An Experimental study for Stabilization of Inverted Pendulum

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Submitted by-

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CERTIFICATE

*This is to certify that the thesis titled "**An Experimental study for Stabilization of Inverted Pendulum**" by **Mr. Sudipta Chakraborty,** submitted to the National Institute of Technology-Rourkela for the award of Master of Technology in Electrical Engineering is a record of bona fide research work carried out by him in the Department of Electrical Engineering, under my supervision. We believe that this thesis fulfils part of the requirements for the award of degree of Master of Technology. The results embodied in this thesis have not been submitted for the award of any degree elsewhere.*

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Prof. Sandip Ghosh

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Date: 03-05-2014 **Sudipta Chakraborty**
Place: Rourkela

ABSTRACT

Stabilization of Inverted Pendulum is defined as a very basic classical control problem in Control System. The Dynamics of Cart Inverted Pendulum is related to many real life applications like missile launching system, balancing systems like human walking, Aircraft landing pad in sea etc. This is a highly Unstable and non-linear system. This system is a under actuated system and also a non-minimum phase system so design a Controller for Inverted Pendulum System is very complex.

This thesis includes system and hardware description of Inverted Pendulum System, Dynamics of the system, State space model, Derivation of Transfer Functions. In Past a lot of research work has already been done in Inverted Pendulum for development of Control Strategy. Here in this thesis we have done a very small work to design Control Strategy and also validate them with real-time experiments.

In this thesis two-loop PID, PID+PI & LQR control have been implemented for Inverted Pendulum System and this control strategies gives satisfactory response.

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CHAPTER
1
INTRODUCTION

1.1 Introduction on Inverted Pendulum

The problem associated with stabilization of Inverted Pendulum is a very basic and benchmark problem of Control System. The design of Inverted Pendulum consists of a DC motor, Cart, Pendulum and Cart driving mechanism.

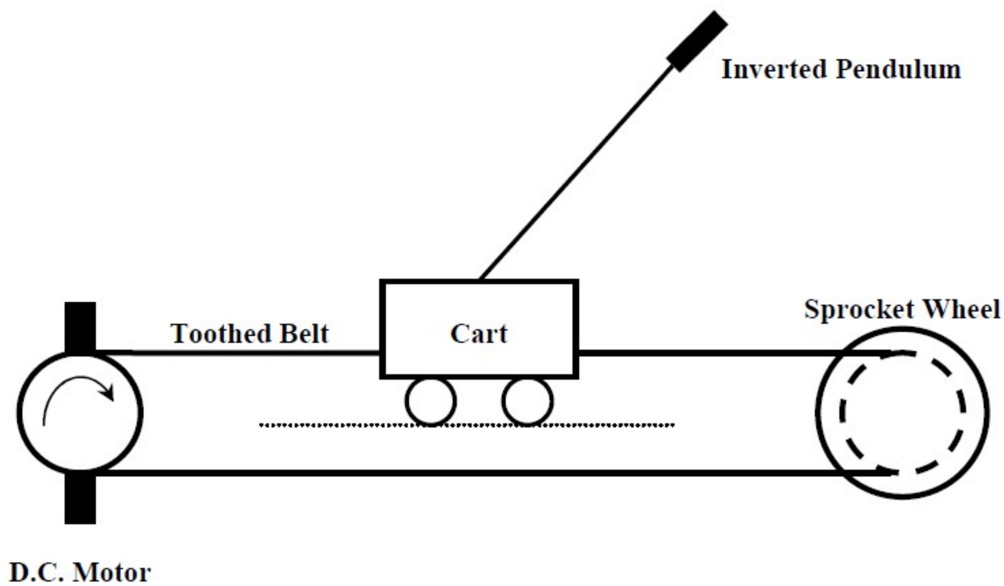


Figure 1.1 Simple Inverted Pendulum Setup

The nature of this system is single input and multi output system where Control voltage act as input and the output of the system are cart position and angle. Here we have to stabilize the pendulum angle to Inverted position which is a challenging work to do as the Inverted position is a highly unstable equilibrium. The main characteristics of the system are Highly Unstable as we have to stabilize the pendulum angle to Inverted position, it is a highly non-linear system as because the dynamics of inverted pendulum consists non-linear terms, as the system have a pole on its right hand it is a non-minimum phase system and the system is also under actuated because the system have only one actuator (the DC Motor) and two degree of freedom.

1.2 System Description

1.2.1 Dynamics of the system

In this section defines the system dynamics of Inverted Pendulum with the help of Newton's law of motion. According to the system dynamics the system has two degree of freedom the one is for cart movement and the other one is for Pendulums rotational motion.

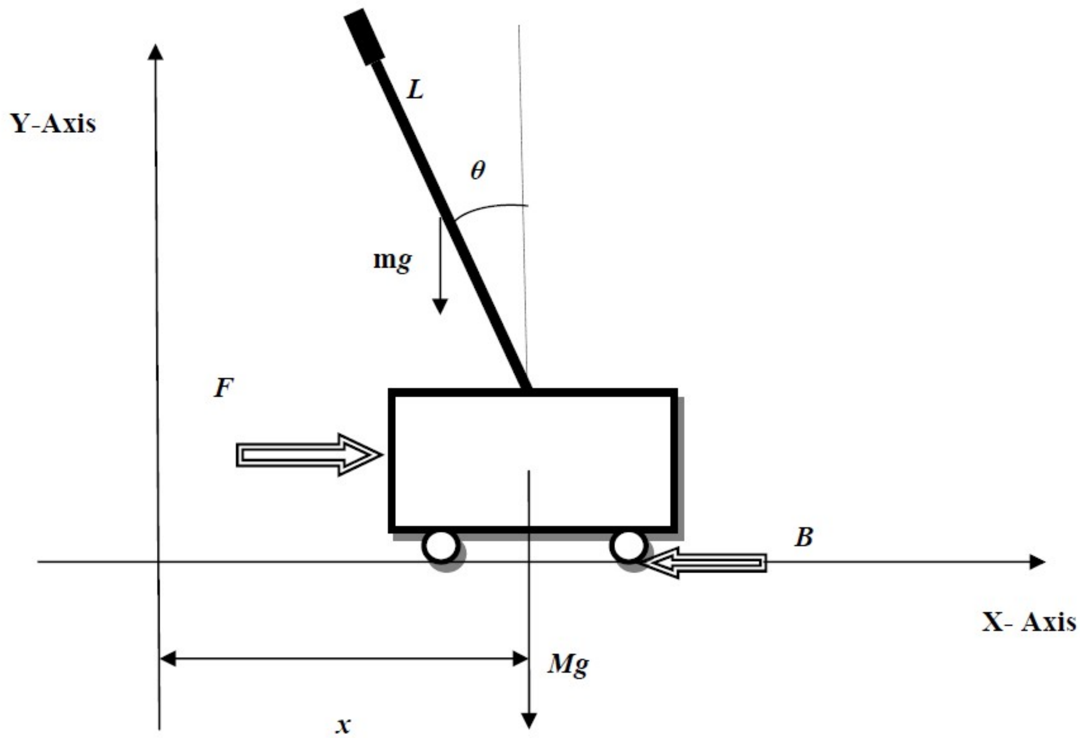


Figure 1.2 Inverted Pendulums parametric presentation

M- Cart mass

m- Pendulum mass

J- Moment of inertia

L- Pendulum length

b- Cart friction co-efficient

g – Gravitational force

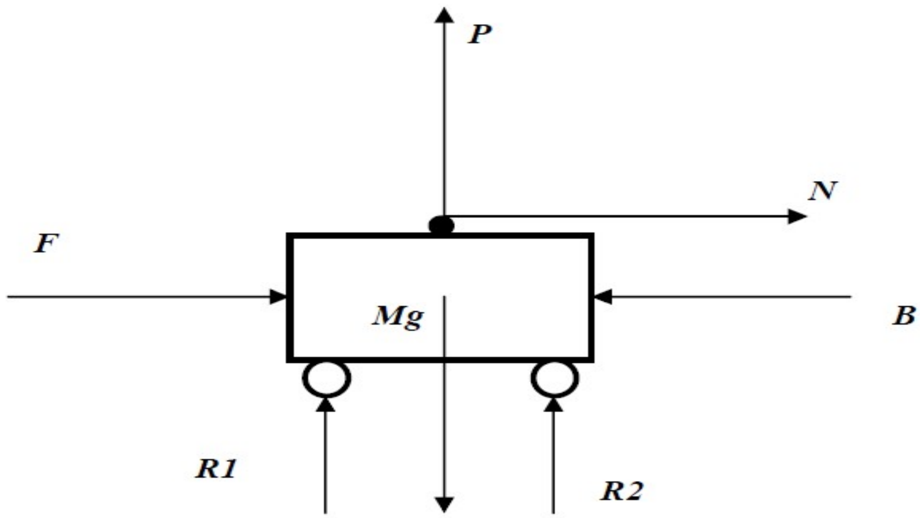


Figure 1.3 Inverted Pendulums free body diagram

We are considering only horizontal forces for analysis as because the motion of the cart is linear in nature. Thus

$$Ma_X = F + N - B \text{ , Where acceleration in Horizontal plane is denoted by } a_X \tag{1.1}$$

N is the Horizontal reaction force and N is given by the equation

$$N = m\frac{d^2}{dt^2}(x + L\sin\theta) = m\ddot{x} + m\ddot{\theta}L\cos\theta - m(\dot{\theta})^2 L\sin\theta \tag{1.2}$$

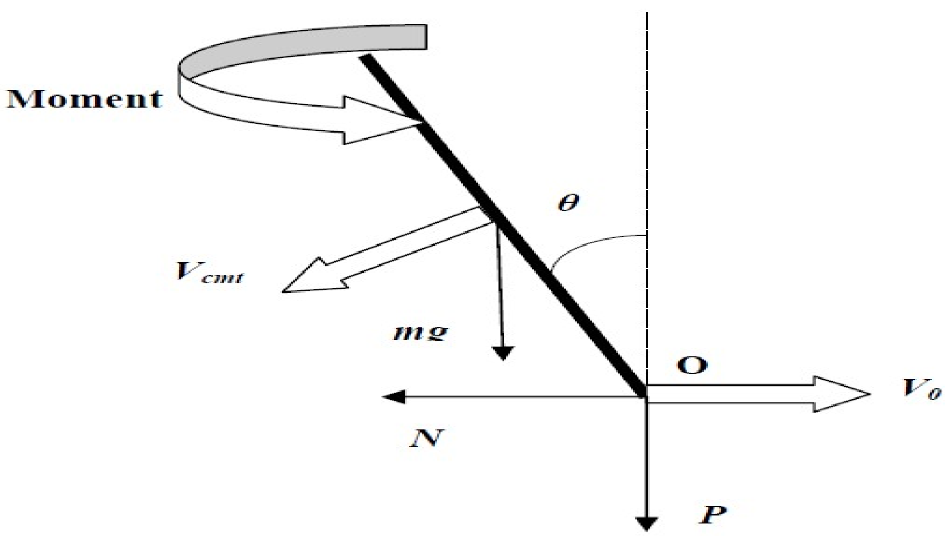


Figure 1.4 Pendulums free body diagram

Here vertical reaction force is defined by P and this given by weight given by the pendulum on cart. Here $L\cos\theta$ is given by the Pendulums displacement from the pivot. So

$$P + mg = m\frac{d^2}{dt^2}\,(x + L\cos\theta) = mL\ddot{\theta}\sin\theta + m(\dot{\theta})^2 L\cos\theta \tag{1.3}$$

Velocity of centre of mass is denoted by V_{cmt} now if will taking the sum of moments we get

$$-NL\cos\theta - PL\sin\theta = J\ddot{\theta} \tag{1.4}$$

Now put the values of (1.2) and (1.3) in equation no (1.4) we get

$$mL\ddot{x}\cos\theta - (mL^2 + J)\ddot{\theta} = -mgL\sin\theta \tag{1.5}$$

Now if we will substitute the equation (1.2) in equation no (1.1) we get

$$\ddot{\theta} = \frac{mL}{\sigma}[(F - b\dot{x})\cos\theta - m(\dot{\theta})^2 L\cos\theta\sin\theta + (m + M)g\sin\theta] \tag{1.6}$$

If we solve (1.5) and (1.6) and simplify we will get

$$\ddot{x} = \frac{1}{\sigma}[(J + mL^2)(F - b\dot{x} - mL\dot{\theta}^2\sin\theta) + mL^2 g\sin\theta\cos\theta] \tag{1.7}$$

Here σ is given by $\sigma = mL^2(M + m\cos^2\theta) + J(M + m)$ (1.8)

These two equations describe the dynamics of the system.

