The influence of stress state on the exponent in the power law equation of fatigue crack growth

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Problem area

In 1963 Paris et al. were the first to show that the fatigue crack growth rate (FCGR) has a power law relationship with the stress intensity factor range, \( \Delta K \). However, linear elastic fracture mechanics (LEFM) has not been able to deduce the Paris equation from basic principles or a physical model, and so it is still an empirical equation. Another major issue is the fact that this equation is dimensionally correct only when the dimensions of the constant are changed with a change in the exponent.

From 1963, many adaptations of the Paris equation have been proposed to incorporate the experimental results at low, high \( \Delta K \) and at different stress ratios. However, recently it has been shown that the correction function attributed to crack closure can be regarded mostly as a correction for energy and for a small part for crack closure that occurs at low stresses. This also indicates that the fundamentals of fatigue crack growth are not so well understood as one would expect after more than 50 years of extensive research after introduction of the Paris equation.
Description of work

Constant amplitude crack growth tests were performed on 29 middle tension specimens according to ASTM E647-00. The specimen were obtained from 6.35 mm thick aluminium alloy AA7075-T7351 plate material. The crack length was measured optically and by direct current potential drop (DCPD). A constant amplitude 13.5 Hz sinusoidal load was introduced and the maximum and minimum stresses during the load cycles were 80 MPa and 8 MPa, corresponding to a stress ratio R = 0.1.

Results and conclusions

Fractography and the crack length measurements show that the exponent is higher and varies at crack lengths where crack growth is dominated by plane strain conditions. The power law exponent decreases and is similar for all specimens after the transition to plane stress conditions at higher crack lengths. The mathematical concept of a pivot point is used to model crack growth with two different exponents using a dimensionally correct equation. It also allows modelling the crack growth variation in all specimens by varying only one parameter, the power law exponent for the plane strain condition.

Applicability

Since the observed crack growth behavior can be described by the model very accurately, it is expected that the model captures some of the physics of crack growth. Therefore, the results can lead to a physics based model and a decrease in the future number of crack growth tests that are necessary for alloy characterization, while getting increased accuracy. This should all lead to increased accuracy of crack growth predictions, decreasing scatter and ultimately to lower design weight of structures.
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Summary

Paris et al. were the first to show that the fatigue crack growth rate in metals shows a power law relationship with the stress intensity factor range. It is generally accepted that the exponent is material-dependent. However, it is also true that the empirical Paris equation is dimensionally correct only when the dimensions of the constant in the equation are changed with the power law exponent. In the present paper it will be shown that for 29 identical fatigue crack growth tests on aluminium alloy 7075-T7351 the exponent changes between specimens and crack lengths. Fractography and the crack length measurements show that the exponent is higher and varies at crack lengths where crack growth is dominated by plane strain conditions. The power law exponent decreases and is similar for all specimens after the transition to plane stress conditions at higher crack lengths. The mathematical concept of a pivot point is used to model crack growth with two different exponents using a dimensionally correct equation. It also allows modelling the crack growth variation in all specimens by varying only one parameter, the power law exponent for the plane strain condition.
## Contents

Abbreviations .................................................................................................................. 6

1 Introduction.................................................................................................................... 7
   1.1 Fatigue crack growth rate equations ...................................................................... 7
   1.2 Pivot point ............................................................................................................. 8

2 Experimental set-up and results .................................................................................. 10

3 Discussion..................................................................................................................... 11
   3.1 Plastic zone size .................................................................................................. 13

4 Conclusions.................................................................................................................. 15

References ...................................................................................................................... 16
# Abbreviations

