A vision of an adaptive geometry model for computer-assisted building surveying

T. Thurow, Bauhaus University Weimar, 99421 Weimar, Germany
(torsten.thurow@informatik.uni-weimar.de)

Summary
The paper describes a concept for the step-by-step computer-aided capture and representation of geometric building data in the context of planning-oriented building surveying. Selected aspects of the concept have been implemented and tested as prototypes. The process of step-by-step capture and representation is determined by the order in which the user experiences the building. Only the information that the user knows (can see) or can reasonably deduce is represented. In addition approaches to the flexible combination of different measuring techniques and geometric abstractions are described which are based upon geodetic computational adjustment.

1 The context and deficit analysis

1.1 The aim of a planning-oriented building survey
An essential prerequisite for planning within existing contexts is reliable and informative planning data. In most cases this is not available or has not been kept up to date with the current situation. As a result a building survey is necessary, either as an extension or validation of existing building documentation or to provide new documentation. A building survey which fulfils the needs of a planning task will be described as a planning-oriented building survey /Donath 2003/.

Despite the many different fields of application for building surveying and the resulting different demands, the representation of the building geometry is typically the most important aspect of a building survey. The gradual process of getting to know an existing building is mirrored in the capture and representation of geometric data. This paper concentrates on this process.

Without relationships to other kinds of information, geometric information on its own can only provide limited information to the planner. The chair “Computer Science in Architecture” has developed a concept which is described in the paper “The building as a container of information” /Petzold 2004/. This paper examines only one element of this concept: the building geometry. The above article describes the context for this paper.

1.2 The deficits of commercially available surveying systems
Current practice-oriented applications in the field of building surveying only provide insular support of specific aspects of the building survey. The majority of available systems deal with the building survey, and in most cases only specific surveying techniques are supported. Complete solutions supporting the entire surveying process are not available. As a result different surveying techniques can only be combined according to particular constructive approaches.
### 1.3 A different approach to the step-by-step capture and representation of building geometry

In comparison to commercially available solutions we have chosen an approach that is oriented towards the way in which a user gradually gets to know and surveys a building. In the following aspects we will focus on the surveying of geometric information.

In the initial phase of a building survey, the surveyor gathers a ‘rough’ impression of the building as a whole. The surveyor can see the exterior elevations of the building and the surfaces of rooms indoors. As the surveyor goes from room to room he or she gradually takes in more and more information. Particular building elements within the rooms such as doors, windows, columns etc. can be identified immediately. Other elements such as the construction of walls, the elements of the structural system remain hidden from view. The surveyor then returns and begins to measure the dimensions of rooms from indoor surface to indoor surface, as well as the dimensions of building elements that are visible. The planner or surveyor can then deduce or infer how the building is constructed based upon the geometry of the building, his or her knowledge of (historic) building construction and possibly through localised examination of particular elements of the building.

The computer-aided support of this gradual process takes the approach “from sketch to detail”. In the first phase of the building survey, the surveyor makes a sketch-like overview of the arrangement of rooms, their surfaces and the external surfaces of the building (elevations etc.). The sketches are not to scale. The individual surfaces can be ordered in a form of semantic i.e. differentiated in walls, floors and ceilings. In addition further formal or informal information about the surfaces (material, appearance, damages etc.) are recorded. A commercial example of a sketch-based approach is the tool “SketchUp”.

During the second stage, the ‘model’ of the building that has resulted from the first stage is ‘enriched’ with geometric dimensions. A variety of different surveying techniques can be applied and combined according to the needs and tools at the surveyor’s disposal. The surveyor must also be free to choose where to begin and in which order the survey will be undertaken. The realisation of this gradual process of ‘enriching’ and correcting the geometric model employs techniques from geodetic computational adjustment.