1.2.2 Linearization

Here in this section the description about the linearization of the non-linear equations are given. Here we use tailors series expansion for Linearize the non-linear equations. We have to stabilize the pendulum angle at the Inverted position so assume

$$\theta \approx 0$$

$$\sin\theta = \theta$$

$$\cos\theta = 1 \tag{1.9}$$

$$\text{And } \dot{\theta}^2 = 0$$

After linearizing the equations we get

$$\ddot{\theta} = \frac{mL}{\sigma'}[(F - b\dot{x}) + (m + M)g\theta] \tag{1.10}$$

$$\ddot{x} = \frac{1}{\sigma'}[(J + mL^2)(F - b\dot{x}) + mL^2 g\theta] \tag{1.11}$$

Here σ' denoted by $\sigma' = \mathrm{MmL}^2 + \mathrm{J}(\mathrm{M}+\mathrm{m})$

Now converting in to state space form we get

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-\left(J+mL^2\right)}{\sigma'} & \dfrac{-m^2L^2g}{\sigma'} & \dfrac{mLb_t}{\sigma'} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{mlb}{\sigma'} & \dfrac{\left(M+m\right)mgL}{\sigma'} & \dfrac{-\left(M+m\right)b_t}{\sigma'} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\left(J+mL^2\right)}{\sigma'} \\ 0 \\ \dfrac{-mL}{\sigma'} \end{bmatrix} F \tag{1.12}$$

And the output equation is given by

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \tag{1.13}$$

Now if we neglect the cart friction co-efficient then the transfer functions will be as

$$\frac{X(s)}{F(s)} = \frac{\left(J + mL^2\right)s^2 - mgL}{s^2\left(\left(MmL^2 + \left(M+m\right)J\right)s^2 - mgL(m+M\right)} \tag{1.14}$$

$$\frac{X(s)}{F(s)} = \frac{mLs^2}{s^2\left(\left(MmL^2 + \left(M+m\right)J\right)s^2 - mgL(m+M\right)} \tag{1.15}$$

Now substituting the values

Parameters	**Values**
Mass of Cart M	2.4kg
Mass of Pendulum m	0.23kg
Moment if Inertia J	0.099kg-m^2
Length of Pendulum L	0.4m
Cart friction Co-efficient b	0.05Ns/m
Acceleration due to gravity g	9.81m/s^2

$$\frac{X(s)}{F(s)} = \frac{0.3894s^2 - 2.6506}{s^2\left(s^2 - 6.807\right)} = \frac{0.3894}{s^2} \tag{1.16}$$

$$\frac{\theta(s)}{F(s)} = \frac{0.2638s^2}{s^2\left(s^2 - 6.807\right)} \approx \frac{0.2638}{\left(s^2 - 6.807\right)} \tag{1.17}$$

1.2.3 Setup for Experiment

Experimental setup for Inverted Pendulum consists of PC with PCI-1711 card, Digital Controller for Pendulum, SCSI cable adaptor, DC motor which act as an actuator device, Cart, Optical encoders, pendent pendulum, Adjustable feet, track with a length of 1 m, Connecting cables and Matlab software.

Cart and Pendulum system is the main thing of this setup. The cart driven by four wheels and a pulley chain mechanism which is connected to a DC motor. The cart is driven by the DC motor according to the Control voltage.

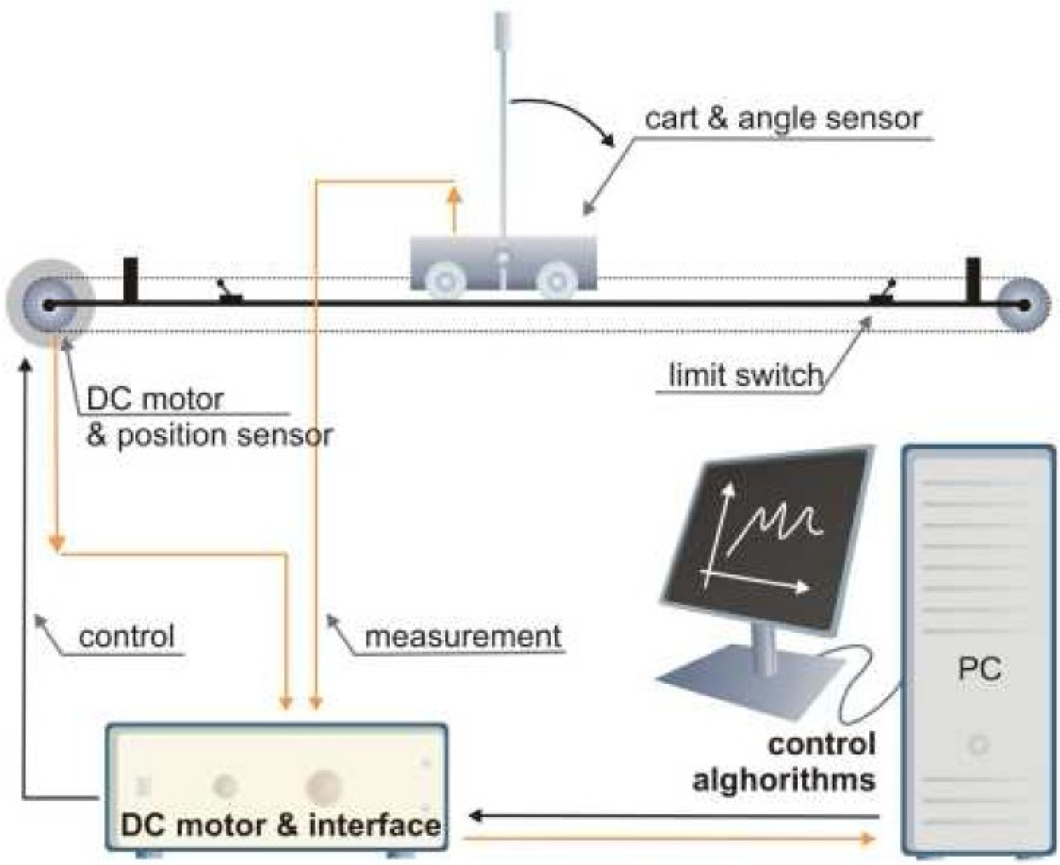


Figure 1.5 Experimental Setup for Inverted Pendulum

In the track there are two limit switches which will cut the power whenever the cart is going to exceed the track limit. The optical encoder consists of a light source and decoder.

