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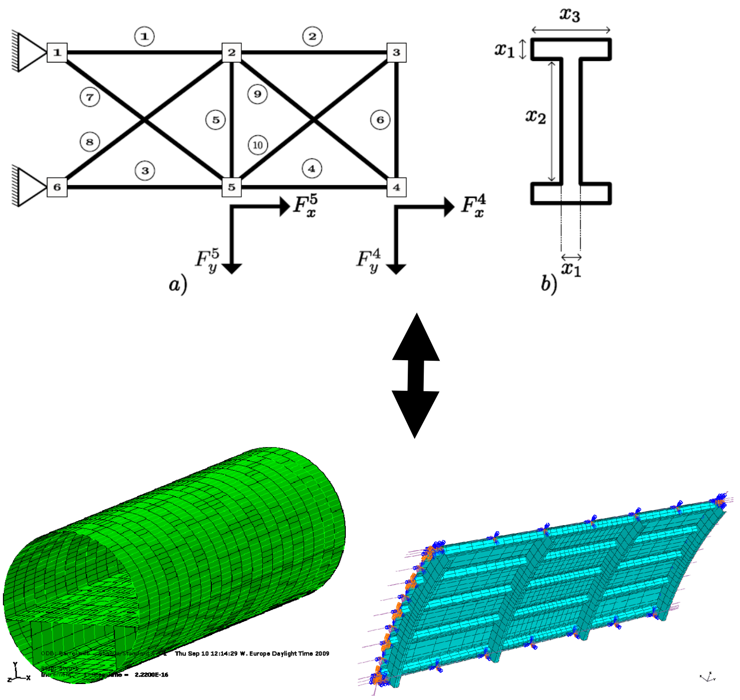
Executive summary

Assessment of optimisation strategies suited for application on composite fuselage panels



The ten-bar truss system (upper), used in structural optimisation test problems and as simple analogy to a fuselage barrel (lower).

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Problem area

International competition urges aeronautic industry in the Netherlands, as supplier for Airbus, to continuously enhance its performance in the engineering design process. The application of novel materials and innovative design methods is of key importance for the further reduction of design time and increased design confidence level.

Composite materials are increasingly used on business jets, regional and commercial aircraft. Composite materials provide higher stiffness and strength to density ratios than metallic ones. They permit for example the design of more integrated structures, with fewer fasteners. They are less prone to progressive damage under in-

This report is based on a presentation held at the fourth European Conference on Computational Mechanics (ECCM 2010), Paris, France, May 16-21, 2010.

service fatigue loads and are also less sensitive to corrosion. Therefore, composite solutions can deliver lighter structures with less maintenance.

The aim of the MAAXIMUS project (More Affordable Aircraft structure through eXtended, Integrated, & Mature nUmerical Sizing) is to demonstrate the fast development and right-first-time validation of a highly-optimised composite airframe. This will be achieved through co-ordinated developments on a physical platform, to develop and validate the appropriate composite technologies for low weight aircraft, and on a virtual platform, to identify faster and validate earlier the best solutions.

Description of work

As part of the virtual platform, a multi-level optimisation framework is developed for co-ordinated design optimisation of composite fuselage panels. Benefits of multi-level optimisation algorithms are expected for very large scale optimisation problems that may become infeasible for standard (single-level or all-in-one) optimisation algorithms.

In this paper we investigate the applicability and efficiency of various multi-level optimisation strategies for the fuselage barrel and panel design optimisation problem. This investigation is based, among others, on simplified test cases such as the commonly used ten-bar truss design optimisation problem. These test problems allow for relatively simple and quick implementation and to assess in detail the behaviour of the considered optimisation processes.

Results and conclusions

From literature surveys some of the most relevant multi-level optimisation algorithms were selected. These algorithms were assessed on their efficiency and accuracy. Two algorithms (Bi-Level Integrated System Synthesis - BLISS and Analytical Target Cascading - ATC-AL) were then selected for further developments and some additional assessments were done with the ten-bar truss test problem. These algorithms were shown to yield comparable results for these relatively simple test-problems as the standard (single-level or all-in-one) optimisation solutions. The total numbers of multi-level optimisation function evaluations were (sometimes much) higher than for all-in-one, but the system-level function evaluations were of the same order as in all-in-one.

Applicability

The multi-level optimisation algorithms investigated here will be further developed in order to be applied to the multi-level optimisation of composite fuselage barrels and panels in the Maaximus project.

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Assessment of optimisation strategies suited for application on composite fuselage panels

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Summary

The optimisation of large composite structures, for example an aircraft fuselage, involves a very high dimensional design space and numerous non-linear constraint functions. One approach to deal with such large optimisation problems is to decompose the overall problem into a number of smaller optimisation problems. These smaller optimisation problems typically consider a series of aspects or sub-systems in various levels of detail. This approach is referred to as multi-level optimisation.

In this study we investigate the applicability and efficiency of various multi-level optimisation strategies for the fuselage barrel-panel level design optimisation problem described above. This investigation is based, among others, on simplified test cases such as the commonly used ten-bar truss design optimisation problem. These test problems allow for relatively simple and quick implementation and to assess in detail the behaviour of the considered optimisation processes.

From literature surveys some of the most relevant multi-level optimisation algorithms were selected. These algorithms were assessed on their efficiency and accuracy. Two algorithms (BLISS and ATC-AL) were then selected for further developments and some additional assessments were done with the ten-bar truss test problem. These algorithms were shown to yield comparable results for these relatively simple test-problems as the standard (single-level or all-in-one) optimisation solutions. The total numbers of multi-level optimisation function evaluations were (sometimes much) higher than for all-in-one, but the system-level function evaluations were of the same order as in all-in-one.

