Application of DEA in the Taguchi Method for Multi-Response Optimization

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In

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By

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Certificate of Approval

This is to certify that the thesis entitled **APPLICATION OF DEA IN THE TAGUCHI METHOD FOR MULTI-RESPONSE OPTIMIZATION** submitted by ***Sri Rahul Ranjan*** has been carried out under my supervision in partial fulfilment of the requirements for the Degree of ***Bachelor of Technology*** in ***Mechanical Engineering*** at National Institute of Technology, NIT Rourkela, and this work has not been submitted elsewhere before for any other academic degree/diploma.

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Abstract

Quality and productivity are two important but conflicting criteria in any machining operations. In order to ensure high productivity, extent of quality is to be compromised. It is, therefore, essential to optimize quality and productivity simultaneously. Productivity can be interpreted in terms of material removal rate in the machining operation and quality represents satisfactory yield in terms of product characteristics as desired by the customers. Dimensional accuracy, form stability, surface smoothness, fulfilment of functional requirements in prescribed area of application etc. are important quality attributes of the product. Increase in productivity results in reduction in machining time which may result in quality loss. On the contrary, an improvement in quality results in increasing machining time thereby, reducing productivity. Therefore, there is a need to optimize quality as well as productivity. Optimizing a single response may yield positively in some aspects but it may affect adversely in other aspects. The problem can be overcome if multiple objectives are optimized simultaneously. It is, therefore, required to maximize material removal rate (MRR), and to improve product quality simultaneously by selecting an appropriate (optimal) process environment. To this end, the present work deals with multi-objective optimization philosophy based on Data Envelopment Analysis (DEA) and Taguchi method applied in CNC end milling operation.

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1. Introduction and State of Art

Milling is a versatile and useful machining operation. End milling is the most important milling operation and it is widely used in most of the manufacturing industries due to its capability of producing complex geometric surfaces with reasonable accuracy and surface finish. However, with the inventions of CNC milling machine, the flexibility has been adopted along with versatility in end milling process.

In CNC end milling precise understanding in controlling of process parameters is indeed required to provide good surface finish as well as high material removal rate (MRR). The surface finish may be viewed as product quality attribute and material removal rate directly related to productivity.

In the present research work, material removal rate (MRR) and surface roughness of the product prepared by CNC end milling operation have been studied experimentally and the results, thereof, obtained have been interpreted analytically.

Yang and Chen (2001) attempted to determine optimal machining parameters for improving surface roughness performance of machined Al 6061 in end-milling operation. The analysis of confirmation experiments established that Taguchi parameter design could successfully verify the predicted optimum cutting parameters consisting of depth of cut, cutting speed, feed rate, and tool diameter. **Ginta et al. (2008)** presented an approach to establish models and the efforts in optimization of tool life and surface roughness in end milling of titanium alloy Ti–6Al–4V using uncoated WCCo inserts under dry conditions. Response surface methodology coupled with small central composite design (CCD) was employed in developing the tool life and surface roughness models in relation to primary cutting parameters such as cutting speed, axial depth of cut and feed.

Kadirgama (2008) attempted optimization of the surface roughness when milling mould aluminium alloys (AA6061-T6) with carbide coated inserts. The approach was based on Response Surface Method (RSM) and Radian Basis Function Network (RBFN). The work aimed to determine the optimized parameters, as well as to find out the most dominant variables (cutting speed, feed rate, axial depth and radial depth). The first order model and RBFN indicated that the feed rate seemed to be the most significant factors effecting surface roughness. RBFN predicted surface roughness more accurately compared to RSM.

Routara et al. (2009) considered five roughness parameters, viz., centre line average roughness, root mean square roughness, skewness, kurtosis and mean line peak spacing for modeling and optimization in CNC end milling using response surface method. **Kadirgama et al. (2010)** presented a study on determination of optimum surface roughness by using milling mould aluminium alloys (AA6061-T6) with Response Ant Colony Optimization (RACO). The approach is based on Response Surface Method (RSM) and Ant Colony Optimization (ACO). **Reddy et al. (2011)** described the development of predictive model for the surface roughness of machinable glass ceramic in terms of speed, feed rate by using micro end-milling operation.

Literature highlights an increasing need towards quality-productivity optimization in milling operation. It is felt that an efficient technique should be established to predict output features of a product before milling in order to evaluate the fitness of machining parameters such as feed rate, spindle speed or depth of cut for keeping a desired quality and increased productivity. It is also important that the prediction technique should be accurate, reliable, low-cost, and non-destructive.