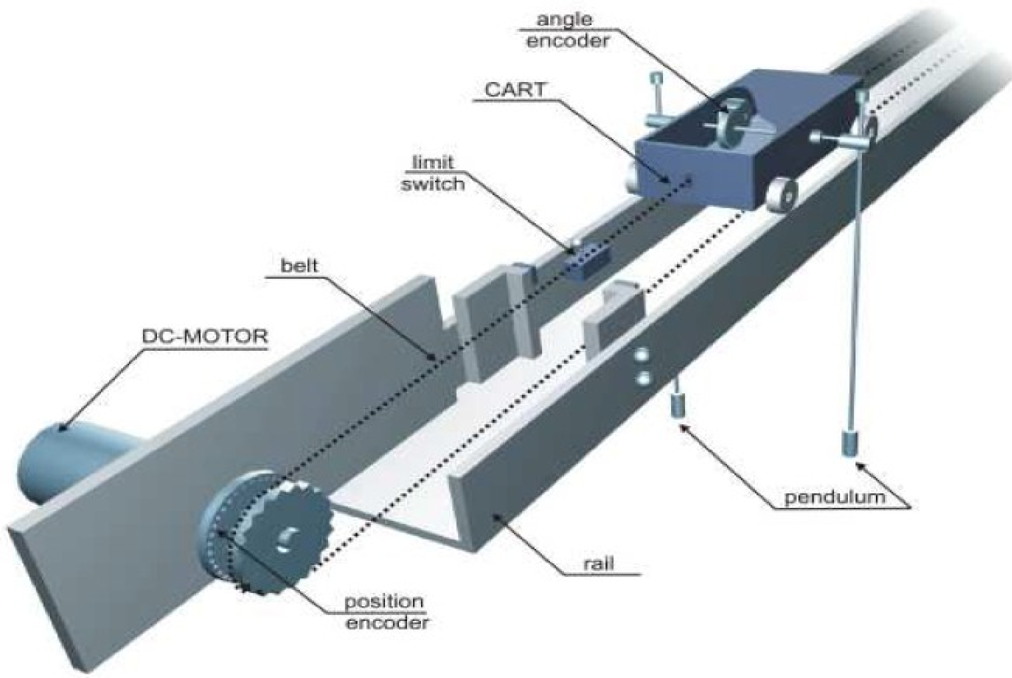


Figure 1.6 Mounting of Sensors

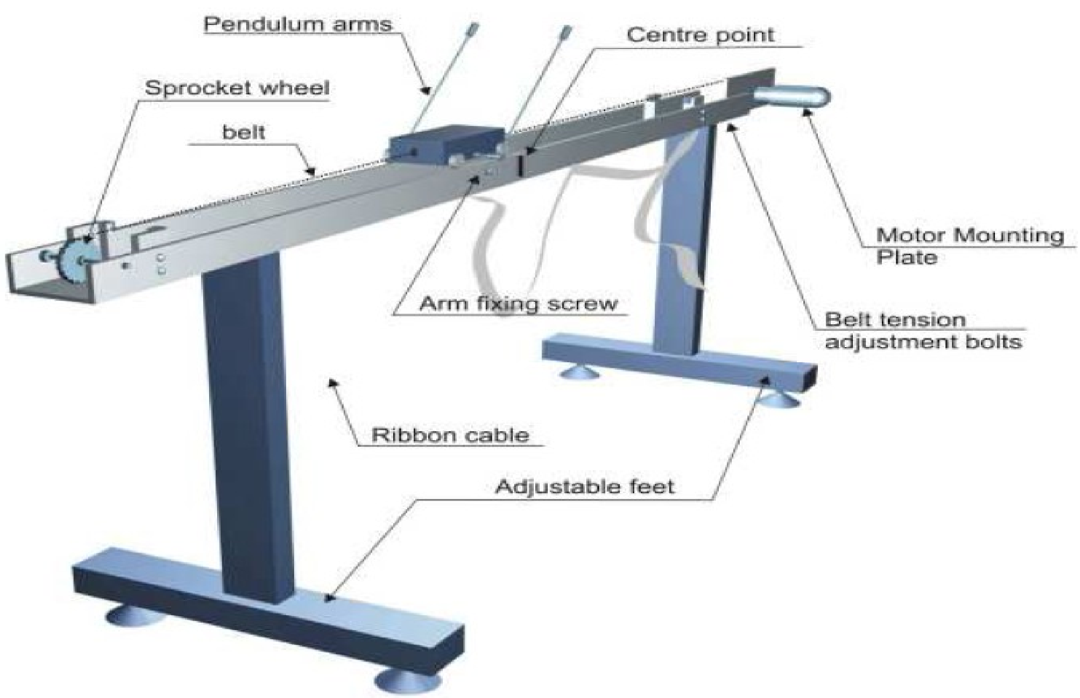


Figure 1.7 Mechanical Setup for Inverted Pendulum

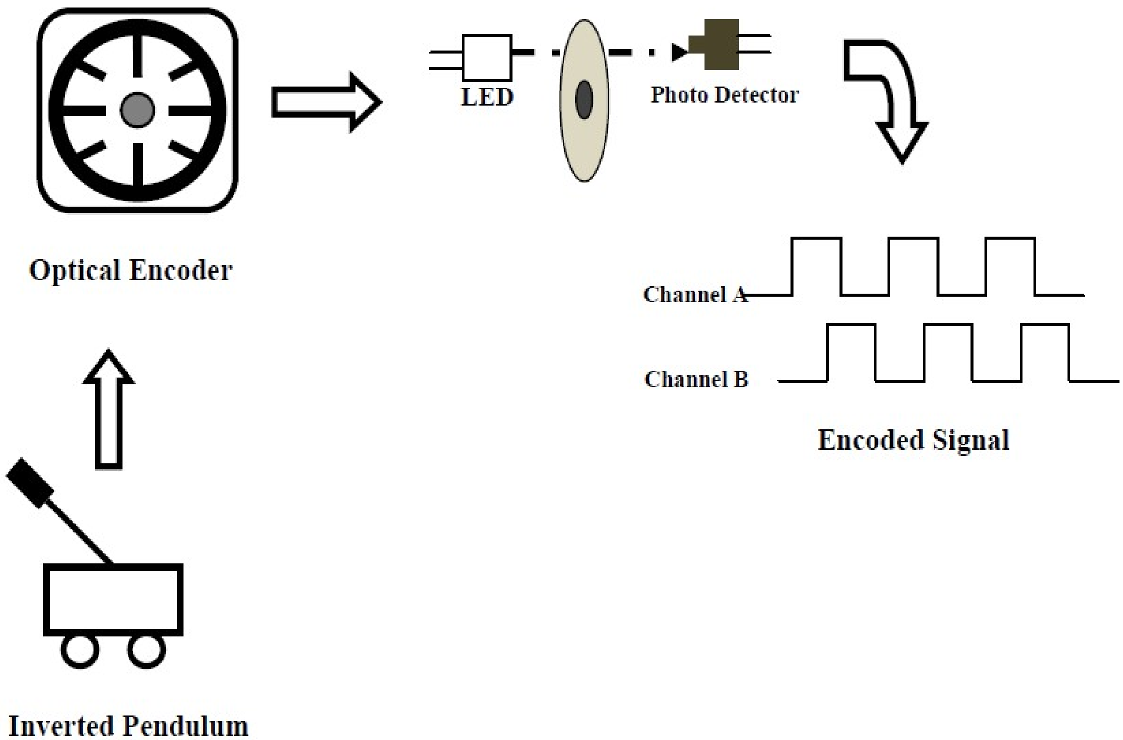


Figure 1.8 Operating Principle of Encoders

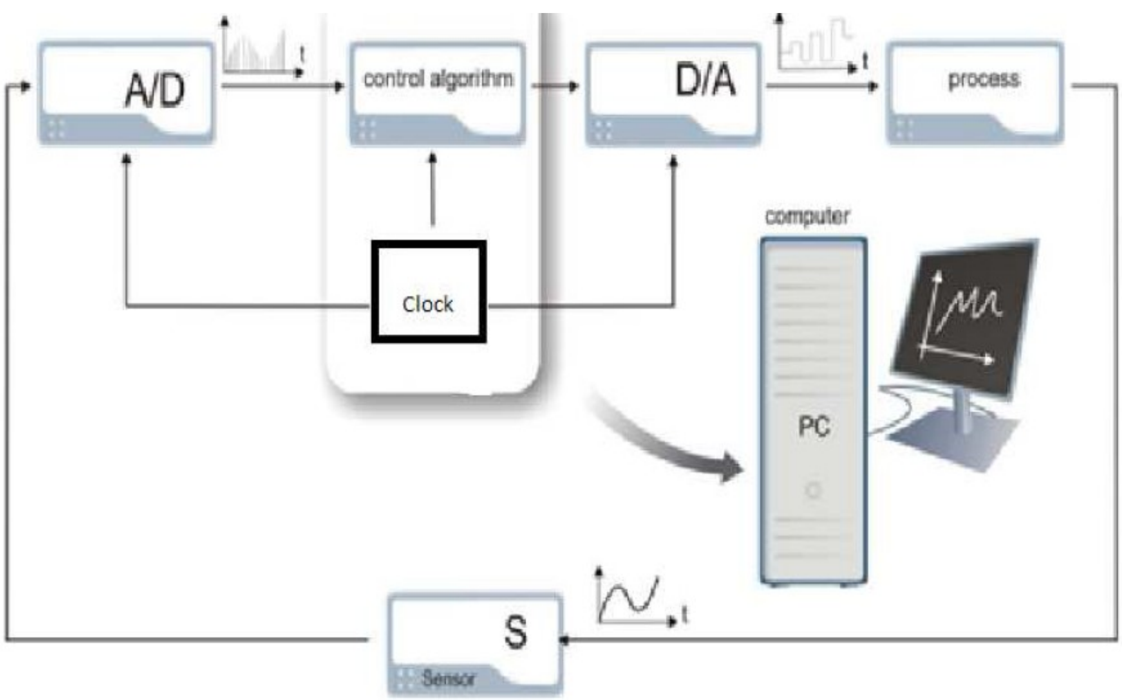


Figure 1.9 Control Algorithms for Inverted Pendulum

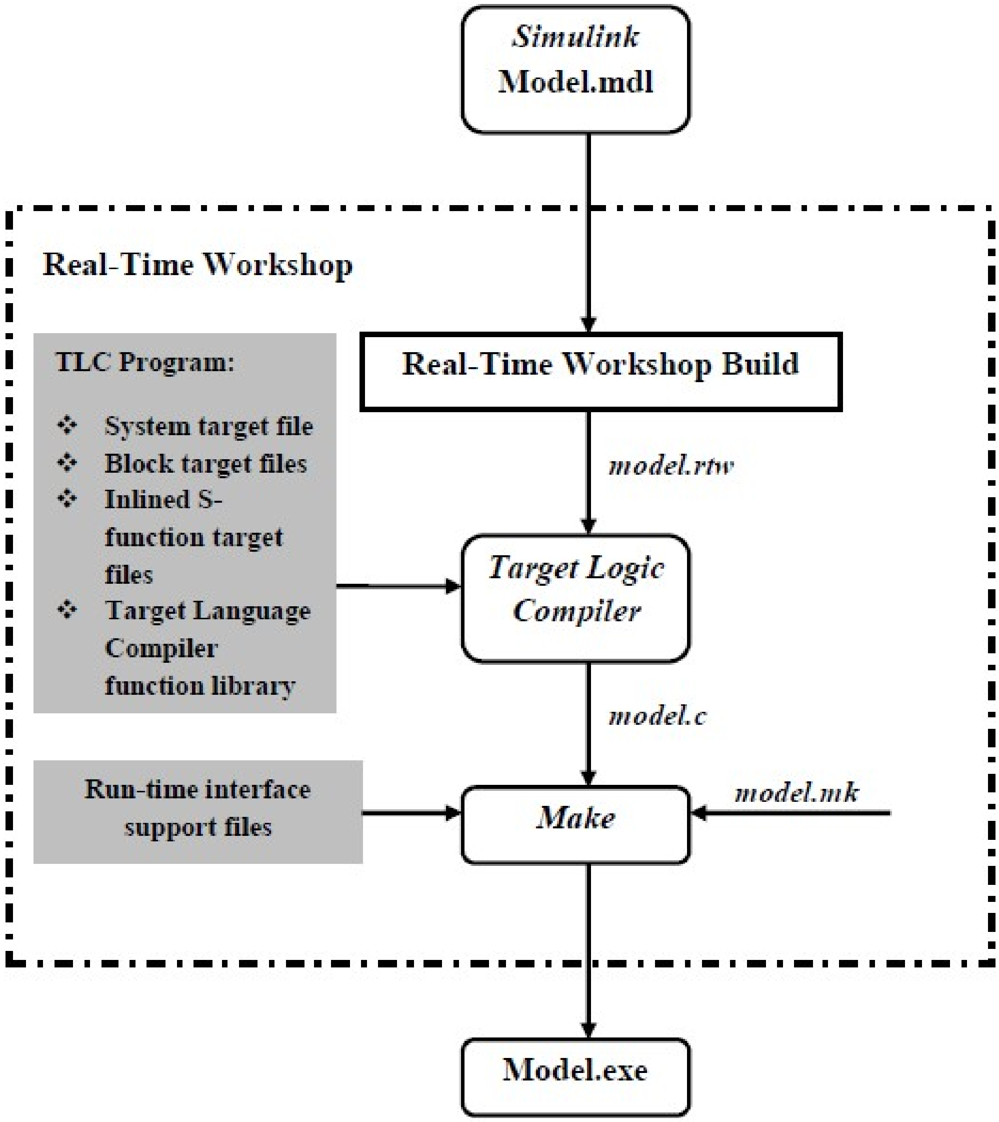


Figure 1.10 Working scheme in real-time

The steps for real-time built are as follows

1. At first analyzing block diagram and after that this is compiled to intermediate hierarchical representation in the form of model.rtw.
2. Now model.rtw is read by TCL and converted into C code.
3. A make file is now constructed by TCL and that placed into built directory.
4. For compiling the source code the make file is now read by system make utility and an executable file model.exe is generated.

Now system can easily understand the file as because the file is in binary format.

The operating voltage of the setup is +2.5V to -2.5V so for the safety purpose we should use a saturation block in it.

1.3 Literature Review on Inverted Pendulum

Inverted Pendulum is a very important control problem in the application areas of Control System. The use of Inverted Pendulum take place first time in Great Britain in the year of 1844 for design purpose of a Seismometer.

Among the all Teqniques the LQR design is the simplest Teqnique to stabilize the Inverted Pendulum System. This is similar to two-loop PD controller Design for stabilizes the system. Stabilization of the system is followed by linearization, finding state feedback gain for LQR and Swing by an energy based Controller. Position states are more penalized in compare to Velocity states.

There are two set of pole in Inverted Pendulum system Fast & Slow. Angle Dynamics is determined by fast set of poles and position Dynamics is determined by other set of poles. Performances of different controllers like Sliding Mode, PD, Fuzzy; Neural Network is shown in [6]. Comparison for different Energy based Controllers to stabilization of Inverted Pendulum is mentioned in [7]. An Energy based gradient method is described in [8]. A feedback control law is derived in [9]. A method for Controlled Lagrangians described in [10]. A combined Controller is described in [11]. For global stabilization of Inverted pendulum a hybrid Controller is designed in [12]. A non-linear controller is designed in [13] considering non-linearity for stabilization of Inverted Pendulum. A simple design to stabilize the system is described in [14].

1.4 Thesis Objective

1. Stabilize the Inverted Pendulum System which is highly unstable.

2. Identification of Cart friction which is non-linear.

3. Develop controllers to stabilize the system.

1.5 Thesis Organization

Chapter 1 this chapter includes brief introduction on Inverted Pendulum and Experimental setup. A literature review has done in this chapter.

Chapter 2 this chapter includes LQR control design for stabilization of Inverted Pendulum along with Simulation and Experimental results.

Chapter 3 this chapter includes two-loop PID controller design for Inverted Pendulum along with Simulation and Experimental results.

Chapter 4 this chapter includes PID+PI controller design for Inverted Pendulum along with Simulation and Experimental results.

Chapter 5 this chapter includes comparison between all designed controllers, conclusion and suggested future work.

CHAPTER

2

LQR CONTROL DESIGN FOR INVERTED PENDULUM

2.1 Introduction

In this chapter a brief description and LQR control design for Inverted Pendulum is given. LQR is a most commonly used state feedback controller in Control System. This is based on pole-placement method. Inverted Pendulum system consists of many physical constraints thus LQR is chosen to stabilize the system. A LTI system is given by

$$\dot{x} = Ax + Bu \tag{2.1}$$

$$y = Cx$$

If for feedback all the n states available and they are completely controllable then we will get a gain matrix K and the input is given by

$$u = -K(x - x_d) \tag{2.2}$$

Here vector of desired states is denoted by x_d. Now if we go for close loop dynamics of the system then we get

$$\dot{x} = (A - BK)x + BKx_d \tag{2.3}$$

Choice of K always depends upon the desired pole locations. For LQR Control always there is a cost function and the cost function is given by

$$J = \frac{1}{2}[Z(t_f) - Y(t_f)]^T F(t_f)[Z(t_f) - Y(t_f)] + \frac{1}{2}\int_{t_0}^{t_f}\{[Z - Y]^T Q[Z - Y] + u^T Ru\}dt \tag{2.4}$$

M dimensional ref vector is denoted by Z, control input is defined by u, error weighted matrix is denoted by Q and control weighted matrix is denoted by R.

The main things which we have to consider for LQR designing purpose is as follows

1. All of the weighted matrices should have to be symmetric in nature.
2. Q should have to be positive semi definite in nature.
3. R should have to be positive definite in nature.

After simplification the cost function becomes

$$J = \int_0^\infty \left(x(t)' Qx(t) + u(t)' Ru(t)\right)dt \tag{2.5}$$

Now if we apply PMP to the function we obtain

$$Ru + B^T\lambda = 0 \tag{2.6}$$

$$\lambda = Px \tag{2.7}$$

$$PAx + A^T Px + Qx - PBR^{-1}B^T Px + P = 0 \tag{2.8}$$

$$PA + A^T P + Q - PBR^{-1} B^T P + P = 0 \tag{2.9}$$

This equation is named as matrix Riccati equation.

$$u = -R^{-1} B^T P x = -Kx \tag{2.10}$$

2.2 Design LQR to stabilize the System

LQR state feedback solution is totally depends upon how to choose the value of Q and R. here we choose the Q matrix according to the system dynamics and as the system is a single input system we choose the value of R as one.

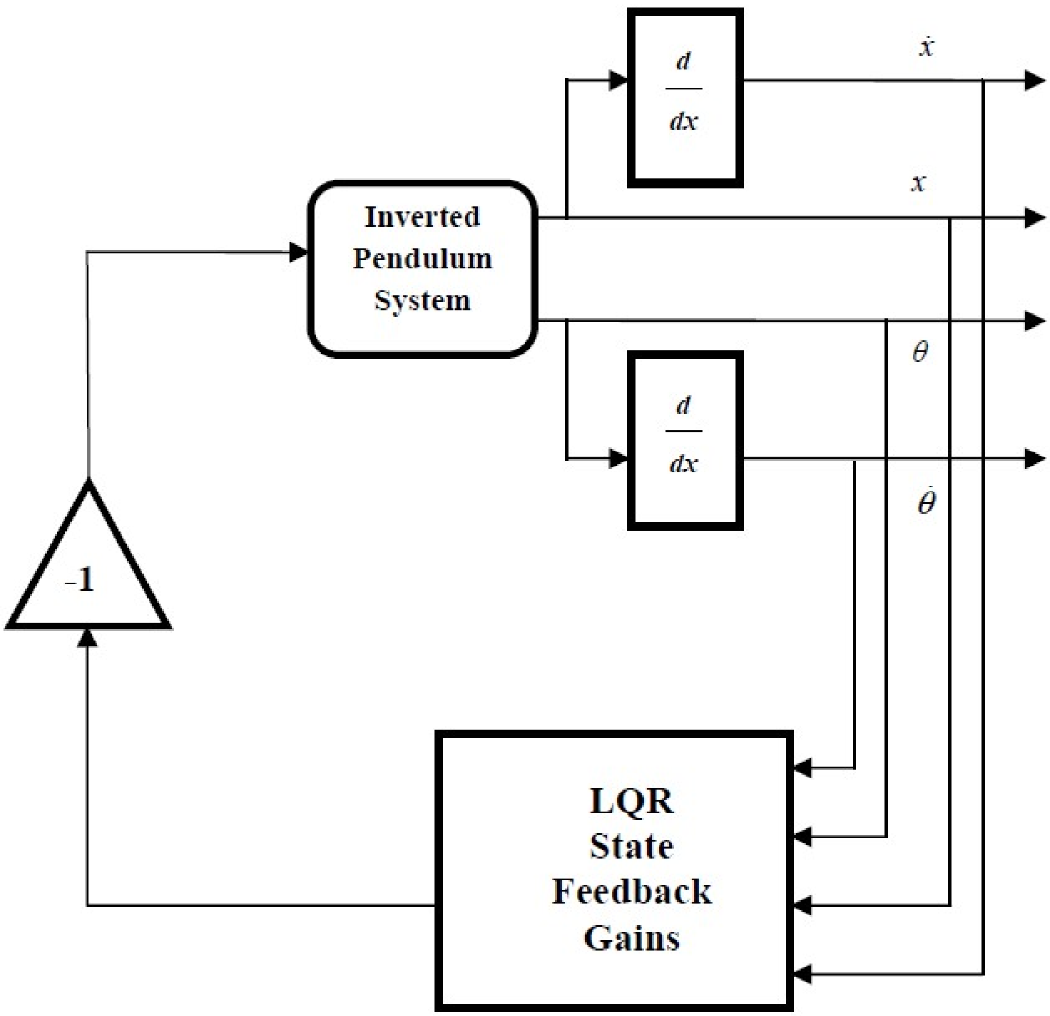


Figure 2.1 Control scheme for LQR

Algorithm for choose the value for Q as follows

1. Q matrix is a diagonal matrix and the elements of this matrix are $q_{1,}$ $q_{2,}$ $q_{3,}$ and q_4. Here the weight act on cart position is denoted by $q_{1.}$ The cart has its linear velocity and for this there act some weight and this weight is denoted by $q_{2.}$ For pendulum angle there will be some weight and this weight is denoted by $q_{3.}$ Pendulum has its angular velocity and for that a weight act and that we denote by q_4.
2. Here weight on cart position is very large than the other weights so the value of q_1 should be very large among these four.
3. Here $q_2>>q_4$
4. So at last the obtained sequence is found as $q_1>>q_2$, $q_3>>q_4$

Here we choose R as one.

