Spatial Population Analysis: Methods and Computer Programs

Willekens, F. and Rogers, A.

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December 1978
SPATIAL POPULATION ANALYSIS:
METHODS AND COMPUTER PROGRAMS

Frans Willekens and Andrei Rogers

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November 1978

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International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria
Preface

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. During the past three years this interest has given rise to a concentrated research activity focusing on migration dynamics and settlement patterns. Four subtasks have formed the core of this research effort:

- the study of spatial population dynamics;
- the definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;
- the analysis and design of migration and settlement policy; and
- a comparative study of national migration and settlement patterns and policies.

This publication presents the computer programs that have been developed for use in the comparative study of migration and redistribution patterns in IIASA’s seventeen NMOs. Together with the special issue of the journal Environment and Planning, A, published as RR-78-06, and the collection of four journal articles, published as RR-78-13, it forms the Migration and Settlement Task’s series of final reports.

Andrei Rogers
Chairman
Human Settlements
and Services Area

November 1978
Summary

This report reviews the integrated methodology for spatial (or multi-regional) demographic analysis developed at IIASA, presents the FORTRAN IV codes of the computer programs, and includes a user's manual for implementing this methodology. Programs included are the multiregional life table; multiregional demographic projections; fertility and mobility analyses of both life table and stable populations; stable population analyses; the spatial reproductive value; and the analysis of alternative paths to spatial zero population growth. The focus of the report is on the interpretation of the output. A user's manual describes steps to be taken in the preparation of the data deck.
Acknowledgements

The development of these computer programs for spatial demographic analysis began at Northwestern University, Evanston, Illinois, in 1971. A number of former graduate students have collaborated in the project. In particular, we are indebted to Jacques Ledent, Richard Walz, and Richard Raquillet who wrote earlier versions of the programs.

The programs listed at the end of this report have been written at IIASA. We made intensive use of IIASA’s in-house computing facilities, a PDP-11/45, and benefited from some of the convenient features of the UNIX time-sharing system. We are most grateful to Computer Services, in particular to Jim Curry and Mark Pearson for their advice and assistance in solving our software problems.

Earlier versions of these computer programs were published as IIASA Research Memoranda RM-76-58 and RM-77-30. The numerous reactions to these reports were extremely helpful in preparing this volume. In particular, we acknowledge the detailed comments of Tom Carroll, Luis Castro, Jacques Ledent, William Orchard-Hayes, Dimiter Philipov, Richard Raquillet, Philip Rees, and Mahendra Shah.

The computer programs for spatial demographic analysis have been extensively used in the Comparative Migration Study, carried out jointly by IIASA’s Migration and Settlement Task and scholars in all of the seventeen IIASA National Member Organizations. The comments on the Research Memoranda and the suggestions of the contributors to the Comparative Migration and Settlement study helped us to restructure the report, add some new subroutines, and completely revise several of the previously published subroutines. More recent methodological innovations have also been introduced.

The manuscript was edited by Jeanne Anderer and typed by Margaret Leggett, who performed her task with great skill and managed to keep her good humor even when the final version was not really final.
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REFERENCES

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C1. Great Britain, 1970
C2. Hungary, 1974

PAPERS OF THE MIGRATION AND SETTLEMENT STUDY
There is a growing awareness among researchers, planners and governments that population growth should be viewed in its spatial dimension. Population declines in major central cities of the more developed world, continuing depopulations of rural areas in the less developed countries, and accelerating suburbanization everywhere have led governments to examine the desirability of population distribution policies.

A fundamental requirement for an effective policy regarding population redistribution is a well-developed understanding of spatial population dynamics. The basic mathematics of spatial demographic growth, recently the subject of study at the International Institute for Applied Systems Analysis (IIASA), has been elaborated as a set of FORTRAN computer programs to provide users with a ready tool for population analysis. These programs are being published in the hope that they may help researchers, students, planners, and policy makers to better understand the dynamic behavior of spatial demographic systems.

Although a number of publications of computer programs for population analysis and for operations research methods have guided us in our work, by far the most influential in this regard has been the book of Keyfitz and Flieger (1971). It has served as our basic reference. Other references were Arriaga (1976), Greenberg, Krueckeberg and Mautner (1973), and Land and Powell (1973).

This report consists of two parts. The first reviews the methodology of multiregional demography that is embodied in the programs. The emphasis, however, is not on methodology but on the interpretation of the output of the computer programs. The output consists of a set of tables, all of which are produced
directly by the computer and described in this part. The numerical illustrations refer to the same two-region system: Slovenia and the Rest of Yugoslavia. The demographic data on which the computations are based refer to the female population in the year 1961 and are given in Rogers (1975a). Data on two other multi-regional systems are presented in Appendix C.

The second part of this report clarifies our general approach to computer programming, gives a user-oriented description of the various subroutines and of the main program, and explains the format in which the input data must be provided. The FORTRAN listings of computer programs are presented in this part. A glossary of mathematical symbols and FORTRAN names of demographic variables used is given in Appendix A.
Part I

Methodology
PART I: METHODOLOGY

The dynamics of a multiregional population system are governed by its age-specific fertility, mortality and migration rates. These fundamental components of demographic analysis determine not only the growth of the population, but also its age composition, spatial distribution, and crude rates.

The observation that a particular combination of age-specific rates results in a unique age and regional composition has induced demographers to read into every population distribution a particular sequence of vital rates. "The demographic history of a population is inscribed in its age distribution" (Keyfitz, et al., 1967, p. 862). For example, an observed population distribution (population pyramid) may reflect periods of high fertility (baby boom) and high mortality (wars). A particularly useful way for understanding how the age and regional structure of a population is determined is to imagine a particular distribution as describing a population that has been subjected to constant fertility,
mortality and migration schedules for a prolonged period of time. The population that ultimately develops under such circumstances is called a stable multiregional population.

We also may view sequences of rates prospectively and derive the population distribution that would evolve if the actual observed schedules would remain unchanged for a prolonged period of time. This is the stable population associated with an observed demographic growth regime. The age-specific rates, of course, do not remain constant and therefore the stable population never will be realized. However, the stable population is a concept that enables one to look behind observed rates to explore what may be hidden in current patterns of fertility, mortality, and migration. It shows where the system is heading, in the long run, under current demographic forces. Keyfitz (1972, p. 347) compares stable population analyses to "microscopic examinations", because they magnify the effects of differences in current rates and therefore show more clearly their true meaning. Rogers (1971, p. 426) and Coale (1972, p. 52) compare them to "speedometer readings" to emphasize their monitoring function and hypothetical nature.

In addition to observed and stable population distributions that may be associated with observed fertility, mortality and migration schedules, demographers usually consider a third population distribution, namely, the distribution of the life table population. This stationary, or zero-growth, population describes the mortality and migration experience of a hypothetical population, with an equal number of births and deaths, that is subjected to the observed set of age-specific mortality and migration rates. The demographic picture shown by the life table, therefore, is the outcome of the observed mortality and migration schedules only and is not affected by the age composition and the regional distribution of the observed population. As in stable population theory, life table analysis enables one to separate out the effects of demographic behavior and of age and regional compositions. The latter act as weights.
The first half of this report consists of eight sections. The first section focuses on the observed population and derives several demographic measures directly from the data. Multiregional life table statistics are then computed. The multiregional demographic growth model and population projection to stability under constant schedules of fertility, mortality and migration are treated next. This produces the stable growth ratio and the age- and region-distribution of the stable population. Stable population analysis is considered further in the following sections and is complemented by additional life table population analysis. In particular, attention is devoted to fertility and mobility analysis in both stable and stationary populations. This brings in the application of the concept of spatial reproductive value, the calculation of the impact of alternative patterns of fertility reduction to replacement level on spatial population characteristics, and the evaluation of the "momentum" of spatial demographic growth.
1. OBSERVED POPULATION CHARACTERISTICS

The first outputs of this collection of computer programs for demographic analysis describe summary characteristics of the observed population. The data inputs are given in Table 1.1. Table 1.2 gives the percentage age distributions of the population, the parents at time of childbearing, deaths, and migrants. The mean age, in each instance, is defined as

\[ \bar{m}_i = \sum_{x} (x + \frac{NY}{2}) \cdot c_i(x) / 100 \]  

(1.1)

where \( c_i(x) \) is the percentage distribution, 
NY is the age interval\(^+\), and 
\( (x + \frac{NY}{2}) \) is the average of the interval.

The direct inputs to the life table program consist of observed age-specific rates (Table 1.3). Death rates are computed by dividing the annual number of deaths by the mid-year population in each age group. Fertility and migration rates are derived in a similar fashion. If death, birth, or migration data are not available on an annual basis, but are given for a five-year period, say, then the program reduces the data to an annual basis.\(^{++}\)

The population must in this case be the population at the mid-period. The sum of the age-specific rates multiplied by the age interval is called the gross rate. The gross fertility rate (gross reproduction rate) (GRR) of Slovenia is 1.1128. The gross migration rate (GMR) is derived in a similar way. The crude rate is the total number of births, deaths or outmigrants divided by the total mid-year population. For example, the crude birth rate of Slovenia is

\[ 0.017 = \frac{14,159}{832,800} \]

\(^{++}\)Annual data are obtained by dividing five-year data by five. This procedure is not a satisfactory one for migration data and should be used only as a first approximation.
Table 1.1. Observed population characteristics.

<table>
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<tr>
<th>region</th>
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</tr>
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<td>7100.</td>
</tr>
<tr>
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<td>-----------------</td>
<td>------------</td>
</tr>
<tr>
<td>slovenia</td>
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<tr>
<td>m.age</td>
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</tr>
<tr>
<td>m.age</td>
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- Table 1.2. Percentage distributions. -
Table 1.3. Observed rates.

### Death Rates

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**Gross** 2.710552 2.369808  
**Crude** 0.008159 0.008823  
**M. Age** 79.1635 74.4001

### Fertility Rates

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**Gross** 1.112808 1.357504  
**Crude** 0.017002 0.022012  
**M. Age** 27.7683 27.4740
Table 1.3. (cont’d)

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|                | migration from r.yugos. to total slovenia r.yugos. |
|                | gross      | crude   | m. age   |
| 0              | 0.002292   | 0.000000| 34.1303  |
| 5              | 0.000166   | 0.000000| 34.1303  |
| 10             | 0.000157   | 0.000000| 34.1303  |
| 15             | 0.000079   | 0.000000| 34.1303  |
| 20             | 0.000037   | 0.000000| 34.1303  |
| 25             | 0.000050   | 0.000000| 34.1303  |
| 30             | 0.000035   | 0.000000| 34.1303  |
| 35             | 0.000022   | 0.000000| 34.1303  |
| 40             | 0.000018   | 0.000000| 34.1303  |
| 45             | 0.000029   | 0.000000| 34.1303  |
| 50             | 0.000130   | 0.000000| 34.1303  |
| 55             | 0.000205   | 0.000000| 34.1303  |
| 60             | 0.000020   | 0.000000| 34.1303  |
| 65             | 0.000156   | 0.000000| 34.1303  |
| 70             | 0.000078   | 0.000000| 34.1303  |
| 75             | 0.000099   | 0.000000| 34.1303  |
| 80             | 0.000196   | 0.000000| 34.1303  |
| 85             | 0.000076   | 0.000000| 34.1303  |

**g.** gross
**c.** crude
**m.** m. age
The mean age given in Table 1.3 is the mean age of the schedule. The mean age of the fertility schedule of Slovenia, for example, is

\[
\bar{m}_1 = \frac{\sum_x (x + \frac{NY}{2}) F_1(x)}{\sum_x F_1(x)} = 27.77
\]

where the \( F_1(x) \) are age-specific fertility rates of Slovenia, and \( NY \) is five.

The mean age of the Slovenia to Rest-of-Yugoslavia migration schedule is 31.16 years. The mean age of the migrants is considerably less (25.04 years). This is due to the relatively young age composition of Slovenia's population. The age composition does not affect the migration schedule or its mean age.

Tables 1.4 and 1.5 repeat the basic data for each region, arranged in a different format and give the single-region life table for each region. The gross rates obtained are based on the regional schedules of fertility, mortality and migration only. The life table statistics, in particular the life expectancy at birth \([e(0)]\), depend only on the regional mortality schedule. The life expectancy is therefore the average number of years a person may expect to live if he remains in the region of birth during his whole lifetime (i.e., if the region is closed to outmigration). The net reproduction rate (NRR) is obtained as follows:

\[
NRR_i = \sum_x F_i(x) L_i(x)
\]

where \( F_i(x) \) is the regional fertility rate of age group \( x \) to \( x + 5 \), and \( L_i(x) \) is an element of the \( LL(x) \)-column of the single-region life table (number of years lived in age group \( x \) to \( x + 5 \)).

The net migraproduction rate (NMR) is determined analogously. NMR is the weighted sum of the age-specific outmigration rates, the weights being the elements of the \( LL(x) \)-column of the single-region life table. The national NMR (Table 1.6b) is called the Wilber-index (Wilber, 1963; Rogers, 1975b).
<table>
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<tr>
<th>Age</th>
<th>Population</th>
<th>Births</th>
<th>Deaths</th>
<th>Arrivals</th>
<th>Departures</th>
<th>Observed Rates (x 1000)</th>
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</thead>
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**Table 1.4a.**
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<th>( l(x) )</th>
<th>( d(x) )</th>
<th>( l(x)^\dagger )</th>
<th>( m(x) )</th>
<th>( s(x) )</th>
<th>( t(x) )</th>
<th>( e(x) )</th>
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</table>

**Table 1.4b.**

Net reproduction rate: 1.065615

Net migration rate: 0.172110

†Expressed in terms of unit radix (birth cohort of a single person).
Table 1.4b (continued)

| p(x): probability of survival from age x to age x + 5. |
| q(x): probability that an individual of age x dies before reaching age x + 5. |
| l(x): number surviving at exact age x, of 100,000 born. |
| d(x): number dying between ages x and x + 5, of 100,000 born. |
| ll(x): number of years lived between ages x and x + 5 per unit born. |
| m(x): age-specific death rate. |
| s(x): survivorship proportion: proportion of people x to x + 4 years old, that will survive to be x + 5 to x + 9 years old, 5 years later. |
| t(x): number of years expected to be lived beyond age x by a newly-born baby. |
| e(x): expectation of life at age x: number of years expected to be lived beyond age x by a person of age x. |
Table 1.5a.

<table>
<thead>
<tr>
<th>age</th>
<th>population</th>
<th>births</th>
<th>deaths</th>
<th>arrivals</th>
<th>departures</th>
<th>observed rates (x 1000)</th>
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<td>$ number - $</td>
<td>$ number - $</td>
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<th>d(x)</th>
<th>l(x)</th>
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net reproduction rate $1.177450$

net migraproduction rate $0.031050$
2. THE MULTIREGIONAL LIFE TABLE

The multiregional life table is a device for exhibiting the mortality and mobility history of an artificial population, called a cohort. Methods for constructing such a life table are treated in detail in Rogers (1975a, Chapter 3).

The cohort we deal with is a birth cohort, or radix. It represents a group of people born at the same moment in time and in the same region. Their life history is of special interest because it provides the necessary input information for numerical computations with multiregional demographic growth models. In multiregional demography, it is convenient to work with unit radices, i.e., birth cohorts of single persons. This allows a separation of the calculation of life table and other demographic statistics from the radix problem. Unless stated otherwise, the figures presented in this report will be per unit radix.

The computation of the multiregional life table begins with the estimation of age-specific death and outmigration probabilities. The probabilities are derived from observed schedules or rates of mortality and migration. The procedure is described at the end of this section. The probabilities of dying and outmigrating of the female population of the two-region system of Yugoslavia are given in Table 2.1. Note that they differ slightly from the probabilities presented in Rogers (1975a, p. 66), due to a small difference in the estimation method. As a consequence, all life-table statistics deviate slightly from those in Rogers (1975a). Probabilities and the two-region life table, consistent with Rogers', are given in Appendix B.

Probabilities of dying and migrating are the inputs for calculating life table statistics. The following statistics are computed by the program and are reviewed in the subsequent sections:

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2. number of survivors at exact age x,
Table 2.1. Probabilities of dying and outmigrating.

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</tr>
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<td>75</td>
<td>0.305390</td>
</tr>
<tr>
<td>80</td>
<td>0.436969</td>
</tr>
<tr>
<td>85</td>
<td>1.000000</td>
</tr>
</tbody>
</table>
3. number of years lived between two consecutive ages, or, the age composition of stationary population,

4. number of years lived beyond age x,

5. life expectancies by region of birth,

6. life expectancies by region of residence,

7. survivorship proportions.

2.1 Life Histories

The life histories of the hypothetical population are computed by applying the age-specific probabilities of dying and outmigrating to the regional radices. Any set of birth cohorts may be used. In this section, birth cohorts of 100,000 in each region of Slovenia and the Rest of Yugoslavia will be used.

We adopt the following notation:

\[ q_{i}(x) \]: the probability that a person in region i at exact age x dies before reaching age x + 5.

\[ p_{ij}(x) \]: the probability that a person in region i at exact age x will reside in region j at exact age x + 5.

\[ j_{0}^{i}(x) \]: the number of people in region i at exact age x, who are born in region j. Note that the radix or birth cohort of region j may be represented by \( j_{0}^{j}(0) \).

\[ j_{0}^{i} \delta(x) \]: the expected number of people alive in region i at exact age x, born in region j, who will die before reaching x + 5.

\[ j_{0}^{i} \kappa(x) \]: the expected number of migrants from i to k between ages x and x + 5 among the people living in i at age x and born in j.

\[ \dagger \text{A glossary of mathematical symbols and the associated FORTRAN names is given in Appendix A.} \]
The quantities $jO_{i}(x)$, $jO_{i\delta}(x)$ and $jO_{ik}(x)$ may also be expressed per unit born, i.e., for a cohort of a single person. They then may be interpreted as probabilities. For instance, $jO_{i}(x)$ is the probability that a $j$-born person is in region $i$ at exact age $x$, and $jO_{ik}(x)$ is the probability that a $j$-born person changes his residence from $i$ to $k$ between ages $x$ and $x + 5$. The relation between, for example, $jO_{i}(x)$ and $jO_{i\delta}(x)$ is straightforward:

$$jO_{i}(x) = jO_{i\delta}(x) \cdot jO_{i\delta}(0).$$  \hspace{1cm} (2.1)

The probability-interpretation will be particularly useful in fertility and mobility analyses of stationary and stable populations.

The life history of the cohorts is derived by the consecutive multiplication of the birth cohort by the mortality and migration probabilities. For example, of the 100,000 babies born in Slovenia (region 1), 3081 will die before they reach age 5, i.e.,

$$100,000 \times 0.030813 = 3081$$

$$10^{\delta}_{1}(0) \times q_{1}(0) = 10^{\delta}_{1\delta}(0),$$

and 1310 will move to the Rest of Yugoslavia (region 2),

$$100,000 \times 0.013103 = 1310$$

$$10^{\delta}_{1}(0) \times p_{12}(0) = 10^{\delta}_{12}(0).$$

The rest, i.e.,

$$100,000 - 3081 - 1310 = 95,608$$

or
remain in Slovenia, and are there at exact age 5. Therefore, of the females born in Slovenia, only 95.6\% will still be there 5 years later.

Of the 100,000 females born in Slovenia, 96,919 will still be alive at exact age 5. A total of 95,608 will still be in Slovenia and 1,310 will be in the Rest of Yugoslavia. Of these 95,608, the number of girls dying before reaching age 10 is

\[ 95,608 \times 0.002164 = 207 \]

\[ 10^{l_1(5)} \times q_1(5) = 10^{l_1(5)} \]

and the number migrating to the Rest of Yugoslavia is

\[ 95,608 \times 0.011370 = 1087 \]

\[ 10^{l_1(5)} \times p_{12}(5) = 10^{l_1(5)} \]

The residual is the number of girls who were in Slovenia at age 5 and are still there at age 10:

\[ 95,608 - 207 - 1087 = 94,314 \]

or

\[ 95,608 \times 0.986467 \]

\[ 10^{l_1(5)} \times p_{11}(5) = 10^{l_1(5)} \]

Note that $\hat{r}_{10}^{5}(5) = 10^{5}_{10}(5)/10^{0}_{1}(0) = 0.00207$ is the probability that a girl born in Slovenia dies in that region between ages 5 and 10. An analogous interpretation may be given to $\hat{r}_{12}^{5}(5)$ and $\hat{r}_{11}^{5}(5)$. Expressing the life histories per unit born yields a set of unconditional probabilities.

What happens to the 1310 migrants born in Slovenia, but who are in the Rest of Yugoslavia at exact age 5? They die, move back to Slovenia or stay in the Rest of Yugoslavia. If one assumes that the mortality and migration behavior depends on the region of residence at the beginning of the interval, then

$$1310 \times 0.003341 = 4$$

$$10^{2}_{2}(5) \times q_{2}(5) = 10^{2}_{20}(5)$$
girls die before reaching age 10, and

$$1310 \times 0.000821 = 1$$

$$10^{2}_{2}(5) \times p_{21}(5) = 10^{2}_{21}(5)$$
move back to Slovenia, while

$$1310 \times 0.995838 = 1305$$
remain in the Rest of Yugoslavia.

Pursuing this procedure until the last age group, we obtain a detailed description of the life history of the people born in Slovenia. The last age group is open-ended; therefore all people who reach age 85 are expected to die in that age group, i.e. $q_{i}(85) = 1.0$, and hence

+This is the Markovian assumption. It is a fundamental hypothesis underlying multiregional and other increment-decrement life tables.
Note that the total number of deaths is equal to the total number of births. For example, of the 100,000 babies born in Slovenia, 84,721 die in Slovenia and 15,279 die in the Rest of Yugoslavia.

An analogous procedure is followed to derive the life history of the females born in the Rest of Yugoslavia (Table 2.2).

### 2.2 Expected Number of Survivors at Exact Age x

Table 2.3 is an aggregation of Table 2.2. We noted earlier that of the 100,000 girls born in Slovenia, there are 1310 who at exact age 5 reside in the Rest of Yugoslavia. This number may also be found in Table 2.3. Of the people born in Slovenia and residing in the Rest of Yugoslavia at age 10, for example, some were there already at age 5 and stayed there, while others moved in from Slovenia, i.e.

\[
2392 = 1305 + 1087
\]

\[
10^{t_2}_2(10) = 10^{t_2}_2(5) + 10^{t_1}_2(5)
\]

where \(j^0_{t_i}(x)\) is the number of people in region i at exact age \(x\), who were born in region j. This expression is equivalent to:

\[
10^{t_2}_2(10) = 10^{t_2}_2(5)p_{22}(5) + 10^{t_1}_1(5)p_{12}(5)
\]

The total of 2392 is given in Table 2.3, its components may be found in Table 2.2.

Table 2.3 gives the number of people by place of birth and place of residence. Hence, it measures the age structure of the life table population, although only people at exact ages are considered. A more complete expression of the age structure is given in the next section.
Table 2.2. Life history of initial cohort.

<table>
<thead>
<tr>
<th>Region of cohort</th>
<th>Slovenia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
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<tr>
<td></td>
<td>deaths</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td></td>
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<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>total</td>
</tr>
</tbody>
</table>

2. Region of residence r.yugos.

<table>
<thead>
<tr>
<th>Region of cohort</th>
<th>Slovenia</th>
</tr>
</thead>
<tbody>
<tr>
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<td>age</td>
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<td>0</td>
</tr>
<tr>
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<td>35</td>
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<td>40</td>
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<td>55</td>
</tr>
<tr>
<td></td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>total</td>
</tr>
</tbody>
</table>
Table 2.2. (cont'd)

**initial region of cohort r.yugos.**

<table>
<thead>
<tr>
<th>age</th>
<th>deaths</th>
<th>migrants to slovenia</th>
<th>migrants to r.yugos.</th>
</tr>
</thead>
<tbody>
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<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
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<td>5</td>
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<td>124.</td>
<td>1.</td>
</tr>
<tr>
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<td>196.</td>
<td>1.</td>
</tr>
<tr>
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<td>1.</td>
<td>258.</td>
<td>7.</td>
</tr>
<tr>
<td>20</td>
<td>2.</td>
<td>532.</td>
<td>19.</td>
</tr>
<tr>
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<td>3.</td>
<td>905.</td>
<td>25.</td>
</tr>
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<td>30</td>
<td>6.</td>
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<td>21.</td>
</tr>
<tr>
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<td>6.</td>
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<td>11.</td>
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<td>13.</td>
<td>1300.</td>
<td>7.</td>
</tr>
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<td>24.</td>
<td>1348.</td>
<td>4.</td>
</tr>
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<td>36.</td>
<td>1346.</td>
<td>4.</td>
</tr>
<tr>
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<td>55.</td>
<td>1339.</td>
<td>4.</td>
</tr>
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</tr>
<tr>
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<td>1713.</td>
<td>14203.</td>
<td>126.</td>
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</table>

2.- region of residence r.yugos.

<table>
<thead>
<tr>
<th>age</th>
<th>deaths</th>
<th>migrants to slovenia</th>
<th>migrants to r.yugos.</th>
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</thead>
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<td>0</td>
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<td>126.</td>
<td>89242.</td>
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<td>69.</td>
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<td>18.</td>
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<td>18367.</td>
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<td>0.</td>
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<td>1839.</td>
<td>1242163.</td>
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</table>
Table 2.3. Expected number of survivors at exact age x in each region.

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort slovenia</th>
<th>r.yugos.</th>
<th>total slovenia r.yugos.</th>
</tr>
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<tbody>
<tr>
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<tr>
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<td>96707.  94316.  2392.</td>
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<td>96561.  93481.  3080.</td>
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<td>96305.  90880.  5425.</td>
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<td>95042.  82766.  12276.</td>
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<tr>
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<td>94409.  81552.  12857.</td>
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<td>93444.  80376.  13069.</td>
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<tr>
<td>85</td>
<td>18139.  14669.  3469.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort r.yugos.</th>
<th>total slovenia r.yugos.</th>
</tr>
</thead>
<tbody>
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<tr>
<td>85</td>
<td>18709.  342.  18367.</td>
<td></td>
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</tbody>
</table>
The computation of the expected number of survivors at exact age $x$ in a multiregional system is more conveniently performed using matrix notation. For our two-region example, let

$$
\ell(x) = \begin{pmatrix} 10^{\ell_1(x)} & 20^{\ell_1(x)} \\ 10^{\ell_2(x)} & 20^{\ell_2(x)} \end{pmatrix}
$$

Note that $\ell(0)$ is a diagonal matrix with the regional radices in the diagonal. The matrix analogue of equation (2.3) is then

$$
P(x) = \begin{pmatrix} p_{11}(x) & p_{21}(x) \\ p_{12}(x) & p_{22}(x) \end{pmatrix}
$$

For $x = 5$, we have

$$
\begin{pmatrix} 94,316 & 198 \\ 2,392 & 98,872 \end{pmatrix} = \begin{pmatrix} 0.986467 & 0.000821 \\ 0.011370 & 0.995838 \end{pmatrix} \begin{pmatrix} 95,608 & 126 \\ 1,310 & 89,242 \end{pmatrix}.
$$

As before, we may express the life history of the hypothetical population in terms of unit born. This yields a set of probabilities. For example, the probability that a person born in region $j$ be in region $i$ $x$ years later is simply $j_0^{\ell_i(x)} = j_0^{\ell_j(x)}/j_0^{\ell_j(0)}$, which is easily derived from Table 2.3. The probability of surviving to age $x$ is the product of conditional probabilities:
The probability of surviving from \( x \) to \( x + n \) is also easily computed from Table 2.3. It is equal to the product

\[
P(x + n - 5) \ p(x + n - 10) \ \ldots \ p(x) .
\]

It follows from (2.6) that

\[
\hat{P}(x + n) = P(x + n - 5) \ p(x + n - 10) \ \ldots \ p(x) \ \hat{\ell}(x) .
\]

Hence,

\[
P(x + n - 5) \ p(x + n - 10) \ \ldots \ p(x) = \hat{P}(x + n) \ \hat{\ell}^{-1}(x) .
\]

The probability that an individual in region \( i \) at age \( x \) will be in \( j \), \( n \) years later, is therefore given by

\[
\hat{P}(x + n) \ \hat{\ell}^{-1}(x) \text{ or } \hat{P}(x + n) \left[ \hat{\ell}(x) \right]^{-1}
\]

where the entries of \( \hat{\ell}(x + n) \) and \( \hat{\ell}(x) \) are found in Table 2.3 and \( \hat{\ell}(x + n) \) and \( \hat{\ell}(x) \) are the entries divided by the regional radices. For example, if one knows the distribution of people at the time they enter the labor force or marriage, age 20 say, and denote this by \{\( w(20) \)\}, then their distribution at retirement age, 60 say, is given by

\[
\{w(60)\} = \hat{\ell}(60) \ \hat{\ell}^{-1}(20) \{w(20)\}
\]

\[
= \begin{bmatrix} 73,251 & 1,416 \\ 12,474 & 75,008 \end{bmatrix} \begin{bmatrix} 90,880 & 553 \\ 5,426 & 87,922 \end{bmatrix}^{-1} \{w(20)\}
\]

\[
= \begin{bmatrix} 0.805532 & 0.005043 \\ 0.137338 & 0.852580 \end{bmatrix} \{w(20)\} .
\]
The probability that an individual in Slovenia at age 20 will be in the Rest of Yugoslavia at retirement age is quite high, almost one-seventh.

2.3 Duration of Residence and Age Composition of the Life Table Population

The knowledge of the probability that a person born in a given region survives to age $x$ and is then in another given region leads us to ask: how long will the person stay in that region? This duration-of-residence question may be answered for persons born in a given region and for persons living in a specific region at age $x$.

a. Duration of Residence by Place of Birth

The number of years individuals at age $x$ may expect to live in the next five years, on the average, is

$$L(x) = \int_{0}^{5} \hat{\ell}(x + t) dt \tag{2.9}$$

where in the two-region case

$$L(x) = \begin{bmatrix} 10L_1(x) & 20L_1(x) \\ 10L_2(x) & 20L_2(x) \end{bmatrix} \tag{2.10}$$

with $L_j(x)$ being the expected number of person-years lived in region $i$ between $x$ and $x + 5$, by an individual born in region $j$. It denotes the average duration of residence in region $i$ by a $j$-born person and depends on two components: (i) the probability of surviving to age $x$ and (ii) the average time spent in region $i$ in a 5-year interval by a person of age $x$ at the beginning of the interval.
The numerical approximation of (2.9) has given rise to a number of variants of life table construction (Keyfitz, 1968, p. 228). A simple approximation of $\hat{L}(x)$ is a linear combination of the probabilities of surviving to exact ages $x$ and $x + 5$:

$$\hat{L}(x) = 5[a \cdot \hat{L}(x) + (1 - a) \cdot \hat{L}(x + 5)]$$

In the computer program, $a$ is set equal to 0.5. Therefore,

$$\hat{L}(x) = \frac{5}{2}[\hat{L}(x) + \hat{L}(x + 5)]$$  \hspace{1cm} (2.11)

For example, $\hat{L}(10)$ given in Table 2.4 is computed from Table 2.3 as follows:

$$\hat{L}(10) = \frac{5}{2}[\hat{L}(10) + \hat{L}(15)] \hat{L}^{-1}(0)$$

$$\begin{pmatrix} 4.69491 & 0.01157 \\ 0.13681 & 4.43660 \end{pmatrix} = \frac{5}{2} \begin{pmatrix} 94,316 & 198 \\ 2,392 & 88,872 \end{pmatrix} + \begin{pmatrix} 93,481 & 265 \\ 3,080 & 88,592 \end{pmatrix} \begin{pmatrix} 100,000 & 0 \\ 0 & 100,000 \end{pmatrix}^{-1}.$$ 

The terminal age interval in a life table is a half-open interval: $z$ years and over. The probability of dying in this interval therefore is unity. Since the length of the interval is infinite, $\hat{L}(z + 5)$ is not available and (2.11) cannot be used to compute $\hat{L}(z)$. The number of years lived in the last age group is given by:

$$\hat{L}(z) = [M(z)]^{-1}\hat{L}(z)$$  \hspace{1cm} (2.12)

where $M(z)$ is a matrix with observed regional death and migration rates of the last age group (see Section 2.7).

The duration of residence or person-years-lived interpretation of $\hat{L}(x)$ is one of several possible perspectives. It also may be viewed as a measure of the age composition of the multiregional life table population. In this perspective, an element
Table 2.4. Number of years lived in each region by a unit birth cohort.

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>total</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>slovenia</td>
<td>r.yugos.</td>
</tr>
<tr>
<td>0</td>
<td>4.92297</td>
<td>4.89021</td>
<td>0.03276</td>
</tr>
<tr>
<td>5</td>
<td>4.84065</td>
<td>4.74810</td>
<td>0.09256</td>
</tr>
<tr>
<td>10</td>
<td>4.83172</td>
<td>4.69491</td>
<td>0.13681</td>
</tr>
<tr>
<td>15</td>
<td>4.82167</td>
<td>4.60903</td>
<td>0.21264</td>
</tr>
<tr>
<td>20</td>
<td>4.80588</td>
<td>4.45662</td>
<td>0.34926</td>
</tr>
<tr>
<td>25</td>
<td>4.78729</td>
<td>4.30303</td>
<td>0.48426</td>
</tr>
<tr>
<td>30</td>
<td>4.76510</td>
<td>4.18757</td>
<td>0.57753</td>
</tr>
<tr>
<td>35</td>
<td>4.73629</td>
<td>4.10796</td>
<td>0.62833</td>
</tr>
<tr>
<td>40</td>
<td>4.69634</td>
<td>4.04820</td>
<td>0.64814</td>
</tr>
<tr>
<td>45</td>
<td>4.63036</td>
<td>3.97803</td>
<td>0.65234</td>
</tr>
<tr>
<td>50</td>
<td>4.52751</td>
<td>3.88038</td>
<td>0.64713</td>
</tr>
<tr>
<td>55</td>
<td>4.37639</td>
<td>3.74302</td>
<td>0.63337</td>
</tr>
<tr>
<td>60</td>
<td>4.14608</td>
<td>3.54038</td>
<td>0.60569</td>
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<tr>
<td>65</td>
<td>3.76009</td>
<td>3.20367</td>
<td>0.55642</td>
</tr>
<tr>
<td>70</td>
<td>3.13922</td>
<td>2.66528</td>
<td>0.47394</td>
</tr>
<tr>
<td>75</td>
<td>2.32764</td>
<td>1.96490</td>
<td>0.36273</td>
</tr>
<tr>
<td>80</td>
<td>1.39903</td>
<td>1.16092</td>
<td>0.23810</td>
</tr>
<tr>
<td>85</td>
<td>0.96385</td>
<td>0.71663</td>
<td>0.24721</td>
</tr>
</tbody>
</table>
\( L_i(x) \) denotes the number of \( j \)-born people in region \( i \) of age \( x \) to \( x + 5 \), per unit born. The product \( \int_0^L_i(x) \times \int_0^L_j(0) \) is the total number of \( j \)-born people living in region \( i \) and \( x \) to \( x + 5 \) years old. Note that \( L(x) \) represents the relative population distribution by place of residence and place of birth. Instead of being expressed in percentages (fractions of the total), or in some other manner, the population is given in unit births. This is a logical procedure in demography since it separates the fertility component from the survivorship (mortality and migration) component. This will be seen to be a very convenient way of "scaling" in spatial population analysis.

b. Duration of Residence by Place of Residence

As mentioned above, the duration of residence in each region depends on two components: (i) the probability of surviving to age \( x \), and (ii) the average time spent in each region during the 5-year interval by a person of age \( x \) at the beginning of the interval. The latter component is the person-years lived between \( x \) and \( x + 5 \) by region of residence at age \( x \) and is equal to

\[
L_r(x) = L(x)[\hat{\gamma}(x)]^{-1}.
\]

Note that \( L_r(x) \) is a conditional measure, since it gives the duration of residence in each region between ages \( x \) and \( x + 5 \), given that the person reaches age \( x \) and is in a specific region at that time. Using the linear approximation of \( L(z) \) we may reduce this expression to

\[
L_r(x) = \frac{5}{2}[\hat{\gamma}(x + 5) + \hat{\gamma}(x)][\hat{\gamma}(x)]^{-1} = \frac{5}{2}[P(x) + I] \quad (2.13)
\]

The number of years lived in the last age group is

\[
L_r(z) = L(z)[\hat{\gamma}(z)]^{-1}.
\]
which is simply

\[ L_r(z) = \left[ M(z) \right]^{-1}. \]  

(2.14)

Numerical values for \( L_r(x) \) are given in Table 2.5.