<table>
<thead>
<tr>
<th>ACRONYM</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>stress intensity factor range</td>
</tr>
<tr>
<td>$\Delta K_{\text{eff}}$</td>
<td>effective stress intensity factor range</td>
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<td>$a$</td>
<td>crack length</td>
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<tr>
<td>AA</td>
<td>Aluminium Alloy</td>
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<td>$b$</td>
<td>intercept on the ordinate axis</td>
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<tr>
<td>$C$</td>
<td>Material constant</td>
</tr>
<tr>
<td>$C_k$</td>
<td>material constant</td>
</tr>
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<td>$da/dN$</td>
<td>fatigue crack growth rate</td>
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<td>DCPD</td>
<td>Direct Current Potential Drop</td>
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<tr>
<td>$f$</td>
<td>correction function</td>
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<tr>
<td>FCGR</td>
<td>Fatigue Crack Growth Rate</td>
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<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
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<tr>
<td>$m$</td>
<td>power law exponent (half of $n$)</td>
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<td>$n$</td>
<td>power law exponent</td>
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<td>NLR</td>
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<td>$R$</td>
<td>stress ratio</td>
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<td>$S$</td>
<td>applied stress</td>
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<td>$W$</td>
<td>Specimen width</td>
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<tr>
<td>$z$</td>
<td>slope</td>
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<tr>
<td>$\beta$</td>
<td>geometry correction factor</td>
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1 Introduction

1.1 Fatigue crack growth rate equations

Paris et al. were the first to show that the fatigue crack growth rate (FCGR) has a power law relationship with the stress intensity factor range, \( \Delta K \) [1]:

\[
\frac{da}{dN} = C_R \cdot \Delta K^n \tag{Eq. 1}
\]

where \( da/dN \) is the FCGR, \( C_R \) is a constant and \( n \) is the power law exponent. \( C_R \) and \( n \) are both regarded as material parameters. The stress intensity factor range is defined as:

\[
\Delta K = \beta(a)(S_{\text{max}} - S_{\text{min}})\sqrt{a} \tag{Eq. 2}
\]

where \( S \) is the applied stress and \( a \) is the crack length. The geometry correction factor \( \beta(a) \) depends on the geometry and the crack length. From 1963, many adaptations of Eq. 1 (the Paris equation) have been proposed to incorporate the experimental results at low \( \Delta K \), near the supposed threshold, and at high \( \Delta K \), near the fracture toughness regime. Also the effective stress intensity factor range, \( \Delta K_{\text{eff}} \), was introduced to account for different FCGRs at different stress ratios, \( R \):

\[
\Delta K_{\text{eff}} = f(R) \cdot \Delta K \tag{Eq. 3}
\]

where \( f(R) \) is a function of the stress ratio, \( R \):

\[
R = \frac{S_{\text{min}}}{S_{\text{max}}} \tag{Eq. 4}
\]

\[
\frac{da}{dN} = C \cdot \Delta K_{\text{eff}}^n \tag{Eq. 5}
\]

Different correction functions, \( f(R) \), have been proposed, e.g. by Elber, Newman, and Schijve, and they all attributed the correction function entirely to crack closure [2-7]. However, Alderliesten recently showed that the correction function can be regarded mostly as a correction for energy and for a small part for crack closure that occurs at low stresses [8]. This also indicates that the fundamentals of fatigue crack growth are not so well understood as one would expect after more than 50 years of extensive research after introduction of the Paris equation [1]. Linear elastic fracture mechanics (LEFM) has not been able to deduce the Paris equation from basic principles or a physical model, and so it is still an empirical equation. Another major issue is the fact that this equation is dimensionally correct only when the dimensions of the constant \( C_R \) are changed with a change in the exponent, \( n \). In the past many people have introduced various crack growth rate equations that can be grouped together by the following general formulation [8 -10]:

\[
\frac{da}{dN} \propto \sigma^m a^n \tag{Eq. 6}
\]

In the present investigation the stress amplitude has been held constant during the test and equal for all specimens. Therefore the only change in crack growth rate results from a change in crack length. In the case of Eq. 5, \( m \) is equal to \( n/2 \) and \( l \) is equal to \( n \) and therefore \( 2m \):

\[
\frac{da}{dN} \propto \Delta S^m \sqrt{a} = \Delta S^{2m} a^m \tag{Eq. 7}
\]
1.2 Pivot point