In this context, the present work aimed at evaluating an optimal setting to be used for mass production with desired quality level as well as enhanced productivity. DEA based Taguchi philosophy has been adopted in this study. DEA method has been given immense importance in literature to solve decision-making problems. It can also be applied for multi-response optimization. **Liao and Chen (2002)** used data envelopment analysis method to solve multi-response optimization problems. **Gutiérrez and Lozano (2010)** used an Artificial Neural Network to estimate the responses for all factor level combinations. Data Envelopment Analysis (DEA) was used first to select the efficient factor level combinations and then for choosing among them the one which lead to a most robust quality loss penalization. Mean Square Deviations of the quality characteristics were used as DEA inputs. Among the advantages of the proposed approach over traditional Taguchi method were the non-parametric, non-linear way of estimating quality loss measures for unobserved factor combinations and the non-parametric character of the performance evaluation of all the factor combinations.

2. Experimentation

Procedural steps for the present work have been listed below.

1. Selection of process parameters and domain of experiment. (Range of parameter variation available in the machine).
2. Selection of an appropriate design of experiment (DOE).
3. Material Selection.
4. Experimentation.

5. Measurement of MRR.
6. Collection of experimental data related to surface roughness of the machined product.
7. Data Analysis using proposed methodology.
8. Conclusion and recommendation.

Samples of copper bars (Ø25x10mm) have been used as work material. Taguchi's L_9 orthogonal array has been used here **(Table1)**. **Table 2** indicates selected process control parameters and their limits. Three machining parameters: cutting speed, feed rate and depth of cut has been considered to be varied into three different levels within experimental domain. HSS tool (C00662D, 12 HSS, TYPE A & N) has been used during experiments. Milling has been performed in CNC MAXMILL set up. Corresponding to each experimental run MRR and average surface roughness values (R_a) have been computed **(Table 3)**. The surface roughness has been measured by the Talysurf (Taylor Hobson, Subtronic 3+).

3. Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) is receiving increasing importance as a tool for evaluating and improving the performance of manufacturing and service operations. It has been extensively applied in performance evaluation and benchmarking of schools, hospitals, bank branches, production plants, etc. **(Charnes et al., 1994)**. This paper provides an introduction to DEA and some important methodological extensions that have improved its effectiveness as a productivity analysis tool.

DEA is a multi-factor productivity analysis model for measuring the relative efficiencies of a homogenous set of decision making units (DMUs). The efficiency score in the presence of multiple input and output factors is defined as:

$$Efficiency = \frac{weighted\ \ sum\ of\ \ outputs}{weighted\ \ sum\ of\ \ inputs} \tag{3.1}$$

Assuming that there are n DMUs, each with m inputs and s outputs, the relative efficiency score of a test DMU p is obtained by solving the following model proposed by **Charnes et al. (1978)**:

$$\max \frac{\sum_{k=1}^{s} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{jp}}$$

$$s.t. \frac{\sum_{k=1}^{s} v_k y_{ki}}{\sum_{j=1}^{m} u_j x_{ji}} \le 1 \quad \forall\, i \tag{3.2}$$

$$v_k, u_j \ge 0 \quad \forall k,\ j,$$

where,

$k = 1\ to\ s$

$j = 1\ to\ m$

$i = 1\ to\ n$

y_{ki} = amount of output k produced by DMU i,

x_{ji} = amount of input j utilized by DMU i,

v_k = weight given to output k,

u_j = weight given to input j,

The fractional program shown as (2) can be converted to a linear program as shown in (3). For more details on model development see **Charnes et al. (1978)**.

$$\max \sum_{k=1}^{s} v_k y_{kp}$$

$$s.t. \sum_{j=1}^{m} u_j x_{jp} = 1$$

$$\sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \leq 0 \quad \forall i$$

$$v_k, u_j \geq 0 \quad \forall k,\ j, \tag{3.3}$$

The above problem is run *n* times in identifying the relative efficiency scores of all the DMUs. Each DMU selects input and output weights that maximize its efficiency score. In general, DMU is considered to be efficient if it obtains a score of 1 and a score of less than 1 implies that it is inefficient.

Benchmarking in DEA

For every inefficient DMU, DEA identifies a set of corresponding efficient units that can be utilized as benchmarks for improvement. The benchmarks can be obtained from the dual problem shown as (4).

$$\min \theta$$

$$s.t. \ \sum_{i=1}^{n} \lambda_i x_{ji} - \theta x_{jp} \leq 0 \quad \forall j$$

$$\sum_{i=1}^{n} \lambda_i y_{ki} - y_{kp} \geq 0 \quad \forall k$$

$$\lambda_i \geq 0 \quad \forall i \tag{3.4}$$

where,

θ = efficiency score, and

λs = dual variables.