2.4 Total Number of Years Lived Beyond Age \( x \)

The total number of years newly born babies may expect to live beyond age \( x \) is

\[ T(x) = \sum_{y=x}^{z} L(y) \]  

(2.15)

where \( z \) is the oldest age group. For example, the value of \( T(10) \) in Table 2.6 is

\[ T(10) = \begin{bmatrix} 55.26056 & 0.79946 \\ 7.45390 & 56.25090 \\ 62.71446 & 57.05036 \end{bmatrix}. \]

The number of years that a girl, just born in Slovenia, may expect to live beyond age 10 is 62.71. From this total, 55.26 years are expected to be lived in Slovenia and 7.45 years in the Rest of Yugoslavia. Similarly, a new-born Slovenian baby girl has \( T(60) \) or 15.74 years of retirement to look forward to, 2.48 years of which will be spent in the Rest of Yugoslavia.

2.5 Expectation of Life

The most important life table statistic is the life expectancy. The expectation of life at age \( x \) is the number of years an individual may expect to live beyond age \( x \), given that he
Table 2.5. Number of years lived in each region by a person of age x.

<table>
<thead>
<tr>
<th>age</th>
<th>region of residence at age x</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.92297</td>
<td>4.89021</td>
<td>0.03276</td>
</tr>
<tr>
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<td>4.99459</td>
<td>4.96617</td>
<td>0.02842</td>
</tr>
<tr>
<td>10</td>
<td>4.99628</td>
<td>4.97783</td>
<td>0.01845</td>
</tr>
<tr>
<td>15</td>
<td>4.99351</td>
<td>4.93017</td>
<td>0.00333</td>
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<tr>
<td>20</td>
<td>4.99057</td>
<td>4.92364</td>
<td>0.00776</td>
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<td>4.92364</td>
<td>0.06776</td>
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<td>30</td>
<td>4.98746</td>
<td>4.94131</td>
<td>0.04615</td>
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<tr>
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<td>4.98470</td>
<td>4.96293</td>
<td>0.02177</td>
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<tr>
<td>40</td>
<td>4.97604</td>
<td>4.96356</td>
<td>0.01247</td>
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<td>4.95577</td>
<td>4.94912</td>
<td>0.00665</td>
</tr>
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<td>4.92748</td>
<td>0.00803</td>
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<td>4.92249</td>
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</table>

<table>
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<th>r.yugos.</th>
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</thead>
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<td>0.00620</td>
</tr>
<tr>
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<td>4.46362</td>
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</tr>
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</tr>
<tr>
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<td>0.00260</td>
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</tbody>
</table>
Table 2.6. Total number of years lived beyond age x.

<table>
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<tr>
<th>age</th>
<th>initial region of cohort</th>
<th>slovenia</th>
<th>total slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
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<td></td>
<td></td>
</tr>
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<td></td>
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<tr>
<td>85</td>
<td>0.96385</td>
<td>0.71663</td>
<td>0.24721</td>
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</table>

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
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<td>0.61980</td>
</tr>
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<td>0.41928</td>
</tr>
<tr>
<td>55</td>
<td>18.23044</td>
<td>0.34967</td>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.08596</td>
</tr>
<tr>
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<td>2.59823</td>
<td>0.04323</td>
</tr>
<tr>
<td>85</td>
<td>1.29669</td>
<td>0.01717</td>
</tr>
</tbody>
</table>
reaches age $x$. In multiregional demography, two types of life expectancies may be distinguished: life expectancy by place of residence and life expectancy by place of birth.

a. Life Expectancy by Place of Residence

The life expectancy by place of residence gives the expectation of life at age $x$ of a person residing in a specific region at that age. It is computed as follows:

$$x_e(x) = \frac{T(x) [\hat{\ell}(x)]^{-1}}{\hat{\ell}(x)} = \frac{\sum_{y=x}^{\infty} L(y)[\hat{\ell}(x)]^{-1}}{\hat{\ell}(x)} , \quad (2.16)$$

where in the two-region case

$$x_e(x) = \begin{bmatrix} 1_x e_1(x) & 2_x e_1(x) \\ 1_x e_2(x) & 2_x e_2(x) \end{bmatrix} \quad (2.17)$$

and $i_x e_j(x)$ is the average number of years lived in region $j$ beyond age $x$ by an individual residing in region $i$ and $x$ years of age (whatever the region of birth). The life expectancy at each age except the first is higher than $T(x)$, since it is a conditional measure. Note that for the last age group $e(z) = L_r(z) = [M(z)]^{-1}$.

The expectations of life by place of residence for 10-year old girls, for example, are (Table 2.7)

$$e(10) = \frac{T(10) [\hat{\ell}(10)]^{-1}}{\hat{\ell}(10)} , \quad (2.18)$$

$$\begin{bmatrix} 58.57163 & 0.76931 \\ 6.29832 & 63.28025 \end{bmatrix} = \begin{bmatrix} 55.26056 & 0.79946 \\ 7.45390 & 56.25090 \end{bmatrix} \begin{bmatrix} 0.94316 & 0.00198 \\ 0.02392 & 0.88872 \end{bmatrix}^{-1}$$
Table 2.7. Expectations of life by place of residence.

<table>
<thead>
<tr>
<th>age</th>
<th>region of residence at age x slovenia</th>
<th>total slovenia r.yugos.</th>
<th><strong><strong>n</strong></strong>n*************************m</th>
<th>total slovenia r.yugos.</th>
</tr>
</thead>
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<td>0.81071 65.43480</td>
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<td></td>
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<td>0.21728 44.91151</td>
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<td>0.14591 40.35762</td>
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<td>0.03102 18.70902</td>
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<td>0.01671 15.15303</td>
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<td></td>
</tr>
<tr>
<td>70</td>
<td>10.97786 10.88414 0.09371</td>
<td>0.00869 12.05331</td>
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<td></td>
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<td>75</td>
<td>8.26969 8.20417 0.06552</td>
<td>0.00685 9.66764</td>
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<td></td>
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<tr>
<td>80</td>
<td>5.94569 5.90220 0.03648</td>
<td>0.00559 7.82416</td>
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<td></td>
</tr>
<tr>
<td>85</td>
<td>4.92249 4.88468 0.03781</td>
<td>0.00260 6.96564</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A 10-year old girl, living in Slovenia, may expect to live another 64.87 years. Of this, 6.30 years will be spent in the Rest of Yugoslavia, i.e. 10%. A girl of the same age in the Rest of Yugoslavia may expect to spend 0.77 years in Slovenia.

b. Life Expectancy by Place of Birth

This measure gives the expectation of life at age x by region of birth of the person. The region of residence at age x is not taken into account. Define the diagonal matrix \( \mathbf{\bar{T}}(x) \), with the elements of the vector \( \{1\}' \hat{T}(x) \) in the diagonal (\( \{1\}' \) is a row vector of ones). For the two-region case this gives

\[
\mathbf{\bar{T}}(x) = \begin{bmatrix}
\sum_i 10 \hat{T}_i(x) & 0 \\
0 & \sum_i 20 \hat{T}_i(x)
\end{bmatrix}.
\]  \tag{2.19}

The matrix of life expectancies by place of birth is obtained as follows:

\[
\mathbf{\bar{e}}(x) = \mathbf{T}(x) [\mathbf{\bar{T}}(x)]^{-1}.
\]  \tag{2.20}

Life expectancies of 10-year old girls are (Table 2.8)

\[
\mathbf{\bar{e}}(10) = \mathbf{T}(10) [\mathbf{\bar{T}}(10)]^{-1}
\]

\[
\begin{bmatrix}
57.14198 & 0.89757 \\
7.70768 & 63.15382
\end{bmatrix}
= 
\begin{bmatrix}
55.26056 & 0.79946 \\
7.45390 & 56.25090
\end{bmatrix}
\begin{bmatrix}
0.96707 & 0 \\
0 & 0.89070
\end{bmatrix}^{-1}
\]

\[
\begin{bmatrix}
64.84966 & 64.05138
\end{bmatrix}.
\]

A girl born in Slovenia may expect to live another 64.85 years, when reaching 10 years of age. Of this, 7.70 years will be spent
in the Rest of Yugoslavia, i.e. 12%. At age 65, however, 2.34 years of the future lifetime of 14.47 years will be spent in the Rest of Yugoslavia, i.e. 16% (Table 2.8).

It is the special feature of the multiregional life table that the demographic measure of the expectation of life is decomposed according to where that life is spent. It introduces the spatial dimension into classical demographic analysis.

2.6 Survivorship and Outmigration Proportions

A useful application of the multiregional life table is found in multiregional population projection. The assumption is that the survivorship and migration behavior exhibited by the stationary life table population adequately represents the survivorship and migration experience of the empirical population for which the life table was developed.

The necessary information for the projection of age groups beyond the first one is given by age-specific matrices of survivorship proportions. The number of people in age group \((x + 5, x + 10)\) in the stationary population is

\[
L(x + 5) = L(x) \tilde{S}(x)
\]

where, in the two-region case,

\[
\tilde{S}(x) = \begin{bmatrix}
    s_{11}(x) & s_{21}(x) \\
    s_{12}(x) & s_{22}(x)
\end{bmatrix}
\]

with \(s_{ij}(x)\) being the proportion of individuals aged \(x\) to \(x + 4\) who survive to be \(x + 5\) to \(x + 9\) years old 5 years later, by new places of residence.
Table 2.8. Expectations of life by place of birth.

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort Slovenia</th>
<th>total Slovenia r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
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<td>72.47807 64.89386 7.57922</td>
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<td>69.70287 61.91648 7.78638</td>
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<td>64.84966 57.14198 7.70768</td>
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<tr>
<td>15</td>
<td>59.94392 52.36627 7.57765</td>
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<tr>
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<td>55.09670 47.71969 7.37701</td>
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<tr>
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<td>50.30264 43.26082 7.04182</td>
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<tr>
<td>30</td>
<td>45.48670 38.92451 6.56219</td>
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</tr>
<tr>
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<td>40.72161 34.73123 5.99339</td>
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</tr>
<tr>
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<td>35.97794 30.61291 5.35503</td>
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</tr>
<tr>
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<td>31.32368 26.59686 4.72582</td>
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<td>26.84941 22.74720 4.10220</td>
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</tr>
<tr>
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<tr>
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<td>14.46598 12.12141 2.34457</td>
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<tr>
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<tr>
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<td>5.31378 3.95087 1.36291</td>
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</table>

<table>
<thead>
<tr>
<th>age</th>
<th>initial region of cohort r.yugos.</th>
</tr>
</thead>
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<tr>
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<td>49.76043 0.83042 48.93001</td>
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<td>40.50714 0.71560 39.79054</td>
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<tr>
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<td>22.73793 0.43613 22.30180</td>
</tr>
<tr>
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<td>18.73173 0.36551 18.36621</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>85</td>
<td>6.93036 0.09177 6.83909</td>
</tr>
</tbody>
</table>
For example, the number of people in the Rest of Yugoslavia at ages 15 to 19, who were born in Slovenia, per unit radix is (Tables 2.4 and 2.9)

\[ 10^{\begin{array}{l} L \end{array}}_{2}(15) = s_{12}(10)_{10}L_{1}(10) + s_{22}(10)_{10}L_{2}(10) \]

\[ 0.21264 = 0.01631 \times 4.69491 + 0.99460 \times 0.13681 \, . \]

The computation of \( L(x) \) in the life table is not performed using (2.21) but by (2.11). In (2.21), the unknown is \( S(x) \); therefore

\[ S(x) = L(x + 5)[L(x)]^{-1} \, . \]  

(2.23)

For \( x = 10 \) in the Yugoslavian example, \( S(x) \) is

\[
\begin{bmatrix}
0.98165 & 0.00205 \\
0.01631 & 0.99460
\end{bmatrix}
\begin{bmatrix}
4.60903 & 0.02046 \\
0.21264 & 4.41284
\end{bmatrix}
\begin{bmatrix}
4.69491 & 0.01157 \\
0.13681 & 0.43660
\end{bmatrix}^{-1}
\]

The number 0.01631, for instance, is the proportion of the girls residing in Slovenia and 10 to 14 years old that will be alive and in the Rest of Yugoslavia 5 years from now.

With the survivorship proportions, all the life table statistics are derived. They are summarized in Table 2.10.†

‡A summary table is produced by the computer for a system of two regions only.
Table 2.9. Survivorship proportions.

**region slovenia**

<table>
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<tr>
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<th>r.yugos.</th>
</tr>
</thead>
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<tr>
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**region r.yugos.**

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<th>total slovenia</th>
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Table 2.10. Multiregional (two-region) life table option 3.†

<table>
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<th>age</th>
<th>( q(x,1) )</th>
<th>( p(x,1,1) )</th>
<th>( p(x,1,2) )</th>
<th>( l(x,1) )</th>
<th>( l(x,2) )</th>
<th>( l(x,1,1) )</th>
<th>( l(x,1,2) )</th>
<th>( m(x,1,2) )</th>
<th>( m(x,1) )</th>
<th>( s(x,1,1) )</th>
<th>( s(x,1,2) )</th>
<th>( w(x,1,1) )</th>
<th>( w(x,1,2) )</th>
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<td>109.0</td>
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<td>0.000009</td>
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<td>0.000356</td>
<td>47026.0</td>
<td>1008.1</td>
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<td>0.04273</td>
<td>0.000009</td>
<td>0.680255</td>
<td>0.000411</td>
<td>9.47</td>
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<td>0.000560</td>
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<td>70.1</td>
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<td>0.000016</td>
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<td>7.66</td>
<td>0.13</td>
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</tr>
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<td>0.000000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
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<td>0.00000</td>
<td>6.84</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

†Column variables are defined on the next page.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(x,i)</td>
<td>probability that an individual at age x in region i will die before reaching age x + 5.</td>
</tr>
<tr>
<td>p(x,j,i)</td>
<td>probability that an individual at age x in region i will be in region j at age x + 5, i.e. 5 years later.</td>
</tr>
<tr>
<td>l(x,j,i)</td>
<td>number surviving at exact age x in region j, of 100,000 born in region i. This is also the probability that a baby born in region i, will survive and be in region j at exact age x, multiplied by 100,000.</td>
</tr>
<tr>
<td>l1(x,j,i)</td>
<td>total years lived between ages x to x + 5 in region j, per unit born in region i.</td>
</tr>
<tr>
<td>m(x,j,i)</td>
<td>age-specific migration rate from region i to j (equal to observed value).</td>
</tr>
<tr>
<td>md(x,i)</td>
<td>age-specific death rates in region i (equal to observed value).</td>
</tr>
<tr>
<td>s(x,j,i)</td>
<td>proportion of people in region i and aged x to x + 4 that will survive to be in region j and aged x + 5 to x + 9, five years later.</td>
</tr>
<tr>
<td>e(x,j,i)</td>
<td>part of expectation of life of i-born people at age x, that will be lived in region j, i.e. the average number of years lived in region j by i-born people, subsequent to age x.</td>
</tr>
</tbody>
</table>
2.7 Estimation of Age-Specific Outmigration and Death Probabilities

Life table probabilities are derived from observed annual age-specific rates of outmigration and death. The rates are computed by dividing the number of outmigrants or deaths in a certain age group by the mid-year population in that age group. Death and outmigration rates for Yugoslavia, are given in Table 1.3.

Starting from the observed rates, the probabilities of dying and outmigrating may be computed along two lines. The basic difference is the assumption about multiple transitions. Early formulations of the probability estimation procedure permitted no multiple transitions (Rogers, 1975a, p. 82) (Option 1). It was assumed that an individual only makes one move during a unit time period, five years say. Later formulations relaxed this assumption (Schoen, 1975; Rogers and Ledent, 1976) (Option 3). Option 2 is treated in Rogers (1975a, p. 85), but is not used in this report.

a. Estimation Under Option 3

This formulation begins by arranging the observed outmigration and death rates into the following matrix:

\[
M(x) = \begin{bmatrix}
M_{16}(x) + \sum_{j \neq 1} M_{ij}(x) & -M_{21}(x) & \cdots & -M_{n1}(x) \\
-M_{12}(x) & M_{26}(x) + \sum_{j \neq 2} M_{2j}(x) & \cdots & -M_{n2}(x) \\
& \ddots & \ddots & \ddots \\
& & \ddots & \ddots \\
& & & -M_{1n}(x) & -M_{2n}(x) & \cdots & M_{n6}(x) + \sum_{j \neq n} M_{nj}(x)
\end{bmatrix}
\]

where \( n \) is the number of regions;

\( M_{16}(x) \) is the age-specific mortality rate in region \( i \);

\( M_{ij}(x) \) is the age-specific migration rate from region \( i \) to region \( j \).
It can be shown that the probability matrix $P(x)$ is (Rogers and Ledent, 1976)

$$P(x) = [I + \frac{5}{2} M(x)]^{-1} [I - \frac{5}{2} M(x)]$$

(2.25)

where, for a two-region model,

$$P(x) = \begin{bmatrix} P_{11}(x) & P_{21}(x) \\ P_{12}(x) & P_{22}(x) \end{bmatrix}$$

with $p_{ij}(x)$ being the probability that an individual in region $i$ at exact age $x$ will survive and be in region $j$ five years later. The off-diagonal elements are migration probabilities analogous to transition probabilities in Markov theory. The diagonal element $p_{ii}(x)$ denotes the probability of surviving and remaining in (or returning to) region $i$. The elements of each column in $P(x)$ do not sum up to unity since the effects of mortality are included. Rather, $P(x)$ is analogous to the transition matrix of an absorbing Markov chain. Note than an element $p_{ij}(x)$ does not denote the probability of making a move from $i$ to $j$ by a person living in $i$ at the beginning of the transition period. What it represents is the probability that an individual in region $i$ at the beginning of the time period is in region $j$ at the beginning of the next period. During the period, several moves may have been made.

For example, the matrix of probabilities at age 10 is (Table 2.1)

$$P(10) = \begin{bmatrix} 0.991131 & 0.000781 \\ 0.007381 & 0.996834 \\ 0.998512 & 0.997615 \end{bmatrix}.$$
The probability that a female in Slovenia at age 10 will survive to age 15 is 0.998512. The probability that she will be in the Rest of Yugoslavia at age 15 is 0.007381.

The probabilities of dying are found by subtraction. The probability that an individual in region $i$ at age $x$ dies before reaching $x + 5$ is

$$q_i(x) = 1 - \sum_{j=1}^{n} p_{ij}.$$ \hspace{1cm} (2.26)

The probability of dying in the next five years for a 10 year old in Slovenia is (Table 2.1)

$$1 - 0.991131 - 0.007381 = 0.001487.$$ 

Note that (2.25) is analogous to the single region formula

$$p(x) = [1 + \frac{5}{2} M_0(x)]^{-1}[1 - \frac{5}{2} M_0(x)]$$

$$= \frac{1 - \frac{5}{2} M_0(x)}{1 + \frac{5}{2} M_0(x)}. \hspace{1cm} (2.27)$$

Formula (2.27) is equivalent to equation (1.1.9) of Keyfitz (1968, p. 14) and Keyfitz and Flieger (1971, p. 135). The probability of dying is then

$$q(x) = 1 - p(x) = \frac{5 M_0(x)}{1 + \frac{5}{2} M_0(x)}. \hspace{1cm} (2.28)$$

b. **Estimation Under Option 1**

On the assumption of no multiple transitions, the outmigration probability $p_{ij}(x)$ is given by (Rogers, 1975a, p. 82)
The probability of dying in region \( i \) is

\[ q_i(x) = \frac{5 M_{i0}(x)}{1 + \frac{5}{2} M_{i0}(x) + \frac{5}{2} \sum_{j \neq i} M_{ij}(x)} \]  

The probability of surviving and remaining in the region is found as a residual

\[ p_{ii}(x) = 1 - q_i(x) - \sum_{j \neq i} p_{ij}(x) \]  

Probabilities computed by this method are given in Appendix B as the first three columns of the two-region life table. The matrix of probabilities at age 10 for our example is

\[
\begin{bmatrix}
0.991129 & 0.000785 \\
0.007393 & 0.996831
\end{bmatrix}
\]

For a single region case, \( p_{ij}(x) = 0 \), and formula (2.25) reduces to (2.27). The distinction between multiple transitions and no multiple transitions is irrelevant in a single-region situation, since one can die only once.

The assumption of multiple versus no multiple transitions affects not only the probabilities directly, but also the person-years lived in the last open-ended age group. Recall (2.12),

\[ L(z) = M^{-1}(z) \hat{\chi}(z) \]  

Under the assumption of no multiple transitions, people cannot migrate and die during the same time-interval. Since all people
die in the last age group, the off-diagonal elements of $M(z)$ are zero and the diagonal consists of regional death rates. Hence

$$j_0^1 p_i(z) = j_0^1 p_i(z)/M_{i0}(z) \quad \text{(Rogers, 1975a, p. 64)}.$$ 

### 2.8 Aggregated Life Table Statistics

The life table statistics considered thus far refer to a multiregional system. The life table functions are basically matrix equations and give regional statistics. In order to aggregate the regional measures to yield the life table statistics for the whole system (country), regional weights must be introduced. The weights are the regional radices specified by the user.

The life table of the aggregate system is a single-region table derived from a set of age-specific mortality rates, which are computed as follows:

$$M_{i0}(x) = \frac{\hat{\ell}_i(x) - \hat{\ell}_i(x + 5)}{L_i(x)} \quad (2.32)$$

where

$$\hat{\ell}_i(x) = \sum_j RR_j^{(0)} \sum_l j_i l_i(x)$$

and

$$L_i(x) = \sum_j RR_j^{(0)} \sum_l j_i L_i(x)$$

with $RR_j^{(0)}$ the radix ratio in the life table or stationary population.

$$RR_j^{(0)} = \frac{\sum_i j_i^{(0)}}{\sum_k k_i^{(0)}} \quad (2.33)$$
The death rate of the last age group is

$$M_{y}(z) = \frac{\hat{i}_{y}(z)}{L_{y}(z)}.$$ 

Table 2.11 gives the aggregated life-table statistics for Yugoslavia. Equal radices are specified for both regions. In interpreting the results, one must keep in mind that unless regional radices are set in proportion to an estimate of appropriate life table births, the aggregated life table values will be incorrect. Setting all radices equal implies that regional births in the life table population are all equal in number. If in the observed population they are not, then obviously the life table statistics in the aggregated table are not realistic.

Note the difference between Table 2.11 and Table 1.6b. The latter is derived from a set of average national age-specific death rates. Regional differences are not accounted for and internal migration is not considered. Table 2.11, on the other hand, is aggregated from a multiregional life table, which explicitly considers regionally deviating mortality and migration. If mortality is the same in all regions, then Table 2.11 and Table 1.6b coincide.
<table>
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<th>age</th>
<th>p(x)</th>
<th>q(x)</th>
<th>l(x)</th>
<th>d(x)</th>
<th>l1(x)</th>
<th>m(x)</th>
<th>s(x)</th>
<th>t(x)</th>
<th>e(x)</th>
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<td>0.068566</td>
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<td>6857.</td>
<td>4.828535</td>
<td>0.014200</td>
<td>0.963181</td>
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<td>69.3618</td>
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<tr>
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<tr>
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</tr>
<tr>
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<td>0.004935</td>
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<td>455.</td>
<td>4.608102</td>
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<tr>
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<td>529.</td>
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<td>0.001154</td>
<td>0.993635</td>
<td>46.0069</td>
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</tr>
<tr>
<td>30</td>
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<td>0.006932</td>
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<tr>
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</table>

+Column variables are defined on page 16.
3. MULTIREGIONAL POPULATION PROJECTION

The population growth process has been represented by demographers as a matrix multiplication or, equivalently, as a system of linear, first-order, homogeneous difference equations with constant coefficients. This approach was used by Leslie in 1945 to project populations composed of a number of age groups. Rogers (1966) and also Feeney (1970) generalized Leslie's idea to include multiregional population systems.

The general matrix expression of the multiregional growth process is (Rogers, 1975a, pp. 122-123):

\[
\{K(t+1)\} = G\{K(t)\}
\] (3.1)

where \(\{K(t)\}\) is the age and regional distribution of the population at time \(t\),

\(G\) is the multiregional matrix growth operator or generalized Leslie matrix.

The vector \(\{K(t)\}\) is partitioned as follows:

\[
\{K(t)\} = \begin{bmatrix}
\{K(t)(0)\} \\
\{K(t)(5)\} \\
\vdots \\
\{K(t)(z)\}
\end{bmatrix}
\]

and \(\{K(t)(x)\}\) = \[
\begin{bmatrix}
K_1(t)(x) \\
K_2(t)(x) \\
\vdots \\
K_n(t)(x)
\end{bmatrix}
\] (3.2)

where \(K_i(t)(x)\) denotes the number of people in region \(i\) at time \(t\), who are \(x\) to \(x+4\) years of age, and \(\{K(t)(x)\}\) is the regional distribution of the population in age group \(x\) to \(x+4\).
3.1 The Growth Matrix

The arrangement of the growth matrix G is

\[
G = \begin{bmatrix}
0 & 0 & \tilde{B}(a - 5) & \ldots & \tilde{B}(\beta - 5) & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
S(0) & 0 & \tilde{S}(5) & \ldots & \tilde{S}(z - 5) & \ldots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \tilde{B}(\beta - 5) \\
\end{bmatrix}
\]

(3.3)

where \(a\) and \(\beta\) are the first and last ages of childbearing respectively. The matrix of survivorship proportions \(S(x)\) in a two-region model is

\[
S(x) = \begin{bmatrix}
s_{11}(x) & s_{21}(x) \\
s_{12}(x) & s_{22}(x)
\end{bmatrix}
\]

(3.4)

where \(s_{ij}(x)\) is the proportion of \(x\) to \((x + 4)\)-year-old residents of region \(i\) at time \(t\) who are alive and \((x + 5)\)-to \((x + 9)\)-years-old in region \(j\) five years later at time \(t + 1\). The survivorship matrix is computed as part of the life table equation (2.23):

\[
S(x) = L(x + 5)[L(x)]^{-1}
\]

or in terms of probabilities

\[
S(x) = [I + P(x + 5)]P(x)[I + P(x)]^{-1}
\]

(3.5)
Allowing multiple transitions, it may also be expressed directly in terms of the observed age-specific rates:

\[
S(x) = \left[ I + \frac{5}{2} M(x + 5) \right]^{-1} \left[ I - \frac{5}{2} M(x) \right]. \tag{3.6}
\]

The survivorship matrix for the next to last age group is, by (2.12),

\[
S(z - 5) = L(z) L^{-1}(z - 5) = \frac{2}{5} M^{-1}(z) P(z - 5) [I - \frac{5}{2} M(z - 5)]^{-1} \tag{3.7}
\]
or

\[
S(z - 5) = \frac{1}{5} M^{-1}(z) [I - \frac{5}{2} M(z - 5)] \tag{3.8}
\]

The first row of \( S \) is composed of matrices \( \hat{B}(x) \):

\[
\hat{B}(x) = \begin{bmatrix}
    b_{11}(x) & b_{21}(x) \\
    b_{12}(x) & b_{22}(x)
\end{bmatrix} \tag{3.9}
\]

where \( b_{ij}(x) \) is the average number of babies born during the unit time interval and alive in region \( j \) at the end of that interval, per \( x \) to \((x + 4)\)-year-old resident of region \( i \) at the beginning of that interval. The off-diagonal elements of \( \hat{B}(x) \) are measures of the mobility of children 0 to 4 years old, who were born to \( x \) to \((x + 4)\)-year-old parents.

It can be shown that \( \hat{B}(x) \) obeys the relationship (Rogers, 1975a, pp. 120-121):

\[
\hat{B}(x) = \frac{5}{2} L(0) \left[ 5 \hat{L}(0) \right]^{-1} [F(x) + F(x + 5) S(x)]
\]
whence

\[ B(x) = \frac{5}{4} \left[ P(0) + I \right] \left[ F(x) + F(x + 5)S(x) \right] \]  

(3.10)

since

\[ L(0) = \frac{5}{2} \left[ \hat{F}(5) + \hat{\lambda}(0) \right] = \frac{5}{2} \left[ P(0) + I \right] \hat{\lambda}(0) \]

where \( L(0), P(0), \) and \( S(x) \) are life table statistics, and \( \hat{\lambda}(0) \) is the identity matrix. \( F(x) \) is a diagonal matrix containing the annual regional birthrates of people aged \( x \) to \( x + 4 \). If multiple transitions are allowed, \( \left[ P(0) + I \right] \) may be replaced by \( 2\left[ \frac{5}{2}M(0) + I \right] \) and \( B(x) \) may be computed directly from the observed rates.

The number of births in year \( t \) from people aged \( x \) to \( x + 4 \) at \( t \) is \( F(x)\{K(t)(x)\} \). The number of births during a five year period starting at \( t \), among people aged \( x \) to \( x + 4 \) at \( t \), is

\[ \frac{5}{2} \left[ F(x)\{K(t)(x)\} + F(x + 5)\{K(t + 1)(x + 5)\} \right] \]

\[ = \frac{5}{2} \left[ F(x) + F(x + 5)S(x) \right]\{K(t)(x)\} \]

Of these births, a proportion \( L(0)\{S(0)\}^{-1} \) will be surviving in the various regions at the end of the time interval. The elements of the matrices \( B(x) \) and \( S(x) \) for Yugoslavia are given in Table 3.1. For example

\[
\begin{bmatrix}
0.321959 & 0.000802 \\
0.007623 & 0.381933
\end{bmatrix}
= \frac{5}{4}
\begin{bmatrix}
1.956084 & 0.001261 \\
0.013103 & 1.892421
\end{bmatrix}
\begin{bmatrix}
0.070652 & 0 \\
0 & 0.087978
\end{bmatrix}
+ \begin{bmatrix}
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0 & 0.074260
\end{bmatrix}
\begin{bmatrix}
0.965259 & 0.003541 \\
0.031117 & 0.989482
\end{bmatrix}
\]
Table 3.1. The discrete model of multiregional demographic growth.

**Multiregional projection matrix**

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**Age** | **Survivorship proportions** | Slovenia | R. Yugoslavia |
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3.2 The Projection Process

The demographic projection model is given by (3.1). Because of the special structure of the generalized Leslie matrix, (3.1) may be expressed in the form of two equation systems:

\[
\begin{align*}
\langle K(t+1)(0) \rangle &= \beta^{-5} \int_{a-5}^{b-5} B(x) \langle K(t)(x) \rangle, \\
\langle K(t+1)(x+5) \rangle &= S(x) \langle K(t)(x) \rangle,
\end{align*}
\]  

for $5 \leq x \leq z - 5$.

The age- and region-specific population is projected forward in time by the equation systems (3.11) and (3.12) using constant coefficients. The initial population is the observed base-year population. The projections are for unit time intervals of $NY$ years (five, say) that are equal to the age-interval (Table 3.2).

Projection should not be confused with forecasting. Forecasting requires the consideration of the effects that possible future events may have on the demographic parameters. The purpose of projecting the population with a constant growth matrix is to study the future impact of current patterns of behavior.

3.3 The Stable Equivalent Population

In the long run, the age and spatial distribution of a population is independent of the current distribution, and is uniquely determined by the schedules of fertility, mortality and migration represented in the growth matrix. Therefore, if one projects the population with a constant growth matrix for a long enough period of time, then the ultimate (stable) growth ratio and the ultimate (stable) distribution are independent of the current growth rate and population distribution. For constant growth matrices, we may write

\[
\langle K(t) \rangle = G^t \langle K(0) \rangle.
\]  

(3.13)
### Table 3.2. Multiregional population projection.

**Year 1961**

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Table 3.2 (cont'd)

**year 1966**

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### population

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In the limit, we have

\[
\{K^{(\infty)}\}_t = \lim_{t \to \infty} G^t \{K(0)\}_t ,
\]

(3.14)

with \(\{K^{(m)}\}\) denoting the stable population by age and region. This procedure for computing the stable population is equivalent to the power method for eigenvalue determination.

Once stability is achieved, the age by region composition of the population remains constant. All regions grow at the same constant ratio, \(\lambda\). (The stable growth ratio is the dominant eigenvalue of \(G\).) The relative stable distribution \(\{x\}\) is the characteristic vector associated with \(\lambda\), the dominant eigenvalue of \(G\). In other words, \(\{x\}\) is the solution of the following system:

\[
G\{x\}_r = \lambda\{x\}_r .
\]

(3.15)

The eigenvector \(\{x\}\) is unique up to a scalar; therefore we may choose \(\{x\}\) such that its elements sum up to unity, i.e.

\[
\{1\}'\{x\}_r = 1
\]

where \(\{1\}\) is a vector of ones.

The population for large values of \(t\), equal to \(n\), say, may be expressed as

\[
\{K^{(\infty)}\}_t = G^n \{K(0)\}_t = G^n Y\{x\}_r
\]

\[
= \lambda^n Y\{x\}_r = \lambda^n\{y\}_r .
\]

(3.16)

The scalar \(Y\) is called the stable equivalent of the observed population (Keyfitz, 1969; Rogers, 1975a, p. 38). It is the total population which, if distributed as the stable population, would increase at the same rate and lead toward the same population as would, in the long run, the observed population under projection (3.13).
From (3.16) it follows that

$$\{1, \ldots, n\} \cap \{k(0)\} = \lambda^{n} Y(1, \ldots, n) = \lambda^{n} Y$$

where

$$Y = \frac{1}{\lambda^{n}} \{1, \ldots, n\} \cap \{k(0)\} .$$

(3.17)

Table 3.3 shows that the stable equivalent population of Yugoslavia is 10.718 million people, 5.58% of which reside in Slovenia and 94.42% in the Rest of Yugoslavia. The age structure of the stable population is considerably older than that of the base year (1961) population. As a consequence, the stable growth rate \(r = 0.006099\) is about half of the average current growth rate.

The stable equivalent population represents the size, age composition and regional distribution of the 1961 population that would be consistent with the observed mortality, fertility and migration schedules. Hence the concept of stable equivalent population enables one to separate the projected population into two components:

i. that part of the change due to the fundamental demographic parameters (schedules), and

ii. that part of the change due to the age and regional structure of the base-year population.

For example, the part of the projected 1966 population that is due to the 1961 demographic schedules is \(e^{5r} Y\) or \(\lambda Y\), and the part due to the 1961 age and regional composition is

$$G(k(0)) - e^{5r} Y .$$

(3.18)

The percentage distribution is only one of the possible ways of expressing the age composition of the stable population. Another expression, which is particularly convenient for further analysis, is in terms of unit born, i.e. stable birth cohorts of a single person. This approach is analogous to the one followed in the
Table 3.3. Stable equivalent to original population.