It is generally accepted that the exponent is material-dependent. Zuidema et al. also observed a decrease in the exponent for aluminium alloy AA 2024 when shear lips develop [11-13]. Usually, fatigue cracks start flat and grow in mode I (tensile mode), until at a certain $\Delta K_{eff}$ the specimen (or component) starts to develop shear lips. The shear lip development in aluminium alloys AA 2024-T3 and AA 7075-T6 starts at a critical growth rate of about 1·$10^7$ m/cycle [14]. The shear lip thickness increases with increasing $\Delta K_{eff}$, but it also depends on the cycle frequency and environment: for constant $\Delta K_{eff}$ fatigue tests the shear lip thickness is constant and proportional to the logarithm of the test frequency [14]; a more aggressive environment decreases the shear lip thickness [14].

Development of shear lips implies that fatigue crack growth occurs in mixed mode, I + II + III [14]. However, conventionally the mode I $\Delta K_{eff}$ is then still used for FCGR modelling. In the present investigation this will also be the case. Zuidema et al. indicate that the shear lips are not responsible for a decrease in $n$, but that they, like $n$, are an effect from an underlying phenomenon [14].

When crack growth rate data are equation-fitted, then a change in $n$ also gives rise to a change in the constant $C$. Many people have reported a linear relationship between $\log C$ and the exponent, $n$ [15-29]:

$$\log C = -zn + b$$  \hspace{1cm} (Eq. 8)

where constant $z$ is the slope and constant $b$ the intercept on the ordinate axis. This indicates that the constant $C$ is not independent of the exponent $n$. Taking the logarithm of Eq. 5 results in;

$$\log C = -\log(\Delta K_{eff}) + \log \left( \frac{da}{dN} \right)$$  \hspace{1cm} (Eq. 9)

If there is a linear relationship between $\log C$ and $n$, the constants $z$ and $b$ result in particular values of $\Delta K_{eff,z}$ and $\frac{da}{dN}$:

$$\Delta K_{eff,z} = 10^z \quad \text{and} \quad \frac{da}{dN_b} = 10^b$$  \hspace{1cm} (Eqs. 10 & 11)

Combining Eqs. 10 & 11 with Eq. 8 results in the following expression for the constant $C$:

$$C = \frac{da}{dN} \cdot \Delta K_{eff,z}^{-n}$$  \hspace{1cm} (Eq. 12)

Substituting Eq. 12 for the constant in Eq. 5 results in a relative crack growth rate equation that is dimensionally correct:

$$\frac{da}{dN} = \frac{da}{dN_b} \left( \frac{\Delta K}{\Delta K_{eff,z}} \right)^n$$  \hspace{1cm} (Eq. 13)

When $\Delta K_{eff,z}$ is equal to 1 Eq. 13 is equal to Eq. 5. The linear relationship between $\log C$ and the exponent, $n$, introduces a so-called pivot point in the $\frac{da}{dN} vs. \Delta K_{eff}$ plot. The pivot point is located at the coordinates $\Delta K_{eff,z}, \frac{da}{dN_b}$ and allows describing the variation between specimens tested under similar conditions by only a variation in $n$.

A number of papers have been written about the pivot point. However, there is much doubt about there being a physical basis for a pivot point [24-28]. Kohout states that it is a mathematical consequence of the fitting process, whereby a pivot point represents only the centre of gravity of the data points [28]. Reviewing several papers for different materials [23-25], lost concludes that the pivot point is material-dependent and may be associated with a transition mechanism in the FCGR curve [25].
In the present paper it will be shown that $n$ not only changes between materials and specimens, but also with crack length: $n$ is higher and varies at crack lengths where crack growth is dominated by plane strain conditions. After the transition to plane stress conditions, at greater crack lengths, $n$ decreases and is similar for all specimens. The change in $n$ causes a pivot point transition in the FCGR curve. Therefore, the mathematical concept of a pivot point will be used to model crack growth with two different exponents using a dimensionally correct equation. This allows modelling the crack growth variation in all specimens by varying only one parameter, the power law exponent $n$ derived from the plane strain condition.
2 Experimental set-up and results