Contents

Abbreviations

AiO	All-in-One
ATC	Analytical Target Cascading
ATC-AL	Analytical Target Cascading via Augmented Lagrangian relaxation
BLISS	Bi-Level Integrated System Synthesis
c	constraint function
CO	Collaborative Optimisation
CSSO	Concurrent Sub System Optimisation
dofs	degrees of freedom
F	external load
$f^{(i)}$	internal force of bar i
$\boldsymbol{f}$	vector of internal forces in equilibrium
GSE	Global Sensitivity Equations
I	the second moment of area
$L^{(i)}$	length of bar in the ten-bar truss problem
LC	loadcase
M	total mass
m^i	mass of bar i
MLO	multi-level optimisation
MLS	moving least squares
MoE	mixture of experts
OLD	Optimisation by Linear Decomposition
OSA	Optimum Sensitivity Analysis
P	External load
QSD	Quasi-separable Subsystem Decomposition
RSM	Response Surface Method
s_i	penalty parameter
$\boldsymbol{v}$	vector of input variables
WLS	weighted least squares
$\boldsymbol{x}$	vector of sub-system level sizing parameters
x	sub-system level sizing parameters
$\boldsymbol{y}$	vector of system level sizing parameters
y	system level sizing parameters
λ	Lagrange multiplier
ν	Poisson's ratio

1 Introduction

Composite materials are increasingly used on business jets, regional and commercial aircraft, representing for example up to 50% of the structural weight for the Airbus A350 XWB [1]. Due to their laminate nature and the wide range of possible fibre reinforcements, composite materials offer a huge range of design variables. Hence these new materials provide much extended design freedom, but also additional complications like more non-linear behaviour, to the design and development of structural components.

The optimisation of large composite structures, for example an aircraft fuselage, involves a very high dimensional design space and numerous non-linear constraint functions. One approach to deal with such large optimisation problems is to decompose the overall problem into a number of smaller optimisation problems. These smaller optimisation problems typically consider a series of aspects or sub-systems in various levels of detail. This approach is referred to as multi-level optimisation (MLO) [2].

Multi-level optimisation decomposes the initial standard (single-level or all-in-one(AiO)) optimisation problem into a hierarchy of smaller more manageable optimisation problems via introducing consistency constraints. A coordination technique is applied to coordinate the individual coupled optimisation problems to the solution of the all-in-one optimisation problem. In the past decades, various methods, such as simultaneous analysis and design (SAND), Concurrent Subspace Optimisation (CSSO) and Collaborative Optimisation (CO) amongst others, have been developed for the decomposition and coordination of multi-level optimisation applied to complex systems [3]. These methods originate predominantly from the field of Multidisciplinary Design Optimisation (MDO), where an intrinsic decomposition of the overall design problem is required due to the multiple specific disciplinary analyses that are applied.

In this study we focus on multi-level optimisation strategies for large scale design optimisation problems with application to the optimisation of an aircraft fuselage barrel. The design of such a large aircraft structure involves structural design analyses, typically by application of finite element method (FEM) models, of the aircraft fuselage barrel. A natural decomposition of the overall design optimisation problem into two levels exists in these analyses. These two levels are the whole fuselage barrel level and the level of the individual fuselage panels. Therefore we will focus in this study on multi-level optimisation strategies for two level optimisation problems, for example the multi-level optimisation method known as BLISS (Bi-level integrated system synthesis) [4]. The decomposition into two levels allows for fast analysis with relatively coarse models of the whole fuselage barrel, while much more detailed models are used for the panel level analyses. These detailed panel models for example may include specific composite material properties like lay-ups and fibre orientations and detailed geometric aspects of frames and stringers.

In this study we investigate the applicability and efficiency of various multi-level optimisation strategies for the fuselage barrel-panel level design optimisation problem described above. This investigation is based, among others, on simplified test cases such as the commonly used ten-bar truss design optimisation problem. These test problems allow for relatively simple and quick implementation and to assess in detail the behaviour of the considered optimisation processes.

This paper presents the assessments of the efficiency and accuracy of some of the most relevant multi-level optimisation algorithms, which were selected from literature. Two algorithms (BLISS and ATC-AL) were then selected for further developments and some additional assessments were done with the ten-bar truss test problem.

2 Overview of multi-level optimisation algorithms

Complex systems such as aircraft structures can typically be considered a hierarchy of individual coupled components. This hierarchy is reflected in the analysis techniques that are used to capture the behaviour of the structure. Consequently, a hierarchy of coupled analysis models is used that belongs to different disciplines, analyzes different physical phenomena or varies in capturing geometric detail of the structure. Optimisation of complex systems with embedded hierarchy is accomplished via so-called multi-level optimisation methods. Closely related are optimisation methods that consider the individual disciplines that are coupled, embedded within a complex system. The latter approaches are called multi-disciplinary optimisation methods.

Over the last decades a large number of methodologies have been published for the optimisation of complex systems that originated from the fields of multi-level optimisation or multi-disciplinary optimisation. A literature study [5] revealed six main stream approaches, namely Optimisation by Linear Decomposition (OLD) [6], Concurrent Sub System Optimisation (CSSO) [7], Collaborative Optimisation (CO) [8], Bi-Level Integrated System Synthesis (BLISS) [9], Analytical Target Cascading (ATC) [10] and Quasi-separable Subsystem Decomposition (QSD) [11]. These approaches differ in how coupled components and/or disciplines are temporarily decomposed and how the solution to the temporarily decoupled problem is coordinated. Although the various MLO approaches differ in their decomposition and solution methods, they share the potential of the main advantages of MLO algorithms [12]:

- the number of design variables on system-level may be (much) reduced, compared to AiO;
- the lower number of design variables on system-level allows for (much) cheaper system-level optimisation in MLO, compared to AiO, in particular if finite difference gradients are used in the optimisation;
- the number of dofs on system-level may be (much) reduced, compared to AiO (e.g. for complete composite fuselage model).