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<td>43391</td>
<td>819138</td>
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<td>42253</td>
<td>792016</td>
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<td>41807</td>
<td>764748</td>
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<td>778349</td>
<td>42126</td>
<td>736223</td>
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<td>749839</td>
<td>41970</td>
<td>707869</td>
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<td>721442</td>
<td>41054</td>
<td>680388</td>
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<td>39995</td>
<td>653277</td>
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<td>38863</td>
<td>625504</td>
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<td>45</td>
<td>633742</td>
<td>37452</td>
<td>596290</td>
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<td>599301</td>
<td>35747</td>
<td>563554</td>
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<td>558830</td>
<td>33879</td>
<td>524951</td>
</tr>
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</tr>
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<td>65</td>
<td>441766</td>
<td>28070</td>
<td>413697</td>
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<td>22846</td>
<td>33246</td>
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<td>75</td>
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<td>16438</td>
<td>240945</td>
</tr>
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<td>80</td>
<td>159195</td>
<td>9515</td>
<td>149630</td>
</tr>
<tr>
<td>85</td>
<td>151509</td>
<td>5820</td>
<td>145690</td>
</tr>
<tr>
<td>total</td>
<td>10718246</td>
<td>597879</td>
<td>10120367</td>
</tr>
</tbody>
</table>

Percentage distribution:

<table>
<thead>
<tr>
<th>age</th>
<th>total</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.7865</td>
<td>7.5418</td>
<td>8.8601</td>
</tr>
<tr>
<td>5</td>
<td>8.0473</td>
<td>7.2574</td>
<td>8.0940</td>
</tr>
<tr>
<td>10</td>
<td>7.7836</td>
<td>7.0671</td>
<td>8.2860</td>
</tr>
<tr>
<td>15</td>
<td>7.5251</td>
<td>6.9926</td>
<td>7.5565</td>
</tr>
<tr>
<td>20</td>
<td>7.2619</td>
<td>7.0459</td>
<td>7.2747</td>
</tr>
<tr>
<td>25</td>
<td>6.9959</td>
<td>7.0193</td>
<td>6.9945</td>
</tr>
<tr>
<td>30</td>
<td>6.7310</td>
<td>6.8666</td>
<td>6.7230</td>
</tr>
<tr>
<td>35</td>
<td>6.4681</td>
<td>6.6895</td>
<td>6.4551</td>
</tr>
<tr>
<td>40</td>
<td>6.1985</td>
<td>6.5002</td>
<td>6.1836</td>
</tr>
<tr>
<td>45</td>
<td>5.9127</td>
<td>6.2642</td>
<td>5.8920</td>
</tr>
<tr>
<td>50</td>
<td>5.5914</td>
<td>5.9789</td>
<td>5.5635</td>
</tr>
<tr>
<td>55</td>
<td>5.2138</td>
<td>5.6665</td>
<td>5.1871</td>
</tr>
<tr>
<td>60</td>
<td>4.7400</td>
<td>5.2791</td>
<td>4.7032</td>
</tr>
<tr>
<td>65</td>
<td>4.1216</td>
<td>4.6949</td>
<td>4.0878</td>
</tr>
<tr>
<td>70</td>
<td>3.3223</td>
<td>3.8212</td>
<td>3.2928</td>
</tr>
<tr>
<td>75</td>
<td>2.4314</td>
<td>2.7494</td>
<td>2.3803</td>
</tr>
<tr>
<td>80</td>
<td>1.4853</td>
<td>1.5915</td>
<td>1.4790</td>
</tr>
<tr>
<td>85</td>
<td>1.4136</td>
<td>0.9734</td>
<td>1.4396</td>
</tr>
<tr>
<td>total</td>
<td>100.000</td>
<td>100.000</td>
<td>100.000</td>
</tr>
</tbody>
</table>

male  | 35.1363 | 36.7655 | 35.0406 |

female | 100.000 | 5.5781 | 94.4219 |

Lam  | 1.030967 | 1.030967 | 1.030967 |

r    | 0.006099 | 0.005099 | 0.006099 |
multiregional life table. Recall that \( \hat{L}(x) \) denotes the number of people of exact age \( x \) by place of birth and place of residence, and \( L(x) \) is the number of people in age group \( (x, x+5) \) by place of birth and residence. In both measures, the number of people is expressed in unit born. Analogously, we may define matrices \( \hat{L}(r)(x) \) and \( L(r)(x) \), representing respectively the number of people of exact age \( x \) and in age group \( (x, x+5) \) by place of birth and residence in a situation of stability.

The expression of the stable population in terms of unit born has an additional advantage; namely, its relation to the life table population. The stable population by place of birth and place of residence, per unit born, is given by

\[
\hat{L}(r)(x) = e^{-rx} \hat{L}(x) \quad (3.19)
\]

and

\[
L(r)(x) = e^{-r(x+2.5)}L(x) \quad ,
\]

where, for example, in the case of the two-region model,

\[
\begin{bmatrix}
10L_1^{(r)}(x) & 20L_1^{(r)}(x) \\
10L_2^{(r)}(x) & 20L_2^{(r)}(x)
\end{bmatrix}
= \begin{bmatrix}
e^{-r(x+2.5)} & e^{-r(x+2.5)} & 0 & 0 \\
e^{-r(x+2.5)} & 0 & e^{-r(x+2.5)} & 0 \\
e^{-r(x+2.5)} & 0 & 0 & e^{-r(x+2.5)}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
e^{-r(x+2.5)} & e^{-r(x+2.5)} & 0 & 0 \\
e^{-r(x+2.5)} & 0 & e^{-r(x+2.5)} & 0 \\
e^{-r(x+2.5)} & 0 & 0 & e^{-r(x+2.5)}
\end{bmatrix}
\]

(3.21)

where \( r \) is the annual growth rate of the stable population, i.e. the intrinsic growth rate. The rate \( r \) only depends on the observed schedules and is independent of the observed population distribution. It is computed as follows:

\[
r = \frac{1}{h} \ln \lambda
\]
with \( h \) being the age interval (5 years), and \( \lambda \) the dominant eigenvalue of the population growth matrix. The numerical evaluation of \( L(\tau)(x) \) for the system Slovenia – Rest of Yugoslavia is given in Table 3.4.

The absolute number of people in each age group by place of residence is

\[
K(x) = e^{-r(x+2.5)} L(x) \{Q\} ,
\]

where \( \{Q\} \) is the stable distribution of births, a variable defined in Section 7 of this report. Expression (3.22) is the numerical evaluation of the continuous formula

\[
K(x) = e^{-rX} L(x) \{Q\} .
\]

The numerical values of \( K(x) \) using (3.22) will be computed in Section 7. The results are comparable to Table 3.3.

At this point it is useful to stress that:

i. The life table population distribution is a special case of (3.19) and (3.20), i.e., with \( r = 0 \).

ii. Any stationary population, i.e. stable population with zero growth rate, is distributed according to a life table-population. Its relative distribution (in terms of unit births) is therefore independent of how fertility is reduced to replacement level (Table 3.5; Table 2.4).

iii. The column totals in Table 3.5 are the number of people in the life table population, per baby born. Adopting the "person-years lived" interpretation of \( L(x) \), the total would be the life expectancies at birth by place of birth and place of residence

\[
e(0) = \int \limits_x L(x) .
\]

For example, the total life expectancy of a baby girl
Table 3.4. Stable population (growth rate = 0.006099).

<table>
<thead>
<tr>
<th>initial region of cohort</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>57.857544</td>
<td>52.225361</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>initial region of cohort</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>52.936321</td>
</tr>
</tbody>
</table>
Table 3.5. Life table population.

<table>
<thead>
<tr>
<th>Initial region of cohort</th>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Slovenia</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4.922967</td>
<td>4.890209</td>
</tr>
<tr>
<td>5</td>
<td>4.840654</td>
<td>4.748097</td>
</tr>
<tr>
<td>10</td>
<td>4.837173</td>
<td>4.694913</td>
</tr>
<tr>
<td>15</td>
<td>4.821670</td>
<td>4.609031</td>
</tr>
<tr>
<td>20</td>
<td>4.805877</td>
<td>4.566222</td>
</tr>
<tr>
<td>25</td>
<td>4.787287</td>
<td>4.503030</td>
</tr>
<tr>
<td>30</td>
<td>4.765104</td>
<td>4.417570</td>
</tr>
<tr>
<td>35</td>
<td>4.736291</td>
<td>4.107964</td>
</tr>
<tr>
<td>40</td>
<td>4.696336</td>
<td>4.048200</td>
</tr>
<tr>
<td>45</td>
<td>4.630364</td>
<td>3.978028</td>
</tr>
<tr>
<td>50</td>
<td>4.527514</td>
<td>3.880381</td>
</tr>
<tr>
<td>55</td>
<td>4.376386</td>
<td>3.753020</td>
</tr>
<tr>
<td>60</td>
<td>4.146075</td>
<td>3.540382</td>
</tr>
<tr>
<td>65</td>
<td>3.760088</td>
<td>3.203669</td>
</tr>
<tr>
<td>70</td>
<td>3.139221</td>
<td>2.665276</td>
</tr>
<tr>
<td>75</td>
<td>2.327638</td>
<td>1.964904</td>
</tr>
<tr>
<td>80</td>
<td>1.390027</td>
<td>1.160924</td>
</tr>
<tr>
<td>85</td>
<td>0.963849</td>
<td>0.716635</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>72.478081</td>
<td>64.898865</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial region of cohort</th>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total R. Yugos.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4.734203</td>
<td>0.003152</td>
</tr>
<tr>
<td>5</td>
<td>4.460945</td>
<td>0.008093</td>
</tr>
<tr>
<td>10</td>
<td>4.448177</td>
<td>0.011574</td>
</tr>
<tr>
<td>15</td>
<td>4.433302</td>
<td>0.020462</td>
</tr>
<tr>
<td>20</td>
<td>4.410329</td>
<td>0.037170</td>
</tr>
<tr>
<td>25</td>
<td>4.379682</td>
<td>0.051365</td>
</tr>
<tr>
<td>30</td>
<td>4.343516</td>
<td>0.059095</td>
</tr>
<tr>
<td>35</td>
<td>4.302426</td>
<td>0.064056</td>
</tr>
<tr>
<td>40</td>
<td>4.249906</td>
<td>0.067394</td>
</tr>
<tr>
<td>45</td>
<td>4.178973</td>
<td>0.069071</td>
</tr>
<tr>
<td>50</td>
<td>4.073606</td>
<td>0.069602</td>
</tr>
<tr>
<td>55</td>
<td>3.914936</td>
<td>0.070336</td>
</tr>
<tr>
<td>60</td>
<td>3.667311</td>
<td>0.070187</td>
</tr>
<tr>
<td>65</td>
<td>3.285550</td>
<td>0.066386</td>
</tr>
<tr>
<td>70</td>
<td>2.729683</td>
<td>0.056808</td>
</tr>
<tr>
<td>75</td>
<td>2.034672</td>
<td>0.042728</td>
</tr>
<tr>
<td>80</td>
<td>1.301538</td>
<td>0.026060</td>
</tr>
<tr>
<td>85</td>
<td>1.296690</td>
<td>0.017170</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>86.245514</td>
<td>80.10706</td>
</tr>
</tbody>
</table>
in Slovenia is 72.48 years. A total of 64.90 years are expected to be lived in Slovenia and 7.57 years in the Rest of Yugoslavia.

iv. The column totals in Table 3.4 are the number of people in the stable population per baby born. If the growth rate \( r \) is positive, then the stable population is growing and the share of the births in the total population is greater than in the stationary population. Therefore, for \( r > 0 \)

\[
\sum_{x} e^{-r(x+2.5)} L(x) < \sum_{x} L(x)
\]

or

\[
e^{-r}(0) < e(0) .
\]  

(3.25)

For example, for each baby born in Slovenia, there are 57.86 persons living in Yugoslavia who were born in Slovenia. Of these, 52.23 are living in Slovenia and 5.63 in the Rest of Yugoslavia. Analogous to the expectation of life at birth-interpretation of \( e(0) \), the matrix \( e^{(r)}(0) \) may be considered as the discounted life expectancy matrix, with \( r \) being the rate of discount (Willekens, 1977). The meaning and relevance of this interpretation will be discussed in Section 6.

v. The stable equivalent number of births may be obtained from equation (3.22):

\[
\{Q\} = [L^{(r)}(x)]^{-1}\{K(x)\}
\]

where the quantities \( \{K(x)\} \) and \( L^{(r)}(x) \) are given in Tables 3.3 and 3.4 respectively. For example, the relation between the number of births and the number of people in the first age group is \( \{Q\} = [L^{(r)}(0)]^{-1}\{K(0)\} \).
Equivalently, \( \{Q\} \) may be derived by means of the following expression:

\[
\{Y\} = \bigg\{ \sum_{x} L(x) \bigg\}_{\{Q\}} = \bigg\{ \sum_{x} L^{(r)}(x) \bigg\}_{\{Q\}} .
\]

Therefore

\[
\{Q\} = \bigg[ \sum_{x} L^{(r)}(x) \bigg]^{-1}\{Y\} .
\]

\[
= [e^{(r)}(0)]^{-1}\{Y\} .
\]

In our two-region illustration, \( \{Q\} \) is

\[
\begin{bmatrix}
52.225361 & 0.599859 \\
5.632182 & 52.336464 \\
\end{bmatrix}
\begin{bmatrix}
597879 \\
10120367 \\
\end{bmatrix}
= \begin{bmatrix}
9235 \\
192379 \\
\end{bmatrix} .
\]

An alternative procedure to compute the stable equivalent number of births will be presented in Section 7 of this report.

The age distribution in terms of unit born is fundamental to further demographic analysis. Fertility analysis is performed by applying age-specific fertility rates to the life table and stable age distributions. For mobility analysis, age-specific outmigration rates are used instead.
4. FERTILITY ANALYSIS

The analysis of fertility begins with the application of the fertility schedule to an age distribution. Let the diagonal matrix \( m(x) \) contain the annual regional fertility rates of women at exact age \( x \), and let \( F(x) \) be the diagonal matrix of annual regional fertility rates of age group \( x \) to \( x + 4 \), e.g., in a two-region model,

\[
F(x) = \begin{bmatrix}
F_1(x) & 0 \\
0 & F_2(x)
\end{bmatrix}.
\]

(4.1)

The integral of the matrices of the age-specific fertility rates over all ages is the gross reproduction rate matrix,

\[
GRR = \int_0^\omega m(x)dx = 5 \sum_x F(x).
\]

The GRR-matrix is a diagonal matrix with the regional gross rates of reproduction as its elements. Age-specific fertility rates for Slovenia and the Rest of Yugoslavia are given in Table 1.3 and repeated in Table 4.1. The column totals denote the regional gross reproduction rates.

Regional crude birth rates may be derived by multiplying the age-specific fertility rates by the observed population distribution, in fractions of the total, and summing over all age groups. Denoting the regional distribution of the people aged \( x \) to \( x + 4 \) by the diagonal matrix \( K(x) \), the regional crude birth rates are given by the vector

\[
\{b^0\} = [\sum_x F(x) K(x)] [\sum_x K(x)]^{-1} \{1\}.
\]

(4.2)
Table 4.1. Age-specific fertility rates.

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000071</td>
<td>0.000057</td>
</tr>
<tr>
<td>15</td>
<td>0.015857</td>
<td>0.026458</td>
</tr>
<tr>
<td>20</td>
<td>0.070652</td>
<td>0.087978</td>
</tr>
<tr>
<td>25</td>
<td>0.063218</td>
<td>0.074260</td>
</tr>
<tr>
<td>30</td>
<td>0.041103</td>
<td>0.044290</td>
</tr>
<tr>
<td>35</td>
<td>0.022862</td>
<td>0.023532</td>
</tr>
<tr>
<td>40</td>
<td>0.007797</td>
<td>0.012051</td>
</tr>
<tr>
<td>45</td>
<td>0.000710</td>
<td>0.002151</td>
</tr>
<tr>
<td>50</td>
<td>0.000292</td>
<td>0.000714</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>60</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>75</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

\( g_{rr} \) 1.112809 1.357504
The product $F(x)$ $K(x)$ represents the observed regional number of births to mothers aged $x$ to $x + 4$.

4.1 The Generalized Net Maternity Function

The generalized net maternity (GNM) function is defined as the product (Rogers, 1975a, p. 93)

$$\phi(x) = m(x) \hat{k}(x)$$

(4.3)

where, for example, in a two-region model,

$$\phi(x) = \begin{bmatrix} 1\phi_1(x) & 2\phi_1(x) \\ 1\phi_2(x) & 2\phi_2(x) \end{bmatrix}$$

An element $\phi_{ij}(x)$ denotes the expected number of children to be born during a unit time interval in region $j$ to a woman of exact age $x$, who was born in region $i$, and who is part of a stationary (life table) population. The fertility rates applied to this stationary population are the observed fertility rates.

Since the actual population data are usually given for five-year age groups, one normally evaluates (4.3) with the numerical approximation

$$\bar{\phi}(x) = F(x) L(x)$$

(4.4)
in which the integral \[ \int_0^5 m(x + t) \tilde{\phi}(x + t) \, dt \] is replaced by the product \( F(x) \) and \( L(x) \). Illustrative numerical evaluations (the integrals of the generalized net maternity function) are given in Table 4.2. They were obtained by multiplying the fertility rates of Table 4.1 by the age composition of the life table population in Table 2.4 (or Table 3.5). For example, \( \tilde{\phi}(20) \) is

\[
\tilde{\phi}(20) = \begin{bmatrix}
0.070652 & 0 \\
0 & 0.087978 \\
0.314868 & 0.002626 \\
0.030727 & 0.384741
\end{bmatrix} \quad \begin{bmatrix}
4.456622 & 0.037170 \\
0.349255 & 4.373158
\end{bmatrix}
\]

The GNM function gives the number of offspring, by age, of a population that is distributed according to the life table (stationary) population, and which is subjected to the observed regional fertility schedules. The total number of offspring per unit birth is

\[
NRR = \sum_x \tilde{\phi}(x). \tag{4.5}
\]

An element

\[
i \, NRR \, j = \sum_x i \tilde{\phi}(x)
\]

denotes the total number of children expected to be born in region \( j \) to a woman who was born in region \( i \), and who is a member of a
life table population.† The matrix NRR is the net reproduction rate matrix, and is the multiregional generalization of the net reproduction rate (NRR) (Rogers, 1975a, p. 106). The elements of NRR are the totals set out in Table 4.2.

The matrix NRR gives the regional distribution of the offspring per unit birth in each region. It has been computed using unit radices. From the discussion of the life table in the previous section it is clear that a birth cohort of \( \{Q_1\} \) would lead to a regional number of offspring, after a generation, of

\[
\{Q_2\} = \text{NRR}\{Q_1\}.
\]  

(4.6)

Note that (4.6) is a growth model of generations.

The GNM function contains additional useful information for fertility analysis. Define the \( n \)-th moment of the GNM function (4.3) as

\[
R(n) = \int_{\alpha}^{\beta} x^n \phi(x) \, dx
\]  

(4.7)

where \( \alpha \) and \( \beta \) are the lowest and highest reproductive ages respectively, and where, for example, in the two-region case,

\[
R(n) = \begin{bmatrix}
R_1(n) & R_1(n) \\
R_2(n) & R_2(n)
\end{bmatrix}
\]

†Recall that a life table population is a stationary population that would result if the mortality and migration schedules were applied to arbitrary regional radices.
Table 4.2. Integrals of generalized net maternity function.

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000324</td>
<td>0.116754</td>
</tr>
<tr>
<td>15</td>
<td>0.000626</td>
<td>0.384741</td>
</tr>
<tr>
<td>20</td>
<td>0.000247</td>
<td>0.321420</td>
</tr>
<tr>
<td>25</td>
<td>0.000249</td>
<td>0.189758</td>
</tr>
<tr>
<td>30</td>
<td>0.000146</td>
<td>0.099739</td>
</tr>
<tr>
<td>35</td>
<td>0.000526</td>
<td>0.050405</td>
</tr>
<tr>
<td>40</td>
<td>0.000049</td>
<td>0.000333</td>
</tr>
<tr>
<td>45</td>
<td>0.000020</td>
<td>0.000259</td>
</tr>
<tr>
<td>50</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>60</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>75</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.961875</td>
<td>0.122364</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>15</td>
<td>0.000001</td>
<td>0.000030</td>
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<tr>
<td>20</td>
<td>0.000324</td>
<td>0.000359</td>
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<td>25</td>
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<td>0.000031</td>
<td>0.000078</td>
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<tr>
<td>45</td>
<td>0.000028</td>
<td>0.000140</td>
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<tr>
<td>50</td>
<td>0.000011</td>
<td>0.000046</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>60</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
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<td>75</td>
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<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.010587</td>
<td>1.174811</td>
</tr>
</tbody>
</table>
The numerical approximation of \(4.7\) is

\[
\tilde{R}(n) = \sum_{x} \left( x + 2.5 \right)^n \tilde{f}(x)
\]

\[
= \sum_{x} \left( x + 2.5 \right)^n \tilde{F}(x) \tilde{L}(x) .
\]  

(4.8)

Observe that the 0-th moment, \(\tilde{R}(0)\), is identical to \(\text{NRR}\).

The 0-th, first and second moments of the GNM function of the two-region system Slovenia-Rest of Yugoslavia are given in Table 4.3. The column totals of \(\tilde{R}(0)\) represent the total number of offspring per woman born in a certain region, e.g.

\[
\sum_{i} \tilde{R}(0) = \sum_{j} \sum_{i} \tilde{R}_{ij}(0) .
\]  

(4.9)

The row totals of \(\tilde{R}(0)\) give the total number of children born in a certain region during one generation, per woman born in that region. It is the number of daughters by which a girl baby in a region is replaced. Noting that \(\tilde{R}(0) = \text{NRR}\), the total number of children born in region \(j\) during one generation is

\[
Q_{2j} = \sum_{i} \tilde{R}_{ij}(0) Q_{1i}
\]  

(4.10)

and the row total of the \(j\)-th region is

\[
\tilde{R}(0) = \frac{Q_{2j}}{Q_{1j}} = \sum_{i} \frac{Q_{1i}}{Q_{1j}} \tilde{R}_{ij}(0) .
\]  

(4.11)

The value of \(\tilde{R}(0)\) depends on the radix ratio \(Q_{1i}/Q_{1j}\) of the life table population. Because the radix ratio is not unique, row totals are not given in Table 4.3.
Table 4.3. Moments of integral function.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.961875</td>
<td>0.010687</td>
</tr>
<tr>
<td>R.yugos.</td>
<td>0.122364</td>
<td>1.174811</td>
</tr>
<tr>
<td>Total</td>
<td>1.084238</td>
<td>1.185497</td>
</tr>
<tr>
<td>1 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>26.499458</td>
<td>0.313662</td>
</tr>
<tr>
<td>R.yugos.</td>
<td>3.587497</td>
<td>32.162094</td>
</tr>
<tr>
<td>Total</td>
<td>30.086954</td>
<td>32.475753</td>
</tr>
<tr>
<td>2 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>767.9428</td>
<td>9.6255</td>
</tr>
<tr>
<td>R.yugos.</td>
<td>110.8332</td>
<td>933.2065</td>
</tr>
<tr>
<td>Total</td>
<td>878.7760</td>
<td>942.8320</td>
</tr>
</tbody>
</table>
Table 4.4 repeats \( R(0) \) or NRR and gives the dominant eigenvalue and associated eigenvectors of \( R(0) \). The eigenvalue of \( \lambda_i[R(0)] \), reflects the net reproduction rate of the whole system or country (Rogers and Willekens, 1976c, p. 28). A life table radix ratio that yields a global NRR equal to \( \lambda_i[R(0)] \) is given by the right eigenvector of \( R(0) \).

The net reproduction allocation \( i^0_j \) denotes the fraction of the offspring of the i-born women born in region j.\(^\dagger\) For example,

\[
\text{i}^0_2 = \frac{i^\text{NRR}_2}{i^\text{NRR}} = \frac{0.122364}{1.084238} = 0.1129,
\]

i.e. 11.29% of the daughters of Slovenia-born women are born in the Rest of Yugoslavia.

The moments of the \( GN M \) function give rise to other demographically meaningful statistics: the mean and the variance of the \( GN M \) function. In the single region case, the mean of the net maternity function is defined as (Keyfitz, 1968, p. 102)

\[
\mu = \frac{\sum_x (x + 2.5) F(x) L(x)}{x F(x) L(x)} = \frac{\bar{R}(1)}{\bar{R}(0)}.
\]

It represents the mean age of childbearing of the life table population (given the observed fertility schedule). The variance of the net maternity function is

\[
\sigma^2 = \frac{\sum_x (x + 2.5 - \mu)^2 F(x) L(x)}{x F(x) L(x)} = \frac{\bar{R}(2)}{\bar{R}(0)} - \mu^2,
\]

and represents the variance of the mean age of childbearing.

\(^\dagger\)The arrangement of the elements in Table 4.4 is the transpose of Table 2 in Rogers (1975b, p. 5).
Table 4.4. Spatial fertility expectancies.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugo.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net reproduction rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.951875</td>
<td>0.010687</td>
<td></td>
</tr>
<tr>
<td>R. Yugo.</td>
<td>0.122364</td>
<td>1.174811</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.034238</td>
<td>1.185497</td>
<td></td>
</tr>
<tr>
<td><strong>Eigenvalue</strong></td>
<td></td>
<td></td>
<td>1.180784</td>
</tr>
<tr>
<td><strong>Eigenvector</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Right</td>
<td>1.000000</td>
<td>20.483990</td>
<td></td>
</tr>
<tr>
<td>- Left</td>
<td>1.000000</td>
<td>1.789021</td>
<td></td>
</tr>
</tbody>
</table>

**Net reproduction allocations**

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugo.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>0.887143</td>
<td>0.009015</td>
<td></td>
</tr>
<tr>
<td>R. Yugo.</td>
<td>0.112857</td>
<td>0.990985</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.000000</td>
<td>1.000000</td>
<td></td>
</tr>
</tbody>
</table>
Multiregional generalizations of (4.12) and (4.13) are (Rogers, 1975a, p. 106):

\[ i^\nu_j = \frac{\sum (x + 2.5) F_j(x) \; i^0 L_j(x)}{\sum x F_j(x) \; i^0 L_j(x)} = \frac{i \tilde{R}_j(1)}{i \tilde{R}_j(0)} \quad (v.14) \]

and

\[ i^\sigma_j = \frac{\sum (x + 2.5 - i^\nu_j)^2 F_j(x) \; i^0 L_j(x)}{\sum x F_j(x) \; i^0 L_j(x)} = \frac{i \tilde{R}_j(2) - i^\nu_j^2}{i \tilde{R}_j(0)} \quad (4.15) \]

respectively.

The matrix of mean ages of childbearing of the life table population is given in Table 4.5 as Alternative 1. For example, the mean age of childbearing among Slovenia-born women who are living in the Rest of Yugoslavia is 29.32 years. The mean age for women living in Slovenia is lower, namely 27.55 years. This is consistent with the observation that mothers who have migrated are normally older.

The single-region measures (4.12) and (4.13) may be generalized to a multiregional system in a different way, one which is analogous to the extension of the single-region survivorship proportion to the multiregional survivorship matrix in the life table. The mean age of childbearing in this case is

\[ \mu = \left[ \sum_{x} (x + 2.5) F(x) \; L(x) \right] \left[ \sum_{x} F(x) \; L(x) \right]^{-1} \quad (4.16) \]

\[ = \left[ \tilde{R}(1) \right] \left[ \tilde{R}(0) \right]^{-1} \]

and the variance matrix is

\[ \sigma^2 = \left[ \tilde{R}(2) \right] \left[ \tilde{R}(0) \right]^{-1} - \mu^2 \quad . \quad (4.17) \]
Table 4.5. Matrices of mean ages and variances.

**alternative 1**

### means

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>27.549805</td>
<td>29.350246</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>29.318336</td>
<td>27.376410</td>
</tr>
<tr>
<td>total</td>
<td>28.434071</td>
<td>28.363329</td>
</tr>
</tbody>
</table>

### variances

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>39.389587</td>
<td>39.247681</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>45.204407</td>
<td>44.878601</td>
</tr>
<tr>
<td>total</td>
<td>42.796997</td>
<td>42.063141</td>
</tr>
</tbody>
</table>

**alternative 2**

### means

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>27.547718</td>
<td>0.016397</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.247326</td>
<td>27.374157</td>
</tr>
<tr>
<td>total</td>
<td>27.795044</td>
<td>27.390554</td>
</tr>
</tbody>
</table>

### variances

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>39.382019</td>
<td>0.031142</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.607339</td>
<td>44.868774</td>
</tr>
<tr>
<td>total</td>
<td>39.989357</td>
<td>44.899918</td>
</tr>
</tbody>
</table>
These matrices are given in Table 4.5 as Alternative 2. The average age at childbearing of a woman who conceived in Slovenia is 27.795 years. Of this total 27.548 have been lived in Slovenia and 0.247 in the Rest of Yugoslavia.

4.2 The Weighted Generalized Net Maternity Function

Thus far we have limited our discussion to the fertility analysis of a population distributed as in the multiregional life table. It is a stationary population that is generated by observed mortality and migration schedules. The life table population was augmented by observed fertility schedules to give the GNM function and the derived statistics discussed above. We now replace the life table population by the stable population, given in Table 3.4, and perform an analogous analysis. As before we assume unit birth cohorts (now birth cohorts in the stable population).

Computationally, fertility analysis in the stable population is completely analogous to the one described above. The only difference is that \( \hat{L}(x) \) now is replaced by

\[
\hat{L}^r(x) = e^{-rx} \hat{L}(x)
\]  

(3.19)

and \( \hat{L}(x) \) by

\[
\hat{L}^r(x) = e^{-r(x+2.5)} \hat{L}(x) .
\]  

(3.20)

Define the weighted generalized net maternity (WGNM) function as the product

\[
\hat{\phi}^r(x) = m(x) \hat{L}^r(x) = e^{-rx} m(x) \hat{L}(x) .
\]  

(11.18)

The weight applied is \( e^{-rx} \). Since this may be considered as a discounting to birth, with \( r \) being the rate of discount, we may denote the WGNM function as a GNM function with discounting. The usefulness of the notion of discounting for demographic analysis becomes clear in the treatment of the reproductive value (Section 6).
An element \( \phi^{(r)}_{i,j}(x) \) denotes the expected number of children to be born in region \( j \) to an \( i \)-born woman who is of exact age \( x \) and part of the stable population. It may also be considered as the number of children discounted back to the time of birth of the mother.

The numerical approximation of (4.18) is

\[
\tilde{\phi}^{(r)}(x) = P(x) \cdot L^{(r)}(x) ,
\]

and the result is given in Table 4.6. Table 4.6 is obtained by multiplying the fertility rates of Table 4.1 by the age composition of the stable population (Table 3.4). For example,

\[
\tilde{\phi}^{(r)}(20) = \begin{bmatrix}
0.070652 & 0 \\
0 & 0.087978 \\
0.274491 & 0.002289 \\
0.026786 & 0.335403
\end{bmatrix} \begin{bmatrix}
3.885118 & 0.032404 \\
0.304468 & 3.812358
\end{bmatrix}
\]

The WGNM function gives the number of offspring, by age of mother, of a unit birth in the stable population. Summing over all groups we get

\[
\bar{\psi}^{(r)} = \sum_{x} \tilde{\phi}^{(r)}(x) .
\]

The matrix \( \bar{\psi}^{(r)} \) is the characteristic matrix of the multiregional population system (Rogers, 1975a, p. 93). An element \( \bar{\psi}_{i,j}^{(r)} \) denotes the total number of children expected to be born in region
j to a woman born in region i, who is a member of the stable population. The characteristic matrix is the stable analogue of the NRR matrix. It gives the regional distribution of the offspring per unit birth in each region of the stable population. For example, Table 4.6 shows that a woman born in the stable population in Slovenia gives birth to a total of 0.916098 children on the average. Of them, 0.813684 are born in Slovenia and 0.102414 in the Rest of Yugoslavia.

If the stable distribution of births is \( \{Q^S\} \), then the distribution of offspring is also \( \{Q^S\} \) (Rogers, 1975a, p. 93):

\[
\{Q^S\} = \bar{\psi}(r) \{Q^S\} .
\]  

Equation (4.21) is the multiregional characteristic equation. It can be seen from (4.21) that the relative distribution of births is given by the right eigenvector of \( \bar{\psi}(r) \). In our numerical example,

\[
\{Q^S\} = \begin{bmatrix} 1 \\ 20.8237 \end{bmatrix},
\]

where the subscript denotes "arbitrary scaling". Since the eigenvector of a matrix is fixed only up to a scalar, we may choose any scaling that is convenient. Equation (4.22) indicates that at stability 4.58% of the births will occur in Slovenia and 95.42% in the Rest of Yugoslavia (in the observed population it was 6.91% and 93.09%, respectively).

As with the GNM function, we define the n-th moment of the WGNM function (4.18) to be
Table 4.6. Integrals of weighted generalized net maternity function.

<table>
<thead>
<tr>
<th>age</th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>10</td>
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<td>0.030408</td>
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<td>0.141170</td>
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</tr>
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<td>35</td>
<td>0.074713</td>
<td>0.011763</td>
</tr>
<tr>
<td>40</td>
<td>0.024358</td>
<td>0.006027</td>
</tr>
<tr>
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<td>0.002113</td>
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<td>50</td>
<td>0.000824</td>
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<tr>
<td>60</td>
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<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>75</td>
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<tr>
<td>80</td>
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<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.813694</td>
<td>0.102414</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>r.yugos.</th>
</tr>
</thead>
<tbody>
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<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>15</td>
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</tr>
<tr>
<td>20</td>
<td>0.002289</td>
<td>0.035403</td>
</tr>
<tr>
<td>25</td>
<td>0.002745</td>
<td>0.271785</td>
</tr>
<tr>
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<td>0.001932</td>
<td>0.079346</td>
</tr>
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<td>0.079346</td>
</tr>
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<td>0.038895</td>
</tr>
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<td>0.000037</td>
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</tr>
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<td>0.000275</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
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<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>75</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.003942</td>
<td>0.994955</td>
</tr>
</tbody>
</table>
and evaluate it numerically as

\[ R(r)(n) = \int_a^\beta x^n \psi(r)(x) \, dx \]
\[ = \int_a^\beta x^n e^{-rx} \phi(x) \, dx , \quad (4.23) \]

and evaluate it numerically as

\[ R(r)(n) = \sum_{a}^{\beta-5} (x + 2.5)^n \phi(r)(x) \]
\[ = \sum_{a}^{\beta-5} (x + 2.5)^n e^{-r(x+2.5)} \phi(x) \, L(x) . \quad (4.24) \]

The moments are given in Table 4.7. Note that the 0-th moment of the WGNM function coincides with \( \psi(r) \). The column totals of \( \psi(r) \) represent the total number of offspring in the stable population per woman by her place of birth, e.g.

\[ \sum_j \psi_j(r) = \sum_j \psi_j(r) . \quad (4.25) \]

The row totals give the total number of daughters by which a female baby is replaced in her region of birth in the stable population. It depends, of course, on the stable ratio of births:

\[ \psi_j(r) = \sum_i \frac{Q^S_{1i}}{Q^S_{1j}} \psi_i(r) , \quad (4.26) \]

where \( Q^S_{1i} \) is an element of the right eigenvector of \( \psi(r) \).

