Research Objectives
Leveraged exchange-traded funds (ETFs) are relatively new to the world of investments but have become increasingly popular to aggressive investors. While a regular ETF tracks the value of a specific index of stocks, a leveraged ETF attempts to achieve a multiple of the return of the underlying index on a daily basis. This multiple can be positive in the case of bull ETFs or negative in the case of bear (or inverse) ETFs. To accomplish these objectives, leveraged and inverse funds pursue a range of investment strategies through the use of swaps, futures contracts, options and other derivative instruments. Due to the effect of compounding, operating expenses and daily resets, not to mention tracking errors, the performance of leveraged funds over longer periods of time can differ substantially from the performance of the underlying index. This study attempts to analyze the performance of the S&P 500 index as the underlying index. The S&P 500 index is essentially a large-cap index while the Russell 2000 is a small-cap index. We collect daily returns for these indices and ETFs from inception to year-end 2013.

This paper provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time. Despite the price decay associated with leveraged ETFs, their long-run performance.

Statistical Properties of Leveraged ETFs and the Underlying Index: A Summary
The underlying index (referred to as 1X) has an arithmetic average of Rₐ, a geometric average of R₉G, and standard deviation of σ. The leveraged ETFs are assumed to achieve exactly n times the return of the index on the daily basis, where n is positive for bull ETFs and negative for bear ETFs.

Regression Analyses
Another approach to access the tracking effectiveness is regress the daily return of an ETF on the daily return of its underlying index as follows:

\[ R_{LETF} = \beta \cdot R_{Index} + \epsilon \]

where \( R_{LETF} \) is the daily return of the LETF, and \( R_{Index} \) is the daily return of the index. The coefficient estimate, \( \beta \), can be used to test the hypothesis that the leveraged ETF achieves its stated objective, i.e., n times the underlying index's return.

The regression results, presented in the following table, show that the adjusted R² is very high and close to 1.0 in all regressions, indicating strong tracking effectiveness. Note that a high R² value of 1.0 indicates perfect tracking. Additionally, the estimated \( \beta \) is very close to its expected value.

Data and Sample
This paper examines two groups of leveraged ETFs which are offered from ProShares, the pioneer of leveraged ETFs. The first panel in the following table presents the five ETFs using the S&P 500 index as the underlying index. The second panel lists the five ETFs that are based on the Russell 2000 index. The S&P 500 index is essentially a large-cap index while the Russell 2000 is a small-cap index. We collect daily returns for these indices and ETFs from inception to year-end 2013.

Tracking Errors and Summary Statistics
The main objective of this study is examine how well the leveraged ETF (LETF) tracks its underlying index. For bull ETFs, tracking error is defined as \( (R_{LETF} - n \cdot R_{Index}) \). For bear ETFs, tracking error is defined as \( (R_{LETF} + n \cdot R_{Index}) \). The daily return of the LETF and \( R_{Index} \) is the daily return of the index, and \( n \) is the stated multiple.

For the S&P500-based LETFs, both the mean and median tracking errors are small. However, the minimum (0.0667) and the maximum (0.0586) suggest the existence of substantial tracking errors.

The results are similar for the Russell2000-based LETFs. Again, both the mean and median tracking errors are small. However, the minimum (0.06011) and the maximum (0.05407) suggest the existence of substantial tracking errors.

Price Decay and Long-Run Performance
The statistical properties of LETFs suggest a price decay. Consider the case of the 2X bull ETF, which aims to achieve twice the daily return of the underlying index on a daily basis. Suppose the underlying index (i.e., the S&P 500 index) has zero one-year total return. Due to the price decay caused by daily resets and volatility, the one-year total return for the bull ETF is -6.1%, even under perfect tracking.

The long-run performance is calculated as the cumulative daily return. The table below presents the year-end 2013 value of one dollar invested at the LETF at inception. For instance, one dollar invested in the UltraPro S&P500 (3X) bull ETF on June 23, 2009 would become $6.697 by the end of 2013, compared to only $2.177 if invested in the benchmark index fund. However, the same dollar would be only $0.041 if invested in the 3X bear ETF.

Summary
The study attempts to understand the statistical properties of leveraged and inverse ETFs. It also provides an empirical assessment of how well the leveraged ETFs track their underlying index. The results show that the tracking errors on average are small. However, substantial tracking errors do occur from time to time.

In conclusion, leveraged ETFs may be appropriate for aggressive investors who want to double or triple their short-term returns, but buy-and-hold investors must be warned of the long-run impacts of price decays.