It should be noted that this paper is only a very brief overview of multi-level optimisation algorithms, and a more complete overview can be found in [12]. Also, not all of the MLO advantages mentioned above will be explicitly demonstrated in the test cases considered here.

3 Assessment of MLO schemes

To explore possibilities of applying an MLO scheme to a complex structural problem such as a composite panel within a fuselage barrel optimisation a representative structural test case based on the two-bar truss optimisation problem has been developed [12]. This test case is simple, however it captures the main essence of each individual MLO scheme. This test case was used to assess the different MLO schemes on the following criteria: the accuracy of the optimal solution obtained with respect to an all-in-one optimisation approach; the computational costs of each methodology to find a solution; the efforts required to implement each methodology into a software code.

The structural test-case of the two-bar truss optimisation problem is shown in Figure 1. The objective is to reduce the total mass of the system that is loaded by a horizontal force *P* in the top node, while taking into account constraints on maximum tip-displacement, buckling of the bars and stresses in the bars. Two levels can be distinguished within the problem shown in Figure 2. At the top-level two design parameters ($^0\mathbf{x} = [^0x_1\ ,\ ^0x_2]$) are used to describe overall dimensions of the two-bar truss system. At the second (lower) level for each bar *i* two design parameters ($^i\mathbf{x} = [^ix_1\ ,\ ^ix_2]$) describe the geometric lay-out (cross-sections) of the bars.

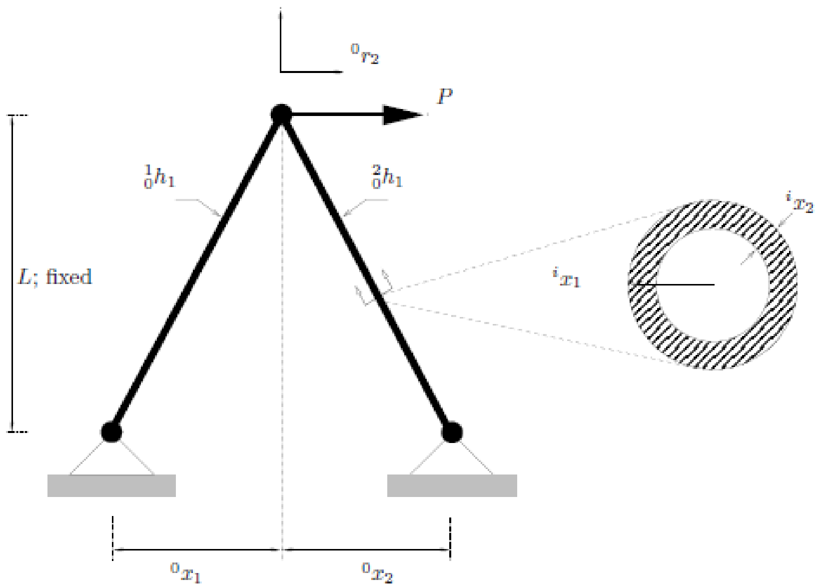

Figure 1: The two-bar truss optimisation problem. [12].

For the problem decomposition in the MLO approach we can consider the cross-sectional area of the bars, expressed as ${}^0y_i=\pi(({}^ix_1)^2 - ({}^ix_1-{}^ix_2)^2)$, as additional independent variables on system level, expressed by the $\boldsymbol{R}^2$ vector ${}^0\boldsymbol{y}=[{}^0y_1 , {}^0y_2]$. The mass of each bar depends only on the system level variables ${}^0\boldsymbol{x}$ and ${}^0\boldsymbol{y}$ and not explicitly on the bar design variables $[{}^1\boldsymbol{x}, {}^2\boldsymbol{x}]$. The forces in the bars (${}^if$) in static equilibrium of the system under the external force P depend on the stiffness of each bar, which is proportional to the bar cross-sectional area ${}^0y_i$. Hence the force vector $\boldsymbol{f}=[{}^1f , {}^2f]$ of the system level equilibrium (where only small displacements are assumed) can be fully expressed in the system level variables ${}^0\boldsymbol{x}$ and ${}^0\boldsymbol{y}$.

The two-bar truss optimization problem can be expressed as:

$$\min_{({}^0x,{}^0y)} M({}^0\boldsymbol{x},{}^0\boldsymbol{y}) = \sum_{i=1}^{2} {}^0m_i({}^0\boldsymbol{x},{}^0\boldsymbol{y}) \quad (1)$$

subject to the constraints:

$$c_{displ}(\boldsymbol{f}({}^0\boldsymbol{x},{}^0\boldsymbol{y})) < 0 \quad (2)$$

$$c_{stress}(\boldsymbol{f}({}^0\boldsymbol{x},{}^0\boldsymbol{y})) < 0 \quad (3)$$

$$c_{buckling}(\boldsymbol{f}({}^0\boldsymbol{x},{}^0\boldsymbol{y}),{}^0\boldsymbol{x},{}^1\boldsymbol{x},{}^2\boldsymbol{x}) < 0 \quad (4)$$

$${}^ic_{bound}({}^i\boldsymbol{x}) \le 0 \quad ; \quad \forall i \quad (5)$$

where ${}^0m_i$ is the mass of truss bar i and M is the total mass of the system (eq. 1). The horizontal displacement of the top node of the system, which depends on the system level force equilibrium, is constrained below a given value (eq. 2). Tension force in the bar is constrained such that stresses remain below 90% of the yield stress (eq. 3), and compressive force in the bar is constrained below 50% of the Euler buckling load (eq. 4). It should be noted that the Euler buckling load explicitly depends on the length and cross-section of the bars. The bounds on the bar design variables are explicitly expressed as a constraint function for each of the variables ${}^i\boldsymbol{x}$ (eq. 5).