Table 4.8 repeats the \( \psi(r) \) matrix. In addition, it shows the net reproduction allocations \( i \circ_j(r) \), with
Table 4.7. Moments of integral function.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.813684</td>
<td>0.008942</td>
</tr>
<tr>
<td></td>
<td>0.102414</td>
<td>0.994965</td>
</tr>
<tr>
<td>total</td>
<td>0.916098</td>
<td>1.003900</td>
</tr>
<tr>
<td>1 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.223692</td>
<td>0.260318</td>
</tr>
<tr>
<td></td>
<td>2.974095</td>
<td>26.970327</td>
</tr>
<tr>
<td>total</td>
<td>25.197687</td>
<td>27.230646</td>
</tr>
<tr>
<td>2 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>638.2903</td>
<td>7.9234</td>
</tr>
<tr>
<td></td>
<td>90.9879</td>
<td>774.3862</td>
</tr>
<tr>
<td>total</td>
<td>729.2787</td>
<td>782.3096</td>
</tr>
</tbody>
</table>
Table 4.8. Spatial fertility expectancies.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r. Jugos.</th>
<th>Slovenia</th>
<th>r. Jugos.</th>
<th>Total</th>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net reproduction rate</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.813684</td>
<td>0.008942</td>
<td>0.102414</td>
<td>0.994965</td>
<td>0.916098</td>
<td>1.003906</td>
<td></td>
</tr>
<tr>
<td>r. Jugos.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.916098</td>
<td>1.003906</td>
<td>0.999883</td>
<td>1.000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenvalue†</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenvector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right</td>
<td>1.00000</td>
<td>20.823744</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td>1.00000</td>
<td>1.818089</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

†The eigenvalue should be equal to one. The deviation is due to the rounding of the intrinsic growth rate r to six decimal places. The growth rate has been computed by projecting the population growth matrix to stability.
For example,

\[ i_{\rho j} = \frac{i_{\psi}(r)}{i_{\psi}(r)} \quad (4.27) \]

\[ i_{1\rho 2} = \frac{i_{\psi}(r)}{i_{\psi}(r)} = 0.102414 \quad 0.916098 = 0.1118 \]

i.e. 11.18% of the daughters born to Slovenia-born women are born in the Rest of Yugoslavia.

The mean and the variance of the WGNM function are given in Table 4.9. Once again, two alternative expressions are distinguished.

Alternative 1:

The matrix of mean ages of childbearing in the stable population, \( A \), has elements:

\[ iA_{j} = \frac{\int x \ (x + 2.5) e^{-r(x+2.5)} F_{j}(x) \ i0L_{j}(x) \ i_{\psi}(r) \ (1)}{\int x \ e^{-r(x+2.5)} F_{j}(x) \ i0L_{j}(x) \ i_{\psi}(r) \ (0)} \quad (4.28) \]

and the variance matrix \( \sigma^{2} \) has elements

\[ i\sigma_{j} = \frac{\int x \ (x + 2.5 - iA_{j})^{2} e^{-r(x+2.5)} F_{j}(x) \ i0L_{j}(x)}{\int x \ e^{-r(x+2.5)} F_{j}(x) \ i0L_{j}(x)} \]

\[ = \frac{i_{\psi}(r)(2)}{i_{\psi}(r)(0)} - iA_{j}^{2} \quad (4.29) \]
Table 4.9. Matrices of mean ages and variances.

```
** alternative 1 **
************************
means
-----
slovenia  r.yugos.
slovenia  27.312317  29.112961
r.yugos.  29.039822  27.106815
total    28.176069  28.109888

variances
---------
slovenia  r.yugos.
slovenia  38.482665  38.556519
r.yugos.  45.120483  43.525635
total    41.801575  41.041077

** alternative 2 **
************************
means
-----
slovenia  r.yugos.
slovenia  27.310278  0.016201
r.yugos.  0.243572  27.104626
total    27.553850  27.120827

variances
---------
slovenia  r.yugos.
slovenia  38.474976  0.033233
r.yugos.  0.622766  43.515696
total    39.097744  43.548920
```
Alternative 2:

\[
A = \left[ \sum_{x} (x + 2.5) e^{-r(x+2.5)} F(x) L(x) \right] \left[ \sum_{x} e^{-r(x+2.5)} F(x) L(x) \right]^{-1}
\]

\[
= [\tilde{R}^{(r)}(1)]^{\dagger} [\tilde{R}^{(r)}(0)]^{-1}
\] 

(4.30)

\[
\sigma^2 = [\tilde{R}^{(r)}(2)] [\tilde{R}^{(r)}(0)]^{-1} - A^2
\]

(4.31)
5. MOBILITY ANALYSIS

There are two alternative approaches to expressing the level of migration in a multiregional system (Rogers, 1975b). The first expresses the migration level in terms of expected durations, i.e. the fraction of an individual's lifetime that is spent in a particular region. The expectation of life at birth by place of residence is computed in the multiregional life table. The life expectancy matrix

\[
\begin{bmatrix}
1_e_1(0) & 2_e_1(0) \\
1_e_2(0) & 2_e_2(0)
\end{bmatrix}
\]

for the system Slovenia - Rest of Yugoslavia is given in Table 5.1. The total life expectancy of a girl born in Slovenia is 72.48 years, of which 64.90 years are expected to be lived in Slovenia \(1_e_1(0)\) and 7.58 years in the Rest of Yugoslavia \(1_e_2(0)\).

Expressing these expectancies as fractions of the total lifetimes yields the migration levels \(i^0_j\):

\[
i^0_j = \frac{i_e_j(0)}{i_e(0)} .
\]

The second approach adopts a fertility perspective to migration analysis. Unlike death, migration is a recurrent event, analogous to birth. Thus, as in fertility, its level can be measured by counting the events, i.e. the number of moves an average person makes during his lifetime.\(^\dagger\) Such indices have been developed by Wilber (1963) and Long (1973) for a population.

\(^\dagger\)The number of moves is defined here as the number of times a person is in another region at the end of the unit time interval. Back and forth moves during a unit interval are not counted (a similar assumption has been adopted by Wilber (1963) and Long (1973)).
Table 5.1. Expectations of life at birth.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugosl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectations of Life</td>
<td>64.882865</td>
<td>0.810736</td>
</tr>
<tr>
<td></td>
<td>7.579217</td>
<td>65.434807</td>
</tr>
<tr>
<td>Total</td>
<td>72.478081</td>
<td>66.245514</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>67.660038</td>
<td></td>
</tr>
<tr>
<td>Eigenvector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Right</td>
<td>1.0000000</td>
<td>3.405951</td>
</tr>
<tr>
<td>- Left</td>
<td>1.0000000</td>
<td>0.364316</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugosl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Migration Levels</td>
<td>0.895427</td>
<td>0.012238</td>
</tr>
<tr>
<td></td>
<td>0.104573</td>
<td>0.987762</td>
</tr>
<tr>
<td>Total</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
</tbody>
</table>
aggregated at the national level. Rogers (1975b) combines Wilber's and Long's ideas of "expected moves" with the approach generalizing the expected number of children (NRR) to a multi-regional system (NRR).

As before, let \( \hat{\xi}(x) \) be the distribution of the life table population at exact age \( x \), and let \( L(x) \) be the stationary life table population aged \( x \) to \( x + 4 \), by place of birth and residence. Define \( m^O \) as the diagonal matrix of annual regional outmigration rates of people at exact age \( x \), and \( M^O(x) \) as the diagonal matrix of annual outmigration rates of people in age group \( x \) to \( x + 4 \), e.g. in a two-region system,

\[
M^O(x) = \begin{bmatrix}
M^O_1(x) & 0 \\
0 & M^O_2(x)
\end{bmatrix}
\]  

(5.3)

with \( M^O_1(x) = \sum_{j \neq i} M_{ij}(x), M_{ij}(x) \) being the age specific migration rate from region \( i \) to region \( j \). Integration of the matrices of age-specific outmigration rates over all ages gives the gross migration production rate matrix:

\[
GMR = \int_{0}^{\infty} m^O(x) \, dx \equiv 5 \sum_{x} M^O(x) .
\]

The origin-destination migration rates of the two-region system Slovenia - Rest of Yugoslavia are given in Table 1.3. Table 5.2 shows the age-specific regional total outmigration rates (the same in this two-region case). Since the system under consideration contains only two regions, \( M^O_1(x) = M_{ij}(x) \) for \( i \neq j \). The column totals multiplied by five denote the regional gross migration rates.
Table 5.2. Age-specific outmigration rates.

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.002832</td>
<td>0.000272</td>
</tr>
<tr>
<td>5</td>
<td>0.002294</td>
<td>0.000166</td>
</tr>
<tr>
<td>10</td>
<td>0.001485</td>
<td>0.000157</td>
</tr>
<tr>
<td>15</td>
<td>0.005158</td>
<td>0.000679</td>
</tr>
<tr>
<td>20</td>
<td>0.007170</td>
<td>0.000937</td>
</tr>
<tr>
<td>25</td>
<td>0.005534</td>
<td>0.000506</td>
</tr>
<tr>
<td>30</td>
<td>0.003756</td>
<td>0.000350</td>
</tr>
<tr>
<td>35</td>
<td>0.001765</td>
<td>0.000226</td>
</tr>
<tr>
<td>40</td>
<td>0.001013</td>
<td>0.000183</td>
</tr>
<tr>
<td>45</td>
<td>0.000543</td>
<td>0.000094</td>
</tr>
<tr>
<td>50</td>
<td>0.000663</td>
<td>0.000130</td>
</tr>
<tr>
<td>55</td>
<td>0.000629</td>
<td>0.000205</td>
</tr>
<tr>
<td>60</td>
<td>0.000884</td>
<td>0.000203</td>
</tr>
<tr>
<td>65</td>
<td>0.000949</td>
<td>0.000156</td>
</tr>
<tr>
<td>70</td>
<td>0.000576</td>
<td>0.000078</td>
</tr>
<tr>
<td>75</td>
<td>0.001111</td>
<td>0.000099</td>
</tr>
<tr>
<td>80</td>
<td>0.000704</td>
<td>0.000196</td>
</tr>
<tr>
<td>85</td>
<td>0.001111</td>
<td>0.000076</td>
</tr>
<tr>
<td>gmr</td>
<td>0.192379</td>
<td>0.023573</td>
</tr>
</tbody>
</table>
The application of the age-specific outmigration rates to the life table and to the stable population yields, respectively, the generalized and the weighted generalized net mobility functions.

5.1 The Generalized Net Mobility Function

The generalized net mobility (GM) function is the product

$$\gamma(x) = m^O(x) \hat{\lambda}(x), \tag{5.4}$$

or, in the case of a two-region system,

$$\begin{bmatrix}
\gamma_1(x) & 2\gamma_1(x) \\
\gamma_2(x) & 2\gamma_2(x)
\end{bmatrix} =
\begin{bmatrix}
m_1^O(x) & m_1^O(x) \\
m_2^O(x) & m_2^O(x)
\end{bmatrix}
\begin{bmatrix}
\hat{\lambda}_1(x) & m_1^O(x) \hat{\lambda}_1(x) \\
\hat{\lambda}_2(x) & m_2^O(x) \hat{\lambda}_2(x)
\end{bmatrix}$$

The element $\gamma_j(x)$ denotes the expected number of migrations out of region $j$, made during a unit time interval following age $x$, by a woman born in region $i$. Since the system consists only of two regions, $\gamma_j(x)$ measures the return migration of the $x$-year old.

The numerical evaluation of equation (5.4) is

$$\overline{\gamma}(x) = M^O(x) L(x). \tag{5.5}$$

The values of $\overline{\gamma}(x)$ are given in Table 5.3. The computational procedure is completely analogous to the one used in the fertility analysis. The only difference is that $F(x)$ of (2.4) is replaced by $M^O(x)$. For example, $\overline{\gamma}(20)$ is
Table 5.3. Integrals of generalized net mobility function.

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.013848</td>
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<td>0.010393</td>
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</tr>
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<td>15</td>
<td>0.023774</td>
<td>0.000144</td>
</tr>
<tr>
<td>20</td>
<td>0.031954</td>
<td>0.000327</td>
</tr>
<tr>
<td>25</td>
<td>0.023812</td>
<td>0.000245</td>
</tr>
<tr>
<td>30</td>
<td>0.015727</td>
<td>0.000202</td>
</tr>
<tr>
<td>35</td>
<td>0.007249</td>
<td>0.000142</td>
</tr>
<tr>
<td>40</td>
<td>0.004099</td>
<td>0.000119</td>
</tr>
<tr>
<td>45</td>
<td>0.002159</td>
<td>0.000061</td>
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<td>0.003129</td>
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</tr>
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</tr>
<tr>
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<td>0.002183</td>
<td>0.000036</td>
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<td>0.000796</td>
<td>0.000019</td>
</tr>
<tr>
<td>total</td>
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<td>0.001849</td>
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</tbody>
</table>

<table>
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<th>age</th>
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<th>r.yugos.</th>
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</thead>
<tbody>
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<td>0.001289</td>
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<tr>
<td>5</td>
<td>0.000019</td>
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<td>0.000017</td>
<td>0.000697</td>
</tr>
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<td>15</td>
<td>0.000106</td>
<td>0.002935</td>
</tr>
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<td>20</td>
<td>0.000267</td>
<td>0.004099</td>
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<td>0.000284</td>
<td>0.002192</td>
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<td>0.000222</td>
<td>0.001500</td>
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<td>0.000113</td>
<td>0.000957</td>
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<td>0.000068</td>
<td>0.000767</td>
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<td>0.000037</td>
<td>0.000386</td>
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<td>0.000521</td>
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</tr>
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<td>0.000729</td>
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</tr>
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<td>0.000203</td>
</tr>
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<td>75</td>
<td>0.000047</td>
<td>0.000198</td>
</tr>
<tr>
<td>80</td>
<td>0.000018</td>
<td>0.000250</td>
</tr>
<tr>
<td>85</td>
<td>0.000019</td>
<td>0.000098</td>
</tr>
<tr>
<td>total</td>
<td>0.001492</td>
<td>0.018915</td>
</tr>
</tbody>
</table>
The expected number of migrations an individual makes during his lifetime is given by the summation of $\tilde{\gamma}(x)$ over all $x$. The result is the net migration matrix (Rogers, 1975b, p. 8):

$$NMR = \sum_{x} \tilde{\gamma}(x)$$

where, in the case of a two-region system,

$$NMR = \begin{bmatrix}
NMR_1 & 2NMR_1 \\
1NMR_2 & 2NMR_2 \\
\hline
1NMR & 2NMR
\end{bmatrix}.$$ 

The column sum $iNMR$ denotes the total expected number of migrations to be made by a person born in region $i$. Some of these migrations, i.e., $iNMR_j$, are made out of region $j$. In other words, $iNMR_j$ denotes the number of times a person born in region $i$ is expected to leave region $j$. The total number of migrations expected to be made by the current birth cohorts out of region $j$ is of course

$$\begin{bmatrix}
0.007170 & 0 \\
0 & 0.000937 \\
0.031954 & 0.000267 \\
0.000327 & 0.004099
\end{bmatrix}.$$
or in matrix notation

\[
\{E\} = \text{NMR}\{Q_1\}.
\] (5.7)

The moments of the GM-function are completely analogous to those of the GNM-function. The \(n\)-th moment of the GM-function is defined as

\[
D(n) = \int_0^\omega x^n \gamma(x) \, dx
\] (5.8)

where \(\omega\) is the highest age of the population. The numerical approximation of (5.8) is

\[
\tilde{D}(n) = \sum_{x=0}^{z-5} (x + 2.5)^n \tilde{\gamma}(x)
\]

\[
= \sum_{x=0}^{z-5} (x + 2.5)^n M^\circ(x) I(x)
\] (5.9)

with \(z\) being the highest age in the discrete case and \(z-5\) the starting age of the highest age group.

The moments of the GM-function for Yugoslavia are contained in Table 5.4. The zeroth moment, \(\tilde{D}(0)\), is identical to the net migraproduction matrix, which is given in Table 5.5 together with the migraproduction allocations. The row sums of \(\tilde{D}(0)\) represent the elements of \(\{E\}\) for the case of unit regional radices. The net migraproduction allocation \(i\varepsilon_j\) denotes the fraction of the migrations made by an \(i\)-born individual out of region \(j\). For example,

\[
\varepsilon_2 = \frac{NMR_i}{NMR} = 0.001849 / 0.159566 = 0.0116
\].
Table 5.4. Moments of integral function.

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>slovenia</td>
<td>0.157716</td>
<td>0.001492</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.001849</td>
<td>0.018915</td>
</tr>
<tr>
<td>total</td>
<td>0.159566</td>
<td>0.020407</td>
</tr>
<tr>
<td>1 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>slovenia</td>
<td>4.199835</td>
<td>0.055735</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.073292</td>
<td>0.569751</td>
</tr>
<tr>
<td>total</td>
<td>4.273128</td>
<td>0.625486</td>
</tr>
<tr>
<td>2 moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>slovenia</td>
<td>160.327911</td>
<td>2.607642</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>3.575156</td>
<td>23.908083</td>
</tr>
<tr>
<td>total</td>
<td>163.903061</td>
<td>26.515726</td>
</tr>
</tbody>
</table>
Table 5.5. Spatial migration expectancies.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R.Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net Migraproduction Rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.157716</td>
<td>0.001492</td>
</tr>
<tr>
<td>R.Yugos.</td>
<td>0.001849</td>
<td>0.018915</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.159566</td>
<td>0.020407</td>
</tr>
<tr>
<td><strong>Eigenvector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Right</td>
<td>1.000000</td>
<td>0.013320</td>
</tr>
<tr>
<td>- Left</td>
<td>1.000000</td>
<td>0.010746</td>
</tr>
<tr>
<td><strong>Eigenvalue</strong></td>
<td></td>
<td>0.157736</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R.Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net Migraproduction Allocations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.938411</td>
<td>0.073101</td>
</tr>
<tr>
<td>R.Yugos.</td>
<td>0.011589</td>
<td>0.926899</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
</tbody>
</table>
The mean and the variance of the GM-function are given by formulas (4.14) to (4.17) in which $F_j(x)$ is replaced by $M^O_j(x)$ and $F(x)$ by $M^O(x)$.

Table 5.6 lists the means and variances of the generalized mobility function for the Yugoslavian example.

5.2 The Weighted Generalized Net Mobility Function

Mobility analysis of the stable population leads to the concept of the weighted generalized net mobility (WGM) function. The WGM-function is estimated by replacing the life table population in (5.4) and (5.5) by the stable population, i.e.,

$$\gamma^r(x) = m^O(x) \cdot e^{-rx} \cdot \hat{\lambda}(x)$$

(5.10)

and

$$\tau^r(x) = m^O(x) \cdot e^{-r(x+2.5)} \cdot \hat{\lambda}(x)$$

(5.11)

The weights are $e^{-rx}$ and $e^{-r(x+2.5)}$, respectively. Numerical values of $\tau^r(x)$ are given in Table 5.7. Summation of $\tau^r(x)$ over all $x$ yields the characteristic mobility matrix $\Gamma^r$:

$$\Gamma^r = \sum_x \tau^r(x)$$

(5.12)

The element $i \gamma^r_j$ denotes the average number of migrations out of region $j$ in the stable population that an $i$-born person is expected to make during his lifetime. The right eigenvector of $\Gamma^r$ represents the regional distribution of births that would result in an equal distribution of the outmigrants. In other words, if the births were distributed according to the right eigenvector of $\Gamma^r$, say, then the relative regional distribution of the migrants and the births are the same. This can easily be seen by writing the characteristic equation
Table 5.6. Matrices of mean ages and variances.

## alternative 1 ##

### ***************

#### means

---

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>26.629034</td>
<td>37.361927</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>39.636120</td>
<td>30.121689</td>
</tr>
<tr>
<td>total</td>
<td>33.132576</td>
<td>33.741810</td>
</tr>
</tbody>
</table>

#### variances

---

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>307.4528</td>
<td>352.1174</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>362.4001</td>
<td>356.6597</td>
</tr>
<tr>
<td>total</td>
<td>334.9265</td>
<td>354.3385</td>
</tr>
</tbody>
</table>

## alternative 2 ##

### ***************

#### means

---

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>26.619102</td>
<td>0.847250</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.111655</td>
<td>30.112393</td>
</tr>
<tr>
<td>total</td>
<td>26.730757</td>
<td>30.960133</td>
</tr>
</tbody>
</table>

#### variances

---

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>307.2101</td>
<td>9.6760</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>1.5218</td>
<td>356.4759</td>
</tr>
<tr>
<td>total</td>
<td>308.7319</td>
<td>366.1519</td>
</tr>
</tbody>
</table>

-110-
Table 5.7. Integrals of weighted generalized net mobility function.

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.013639</td>
<td>0.00009</td>
</tr>
<tr>
<td>5</td>
<td>0.010406</td>
<td>0.000315</td>
</tr>
<tr>
<td>10</td>
<td>0.006461</td>
<td>0.00020</td>
</tr>
<tr>
<td>15</td>
<td>0.021367</td>
<td>0.000130</td>
</tr>
<tr>
<td>20</td>
<td>0.027857</td>
<td>0.000285</td>
</tr>
<tr>
<td>25</td>
<td>0.020135</td>
<td>0.000207</td>
</tr>
<tr>
<td>30</td>
<td>0.012899</td>
<td>0.000166</td>
</tr>
<tr>
<td>35</td>
<td>0.005767</td>
<td>0.000113</td>
</tr>
<tr>
<td>40</td>
<td>0.003616</td>
<td>0.000092</td>
</tr>
<tr>
<td>45</td>
<td>0.001616</td>
<td>0.000046</td>
</tr>
<tr>
<td>50</td>
<td>0.001867</td>
<td>0.000061</td>
</tr>
<tr>
<td>55</td>
<td>0.001658</td>
<td>0.000092</td>
</tr>
<tr>
<td>60</td>
<td>0.002137</td>
<td>0.000084</td>
</tr>
<tr>
<td>65</td>
<td>0.002015</td>
<td>0.000058</td>
</tr>
<tr>
<td>70</td>
<td>0.001500</td>
<td>0.000024</td>
</tr>
<tr>
<td>75</td>
<td>0.001361</td>
<td>0.000022</td>
</tr>
<tr>
<td>80</td>
<td>0.000494</td>
<td>0.000028</td>
</tr>
<tr>
<td>85</td>
<td>0.000467</td>
<td>0.000011</td>
</tr>
<tr>
<td>total</td>
<td>0.134803</td>
<td>0.002462</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000009</td>
<td>0.001269</td>
</tr>
<tr>
<td>5</td>
<td>0.000018</td>
<td>0.000705</td>
</tr>
<tr>
<td>10</td>
<td>0.000016</td>
<td>0.000646</td>
</tr>
<tr>
<td>15</td>
<td>0.000095</td>
<td>0.000269</td>
</tr>
<tr>
<td>20</td>
<td>0.000032</td>
<td>0.000357</td>
</tr>
<tr>
<td>25</td>
<td>0.000240</td>
<td>0.000154</td>
</tr>
<tr>
<td>30</td>
<td>0.000182</td>
<td>0.000123</td>
</tr>
<tr>
<td>35</td>
<td>0.000090</td>
<td>0.000761</td>
</tr>
<tr>
<td>40</td>
<td>0.000053</td>
<td>0.000592</td>
</tr>
<tr>
<td>45</td>
<td>0.000028</td>
<td>0.000289</td>
</tr>
<tr>
<td>50</td>
<td>0.000033</td>
<td>0.000378</td>
</tr>
<tr>
<td>55</td>
<td>0.000031</td>
<td>0.000556</td>
</tr>
<tr>
<td>60</td>
<td>0.000042</td>
<td>0.000493</td>
</tr>
<tr>
<td>65</td>
<td>0.000042</td>
<td>0.000333</td>
</tr>
<tr>
<td>70</td>
<td>0.000032</td>
<td>0.000134</td>
</tr>
<tr>
<td>75</td>
<td>0.000030</td>
<td>0.000123</td>
</tr>
<tr>
<td>80</td>
<td>0.000011</td>
<td>0.000151</td>
</tr>
<tr>
<td>85</td>
<td>0.000011</td>
<td>0.000057</td>
</tr>
<tr>
<td>total</td>
<td>0.001195</td>
<td>0.015841</td>
</tr>
</tbody>
</table>
where \( L(x) \) is the distribution of the age group \( x \) to \( x + 4 \) in the stable population, by place of residence and by place of birth, and \( \lambda[\Gamma(r)] \) is the dominant eigenvalue of \( r \). In our numerical example (Table 5.9), the equation (5.13) is:

\[
\lambda[\Gamma(r)] \{z\} = \Gamma(r) \{z\}
\]

\[
\lambda[\Gamma(r)] \{z\} = \left[ \sum_x M^0(x) e^{-r(x+2.5)} L(x) \right] \{z\}
\]

\[
= \sum_x M^0(x) L^r(x) \{z\}
\]

where \( L^r(x) \) is the distribution of the age group \( x \) to \( x + 4 \) in the stable population, by place of residence and by place of birth, and \( \lambda[\Gamma(r)] \) is the dominant eigenvalue of \( r \). In our numerical example (Table 5.9), the equation (5.13) is:

\[
\begin{bmatrix}
0.1348 \\
0.1349 \\
0.1349
\end{bmatrix}
= \begin{bmatrix}
0.0100 \\
0.0103 \\
0.0103
\end{bmatrix}.
\]

At stability, the migrants have not only the same relative regional distribution as the births, but they also are proportional to the number of births. If the vector of births is \( \{Q^m\} \), with elements proportional to \( \{z\} \), then the vector of migrants \( \{z\} \) is:

\[
\{z\} = \Gamma(r)\{Q^m\} = \lambda[\Gamma(r)]\{Q^m\}.
\]

For the system Slovenia - Rest of Yugoslavia \( \lambda[\Gamma(r)] = 0.134823 \), i.e. the number of migrants is 13 percent of the number of births. In other words, if the births are distributed according to \( \{Q^m\} \), then the number of people leaving Slovenia during one generation (independent of where they are born) is 13% of the births in Slovenia in the beginning of this generation.

The moments of the WGM-function are defined in a manner analogous to (4.23):
Examples are given in Table 5.8. The 0-th moment is of course equal to \( \Gamma(r) \), which is repeated in Table 5.9. The mean and variance of the WGM-function are derived in a manner analogous to equations (4.28) to (4.31) and are set out in Table 5.10. Finally, the associated discounted life expectancy matrix is presented in Table 5.11.

\[
\begin{align*}
\bar{p}^{(r)}(n) &= \int_0^\omega x^n \gamma^{(r)}(x) \, dx = \int_0^\omega x^n e^{-rx} \gamma(x) \, dx \\
\end{align*}
\]

and

\[
\begin{align*}
\overline{\bar{p}}^{(r)}(n) &= \sum_{x=0}^{x=5} x^n \overline{\gamma}^{(r)}(x) = \int_0^\omega x^n e^{-r(x+2.5)} \overline{\gamma}(x) \\
\end{align*}
\]
Table 5.8. Moments of integral function.

<table>
<thead>
<tr>
<th>0 moment</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>slovenia</strong></td>
<td>0.134808</td>
<td>0.001195</td>
</tr>
<tr>
<td><strong>r.yugos.</strong></td>
<td>0.001462</td>
<td>0.015841</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>0.136270</td>
<td>0.017037</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1 moment</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>slovenia</strong></td>
<td>3.352751</td>
<td>0.042231</td>
</tr>
<tr>
<td><strong>r.yugos.</strong></td>
<td>0.054825</td>
<td>0.444553</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>3.407576</td>
<td>0.486785</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 moment</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>slovenia</strong></td>
<td>119.799019</td>
<td>1.867092</td>
</tr>
<tr>
<td><strong>r.yugos.</strong></td>
<td>2.545094</td>
<td>17.526709</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td>122.344116</td>
<td>19.393801</td>
</tr>
</tbody>
</table>
Table 5.9. Spatial migration expectancies.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Migration Production Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.134808</td>
<td>0.001195</td>
</tr>
<tr>
<td>R. Yugos.</td>
<td>0.001462</td>
<td>0.015841</td>
</tr>
<tr>
<td>Total</td>
<td>0.136270</td>
<td>0.017037</td>
</tr>
</tbody>
</table>

Eigenvalue: 0.134823

Eigenvector:
- Right: 1.000000 0.012284
- Left: 1.000000 0.010046

Net Migration Production Allocations

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>0.989274</td>
<td>0.070159</td>
</tr>
<tr>
<td>R. Yugos.</td>
<td>0.010726</td>
<td>0.929841</td>
</tr>
</tbody>
</table>
Table 5.10. Matrices of mean ages and variances.

** alternative 1 **
***************

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>24.870487</td>
<td>35.331787</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>37.510452</td>
<td>28.062807</td>
</tr>
<tr>
<td>total</td>
<td>31.190470</td>
<td>31.697298</td>
</tr>
</tbody>
</table>

variances
--------

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>270.1201</td>
<td>313.7201</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>334.2782</td>
<td>318.8672</td>
</tr>
<tr>
<td>total</td>
<td>302.1992</td>
<td>316.2936</td>
</tr>
</tbody>
</table>

** alternative 2 **
***************

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>24.861923</td>
<td>0.789983</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.102515</td>
<td>28.055071</td>
</tr>
<tr>
<td>total</td>
<td>24.964439</td>
<td>28.845053</td>
</tr>
</tbody>
</table>

variances
--------

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>269.9137</td>
<td>9.0477</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>1.4647</td>
<td>318.7004</td>
</tr>
<tr>
<td>total</td>
<td>271.3784</td>
<td>327.7481</td>
</tr>
</tbody>
</table>
Table 5.11. Discounted life expectancies at birth.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugoslav.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectations of life</td>
<td>52.225361 0.599859</td>
<td>52.336464 0.523182</td>
</tr>
<tr>
<td>Total</td>
<td>57.857544 0.52936321</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugoslav.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Migration levels</td>
<td>0.902654 0.011332</td>
<td>0.097346 0.988668</td>
</tr>
<tr>
<td>Total</td>
<td>1.000000 1.000000</td>
<td></td>
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</table>
6. FERTILITY ANALYSIS; CONTINUED

In this section we approach fertility analysis from a somewhat different perspective. Although the starting point is the net reproduction rate matrix (NRR) and the characteristic matrix \( \Psi(r) \) or \( R'(0) \), the interpretation is different. This allows us to derive additional useful information regarding the fertility behavior of the population.

Recall that both NRR and \( \Psi(r) \) represent the regional distribution of the offspring by place of birth of the mother. The matrix NRR refers to a life table population and \( \Psi(r) \) to a stable population. The intrinsic or stable growth rate is \( r \). In equation (4.18), the rate \( r \) also may be considered to be a rate of discount. Introducing the notion of discounting, and hence a time preference to the fact of having children, adds an interesting new dimension to fertility analysis.

The central concept here is the reproductive value. It has been developed by Fisher (1929), and studied by Goodman (1967, 1971), Keyfitz (1975) and others. For a reformulation of the concept and a generalization to multiregional demographic systems, see Rogers (1975a), Rogers and Willekens (1976b), and Willekens (1977). In this study we highlight only a few important elements of the theory of spatial reproductive value (Section 6.1), and then focus on computational algorithms (Section 6.2).

6.1 The Theory of the Spatial Reproductive Value

Fisher (1929) looks at life as a debt one has incurred at birth, and at the offspring of a child as the repayment of this debt. Let the debt or loan incurred at birth be equal to unity. At stability, the present value of the subsequent repayment must equal the debt, i.e.,

\[
1 = \int_0^\infty e^{-ra} m(a) \hat{\lambda}(a) da = \Psi(r) \quad (6.1)
\]
where \( m(a) \hat{\lambda}(a) da \) is the expected number of children to be born between ages \( a \) and \( a + da \) to a baby born in a life table population and following the observed fertility schedule, and \( r \) is the rate of discount. Equation (6.1) is of course identical to the characteristic equation of a single-region population system.

The multiregional counterpart of (6.1) is

\[
\{Q^S\} = \psi(r) \{Q^S\},
\]

where \( \{Q^S\} \) is the right eigenvector associated with the dominant eigenvalue of \( \psi(r) \). An alternative generalization of (6.1) is

\[
\{\psi(0)\}' = \{\psi(0)\}' \psi(r)
\]

where \( \{\psi(0)\}' \) is the corresponding left eigenvector of \( \psi(r) \) and where the prime denotes transposition.

Both formulations (6.2) and (6.3) have their demographic significance. Equation (6.2) has already been considered in Section 4 of this paper. The eigenvector \( \{Q^S\} \) gives the regional distribution of births in the stable population. Following the investment approach to life and childbearing, \( \{Q^S\} \) denotes that spatial distribution of the investments (or births) which makes the intrinsic rate of return of each investment equal to \( r \), the equilibrium rate of return.

Whereas \( \{O^S\} \) denotes the number of births, the left eigenvector \( \{\psi(0)\}' \) represents the marginal value of a 0-year old girl. The value is measured in terms of a contribution to the ultimate population of the demographic system. It reflects the capacity to produce new life. (Note that, since the model we consider is linear, the marginal value of one birth is equal to its average value.)

Exploring the investment approach to fertility analysis a little further, we note that if the regional distribution of
births is \( q^S \), then the present value of the offspring must also equal \( q^S \) (equation (6.2)). This implies that

\[
Q_i^S = \sum_j \pi_j (r) Q_j^S .
\]  

(6.4)

In each region, the discounted number of offspring must be equal to the current number of births. In other words, each region must pay back the debt it has incurred by receiving \( Q_i^S \) births. A part of this debt is paid back by people born in another region. People born in region \( j \), for example, contribute a total of \( j_{NNR_i} \times Q_j^S \) to region \( i \), which has a discounted value of \( j_{\pi_i} \times Q_j^S \).

Recall that in the numerical illustration of Slovenia - Rest of Yugoslavia,

\[
NRR = \begin{bmatrix}
0.9619 & 0.0107 \\
0.1224 & 1.1748
\end{bmatrix}.
\]

Equation (6.2) is

\[
\begin{bmatrix}
1.0000 \\
20.8237
\end{bmatrix} = \begin{bmatrix}
0.8137 & 0.0089 \\
0.1024 & 0.9950
\end{bmatrix} \begin{bmatrix}
1.0000 \\
20.8237
\end{bmatrix} .
\]

Therefore, a baby born in Slovenia is replaced by an average of

\[
0.9619 \times 1.0000 + 0.0107 \times 20.8237 = 1.184^a
\]

babies born in the stable population. An average of 0.9619 babies will be born to mothers who are born in Slovenia themselves, and 0.2225 will be born to mothers born in the Rest of Yugoslavia. The present value of 0.9619 babies is 0.8137 and of 0.2225 babies is
\[ 0.0089 \times 20.8237 = 0.1862. \] Hence the average present value of a baby born in Slovenia to a Slovenian-born woman is
\[
\frac{0.8137}{0.9619} = 0.8459,
\]
while that of a baby born in Slovenia to a Rest of Yugoslavia-born woman is
\[
\frac{0.9950}{1.1748} = 0.8469.
\]

The deviation is explained by the difference in mean ages at childbearing in the stable population and in the stationary population.

