

Fast Implementation of a General L/M Rate Changer by a Filter Bank Structure

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Abstract—In this paper, we show that an L/M rate changer can be realized as a discrete time SISO (L, M) shift invariant system in form of a two-dimensional kernel function or a filter bank structure. Based on this realization, we can implement an L/M rate changer by a bank of filters with the average number of the coefficients in the filters in each channel is $1/L$ of the original L/M rate changer. Hence, the system is speed up by L . This helps the designer to design a sharp cutoff discrete time FIR filters in an L/M rate changer for some real time applications in video systems.

Index Terms— L/M rate changer, discrete time SISO linear (L, M) shift invariant system, kernel function, filter bank structure, sharp cutoff discrete time FIR filters

I. INTRODUCTION

An L/M rate changer shown in figure 1 plays an important role in the audio, image and video systems [1]. A good L/M rate changer sometimes requires a sharp cutoff discrete time FIR filter $h[n]$, especially those applied in the digital image and the digital video systems. However, a sharp cutoff discrete time FIR filter always contains a lot of coefficients. As a result, it requires a very long processing time and restricts in some of the real time applications in video systems.

In this paper, a parallel processing technique is proposed to break down a single L/M rate changer into a multi-channel system shown in figure 2, such that the average number of coefficients of the filters in each channel $h_j[n]$, for $j=0, 1, \dots, L-1$, is $1/L$ that of the original L/M rate changer. This will speed up the system by L .

II. REALIZATION OF AN L/M RATE CHANGER AS A DISCRETE TIME SISO LINEAR (L, M) SHIFT INVARIANT SYSTEM

When the input of an L/M rate changer shifts by M samples, the output will shift by L samples. Hence, an L/M rate changer is a discrete time SISO linear (L, M) shift invariant system. As a discrete time SISO linear (L, M) shift invariant system can be characterized by a two-dimensional

kernel function $g[n, k]$, where $g[n, k]=g[n-L, k-M]$, $\forall k, n \in \mathbf{Z}$ [2], or by a filter bank structure shown in figure 2 [3], so we can realize an L/M rate changer $h[n]$ in terms of a two-dimensional kernel function or a filter bank structure. The transformation from an L/M rate changer to a discrete time SISO linear (L, M) shift invariant system is summarized in the following two theorems:

Theorem 1

Given an L/M rate changer shown in figure 1, there exists a discrete time SISO linear (L, M) shift invariant system, characterized by a two-dimensional kernel function $g[n, k]$ with $g[n, k]=h[n \cdot M - k \cdot L]$, $\forall k, n \in \mathbf{Z}$, such that the input output relationship of the discrete time SISO linear (L, M) shift invariant system will be exactly the same as that of an L/M rate changer. The proof is as follows:

The input output relationship of the discrete time SISO linear (L, M) shift invariant system is governed by [2]:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} g[n, k] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (1)$$

Let $g[n, k]=h[n \cdot M - k \cdot L]$, $\forall k, n \in \mathbf{Z}$, we have:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} h[n \cdot M - k \cdot L] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (2)$$

which is the same input output relationship of the L/M rate changer [1], so this proves the theorem. ■

For example, if the upsampler and the downsampler of an L/M rate changer is 2 and 3 respectively, and the filter is:

$$h[n] = \begin{cases} \frac{1}{2} \cdot \text{sinc}\left(\frac{n}{2}\right) & ; \text{for } -10 \leq n \leq 10, \\ 0 & ; \text{otherwise} \end{cases} \quad (3)$$

then the two-dimensional kernel function of the discrete time SISO linear (L, M) shift invariant system is:

$$g[l, k]=0.5 \cdot \delta[k],$$

$$g[l, k] = \begin{cases} \frac{1}{2} \cdot \text{sinc}\left(\frac{3-2 \cdot k}{2}\right) & ; \text{for } -3 \leq k \leq 6, \\ 0 & ; \text{otherwise} \end{cases} \quad (4)$$

and $g[n, k]=g[n-2, k-3]$ for other integer values of n .