Each of the MLO schemes considered in this study has been applied to the two-bar truss problem. Table 1 lists the accuracy of the solution found via each MLO scheme. The results show that all methodologies except Collaborative Optimisation are able to find the same solution as the all-in-one solution with reasonable accuracy. The small deviation of the final solution for BLISS and QSD lies in the use of surrogate models and deviations in ATC are due to the relaxation principle and corresponding numerical cut-off value.

Numerical costs were measured monitoring three different cost criteria:

1. The cumulative number of function evaluations required for each individual sub system. A single function evaluation corresponds to a single analysis of the sub system.
2. The cumulative number of optimisation iterations required for each individual sub system. A single optimisation iteration corresponds to a single Newton step (gradient based optimisation algorithm was used).
3. The total number of hierarchical updates is measured. A single hierarchical update corresponds to a complete update of coupling information within the entire hierarchy.

Table 1: MLO algorithms assessment results: Numerical results obtained for the two-bar truss test problem considering various solution methods (all-in-one (AiO) and MLO schemes). Gradient based optimisation algorithms were always used and sensitivities were evaluated via finite differences. (Details are given in [12]).

Method	Function evaluations	Optimisation Iterations	Hierarchical Updates	Solution Error	Objective value/ Objective value from AIO
AiO	285	95		0.0	1.00
OLD	1045	192	12	7E-6	1.00
OLD OSA	1035	331			
QSD	50	9	1	5E-2	1.02
QSD RSM	1369	272			
CSSO	1104	152	8	2E-1	1.03
CSSO GSE	48				
CSSO C	380	12			
BLISS	328	60	5	1E-1	1.01
BLISS RSM	1907	451			
CO	214	42	2	6E-1	3.19
ATC	2800	506	24	2E-1	1.01
ATC AL	7326	1674	47	2E-1	1.01

The results listed in Table 1 show that in terms of function evaluations the MLO schemes turned out to be expensive compared to the AiO optimisation. However, once surrogate models are available and can be re-used, QSD and BLISS are numerically more efficient then the AiO optimisation. In terms of optimisation iterations a similar trend is spotted. Finally, the number of hierarchical updates is of the same order of magnitude for most of the MLO scheme. A hierarchical update can be considered as a major change of the system equilibrium, and hence can be compared to the number of optimisation iterations in the AiO approach. As such, the number of these updates of system equilibrium that are needed to achieve the optimum configuration is shown to be significantly lower for most of the MLO methods as compared to the AiO approach. Moreover, the MLO approaches based on a relaxation of the consistency constraints were less efficient than approaches based on a strong formulation of the consistency constraints.

With respect to implementation effort of the individual MLO scheme relaxation based methodologies were found straight forward to implement. Methodologies that were based on a strong form of the consistency constraint require additional techniques such as optimum sensitivity analysis and surrogate models approximations. These additional techniques are not straightforward to implement and require a significant amount of additional programming effort.

Based on the results of the assessment study, some MLO schemes were selected to be considered for further developments. Because of its (relative) ease of implementation and because of the current wide interest within the MLO community ([13], [14], [15], [16]), ATC via Augmented Lagrangian (ATC-AL) relaxation is chosen to be further developed. In addition, because of the satisfying results in the assessment study with respect to finding the optimum, the numerical results obtained and results presented in recent literature ([17], [18], [19]) the BLISS scheme was chosen to be further developed.

4 The ten-bar truss optimisation problem

To further analyse the performance of the selected MLO algorithms we make use of a test problem based on a variant of the well-known structural optimization test problem of the ten-bar truss system (Figure 2).

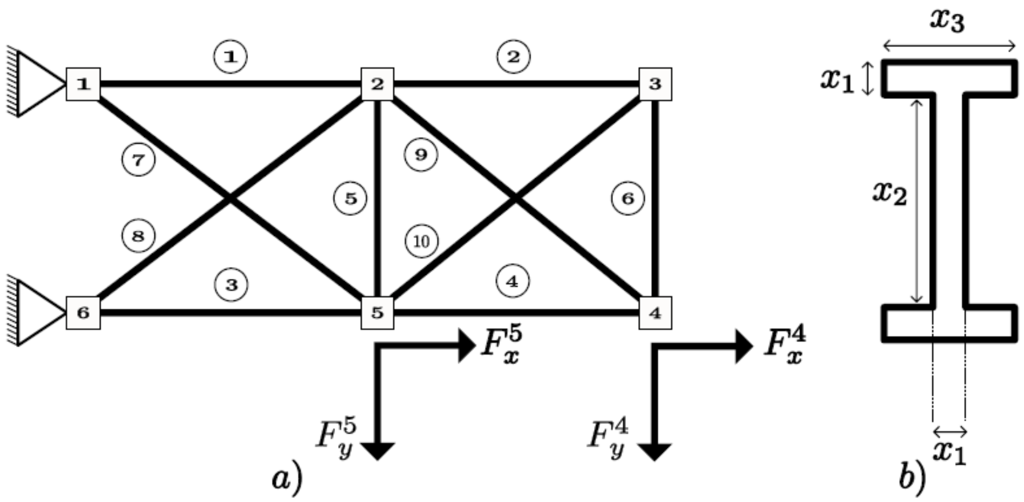

Figure 2: The ten-bar truss system (left) and the cross-section of each bar with the design variables (x1,x2,x3) of truss bar i.