After several cycle we choose Q matrix as follows

$$Q = \begin{bmatrix} 3000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}, \ R = 1$$

And gains calculated as -2.2361, -2.7209, 17.5208 and 6.7791

Closed loop poles are $-2.8862 \pm 2.1606i$, $-2.58 \pm 0.1461i$

2.3 Simulation and Experimental Result

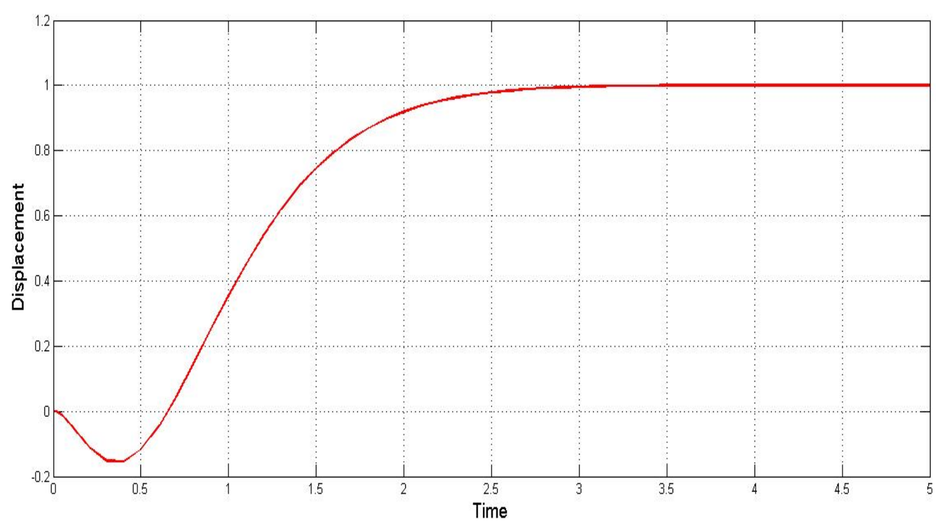


Figure 2.2 Simulation results for displacement (LQR)

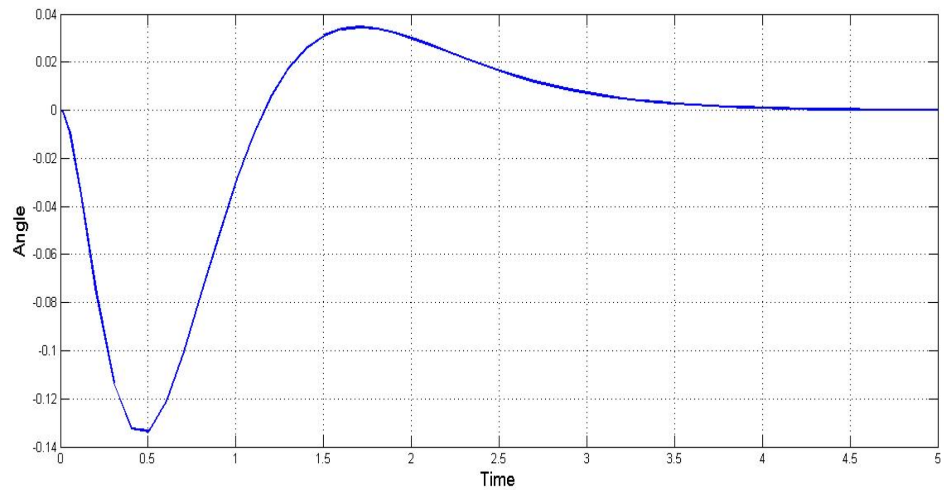


Figure 2.3 Simulation results for Angle (LQR)

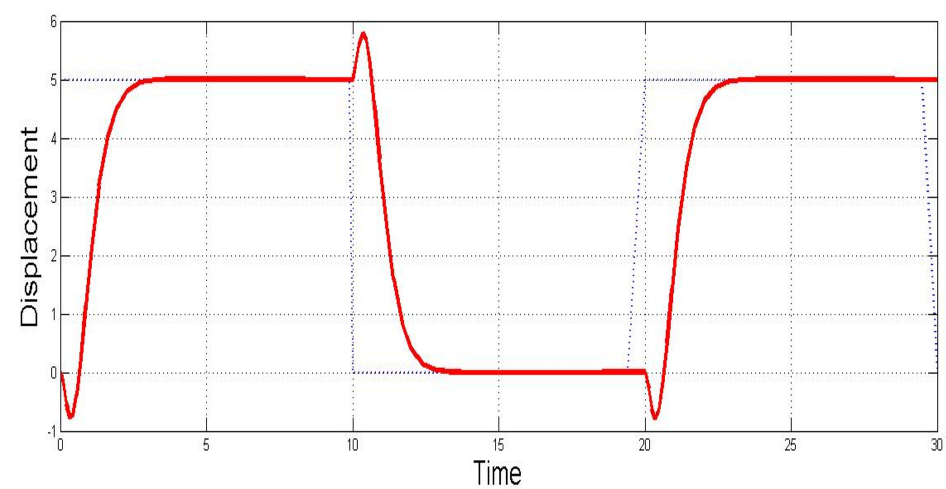


Figure 2.4 tracking ability check for displacement (LQR)

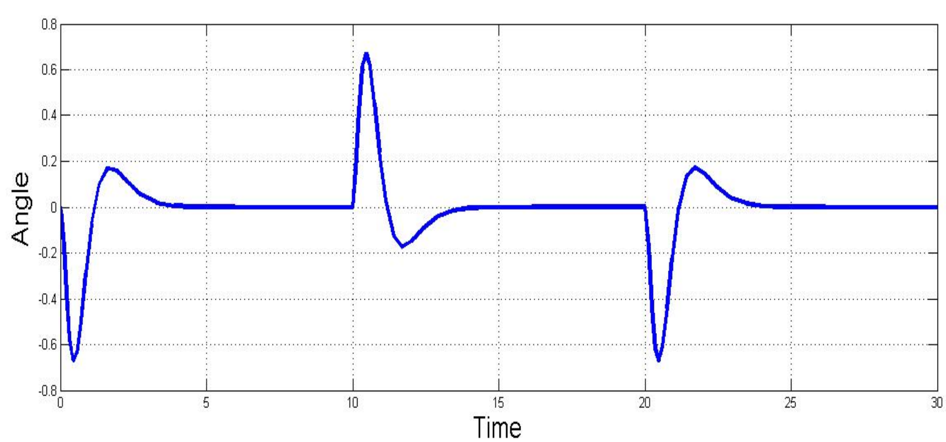


Figure 2.5 tracking ability check for angle (LQR)

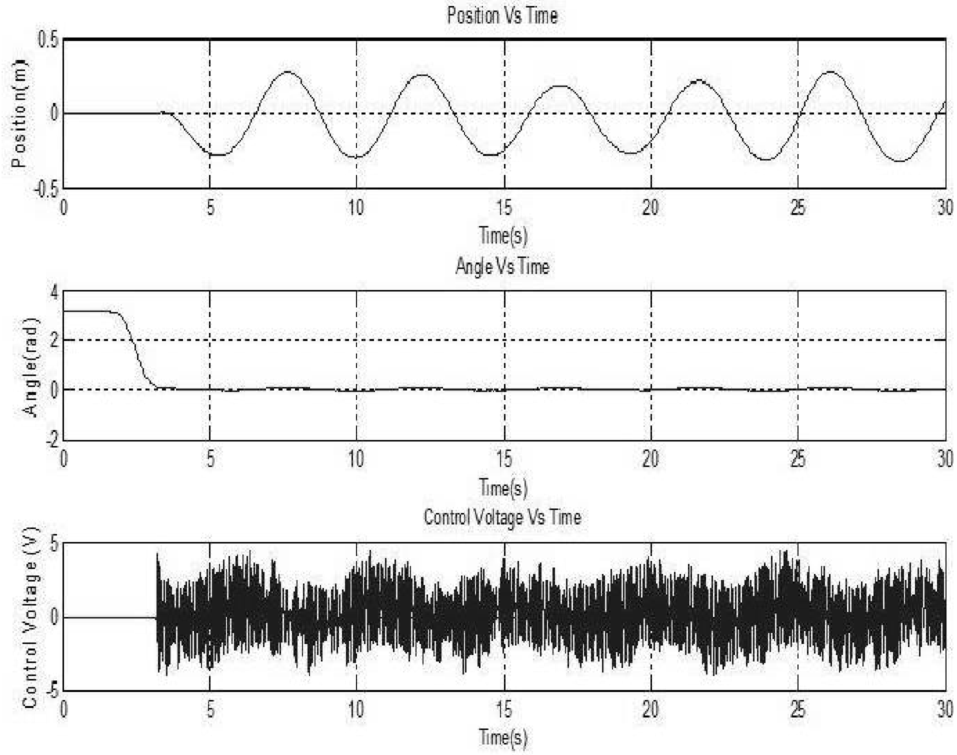


Figure 2.6 Experimental results for LQR

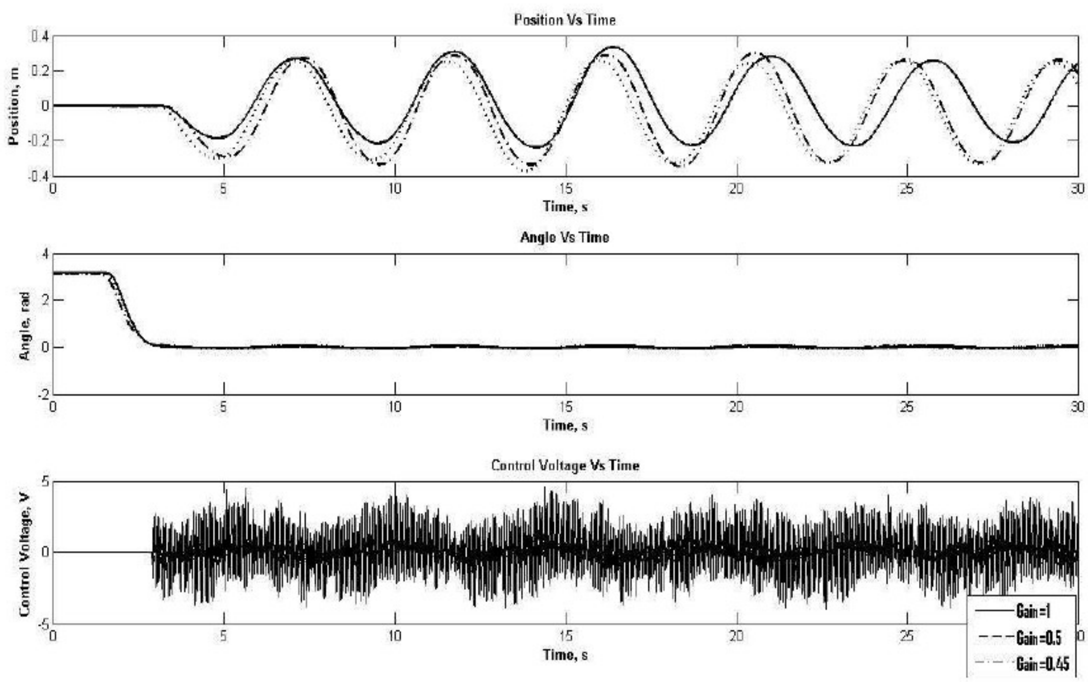


Figure 2.7 Experimental results for Decrease in gain

This is observable that at 0.45 gain the cart seems to exceed the track limit.

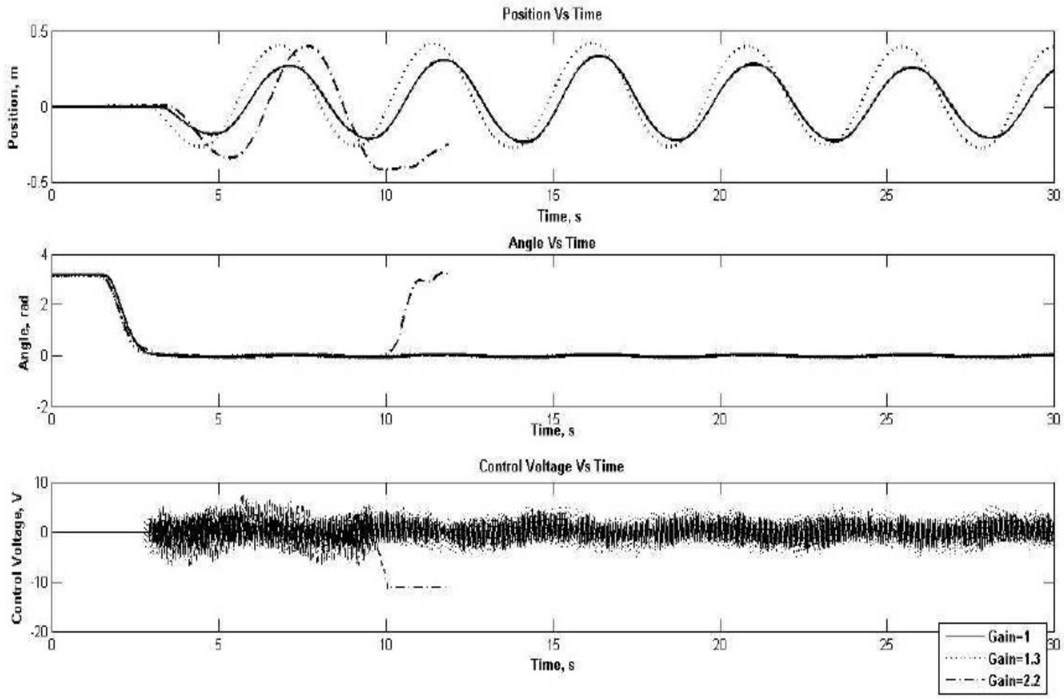


Figure 2.8 Experimental results for increase in gain

It is observable that after a gain of 2.2 the system becomes unstable.