<table>
<thead>
<tr>
<th>Different commercially available surveying systems and the supported surveying techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Manual surveying</strong></td>
</tr>
<tr>
<td><strong>Tacheometry</strong></td>
</tr>
<tr>
<td><strong>Photogrammetry</strong></td>
</tr>
<tr>
<td><strong>Laser scanning</strong></td>
</tr>
</tbody>
</table>
A comparison between conventional approaches to planning and the actual requirements of the surveyor as he or she gradually surveys the building

1.4 Combining different surveying techniques and geometric abstraction

As previously described, the surveyor should be able to use a variety of different surveying techniques. The most common techniques employed are manual measuring by hand, tacheometry and photogrammetry. Each of these surveying methods have their own advantages and disadvantages, as described in /Petzold 2004/. All are able to determine the geometric location of a particular point (or series of points) in relation to another point with slightly different degrees of precision. In order to reconstruct the geometry of a building from these series of measured distances certain geometric abstractions must be applied. For example, wall surfaces are generally regarded as being flat or planar. The use of such abstractions mean that the geometric model will inherently contain an element of error in comparison with the original building. This should be differentiated from errors resulting from low precision surveying techniques. It is the surveyor’s responsibility to take sufficient and intelligently chosen measurements (diagonals, skews, perceived bends in walls etc.) in order to minimise model inaccuracies through geometric abstraction. As a result the survey itself can be regarded as a process of modelling the building.

Geometric abstractions also help to speed up the surveying process and therefore to keep costs down. The pay-off is a reduction in geometric precision. The surveying of window jambs serves as a good example. Depending upon the level of accuracy required and the form of the window, many window jambs can be regarded as being parallel. This level of abstraction means that only
the dimension of the opening, the distance between the jambs, need be measured. This is usually undertaken in the middle of the wall depth to keep model inaccuracies to a minimum. The same principle of geometric abstraction of dependencies applies for other elements in a building. These include:

- Parallel and orthogonal orientation of surfaces
- Horizontal and vertical surfaces
- Symmetry
- Geometric similarity (repetition) of particular elements, e.g. windows, columns etc.

To combine measurements from different surveying techniques and geometric abstractions a computational adjustment model from the field of geodetics is applied. This allows the conscious ‘enrichment’ or validation of the model with a view to improving the model’s accuracy. All measured dimensions and abstractions are recorded in a formalised form in a database and their representation in the form of a geometric model can be recalculated periodically as required.

The basic principle of adjustment of a geometric representation is relatively straightforward. The geometric representation is described using a surface model. Measurements and abstractions are “introduced” into the surface model. These change the dimensions of the representation. Computational adjust attempts to resolve conflicts between the representation in the model and the actual measured data in such a way as these are minimised and the model need be changed as little as possible. Using this approach it is possible to begin with a high level of geometric abstraction and only a few measurements and to gradually add further measurements and reduce the geometric abstraction to increase the accuracy of the model.
1.5 **Formalising measurements and geometric abstractions**

The adjust model chosen is a mediating adjustment on the basis of observation, as this model produces small adjustment systems and because the calculated unknowns are required for the dimensional adjustment of the geometric module. All observations are presumed to be uncorrelated as the automatic determination of correlations is difficult and cannot be expected from a user without experience of geodetics. The following explanation provides an introduction to the computational adjustment model from the field of geodetics as well as experience from concrete applications.

The introduction of observations is applied graphically. In practice, the surveyor models everything he or she has measured. The measurements are quite simply modelled. In a subsequent stage the geometry is modified via computational adjustment algorithms.
Manually measured plan in modelled in sketch form before and after adjustment

For the approach described here, the adjusted model and the geometric model are coupled with one another through the points of the geometric model. The coordinates of the points are unknown in the adjustment model.

Measurements and abstractions are modelled as observations according to the form

$$L + v = \varphi(\hat{X})$$

Here $L$ represents the value measured from the original and $v$ the deviation between $L$ and the adjusted model. A distance between two points would therefore be modelled as:

$$L + v = |p_2 - p_1|.$$  (2)

By way of an example of geometric abstraction, a collection of points $p_1, \ldots, p_m$ are introduced which lie in the same plane. The abstraction is formalised using $m$ observations of the form

$$0 + v = \frac{n \cdot (p_2 - p_s)}{n}.$$  (3)