The objective is to minimize the total mass of the system for given external forces ($F_x^4, F_y^4, F_x^5, F_y^5$), subject to constraints related to maximum stress allowables and Euler buckling and local web buckling for each of the bars, using the design variables in the $\boldsymbol{R}^3$ vector $\boldsymbol{x}^{(i)}=(x_1,x_2,x_3)^{(i)}$ of each of the 10 truss bars as independent variables.

In an AiO optimization approach, the mass minimization of the whole system under the given constraints, is directly performed in the 30-dimensional space of the design variables of all 10 bars.

In the MLO algorithms the problem is decomposed into separate and simplified optimizations on the level of the bar and on the level of the whole system. The advantage for this system is that the bar optimizations are performed in the 3-dimensional space of the bar design variables. The disadvantage here is that we need exchange of constraint information between the bar level and system level, which requires iterations of optimizations on the two levels.

For the problem decomposition in the MLO algorithms we consider the cross-sectional area $y^{(i)}=(x_1x_2+2x_1x_3)^{(i)}$ of each of the 10 bars as an additional set of independent variables, expressed by the $\boldsymbol{R}^{10}$ vector $\boldsymbol{y}$. The mass of each bar depends only on the bar area $y^{(i)}$ and not explicitly on the bar design variables $\boldsymbol{x}^{(i)}$. The forces in the bars ($f^{(i)}$) in static equilibrium of the system under the external forces depend on the stiffness of each bar, which is proportional to the bar cross-sectional area $y^{(i)}$. Hence the allowable tension and compression stresses in each of the bars can be expressed as the stress constraints given in eq. (10). Two types of buckling constraints are considered in this problem: Euler buckling and local web buckling, expressed as:

$$f^{(i)} > -F^{(i)}_{Euler}; F^{(i)}_{Euler} = \frac{\pi^2 E I^{(i)}}{(L^{(i)})^2} \tag{6}$$

$$f^{(i)} > -F^{(i)}_{web}; F^{(i)}_{web} = \frac{4\pi^2 E}{12(1-\upsilon^2)}(R^{(i)})^2 \tag{7}$$

where $L^{(i)}$ is the length, $I^{(i)}$ is the second moment of area and $R^{(i)}$ is the thickness-height ratio (x_1/x_2) of bar i and E is Young's modulus v is Poisson's ratio of the (linear elastic) material of the bars. These buckling constraints depend on the bar forces ($f^{(i)}$), but also have an explicit dependency on the bar design variables $\boldsymbol{x}^{(i)}$ and are therefore expressed as given in eq. (11).

The ten-bar truss optimization problem can therefore be formulated as a system level minimization expressed in $\boldsymbol{y}$ ($\in \boldsymbol{R}^{10}$):

$$\min_y M(\mathbf{y}) = \sum_{i=1}^{10} m^{(i)}(y^{(i)}) \tag{8}$$

subject to the constraints, expressed in $\boldsymbol{x}^{(i)}$ ($\in \boldsymbol{R}^3$) and $\boldsymbol{y}$:

$$c^{(i)}_{bound}(\boldsymbol{x}^{(i)}) \le 0 \quad ; \quad \forall i \tag{9}$$

$$c^{(i)}_{stress}(f^{(i)}(\mathbf{y})) < 0 \quad ; \quad \forall i \tag{10}$$

$$c^{(i)}_{buckling}(\boldsymbol{x}^{(i)}, f^{(i)}(\mathbf{y})) < 0 \quad ; \quad \forall i \tag{11}$$

where $m^{(i)}$ is the mass of truss bar i and M is the total mass of the system. The bounds on the bar design variables are explicitly expressed as a constraint function of $\boldsymbol{x}^{(i)}$ (eq. (9)).

4.1 ATC-AL algorithm

A formal description of ATC-AL (Analytical Target Cascading via Augmented Lagrangian relaxation) can be found in the work of Tosserams (2007) [15]. This section briefly describes the method applied to the ten-bar truss problem.

Decomposition of the ten-bar truss is accomplished according to the initial multi-level problem formulation. The global level (or system-level) optimisation problem is adapted to the ATC AL scheme by introducing a relaxation term in the objective and is mathematically expressed as:

$$\begin{aligned}
&\min_y M(\mathbf{y}) = \sum_{i=1}^{10} m^{(i)}(y^{(i)}) + \sum_{i=1}^{10} \varepsilon^{(i)}(y^{(i)}) \qquad (12)\\
&\textit{subject to}:\\
&c_{glob}^{K}(\mathbf{y}) \le 0\\
&\quad where \quad \varepsilon^{(i)}(y^{(i)}) = \lambda^T\left(y^{(i)} - y^{(i)}\left(\boldsymbol{x}^{(i)}\right)\right) + \left|s_i \circ \left(y^{(i)} - y^{(i)}\left(\boldsymbol{x}^{(i)}\right)\right)\right|_2^2 \quad for\ i = 1\ldots 10
\end{aligned}$$

(on the global level c^K_{glob} represents K global constraint functions.)
The corresponding local-level (or sub element level) problems are mathematically expressed as:

$$\begin{aligned}
&\min_{\boldsymbol{x}^{(i)}} \quad v^{(i)} = \varepsilon^{(i)}\left(\boldsymbol{x}^{(i)}\right) \qquad (13)\\
&\textit{subject to}:\\
&\quad c_{glob-loc}^{(i)}\left(\boldsymbol{x}^{(i)}\right) \le 0\\
&\quad c_{loc}^{(i)}\left(\boldsymbol{x}^{(i)}\right) \le 0\\
&\quad where \quad \varepsilon^{(i)}(y^{(i)}) = \lambda^T\left(y^{(i)} - y^{(i)}\left(\boldsymbol{x}^{(i)}\right)\right) + \left|s_i \circ \left(y^{(i)} - y^{(i)}\left(\boldsymbol{x}^{(i)}\right)\right)\right|_2^2 \quad for\ i = 1\ldots 10
\end{aligned}$$

Two additional parameters have been inserted into the above MLO formulation. Parameter λ_i represents the so-called Lagrange multiplier and s_i a penalty parameter. Parameters λ_i and s_i are determined in a separate step, here called the coordination step. After each individual global and lower element optimisation problem two convergence criteria are checked. The first checks if the individual objectives of the global and sub problems have changed with respect to their previous computed value. The current problem formulation corresponds to so-called hierarchic top-down decomposition. Consistency is formulated via the cross-sectional areas ($y^{(i)}$) of the truss bars.