Constant amplitude crack growth tests were performed on 29 middle tension specimens according to ASTM E647-00. The specimen dimensions were 500 mm x 160 mm x 6.35 mm, and were obtained from 6.35 mm thick aluminium alloy AA7075-T7351 plate material. The rolling direction was in the length of the specimens. Fatigue crack growth starter notches were central holes (1.6 mm diameter) with 0.7 mm deep electric discharge machined slots on either side of the hole (total starter notch length of 3 mm). Holes were drilled at 8 mm above and below the start notch hole for copper pins. These were used for automated crack length measurements by direct current potential drop (DCPD). The current was introduced to the specimen at the specimen clamping. The area next to the starter notch was polished for optical crack growth measurements on both front and rear sides of the specimens. The optical measurements (4 for each crack length measurement) were averaged and used to check the DCPD measurements.

A constant amplitude 13.5 Hz sinusoidal load was introduced by an electrohydraulic test machine with a 200 kN load cell. The maximum and minimum stresses during the load cycles were 80 MPa and 8 MPa, corresponding to a stress ratio $R = 0.1$. The specimens were pre-cracked to a crack length of 2 mm (4 mm in total). Failure was defined as occurring at 40.8 mm (single side) even though actual failure occurred at higher crack lengths. The failure criterion of 40.8 mm was used because all specimens gave crack length data points up to at least 40.8 mm. For higher crack lengths, a single crack length could not always be measured with the DCPD-system, owing to very high crack growth rates.

Fig. 1 shows examples of the obtained fatigue crack growth curves: specimen 1 had the lowest number of cycles to failure and specimen 26 one of the highest number of cycles to failure. In total, the number of cycles to failure ranged between 103,150 and 133,350 cycles, see Fig. 2.
3 Discussion

It was noticed that the number of cycles to grow the crack from about 10 mm to 40.8 mm was approximately the same for all specimens (see Fig. 3) and that the shapes of the curves were similar (see Fig. 4). This indicates that the constant, $C$, and exponent, $n$, from Eq. 5 were both the same for this range of crack growth.

The initial crack lengths were set to 2 mm for all specimens. Fig. 3 clearly shows that the differences in the number of cycles to failure were caused by the differences in crack growth up to about 10 mm. In fact, these differences in earlier crack growth dominated the total lives. For example, Fig. 3 shows that specimen 26 had grown only to a crack length of 11.3 mm when specimen 1 reached the failure length of 40.8 mm.

The differences in crack growth in the crack length range 2-10 mm were evaluated to see if they could be modelled by either a difference in $C$ or $n$. An example is given in Fig. 5, which shows the crack length as a function of cycles for specimens 1, 12 and 26. The crack growth curves for these specimens were modelled using Eq. 5 with a fixed $n = 3.38$ and various values of $C$. The value of $n=3.38$ is obtained from specimen 12. It is apparent that the curve shapes for specimens 1 and 26 could not be modelled correctly using different constants. Obviously, for specimen 12 a good fit was obtained. These results indicate that the curve shapes are different and that the differences can only be modelled correctly, if at all, by varying the exponent $n$ in Eq. 5.

On the other hand, as mentioned above, for crack lengths >10 mm the constant, $C$, and exponent, $n$, were the same for all specimens. The crack length vs. cycles curves are continuous, therefore the fatigue crack growth rate has to be the same at about 10 mm for all the curves. For that reason, the mathematics of a pivot point have been used to model the crack growth in the specimens (Eq. 13), however with two exceptions: (i) different exponents are used before and after the pivot point, and (ii) a single exponent is used for crack lengths larger than about 10 mm, while the crack growth curve variations at shorter crack lengths have been captured by varying the exponent. At the pivot point the crack growth rate is independent of the exponent $n$.