Theorem 2

Given an L/M rate changer shown in figure 1, there

exists a discrete time SISO linear (L, M) shift invariant system, characterized by a filter bank structure shown in figure 2 with $h_j[n]=h[j \cdot M+n \cdot L]$, for $\forall n \in \mathbf{Z}$ and $j=0, 1, \dots, L-1$, such that the input output relationship of the discrete time SISO linear (L, M) shift invariant system will be exactly the same as that of an L/M rate changer. The proof is as follows:

The input output relationship of the discrete time SISO linear (L, M) shift invariant system is governed by:

$$y[n] = \begin{cases} \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_0 \left[\frac{n \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = 0, \\ \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_1 \left[\frac{(n-1) \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = 1, \\ \vdots & \vdots \\ \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_{L-1} \left[\frac{(n-L+1) \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = L-1 \end{cases} \quad (5).$$

Let $h_j[n]=h[j \cdot M+n \cdot L]$, for $\forall n \in \mathbf{Z}$ and $j=0, 1, \dots, L-1$, we have:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} h[n \cdot M - k \cdot L] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (6),$$

which is also the same input output relationship of the L/M rate changer, hence, the theorem is proved. ■

For the same example described after theorem 1, the filters of the filter bank structure of the discrete time SISO linear (L, M) shift invariant system are:
 $h_0[n]=0.5 \cdot \delta[n]$ and

$$h_1[n] = \begin{cases} \frac{1}{2} \cdot \sin c \left(\frac{3 \cdot n + 2}{2} \right) & ; \text{for } -6 \leq n \leq 3, \\ 0 & ; \text{otherwise} \end{cases} \quad (7).$$

III. REALIZATION OF A DISCRETE TIME SISO LINEAR (L, M) SHIFT INVARIANT SYSTEM AS AN L/M RATE CHANGER

It can be shown that an L/M rate changer is input output equivalent to a discrete time SISO linear (L, M) shift invariant system if and only if L and M is co-prime. Hence, only the discrete time SISO linear (L, M) shift invariant system with L and M is co-prime can be realized as an L/M rate changer. Under this condition, the transformation from a discrete time SISO linear (L, M) shift invariant system to an L/M rate changer is summarized in the following two theorems:

Theorem 3

Given a discrete time SISO linear (L, M) shift invariant system with L and M is co-prime, there exists an L/M rate changer shown in figure 1 with $h[n \cdot M - k \cdot L] = g[n, k]$, $\forall n, k \in \mathbf{Z}$, such that the input output relationship of the L/M rate changer will be exactly the same as that of the discrete time SISO linear (L, M) shift invariant system. The proof is as follows:

The input output relationship of the L/M rate changer is governed by [1]:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} h[n \cdot M - k \cdot L] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (8).$$

Since $h[n \cdot M - k \cdot L] = g[n, k]$, $\forall n, k \in \mathbf{Z}$, we have:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} g[n, k] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (9),$$

which is the same input output relationship of the discrete time SISO linear (L, M) shift invariant system, so this proves the theorem. ■

For example, if L and M of a discrete time SISO linear (L, M) shift invariant system are 2 and 3 respectively, and the two-dimensional kernel function is:
 $g[0, k]=k$, $g[1, k]=2 \cdot k$ and $g[n, k]=g[n-2, k-3]$ for other integer values of n , then the filter in an L/M rate changer is:

$$h[n] = \begin{cases} -\frac{n}{2} & ; \text{for } n \text{ is even,} \\ 3-n & ; \text{for } n \text{ is odd} \end{cases} \quad (10).$$

Theorem 4

Given a discrete time SISO linear (L, M) shift invariant system with L and M is co-prime, there exists an L/M rate changer shown in figure 1 with $h[j \cdot M+n \cdot L]=h_j[n]$, for $\forall n \in \mathbf{Z}$ and $j=0, 1, \dots, L-1$, such that the input output relationship of the L/M rate changer will be exactly the same as that of the discrete time SISO linear (L, M) shift invariant system. The proof is as follows:

The input output relationship of the L/M rate changer is governed by [1]:

$$y[n] = \sum_{k \rightarrow -\infty}^{+\infty} h[n \cdot M - k \cdot L] \cdot x[k], \quad \forall n \in \mathbf{Z} \quad (11).$$

Since $h[j \cdot M+n \cdot L]=h_j[n]$, for $\forall n \in \mathbf{Z}$ and $j=0, 1, \dots, L-1$, we have:

$$y[n] = \begin{cases} \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_0 \left[\frac{n \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = 0, \\ \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_1 \left[\frac{(n-1) \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = 1, \\ \vdots & \vdots \\ \sum_{k \rightarrow -\infty}^{+\infty} x[k] \cdot h_{L-1} \left[\frac{(n-L+1) \cdot M}{L} - k \right] & ; \text{if } \text{mod}(n, L) = L-1 \end{cases} \quad (12),$$

which is also the same input output relationship of the discrete time SISO linear (L, M) shift invariant system, hence, the theorem is proved. ■

For example, if the upsampler and the downsampler of the discrete time SISO linear (L, M) shift invariant system are 2 and 3 respectively, and the filters of the filter bank structure are $h_0[n]=n$ and $h_1[n]=2 \cdot n$ respectively, then the filter in an L/M rate changer is:

$$h[n] = \begin{cases} \frac{n}{2} & ; \text{for } n \text{ is even,} \\ n-3 & ; \text{for } n \text{ is odd} \end{cases} \quad (13).$$

IV. APPLICATION ON FAST IMPLEMENTATION OF AN L/M RATE CHNAGER

By realizing an L/M rate changer as a discrete time SISO linear (L, M) shift invariant system in form of a filter bank structure shown in figure 2, the average number of coefficients of the filters in each channel $h_j[n]$, for $j=0, 1, \dots, L-1$, is $1/L$ that of the original L/M rate changer, this will speed up the system by L .

The transformation helps the user to design an L/M rate changer with a sharp cutoff discrete time FIR filter $h[n]$, and can be applied in real time applications in video systems.

V. CONCLUSION

In this paper, we show that a discrete time SISO (L, M) shift invariant system with L and M is co-prime can be realized as an L/M rate changer. Also, we show that an L/M rate changer can be realized as a discrete time SISO (L, M) shift invariant system in form of a two-dimensional kernel function or a filter bank structure shown in figure 2. Based on this realization, we can implement an L/M rate changer by a bank of filters with the average number of the coefficients in the filters in each channel is $1/L$ of the original L/M rate changer. This can speed up the system by L and helps the designer to design a sharp cutoff discrete time FIR filters in an L/M rate changer for some real time applications in video systems.

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REFERENCES

- [1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [2] Tongwen Chen, Li Qiu and Er-Wei Bai, "Generate Multirate Building Structures with Application to Nonuniform Filter Banks," *IEEE Transaction on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 45, No. 8, pp. 948-958, August, 1998.
- [3] Wing-kuen Ling and Kwong-Shun Tam, "Theory of Discrete Time SISO Linear (L, M) Shift Invariant System", *IEEE International Symposium on Intelligent Multimedia, Video and Speech Processing*, May, 2001.

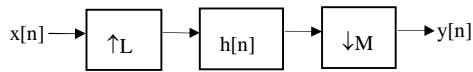


Fig. 1. An L/M rate changer

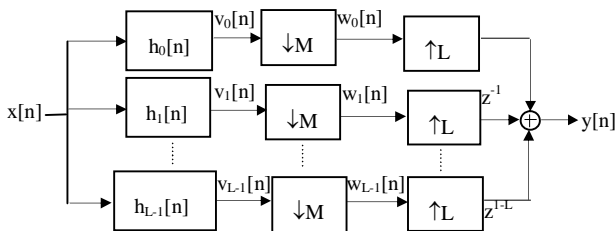


Fig. 2. Filter bank realization of an L/M rate changer