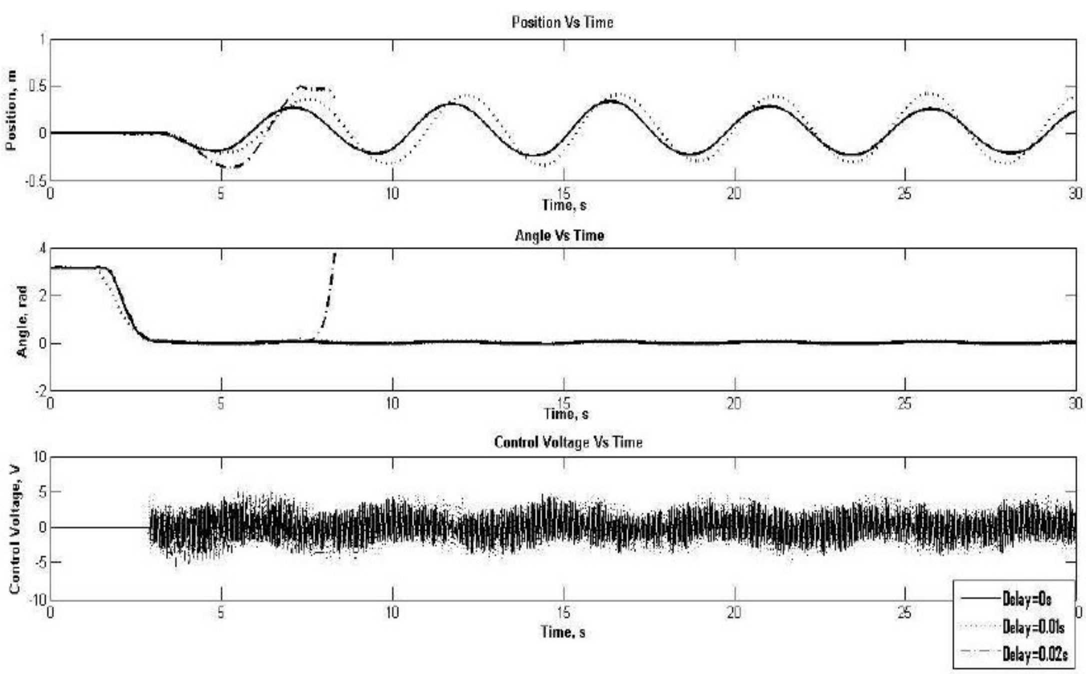


Figure 2.9 experimental results for increase in delay

This is observable that after a gain of 0.02s the system becomes unstable.

2.4 Discussion and Summary

This chapter includes from the very basic of LQR problem, mathematical derivations all fundamental equations. Here we have design a LQR control for Inverted Pendulum System and the design is validated both in Simulation and in Experimental study.

CHAPTER

3

TWO-LOOP PID CONTROLLER DESIGN

3.1 Introduction

The PID controllers are hugely popular owing to their simplicity in working. These controllers are also easy to implement with the help of electronic components. There are several PID tuning Methods available in literature like Ziegler-Nichols method, relay method for non-linear systems, here a pole placement method is presented. The concept of feedback has revolutionized the process control industry. The concept of feedback is really simple. It involves the case when two or more dynamic systems are connected together such that, each system affects the other and the dynamics is strongly coupled. The most important advantage of feedback is that it makes the control insensitive to external disturbances and variation of parameters of system.

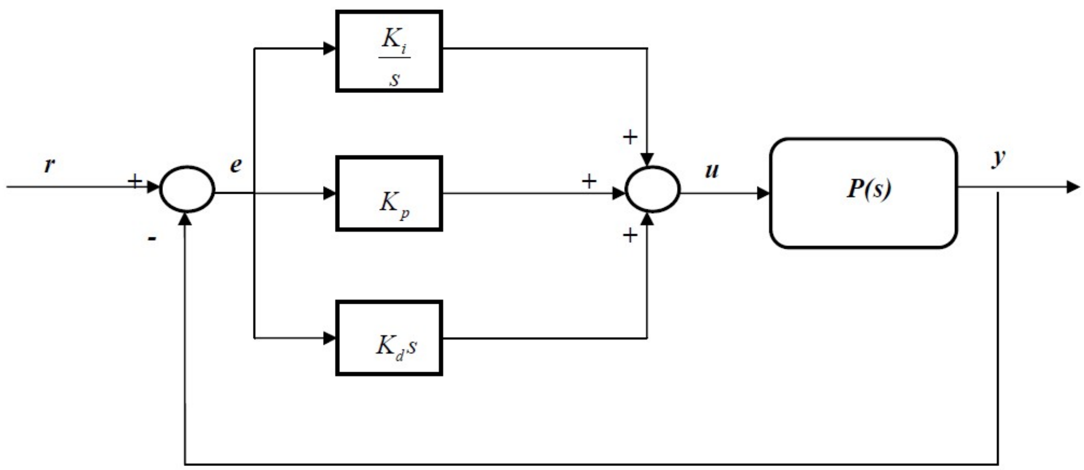


Figure 3.1 PID Controller structure

The control signal u is entirely based on the error generated e . The command input r is also called the set-point weighting in process control literature. The mathematical representation of The control action is

$$e = r - y$$

$$u = K_p e + K_i \int e \, dt + K_d \frac{de}{dt} \tag{3.1}$$

It is seen that with the increase in the value of proportional gain K_p the value of error becomes Greatly reduced but the response becomes highly oscillatory. But, with a constant steady state error. Integral term K_i ensures that the steady state error is zero, i.e. the process output will agree with the reference in its steady state. But, large values of the integral gain would make the control input sluggish leading to unsatisfactory performance. The role of the derivative gain K_d is to damp the oscillatory behaviour of the process output. Use of high value of K_d may lead to instability. So, in order to achieve satisfactory performance we need to choose these values wisely. There exist many tuning rules out of which Ziegler-Nichols tuning is the most popular one.

Initially, the on-off type of feedback control was widely used. But, due to high oscillatory nature of output response the on-off type feedback controller and due to overreaction of control action, gave way to the P type controller. The control action in the case of P type feedback will be directly proportional to the error generated. A large K_p will reduce sensitivity to load disturbance, but increases measurement noise too. Choice of K_p be is a trade off between these two conflicting requirements. It may be noted that the problem of high gain feedback causes instability in closed loop. The upper limit of high gain is determined by the process dynamics.

The Integral action has been a necessary evil in control loops. It has the advantage of guaranteed zero steady state error, but at the cost of sluggish control signal. The derivative action on the other hand improves transient response as it acts on the rate of change of error. It improves the closed loop stability. The choice of K_d is also very crucial, initially increase in its value will increase damping but a high value will eventually decrease the damping.

3.2 Controller Design

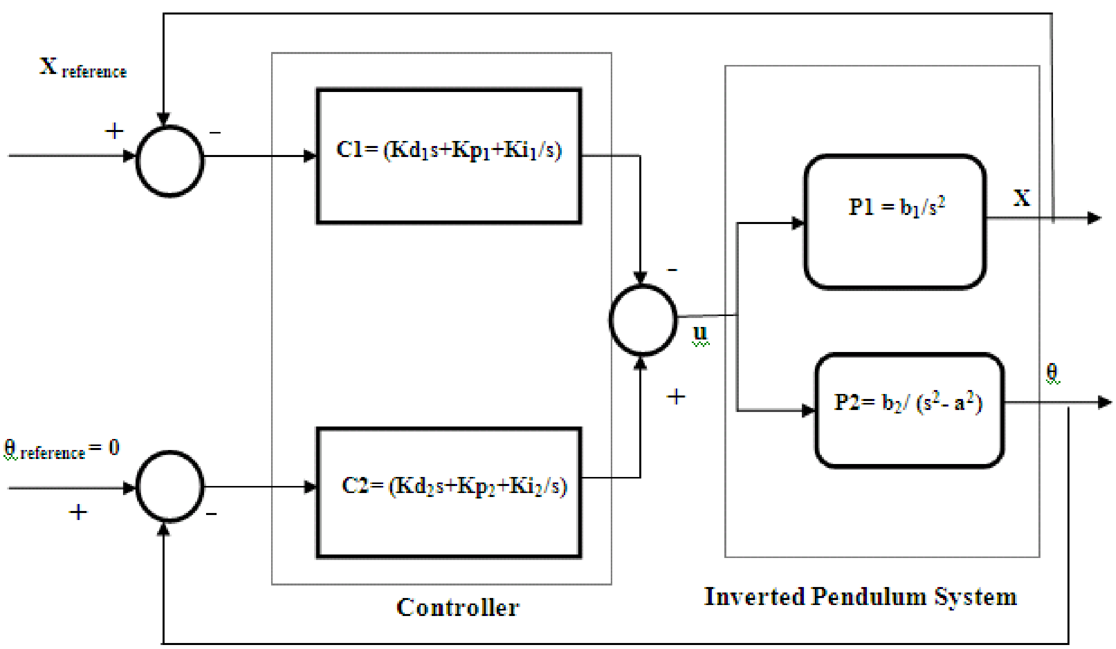


Figure 3.2 Control strategy of two-loop PID Controller for Inverted Pendulum

Here C1 and C2 are the PID Controllers. One is displacement controller and another one is angle controller. P1 and P2 are the Plant transfer functions. We give angle reference as zero as we have to stabilize the Pendulum angle to Zero.

Now if we simplify the block diagram then we get characteristic equation as

$$1 - P_1C_1 + P_2C_2 = 0 \qquad (3.2)$$

$$\Rightarrow 1-\frac{b_1}{s^2}\frac{\left(Kd_1s^2+Kp_1s+Ki_1\right)}{s}+\frac{b_2}{\left(s^2-a^2\right)}\frac{\left(Kd_2s^2+Kp_2s+Ki_2\right)}{s}=0 \tag{3.3}$$

Now from the desired pole location we get the characteristic equation as

$$s^5+26.4s^4+218.6s^3+871.3s^2+1721.8s+1343.7=0 \tag{3.4}$$

Now comparing (3.3) and (3.4) we get

$$\begin{bmatrix} -b_1 & 0 & 0 & -b_2 & 0 & 0 \\ 0 & -b_1 & 0 & 0 & -b_2 & 0 \\ a^2b_1 & 0 & -b_1 & 0 & 0 & -b_2 \\ 0 & a^2b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^2b_1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} Kd_1 \\ Kp_1 \\ Ki_1 \\ Kd_2 \\ Kp_2 \\ Ki_2 \end{bmatrix}=\begin{bmatrix} p_1 \\ p_2+a^2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} \tag{3.5}$$

Substituting values we get

$$\begin{bmatrix} -5.841 & 0 & 0 & -3.957 & 0 & 0 \\ 0 & -5.841 & 0 & 0 & -3.957 & 0 \\ 39.759 & 0 & -5.841 & 0 & 0 & -3.957 \\ 0 & 39.759 & 0 & 0 & 0 & 0 \\ 0 & 0 & 39.759 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} Kd_1 \\ Kp_1 \\ Ki_1 \\ Kd_2 \\ Kp_2 \\ Ki_2 \end{bmatrix}=\begin{bmatrix} 26.4 \\ 225.07 \\ 871.3 \\ 1721.8 \\ 1343.7 \end{bmatrix} \tag{3.6}$$

Here we have six unknown and five equations. Thus we assume K_{d2} as 10.

After Comparing we get the PID gain values as follows.