To determine the pivot point the crack growth rates of the specimens with the least and one of the most number of cycles to failure were compared. By substituting for $n$ by $2m$, Eq. 13 becomes

$$\frac{da}{dN} = \frac{da}{dN} \left( \frac{\beta^2 a}{\beta^2 a} \right)^m$$

(Eq. 14)

where according to ASTM E647 [30] $\beta^2$ is:

$$\beta^2 = \cos^{-1} \left( \frac{ma}{W} \right)$$

(Eq. 15)

and $W$ is the width of the specimen. $\beta^2$ increases from 1.00077 to 1.44 for crack lengths of 2 to 40.8 mm, and for a crack length of 10 mm $\beta^2$ is 1.0196. Therefore, the crack growth rate only depends on the crack length and the exponent $m$, where $m$ ranges from 1 to higher values.

Fig. 6 shows the crack growth rates vs. $\beta^2a$ for specimens 1 and 26. Visual best fit lines of the initial parts of the crack growth rates were used to determine the pivot point. These lines intersect at a crack
length of 9.69 mm ($\beta^2 a = 9.87$ mm, where $\beta^2$ is 1.0191 and the crack length is 9.69 mm). It is worth noting that visual best fit lines were used to find the intersection (pivot point) and no linear least squares method was used, because the visual best fit is less affected by the scatter that is present in $da/dN$ vs. $\Delta K$ data.

Now, $\beta^2 a = 9.87$ mm corresponds to $\Delta K = 12.68$ MPa$\sqrt{m}$ ($\Delta K_{eff} = 7.41$ MPa$\sqrt{m}$ based on the correction function of Schijve [4, 5]) and the crack growth rate at the pivot point is $3.09 \times 10^{-7}$ m/cycle. Thus we may write:

$$\frac{da}{dN} = 3.09 \times 10^{-7} \left(\frac{\beta^2 a}{9.87}\right)^m$$  \hspace{1cm} \text{for } a < 9.69 \text{ mm} \quad \text{(Eq. 16)}$$

For $\beta^2 a > 9.87$ mm, the crack growth rate is equal for both specimens, see Fig. 6b, and can be approximated by Eq. 16 only by changing the value of $m$ and therefore the slope. This allows for a very accurate determination of $m$, because it is not possible to shift the line by changing the constant $C$ (which is normally the case when Eq. 5 is used to fit linear crack growth rates on a logarithmic scale). Then:

$$\frac{da}{dN} = 3.09 \times 10^{-7} \left(\frac{\beta^2 a}{9.87}\right)^{1.285}$$  \hspace{1cm} \text{for } a \geq 9.69 \text{ mm} \quad \text{(Eq. 17)}$$

The entire FCGR curves for all specimens can be described by Eqs. 16 and 17, where $m$ varies between 1.43 and 1.81 for crack lengths smaller than 9.69 mm and $m = 1.285$ ($n = 2.57$) for crack length larger than 9.69 mm.

Fig. 7 shows the measured crack lengths and the model for 5 specimens. The crack length as a function of cycles for each specimen can be perfectly modelled by changing only one parameter that is characteristic for each specimen. For crack lengths smaller than 9.69 mm the value for $m$ is determined from the crack length vs. cycles ($a$-$N$) curve of each specimen. Fitting the $a$-$N$ curve is preferable to using the FCGR curve, which derives from the $a$-$N$ curve and results in increased scatter. At the pivot point the crack growth rate is $3.09 \times 10^{-7}$ m/cycle and is independent of the exponent $m$. Therefore, all model curves in Fig. 7 are continuous.

It is clear from Fig. 6 and Fig. 7 that the number cycles to failure increases if the power law exponent before the pivot point increases. Fig. 8 shows the cumulative distribution function for the exponent $m$. The distribution of the exponent $m$ for $a < 9.69$ mm can be described by a normal distribution with a mean of $m = 1.67$ and a standard deviation of 0.09.