Based on problem (4), a test DMU is inefficient if a composite DMU (linear combination of units in the set) can be identified which utilizes less input than the test DMU while maintaining at least the same output levels. The units involved in the construction of the composite DMU can be utilized as benchmarks for improving the inefficient test DMU. DEA also allows for computing the necessary improvements required in the inefficient unit's inputs and outputs to make it efficient. It should be noted that DEA is primarily a diagnostic tool and does not prescribe any reengineering strategies to make inefficient units efficient. Such improvement strategies must be studied and implemented by managers by understanding the operations of the efficient units.

Although benchmarking in DEA allows for the identification of targets for improvements, it has certain limitations. A difficulty addressed in the literature regarding this process is that an inefficient DMU and its benchmarks may not be inherently similar in their operating practices. This is primarily due to the fact that the composite DMU that dominates the inefficient DMU does not exist in reality. To overcome these problems researchers have utilized performance-based clustering methods for identifying more appropriate benchmarks **(Doyle and Green, 1994; Talluri and Sarkis, 1997)**. These methods cluster inherently similar DMUs into groups, and the best performer in a particular cluster is utilized as a benchmark by other DMUs in the same cluster.

4. Taguchi Method

Taguchi's philosophy, developed by *Dr. Genichi Taguchi*, is an efficient tool for the design of high quality manufacturing system. Taguchi's Orthogonal Array (OA) provides a set of well-balanced experiments (with less number of experimental runs), and Taguchi's signal-to-noise ratios (S/N), which are logarithmic functions of desired output; serve as objective functions in the optimization process. Taguchi method uses a statistical measure of performance called signal-to-noise ratio. The S/N ratio takes both the mean and the variability into account. The S/N ratio is the ratio of the mean (Signal) to the standard deviation (Noise). The ratio depends on the quality characteristics of the product/process to be optimized. The standard S/N ratios generally used are as follows: - Nominal-is-Best (NB), lower-the-better (LB) and Higher-the-Better (HB). The optimal setting is the parameter combination, which has the highest S/N ratio). Because, irrespective of the quality criteria may be (NB, LB, HB) S/N ratio should always be maximized. Once experimental data (quality attribute value) is normalized using NB/LB/HB criteria; normalized value lies in between zero to one. Zero represents worst quality to be rejected and one represents most satisfactory quality. Since S/N ratio is expressed as mean (signal) to the noise (deviation from the target); maximizing S/N ratio ensures minimum deviation and hence it is (S/N ratio) to be maximized.

5. Procedural Steps, Results and Discussions

Data Envelopment Analysis (DEA) first formulated by *Charnes, Cooper and Rhodes* in *1978* has been recognized as a valuable analytical research instrument and a practical decision-making tool. DEA is linear programming based technique which is used to empirically measure the relative efficiency of decision making units (DMUs) when the production process presents a structure of multiple inputs and outputs. The efficiency of ‘multiple inputs and output factors’ can be defined as the following:

E_k = weighted sum of outputs/ weighted sum of inputs

Step 1: Normalization of input-output response pair

It is necessary to normalize responses to ensure that all the attributes are equivalent in terms of value domain as well as units.

The given MRR response has been normalized by the following equations:

$$Z_{ij} = \frac{X_{ij}}{\max X_{ij}}, \text{ for } i = 1,2,........., m \text{ and } j = 1,2,........., n \tag{5.1}$$

For average surface roughness R_a:

$$Z_{ij} = \frac{\min X_{ij}}{X_{ij}}, \text{ for } i = 1,2,........., m \text{ and } j = 1,2,........., n \tag{5.2}$$

Here, X_{ij} is mean for the i^{th} response in the j^{th} experiment.

Step 2: Calculation for relative efficiency

For each experiment the relative efficiency has been computed by the aid of LINGO software package.

Following equation has been used for the calculation of the relative efficiency:

$$\max E_{kk} = \sum_y O_{ky} V_{ky} \tag{5.3}$$

Such that,

$$\sum I_{kx} U_{kx} = 1$$

$E_{ks} \leq 1$ $\forall$ design such that,

$U_{kx}, V_{ky} > 0$

Taguchi has been finally applied on relative efficiency for evaluating the most favorable process environment.