Equation (6.2) expresses births in one generation as a function of the number of births in the previous generation. It denotes the number of daughters by which a woman is replaced in the stable population, or, alternatively, the present value of the daughters replacing a woman, under the mortality and migration regime given by the life table. The regional distribution of births is consistent with the given fertility, mortality and migration schedules and with the growth rate or rate of discount, \( r \). Since these schedules differ from one region to another, whereas \( r \) is unique, a birth in a less fertile region contributes less to the sustainment of the overall \( r \) than a birth in a highly fertile area. The value of a birth for sustaining \( r \) depends on the capacity of the 0-year old to produce new lives. This capacity is measured by the reproductive value.

The vector \( \{ v_i(0) \} \) denotes the reproductive value of a baby or a 0-year old girl, by region of birth. If the reproductive value of a 0-year old in region \( i \) is \( v_i(0) \), then the value of the discounted number of offspring must also be \( v_i(0) \), which, for a two-region system, gives
\[
v_i(0) = v_i(0) \frac{\psi_i(r)}{1 + \psi_i(r)} + v_j(0) \frac{\psi_j(r)}{1 + \psi_j(r)} ,
\]
Equation (6.5) suggests an equivalent formulation: the present worth of the reproductive value of the offspring must equal the reproductive value of the 0-year old. If \( \nu_i(0) \) represents the value (cost) of the life invested in an individual, then that individual must pay back the value of this investment. Since \( \nu_i(0) \neq \nu_j(0) \), \( \sum_j \psi_j(r) \neq 1 \), which means that the discounted number of offspring of an individual does not have to be exactly unity.

Consider the Slovenia - Rest of Yugoslavia example. The matrix \( \psi(r) \) is given in Table 4.8. The left eigenvector is

\[
\{ \nu(0) \} = \begin{bmatrix} 1.0000 \\ 1.8181 \end{bmatrix}
\]

and equation (6.3) becomes

\[
\begin{bmatrix} 0.8137 & 0.0089 \\ 0.1024 & 0.9950 \end{bmatrix} \begin{bmatrix} 1.0000 \\ 1.8181 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 1.8181 \end{bmatrix}.
\]

If the reproductive value of a 0-year old in Slovenia is unity, then the reproductive value of a baby in the Rest of Yugoslavia
is 1.818. Any other scaling may be used since the eigenvector is fixed up to a scalar. Throughout this paper, the regional reproductive values are scaled such that $v_1(0) = 1$.

Note that the discounted number of daughters of a Slovenian-born girl is 0.916098, i.e. less than unity. Therefore, she does not replace herself by one child (discounted). The value of the offspring, however, is equal to her reproductive value at birth:

$$v_1(0) = 1.0000 = 1.0000 \times 0.8137 + 1.8181 \times 0.1024.$$  

6.2 The Computation of the Spatial Reproductive Value

The above interpretation of (6.3) suggests the question: what is the productive capacity of a girl aged $x$? The answer is the expected number of subsequent children discounted back to age $x$ and weighted for the region of birth. The vector of reproductive values of $x$-year old women, differentiated by region of residence, is

$$\{ \hat{v}(x) \}' = \{ \hat{v}(0) \}' \int_{x}^{\infty} e^{-r(a-x)} \frac{m(a)}{\hat{\ell}(a)} \, da \left[ \hat{\ell}(x) \right]^{-1}$$

$$= \{ \hat{v}(0) \}' \hat{n}(x), \text{ say.} \quad (6.7)$$

For example, in a two-region case, the matrix

$$\hat{n}(x) = \begin{bmatrix} n_{11}(x) & n_{21}(x) \\ n_{12}(x) & n_{22}(x) \end{bmatrix} \quad (6.8)$$

represents the expected total number of female offspring per woman at age $x$, discounted back to age $x$. The element $n_{ij}(x)$ gives the discounted number of daughters to be born in region $j$ to a woman now $x$ years of age and a resident of region $i$. 
There exist two approaches to evaluate (6.2) and (6.7) numerically. The first evaluates the reproductive values at exact age $x$:

$$\{z(x)\}' = \{z(0)\}' \sum_{a=x}^{b-5} e^{-r(a+2.5-x)} P(a)o(a)\{z(x)\}'^{-1}$$

$$= \{z(0)\}' \bar{n}_x' \text{, say} \quad (6.9)$$

Both $\bar{n}_x$ and $\{z(x)\}'$ refer to exact age $x$. The values of $\bar{n}_x$ for Slovenia - Rest of Yugoslavia are given in Table 6.1. For example, the discounted number of female descendants of a woman living in Slovenia and 10 years old is 1.0020. A total of 0.9168 are expected to be born in Slovenia and 0.0852 in the Rest of Yugoslavia. On the other hand, a woman of the same age in the Rest of Yugoslavia has an expected discounted number of daughters of 1.1984. Because of the low migration level out of the Rest of Yugoslavia and the relatively low fertility in Slovenia, an average of only 0.0087 daughters will be born to these women in Slovenia.

Reproductive values by age, $\{z(x)\}'$, are presented in Table 6.2. For example, the reproductive value of 10-year old girls is

$$\{z(10)\}' = \{z(0)\}' \bar{n}_{10}$$

or

$$\begin{bmatrix} 0.9168 & 0.0087 \\ 1.0717 & 2.1718 \end{bmatrix} = \begin{bmatrix} 1.0000 & 1.8181 \\ 0.0852 & 1.1998 \end{bmatrix}.$$  

Note that $\bar{n}_0$ is identical to the characteristic matrix $\psi(r)$.  

-120-
Table 6.1. Discounted number of offspring per person of exact age x.

<table>
<thead>
<tr>
<th>age</th>
<th>region of residence</th>
<th>region of birth of offspring</th>
<th>total</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.916098</td>
<td>0.813684</td>
<td>0.102414</td>
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<td></td>
</tr>
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<td>0.042713</td>
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<td>0.004837</td>
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<td>75</td>
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</table>
Table 6.2. Spatial reproductive value per person of exact age $x$.

<table>
<thead>
<tr>
<th>Age</th>
<th>Slovenia</th>
<th>Yugoslavia</th>
</tr>
</thead>
<tbody>
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<td>1.818099</td>
</tr>
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<td>5</td>
<td>1.049434</td>
<td>2.098617</td>
</tr>
<tr>
<td>10</td>
<td>1.071744</td>
<td>2.171764</td>
</tr>
<tr>
<td>15</td>
<td>1.097740</td>
<td>2.244651</td>
</tr>
<tr>
<td>20</td>
<td>1.025120</td>
<td>2.083262</td>
</tr>
<tr>
<td>25</td>
<td>0.669643</td>
<td>1.350547</td>
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<tr>
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<td>0.356510</td>
<td>0.716647</td>
</tr>
<tr>
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<td>0.155004</td>
<td>0.335225</td>
</tr>
<tr>
<td>40</td>
<td>0.043153</td>
<td>0.131025</td>
</tr>
<tr>
<td>45</td>
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<td>0.025046</td>
</tr>
<tr>
<td>50</td>
<td>0.001429</td>
<td>0.006290</td>
</tr>
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<td>60</td>
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<tr>
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<td>0.000000</td>
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<td>0.000000</td>
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</tr>
<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
The second approach computes the average reproductive value for each age group \( x \) to \( x + 4 \). Denoting this by \( \{ \frac{V}{x} \} \), we have

\[
\{ \frac{V}{x} \} = \{ v(0) \}, \frac{5}{2} \sum_{a=x}^{x+4} \left[ e^{-r(a-x)} F(a) L(a) + e^{-r(a+5-x)} F(a + 5) L(a + 5) \right]^{-1}
\]

\[
= \{ v(0) \}, \frac{5}{2} \sum_{a=x}^{x+4} \left[ F(a) + e^{-5r} F(a + 5) S(a) \right] e^{-r(a-x)} L(a) [L(x)]^{-1}
\]

\[
= \{ v(0) \}, \frac{5}{2} N_x , \text{ say.}
\]

The matrix \( N_x \) gives the discounted number of offspring per person in age group \( x \) to \( x + 4 \), and not the number per person at exact age \( x \) (Table 6.3). It has been shown by Willekens (1977, p.14) that \( N_x \) may be expressed in terms of \( N_{x+5} \):

\[
N_x = \frac{5}{2} F(x) + \left[ \frac{5}{2} F(x + 5) + N_{x+5} \right] e^{-5r} S(x) \quad (6.13)
\]

Yugoslavian average reproductive values by age group are listed in Table 6.4.

The discounted number of offspring and the reproductive value in (6.12) and (6.13) are expressed per person in age group \( x \) to \( x + 4 \) of the life table population. To obtain an estimate of the discounted number of offspring of the total observed population (NK) and of its reproductive value, we multiply \( N_x \) and \( V_x \) by the observed population distribution and sum over all age groups:

\[
NK = \sum_{x=0}^{x=5} N_x K(x) \quad (6.14)
\]
Table 6.3. Discounted number of offspring per person in age group $x$ to $x + 4$.

<table>
<thead>
<tr>
<th>region of residence</th>
<th>slovenia</th>
<th>r.yugos.</th>
<th>total slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>region of birth of offspring</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>total</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>slovenia</td>
<td>r.yugos.</td>
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<table>
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<tr>
<th>region of residence</th>
<th>r.yugos.</th>
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<td>region of birth of offspring</td>
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<td>65</td>
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<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
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</tr>
<tr>
<td></td>
<td>Slovenia</td>
<td>r.yugos.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
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<td>1.024976</td>
<td>1.950056</td>
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<td>5</td>
<td>1.060599</td>
<td>2.134870</td>
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<td>1.084696</td>
<td>2.207902</td>
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<td>15</td>
<td>1.061386</td>
<td>2.165788</td>
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<td>20</td>
<td>0.846927</td>
<td>1.724361</td>
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<td>25</td>
<td>0.513473</td>
<td>1.040082</td>
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<td>30</td>
<td>0.256701</td>
<td>0.529859</td>
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<td>35</td>
<td>0.099936</td>
<td>0.235284</td>
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<td>40</td>
<td>0.024307</td>
<td>0.079257</td>
<td></td>
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<td>0.003176</td>
<td>0.015903</td>
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<td>0.000731</td>
<td>0.0033245</td>
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<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.4. Spatial reproductive value per person in age group x to x + 4.
and

\[ \{ V \} = \{ v(0) \}' \sum_{x} K(x) = \{ v(0) \}' NK , \]  

(6.15)

where \( K(x) \) is the diagonal matrix containing the regional populations aged \( x \) to \( x + 4 \).

The value of \( NK \) for Yugoslavia is given in Table 6.5. Under the 1961 regime of fertility, mortality and migration, the total discounted number of female offspring is 5,528,633. Of them, 382,697 or 6.92% will be born in Slovenia. However, the female residents of Slovenia will account for only 379,094 or 6.68% of the total discounted number of births. Of the ultimate discounted 382,697 female children born in Slovenia, 29,936 can be attributed to women now residing in the Rest of Yugoslavia. On the other hand, of the discounted 379,094 daughters born to the female population of Slovenia, 26,333 will be born in the Rest of Yugoslavia and 352,761 in Slovenia.

The reproductive value of the total female population by place of residence is obtained by weighting the discounted number of offspring for the region of birth, as in (6.15). If we attach to a birth in Slovenia the reproductive value of unity, then a birth in the Rest of Yugoslavia has a reproductive value of 1.818. Adopting this scaling, the total reproductive value by region of residence is

\[
\begin{bmatrix}
352,761 & 29,936 \\
26,333 & 5,119,602
\end{bmatrix}
= 
\begin{bmatrix}
400,637 \\
9,337,829
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.0000 & 1.8181
\end{bmatrix}
\cdot
\]

The total reproductive value for the whole of Yugoslavia is (Table 6.6)

\[ V = 400,637 + 9,337,829 = 9,738,466 . \]
Table 6.5. Total discounted number of offspring of observed population.

<table>
<thead>
<tr>
<th></th>
<th>total</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>382697.</td>
<td>352761.</td>
<td>29936.</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>5145935.</td>
<td>26333.</td>
<td>5119602.</td>
</tr>
<tr>
<td>total</td>
<td>5528633.</td>
<td>379094.</td>
<td>5149538.</td>
</tr>
</tbody>
</table>

Table 6.6. Reproductive value of the total population.

<table>
<thead>
<tr>
<th></th>
<th>total percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>400637.</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>9337829.</td>
</tr>
<tr>
<td>total</td>
<td>9738466.</td>
</tr>
</tbody>
</table>
Note that the unit in which $V$ is measured is the reproductive value of a birth or a 0-year old in Slovenia. The choice of the unit is arbitrary, since its only function is that of a "numeraire".
7. FURTHER STABLE POPULATION ANALYSIS

In Sections 4 and 5 of this report, we performed some introductory analyses of the fertility and migration characteristics of stationary populations. In this section, stable population analysis is advanced by means of the notions of spatial reproductive value that was developed in the previous section.

If age-specific birth, death and migration rates remain fixed, then a population exposed to these rates ultimately will evolve into a stable population whose principal characteristics are: unchanging regional age compositions and regional shares; constant regional annual rates of birth, death, and migration; and a fixed multiregional annual rate of growth that also is the annual rate of population growth in each and every region (Rogers and Willekens, 1976c, p. 12). The constant growth rate implies that births and population increase at the same rate and follow an exponential growth path. This trajectory may be expressed in terms of observed population characteristics. This is the topic of the first part of this section. The second part focuses on the calculation of the intrinsic rates of birth, death, out- and immigration.

7.1 The Ultimate Trajectory of Births and Population

When a multiregional population system has been projected to stability, its births grow exponentially and their regional distribution remains constant. The ultimate birth trajectory then is (Willekens, 1977, p. 29):†

\[
\{Q(t)\} = e^{r t} \frac{V}{\{\gamma(0)\}' \times \{Q_1\}} 
\]  

(7.1)

where \( r \) is the stable growth rate, \( V \) is the total reproductive value of the whole population system, \( \{\gamma(0)\}' \) and \( \{Q_1\} \) are, respectively, the left and right eigenvectors of \( \gamma(r) \), associated

†The superscript of \( \{Q^s\} \) is dropped for convenience.
with the dominant eigenvalue, and $\kappa$ is the matrix of mean ages of childbearing in the stable population, defined in (4.30) as:

$$\kappa = [R(1)][R(0)]^{-1}.$$  \hspace{1cm} (7.2)

The expression $(\psi(0))'\kappa Q_1$ is a normalizing factor. Writing

$$\kappa = (\psi(0))'\kappa Q_1,$$  \hspace{1cm} (7.3)

yields the simple expression for the ultimate birth trajectory:

$$\{q(t)\} = e^{rt} \frac{V}{\kappa} Q_1.$$  \hspace{1cm} (7.4)

If $Q_1$ is chosen such that its elements sum up to unity, then the ultimate total number of births is proportional to the total reproductive value. The total number of births is then allocated to the different regions according to $Q_1$. Substituting $V$ of (6.15) into (7.4) shows that the stable number of births in each region $Q(t)$ may also be expressed as a linear combination of the discounted number of offspring by region of birth (see Willekens, 1977, pp. 32-33). The vector of stable equivalent births is:

$$\{q(0)\} = \{q\} = \frac{V}{\kappa} Q_1.$$  \hspace{1cm} (7.5)

Recall our numerical illustration. The matrix of mean ages of childbearing is given in Table 4.9. Since the growth rate $r$ is 0.006099, the normalizing factor $\kappa$ is 1054.256 (Table 7.2). The total reproductive value $V$ has been computed to be 9,738,466; hence the stable equivalent of births is by (7.5):

$$Q = \frac{9,738,466}{1054.256} = \begin{bmatrix} 1.0000 \\ 20.8237 \end{bmatrix} = \begin{bmatrix} 9,237 \\ 192,355 \end{bmatrix}.$$  \hspace{1cm} (7.6)
The total number of births is 201,592. Of this number of babies 4.58% will be born in Slovenia and 95.42% in the Rest of Yugoslavia.†

The stable equivalent population in each age group \( x \) to \( x + 4 \) is easily obtained by the formula (3.22):

\[
\{K(x)\} = e^{-r(x+2.5)} \{L(x)\} . \tag{3.22}
\]

The stable equivalent of the total population is:

\[
\{Y\} = \sum_{x} \{K(x)\} = \left[ \sum_{x} e^{-r(x+2.5)} \{L(x)\} \right] . \tag{7.7}
\]

Defining

\[
\sum_{x} e^{-r(x+2.5)} \{L(x)\} = e^{(r)}(x) \tag{7.8}
\]

as the matrix of discounted life expectancies at birth, reduces equation (7.7) to

\[
\{Y\} = e^{(r)}(0) . \tag{7.9}
\]

The numerical values of Yugoslavia's stable equivalent population are given in Table 7.1. Note that those values are very close to the ones given in Table 3.3, which was obtained by projecting the observed population.‡‡

Equations (3.22) and (7.7) demonstrate that for population analysis it is more convenient to express the relative age composition of the population in unit births instead of in fractions or percentages of the total population. The values of

†Compare this allocation with the observed number of births (205,010) and the regional distribution: 6.90% in Slovenia and 93.10% in the Rest of Yugoslavia.
‡‡Minor deviations are due to rounding error.
Table 7.1. Stable equivalent of total population.

<table>
<thead>
<tr>
<th></th>
<th>total Slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>941653.</td>
<td>45036.</td>
</tr>
<tr>
<td>5</td>
<td>862430.</td>
<td>43385.</td>
</tr>
<tr>
<td>10</td>
<td>834173.</td>
<td>42247.</td>
</tr>
<tr>
<td>15</td>
<td>806462.</td>
<td>41802.</td>
</tr>
<tr>
<td>20</td>
<td>778259.</td>
<td>42121.</td>
</tr>
<tr>
<td>25</td>
<td>749753.</td>
<td>41965.</td>
</tr>
<tr>
<td>30</td>
<td>721359.</td>
<td>41049.</td>
</tr>
<tr>
<td>35</td>
<td>693192.</td>
<td>39990.</td>
</tr>
<tr>
<td>40</td>
<td>664290.</td>
<td>38859.</td>
</tr>
<tr>
<td>45</td>
<td>633668.</td>
<td>37448.</td>
</tr>
<tr>
<td>50</td>
<td>599231.</td>
<td>35742.</td>
</tr>
<tr>
<td>55</td>
<td>558765.</td>
<td>33875.</td>
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<tr>
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<td>507986.</td>
<td>31559.</td>
</tr>
<tr>
<td>65</td>
<td>441715.</td>
<td>28066.</td>
</tr>
<tr>
<td>70</td>
<td>356051.</td>
<td>22843.</td>
</tr>
<tr>
<td>75</td>
<td>257353.</td>
<td>16436.</td>
</tr>
<tr>
<td>80</td>
<td>159177.</td>
<td>9514.</td>
</tr>
<tr>
<td>85</td>
<td>151491.</td>
<td>5819.</td>
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</table>

**Total**: 10717010. 597806. 10119204.

**Percentage distribution**

<table>
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<td>7.542</td>
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<tr>
<td>5</td>
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</tr>
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<td>7.784</td>
<td>7.057</td>
</tr>
<tr>
<td>15</td>
<td>7.525</td>
<td>6.993</td>
</tr>
<tr>
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<td>7.262</td>
<td>7.046</td>
</tr>
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<td>25</td>
<td>6.996</td>
<td>7.020</td>
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<td>6.867</td>
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<td>6.468</td>
<td>6.690</td>
</tr>
<tr>
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<td>6.193</td>
<td>6.500</td>
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<tr>
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<td>5.913</td>
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<tr>
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<td>5.591</td>
<td>5.979</td>
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<td>4.695</td>
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<tr>
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<td>3.821</td>
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<tr>
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<td>2.749</td>
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<tr>
<td>80</td>
<td>1.485</td>
<td>1.592</td>
</tr>
<tr>
<td>85</td>
<td>1.414</td>
<td>0.973</td>
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</table>

**Total**: 100.000 100.000 100.000

**Share**: 100.000 5.578 94.422
are given in Table 3.4.

7.2 Stable Equivalents and Intrinsic Rates

The fertility, mortality and migration characteristics of a stable population may be described by a small number of parameters, e.g., the intrinsic rates (Rogers, 1975a, pp. 109-115). The intrinsic rates are directly related to the stable equivalents of births, deaths, and migrants. Therefore, we treat both rates and equivalents simultaneously.

Applying the fixed age-specific schedules of fertility, mortality and migration to the stable equivalent of the population gives the stable equivalent of births, deaths and migrants. The stable equivalent of births has already been computed. Applying the fertility schedule to the population distribution of (3.22) and summing over all age groups yields, of course, the characteristic equation:

\[ \{Q\} = \sum_x F(x)\{K(x)\} = \sum_x F(x) e^{-r(x+2.5)} L(x) \{Q\} = \overline{v}(r)\{Q\} \]

The intrinsic birth rate of region i is the ratio between \(Q_i\) and the stable equivalent population \(Y_i\), which may be written as (Rogers, 1975a, p. 115):

\[ b_i = \frac{Q_i}{Y_i} = \frac{Q_i}{\sum_x e^{-r(x+2.5)} \sum_j j_0 b_i(x) Q_j} = \frac{1}{\sum_x e^{-r(x+2.5)} \sum_j \frac{Q_j}{Q_i} j_0 L_i(x)} \]
The vector of intrinsic birth rates is

\[
\{b\} = Y^{-1} \{Q\} ,
\]

(7.10)

where \( Y \) is the diagonal matrix of stable equivalent total populations, i.e.

\[
Y \{1\} = \{Y\} .
\]

The vector \( \{b\} \) also may be expressed as

\[
\{b\} = \sum \frac{P(x) \{C(x)\}}{x} ,
\]

(7.11)

where \( \{C(x)\} \) denotes the age composition of the population as fractions of the total, i.e.

\[
\{C(x)\} = Y^{-1} \{K(x)\} .
\]

(7.12)

The proportion of the regional population, which is aged \( x \) to \( x + 4 \), may be expressed as

\[
\{C(x)\} = \Phi^{-1} b \ e^{-r(x+2.5)} L(x) \{Q\} ,
\]

(7.13)

since by (7.10) \( Y^{-1} \) is equal to \( \Phi^{-1} b \), where both \( \Phi \) and \( b \) are diagonal matrices. Defining \( \{C(x)\} \) as

\[
\{C(x)\} = b \ e^{-r(x+2.5)} L(x) \ {Q},
\]

(7.14)

gives

\[
\{C(x)\} = \Phi^{-1} \{C(x)\} \{Q\} .
\]

(7.15)

To compute the stable equivalents of deaths, outmigrants and inmigrants, we must reconsider the age-specific death and migration rates (Ledent, 1977). The deaths and outmigrants in age
group \( x \) to \( x + 4 \) in a life table population are given by (Rogers and Ledent, 1976, p. 289),

\[ \hat{\ell}(x) - \hat{\ell}(x + 5) = M(x) \hat{L}(x) , \]  

(7.16)

where \( \hat{\ell}(x) \) represents the distribution of the life table population at exact age \( x \) by place of birth and place of residence (in terms of unit born),

\( L(x) \) is given in (2.10) and represents the distribution of the life table population aged \( x \) to \( x + 4 \) by place of birth and place of residence (in terms of unit born), and

\( M(x) \) is the matrix in (2.24).

Equation (7.16) is the discrete approximation of the continuous relation

\[ \hat{\ell}(x) - \hat{\ell}(x + 5) = \int_0^5 \mu(x + t) \hat{\ell}(x + t) dt , \]  

(7.17)

where \( \mu(x) \) is a matrix of the format \( M(x) \). Its elements are the age-specific forces of mortality \( \mu_{i0}(x) \) and of migration \( \mu_{ij}(x) \), i.e.

\[ \mu(x) = - \frac{1}{dx} [d\hat{\ell}(x)] [\hat{\ell}(x)]^{-1} = - \frac{d \ln \hat{\ell}(x)}{dx} . \]

Equation (7.17) represents the decrements due to death and outmigrations in a stationary population. To derive the decrements in a population growing at rate \( r \), we write

\[ \hat{\ell}(r)(x) - \hat{\ell}(r)(x + 5) = \int_0^5 \mu(x + t) \hat{\ell}(r)(x + t) dt , \]
with \( \hat{\lambda}(r)(x) = e^{-rx} \hat{\lambda}(x) \). Hence

\[
\hat{\lambda}(r)(x) - \hat{\lambda}(r)(x + 5) = \int_{0}^{5} \mu(x + t) e^{-r(x+t)} \hat{\lambda}(x + t) dt ,
\]

\[
= \int_{0}^{5} e^{-r(x+t)} d \hat{\lambda}(x + t) . \tag{7.18}
\]

Integration by parts yields

\[
\hat{\lambda}(r)(x) - \hat{\lambda}(r)(x + 5) = e^{-rx} \hat{\lambda}(x) - e^{-r(x+5)} \hat{\lambda}(x + 5)
\]

\[
- r \int_{0}^{5} e^{-r(x+t)} \hat{\lambda}(x + t) dt .
\]

\[
= e^{-rx} \hat{\lambda}(x) - e^{-r(x+5)} \hat{\lambda}(x + 5)
\]

\[
- r \hat{L}_{r}(x) .
\]

The age-specific death and outmigrant rates in the stable population are given by the matrix

\[
M^{(r)}(x) = [\hat{\lambda}(r)(x) - \hat{\lambda}(r)(x + 5)] [\hat{L}_{r}(x)]^{-1}
\]

\[
= [e^{-rx} \hat{\lambda}(x) - e^{-r(x+5)} \hat{\lambda}(x + 5) - r \hat{L}_{r}(x)] [\hat{L}_{r}(x)]^{-1}
\]

\[
= [e^{-rx} \hat{\lambda}(x) - e^{-r(x+5)} \hat{\lambda}(x + 5)] \left[ e^{-r(x+2.5)} \hat{L}(x) \right]^{-1} .
\]
which after substitution yields

$$M_r^z(x) = \frac{2}{5} e^{2.5r} [I - e^{-5r} \rho(x)][I + \rho(x)]^{-1} - rI. \quad (7.19)$$

For the last age group $z$, the rates are:

$$M_r^z(z) = \hat{\lambda}_r^z(z)[L_r^z(z)]^{-1} - rI$$

$$= e^{2.5r} \hat{\lambda}_r^z(z)[L(z)]^{-1} - rI$$

$$= e^{2.5r} \lambda_r(z) - rI. \quad (7.20)$$

The outmigration rates $M_r^z(x)$ are contained in the off-diagonal elements of $M_r^z(x)$. The death rates $M_r^z(x)$ are equal to the diagonal elements minus the outmigration rates, i.e. plus the off-diagonal elements in the same column.

To facilitate further analysis, define the diagonal matrix $\delta M_r^z(x)$ of regional death rates, and the diagonal matrix $\delta O_r^z(x)$ of total regional outmigration rates, i.e.

$$\delta O_r^z(x) = \sum_{j \neq i}^z M_r^z(x). \quad (7.22)$$

Let $\delta M_r^z(x)$ be the matrix of outmigration rates, i.e.

---

+ Compare this with the formula for the life table (= observed) rates. Solving equation (2.25)

$$P_r(x) = [I - \frac{5}{2} M_r(x)]^{-1} [I - \frac{5}{2} \bar{M_i}(x)]$$

for $\bar{M_i}(x)$ yields

$$\bar{M_i}(x) = \frac{2}{5} [I - \rho(x)][I + \rho(x)]^{-1} \quad (7.21)$$
Once consistent age-specific death and migration rates are derived, we may proceed with the computation of the stable equivalents of deaths and out- and inmigrants, and of the associated intrinsic rates. The stable equivalent of deaths is

\[ \{D\} = \sum_x \delta \tilde{M}^{(r)}(x) \{K(x)\} \]

\[ = \left[ \sum_x \delta \tilde{M}^{(r)}(x) e^{-r(x+2.5)\tilde{L}(x)} \right] \{Q\} . \]  

(7.24)

The intrinsic death rates follow immediately:

\[ \{d\} = \gamma^{-1} \{D\} \]  

(7.25)

or

\[ \{d\} = \sum_x \delta \tilde{M}^{(r)}(x) \{C(x)\} . \]  

(7.26)

The stable equivalent of the outmigrants from region i to region j is

\[ O_{ij} = \sum_x \tilde{M}_{ij}^{(r)}(x) K_i(x) \]  

(7.27)

where \( \tilde{M}_{ij}^{(r)}(x) \) is the age-specific migration rate and \( K_i(x) \) is the stable population of region i aged x to x + 4. In general, we
may write the origin destination flow of stable equivalent migrations as

\[ \Phi = \sum_{x} O_{oM}^{x}(r) K(x) \tag{7.28} \]

where \( O_{oM}^{x}(r) \) is defined in (7.23) and \( K(x) \) is a diagonal matrix of stable regional populations of ages \( x \) to \( x + 4 \). The outmigration rates are simply

\[ \dot{\Phi} = \Phi \gamma^{-1} \tag{7.29} \]

or

\[ \dot{\Phi} = \sum_{x} O_{oM}^{x}(r) \bar{C}(x) \tag{7.30} \]

where \( \bar{C}(x) = K(x) \gamma^{-1} \), i.e. \( \bar{C}(x)^{(1)} = \{C(x)\} \).

The stable equivalent of the total number of outmigrants is

\[ \{O\}' = \{1\}' \Phi \tag{7.31} \]

and the total outmigration rates are:

\[ \{\dot{\Phi}\}' = \{1\}' \Phi \gamma^{-1} = \{\dot{\Phi}\}' \gamma^{-1} \tag{7.32} \]

or

\[ \{\Phi\}' = \{1\}' \Phi \gamma^{-1} = \{\Phi\}' \gamma^{-1} . \]

Another expression for (7.31) is:

\[ \{O\}' = \sum_{x} \{1\}' O_{oM}^{x}(r) K(x) \]

\[ = \sum_{x} \{O_{oM}^{x}(r)^{'} K(x) \}. \tag{7.33} \]
where \( \{ O_M(x) \} \) is the vector of total outmigration rates, defined in (7.22).

The stable equivalent of the total number of immigrants by region is

\[
\{ I \} = O(1)
\]

(7.34)

and the immigration rates are

\[
\{ i \} = Y^{-1} \{ I \}
\]

\[
= Y^{-1} O(1)
\]

(7.35)

The matrix \( h = Y^{-1} O \) contains immigration rates by region of origin and region of destination. An element \( h_{ij} \) describes the migrants from region \( i \) to \( j \) as a fraction of the population in \( j \).

There exists a unique relationship between immigration rates and outmigration rates. Since by (7.29)

\[
O = O Y
\]

we have

\[
\{ i \} = Y^{-1} O Y
\]

(7.36)

and the total immigration rates

\[
\{ i \} = \{ i \} = Y^{-1} O Y (1)
\]

(7.37)

The stable equivalents of births, deaths and outmigrants and inmigrants for Yugoslavia are given in Table 7.2, together with the intrinsic rates. Note that the intrinsic rates obey the following definitional relationship:

\[
r = b_i - d_i - o_i + i_i
\]

(7.38)

Thus equation (7.38) provides an independent check of the results.
Table 7.2. Stable equivalents and intrinsic rates.

<table>
<thead>
<tr>
<th>Region</th>
<th>Births</th>
<th></th>
<th>Deaths</th>
<th></th>
<th>Outmigration</th>
<th></th>
<th>Immigration</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>number</td>
<td>rate</td>
<td>number</td>
<td>rate</td>
<td>number</td>
<td>rate</td>
<td>number</td>
<td>rate</td>
</tr>
<tr>
<td>slovenia</td>
<td>9237</td>
<td>0.015452</td>
<td>7177</td>
<td>0.012005</td>
<td>1469</td>
<td>0.002457</td>
<td>3050</td>
<td>0.005102</td>
</tr>
<tr>
<td>R.yugos.</td>
<td>192355</td>
<td>0.019009</td>
<td>129044</td>
<td>0.012753</td>
<td>3050</td>
<td>0.000301</td>
<td>1469</td>
<td>0.000145</td>
</tr>
<tr>
<td>total</td>
<td>201592</td>
<td>0.018810</td>
<td>136224</td>
<td>0.012711</td>
<td>4519</td>
<td>0.003422</td>
<td>4519</td>
<td>0.000422</td>
</tr>
</tbody>
</table>

stable growth rate 0.006039
normalizing factor 1054.2560
8. SPATIAL ZERO POPULATION GROWTH

The demographic system we have considered thus far is one that is characterized by constant fertility, mortality and migration schedules. The ultimate population evolving under these conditions is a stable population, with the following features: fixed age and regional structures, unchanging regional birth, death, and migration rates, and a unique and constant growth rate.

The growing public concern about rapid population increase has generated a vast literature on the social and economic impacts of high fertility and has focused attention on fertility decline as a means for relieving socio-economic problems. An immediate drop of fertility to replacement level, however, would not stop population growth. Children outnumber parents in a growing population. Consequently, the number of potential parents in the next generation will be larger than at present. This built-in tendency for continued growth causes the number of people to increase for some time before the population becomes stationary (i.e. stable, but with zero growth). The ratio by which the ultimate stationary population exceeds a current population undisturbed by migration has been studied by Keyfitz (1971).

Although population growth is an important concern, where people choose to live in the future presents issues and problems that are potentially as serious as those posed by the number of children they choose to have. A drop in fertility, for example, not only causes the population to continue to grow for a while, but, in the presence of migration, also affects the regional distribution of this population. The spatial impact of fertility reduction has been studied by Rogers and Willekens (1976a, 1976b).

The spatial momentum of zero population growth may be computed numerically and, if the initial population is stable, analytically. In the first section, the numerical approach is discussed. The analytical approach is examined in the following section.
8.1 The Numerical Approach

The numerical approach to spatial zero population growth analysis substitutes fertility schedules representing fertility at replacement level in place of observed fertility schedules. All the computations for population projection and stable population analysis are repeated, and the new results are then compared with the results obtained using the original fertility schedules.

Many alternative fertility reduction schemes are possible. Some age groups may have a proportionally greater decline than others because of differences in birth control practices or because of shifts in the patterns of marriage and divorce. Alternatively, the decline may be age-independent, i.e. the proportional decline of age-specific fertility rates is the same at all ages. Keyfitz (1971) considers a fertility drop that is age-independent. Most demographers have followed this practice and it is also adopted in the discussion that follows.

Regional differences in fertility decline are introduced through two alternative schemes:

Alternative 1: the cohort replacement alternative: the fertility of each female cohort is reduced to bare replacement level in each region, i.e. to a level of one daughter (net) per woman born there.

Alternative 2: the proportional reduction alternative: every regional fertility schedule is reduced by the same proportion at all ages.

To derive mathematical expressions for both alternatives, recall (4.6), which may be written as

\[
\{Q_2\} = R(0)\{Q_1\} = \sum_x F(x) L(x)\{Q_1\},
\]

where \(\{Q_1\}\) is the vector of births and \(\{Q_2\}\) the vector of their offspring, i.e. births in the next generation. Equation (8.1)
expresses the births in one generation as a function of births in the previous generation.

A multiregional population system that is growing exhibits a net reproduction matrix $R(0)$ with a dominant characteristic root $\lambda_1[R(0)]$ that is greater than unity. The total number of offspring per woman born in a certain region is given by the column totals of $R(0)$, i.e.

$$iR(0) = \sum_j iR_j(0) .$$

If fertility is reduced according to the cohort replacement alternative, then

$$iR(0) = 1 \quad \text{for all } i ;$$

or, in matrix form,

$$R(0)^{-1} = \{1\} . \quad (8.2)$$

This means that every woman would have a net reproduction rate of unity. The problem is now to determine by how much the observed age-specific fertility rates must be altered for this to occur.