$\mathbf{C_1}$:–

$Kp_1 = 43.3$

$Ki_1 = 33.796$

$Kd_1 = 2.254$

$\mathbf{C_2}$:–

$Kp_2 = 120.9$

$Ki_2 = 247.43$

$Kd_2 = 10$

3.3 Simulation and Experimental Results

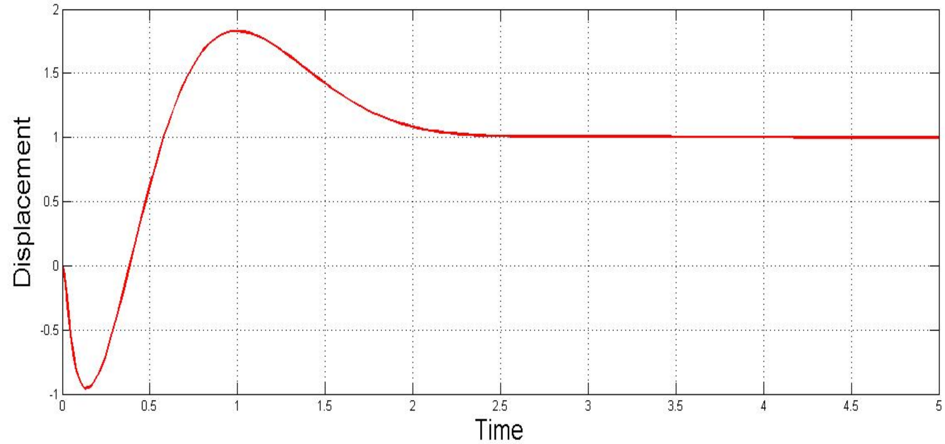


Figure 3.3 Simulation result for Displacement (two-loop PID Controller)

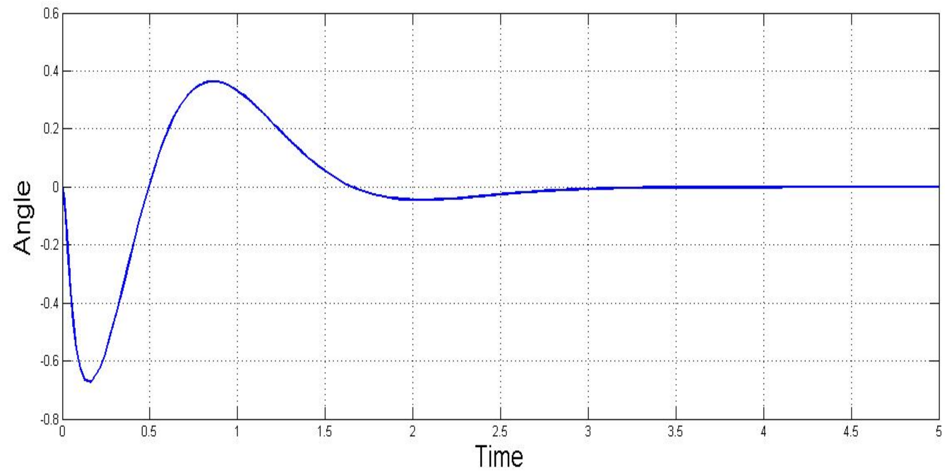


Figure 3.4 Simulation result for angle (two-loop PID Controller)

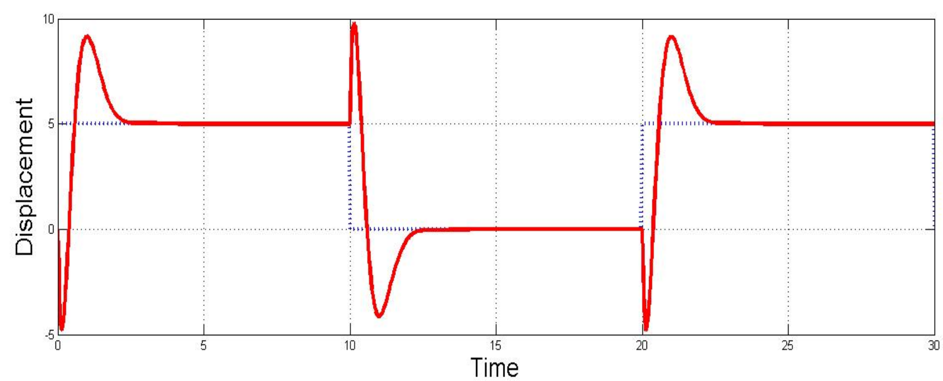


Figure 3.5 Simulation result for tracking ability (displacement)

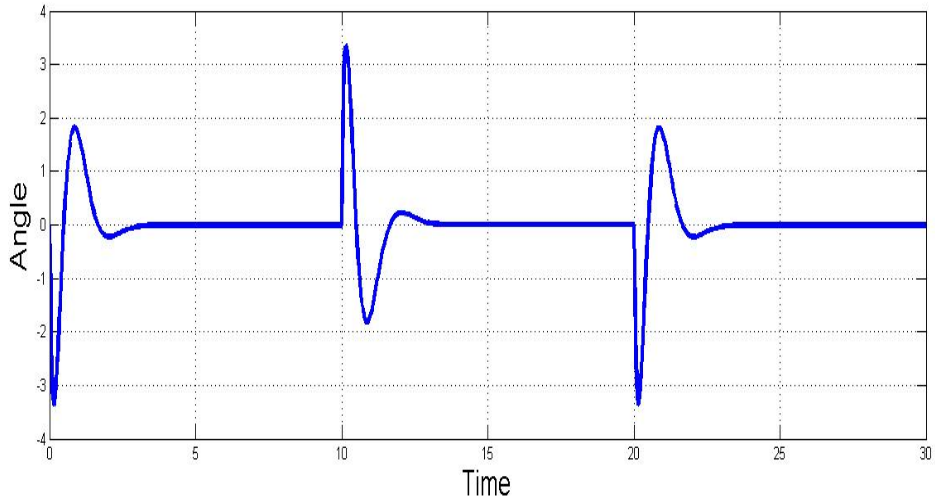


Figure 3.6 Simulation result for tracking ability (angle)

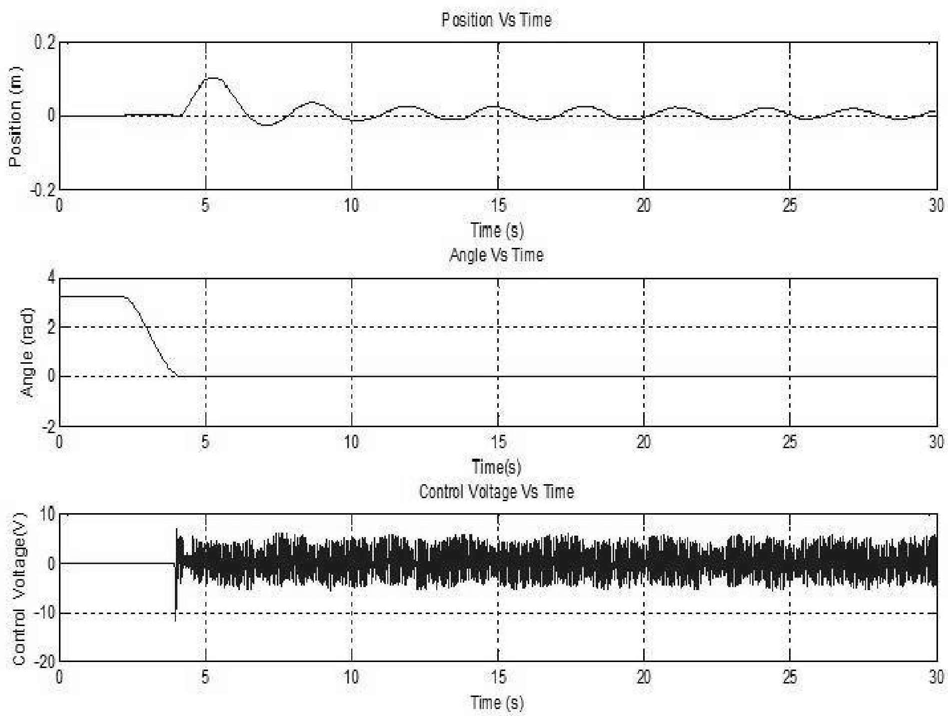


Figure 3.7 Experimental results for two-loop PID Controller

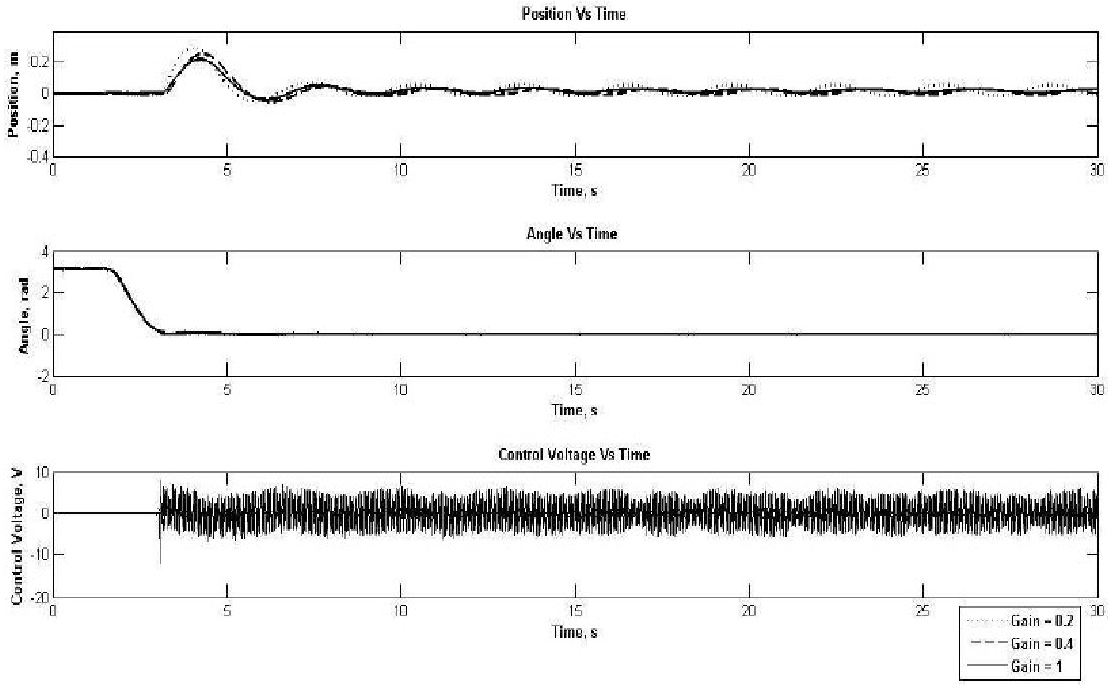


Figure 3.8 Experimental results for decrease in Gain

It is observable that exceed track limit after a gain of 0.2

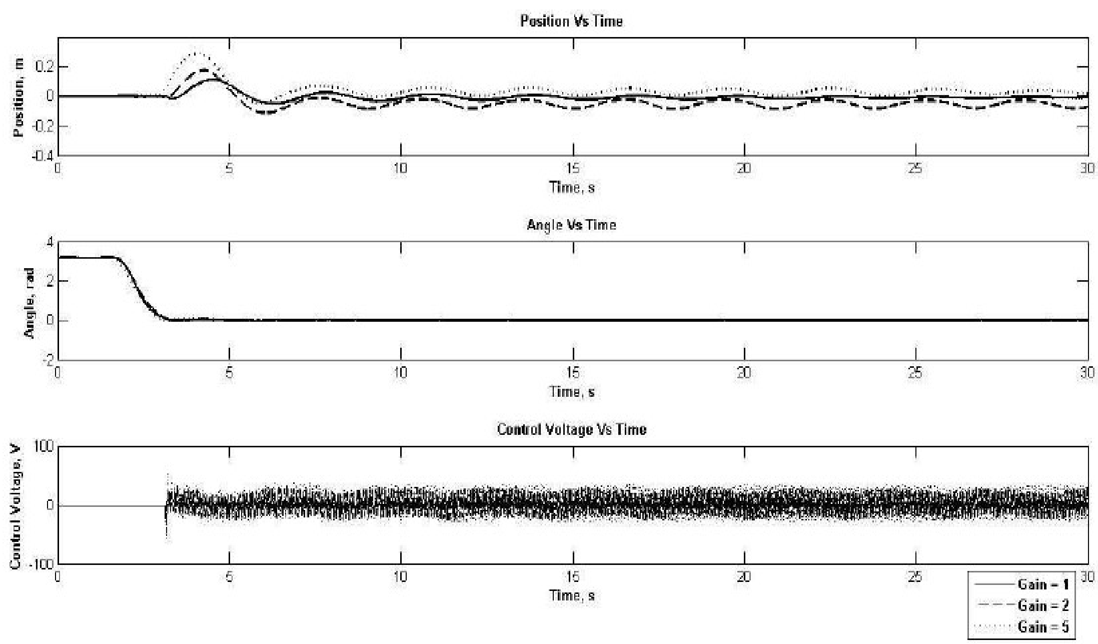


Figure 3.9 experimental results for decrease in gain

It is observable that exceed track limit after a gain 5.

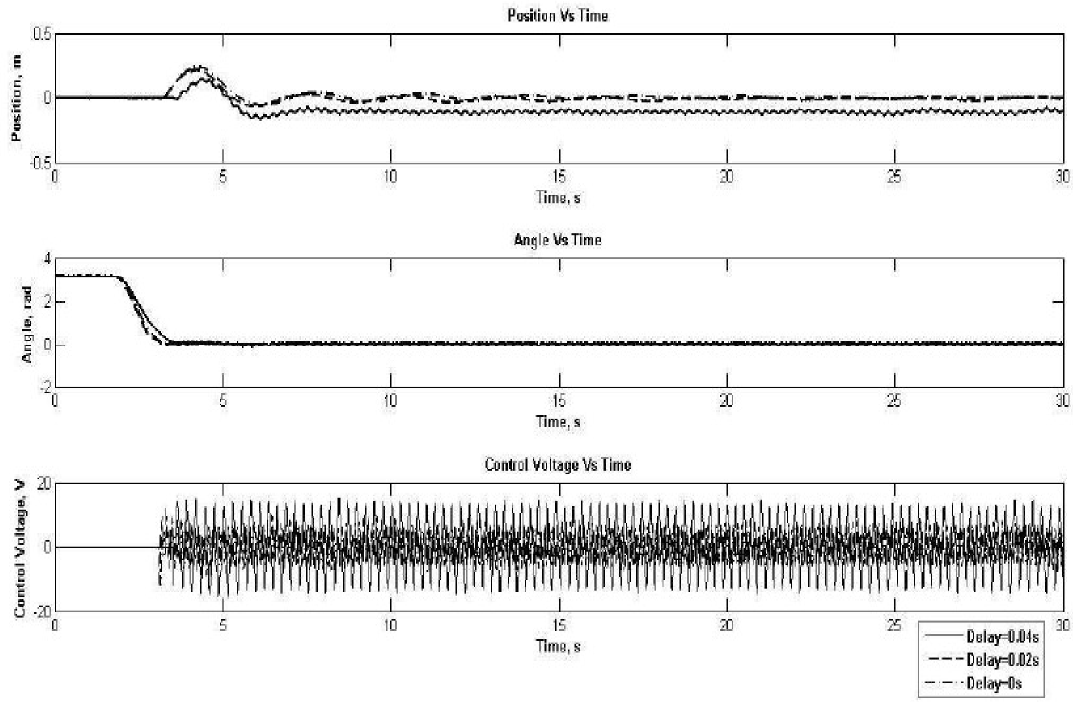


Figure 3.10 Experimental results for increase in delay

As a result more oscillation occurs in Pendulum angle

3.3 Discussion and Summary

This chapter describes very basic of PID controller to solve a practical problem associated with Inverted Pendulum. Here pole-placement Teqnique is used for PID controller design and designed controller gives satisfactory response both in Simulation and Real-time.