Experimental data presented in **Table 3** have been analysed by DEA technique as described by **Liao and Chen, 2002**. Data have been normalized first by using **Eq. 5.1-5.2** respectively. Normalized data has been furnished in **Table 4**. Normalized data of average surface roughness has been treated as input factor whereas normalized data of MRR has been considered as output factor in *LINGO* software for assessing the relative efficiency **(Table 5)** corresponding to each experimental run. Finally, Taguchi has been adopted on relative efficiency for assessing optimal condition and $N_3f_3d_3$ has been predicted **(Figure 1)** as the most favourable machining condition. Predicted result has been verified through confirmatory test. **Table 6** represents factor ranking in accordance with their degree of significance.

6. Conclusions

In the present study DEA coupled with Taguchi's optimization technique has been proposed for determining favourable machining conditions in machining of copper. DEA can combine multiple objectives into single objective by computing relative efficiency of each experiment run which can further be optimized using Taguchi method.

This approach can be recommended for continuous quality improvement and off-line quality of any production process.

Table 1: Design of experiment

Sl. No.	Factorial combination (Coded form)		
	N	f	d
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

Table 2: Domain of experiments

Factors	Unit	Level 1	Level 2	Level 3
Cutting Speed, N	RPM	750	1000	1500
Feed Rate, f	mm/min	50	150	200
Depth of Cut, d	mm	0.2	0.4	0.6

Table 3: Experimental data

Sl. No.	MRR (mm^3/min)	R_a (μm)
1	124.7841834	3.93
2	536.2259115	2.22
3	1371.112109	2.66
4	183.4474117	3.8
5	924.5593027	3.0
6	360.6859312	3.133
7	375.2652723	3.066
8	694.6226812	3.2
9	1231.741019	3.133

Table 4: Normalized data

Sl. No.	Normalized Data	
	Surface roughness	MRR
1	0.559796	0.091009
2	1	0.391088
3	0.827068	1
4	0.578947	0.133789
5	0.733333	0.674313
6	0.702202	0.263061
7	0.717547	0.273694
8	0.6875	0.506613
9	0.702202	0.898352

Table 5: Relative efficiency with S/N ratios

Sl. No.	Relative efficiency	S/N Ratios
1	0.12708	-17.9186
2	0.30570	-10.2942
3	0.94509	-0.4905
4	0.18063	-14.8641
5	0.71874	-2.8686
6	0.29283	-10.6678
7	0.29815	-10.5114
8	0.57600	-4.7916
9	1.00000	0.0000

Table 6: Mean response table

Level	N	f	D
1	-9.568	-14.431	-11.126
2	-9.467	-5.985	-8.386
3	-5.101	-3.719	-4.623
Delta	**4.467**	**10.712**	**6.503**
Rank	**3**	**1**	**2**