Let $\gamma_i$ be the required fertility adjustment factor for region $i$, i.e.

$$iR(0) = 1 = \gamma_i iR(0) .$$

In general, we have

$$\hat{R}(0) = \gamma R(0) , \quad (8.3)$$

where $\gamma$ is a diagonal matrix of regional fertility adjustment factors. Substituting (8.3) into (8.2) gives

$$R(0)^{-1} \gamma \{1\} = \{1\} ,$$
whence

\[ \{ \gamma \} = \{ R(0) \}^{-1} \{ 1 \} \quad (8.4) \]

Therefore, the cohort replacement alternative yields the replacement fertility rates \( \hat{F}(x) \):

\[ \hat{F}(x) = \gamma \hat{F}(x) \quad (8.5) \]

where \( \hat{F}(x) \) is the diagonal matrix of observed regional fertility rates of age group \( x \) to \( x + 4 \), and \( \gamma \) is the diagonal matrix with the elements of \( \{ \gamma \} \) in the diagonal.

Recall our numerical illustration: the two-region system of Slovenia and the Rest of Yugoslavia. The matrix of fertility adjustment factors is given in Table 8.1. Since the women of both regions originally had a net reproduction rate greater than unity, the fertility adjustment factors are less than one, causing a fertility drop in both regions. In Slovenia, fertility rates drop to 93.24% of their original values, whereas in the Rest of Yugoslavia they decline to 84.27% of their previous levels. The difference is caused by differences in the initial fertility levels. The new fertility rates \( \hat{F}(x) \) are also given in Table 8.1. Note that the gross rates of reproduction must decrease in the same proportion as the age-specific fertility rates.

With these new rates, fertility analysis is performed as before (see Sections 4, 6 and 7). The results are listed in Tables 8.2 to 8.13. A comparison of these results with Tables 4.2 to 4.9 reveals the impact of the fertility drop to replacement level.

In the proportional reduction alternative, the age-specific fertility rates of each region are reduced by the same proportion. The fertility adjustment factor is identical for each region and is equal to

\[ \gamma_i = \gamma_j = \gamma = \frac{1}{\lambda_1} \quad (8.6) \]
Table 8.1. Zero population growth alternative 1.

matrix of fertility adjustment factors
-------------------------------------
slovenia    r.yugos.

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>0.932431</td>
<td>0.000000</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.000000</td>
<td>0.842719</td>
</tr>
<tr>
<td>total</td>
<td>0.932431</td>
<td>0.842719</td>
</tr>
</tbody>
</table>

fertility analysis

age-specific rates

<table>
<thead>
<tr>
<th>age</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>15</td>
<td>0.014785</td>
<td>0.022295</td>
</tr>
<tr>
<td>20</td>
<td>0.059878</td>
<td>0.074141</td>
</tr>
<tr>
<td>25</td>
<td>0.059940</td>
<td>0.052580</td>
</tr>
<tr>
<td>30</td>
<td>0.033326</td>
<td>0.037324</td>
</tr>
<tr>
<td>35</td>
<td>0.021317</td>
<td>0.019631</td>
</tr>
<tr>
<td>40</td>
<td>0.007271</td>
<td>0.010156</td>
</tr>
<tr>
<td>45</td>
<td>0.005682</td>
<td>0.001812</td>
</tr>
<tr>
<td>50</td>
<td>0.00273</td>
<td>0.000632</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>60</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
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</tr>
<tr>
<td>75</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>80</td>
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<tr>
<td>85</td>
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<td>0.000000</td>
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<tr>
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<td>0.000000</td>
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<tr>
<td>95</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
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<td>0.000000</td>
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</tr>
<tr>
<td>crr</td>
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<td>1.143994</td>
</tr>
</tbody>
</table>
Table 8.2. Integrals of generalized net maternity function.

<table>
<thead>
<tr>
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<th>r.yugos.</th>
</tr>
</thead>
<tbody>
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<td>0.000000</td>
</tr>
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<tr>
<td>25</td>
<td>0.253648</td>
<td>0.030305</td>
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<tr>
<td>30</td>
<td>0.160491</td>
<td>0.021555</td>
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<td>35</td>
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<tr>
<td>40</td>
<td>0.029433</td>
<td>0.006582</td>
</tr>
<tr>
<td>45</td>
<td>0.002633</td>
<td>0.001182</td>
</tr>
<tr>
<td>50</td>
<td>0.001058</td>
<td>0.000389</td>
</tr>
<tr>
<td>55</td>
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<td>0.000000</td>
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<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
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<td>0.000000</td>
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<tr>
<td>75</td>
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<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.896882</td>
<td>0.103118</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>age</th>
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<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>0.000000</td>
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<td>10</td>
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<td>0.000250</td>
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<td>15</td>
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<td>0.002449</td>
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<td>25</td>
<td>0.003028</td>
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<td>30</td>
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<tr>
<td>80</td>
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<td>0.000000</td>
</tr>
<tr>
<td>85</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>total</td>
<td>0.009965</td>
<td>0.990035</td>
</tr>
</tbody>
</table>
Table 8.3. Moments of integral function.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 moment</td>
<td>0.896882</td>
<td>0.009965</td>
</tr>
<tr>
<td></td>
<td>0.103118</td>
<td>0.990035</td>
</tr>
<tr>
<td></td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 moment</td>
<td>24.708921</td>
<td>0.292468</td>
</tr>
<tr>
<td></td>
<td>3.023252</td>
<td>27.103607</td>
</tr>
<tr>
<td></td>
<td>27.732174</td>
<td>27.396076</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
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<th>R.yugos.</th>
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</thead>
<tbody>
<tr>
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<td>8.9751</td>
</tr>
<tr>
<td></td>
<td>93.4012</td>
<td>786.4307</td>
</tr>
<tr>
<td></td>
<td>809.4550</td>
<td>795.4058</td>
</tr>
</tbody>
</table>
Table 8.4. Spatial fertility expectancies.

<table>
<thead>
<tr>
<th></th>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>0.896882</td>
<td>0.009965</td>
</tr>
<tr>
<td>R. Yugos.</td>
<td>0.103118</td>
<td>0.990035</td>
</tr>
<tr>
<td>Total</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

**Net reproduction rate**

<p>| | |</p>
<table>
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</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
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<tr>
<td>Eigenvector</td>
<td></td>
</tr>
<tr>
<td>- Right</td>
<td>1.000000  10.348280</td>
</tr>
<tr>
<td>- Left</td>
<td>1.000000  1.000005</td>
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</tbody>
</table>

**Net reproduction allocations**

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>0.896882  0.009965</td>
</tr>
<tr>
<td>R. Yugos.</td>
<td>0.103118  0.990035</td>
</tr>
<tr>
<td>Total</td>
<td>1.000000  1.000000</td>
</tr>
</tbody>
</table>
Table 8.5. Matrices of mean ages and variances.

** alternative 1 **

means

<table>
<thead>
<tr>
<th></th>
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<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>27.549809</td>
<td>29.350248</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>29.318338</td>
<td>27.376410</td>
</tr>
<tr>
<td>total</td>
<td>28.434074</td>
<td>28.363329</td>
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</tbody>
</table>

variances

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>39.389526</td>
<td>39.247437</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>46.204285</td>
<td>44.878540</td>
</tr>
<tr>
<td>total</td>
<td>42.796906</td>
<td>42.062988</td>
</tr>
</tbody>
</table>

** alternative 2 **

means

<table>
<thead>
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<th>r.yugos.</th>
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</thead>
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<tr>
<td>slovenia</td>
<td>27.547722</td>
<td>0.018143</td>
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<tr>
<td>r.yugos.</td>
<td>0.223530</td>
<td>27.374157</td>
</tr>
<tr>
<td>total</td>
<td>27.771252</td>
<td>27.392300</td>
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</tbody>
</table>

variances

<table>
<thead>
<tr>
<th></th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>39.381897</td>
<td>0.034458</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>0.548902</td>
<td>44.863652</td>
</tr>
<tr>
<td>total</td>
<td>39.930798</td>
<td>44.903111</td>
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</tbody>
</table>
Table 8.6. Discounted number of offspring per person of exact age x.

<table>
<thead>
<tr>
<th>region of residence</th>
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<th>r.yugos</th>
</tr>
</thead>
<tbody>
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<td>region of birth of offspring</td>
<td>total</td>
<td>slovenia</td>
</tr>
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<td>1.000000</td>
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</tr>
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<td>0.937944</td>
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<td>15</td>
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</tr>
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<td>20</td>
<td>0.958980</td>
<td>0.911243</td>
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<td>25</td>
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<td>0.811876</td>
</tr>
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Table 8.7. Spatial reproductive value per person of exact age \( x \).

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Table 8.8. Discounted number of offspring per person in age group \( x \) to \( x + 4 \).

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<th>r.yugos.</th>
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<table>
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Table 8.9. Spatial reproductive value per person in age group \( x \) to \( x + 4 \).

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Table 8.10. Total discounted number of offspring of observed population.

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The small deviation from the total discounted number of offspring of the observed population is due to rounding error.
Table 8.12. Stable equivalent of total population.

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Table 8.13. Stable equivalents and intrinsic rates.

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</table>
The matrix of fertility adjustment factors for Yugoslavia is given in Table 8.14 together with the new fertility rates:

\[
\hat{F}(x) = \gamma \hat{F}(x), \quad (8.7)
\]

where \( \gamma = \gamma I \).

This reduction scheme produces a different stationary population. A baby girl born in Slovenia is replaced by only 0.918 daughters on the average, while a girl born in the Rest of Yugoslavia replaces herself with 1.004 daughters. Further results of this replacement alternative are given in Tables 8.15 to 8.26.

8.2 The Analytical Approach

If the initial population is stable, the momentum of spatial zero population growth may be expressed as a simple analytical formula. The ultimate number of stationary equivalent births is by (7.1)

\[
\{\hat{Q}\} = \frac{1}{\{\hat{v}(0)\} \hat{r}\{\hat{Q}_1\}} \int_0^\omega \{\hat{v}(x)\}'\{k(x)\} dx \{\hat{Q}_1\} \quad (7.1)
\]

where the caret designates a stationary population. The total reproductive value \( \hat{V} \) is

\[
\hat{V} = \int_0^\omega \{\hat{v}(x)\}'\{k(x)\} dx ,
\]

with \( \{k(x)\} \) being the vector defining the regional distribution of people at exact age \( x \). If the distribution \( \{k(x)\} \) is stable, then by (3.23)

\[
\{k(x)\} = e^{-\Gamma x} \hat{k}(x) \{Q\} , \quad (8.8)
\]

### Matrix of Fertility Adjustment Factors

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### Fertility Analysis

#### Age-Specific Rates

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| GRR | 0.942432 | 1.149663 |
Table 8.15. Integrals of generalized net maternity function.

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Table 8.17. Spatial fertility expectancies.

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Table 8.18. Matrices of mean ages and variances.

** alternative 1 **
***************

means
-----
slovenia r.yugos.
slovenia 27.549803 29.350245
r.yugos. 29.318340 27.376406
total 28.434072 28.363325

variances
-------
slovenia r.yugos.
slovenia 39.389832 39.247742
r.yugos. 46.204163 44.878723
total 42.796997 42.063232

** alternative 2 **
***************

means
-----
slovenia r.yugos.
slovenia 27.547716 0.016397
r.yugos. 0.247327 27.374157
total 27.795044 27.390554

variances
-------
slovenia r.yugos.
slovenia 39.382263 0.031142
r.yugos. 0.607282 44.868713
total 39.989544 44.899857
Table 8.19. Discounted number of offspring per person of exact age \( x \).

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Table 8.20. Spatial reproductive value per person of exact age $x$.

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Table 8.21. Discounted number of offspring per person in age group $x$ to $x + 4$.

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<table>
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<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</tr>
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<td>0.002300</td>
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<td></td>
</tr>
<tr>
<td>35</td>
<td>0.111903</td>
<td>0.000341</td>
<td>0.111852</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.037361</td>
<td>0.000004</td>
<td>0.037359</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>0.007499</td>
<td>0.000031</td>
<td>0.007498</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.001512</td>
<td>0.000000</td>
<td>0.001512</td>
<td></td>
</tr>
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<tr>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>
Table 8.22. Spatial reproductive value per person in age group $x$ to $x + 4$.

<table>
<thead>
<tr>
<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.009791</td>
</tr>
<tr>
<td>5</td>
<td>1.013931</td>
</tr>
<tr>
<td>10</td>
<td>1.006158</td>
</tr>
<tr>
<td>15</td>
<td>0.957651</td>
</tr>
<tr>
<td>20</td>
<td>0.749220</td>
</tr>
<tr>
<td>25</td>
<td>0.449374</td>
</tr>
<tr>
<td>30</td>
<td>0.222877</td>
</tr>
<tr>
<td>35</td>
<td>0.035883</td>
</tr>
<tr>
<td>40</td>
<td>0.020747</td>
</tr>
<tr>
<td>45</td>
<td>0.002726</td>
</tr>
<tr>
<td>50</td>
<td>0.000619</td>
</tr>
<tr>
<td>55</td>
<td>0.000000</td>
</tr>
<tr>
<td>60</td>
<td>0.000000</td>
</tr>
<tr>
<td>65</td>
<td>0.000000</td>
</tr>
<tr>
<td>70</td>
<td>0.000000</td>
</tr>
<tr>
<td>75</td>
<td>0.000000</td>
</tr>
<tr>
<td>80</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
Table 8.23. Total discounted number of offspring of observed population.

<table>
<thead>
<tr>
<th></th>
<th>total</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>354222.</td>
<td>325533.</td>
<td>28539.</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>4764538.</td>
<td>25147.</td>
<td>4739390.</td>
</tr>
<tr>
<td>total</td>
<td>5118760.</td>
<td>350780.</td>
<td>4767980.</td>
</tr>
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</table>

Table 8.24. Reproductive value of the total population.

<table>
<thead>
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<th>total percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>slovenia</td>
<td>370621.</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>8507440.</td>
</tr>
<tr>
<td>total</td>
<td>8878061.</td>
</tr>
</tbody>
</table>
Table 8.25. Stable equivalent of total population.

<table>
<thead>
<tr>
<th></th>
<th>total</th>
<th>slovenia</th>
<th>r.yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>42646</td>
<td>834400</td>
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<td>5</td>
<td>628160</td>
<td>42294</td>
<td>785866</td>
</tr>
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<td>10</td>
<td>825832</td>
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</tr>
<tr>
<td>15</td>
<td>823123</td>
<td>43278</td>
<td>779845</td>
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<tr>
<td>20</td>
<td>815937</td>
<td>44912</td>
<td>774025</td>
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<td>25</td>
<td>813373</td>
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<td>46462</td>
<td>760345</td>
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<td>35</td>
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<td>742955</td>
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<tr>
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<td>776636</td>
<td>46417</td>
<td>730219</td>
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<tr>
<td>50</td>
<td>757174</td>
<td>45670</td>
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<td>727909</td>
<td>44617</td>
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<td>60</td>
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<td>42847</td>
<td>639412</td>
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<td>611629</td>
<td>39279</td>
<td>572350</td>
</tr>
<tr>
<td>70</td>
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<td>32955</td>
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<td>75</td>
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<td>80</td>
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<td>85</td>
<td>236912</td>
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</table>

Total: 12303373  701522  11601851

Percentage Distribution

<table>
<thead>
<tr>
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<th>slovenia</th>
<th>r.yugos.</th>
</tr>
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<tbody>
<tr>
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<td>7.192</td>
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<td>6.731</td>
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<td>6.774</td>
</tr>
<tr>
<td>10</td>
<td>6.712</td>
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<td>6.690</td>
<td>6.169</td>
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<td>6.656</td>
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<td>6.611</td>
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<td>5.558</td>
<td>6.623</td>
<td>6.554</td>
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<td>5.133</td>
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<td>5.599</td>
<td>4.933</td>
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<td>4.131</td>
<td>4.698</td>
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<td>3.485</td>
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<td>2.079</td>
<td>1.956</td>
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<tr>
<td>85</td>
<td>1.926</td>
<td>1.311</td>
<td>1.963</td>
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Total: 100.000  100.000  100.000
Share: 100.000  5.702   94.298
### Table 8.26. Stable equivalents and intrinsic rates.

<table>
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<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Rate</td>
<td>Number</td>
<td>Rate</td>
<td>Number</td>
<td>Rate</td>
<td>Number</td>
<td>Rate</td>
</tr>
<tr>
<td>Slovenia</td>
<td>8507.0</td>
<td>0.012269</td>
<td>10337.0</td>
<td>0.014736</td>
<td>1610.0</td>
<td>0.002295</td>
<td>3333.0</td>
<td>0.004752</td>
</tr>
<tr>
<td>r.yugos.</td>
<td>176397.0</td>
<td>0.015196</td>
<td>174577.0</td>
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<td>3333.0</td>
<td>0.000287</td>
<td>1610.0</td>
<td>0.000139</td>
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<td>0.015030</td>
<td>184914.0</td>
<td>0.015030</td>
<td>9944.0</td>
<td>0.000402</td>
<td>9944.0</td>
<td>0.000402</td>
</tr>
</tbody>
</table>

**Stable growth rate**: 0.000000  
**Normalizing factor**: 1031.4851
where \( \{Q \} \) represents the regional distribution of births before the drop in fertility. Substituting \( \{k(x)\} \) in (8.8) into (7.1) and simplifying gives (Rogers and Willekens, 1976b, p. 22):

\[
\{\hat{Q}\} = \frac{1}{\mu} \left[ (\tilde{v}(0))' \gamma [R(0) - \gamma (r)] \{Q\} \right] \{\hat{Q}_1\} ,
\]

where \( \mu = (\tilde{v}(0))' \kappa \{\hat{Q}_1\} \), with \( \kappa = \mu = \gamma R(1) R^{-1}(0) \gamma^{-1} \) being the matrix of mean ages of childbearing in the stationary population after the decline in fertility. The matrices \( R(0) \) and \( \gamma (r) \) and the vector of stable equivalent births refer to the stable population before the drop in fertility. The matrix of fertility adjustment factors is \( \gamma \).

It can be shown that equation (8.9) is equivalent to

\[
\{\hat{Q}\} = \frac{1}{\mu} R(1)^{-1} [\tilde{R}(0) - \gamma (r)] \{Q\} = S^0 \{Q\} , \text{ say.}
\]

The stationary births are therefore a linear combination of the stable births, before the drop in fertility. The conversion matrix is \( S^0 \). A numerical evaluation is given in Table 8.27.

The ultimate stationary population is

\[
\{\hat{Y}\} = \left( \int_0^{\infty} \hat{t}(x)dx \right) \{\hat{Q}\} = \tilde{e}(0) \{\hat{Q}\}
\]

and the total reproductive value is

\[
\hat{V} = (\hat{v}(0)) \frac{1}{\mu} \gamma [R(0) - \gamma (r)] \{Q\} .
\]

Let \( \hat{Y} \) be the diagonal matrix of the total population before the drop in fertility, then

\[
\hat{Y}(1) = \left( \int_0^{\infty} e^{-rx} \hat{t}(x)dx \right) \{Q\} = e^{(r)}(0) \{Q\} ,
\]
where \( e^{(r)}(0) \) has been labeled the matrix of discounted life expectancies. Recalling the characteristic equation, (8.13) also may be written as

\[
\{Y\} = e^{(r)}(0) \{\Psi(r)\}^{-1} \{Q\},
\]

whence

\[
\{Q\} = \Psi(r) \{e^{(r)}(0)\}^{-1} \{Y\} = b \{Y\}.
\]

The spatial momentum of zero population growth is then

\[
\gamma^{-1} \{\hat{Y}\} = \frac{1}{\mu} \{[\nu(0)']_\gamma [R(0) - \Psi(r)] \{Q\}\} \gamma^{-1} e(0) \{\hat{Q}_1\}
\]

\[
= \frac{1}{\mu} \{[\nu(0)']_\gamma [R(0) - \Psi(r)] \{b\}\} e(0) \{\hat{Q}_1\}
\]

(8.16)

(8.17)

where \( \{b\} \) is the vector of regional intrinsic birth rates before the drop in fertility. Applying (8.10) the momentum becomes

\[
\gamma^{-1} \{\hat{Y}\} = e(0) \frac{1}{\mu} \gamma [R(1)]^{-1} [R(0) - \Psi(r)] \{b\}.
\]

(8.18)

Introducing (8.15) into (8.16) gives yet another expression for the momentum

\[
\gamma^{-1} \{\hat{Y}\} = \frac{1}{\mu} \{[\nu(0)']_\gamma [R(0) - \Psi(r)] b \{1\}\} e(0) \{\hat{Q}_1\}.
\]

(8.19)

The analytical approach is illustrated in Table 8.27. It is assumed that the initial population coincides with the stable equivalent population of Slovenia and the Rest of Yugoslavia. Hence the regional births are contained in the vector
Table 8.27 reveals that, given a population of 597,806 in Slovenia and 10,119,204 in the Rest of Yugoslavia, an immediate drop of fertility to replacement level would result in an ultimate population increase of 15.74% in Slovenia and of 14.66% in the Rest of Yugoslavia. The momentum is a consequence of the growth potential in the initial age and regional distribution of the population.†

and the population by age-group and region is given in Table 7.1.

\[ \{Q\} = \begin{bmatrix} 9,237 \\ 192,355 \end{bmatrix} \]

†Note that the stationary population distribution in unit births was given in Tables 2.4 and 3.5.
Table 8.27. Spatial momentum of zero population growth.

<table>
<thead>
<tr>
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<th>Slovenia</th>
<th>R. Yugos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>0.916844</td>
<td>-0.000054</td>
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<tr>
<td>R. Yugos.</td>
<td>-0.000574</td>
<td>0.916786</td>
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<tr>
<td>Total</td>
<td>0.916270</td>
<td>0.916732</td>
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</table>

stable and stationary equivalents

<table>
<thead>
<tr>
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<th>Stable</th>
<th>Stationary</th>
</tr>
</thead>
<tbody>
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<td>176343.</td>
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<tr>
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<td>201592.</td>
<td>184802.</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Stable</th>
<th>Stationary</th>
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</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>597806.</td>
<td>691925.</td>
</tr>
<tr>
<td>R. Yugos.</td>
<td>10119204.</td>
<td>11603081.</td>
</tr>
<tr>
<td>Total</td>
<td>10717010.</td>
<td>12295006.</td>
</tr>
</tbody>
</table>

|                  |            | Population |
|------------------|------------|
| Slovenia         | 1.1574     |
| R. Yugos.        | 1.1465     |
| Total            | 1.1472     |
Part II

User's Manual
9. PROGRAM DESCRIPTION

The concept underlying the programs is that of a modular system. It consists of a set of subroutines, each of which performs a specific task, such as matrix inversion, calculating the dominant eigenvalue and associated eigenvectors, computing the integral functions and their moments, and so on. The main program is kept very short; it coordinates the computations through CALL statements. Information is transmitted from one subroutine to another as follows:

- arrays: labeled COMMON statements,
- parameters: argument string in the CALL statement.
  labeled COMMON/CPAR/ for projection parameters.

No computations are performed in the main program.

The subroutines consist of the frequently used general purpose subroutines and special purpose subroutines.
i. General purpose subroutines:

MULTIP: matrix multiplication.
INVERT: matrix inversion.
EIGEN: computation of dominant eigenvalue and associated right and left eigenvectors.

ii. Special purpose subroutines:

DATAS: reads and prints the data as they are read in; computes the observed rates.
PRELIM: performs a preliminary analysis with the data.
TOSY: computes and prints demographic features of a single region; computes and prints the single region life table.
PROBR: computes and prints the probabilities of dying and outmigrating, following Option 1 (see p. 51).
PROBSC: computes and prints the probabilities of dying and outmigrating, following Option 3 (see p. 49).
HIST: computes and prints the complete life histories of the regional birth cohorts.
LIFE: computes and prints the multiregional life table.
PRLIF2: prints the summary life table for a two-region system.
WHOLE: computes aggregate age-specific death rates that are consistent with the aggregation of the multiregional life table.
GROWTH: computes and prints the generalized Leslie matrix.
PROJEC: projects the population until stability is reached.
LMAT: computes the age composition of the stationary (life table) population in terms of unit born (the L(x) matrices).
RELAM: reads in the stable growth ratio.
FERMOB: performs the fertility and mobility analysis; computes the integral functions, i.e., the
(weighted) generalized maternity and mobility functions, and their zero-th, first and second moments. In addition, it calculates the matrices of mean ages at childbearing and mobility and the matrices of the variances of the ages at childbearing and mobility.

AGEDIS: generates the stationary and stable population distributions by age and region and in terms of unit radices.

RVALUE: computes the discounted number of offspring and computes the spatial reproductive values by age and region.

RINTR: computes the stable equivalents of births, deaths, outmigrants and inmigrants, and the intrinsic rates.

MOMENT: computes the spatial momentum of zero population growth (analytical approach).

ZERO: replaces the observed regional fertility schedules with fertility schedules at replacement level. Two alternative fertility reduction schemes are possible.

The purpose of separating each major task into subroutines is to keep the whole structure of the programs very clear and to enable the user to change parts of the programs according to his needs. Clarity and flexibility are major objectives which we tried to keep in mind while writing the programs. Computational efficiency was of secondary importance. In a rapidly growing field such as multiregional demographic analysis, computer programs must be flexible and easy to adapt to new theoretical or methodological developments. The computer programs published here are not final fixed products; they are working tools to produce useful numerical demographic results. The user is urged to adapt them to fit his own needs in order to get the most out of them.
Each subroutine that performs a major computational task is covered in detail. In Part I, we focused on the clarification of the output. For a detailed mathematical treatment of the various topics, the user is referred to Rogers (1975a) and to the papers of the dynamics subtask of IIASA's Migration and Settlement Task (see list at the back of this report).

9.1 The General Purpose Subroutines

a. MULTIP:

SUBROUTINE MULTIP (N,K,L)

Task: multiplication of two matrices $A_1$ and $B$.

\[ C = A_1 \times B. \]

Parameters:
- $N$: number of rows of $A_1$.
- $K$: number of columns of $A_1$ (and consequently, number of rows of $B$).
- $L$: number of columns of $B$.

Input:
- parameters in the CALL statement.
- matrices $A_1$ and $B$ in a labeled COMMON:
  \[ \text{COMMON/CMUL/A1(N,K), B(K,L), C(N,L)}. \]

Output: the result of the matrix multiplication is stored in the $N \times L$ matrix $C$.

Printing: none.

b. INVERT:

SUBROUTINE INVERT (NR)

Task: inversion of the matrix $CC$.

Parameters:
- $NR$: rank of $CC$.

Input:
- parameter $NR$ in the CALL statement. The subroutine assumes that $CC$ is nonsingular, and that all the diagonal elements are nonzero.
- matrix $CC$ in labeled COMMON:
  \[ \text{COMMON/CINV/CC(NR,NR)}. \]

Output: the original matrix $CC$ is replaced by the inverted matrix.

Printing: none.
c. EIGEN:

SUBROUTINE EIGEN (NR, NP, NEIG)

Task: calculation of the dominant eigenvalue of the matrix CE and of the associated right and left eigenvectors. EIGEN may also be used to compute row and column totals and to print a matrix.

Parameters: NR: dimension of the matrix.
NEIG: parameter related to the computation.
   NEIG = 1: the complete computation procedure is performed: row and column totals, dominant eigenvalue and associated eigenvectors.
   NEIG = 0: only the column sums are computed and printed. By using this option, EIGEN may be used to print a matrix.
NP: parameter related to printing.
   NP = 1: EIGEN prints the original matrix and its column sums. The dominant eigenvalue and its right and left eigenvectors are printed if NEIG = 1.
   NP = 0: nothing is printed.
   NP = 2: row and column sums are printed together with the matrix.

Input: - parameters in CALL statement.
   - the matrix CE in labeled COMMON:
     COMMON/CEIGEN/CE(NR,NR),ROOT,VECT(NR),VECTL(NR).

Output: the dominant eigenvalue ROOT, the right eigenvector VECT(I) and the left eigenvector VECTL(I) are stored in labeled COMMON.

Printing: according to the specification of the parameter NP.
Algorithm: let the original matrix be $\mathbf{A}$:

$$
\mathbf{A} = \begin{bmatrix}
\hat{a}_{11} & \hat{a}_{21} & \ldots & \hat{a}_{n1} \\
\hat{a}_{12} & \hat{a}_{22} & \ldots & \hat{a}_{n2} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{a}_{1n} & \hat{a}_{2n} & \ldots & \hat{a}_{nn}
\end{bmatrix}
$$

The dominant eigenvalue of $\mathbf{A}$ is obtained by the power method (for details see Rogers 1971, Chapter 7):

$$
\lambda^{(n)} = \frac{a_{11}^{(n+1)}}{a_{11}^{(n)}},
$$

where the superscript denotes the iteration and $a_{11}^{(n)}$ is the first element of the matrix $\mathbf{A}^n$. As $n$ becomes large, $\lambda^{(n)}$ converges to the true eigenvalue. The iteration terminates when

$$
-\varepsilon < \left| \frac{a_{12}^{(n+1)}}{a_{11}^{(n+1)}} - \frac{a_{12}^{(n)}}{a_{11}^{(n)}} \right| < \varepsilon,
$$

with $\varepsilon = 0.000001$.

The right eigenvector $\{\xi\}$ associated with $\lambda$ is proportional to any column of $\mathbf{A}^n$ for $n$ large. In the program, $\{\xi\}$ is taken to be the first column of $\mathbf{A}^n$, scaled such that $\xi_1 = 1$. The scaling selected is arbitrary, since an eigenvector is constant up to a scalar. For convenience, we have retained the scaling $\xi_1 = 1$, i.e. the first element of $\{\xi\}$ is unity.
The left eigenvector is the solution to the system:

\[ (v)' [A - \lambda I] = (0)' \]

As before, we take \( v_1 \) to be unity. Therefore

\[
\begin{bmatrix}
1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix}
\begin{bmatrix}
a_{11} - \lambda \\
a_{12} \\
\vdots \\
a_{1n}
\end{bmatrix}
\begin{bmatrix}
a_{21} \\
a_{22} - \lambda \\
\vdots \\
a_{2n}
\end{bmatrix}
\begin{bmatrix}
\cdots \\
\cdots \\
\vdots \\
\cdots \\
a_{n1} \\
a_{n2} - \lambda \\
\vdots \\
a_{nn}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
v_2 \\
\vdots \\
v_n
\end{bmatrix}'
\begin{bmatrix}
a_{21} - \lambda \\
a_{22} \\
\vdots \\
a_{2n}
\end{bmatrix}
\begin{bmatrix}
a_{n1} \\
a_{n2} - \lambda \\
\vdots \\
a_{nn}
\end{bmatrix}
= -1
\]

9.2 The Special Purpose Subroutines

a. DATAS:

**SUBROUTINE DATAS** (NPR,NA,NY,ZFNY,NR,XZBIXZDI
XZO,IPROB,NEIG)

**Task:**
- reads data and prints them as they are read in (for details, see Section 10 on preparation of data deck).
- computes observed rates.

**Parameters:**
see Section 10.

**Input:**
see Section 10.

**Output:**
data as they are read in. The data are stored in labeled COMMON.
b. PRELIM:  
SUBROUTINE PRELIM (NA,NY,ZNY,NR,XZB,XZD,XZO)  
Task:  
performs a preliminary analysis with the data:  
a. computes and prints age compositions,  
rates, mean ages, etc.;  
b. calls TOTSY for each region separately  
and for the country as a whole.  
Parameters: see Section 10.  
Input: DATAS must precede PRELIM.  
Output: Tables of Section 1.

c. TOTSY:  
SUBROUTINE TOTSY (NA,ZNY,XZB,XZD,XZO,NWHOL,REGL)  
Task:  
single region analysis: computes and prints  
demographic features of each region, includ-  
ing a single region life table.  
The subroutine TOTSY is called on two  
occasions:  
- by PRELIM to compute life tables for each  
region separately and for the country;  
- by WHOLE to compute the life table for the  
aggregate (sum of regions) system.  
Parameters: NA,ZNY,XZB,XZD,XZO: see Section 10.  
NWHOL = 0: aggregate system (aggregate  
death rates are derived from  
multiregional life table).  
The table with basic demograph-  
ic features is not printed. Only  
the life table of the aggregate  
(sum of regions) system is com-  
puted and printed. (NWHOL = 0  
if TOTSY is called from WHOLE).  
NWHOL = 1: both basic features - table  
and life table are printed  
(NWHOL = 1 if TOTSY is called  
from PRELIM).
Input: PRELIM or WHOLE must precede TOTSY.
Output: Tables 1.4 to 1.6; Table 2.11

d. PROBSC:
SUBROUTINE PROBSC (NA,ZFNY,NR,IPROB)
Task: computes and prints the probabilities of dying and outmigrating following Option 3.
Parameters: NA,ZFNY,NR,IPROB: see Section 10.
           IPROB must be 3.
Input: DATAS must precede PROBSC.
Output: Table 2.1.
Algorithm: see Section 2.7.

e. PROBR:
SUBROUTINE PROBR (NA,ZFNY,NR,IPROB)
Task: computes and prints the probabilities of dying and outmigrating following Option 1.
Parameters: NA,ZFNY,NR,IPROB: see Section 10.
           IPROB must be 1.
Input: DATAS must precede PROBR.
Output: Appendix B.
Algorithm: see Section 2.7.

f. HIST:
SUBROUTINE HIST (NA,NR,IHIST)
Task: computes and prints the complete life histories of each regional birth cohort.
Parameters: NA,NR: see Section 10.
           IHIST: parameter indicating that life histories are computed. If subroutine HIST is called, IHIST takes the value of one and the computation of the matrices \( \hat{p}(x) \) is skipped in the subroutine LIFE. If HIST is not called before LIFE, then IHIST is zero and \( \hat{p}(x) \) is computed in LIFE.
Input: PROBSC or PROBR must precede HIST.
Output: Table 2.2.
Algorithm: see Section 2.1.
g. LIFE:

SUBROUTINE LIFE (NA, ZFNY, NR, IPROB, IHIST, ILIF)

Task: computes and prints the multiregional life table.

Parameters:
- NA, ZFNY, NR, IPROB: see Section 10.
- IHIST: see HIST
- ILIF: parameter indicating that life table is computed.
  If subroutine LIFE is called, ILIF takes the value of one and the computation of \( L(x) \) and \( S(x) \) (needed later) is skipped in subroutines following LIFE.
  If LIFE is not called, ILIF is zero and \( L(x) \) is computed in a special subroutine LMAT while \( S(x) \) is computed in GROWTH.

Input: PROBSC or PROBR must precede LIFE.
HIST may precede LIFE but this is not necessary.

Output: multiregional life table (Tables 2.3 to 2.9).

Algorithm: see Sections 2.2 to 2.6.

h. PRLIF2:

SUBROUTINE PRLIF2 (NA, NY, IPROB)

Task: prints the summary life table for a two-region system.

Parameters:
- NA, NY, IPROB: see Section 10.

Input: subroutine LIFE must precede PRLIF2.

Output: Table 2.10.

i. WHOLE:

SUBROUTINE WHOLE (NA, ZFNY, NR)

Task: computes aggregate age-specific death rates from multiregional life table statistics, applying the radix ratio provided by the user. These death rates are transmitted to TOTSY which computes and prints the single region life table of the aggregated system.
The columns of this table are identical to the weighted sums of the results of the multiregional life table, the weights being the regional radices given by the user.