Comparing the reference optimum solution and the results obtained via ATC-AL the following observations were made. The ATC-AL algorithm is able to find the same solution as the reference solution. However, numerical costs of the algorithm are much higher than those found via an AiO implementation. Therefore, efficient use of the method poses a challenge to the user. Suggestions for modifications to the algorithm are non-hierarchic decomposition and parallel execution via a Diagonal Quadratic Approximation. The effect of such changes are problem depended, see (De Wit and Van Keulen 2008 [20], 2010 [21]) and is outside the scope of the present study.

4.2 BLISS algorithm

In the BLISS algorithm we minimize on the bar-level the cross-sectional area ($y^{(i)}$) of the bar under the given constraints (eqs. (9,10,11)) in the $\boldsymbol{R}^3$ space of the bar design variables for a series of prescribed force values $f^{(i)*}$. This minimum bar area ($y^{(i)}_{min}(f^{(i)*})$) is driven by the constraints: either the bound, stress or buckling constraint is active in the minimum, as illustrated in the Figure 3 below. Obviously the buckling behaviour of the long (diagonal) bars is slightly different from the short bars.

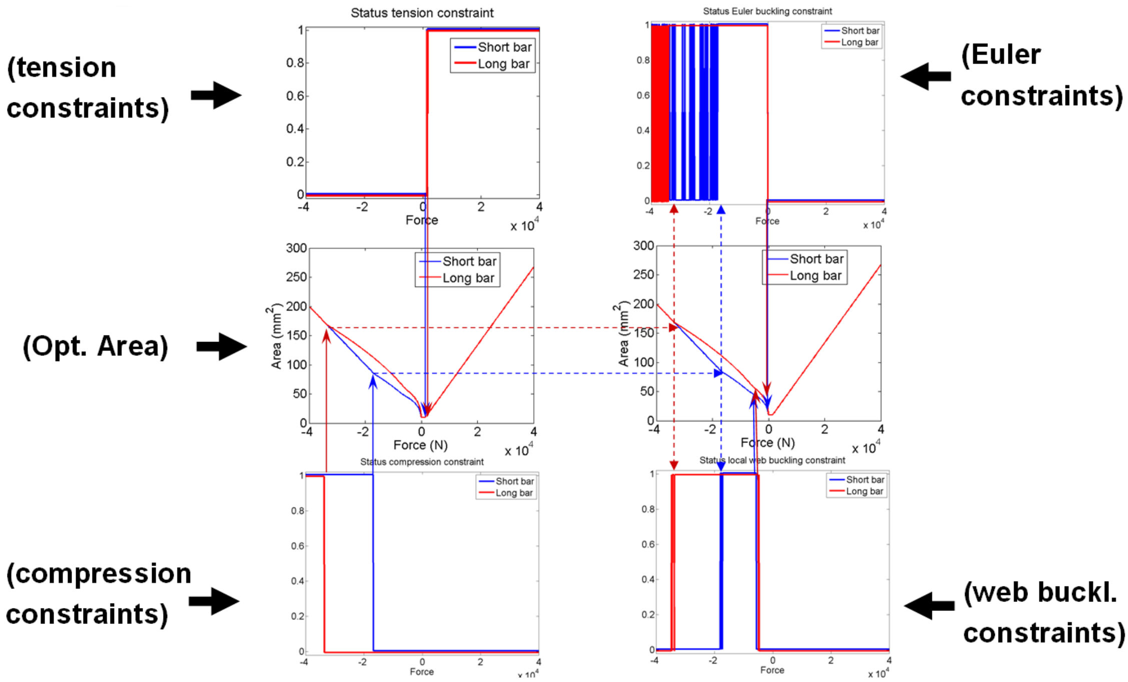

Figure 3: The optimal cross-sectional areas (2 middle graphs) of the long (diagonal) bars (red curves) and short bars (blue curves) as a function of prescribed bar force (y(i)min(f(i))) for the bars in the ten-bar truss problem. These optimal bar areas depend on the various constraints that are active at each force value (as illustrated by the 2 upper and 2 lower graphs).*

For very low tension force only the bound constraint is active (small horizontal part of the curves in the graphs), i.e., the minimum bar area is determined by the lower bounds of the bar design variables (x_1,x_2,x_3). For higher tension force only the stress constraints are active. For low compressive force the Euler buckling constraints are active and for slightly higher compressive force also the web buckling constraints become active. For further increased compressive force the stress constraints become active, while the Euler constraint is nearly active ("switching on and off"). For the long bars (i.e., the diagonal bars in the ten-bar truss system, indicated by the red lines in the graphs) the buckling constraints remain active until higher compressive force values than for the short bars (i.e., the horizontal and vertical bars in the ten-bar truss system, indicated by the blue lines in the graphs).