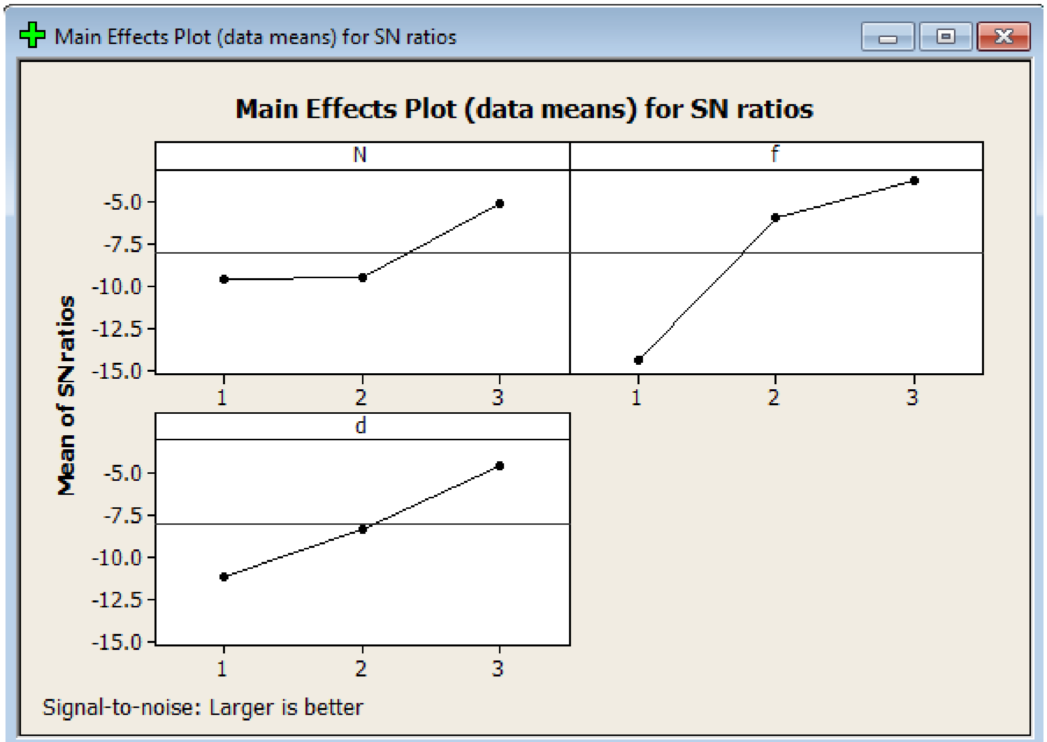

Figure 1: Evaluation of optimal setting

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Appendix

MAXMILL is a numerically controlled machine tool used for machining parts in every industrial field, featuring high speed, high accuracy, and high productivity.

MAXMILL performs drilling, parting, boring, roughing, chamfering, tapping of circular and rectangular work pieces , using CNC programming and operating software.

Standard Equipment:

- MAXMILL 3 axis CNC milling machine with Fanuc Oi Mate MC Controller.
- Machine Operator Panel
- Central Automatic Lubrication system
- Flood Coolant system

Optional equipment:

- ATC (Automatic Tool Changer)
- Pneumatic Vice
- Panel Cooler
- Auto Door
- Servo Stabilizer

Machine specifications:

X Axis travel (Longitudinal Travel)	300 mm
Y Axis travel (Cross Travel)	250 mm
Z Axis travel (Vertical Travel)	250 mm
Table Dimension	
Clamping surface	500 x 350 mm
T- slots (No. x Size)	3 x 14 mm
Accuracy	
Repeatability	± 0.005 mm
Positional Accuracy	0.010 mm
Coolant	
Coolant Motor	RKM 02505 (Rajamane)
Motor Power	0.37 kW
Tank Capacity	110 liter (Filter & Tray)
X-Axis dive data	
Table Size	500 x 350 mm
Weight of the table	35 kg
Load on Table	200 kg

Rapid Feed			10 m / min
Stroke			300 mm
T Slots			14 – 3 No’s
Ball screw			R20-5B2-FDW (Hiwin)
Bearings			BSB 017047 DUMP3 (RHP)
Servo Motor			FANUC β 4/4000 i s
L M Guide			HGH 20 HA (Hiwin)
Coupling			SFC-SA-050-14H7-15H7 (Mikipulley)
Y-Axis drive data			
Saddle size			468 x 350 mm
Weight of the Saddle			50 kg
Load on Saddle			300 kg
Rapid Feed			10 m / min
Stroke			250 mm
Ball Screw			R20-5B2-FDW (Hiwin)
Bearings			BSB 020047 (RHP)
Servo Motor			FANUC β 4/4000 i s
L M Guide			HGH 25 HA (Hiwin)
Coupling			SFC-SA-050-14H7-15H7 (Mikipulley)
Electrical Specification			
Power ratings			415 V, 3 ϕ , 15 k VA
Axes motor			Fanuc Servo Motor β 4i Series
Spindle motor			Fanuc Spindle Motor β 4i Series
Spindle motor			
Model			FANUC β 3/10000 i
Rated Output	Cont. rated		3.7 kW
	15 min rated 60 min rated		5.5 k W 3.7 k W
Speed	Base speed	Cont. rated	2000 rpm
		15 min rated 60 min rated	1500 rpm
	Power cons Range	Cont. rated 15 min rated	4500 rpm
		60 min rated	2000 rpm
	Max. Speed		10000 rpm
ATC make			MACO
Tool Type			BT 30
Control System			
3 Axis continuous path system			Fanuc Oi Mate MC

Lubrication		
Automatic centralized lubrication for slides and ball screws		DMCLS-2800 DX (Dropco)
Axis Drive		
X,Y & Z Axis	Motor model	FANUC β 4/4000i s
	Rated Output	0.75 k W
	Stalling Torque	3.5 Nm
	Max. Speed	4000 rpm

CNC Program

```
02000
G00 G53 G90 G40 G69 G80 G94;
G00 G53 Z0;
G00 G53 X0 Y0;
G00 G59 X0 Y0;
G00 G43 H10 Z50;
M03 S1250;
G00 Z1;
G01 Z-0.6 F50;
G01 X-50
M05;
G00 G90 Z50;
G00 G53 Y0;
M30
%
```

Communication

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