Parameters: NA, ZNFY, NR: see Section 10.

Input: LIFE must precede WHOLE.

Output: TOTS, called by WHOLE, produces Table 2.11.

Algorithm: see Section 2.8.

j. GROWTH:

SUBROUTINE GROWTH (NA, ZFN, NR, ILIF)

Task: computes and prints the generalized Leslie matrix.

Parameters: NA, ZFN, NR: see Section 10.

The value of ILIF depends on whether LIFE is called or not: if LIFE precedes GROWTH, ILIF = 1; if LIFE does not precede GROWTH, ILIF = 0 and S(x) is computed in GROWTH.

Input: PROBSC or PROBR must precede GROWTH.

Output: the nonzero submatrices of the generalized Leslie matrix (Table 3.1).

Algorithm: see Section 3.1.

k. PROJEC:

SUBROUTINE PROJEC (NA, NY, ZFN, NR, ZLAMDK, IPROJ)

Task: projects the population over an interval of NY years (usually 5). The projection interval is the same as the age interval.

Parameters: NA, NY, ZFN, NR: see Section 10.

INIT, NHORIZ, INTV, ITOLX, NTOLL in labeled COMMON/CPAR/: see Section 10.

ZLAMDK: the stable growth ratio $\lambda$. To derive the stable growth rate $r$, apply the formula $r = [\ln \lambda]/NY$.

IPROJ: parameter indicating that PROJEC is called and stable growth ratio is computed. If PROJEC is called, IPROJ = 1.

Input: GROWTH must precede PROJEC.
Output: population projections by age and region (Tables 3.2 and 3.3).

For pragmatic reasons, a distinction is made between short-term and long-term projections. Short-term projection outputs are given for every NY years, whereas long-term projection outputs are listed for every NHORIZ years (100 or 200, say). The limit between short and long-term projections is specified by the user. The purpose of the long-term projection is to identify the stable characteristics of the population system.

In addition to population totals by age and region, the output also contains the age composition and mean (M.AGE) of each regional population, the regional share (SHA) of the total population, and the growth ratio (LAM) of the previous period, i.e., from \( (t - 1) \) to \( t \), and the average annual growth rate

\[
r = \frac{\ln \text{LAM}}{\text{NY}}.
\]

Algorithm: see Sections 3.2 and 3.3.

The stable population is characterized by a constant growth rate and age-by-region distribution. This feature underlies the stopping criterion for the projection process (or iteration in the power method).

The user may choose between two options:

a. \( \lambda_1(t) - \lambda_1(t - 1) \leq \text{TOLX} \) (ITOLX = 3)

b. \( \lambda_{NR}(t) - \lambda_1(t) \leq \text{TOLX} \) (ITOLX = 2)

where \( NR \) is the number of regions,

\( \lambda_1(t) \) is the growth ratio of region 1 in the period from \( (t - 1) \) to \( t \),
TOLX is the tolerance level for the eigenvalue computation.

The user makes his choice by specifying the parameter ITOLX (default value ITOLX = 2). The tolerance level TOLX is

\[ TOLX = 10^{-NTOLL} \]

where NTOLL is given by the user (default value NTOLL = 7).

1. **LMAT**

   SUBROUTINE LMAT (NA,NR,ZFNY,ILIF)

   **Task:**
   computes the life table (stationary) population distribution by region of birth and region of residence, in terms of unit birth cohorts \( L(x) \).

   **Parameters:**
   NA,NR,ZFNY: see Section 10.
   ILIF: see description of LIFE. If life table is computed (ILIF = 1), LMAT is not called.

   **Input:**
   PROBSC or PROBR must precede LMAT.

   **Output:**
   nothing printed.

   **Algorithm:**
   see life table, Sections 2.2 and 2.3.

m. **RELAM**

   SUBROUTINE RELAM (ZFNY,ZLAMDK,RSTAB)

   **Task:**
   reads the stable population growth ratio; computes the stable growth rate and prints both.

   **Parameters:**
   ZFNY: see Section 10.
   ZLAMDK: stable population growth ratio (over period NY).
   RSTAB: stable population growth rate (annual).

   **Input:**
   ZLAMDK, added to the input data set (see Section 10).

   RELAM is called only if IPROJ = 0, i.e., if PROJEC is not called.
Output: RSTAB.
Algorithm: \( \text{RSTAB} = [\ln ZLAMDK]/ZFN\).

**FERMOB:**

This subroutine performs fertility and mobility analyses. It computes and prints the integral functions (generalized maternity and mobility functions and the weighted functions), and their moments. From this information it derives the matrices of mean ages at childbearing and mobility and their variances.

**Task:**

Subroutine FERMOB (NA, ZFNY, NR, NOPMOB, NEIG, R)

**Parameters:**

- **NA, ZFNY, NR:** See Section 10.
- **NOPMOB:** Parameter indicating the type of population analyzed. This parameter is already defined in the argument string.
  - NOPMOB = 1: life table population.
  - NOPMOB = 2: stationary (ZPG) population.
  - NOPMOB = 3: stable population.
- **NEIG:** See description of EIGEN.
- **R:** Relevant annual growth rate:
  - Life table and ZPG analysis: \( R = 0 \).
  - Stable population analysis: \( R = \text{RSTAB} \), i.e., the stable growth rate.

**Input:**

- **LIFE OR LMAT** must precede FERMOB.
- If NOPMOB = 3 (stable population), the stable growth rate RSTAB must be given.

**NOTE:** RSTAB is derived from the stable growth ratio \( \lambda \) or ZLAMDK.

The value of ZLAMDK is given by subroutine PROJEC. If PROJEC does not precede FERMOB (IPROJ = 0), ZLAMDK must be read in by RELAM.
Output: tables of fertility and mobility analyses (Tables of Sections 4 and 5 and equivalent tables of Section 8.1).

Algorithm: see Sections 4 and 5.

The integral functions are computed by (4.4), (4.19), (5.5) and (5.11). In the program, these reduce to a single expression:

$$ HU(X) \times L(X,I,J) \times ZGRAL(X,J), $$

where $HU(X)$ is the weighting factor:

- $HU(X) = 1$ in the life table and ZPG population, and is equal to $EX(X)$ in the stable population,
- $L(X,I,J)$ is the number of people in region $J$ aged $X$ to $X+NY$, who are born in region $I$,
- $ZGRAL(X,J)$ contains the age and region-specific rates applied to the population distribution to give the integral function. In the case of the maternity function, $ZGRAL(X,J) = RATF(X,J)$, i.e., the regional age-specific fertility rates. In the mobility analysis, $ZGRAL(X,J) = \sum I RATM(X,I,J)$, i.e., the regional age-specific total out-migration rates.

0. AGEDIS:

SUBROUTINE AGEDIS (NA, ZFNY, NR, RSTAB)

Task: computes and prints the three types of population distribution: observed population, life table population and stable population.

Parameters: see Section 10.

Input:
- LIFE or LMAT must precede AGEDIS.
- PROJEC or RELAM must precede AGEDIS

Output: Tables 3.4 and 3.5.

Algorithm: the observed population is printed as read in. The life table population is computed in LIFE or LMAT. The stable population is
computed by equation (3.20) of the text:

\[ L(r)(x) = e^{-(x+NY)/Z} L(x), \]

or, in FORTRAN:

\[ EX(X) \times L(X, I, J), \]

where

\[ EX(X) = \exp(-Z \times RSTAB), \]

with

\[ Z = \text{FLOAT}(\text{NAGE}(X)) + \text{ZFNY} \times 0.5. \]

P. RVALUE:

SUBROUTINE RVALUE (NA, ZFNY, NR, R, ZVT)

Task: computes and prints the discounted number of offspring and the reproductive values by age and region.

Parameters: NA, ZFNY, NR: see Section 10.

R: the ultimate growth rate of the population under consideration:

\[ R = 0 \] in the ZPG population.

\[ R = \text{RSTAB} \] in the stable population.

ZVT: total reproductive value of whole system (all regions), (e.g., column total of Table 6.6).

Input: LIFE or LMAT must precede RVALUE.

- in stable population analysis \( R = \text{RSTAB} \), which must be known through PROJEC or RELAM.

- in stationary (ZPG) population analysis, \( R = 0 \).

Output: tables of Section 6 and equivalent tables of Section 8.1.
Algorithm: see Section 6.

q. RINTR:

**SUBROUTINE RINTR (NA,ZFNY,NR,R,ZVT)**

**Task:** computes and prints the stable equivalents and the intrinsic rates.

**Parameters:** NA,ZFNY,NR: see Section 10. 
R and ZVT: see description of RVALUE.

**Input:** RVALUE must precede RINTR.

**Output:** Tables 7.1 and 7.2. The stable equivalents of births and total population are saved for later use and stored in the arrays QQ(I) and YY(I) respectively. The stable equivalent population by age and region is contained in POPST(X,I).

Algorithm: see Section 7.

r. MOMENT:

**SUBROUTINE MOMENT (NA,ZFNY,NR,RSTAB)**

**Task:** spatial ZPG-analysis following the analytical approach.

**Parameters:** NA,ZFNY,NR: see Section 10. 
RSTAB: stable growth rate.

**Input:**  
- it is assumed that the initial population is stable and that the births are given by the vector QQ(I).
- subroutine RINTR must precede MOMENT.

**Output:** table 8.27.

Algorithm: see Section 8.2.

s. ZERO:

**SUBROUTINE ZERO (NA,NR,NZERO,RONRR)**

**Task:** ZERO replaces the observed regional fertility schedules with fertility schedules at replacement level. The new fertility rates are computed according to two alternative fertility reduction schemes described in Section 8.1.
Parameters:

NA, NR: see Section 10.

NZERO: denotes the alternative fertility reduction scheme

NZERO = 1: the cohort-replacement alternative.
NZERO = 2: the proportional reduction alternative.

RONRR: dominant eigenvalue of the net reproduction rate matrix (NRR). It is computed when ZERO is called for the first time (NZERO = 1) and is used to compute the matrix of fertility adjustment factors in the proportional reduction alternative (NZERO = 2).

Input:
LIFE or LMAT must precede ZERO.

Output:
- diagonal matrix of fertility adjustment factors. The diagonal elements are stored in the vector VI(I) and printed (Tables 8.1 and 8.14).

Algorithm: see Section 8.1.

9.3 Main Program

The main program is kept very short. Its function is to co-ordinate the calculations, and it therefore consists merely of CALL statements.

The subroutines do not have to be called in sequence. Multiregional life table computation, population projection, fertility-mobility analysis, ZPG-analysis (numerical approach) and the computation of reproductive values all may be performed independently, starting from the raw input data. For instance, the user may want the fertility-mobility analysis of a stationary population without first calculating the whole life table. This is easily done by combining the subroutines DATAS, PROBSC (or PROBR), LMAT and FERMOB. Other possible combinations are given in Table 9.1. The subroutines in parentheses are not
called by the main program but by other subroutines. They are mostly general purpose subroutines that must be linked with the special purpose routines.

10. PREPARATION OF THE DATA DECK

There is a single data deck for the computation of all programs. All data are read in at the beginning of the set of programs by the subroutine DATAS. The advantage of concentrating all the READ statements in a single subroutine is that it enables the user to easily change the FORMAT statements to fit particular data sets.

The data are read in fixed format from unit 5 (the conventional unit for cards with most computers). Only the first 72 columns are used. The last columns are for identification and are not read.

The data deck used to produce the tables in this paper is given in Table 10.1. (See also Figure 10.1). The card sequence is as follows:

1. Identification card
2. Parameter card
3. Title cards
4. Names of the regions
5. Regional radices
6. For each region:
   a. population
   b. births
   c. deaths
   d. migrants
7. Stable growth ratio (optional)
8. The last card of the deck is an "END" card. It may be a colored card to identify the end of the deck to the user.
Table 9.1 Alternative combinations of subroutines for spatial population analysis.

<table>
<thead>
<tr>
<th>Function</th>
<th>Subroutines Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Analysis</td>
<td>DATAS, PRELIM, (TOSY)</td>
</tr>
<tr>
<td>Complete Life Histories of Birth Cohorts</td>
<td>DATAS, PROBSC or PROBR, HIST, (MULTIP, INVERT)</td>
</tr>
<tr>
<td>Life Table</td>
<td>DATAS, PROBSC or PROBR, LIFE, (TOSY, MULTIP, INVERT)</td>
</tr>
<tr>
<td></td>
<td>(optional) PRLIF2</td>
</tr>
<tr>
<td>Projection</td>
<td>DATAS, PROBSC or PROBR, GROWTH, PROJEC (MULTIP, INVERT)</td>
</tr>
<tr>
<td>Fertility-Mobility Analysis</td>
<td>DATAS, PROBSC or PROBR, LMAT, FERMOB (MULTIP, INVERT, EIGEN)</td>
</tr>
<tr>
<td></td>
<td>+ RELAM</td>
</tr>
<tr>
<td>Age and Regional Distribution of Observed Life Table and Stable Population</td>
<td>DATAS, PROBSC or PROBR, LMAT, RELAM, AGEDIS, (MULTIP, INVERT)</td>
</tr>
<tr>
<td>Reproductive Value</td>
<td>DATAS, PROBSC or PROBR, LMAT, RELAM, RVALUE (MULTIP, INVERT, EIGEN)</td>
</tr>
<tr>
<td>Further Stable Population Analysis</td>
<td>DATAS, PROBSC (not PROBR), LMAT, RELAM, RVALUE, RINTR (MULTIP, INVERT, EIGEN)</td>
</tr>
<tr>
<td>Zero Population Growth Analysis: Analytical Approach (Momentum)</td>
<td>DATAS, PROBSC (not PROBR), LMAT, RELAM, RVALUE, RINTR, MOMENT, (MULTIP, INVERT, EIGEN)</td>
</tr>
<tr>
<td>Zero Population Growth Analysis: Numerical Approach (Two Alternatives)</td>
<td>DATAS, PROBSC or PROBR, LMAT, ZERO, FERMOB, (MULTIP, INVERT, EIGEN)</td>
</tr>
<tr>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>DATAS, PROBSC, LMAT, ZERO, [FERMOB], RVALUE, RINTR (MULTIP, INVERT, EIGEN)</td>
</tr>
</tbody>
</table>
### Table 10.1.

<table>
<thead>
<tr>
<th>Region</th>
<th>Population</th>
<th>Births</th>
<th>Deaths</th>
<th>Migrants from Slovenia to Slovenia</th>
<th>Migrants from Slovenia to Rest of Yugoslavia</th>
<th>Migrants from Rest of Yugoslavia to Slovenia</th>
<th>Migrants from Rest of Yugoslavia to Rest of Yugoslavia</th>
<th>Stable Growth Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slovenia</td>
<td>847900.</td>
<td>808100.</td>
<td>725000.</td>
<td>774000.</td>
<td>728400.</td>
<td></td>
<td>940.</td>
<td>1.030967</td>
</tr>
<tr>
<td>Rest of Yugoslavia</td>
<td>180000.</td>
<td>126800.</td>
<td>112000.</td>
<td>391000.</td>
<td>268000.</td>
<td>110000.</td>
<td>339000.</td>
<td>1.030967</td>
</tr>
</tbody>
</table>

**Population of Slovenia**

**Births in Slovenia**

**Deaths in Slovenia**

**Migrants from Slovenia to Slovenia**

**Migrants from Slovenia to Rest of Yugoslavia**

**Population of Rest of Yugoslavia**

**Births in Rest of Yugoslavia**

**Deaths in Rest of Yugoslavia**

**Migrants from Rest of Yugoslavia to Slovenia**

**Migrants from Rest of Yugoslavia to Rest of Yugoslavia**

= Stable Growth Ratio

-- "END" Card
Figure 10.1 Data deck for multiregional population model.
1. **Identification card**
The first card of the deck is an identification card. It may contain any information for the user. The identification card is read in and saved for the page heading. In zero population growth analysis (subroutine ZERO), the identification card is supplemented automatically with the identification ZPG1 for alternative 1 and ZPG2 for alternative 2.

2. **Parameter card**
The parameter card contains instructions to the program concerning some general characteristics of the data and concerning the desired computations. The parameter card is composed of 14 "fields", each requiring a specific piece of information. The parameter names, their interpretation, required format and default values are given in Table 10.2.

3. **Title cards**
There are NU title cards. There is no limit on NU, as long as it is greater than zero. Each title card is printed out as it is read in. The first 72 columns of the card may be used. The title is not stored.

4. **Names of the regions**
In listing the output, each region is identified by its name. Each name consists of a maximum of eight characters. Any character can be used. The names appear in sequence on the same card. The last name in the sequence is that of the country or total system.

<table>
<thead>
<tr>
<th>COLS</th>
<th>FORMAT</th>
<th>VAR. NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-72</td>
<td>9A8</td>
<td>REG(I),I = 1,NR1</td>
</tr>
</tbody>
</table>

where NR1 = NR + 1. The name of the country is contained in REG(NR1).

5. **Regional radices**
The radices appear in sequence on one card.

<table>
<thead>
<tr>
<th>COLS</th>
<th>FORMAT</th>
<th>VAR. NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-72</td>
<td>9F8.0</td>
<td>RADIX(I),I = 1,NR</td>
</tr>
</tbody>
</table>
### Table 10.2. Parameter specification.

<table>
<thead>
<tr>
<th>COLUMNS</th>
<th>FORMAT</th>
<th>NAME</th>
<th>INTERPRETATION</th>
<th>DEFAULT VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>12</td>
<td>NA</td>
<td>Number of age groups</td>
<td>---</td>
</tr>
<tr>
<td>3-4</td>
<td>12</td>
<td>NR</td>
<td>Number of regions. The upper limit is 12 (for output reasons)</td>
<td>---</td>
</tr>
<tr>
<td>5-6</td>
<td>12</td>
<td>NW</td>
<td>Width of age group (e.g., 5 years)</td>
<td>---</td>
</tr>
<tr>
<td>7-8</td>
<td>12</td>
<td>NU</td>
<td>Number of title cards</td>
<td>---</td>
</tr>
<tr>
<td>9-10</td>
<td>12</td>
<td>NZB</td>
<td>Time interval for which birth data are given, e.g., NZB = 1: one-year birth data, NZB = 5: five-year birth data</td>
<td>1</td>
</tr>
<tr>
<td>11-12</td>
<td>12</td>
<td>NZD</td>
<td>Time interval of death data</td>
<td>1</td>
</tr>
<tr>
<td>13-14</td>
<td>12</td>
<td>NZO</td>
<td>Time interval of migration data</td>
<td>1</td>
</tr>
<tr>
<td>15-16</td>
<td>12</td>
<td>IPROB</td>
<td>Option for estimating death and outmigration probabilities IPROB = 1: Option 1 method (PROBR), IPROB = 3: Option 3 method (PROBSC)</td>
<td>3</td>
</tr>
<tr>
<td>17-20</td>
<td>4</td>
<td>INIT</td>
<td>Base (initial) year</td>
<td>---</td>
</tr>
<tr>
<td>21-24</td>
<td>4</td>
<td>NHORZ</td>
<td>Time horizon of short term projections: year until which the detailed projection output is given at each time period</td>
<td>INIT</td>
</tr>
<tr>
<td>25-28</td>
<td>4</td>
<td>INTV</td>
<td>Time interval in years, for long-term projection output, e.g., INTV = 100 = detailed projection output is given every 100 years (starting from base year)</td>
<td>200</td>
</tr>
<tr>
<td>29-30</td>
<td>12</td>
<td>ITOLX</td>
<td>Choice of stopping criterion for stable population analysis ITOLX = 2: criterion is difference in growth ratio between the first and the last region, ITOLX = 3: criterion is difference in growth ratio of the first region between the actual and the previous time period</td>
<td>2</td>
</tr>
<tr>
<td>31-32</td>
<td>12</td>
<td>NTOLL</td>
<td>Tolerance level for stopping criterion</td>
<td>7</td>
</tr>
<tr>
<td>33-34</td>
<td>12</td>
<td>NEIG</td>
<td>Option to compute in FERMOD the dominant eigenvalue and associated right and left eigenvectors of all the moments of the (weighted) generalized maternity and mobility functions and of the matrices of mean ages and variances. NEIG = 0: no computation (generally NEIG = 0), NEIG = 1: computation</td>
<td>---</td>
</tr>
</tbody>
</table>
6. **Population data, births, deaths and migrants**

The data related to each region are given sequentially, i.e.,
- observations for region 1
- observations for region 2
  ...
- observations for region NR

The age structure is contained on one card followed by continuation cards. If no observations on births are available or required (as for life table computations), the number of births by age of mother is replaced by zero (i.e., 0.).

Intra-regional migration data are read in and must be replaced by zeros if not available.

The sequence of cards and the formats are as follows (in the case of 18 age groups):

**Observations for Region I:**

<table>
<thead>
<tr>
<th>CARD #</th>
<th>COLS</th>
<th>FORMAT</th>
<th>VAR. NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>1-70</td>
<td>7F10.0</td>
<td>POP(X,I), X = 1,7</td>
</tr>
<tr>
<td>1b</td>
<td>1-70</td>
<td>7F10.0</td>
<td>POP(X,I), X = 8,13</td>
</tr>
<tr>
<td>1c</td>
<td>1-40</td>
<td>4F10.0</td>
<td>POP(X,I), X = 14,18</td>
</tr>
<tr>
<td>2a</td>
<td>1-72</td>
<td>9F8.0</td>
<td>BIRTH(X,I), X = 1,9</td>
</tr>
<tr>
<td>2b</td>
<td>1-72</td>
<td>9F8.0</td>
<td>BIRTH(X,I), X = 10,18</td>
</tr>
<tr>
<td>3a</td>
<td>1-72</td>
<td>9F8.0</td>
<td>DEATH(X,I), X = 1,9</td>
</tr>
<tr>
<td>3b</td>
<td>1-72</td>
<td>9F8.0</td>
<td>DEATH(X,I), X = 10,18</td>
</tr>
<tr>
<td>4a</td>
<td>1-72</td>
<td>9F8.0</td>
<td>OMIG(X,J,I), X = 1,9</td>
</tr>
<tr>
<td>4b</td>
<td>1-72</td>
<td>9F8.0</td>
<td>OMIG(X,J,I), X = 10,18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ J = 1, NR \]
11. FORTRAN Listing of Computer Programs
11.1 General Purpose Subroutines
SUBROUTINE EIGEN (NR,NP,NEIG)
DIMENSION ZMOMT(7),HU(7)
COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
COMMON /CINV/ CC(7,7)
COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
COMMON /CRAD/ RADIX(7),RADIX
COMMON /CREC/ REG(13)
DOUBLE PRECISION REG

C NR : NUMBER OF ROWS (COLUMNS)
C NP = 0 : DOMINANT EIGENVALUE AND EIGENVECTORS COMPUTED
C BUT NOT PRINTED
C NP = 1 : DOMINANT EIGENVALUE AND EIGENVECTORS COMPUTED
C AND PRINTED
C NEIG = 0 : DOMINANT EIGENVALUE AND EIGENVECTORS NOT COMPUTED,
C AND MATRIX ONLY IS PRINTED
C
C COMPUTE DOMINANT EIGENVALUE BY POWER METHOD

IF (NEIG.EQ.0) GO TO 820
DO 21 I=1,NR
DO 21 J=1,NR
A1(J,I)=CE(J,I)
B(J,I)=CE(J,I)
21 CONTINUE
CALL MULTIP (NR,NR,NR)

24=C(1,1)
23=10000.
25=24
Z=C(1,1)
DO 22 I=1,NR
DO 22 J=1,NR
A1(J,I)=C(J,I)/Z
B(J,I)=C(J,I)
22 CONTINUE
CALL MULTIP (NR,NR,NR)
ROOT=C(1,1)/A1(1,1)

C COMPUTE RIGHT AND LEFT EIGENVECTOR (VECT AND VECTL)

DO 25 J=1,NR
25 VECT(J)=C(J,1)/C(1,1)
26 CONTINUE
NR=NR-1
DO 11 I=1,NR1
DO 11 J=1,NR1
I1=I+1
J1=J+1
IF (I.EQ.J) CC(I,J)=CE(J1,I1)-ROOT
IF (I.NE.J) CC(I,J)=CE(J1,I1)
11 CONTINUE
CALL INVERT (NR1)
DO 12 I=1,NR1
I1=I+1
B(I,1)=-CE(1,I1)
DO 12 J=1,NR1
2 A(I,J)=CC(I,J)
CALL MULTIP (NR1,NR1,1)
VECTL(1)=1.
DO 13 I=1,NR1
I1=I+1
13 VECT(I1)=C(I,1)
C PRINT MATRIX, EIGENVALUE AND EIGENVECTORS
C--------------------------------------------------------
IF (NP.EQ.0) GO TO 30
820 CONTINUE
ZTOT=0.
DO 6 J=1,NR
ZMOMT(J)=0.
DO 8 I=1,NR
8 ZMOMT(J)=ZMOMT(J)+CE(I,J)
IF (NP.EQ.2) ZMOMT(J)=ZMOMT(J)/FLOAT(NR)
ZTOT=ZTOT+ZMOMT(J)
6 CONTINUE
PRINT 62, (REG(J),J=1,NR)
62 FORMAT (/11X,12(2X,A8))
PRINT 64
64 FORMAT (/1X)
DO 5 I=1,NR
IF (ZTOT.LT.200.) PRINT 91, REG(I),(CE(I,J),J=1,NR)
5 FORMAT (/1X,A8,2X,12F10.6)
IF (ZTOT.GE.200.) PRINT 80, REG(I),(CE(I,J),J=1,NR)
80 FORMAT (/1X,A8,2X,12F10.4)
7 FORMAT (/1X,5HTOTAL,2X,12F10.6)
IF (ZTOT.GE.200.) PRINT 81, (ZMOMT(J),J=1,NR)
81 FORMAT (/1X,5HTOTAL,2X,12F10.4)
IF (NEIG.EQ.0) GO TO 30
PRINT 64
PRINT 93, ROOT
93 FORMAT (/1X,10HEIGENVALUE,5X,F11.6)
PRINT 94
94 FORMAT (/1X,11HEIGENVECTOR)
PRINT 95, (VECT(J),J=1,NR)
95 FORMAT (/1X,7H- RIGHT ,12F10.6)
PRINT 96, (VECTL(J),J=1,NR)
96 FORMAT (/1X,7H- LEFT ,12F10.6)
30 CONTINUE
RETURN
END
SUBROUTINE MULTIP (N,K,L)
C A1 * B = C
C N : NUMBER OF ROWS OF A1
C K : NUMBER OF COLUMNS OF A1 = NUMBER OF ROWS OF B
C L : NUMBER OF COLUMNS OF B
COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
DO 3 J=1,N
DO 3 J=1,L
C(I,J)=0.
DO 3 JJ=1,K
C(I,J)=C(I,J)+A1(I,JJ)*B(JJ,J)
3 CONTINUE
RETURN
END

SUBROUTINE INVERT (NR)
C NR : DIMENSION OF MATRIX CC TO BE INVERTED
DIMENSION PIVOT(7)
COMMON /CINV/ CC(7,7)
DO 606 I=1,NR
PIVOT(I)=CC(I,I)
CC(I,I)=1.0
DO 607 J=1,NR
CC(I,J)=CC(I,J)/PIVOT(I)
607 CONTINUE
IF (NR.EQ.1) GO TO 10
DO 608 K=1,NR
IF (K.EQ.1) GO TO 608
H=CC(K,I)
CC(K,I)=0.
DO 609 L=1,NR
CC(K,L)=CC(K,L)-CC(I,L)*H
609 CONTINUE
608 CONTINUE
606 CONTINUE
10 CONTINUE
RETURN
END
11.2 Special Purpose Subroutines
SUBROUTINE DATAS (NPR, NA, NY, ZFNY, NR, XZB, XZD, XZO, IPROB, NEIG)

DIMENSION TITLE(10)
COMMON /CPAR/ INIT, NHORIZ, INTV, ITOLX, NTOLL
COMMON /C1/ POP(18,7)
COMMON /CBIR/ BIRTH(18,7), DEATH(18,7), OMIG(18,7,7)
COMMON /C1AG/ NAGE(18)
COMMON /CRAD/ RADIX(7), RADIUS
COMMON /CATE/ RATD(18,7), RATM(18,7,7), RATF(18,7)
COMMON /CREG/ RECA(13)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X
DATA END/4HEND/

C READ AND PRINT INPUT FILE AS IT IS
C REWIND INPUT FILE

C 301 READ (5,1) (TIT(J), J=1,20)
  1 FORMAT (20A4)
  PRINT 356, (TIT(J), J=1,20)
356 FORMAT (1X,20A4)
  IF (TIT(1).EQ.END) GO TO 302
GO TO 301

C REWIND 5
C READ PAGE HEADING CARD
C READ PARAMETER CARD

READ (5,1) (TIT(J), J=1,20)
READ (5,2) NA, NR, NY, NU, NZB, NZD, NZO, IPROB, INIT, NHORIZ,
  INTV, ITOLX, NTOLL, NEIG
  2 FORMAT (612,314,312)

C DEFINE DEFAULT VALUES FOR PARAMETERS
C
DO 6 X=1, NA
  6 NAGE(X)= (X-1)*NY
  IF (NZB.EQ.0) NZB=1
  IF (NZD.EQ.0) NZD=1
  IF (NZO.EQ.0) NZO=1
  IF (NHORIZ.LE.INIT) NHORIZ=INIT
  IF (ITOLX.EQ.0) ITOLX=2
  IF (NTOLL.EQ.0) NTOLL=7
  IF (INTV.EQ.0) INTV=200
  XZB=FLOAT(NZB)
  XZD=FLOAT(NZD)
  XZO=FLOAT(NZO)
  ZFNY=FLOAT(NY)
DO 45 I=1, NR
  45 IF (RADIX(I).EQ.0.) RADIX(I)=100000.

C PRINT TITLE
C -----------------------------------------------------------
    IF (NPR.EQ.0) GO TO 51
  17 FORMAT (1H1,1X)
  304 FORMAT (50X,20A4)
    PRINT 17
    PRINT 304, (TIT(J), J=1,20)
    DO 15 J=1,3
  15 PRINT 16
  16 FORMAT (//1X)
    PRINT 22
  22 FORMAT (1X/4(20X,90(1H*),20X,5(1H*),80X,5(1H*))
    CONTINUE
    DO 4 I=1,NR
      READ (5,14) (TITLE(J),J=1,9)
  14 FORMAT (9A8)
      IF (NPR.NE.0) PRINT 5, (TITLE(J),J=1,9)
        CONTINUE
    5 FORMAT (20X,5(1H*),4X,9A8,4X,5(1H*))
    IF (NPR.NE.0) PRINT 24
    24 FORMAT (20X,5(1H*),80X,5(1H*)/4(20X,90(1H*))
C -----------------------------------------------------------
C LIST PARAMETERS
C READ OTHER INPUT DATA
C -----------------------------------------------------------
C READ NAMES OF REGIONS REG(J)
    NR1=NR+1
    READ (5,14) (REG(J),J=1,NR1)
C READ RADICES RADIX(J)
    READ (5,13) (RADIX(J),J=1,NR)
  13 FORMAT (9F8.0)
    RADIUS=0.
    DO 66 I=1,NR
      RADIUS=RADIUS+RADIX(I)
  66    CONTINUE
C READ POPULATION, BIRTHS, DEATHS, MIGRANTS
    DO 10 I=1,NR
      READ (5,3) (POP(X,I),X=1,NA)
  3 FORMAT (7F10.0)
      READ (5,31) (BIRT(B,X,I),X=1,NA)
  31 FORMAT (9F8.0)
      READ (5,311) (DEATH(X,I),X=1,NA)
      DO 32 J=1,NR
        READ (5,311) (OMIG(X,J,I),X=1,NA)
  32 CONTINUE
C -----------------------------------------------------------
C PRINT THE LIST OF PARAMETERS
C -----------------------------------------------------------
    IF (NPR.EQ.0) GO TO 54
    PRINT 17
    PRINT 304, (TIT(J), J=1,20)
    DO 18 J=1,3
  18 PRINT 16
    PRINT 19
  19 FORMAT (20X,18HLIST OF PARAMETERS/20X,18(1H*))
    PRINT 20, NA, NY, NR
COMPUTE THE OBSERVED MORTALITY, FERTILITY AND MIGRATION RATES

NOTE: RATES ARE ALSO COMPUTED IN PRELIM IF CALLED

DO 35 I=1,NR
   DO 35 X=1,NA
      RATD(X,I)=DEATH(X,I)/(POP(X,I)*XZD)
      RATF(X,I)=BIRTH(X,I)/(POP(X,I)*XZB)
   DO 35 J=1,NR
   RATM(X,J,I)=OMIG(X,J,I)/(POP(X,I)*XZO)
RETURN
END
SUBROUTINE PRELIM(NA, NY, ZFN, NR, XGB, XGD, XIO)

DIMENSION HU(7), HUU(7)
DIMENSION POPT(7), DEATHT(7), BIRHT(7), OMIAT(7, 7)
DIMENSION POFC2(18), DEHCT2(18), BIRCT2(18), ZMIGC2(18)
DIMENSION GRD(7), GRR(7), GRO(7, 7), GRO(7)
DIMENSION CRUDD(7), CRUSD(7), CRUCO(7, 7), CRUCO(7)
DIMENSION AGEF(7), AGED(7), AGEF(7), AGED(7, 7), AGEO(7)
COMMON /C1/ POP(18, 7)
COMMON /CNAG/ NAGE(18)
COMMON /CRATE/ RATD(18, 7), RATM(18, 7, 7), RATF(18, 7)
COMMON /CIT/ TIT(20)
COMMON /COT/ BIRHTC(18), DEATHC(18), CMIGC(18, 18), CMIGA(18)
COMMON /CTOT/ POPC(18), RATDT(18), RATFT(18), RATMT(18)
DOUBLE PRECISION REG, RECL