CHAPTER

4

PID+PI design for Inverted Pendulum system

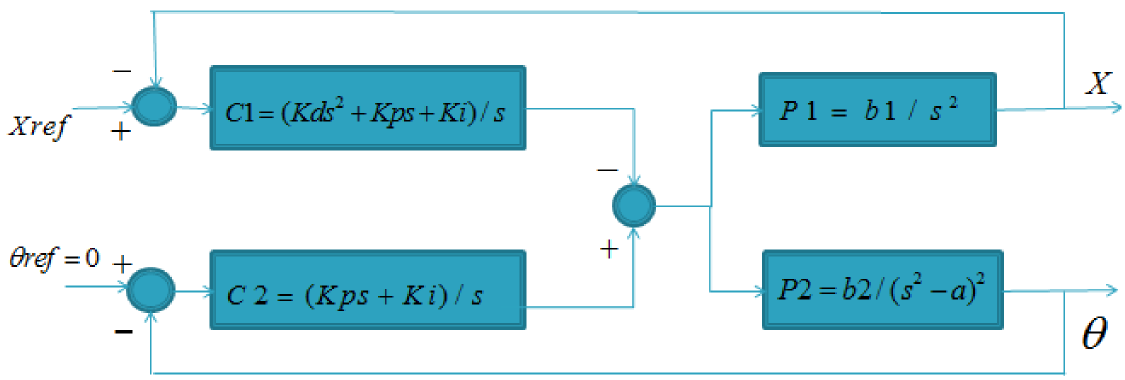


Fig 4.1 Control scheme for PID+PI for Inverted Pendulum

Where $b_1 = 5.841, b_2 = 3.957, a^2 = 6.807$

Here C1 and C2 are the PID Controllers. One is displacement controller and another one is angle controller. P1 and P2 are the Plant transfer functions. We give angle reference as zero as we have to stabilize the Pendulum angle to Zero.

Now if we simplify the block diagram then we get characteristic equation as

$$1 - P_1C_1 + P_2C_2 = 0 \tag{4.1}$$

$$\Rightarrow 1 - \frac{b_1}{s^2}\frac{\left(Kd_1s^2 + Kp_1s + Ki_1\right)}{s} + \frac{b_2}{\left(s^2 - a^2\right)}\frac{\left(Kd_2s^2 + Kp_2s + Ki_2\right)}{s} = 0 \tag{4.2}$$

Now from the desired pole location we get the characteristic equation as

$$s^5 + 26.4s^4 + 218.6s^3 + 871.3s^2 + 1721.8s + 1343.7 = 0 \tag{4.3}$$

Now comparing (4.2) and (4.3) we get

$$\begin{bmatrix} -b_1 & 0 & 0 & 0 & 0 \\ 0 & -b_1 & 0 & b_2 & 0 \\ b_1a^2 & 0 & -b_1 & 0 & -b_2 \\ 0 & b_1a^2 & 0 & 0 & 0 \\ 0 & 0 & a^2b_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Kd_1 \\ Kp_1 \\ Ki_1 \\ Kp_2 \\ Ki_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 + a^2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} \tag{4.4}$$

After substituting values and compare we get PID Gains as follows

$\mathbf{C_1}$: –

$Kp_1 = 43.35$

$Ki_1 = 33.83$

$Kd_1 = -4.52$

$\mathbf{C_2}$: –

$Kp_2 = 121.15$

$Ki_2 = 316.02$

4.3 Simulation and Experimental Results

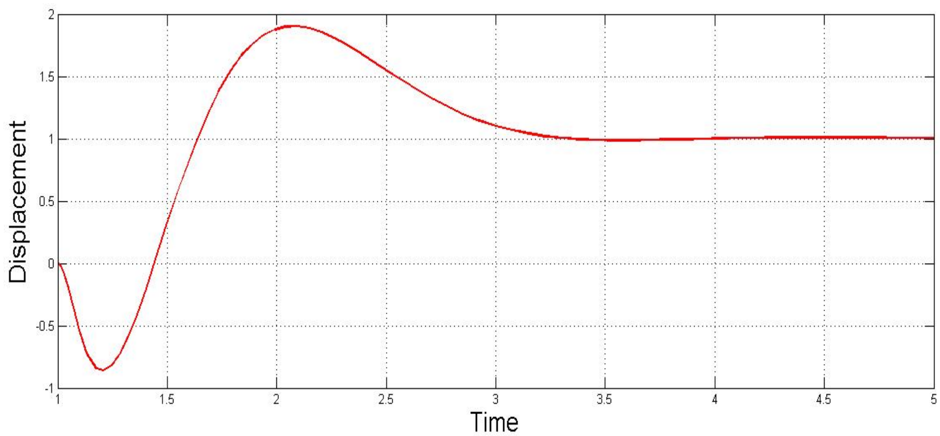


Figure 4.2 Simulation result for Displacement (PID+PI)

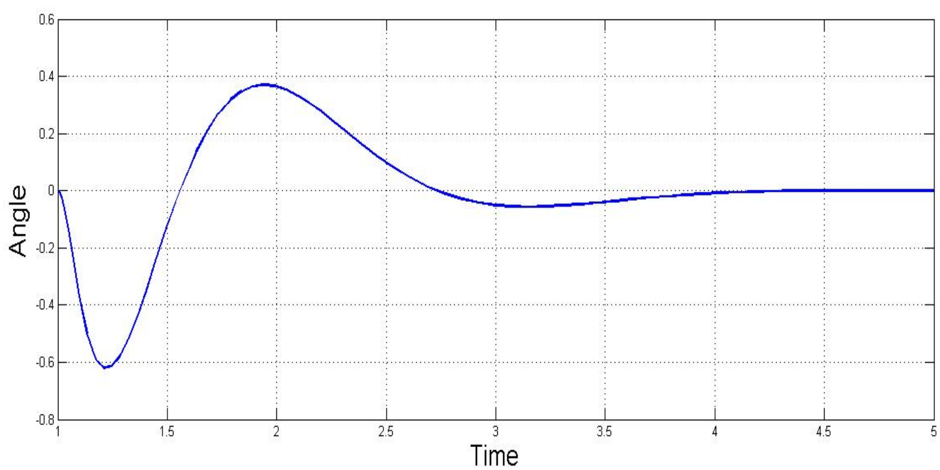


Figure 4.3 Simulation result for Angle (PID+PI)

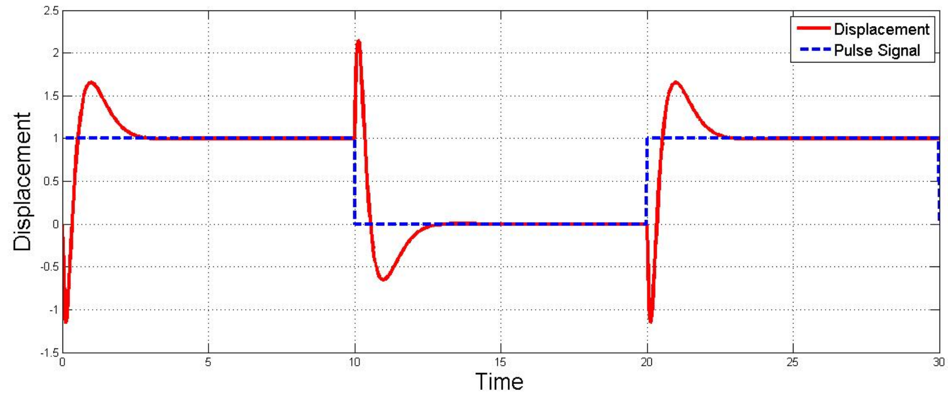


Figure 4.4 Tracking ability of the controller for Displacement

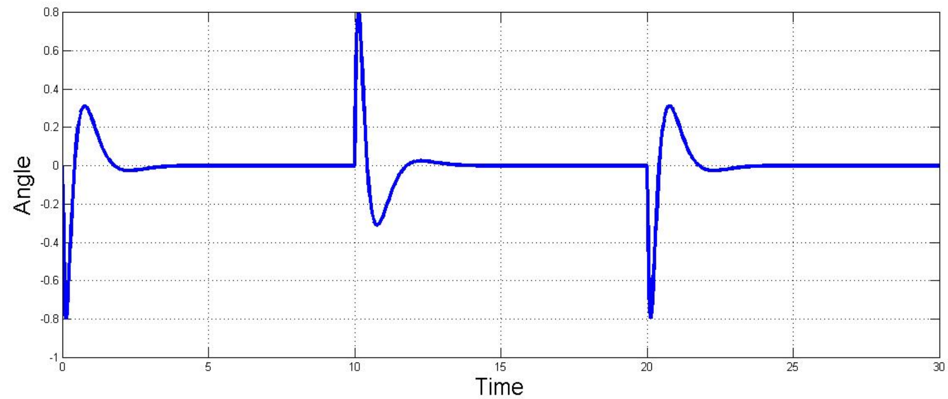


Figure 4.5 Tracking ability of the controller for Angle

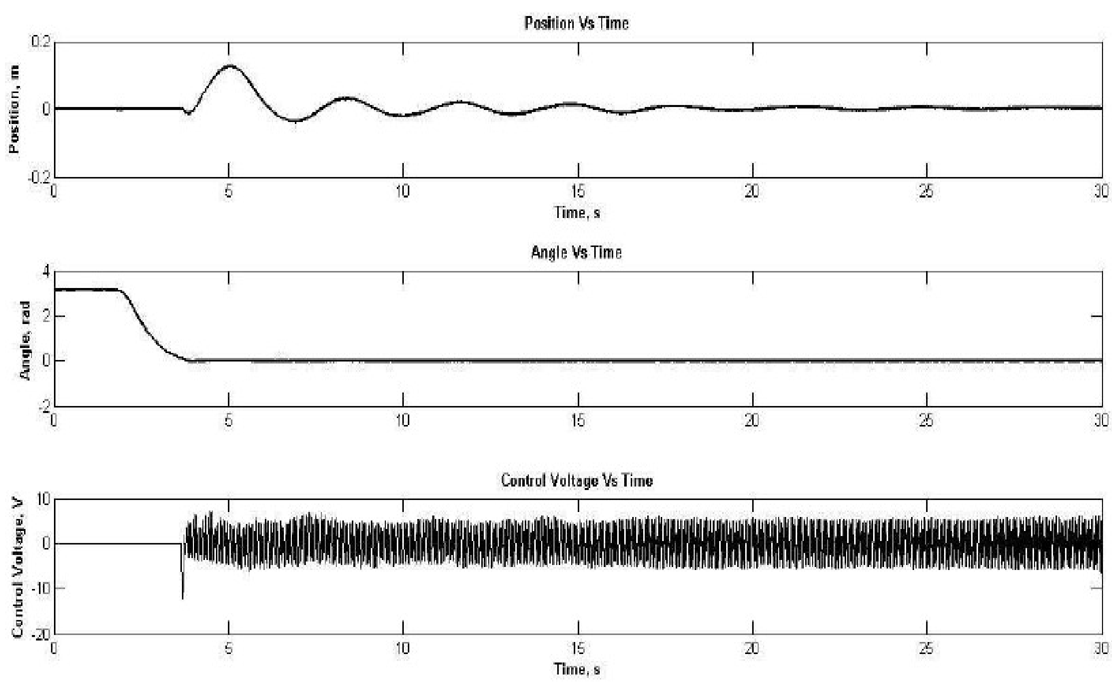


Figure 4.6 Experimental results for PID+PI

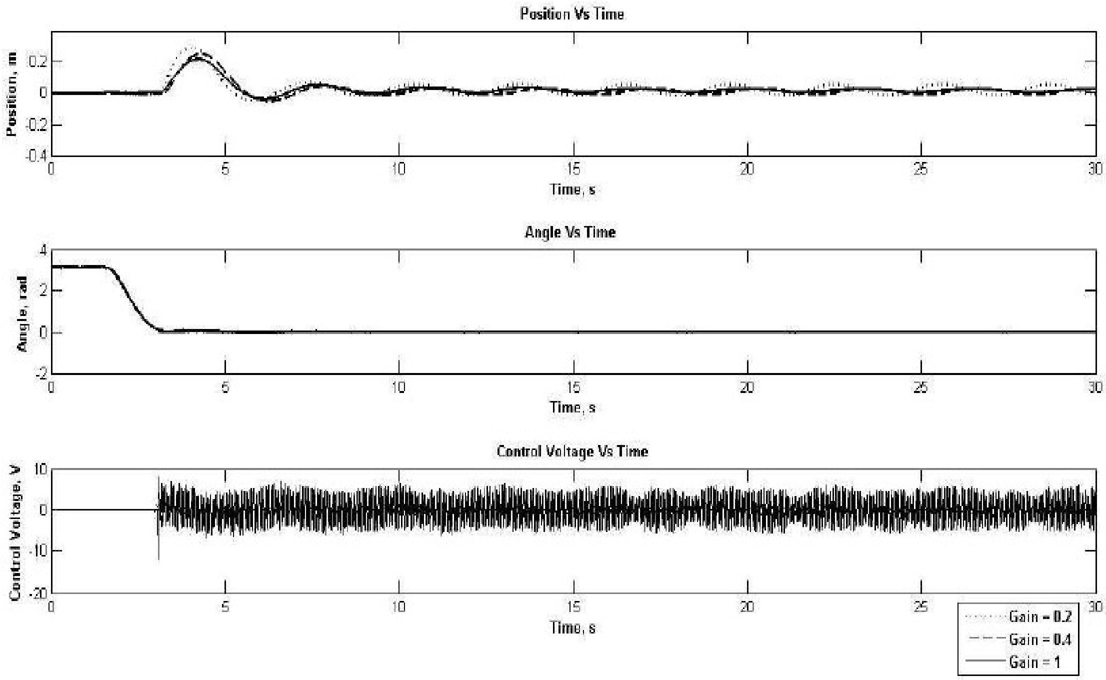


Figure 4.7 Experimental results for decrease in Gain (PID+PI)