From the bar level optimization results of minimum (allowable) bar area values as a function of prescribed bar force ($y^{(i)}_{min}(f^{(i)*})$), we construct a surrogate model where the aim is to achieve optimal accuracy with as few as possible prescribed force sample points. Therefore we applied specific iterative local force sampling and various fitting methods to capture as good as possible the minimum (allowable) bar area (see Figure 4 below).

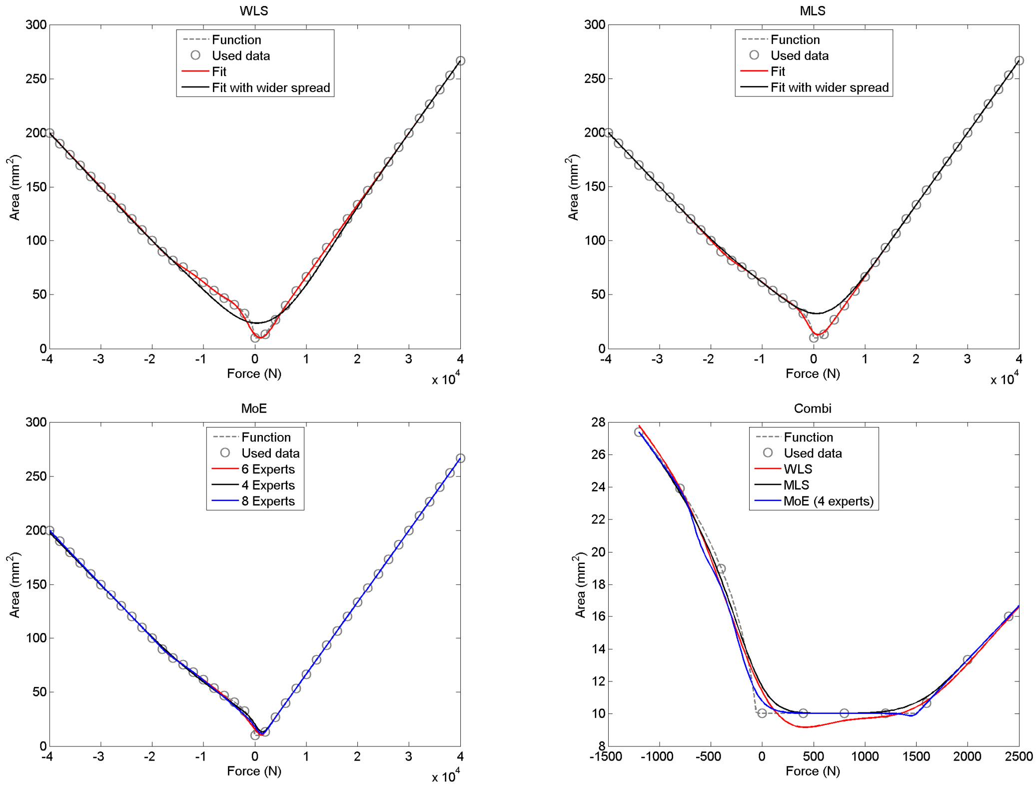

Figure 4: Illustration of the surrogate model accuracy for minimum bar area as function of prescribed bar force obtained with various fitting methods: Weighted Least Squares (WLS), Moving Least Squares (MLS) [22] (used here with different spread values), Mixtures of Experts (MoE) [23], and the results for each of these fits for a specific local force sample in the central part of the curve.

The MoE method appeared to provide the most accurate surrogate model, and was applied with a local sampling in 6 force values per bar. The surrogate model of the minimum bar area as a function of bar force that has been determined on the bar level is subsequently used on the system level as an in-equality constraint function in the minimization of the total mass, where the force in each of the bars is determined from the system equilibrium ($f^{(i)}(\mathbf{y})$):

$$y^{(i)} > y^{(i)}_{min}(f^{(i)}(\mathbf{y})) \tag{14}$$

A number of load-cases, using different external force vectors ($F_x^4, F_y^4, F_x^5, F_y^5$), was evaluated with both the AiO and the MLO approaches. In comparison with the AiO optimizations, the MLO methods yield similar values for the minimum total mass within 1% deviation from the AiO results. However, the computational efficiency, particularly in terms of function evaluations on the bar level, is lower for the MLO methods; see Table 2 below. But it should be noted that the bar evaluations involve only 2 degrees of freedom (dofs), whereas the system evaluations involve 8 dofs. All optimizations in the AiO and the MLO evaluations were run with the non-linear constrained minimization function (fmincon) of Matlab, where finite difference approximations of the gradients were used.

Table 2: Computational efficiency of the ATC-AL and BLISS algorithms compared to the AiO optimization for one load-case of the ten-bar truss problem.

Method	Approximate nr. of Function evaluations on System level / Subsystem level	Approximate nr. of Optimisation iterations on System level / Subsystem level
AiO	400 / not applicable	12 / not applicable
ATC-AL	1e4 / 1e4	1e3 / 2e3
BLISS	200 / 5e3	20 / 1e3

5 Conclusions

An assessment of the efficiency and accuracy of some of the most relevant multi-level optimisation algorithms, which were selected from literature, was performed on a simple test case based on the two-bar truss optimisation problem. Two algorithms (BLISS and ATC-AL) were selected for further developments and some additional assessments were done with the ten-bar truss test problem.

The MLO algorithms were shown to yield comparable results as the AiO solutions. The total numbers of MLO function evaluations were (sometimes much) higher than for AiO, but the system-level function evaluations were of the same order as in AiO.

Concerning the main advantages of MLO algorithms [12], it was demonstrated on a basic level with the presented test cases that:

- the number of design variables on system-level can be reduced in comparison to AiO;
- the lower number of design variables on system-level allows for cheaper system-level optimisation in MLO, compared to AiO, in particular if finite difference gradients are used in the optimisation;
- the number of dofs on system-level can be reduced in comparison to AiO.