The question remains whether it is valid to model the FCGR with two different exponents. There should be a physical reason behind it to make it valid and applicable for other specimens or materials. The fracture surfaces have four different regions of crack propagation. In the first region the crack propagates in the plane strain condition. The second corresponds to crack propagation under combined plane stress and plane strain. The third region corresponds to plane stress conditions and the fourth region is net section yielding of the remaining section.

For the tested specimens there was a gradual transition from plane strain to plane stress between 7-11 mm and 22-25 mm crack lengths. Fig. 9 shows the development of shear lips for specimens 1 and 26; and Fig. 10 shows the fracture surfaces of specimen 1 with markers to show the onset of the transition from plane strain to plane stress. The average onset of the transition was at $a = 9.7$ mm. This
corresponds excellently with the 9.69 mm crack length for the pivot point. Since it is evident from the basic $a-N$ data that the crack growth rates were different for crack lengths up to 9.69 mm (see Fig. 3), the decrease in the exponent of the $da/dN$ vs. $\Delta K_{\text{eff}}$ curve after the pivot point is not a result from, for example, fitting procedures. In fact, this correlation has already been observed in the 1960’s. Wilhem showed that the point of decrease in the exponent of the $da/dN$ vs. $\Delta K$ curve corresponded to the onset of the transition to plane stress conditions for 7075-T6 and 2024-T3 [31]. The transition onset occurred at a certain effective stress intensity factor and corresponding crack growth rate [31, 32].

It is not possible to obtain the correct shape of the crack length vs. cycles curves up to 9.69 mm by changing the crack growth constant $C$ (see Fig. 5); and so the only way to obtain the correct shape is to change the exponent $n$. By changing $n$ one would conventionally also have to change $C$ and its dimensions; but by using the mathematics of the pivot point the variation between specimens could be described by changing only one parameter, $n$.

Why the exponent is different for each specimen in the plane strain region is presently unknown. It should be noted that the pivot point used in this paper is merely a mathematical tool for calculating the crack growth rate. It is expected that the change in exponent is continuous between the plane strain and plane stress regions. The advantage of the mathematical concept of the pivot point (Eqs. 13 and 14) is that it is dimensionally correct and that it remains correct when the exponent is changed. It is possible that the effect of different exponents in plain strain condition is also present in other crack growth rate tests, but that the difference between the maximum and minimum exponent was small due to a small number of tests. In our case the number of specimens is 29, which results in a higher probability to get points in the tails of the normal distribution. The comparison of two extreme specimens allows comparing regions of similar and dissimilar FCGR.

3.1 Plastic zone size

At present it is still unclear what exactly causes shear lip formation. A simple mechanical explanation is that the process is initiated by a situation of plane stress at the specimen surface, which leads to maximum shear stress on planes inclined at 45° to the specimen surface [14]. Materials with face centred cubic or body centred cubic structures have many possible slip systems. There will almost always be a slip possibility near the direction of maximum shear stress and shear will form [14]. A situation of plane stress is considered a necessary, but not a sufficient condition [14]. It was found that shear lips in AA 5083 could be suppressed by making a small scratch along the crack growth direction [33]. Specimen with shear lips (without a scratch) and without shear lips showed the same $da/dN$ vs. $\Delta K$ result. Thus, the shear lip did not have an effect on $da/dN$ and could not be responsible for the change in exponent [14]. This showed that there has to be another mechanism responsible for the change in exponent [14]. It is expected that the mechanism responsible for the change in exponent is also triggered by the change in stress state, just as the formation of shear lips.