Here $n$ represents the normal vector of the place and $p_s$ the geometric centroid of the collection of points. The same pattern applies for the modelling of other measurements and abstractions.
The combination of different measurements and abstractions is achieved through the
linearisation of the observations

\[
A = \left( \frac{\partial \varphi_i(X^0)}{\partial X} \right)_0 = \begin{bmatrix}
\left( \frac{\partial \varphi_1(X^0)}{\partial X_1} \right)_0 & \left( \frac{\partial \varphi_1(X^0)}{\partial X_2} \right)_0 & \cdots & \left( \frac{\partial \varphi_1(X^0)}{\partial X_u} \right)_0 \\
\left( \frac{\partial \varphi_2(X^0)}{\partial X_1} \right)_0 & \left( \frac{\partial \varphi_2(X^0)}{\partial X_2} \right)_0 & \cdots & \left( \frac{\partial \varphi_2(X^0)}{\partial X_u} \right)_0 \\
\vdots & \vdots & \ddots & \vdots \\
\left( \frac{\partial \varphi_n(X^0)}{\partial X_1} \right)_0 & \left( \frac{\partial \varphi_n(X^0)}{\partial X_2} \right)_0 & \cdots & \left( \frac{\partial \varphi_n(X^0)}{\partial X_u} \right)_0
\end{bmatrix}
\]  

(4)

The linearisation method in the field of geodetics is usually analytic. In the prototypical
realisation of this project, a numeric differential is used instead:

\[
\frac{\partial \varphi_i(X^0)}{\partial X_j} = \frac{\varphi_i(X_1 \ldots X_{j-1}, X_j + \epsilon, X_{j+1} \ldots X_n) - \varphi_i(X_1 \ldots X_{j-1}, X_j - \epsilon, X_{j+1} \ldots X_n)}{2\epsilon}
\]  

(5)

The method shown has proven itself in practice. To minimise numeric inaccuracy \( \epsilon \) was chosen
as \( 2^{-16} = 1,52587890625 \cdot 10^{-5} \). The numeric deviation of the prototype, including use of a
FastSolver, remains below that of a millimetre. It should be noted that certain cases necessitate
special care in the calculation of differences. Angles are a case in point. The difference between
2° and 352° is 10° and not -350°.

Each observation \( \varphi_i \) is attributed a weighting, \( P_i \). Through the use of weightings the influence
of the different observations upon each other can be changed. The aim of the computational
adjustment is to keep differences to a minimum, i.e. \( \psi^T P \psi = \min \).

Using the weightings, different levels between measurements and abstractions can be modelled.
This approach has not proven to be optimal. A discrete staggered arrangement of groups of
observations, quasi as “primary and secondary constraints”, is more desirable.

Because of linearisation errors in the observations, the computational adjustment must be
undertaken iteratively. In addition, at the beginning of the adjustment process approximate
values are required for the unknowns. For the approach “from sketch to detail” the coordinate
points from the sketch are taken as the approximate values. Other approximations, for instance
normal vectors of geometric centroids can usually be calculated in the sketch-like introduction
of observations. It should be noted approximations must be recalculated once a sketch-based
modification of the geometric model has taken place, for instance through rotation or movement
through the user.

With each interaction the improvement vector \( x \) of the unknown vector \( X \) is calculated anew
( \( \tilde{X} = X^0 + \hat{x} \)). The calculation results as follows

\[
N\hat{x} = n, \text{ where } N = A^T PA \text{ and } n = A^T Pl. 
\]  

(6)
The vector $l$ represents the difference between the measured observational value $L$ and the theoretical value in the adjustment model before adjustment ($l = L - \varphi(X^0)$). As with the calculation of differences, care should be taken in the numeric differentiation of special cases such as angular values.

1.6 Resolution of adjustment systems

In order to be able to use such a system on site, the system must respond quickly to the user’s actions. This places particular speed requirements on the resolution of adjustment systems.

The matrix $N$ remains singular for the larger proportion of the step-by-step survey. The adjustment computation should nevertheless be applied at this stage so that the user can check the correlation between model and reality both visually as well as by comparing dimensions. As a result the regularisation of $N$ is necessary.

The singularity of $N$ is a condition of datum and configuration defects. Standard approaches to correcting datum defects are not sufficient on their own. Instead a continual regularisation takes place according to the approach $\|Nx - n\|^2 + \alpha \|x\|^2 = \min$. The approach is easily applied using $N_a = N + \alpha l$ and has no noticeable effect on the duration of the calculation. Once again, $\alpha = 2^{-16}$ has been chosen.

