzzy state-feedback controller has been investigated. The fuzzy state-feedback controller using the system state of the response chaotic system and the time-delayed system state of the drive chaotic system has been proposed to realize the synchronization. To overcome the analysis difficulties introduced by the system time delay and parameter uncertainties, first, T-S fuzzy has been employed to represent the chaotic systems subject to parameter uncertainties. Then, the membership functions of both fuzzy model and fuzzy controller have been considered to facilitate the stability analysis and produce less conservative stability analysis result. LMI-based stability conditions have been derived using Lyapunov-based approach to guarantee the system stability and aid the design of the time-delayed fuzzy controller. A simulation example has been given to illustrate the effectiveness of the proposed approach.

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