From linear elastic fracture mechanics (LEFM) it is known that there is a sharp increase in the crack resistance curve when the plane stress condition starts to evolve [34]. The crack resistance is mostly determined by plasticity, it is therefore expected that a change in the plastic zone volume results in a change in crack growth rate. Ritchie and Fleck et al. both have shown the general trend that the power law exponent decreases if the plasticity in a material increases [35, 36]. Serdyuk et al. measured the
plastic zone size (PZS) and FCGR of magnesium MA12 alloy in vacuum at room- and low temperature. The temperature reduction leads to a decrease in PZS and an increase in the exponent [37]. The decrease in PZS was partially caused by the change in yield stress due to the temperature drop. Therefore, a change in exponent is also expected if the volume of plasticity increases within a specimen of the same material. It is therefore expected that the change in PZS due to the transition from plane strain to plane stress is responsible for the change in the power law exponent.
4 Conclusions

Fractography and crack length measurements show that the power law exponent is higher and variable at crack lengths where crack growth is dominated by plane strain conditions (2-9.7 mm). The power law exponent decreases and is similar for all specimens after the transition to plane stress conditions (9.7-40.8 mm).

The mathematical concept of a pivot point allowed 1) to model crack growth with two different exponents using a dimensionally correct equation and 2) to model the crack growth variations in all specimens by varying only one parameter; the power law exponent for the plane strain condition.

Measurements from 29 specimens gave a broad range in number of cycles to failure and allowed a) comparing between regions of similar and dissimilar FCGR. The difference in the number of cycles to failure between specimens originates from a difference in FCGR in plane strain conditions. b) Determining the power law constant accurately, and subsequently the individual power law exponents for each specimen by fits to the crack length vs. cycles curves (this is preferable to obtaining fits from \( \frac{da}{dN} \) vs. \( \Delta K \) curves).

The decrease in FCGR after the transition to plane stress indicates an increase in the crack growth resistance curve. It is expected that the change in plastic zone size due to the transition to plane stress conditions is responsible for the decrease in the exponent for crack growth under plane stress conditions.
References

Figure 1: Crack length vs. cycles for five of the 29 specimens. Specimen 1 had the lowest cycles to failure (defined for all specimens as \( a = 40.8 \text{ mm} \)), which is the number of cycles to failure at 40.8 mm. Specimen 26 had one of the highest number of cycles to failure. Specimens 7, 12 and 24 are a selection with intermediate number of cycles to failure.

Figure 2: Number of cycles to failure vs. the specimen numbers: failure is defined to be at a crack length \( a = 40.8 \text{ mm} \).
Figure 3: Crack length vs. cycles for four of the 29 specimens. Up to $a = 10$ mm the curves have different shapes. Beyond $a \approx 10$ mm the curves are similar (the actual number is $a = 9.69$ mm as determined in Fig. 6).

Figure 4: Crack extension from 9.69 mm crack length. The relative cycles are the number of cycles after the crack has reached 9.69 mm crack length.
Figure 5: Crack length vs. cycles up to 9.69 mm for three specimens and some model predictions. The three specimen crack growth curves were modelled using a power law with a fixed exponent (of specimen 12) and different power law constants. The shapes of the curves for specimens 1 and 26 could not be modelled correctly using different constants; the crack growth curve for specimen 12 was, obviously, accurately modelled.
Figure 6: Crack growth rate vs. $\beta^2a (~ \Delta K^2)$ from a) 2 to 12 mm and b) 9.8 to 40.8 mm. The black lines in a) are visual best fit lines of the data (no fit) and give a pivot point at 9.87 mm and a crack growth rate of $3.09 \times 10^{-7}$ m/cycle. The black line in b) goes through the pivot point and only the slope is changed ($m$) to approximate the data for both specimens (no fit).
Figure 7: Crack length vs. cycles for five of the 29 specimens from 2 mm to a) 9.69 mm and b) 40.8 mm. The black dashed lines are from the model that is used (Eqs. 16 and 17), where the differences between the specimens are captured by changing the power law exponent, m.
Figure 8: Cumulative distribution function for the exponent m. The distribution of the exponent m for $a<9.69$ mm can be described by a normal distribution with a mean of $m=1.67$ and a standard deviation of 0.09.
Figure 9: Combined side and top views of the fracture surfaces of specimens a) 1 and b) 26. Shear lip development can be seen from both the side and top views.

Figure 10: Fracture surface of specimen 1 a) without and b) with white lines indicating the change in fracture surface angle.