This is observable almost exceed track limit at gain 0.2

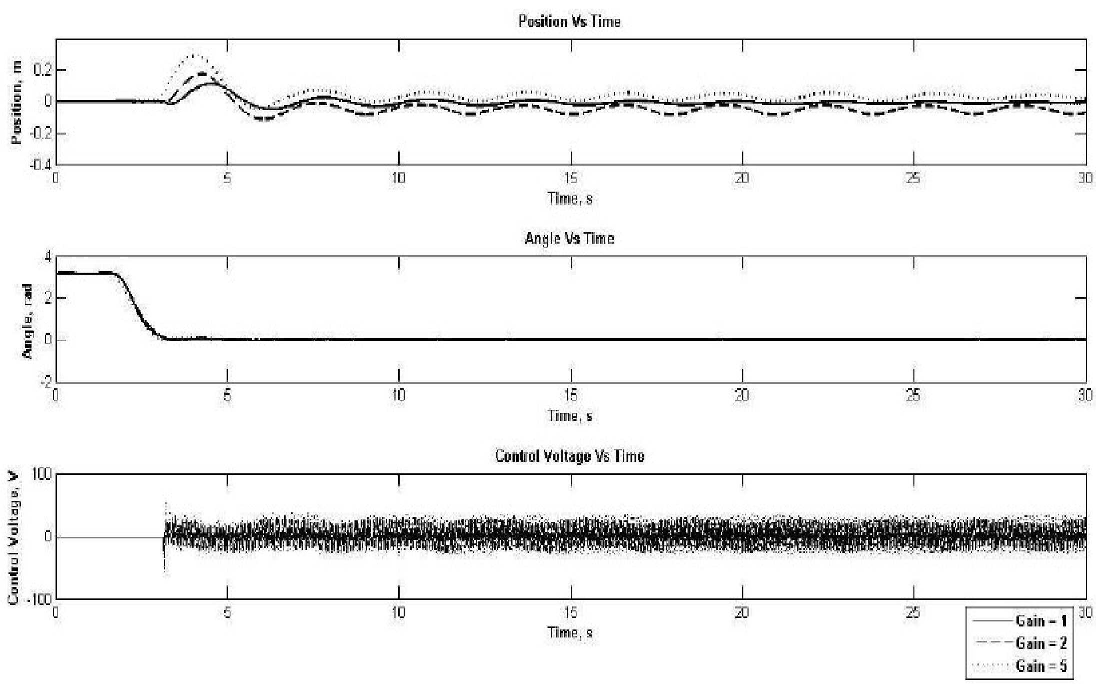


Figure 4.8 Experimental results for increase Gain

This is observable almost exceed track limit at gain 5

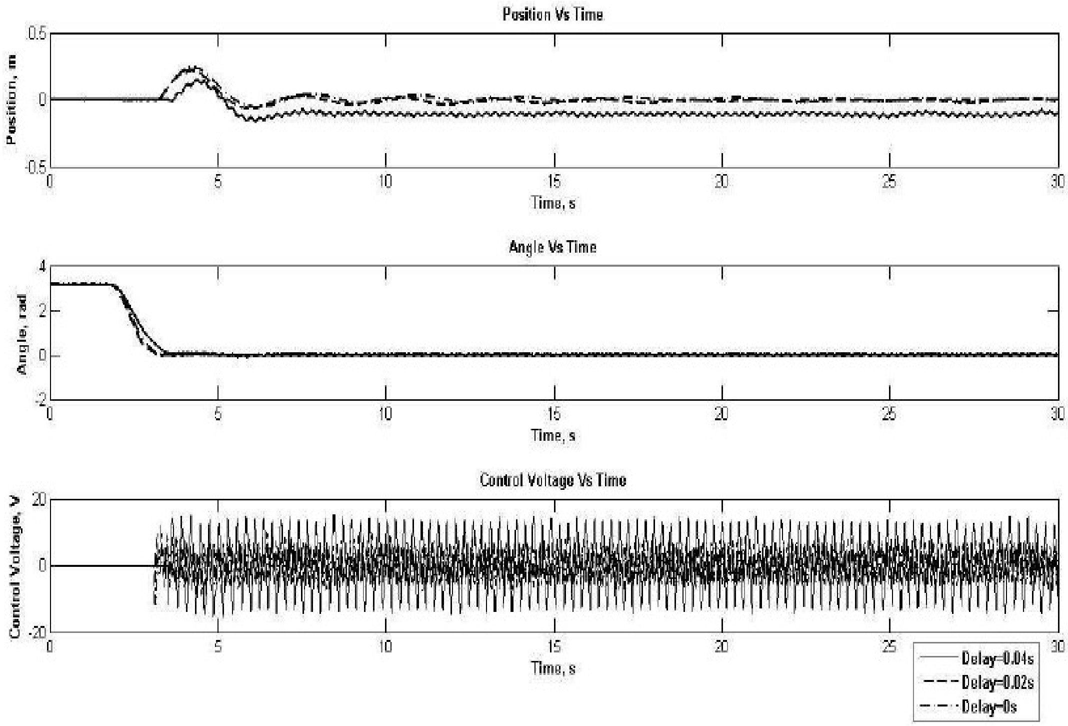


Figure 4.9 Experimental results for increase in Delay

As a result more oscillation occurs in Pendulum angle

4.4 Discussion and Summary

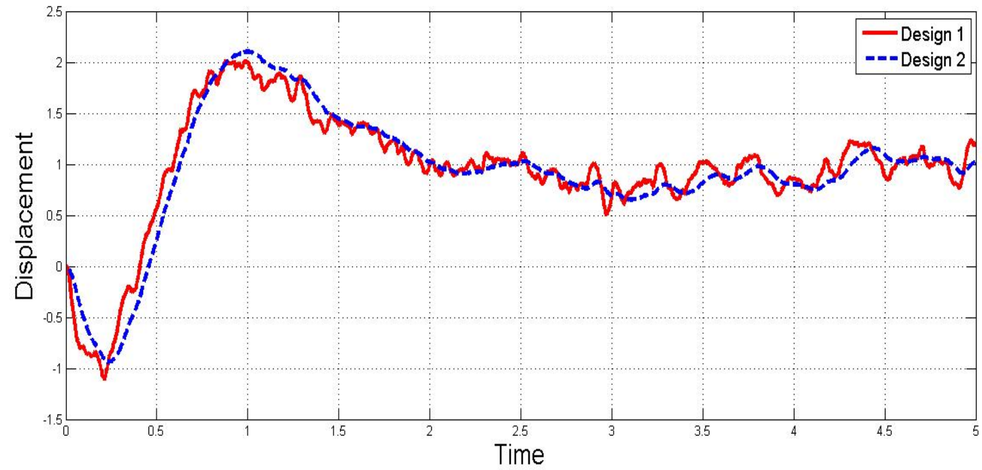
This chapter describes very basic of PID controller to solve a practical problem associated with Inverted Pendulum. Here pole-placement Teqnique is used for PID+PI controller design and designed controller gives satisfactory response both in Simulation and Real-time.

CHAPTER

5

CONCLUSION & SUGGESTED FUTURE WORK

5.1 Conclusion



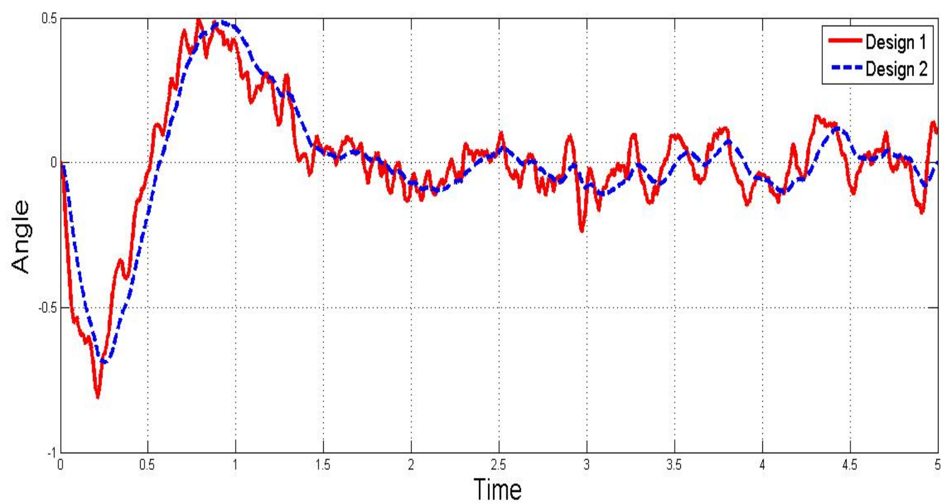


Figure 5.1 Comparison between Two-loop PID and PID+PI with addition of band limited white noise

In presence of disturbance PID+PI (blue line Design 2) Controllers response is better than Two-loop PID Controller (red line Design 1).

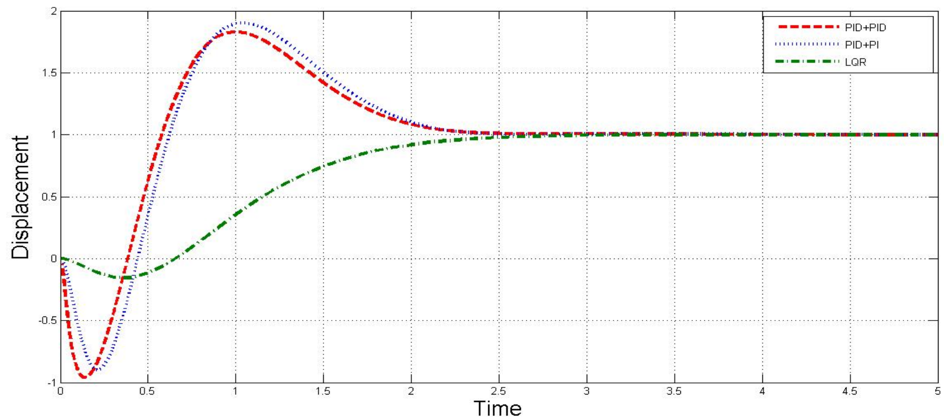


Figure 5.2 Comparison results for Displacement

	Overshoot	Undershoot	Sett time
PID+PID	1.8	-0.9	2.4
PID+PI	1.9	-0.8	2.4
LQR	1.0	-0.18	3.0

Table for comparison of Displacement

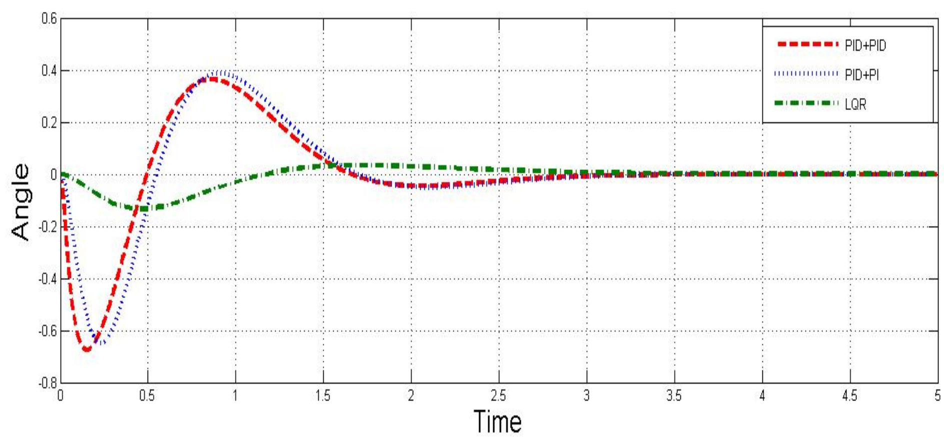


Figure 5.3 Comparison results for Angle

	Overshoot	Undershoot	Sett time
PID+PID	1.8	-0.9	2.4
PID+PI	1.9	-0.8	2.4
LQR	1.0	-0.18	3.0

Table for Comparison of Angle

Here we can see that the Overshoot & Undershoot is less for LQR compare to other two Controllers but the settling time is higher. In simulation study without any disturbance performance of Two-loop PID and PID+PI almost same. In real-time experiment oscillations small occurs in case of LQR in displacement and a little effect goes to angle also. Control voltage requirement for the LQR is much more than the other two controllers. In case of Two-loop PID controller overshoot is minimum as compare to other controllers but control voltage requirement is high than PID+PI controller. In case of PID+PI controller overshoot is slight high than Two-loop PID controller but settling time, control voltage is less than other two controller.

Over all response of PID+PI is satisfactory than the other two controllers.

5.2 Contribution for thesis

1. LQR Control has designed & successfully implemented both in Simulation and real-time.

2. Two-loop PID controller successfully implemented both in real-time & simulation.

3. PID+PI Controller implemented & validate in Simulation and real-time.

4. Robustness analysis made for all Controllers.

5.3 Suggested future work

1. Design a Fuzzy logic & Integral Sliding mode Controller for Inverted Pendulum system.

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