After the regularisation of $N$, this is a low and positive value. Resolution in the prototype employs the Cholesky-approach with modified Cholesky separation $LDL^T$ using a skyline matrix.

2 Observation adjustment

The following details the specialities of observation adjustment as applied in the prototype. A full description of all observation adjustments is available in [Thurow04].

As previously mentioned, a typical geometric abstraction is the adjustment of a collection of points in a planar surface. This is applied to every polygon in the prototype. The parallel nature of two surfaces is described by the normal vectors of the surfaces. In the prototypical realisation...
Several observations proved problematic where the optimum situation concurred with a reversal point. As a result the parallel nature of the normal vectors is compared using the following pragmatic methods:

\[
\begin{align*}
0 + v &= \frac{x_n}{|n|} - \frac{x_m}{|m|} \\
0 + v &= \frac{y_n}{|n|} - \frac{y_m}{|m|} \quad \text{where } \cos \alpha(n, m) \geq 0 \\
0 + v &= \frac{z_n}{|n|} - \frac{z_m}{|m|} \\
0 + v &= \frac{x_n}{|n|} + \frac{x_m}{|m|} \quad \text{where } \cos \alpha(n, m) < 0 \quad (7)
\end{align*}
\]

The same problem also occurs with other observations such as horizontal planes or comparisons with point positions with a distance of 0.

Compared to photogrammetry and tacheometry, manual measurement produces very different values which are often conditioned through a geometric abstraction. An example is the distance between edges and surfaces. It is assumed that the edges or surfaces are (more or less) parallel to one another. In order not to force the condition of parallel surfaces, observational adjustments were introduced which expect approximate parallel arrangement. As a result the distance between two approximately parallel edges is given by:

\[
d + v = \min \left( \left( \frac{p_3 + p_4 - p_1}{2} \right) \times \left( p_2 - p_1 \right), \left( \frac{p_1 + p_2 - p_3}{2} \right) \times \left( p_4 - p_3 \right) \right) \quad (8)
\]

The distance between two approximately parallel surfaces follows the same pattern. This is illustrated in the following diagram.

The geometric centroids (Centroid1 and Centroid2) from two polygons (Polygon1 and Polygon2) are calculated. One of the polygons is then shifted in the imagination so that the geometric centroids of both polygons have the same position. A common adjusted layer is generated using the points from both polygons and arranged to lie on the same position as both of the centroids. The geometric centroid of the (in the imagination) shifted polygon is now shifted from its real position to its perpendicular location in relation to the adjusted layer. The perpendicular distance is the distance between the two polygons. It doesn’t matter which polygon is ‘shifted’ as the end result will be the same (see a and b in the diagram).
In addition observation adjustments were introduced to describe the exemplary similarity between geometric elements based upon their defining perimeter points (block copy).

3 Prototypical realisation
Selected aspects of the concept as described have been realised in a prototypical system. The prototype with the name “experimental platform FREAK” consists of a series of extendable tools which access and work with the same database.

3D and plan-oriented views of a geometry model using the tools “OpenGLviewer” and “PlanarViewer”, showing an example of a manually measured survey of the chair building.

The tools allow the sketch-based, plan-oriented creation of simple building geometries and their adaptation to fit taken measurements. After the building geometry has been entered in sketch form the system looks for likely geometric abstractions. Using various different tools manual measurements or measurements obtained with tachometry or photogrammetry can be introduced into the model. The geometry is then adapted accordingly.
A room containing measurements obtained through manual measuring, tacheometry and photogrammetry

Through the use of a motorised tacheometer with visible laser beam it is possible to compare model and reality in real-scale. The tacheometer rotates to show the location of points in the geometrical model as a laser-beam point in the real building. Another method is the visual comparison between a distortion-corrected photo and the geometry model.

Comparison of coordinate point in the model with the real situation:
- a) using a motorised tacheometer with laser beam
- b) using a distortion-corrected photo as model overlay

As previously noted, a geometric model of building surfaces is only one aspect of a building survey. The chair has developed concepts and will continue to research concepts which allow the user to derive building elements from the geometric survey and to link these with attributes and with further kinds of relevant information in a variety of forms.

4 References