Besides the implementation of these schemes (BLISS and ATC-AL), some specific developments of iterative surrogate modelling techniques were investigated.

The MLO algorithms considered in this study will be further generalised, and will be applied to the multi-level optimisation of composite aircraft fuselage barrel and panels.

References

[1] Engineering News online, 11 May 2009. Airbus to start manufacturing parts for new A350 XWB in late '09. K. Campbell, http://www.engineeringnews.co.za/article/airbus-to-start-manufacturing-parts-for-new-a350-xwb-in-late-09-2009-05-11 (accessed on 18-08-2009).

[2] Proceedings of the 5th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Optimization of Coupled Systems: A Critical Overview of Approaches, AIAA-94-4330-CP, R.J. Balling, J. Sobieszczanski-Sobieski, pp. 697 - 707, Panama City, Florida, September 7-9 1994.

[3] Proceedings of the 34th AIAA Aerospace Sciences Meeting and Exhibit, Multidisciplinary Aerospace Design Optimization: Survey of Recent Developments, AIAA 96-0711, J. Sobieszczanski-Sobieski, and R.T. Haftka, Reno, Nevada, January 15-18 1996.

[4] Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bi-level integrated system synthesis, J. Sobieszczanski-Sobieski, J.S. Agte, R.R. Sandusky, St. Louis, MO, Sept. 2-4, 1998, Collection of Technical Papers. Pt. 3 (A98-39701 10-31), AIAA-1998-4916.

[5] A.J. de Wit and A. van Keulen, Numerical Comparison of Multi-Level Optimization Techniques, 3rd AIAA Multidisciplinary Design Optimization Specialist Conference, Waikiki in Honolulu, Hawaii, 23-26 April, 2007.

[6] J. Sobieszczanski-Sobieski, B.B. James and A.R. Dovi, Structural Optimization by Multilevel Decomposition, American Institute of Aeronautics and Astronautics Journal, 23(11), pp. 124-142, 1985.

[7] J. Sobieszczanski-Sobieski, A step from hierarchic to non-hierarchic systems, 2nd NASA Air Force symposium on recent advances in multidisciplinary analysis and optimization, 1988.

[8] R.D. Braun and I. Kroo, Development and Application of the Collaborative Optimization Architecture in a Multidisciplinary Design environment, in: Multidisciplinary Design Optimization, State of the Art, N.M. Alexandrov and M.Y. Hussaini, eds., Philadelphia: SIAM, 1997.

[9] J. Sobieszczanski-Sobieski, T.D. Altus, M. Phillips and R. Sandusky, Bilevel Integrated System Synthesis for Concurrent and Distibuted Processing, American Institute of Aeronautics and Astronautics Journal, 41(10), pp. 1996-2003, 2003.

[10] H.M. Kim, N.F. Michelena, P.Y. Papalambros and T. Jiang, Target cascading in optimal system design, Journal of Mechanical Design, 125, pp. 474-480, 2003.

[11] R.T. Haftka and L.T. Watson, Multidisciplinary Design Optimization with Quasiseparable Subsystems, Optimization and Engineering, 6(1), pp. 9-20, 2005.

[12] A.J. de Wit, A unified approach towards decomposition and coordination for multi-level optimization, PhD thesis, University of Delft, Delft, the Netherlands, November 2009.

[13] H.M. Kim, W. Chen and M.M. Wiecek, Lagrangian Coordination for Enhancing the Convergence of Analytical Target Cascading, American Institute of Aeronautics and Astronautics Journal, 44(10), pp. 2197--2207, 2006.

[14] Y. Li, Z. Lu and J. Michalek, Diagonal Quadratic Approximation for Parallelization of Analytical Target Cascading, ASME Journal of Mechanical Design, 130(5), 2008.

[15] S. Tosserams, L.F.P. Etman and J.E. Rooda, Augmented Lagrangian coordination for distributed optimal design in MDO, International Journal for Numerical Methods in Engineering, 2007.

[16] S. Tosserams, M. Kokkolaras, L.F.P. Etman, and J.E. Rooda, A Non-Hierarchical Formulation of Analytical Target Cascading, ASME Journal of Mechanical Design, 2010.

[17] H.M. Kim, S. Ragon, G. Soremekun, B. Malone, and J. Sobieszczanski-Sobieski, Flexible Approximation Model Approach for Bi-Level Integrated System Synthesis, AIAA-2004-4545, 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Albany, New York, Aug. 30-1, 2004.

[18] J. Agte, A Tool for Application of Bi-Level Integrated System Synthesis to Multidisciplinary Design Optimization Problems, DGLR-2005-241, German Air and Space Congress, September 2005.

[19] S.I. Yi, J.K. Shin and G.J. Park, Comparison of MDO methods with mathematical examples, Structural Multidisciplinary Optimization, 35, pp. 391—402, 2008.

[20] A.J. de Wit and A. van Keulen, Framework for multilevel optimization, CJK-OSM5, 5th China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems, Jeju Island, South Korea, 16-19 April, 2008.

[21] A.J. de Wit and A. van Keulen, Framework for Multi-Level Optimization of Complex Structures, Lecture Notes in Applied and Computational Mechanics (LNACM), Springer, to be published, 2010.

[22] Nealen, A. An As-Short-As-Possible Introduction to the Least Squares, Weighted Least Squares an Moving Least Squares Methods for Scattered Data Approximation and Interpolation. TU Darmstadt. May 2004.

[23] Jordan, M.I. and Xu, L. Convergence results for the EM approach to mixtures of experts architectures. Neural Networks, 8, 1409-1431, 1995.