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General relativistic effects on the evolution of binary systems

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Abstract. When a radio pulsar brakes down due to magnetodipole emission, its gravitational mass decreases accordingly. If the pulsar is hosted in a binary system, this mass loss will increase the orbital period of the system. We show that this relativistic effect can be indeed observable if the neutron star is fast and magnetized enough and that, if observed, it will help to put tight constraints to the equation of state of ultradense matter. Moreover, in Low Mass X-ray Binaries that evolve towards short periods, the neutron star lights up as a radio pulsar during the “period gap”. As the effect we consider contrasts the orbital period decay, the system spends a longer time in this phase. As a consequence, the neutron star can survive this phase only if it is non-supramassive. Since in such binaries $\sim 0.8M_\odot$ can be accreted onto the neutron star, short period ($P \leq 2\ h$) millisecond X-ray pulsars like SAX J1808.4-3658 can be formed only if either a large part of the accreting matter has been ejected from the system, or the equation of state of ultradense matter is very stiff.

INTRODUCTION

Neutron stars (hereafter NSs) are the densest objects we can observe directly: their density can exceed by an order of magnitude the nuclear density and therefore their binding energy is non-negligible. Constraining the equation of state (EOS) of matter at supernuclear densities is of great interest. Anyway, only a few constraints that are sufficiently model-independent have been found from radio and X–ray observations. Moreover, these constraints did not allow to rule out most of the EOSs proposed in the literature [1].

Millisecond radio pulsars (MSPs) are some of the fastest spinning NSs known to date, and their timing behavior is measurable with great precision thanks to their stability. Of particular interest are millisecond pulsars hosted in binary systems: in particular, timing studies of NS-NS binaries allowed us to test general relativity to an unprecedented precision (see for example [2]).

These pulsars are thought to originate from Low Mass X-ray binaries (LMXBs) once accretion stops [3]. LMXBs in which the mass transfer is driven by orbital angular momentum losses evolve towards short orbital periods, typically below 2 h. It should be expected that these LMXBs evolve through a “period gap” similar to the gap found in the distribution of cataclysmic binaries [4].

Here we present a mechanism that, if observed in binary MSPs, could allow to put strong constraints on the EOS of NSs. We will show also how this mechanism affects the...
evolution of shrinking LMXBs, allowing us to make some predictions on such systems.

**CONSTRAINING THE EOS WITH BINARY RADIO PULSAR TIMING**

In a NS, the variation gravitational mass \( M_G \) per unit time depends on the variation of both the baryonic mass \( M_B \) and the angular momentum \( J \) of the star [5]:

\[
\dot{M}_G = \Phi \dot{M}_B + \frac{\omega}{c^2} \dot{J}.
\]  

(1)

where \( \omega \) is the NS spin frequency, and \( \Phi \) is the energy needed to bring a unit mass from infinity to the surface of the star. Although a pulsar does not lose matter \( \dot{M}_B = 0 \), it loses angular momentum via magnetodipole radiation, and therefore loses gravitational mass. This will have effects on the orbital evolution of the system. We obtain for the evolution of the orbital period \( P \) [6]:

\[
\frac{\dot{P}}{P} = -2 \frac{\dot{M}_G}{M_c} \frac{q}{1 + q} + \frac{\dot{P}_{GW}}{P}
\]  

(2)

where \( q = M_c/M_G \) and \( \dot{P}_{GW} \) is the orbital period derivative due to the emission of gravitational waves, that is in general negative. As the gravitational mass of the NS decreases, the period of the binary system widens, yielding a positive contribution to the orbital period derivative, opposite to the contribution of gravitational waves emission. Since the effect of gravitational mass loss is \( \propto P \), while the effect of gravitational waves emission is \( \propto P^{-5/3} \) [7], the former effect will be dominant in systems with large enough orbital periods (say \( P \geq 6 \) h). To a good approximation we can write:

\[
\dot{M}_G = \frac{I}{c^2} \omega \dot{\omega}
\]  

(3)

where \( I \) is the momentum of inertia of the NS. In a binary MSP it is often possible to measure both the spin frequency and its derivative with high precision. The orbital period derivative depends then only on measured quantities (the spin frequency \( \omega \) and its derivative \( \dot{\omega} \), on the masses of the two stars and of the momentum of inertia of the neutron star (see equation 2). We can extract information on the two masses from the mass function \( f(M) \), that is measurable in binary MSP with very good precision. Using it, we can impose constraints on the momentum of inertia of the NS. Since the momentum of inertia depends strongly on the EOS of the NS [8], the detection of this effects will allow us to discriminate between various EOSs on a solid observational basis. Suppose, for example, that a system with an orbital period \( P = 8 \) h, a spin period \( P_s = 2 \) ms, \( \dot{P}_s = 3 \times 10^{-19} \) and a mass function of 0.005\( M_\odot \) is observed, and that the orbital period derivative has been measured to be \( +2.5 \times 10^{-14} \). In figure 1, we plot the values of the masses of the two stars that are compatible with the value of the orbital period derivative we obtained, in the hypothesis that the NS is governed by EOS A (the pure neutron EOS by Pandharipande [9]) or by EOS BBB (a realistic EOS by Baldo,
Figure 1. Allowed values of the mass of the primary versus the mass of the companion (both in solar masses) for the system described in the text. The two lines are for NSs with EOS A (dashed line) and NSs with EOS BBB (solid line). The dotted line indicates the lower limit on the companion mass obtained if the mass function is $5 \times 10^{-3} M_\odot$.

Bombaci and Burgio, [10]) respectively. The mass function requires that the values of the two masses should be above the dashed line in figure 1. Therefore, this particular system would allow us to rule out EOS A on an almost model-independent basis. This could be a potentially very powerful method for constraining the EOS of NS with very precise measurements and with an almost model-independent method.

Which is, between the known millisecond binary radio pulsars, the best candidate for detecting such an effect? The most promising object we found is PSR J0218+4232, which is constituted of a NS spinning at 2.3 ms in orbit with a white dwarf companion, with an orbital period of 2 days. The mass ratio is measured to be 0.13 ± 0.04 [11].

According to equation (2), we find that the orbital period derivative of the system is $\dot{P}_{\text{orb}} = 2.5 \times 10^{-14} I_{45}$, where $I_{45}$ is the moment of inertia of the NS in units of $10^{45}$ g cm$^2$. The variation in the orbital period can be derived from the measure of the periastron time delay. The effect of an orbital period derivative on the periastron arrival time is given by:

$$\Delta T_{\text{per}} \simeq 0.5 \frac{\dot{P}}{P} \Delta T_{\text{obs}}^2.$$  \hspace{1cm} (4)

We find that we will need $118/I_{45}^{1/2}$ years of observation to detect a delay of 1 second. However, if a pulsar with an higher spin-down energy will be discovered, this effect might become observable in a much shorter observation time.
EVOLUTION OF LOW MASS X-RAY BINARIES TOWARDS SHORT ORBITAL PERIODS

LMXBs that transfer mass due to angular momentum losses evolve towards short orbital periods. As the period becomes smaller and smaller, the companion becomes fully convective (typically at an orbital period $\sim 3$ h), magnetic braking is thought to stop, the companion recovers thermal equilibrium and the mass transfer ceases. The binary system will continue to shrink due to gravitational waves emission, until it reaches an orbital period $\sim 2$ h, when mass transfer resumes.

What happens to the NS during this gap in the accretion process? It will light up as a millisecond radio pulsar since accretion has stopped. It will moreover be spinning very fast, as a considerable amount of matter has been accreted [12]. Two general relativistic effects are relevant during the gap:

1. The widening of the system due to the loss of gravitational mass from the pulsar (see the preceding section). This widening has the effect of increasing the duration of the detached phase of the system. This increase can vary strongly depending on the spin-down energy of the primary and on $q$ (see equation 2).
2. The silent collapse to a black hole if the pulsar is supramassive (i.e. its mass exceeds the maximum non-rotating mass), once it loses enough angular momentum.

The NS survives the gap only if the collapse time (e.g. the time it takes for it to collapse to a black hole, $T_c$) is larger than the gap time (e.g. the time the system needs to cross the period gap, $T_g$). Else, the NS will collapse to a black hole before the mass transfer resumes.

We can define the collapse time via the equation:

$$ J_{\text{in}} - J_{\text{crit}} = - \int_0^{T_c} \dot{J} \, dt $$

(5)

where $J_{\text{in}}$ is the angular momentum at the beginning of the detached phase, $J_{\text{crit}}$ is the critical angular momentum below which the star collapses, and $\dot{J}$ is the angular momentum lost during the pulsar phase. Since we know that $\dot{J} = \dot{E}/\omega$, using the relativistic formulation of the energy released from a rotating dipole [13] we obtain

$$ \dot{J} = -\frac{2}{3c^3} \mu^2 \omega^3 \left( \frac{f}{N^2} \right)^2 $$

(6)

where $\mu$ is the magnetic dipole moment of the NS and

$$ N = (1 - 2\chi)^{1/2}, \quad \chi = \frac{GMG}{c^2 R} $$

$$ f = \frac{3}{8} \chi^{-3} \left[ \log N^2 + 2\chi (1 + 2\chi) \right]. $$

(7)

where $R$ is the equatorial radius of the NS. On the other hand, the time the system spends in the gap, i.e. without accreting, is defined by the equation

$$ \Delta P_{\text{gap}} = - \int_0^{T_g} \dot{P} \, dt $$

(8)
where $\Delta P_{\text{gap}}$ is the amplitude of the period gap and $P$ is defined in equation (2). In most situations $\Delta P_{\text{gap}} \sim 1 \text{ h}$, and the mass of the companion is $\leq 0.25M_\odot$. Integrating these two equations we find that, over a vast range of EOSs and of initial conditions, if the NS is supramassive and an even weak magnetic field ($\sim 10^8 \text{ G}$), $T_g > T_c$. This means that LMXBs that host a NS and have a period shorter than 3 h cannot be supramassive. This is true, obviously, only if the binary system was a NS-main sequence binary: in systems where the companion is a white dwarf, and that evolve from short periods towards long periods and do not evolve though a pulsar phase. For instance, the surface magnetic field of the first millisecond X-ray pulsar discovered, SAX J1808.4-3658 [14], has been estimated to be in the range $(1 - 5) \times 10^8 \text{ G}$ [15]. If this system has a MS companion, so that it evolved from longer orbital periods, it cannot be supramassive, as it survived the period gap. The primary will not be a supramassive NS only if one of the following holds:

1. A relevant part of the accreting matter has been ejected from the system, and the mass transfer has therefore been non-conservative for most of the binary evolution.
2. The maximum non-rotating mass of the NS is very high, $\geq 2M_\odot$ (i.e. the EOS of the NS is very stiff).

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