INTEGER X, XX
REAL L

NAA = NA - 1
PRINT 1, (TIT(J), J = 1, 20)
1 FORMAT (181, 50X, 20A4)
PRINT 65
65 FORMAT (1HOBSERVED POPULATION CHARACTERISTICS/6X, 135(1H=)/)
C ........................................................................................................
C PRINT NUMBER OF PEOPLE, BIRTHS, DEATHS AND MIGRANTS ;
C .................................................................................................
ISKIP = 3
DO 6 I = 1, NR
IF (ISKIP .NE. I) GO TO 164
PRINT 165
165 FORMAT (1H1/1X)
ISKIP = ISKIP + 2
164 CONTINUE
PRINT 15, REG(I)
15 FORMAT (/5X, 6HREGION, 3X, A8/5X, 17(1H-))
PRINT 16, REG(I)
16 FORMAT (3X, 3HAGE, 1X, 10HPOPULATION, 4X, 6HBIRTHS, 4X, 6HDEATHS, 5X, 11HMIGRATION FROM, 1X, A8, 1X, 2HTO)
IF (NR .LE. 10) PRINT 17, (REG(J), J = 1, NR)
IF (NR .GT. 10) PRINT 80, (REG(J), J = 1, NR)
17 FORMAT (37X, 16(1X, A8))
80 FORMAT (36X, 12A8)
PRINT 66
66 FORMAT (1X)
DO 14 X = 1, NA
IF (NR .LE. 10) PRINT 8, NAGE(X), POP(X, I), BIRHT(X, I), DEATH(X, I), 10MI(X, J, I), J = 1, NR
14 IF (NR .GT. 10) PRINT 81, NAGE(X), POP(X, I), BIRHT(X, I), DEATH(X, I), 10MI(X, J, I), J = 1, NR
8 FORMAT (3X, I3, 1X, 3F10.0, 10F9.0)
81 FORMAT (2X, I3, 1X, 3F10.0, 12F8.0)
POPT(I) = 0.
DEATHT(I) = 0.
BIRTHT(I)=0.
DO 41 J=1,NR
OMIGT(J,I)=0.
DO 42 X=1,NA
POPT(I)=POPT(I)+POP(X,I)
DEATHT(I)=DEATHT(I)+DEATH(X,I)
BIRTHT(I)=BIRTHT(I)+BIRTH(X,I)
DO 42 J=1,NR
OMIGT(J,I)=OMIGT(J,I)+OMIG(X,J,I)
CONTINUE
IF (NR.LE.10) PRINT 40, POPT(I),BIRTHT(I),DEATHT(I),
   1(OMIGT(J,I),J=1,NR)
IF (NR.GT.10) PRINT 82, POPT(I),BIRTHT(I),DEATHT(I),
   1(OMIGT(J,I),J=1,NR)
40 FORMAT (/1X,STOTAL,3F10.0,10F9.0)
82 FORMAT (/1X,STOTAL,3F10.0,12F8.0)
CONTINUE
C-----------------------------------------------
C COMPUTE AND PRINT PERCENTAGE DISTRIBUTION
C COMPUTE AND PRINT MEAN AGES
C-----------------------------------------------
PRINT 44
44 FORMAT (1H1,10X,24HPERCENTAGE DISTRIBUTIONS/11X,24(1H*))
ISKIP=3
DO 45 I=1,NR
   IF (ISKIP.NE.1) GO TO 166
   PRINT 165
   ISKIP=ISKIP+2
CONTINUE
PRINT 45, REG(I)
PRINT 16, REG(I)
IF (NR.LE.10) PRINT 17, (REG(J),J=1,NR)
IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
PRINT 66
ZP=0.
ZB=0.
ZD=0.
DO 700 J=1,NR
   HU(J)=0.
68 AGEO(J,I)=0.
   DO 66 X=1,NA
      Z2=0.
      Z3=0.
Z1=100.*POP(X,I)/POPT(I)
IF (DEATHT(I).NE.0.) Z3=100.*DEATH(X,I)/DEATH(I)
IF (BIRTHT(I).NE.0.) Z2=100.*BIRTH(X,I)/BIRTHT(I)
   ZP=ZP+Z1
   ZB=ZB+Z2
   ZD=ZD+Z3
   DO 148 J=1,NR
      HU(J)=0.
IF (OMIGT(J,I).EQ.0.) GO TO 148
HU(J)=100.*OMIG(X,J,I)/OMIGT(J,I)
HU(J)=HU(J)+HU(J)
148 CONTINUE
IF (NR.LE.10) PRINT 47, NAGE(X),Z1,Z2,Z3,(HU(J),J=1,NR)
IF (NR.GT.10) PRINT 84, NAGE(X),Z1,Z2,Z3,(HU(J),J=1,NR)
47 FORMAT (3X,13,1X,3F10.4,10F9.4)
84 FORMAT (2X,I3,1X,3F10.4,12F8.4)
Z=FLOAT(NAGE(X))+FLOAT(NY)*0.5
Z=Z/100.
AGEP(I)=AGEP(I)+Z*Z1
AGED(I)=AGED(I)+Z*Z2
AGEF(I)=AGEF(I)+Z*Z3
DO 67 J=1,NR
AGEO(J,I)=AGEO(J,I)+HU(J)
67 CONTINUE
IF (NR.LE.10) PRINT 147, ZP,ZB,ZD,(HU(J),J=1,NR)
IF (NR.GT.10) PRINT 85, ZP,ZB,ZD,(HU(J),J=1,NR)
147 FORMAT (/1X,5HTOTAL,lX,3F10.4,10F9.4)
85 FORMAT (/1X,5HTOTAL,3F10.4,12F8.4)
DO 60 I=1,NR
DO 5 X=1,NA
RATD(X,I)=DEATH(X,I)/(POP(X,I)*XZD)
RATF(X,I)=BIRTH(X,I)/(POP(X,I)*XZB)
DO 21 J=1,NR
RATM(X,J,I)=OMIG(X,J,I)/(POP(X,I)*XZO)
5 CONTINUE
DO 35 I=1,NR
GRD(I)=0.
GRR(I)=0.
36 GRO(J,I)=0.
DO 35 X=1,NA
GRD(I)=GRD(I)+RATD(X,I)
GRR(I)=GRR(I)+RATF(X,I)
35 CONTINUE
DO 35 I=1,NR
GRO(J,I)=GRO(J,I)+RATM(X,J,I)
35 CONTINUE

C COMPUTE AND PRINT OBSERVED RATES, GROSS RATES, MEAN AGES OF SCHEDULES

C COMPUTE AND PRINT CRUDE RATES

DO 635 I=1,NR
CRUDD(I)=DEATHT(I)/(POPT(I)*XZD)
CRUDF(I)=BIRTHT(I)/(POPT(I)*XZB)
Z=0.
DO 69 J=1,NR
Z=Z+OMIGT(J,I)
69 CRUDO(J,I)=OMIGT(J,I)/(POPT(I)*XZO)
CRUDOT(I)=Z/(POPT(I)*XZO)
635 CONTINUE
DO 5 I=1,NR
DO 5 X=1,NA
RATD(X,I)=DEATH(X,I)/(POP(X,I)*XZD)
RATF(X,I)=BIRTH(X,I)/(POP(X,I)*XZB)
DO 21 J=1,NR
RATM(X,J,I)=OMIG(X,J,I)/(POP(X,I)*XZO)
5 CONTINUE
DO 35 I=1,NR
GRD(I)=0.
GRR(I)=0.
36 GRO(J,I)=0.
DO 35 X=1,NA
GRD(I)=GRD(I)+RATD(X,I)
GRR(I)=GRR(I)+RATF(X,I)
35 CONTINUE
DO 35 I=1,NR
GRO(J,I)=GRO(J,I)+RATM(X,J,I)
35 CONTINUE
PRINT 20
20 FORMAT (1H1,5X,14OBSERVED RATES/6X,14(1H*))
DO 33 I=1,NR
GR(I)=0.
DO 78 J=1,NR
GR(I)+=GROT(I)+GRO(J,I)
78 CONTINUE
AGED(I)=0.
AGEF(I)=0.
DO 30 J=1,NR
AGED(I)=AGEG(I)+2*RATE(X,I)/GRD(I)
IF (C/R(I).GT.0.)
AGED(I)=AGED(I)+Z*RATE(X,I)/GRD(I)
DO 48 J=1,NR
48 CONTINUE
PRINT 31
31 FORMAT (/20X,1HDEATH RATES/20X,11(1H*))
PRINT 32, (REC(J),J=1,NR)
PRINT 66
DO 18 X=1,NA
18 PRINT 19, NAGE(X),(RATE(X,J),J=1,NR)
19 FORMAT (3X,I3,5X,13F9.6)
DO 11 J=1,NR
11 HU(J)=GRD(J)*ZFNY
PRINT 37, (HU(J),J=1,NR)
37 FORMAT (/1X,5HGRAISS,5X,F9.6,12F9.6)
PRINT 39, (CURD(J),J=1,NR)
39 FORMAT (/1X,5HCRUDE,5X,F9.6,12F9.6)
PRINT 49, (AGED(J),J=1,NR)
PRINT 71
71 FORMAT (/20X,15HFERTILITY RATES/20X,15(1H*))
PRINT 32, (REC(J),J=1,NR)
PRINT 66
DO 72 X=1,NA
72 PRINT 19, NAGE(X),(RATEF(X,J),J=1,NR)
DO 12 J=1,NR
12 HU(J)=GRR(J)*ZFNY
PRINT 37, (HU(J),J=1,NR)
37 FORMAT (/1X,5HGRROSS,5X,F9.6,12F9.6)
PRINT 39, (CRDF(J),J=1,NR)
39 FORMAT (/1X,5HCRUDF,5X,F9.6,12F9.6)
PRINT 49, (AGEDF(J),J=1,NR)
PRINT 73
73 FORMAT (/1H1,19X,18HOUTMIGRATION RATES/20X,18(1H*))
ISKIP=3
DO 79 I=1,NR
79 AGEOT(I)=0.
IF (ISKIP.NE.1) GO TO 167
PRINT 165
ISKIP=ISKIP+2
167 CONTINUE
PRINT 74, REG(I)
74 FORMAT (/20X,1AHMIGRATION FROM,1X,A8,1X,2HTO)
PRINT 75, (REG(J), J=1,NR)
75 FORMAT (3X,3HAGE,9X,5HTOTAL,12(1X,A8))
PRINT 66
DO 76 X=1,NA
Z = FLOAT(NAGE(X)) +ZFNY*0.5
Z = 0.
DO 77 J=1,NR
ZZ = ZZ +RATM(X,J,I)
77 CONTINUE
IF (GROT(I).GT.0.)
AGEOT(I) = AGEOT(I) + Z*ZZ/GROT(I)
76 PRINT 19, NAGE(X), ZZ, (RATM(X,J,I), J=1,NR)
HHU = GROT(I)*ZFNY
DO 13 J=1,NR
HU(J) = GROT(J,I)*ZFNY
PRINT 37, HHU, (HU(J), J=1,NR)
PRINT 39, CRUDOT(I), (CRUDO(J,I), J=1,NR)
PRINT 49, AGEOT(I), (AGEO(J,I), J=1,NR)
PRINT 66
79 CONTINUE
C ........................................................
C LIFE TABLE FOR EACH REGION SEPARATELY
C ........................................................
DO 22 I=1,NR
DO 23 X=1,NA
POPC(X) = POP(X,I)
DEATHC(X) = DEATH(X,I)
BIRTHC(X) = BIRTH(X,I)
CMIGC(X) = 0.
CMIGA(X) = 0.
DO 23 J=1,NR
CMIGA(X) = CMIGA(X) + OMIG(X,J,I)
DO 23 CMIGC(X) = CMIGC(X) + OMIG(X,J,I)
REGL = REG(I)
CALL TOTSY (NA, ZFNY, XZB, XZD, XZO, 1, REGL)
22 CONTINUE
C ........................................................
C LIFE TABLE FOR COUNTRY
C ........................................................
DO 53 X=1,NA
POPC(X) = 0.
DEATHC(X) = 0.
BIRTHC(X) = 0.
CMIGC(X) = 0.
CMIGA(X) = 0.
DO 53 I=1,NR
POPC(X) = POPC(X) + POP(X,I)
DEATHC(X) = DEATHC(X) + DEATH(X,I)
BIRTHC(X) = BIRTHC(X) + BIRTH(X,I)
DO 53 J=1,NR
CMIGC(X) = CMIGC(X) + OMIG(X,J,I)
CMIGA(X) = CMIGA(X) + OMIG(X,I,J)
53 CONTINUE
NR1 = NR + 1
REGL = REG(NR1)
CALL TOTSY (NA, ZFNY, XZB, XZD, XZO, 1, REGL)
RETURN
END
SUBROUTINE TOTSY(NA,ZFNY,XZB,XZD,XZO,NWHOL,REGL)

C ........................................................................
C RWHOL = 0 AGGREGATE SYSTEM (TOTAL POPULATION FEATURES ARE NOT PRINTED)
C REGL = NAME OF REGION CONSIDERED
C ........................................................................

DIMENSION ZMIGA(18)
DIMENSION POPC2(18),DETHC2(18),BIRC2(18),ZMIGC2(18)
DIMENSION P(18),Q(18),CL(18),CLL(18),T(18),SU(18),E(18)
COMMON /CNAG/ NAGE(18)
COMMON /CTIT/ TIT(20)
COMMON /CTOT/ BIRTHC(18),DEATHC(18),CMIGC(18),CMIGA(18)
COMMON /CTOTRAT/ POPC(18),RATDT(18),RATFT(18),RATMT(18)
DOUBLE PRECISION REGL
INTEGER X,XX
REAL L
NAA=NA-1
ZFNY=ZFNY*0.5
C ........................................................................
C COMPUTE AND PRINT CHARACTERISTICS OF TOTAL POPULATION SYSTEM
C ........................................................................

IF (NWHOL.EQ.0) GO TO 555
PRINT 30
PRINT 1, (TIT(J),J=1,20)
1 FORMAT (5OX,2OA4)
PRINT 51,REGL
51 FORMAT (1H0,30X,A8/31X,8(1HI)//)
102 CONTINUE
PRINT 52
52 FORMAT (1X,3HAGE,6X,10HPopulation,8X,6HBirths,10X,6HDeaths,5X,14X,8HArrivals,7X,10HDepartures,13X,25Hobserved Rates (X 1000 ))
PRINT 559
559 FORMAT (5X,5(3X,6HNumber,2X,5H- % -),5X,5HBirth,4X,5HDeath,14X,5HmIG,3X,6HOutMIG,2X,7HNet MIG/)
C TOTAL POPULATION, BIRTHS, DEATHS, MIGRANTS
PP4=0.
BB4=0.
DD4=0.
ZMM4=0.
ZMI4=0.
DO 54 X=1,NA
PP4=PP4+POPC(X)
DD4=DD4+DEATHC(X)
BB4=BB4+BIRC(X)
ZMM4=ZMM4+CMIGC(X)
ZMI4=ZMI4+CMIGA(X)
54 CONTINUE
C PERCENTAGE DISTRIBUTION
ZP=0.
ZB=0.
ZD=0.
ZM=0.
ZI=0.
DO 53 X=1,NA
POPC2(X)=100.*POPC(X)/PP4
IF (DD4.NE.0.) DETHC2(X)=100.*DEATHC(X)/DD4
IF (BB4.NE.0.) BIRC2(X)=100.*BIRTHC(X)/BB4
IF (ZMM4.NE.0.) ZMICC2(X)=100.*CMICC(X)/ZMM4
IF (ZMI.NE.0.) ZMICA(X)=100.*CMICA(X)/ZMI
ZP=ZP+POPC2(X)
ZB=ZB+BIRC2(X)
ZD=ZD+DETHC2(X)
ZM=ZM+ZMICC2(X)
ZI=ZI+ZMICA(X)
ZP.+POP5
ZB=ZB+BIRC2(X)
ZD=ZD+DETHC2(X)
ZM=ZM+ZMICC2(X)
ZI=ZI+ZMICA(X)
53 CONTINUE
C RATES
DO 58 X=1,NA
RATDT(X)=DEATHC(X)/(POPC(X)*XZD)
RATFT(X)=BIRTHC(X)/(POPC(X)*XZB)
RATMT(X)=CMICC(X)/(POPC(X)*XZO)
58 CONTINUE
C PRINT OBSERVED POPULATION CHARACTERISTICS
RAMI=0.
DO 59 X=1,NA
RIT=CMICA(X)/(POPC(X)*XZO)
RAMI=RAMI+RIT
Z=1000.
R2D=RATDT(X)*Z
R2F=RATFT(X)*Z
R2M=RATMT(X)*Z
RZI=RIT*Z
RNT=RZI-RZM
PRINT 65, NAGE(X),POPC(X),POPC2(X),BIRTHC(X),BIRC2(X),DEATHC(X),
1 DETHC2(X),CMICA(X),ZMICA(X),CMICC(X),ZMICC2(X),RZD,
1 RZD+ZD,
1 RZD+ZD
63 FORMAT (1X,13,F10.0,F7.2,4(F9.0,F7.2),1X,5F9.3)
59 CONTINUE
C CROSS RATES
RZD4=0.
R2F=L2F*Z
RZD=RADD4*Z
R2M=RAMM4*Z
1 RZD,RZI,RZM
PRINT 18, RZF,RZD,RZI ,RZM
18 FORMAT (1X,5HCROSS,8OX,5F9.3)
C CRUDE RATES
Z=1000.
RAFC4=Z*BB4/(PP4*XZB)
RADC4=Z*DD4/(PP4*XZD)
RAMC4=Z*ZMI/(PP4*XZO)
RIMC4=Z*ZMI/(PP4*XZO)
RNET=RMCH4=RAMC4
PRINT 71, RAFC4,RADC4,RIMC4,RAMC4,RNET
71 FORMAT (1X,SHCRUDE,7H(X1000),73X,5F9.3)
C MEAN AGE
AGEPC4=0.
AGEFC4=0.
AGEDC4=0.
AGEMC4=0.
AGEFR4=0.
AGEMR4=0.
ACEMC4=0.
ACIMC4=0.
ACEFR4=0.
AGEDR4=0.
ACEDC4=0.
ACIMR4=0.
DO 81 X=1,NA
Z=FLOAT(NAGE(X))+ZFNY*0.5
Z1=Z/100.
ACEPC4=ACEPC4+Z1*POPC2(X)
AGEFC4=AGEFC4+Z1*BIRC2(X)
AGEDC4=AGEDC4+Z1*DETHC2(X)
AGEMC4=AGEMC4+Z1*ZMIGC2(X)
AGIMC4=AGIMC4+Z1*ZMICA(X)
IF (RAFF4.CT.O.)
AGEFR4=AGEFR4+Z1*RATFT(X)/RAFF4
IF (RADD4.CT.O.)
AGEDR4=AGEDR4+Z1*RATDT(X)/RADD4
IF (RAMM4.CT.O.)
AGEMR4=AGEMR4+Z1*RATMT(X)/RAMM4
IF (RAMI.CT.0.)
ACIMR4=ACIMR4+Z1*ZMICA(X)/(POPC(X)*XZO*RAMI)
81 CONTINUE
PRINT 82, ACEPC4,AGEFC4,AGEDC4,ACEMC4,AGEFR4,AGEDR4,
ACIMC4,ACIMR4
82 FORMAT (1X,5H.M,ACE,8X,F7.2,4(9~,~7.2),1~,5F9.2)
555
C SINGLE REGION LIFE TABLE OF TOTAL POPULATION SYSTEM
C
DO 10 X=1,NA
P(X)=(1.-ZFNY2*ZATDT(X))/(1.+ZFNY2*ZATDT(X))
10 Q(X)=1.-P(X)
CL(1)=1.
DO 11 X=1,NA
XX=X+1
CL(XX)=CL(X)*P(X)
11 CLL(X)=ZFNY2*(CL(X)+CL(XX))
CLL(NA)=CL(NA)/RATDT(NA)
T(NA)=CLL(NA)
E(NA)=T(NA)/CL(NA)
DO 12 X=1,NA
XX=NA-X
SU(X)=CLL(XX)/CLL(X)
12 IF (NWHOL.EQ.1) PRINT 17, E(1)
17 FORMAT (1H1,9HE(0),90X,F9.2)
DO 13 X=1,NA
XX=NA-X
13 IF (NWHOL.NE.0) PRINT 30
30 FORMAT (1H1/)
C
PRINT 1, (TIT(J), J=1,20)
PRINT 5060, REGL,E(1)
5060 FORMAT (1HO, //10X,5HTABLE,4X,26H- SINGLE REGION LIFE TABLE, 3X
1A6,6X,17HMORTALITY LEVEL = ,F6.2/11X,74(1H-)/
PRINT 5062
5062 FORMAT (1X,3HAGE,7X,4HP(X),7X,4HQ(X),7X,4HL(X),7X,4HD(X),6X,
15HLL(X),7X,4HM(X),7X,4HS(X),6X,4HT(X),6X,4HE(X)/)
DO 5063 X=1,NA
21=CL(X)*100000.
Z2=100000.*CL(X)*Q(X)
5063 PRINT 5064, NAGE(X),P(X),Q(X),Z1,Z2,CLL(X),RATDT(X),SU(X),
1T(X),E(X)
5064 FORMAT (1X,I3,2F11.6,2F11.0,3F11.6,2F10.4)
C COMPUTE AND PRINT NRR AND NMR
IF (NWHOL.EQ.0) GO TO 556
RNM=0.
RNR=0.
DO 14 X=1,NA
RNR=RNR+CLL(X)*RATFT(X)
14 RNM=RNM+CLL(X)*RATMT(X)
PRINT 66
66 FORMAT (//1X)
PRINT 15, RNR
15 FORMAT (/30X,21HNET REPRODUCTION RATE,4X,F10.6)
PRINT 16, RNM
16 FORMAT (/30X,24HNET MIGRATION RATE,1X,F10.6)
556 CONTINUE
RETURN
END
SUBROUTINE PROBSC (NA, ZFNY, NR, IPROB)
DIMENSION RM(7,7)
COMMON /CNAG/ NAGE(18)
COMMON /CINV/ CC(7,7)
COMMON /CMUL/ A(7,7), B(7,7), C(7,7)
COMMON /CFQ/ P(18,7,7)
COMMON /CRADE/ RATD(18,7), RATM(18,7,7), RATF(18,7)
COMMON /CREG/ REG(13)
COMMON /CRLA/ RMLA(7,7)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X

C MATRIX OF OBSERVED RATES M(X)
C
NA=NA-1
ZZZ=ZFN1*0.5
DO 100 X=1,NA
DO 5 I=1, NR
2=RATD(X,I)
DO 4 J=1,NR
IF (1.EQ.J) GO TO 4
2=2+RATM(X,J,I)
4 CONTINUE
RM(I,I)=2
DO 6 J=1, NR
IF (J.EQ.1) GO TO 6
RM(J,I)=-RATM(X,J,I)
6 CONTINUE
5 CONTINUE
IF (X.NE.NA) GO TO 13
DO 14 I=1,NR
DO 14 J=1, NR
14 RMLA(J,I)=RM(J,I)
GO TO 100
13 CONTINUE
C PROBABILITY MATRICES
C
DO 7 I=1, NR
DO 7 J=1, NR
IF (1.EQ.J) CC(J,I)=1+ZZZ*RSM(J,I)
7 IF (1.NE.J) CC(J,I)=ZZZ*RSM(J,I)
CALL INVERT (NR)
DO 8 I=1, NR
DO 8 J=1, NR
A(J,I)=CC(J,I)
8 IF (J.EQ.I) B(J,I)=1-ZZZ*RSM(J,I)
CALL MULTIP (NR, NR, NR)
DO 9 I=1, NR
DO 9 J=1, NR
9 P(X,J,I)=C(J,I)
100 CONTINUE
DO 10 I = 1, NR
DO 10 J = 1, NR
10 P(NA, J, I) = 0.
C -------------------------------------------------------------
C PRINT PROBABILITIES
C -------------------------------------------------------------
9999 FORMAT (1H1, 'X')
PRINT 9999
PRINT 1, (TIT(J), J = 1, 20)
1 FORMAT (50X, 20AN/)
PRINT 4500, IPROB
4500 FORMAT (20X, 36HPROBABILITIES OF DYING AND MIGRATING/
120X, 13(1H*), 7H OPTION, I2, 1X, 13(1H*)/33X, 10(1H*))
ISKIP = 3
DO 726 I = 1, NR
IF (ISKIP .NE. I) GO TO 121
PRINT 9999
ISKIP = ISKIP + 2
121 CONTINUE
PRINT 9001, REG(I)
9001 FORMAT (//20X, 6HREGION, 2X, A8, 1X/20X, 16(1H*)/)
PRINT 9011, REG(I)
9011 FORMAT (5X, 3HAGE, 5X, 5HDEATH, 5X, 14HMIGRATION FROM, 1X, A8, 1X, 2HTO)
PRINT 9020, (REG(J), J = 1, NR)
9020 FORMAT (18X, 12(1X, A8))
PRINT 66
66 FORMAT (1X)
DO 726 X = 1, NA
ZZ = 0.
DO 11 J = 1, NR
ZZ = ZZ + P(X, J, I)
11 CONTINUE
ZQ = 1. - ZZ
PRINT 9103, NAGE(X), ZQ, (P(X, J, I), J = 1, NR)
9103 FORMAT (5X, I3, 1X, 13F9.6)
726 CONTINUE
RETURN
END
SUBROUTINE PROBR (NA, ZFNY, NR, IPROB)

DIMENSION Q(18,7)
COMMON /CNAG/ NAGE(18)
COMMON /CQF/ P(18,7,7)
COMMON /CATE/ RATD(18,7), RATM(18,7,7), RATF(18,7)
COMMON /CREG/ REG(13)
COMMON /CMLA/ RMLA(7,7)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG

INTEGER X
NAA = NA - 1
ZFN2 = ZFNY*0.5

C -------------------------------
C COMPUTE PROBABILITIES
C -------------------------------

DO 8 I = 1, NR
   DO 8 X = 1, NA
      Z2 = 0.
      DO 6 J = 1, NR
         IF (I.EQ.J) GO TO 6
         Z2 = Z2 + RATM(X, J, I)
      6 CONTINUE
      Z = 1. - ZFN2 * RATD(X, I) + ZFN2 * Z2
      Q(X, I) = ZFN2 * RATD(X, I) / Z
      DO 7 J = 1, NR
      7 CONTINUE
   8 CONTINUE

DO 10 X = 1, NAA
   DO 10 I = 1, NR
      PMIGT = 0.
      DO 9 J = 1, NR
         IF (I.EQ.J) GO TO 9
         PMIGT = PMIGT + P(X, J, I)
      9 CONTINUE
      P(X, I, I) = 1. - Q(X, I) - PMIGT
   10 CONTINUE

DO 11 I = 1, NR
   Q(NA, I) = 1.
   DO 11 J = 1, NR
      P(NA, J, I) = 0.
   11 CONTINUE

DO 12 I = 1, NR
   DO 12 J = 1, NR
      IF (I.EQ.J) RMLA(I, I) = RATD(NA, I)
      IF (I.NE.J) RMLA(J, I) = 0.
   12 CONTINUE

C -------------------------------
C PRINT PROBABILITIES
C -------------------------------

9999 FORMAT (1H1, 1X)
PRINT 9999
   PRINT 1, (TIT(J), J = 1, 20)
1 FORMAT (50X, 20A4//)
PRINT 4500, IPROB
4500 FORMAT (2OX, 34/PROBABILITIES OF DYING AND MIGRATING/
11OX, 13(1H*), 7H OPTION, 12, 1X, 13(1H*))/33X, 10(1H*))
   ISKIP=3
   DO 726 I=1, NR
   IF (ISKIP .NE. I) GO TO 121
   PRINT 9999
   ISKIP=ISKIP+2
121   CONTINUE
   PRINT 9001, REG(I)
9001   FORMAT (/2OX, 6/REGION, 2X, A8, 1X/20X, 16(1H*))
   PRINT 9011, REG(I)
9011   FORMAT (5X, 3/HAGE, 5X, 5/HDEATH, 5X, 14/MIGRATION FROM, 1X, A8, 1X, 2HT0)
   PRINT 9020, (REG(J), J=1, NR)
9020   FORMAT (16X, 10(1X, A8))
   PRINT 66
66   FORMAT (1X)
   DO 726 X=1, NA
   PRINT 9103, NAGE(X), Q(X, J), (P(X, J, I), I=1, NR)
9103   FORMAT (5X, 11, 1X, 11F9.6)
726   CONTINUE
   RETURN
END
SUBROUTINE HIST (N, NR, IHIST)
DIMENSION HULP(7), RM(7)
COMMON /CNAG/ NAGE(18)
COMMON /CCL/ CL(18, 7, 7)
COMMON /CMUL/ A(7, 7), B(7, 7), C(7, 7)
COMMON /CPQ/ P(18, 7, 7)
COMMON /CRAD/ RADIX(7), RADIXT
COMMON /CREG/ REG(13)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
REAL L
INTEGER X, XX
66 FORMAT (1X)
IHIST = 1
C
CMPLTE THE NUMBER OF SURVIVORS AT EXACT AGE X
C
DO 5 I = 1, NR
  DO 5 J = 1, NR
    IF (I.NE.J) CL(I, I, J) = 0.
  5 CONTINUE
NAA = NA - 1
DO 14 X = 1, NAA
  XX = X + 1
  DO 15 I = 1, NR
    DO 15 J = 1, NR
      A(J, I) = P(X, J, I)
  15 B(J, I) = CL(I, 1, J)
  CALL MULTIP (NR, NR, NR)
  DO 16 I = 1, NR
    DO 16 J = 1, NR
      CL(XX, I, J) = C(J, I)
  14 CONTINUE
C
COMPUTE AND PRINT THE LIFE HISTORY OF THE INITIAL COHORT
C
PRINT 1, (TIT(J), J = 1, 20)
1 FORMAT (1H1, 50X, 20A4)
PRINT 9201
9201 FORMAT (1H0, /20X, 30HLIFE HISTORY OF INITIAL COHORT/20X, 30(1H*))
DO 250 I = 1, NR
  IF (I0.NE.1) PRINT 9211
9211 FORMAT (1H1, 1X)
PRINT 9202, REG(I0)
9202 FORMAT (1H0, 20X, 24HINITIAL REGION OF COHORT, 2X, A8/21X, 13H(/))
ISLIP = 1
DO 20 I = 1, NR
  IF (ISKIP.NE.I) GO TO 29
PRINT 9211
  ISKIP = ISKIP + 1
20 CONTINUE
PRINT 21, I, REG(I)
21 FORMAT (10X, I2, 2H.-, 1X, 19REGION OF RESIDENCE, 2X, A8/)
PRINT 22
22 FORMAT (9X, 6HDEATHS, 5X, 11HMIGRANTS TO)
PRINT 23, (REG(J), J=1, NR)
23 FORMAT (1X, 3HAGE, 11X, 12(1X, A8))
PRINT 66
CDRT=0.
DO 6 J=1, NR
6 HULP(J)=0.
DO 230 X=1, NA
ZZ=0.
DO 119 J=1, NR
119 ZZ=ZZ+P(X, J, I)
ZZ=CL(X, IO, I)*ZZ
CDR=ZZ*RADIX(IO)
CDRT=CDRT+CDR
DO 24 J=1, NR
ZZ=CL(X, IO, I)*P(X, J, I)
RM(J)=ZZ*RADIX(IO)
24 HULP(J)=HULP(J)+RM(J)
PRINT 25, NAGE(X), CDR, (RM(J), J=1, NR)
25 FORMAT (1X, I3, 2X, 13F9.0)
CONTINUE
PRINT 26, CDRT, (HULP(J), J=1, NR)
26 FORMAT (/1X, 5HTOTAL, 13F9.0)
PRINT 66
CONTINUE
20 CONTINUE
RETURN
END
SUBROUTINE LIFE (NA, ZFNY, NR, IPROB, IHIST, ILIF)
DIMENSION CM(7)
DIMENSION E(18,7,7), T(7)
COMMON /CNAG/ NAGE(18)
COMMON /CCL/ CL(18,7,7)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)
COMMON /CMUL/ A1(7,7), B(7,7), C(7,7)
COMMON /CPQ/ P(18,7,7)
COMMON /CRATE/ RATD(18,7), RATM(18,7,7), RATF(18,7)
COMMON /CRAD/ RADIX(7), RADIXT
COMMON /CREG/ REG(13)
COMMON /CRMLA/ RMLA(7,7)
COMMON /CTIT/ TIT(20)
COMMON /CSU/ SU(18,7,7)
COMMON /CRATE/ RATD(18,7), RATM(18,7,7), RATF(18,7)
DOUBLE PRECISION REG
REAL L
INTEGER X, XX, XY, XZ;
ILIF=1
NAA=NA-1
IPREX=1
66 FORMAT (1X)
9103 FORMAT (1X,13,1X, F10.5, 12P9.5)
9020 FORMAT (15X, 'REGION', 12X, A8)
9001 FORMAT (1H1, '/X)
9011 FORMAT (1X, 'REGION', 6X, SHDEATH, 5X, 14HMIGRATION FROM, 1X, A8, 1X, 2HTO)
C COMPUTE NUMBER OF SURVIVORS
IF (IHIST .NE. 0) GO TO 5
DO 76 I=1, NR
   CL(1,1,I)=1.
   DO 76 J=1, NR
      IF (I .NE. J) CL(1,J,I)=0.
   CONTINUE
DO 77 X=1, NAA
   XX=X+1
   DO 6 I=1, NR
      DO 6 J=1, NR
         A1(I,J)=P(X, J, I)
      CONTINUE
      B(J, I)=CL(X, I, J)
   CALL MULTIP (NR, NR, NR)
   DO 7 I=1, NR
      DO 7 J=1, NR
         CL(XX, I, J)=C(J, I)
   CONTINUE
   CONTINUE
C PRINT NUMBER OF SURVIVORS
   PRINT 1, (TIT(J), J=1, 20)
   1 FORMAT (1H1, 5DX, 20A4)
   PRINT 66
   PRINT 66
PRINT 4831
4831 FORMAT (20X,5HEXPECTED NUMBER OF SURVIVORS AT EXACT AGE X IN EACH
17TH REGION, 20X, 5H(*1H*))
ISKIP = 3
DO 60 IO = 1, NR
IF (ISKIP .NE. IO) GO TO 123
PRINT 9999
ISKIP = ISKIP + 2
123 CONTINUE
PRINT 9502, REG(IO)
9502 FORMAT (/1X, 3HAGE, 6X, 24HINITIAL REGION OF COHORT, 2X, A8/1X,
13(1H*), 6X, 34(1H*))
PRINT 9100, (REG(J), J = 1, NR)
9100 FORMAT (10X, 5HTOTAL, 10(1X, A8))
PRINT 66
DO 60 X = 1, NA
CLT = 0.
DO 8 J = 1, NR
CM(J) = CL(X, IO, J) * RADIX(IO)
8 CLT = CLT + CM(J)
PRINT 9101, NAGE(X), CLT, (CM(J), J = 1, NR)
9101 FORMAT (1X, I3, 1X, F10.0, 12F9.0)
60 CONTINUE
C NUMBER OF YEARS LIVED BETWEEN X AND X+NY
C BY REGION OF BIRTH
DO 10 IO = 1, NR
DO 10 X = 1, NAA
ZZ = ZPNY * 0.5
XX = X + 1
DO 9 I = 1, NR
9 L(X, IO, I) = ZZ * (CL(X, IO, I) + CL(XX, IO, I))
10 CONTINUE
18 CONTINUE
C NUMBER OF YEARS LIVED IN LAST AGE GROUP
DO 2 I = 1, NR
DO 2 J = 1, NR
2 CC(J, I) = RMLA(J, I)
CALL INVERT (NR)
DO 3 I = 1, NR
DO 3 J = 1, NR
A(J, I) = CC(J, I)
3 B(J, I) = CL(NA, I, J)
CALL MULTIP (NR, NR, NR)
DO 4 I = 1, NR
DO 4 J = 1, NR
4 L(NA, I, J) = C(J, I)
120 CONTINUE
50 CONTINUE
PRINT 9999
PRINT 4832
4832 FORMAT (10X, 3HNUMBER OF YEARS LIVED IN EACH REGION BY
1, 20H A UNIT BIRTH COHORT, 10X, 5H(*1H*))
ISKIP=3
DO 34 IO=1,NR
IF (IO.NE.ISKIP) GO TO 124
PRINT 9999
ISKIP=ISKIP+2
124 CONTINUE
PRINT 9502, REG(IO)
PRINT 9100, (REG(J),J=1,NR)
PRINT 66
DO 58 X=1,NA
CLLT=0.
DO 11 J=1,NR
11 CLLT=CLLT+L(X,IO,J)
PRINT 9103, NAGE(X),CLLT,(L(X,IO,J),J=1,NR)
58 CONTINUE
34 CONTINUE
C *---------------------------------------------------------------------
C - BY REGION OF RESIDENCE AT AGE X
C *---------------------------------------------------------------------
ZFNY2=ZFNY*0.5
PRINT 9999
80 FORMAT (10X,39HNUMBER OF YEARS LIVED IN EACH REGION BY 
1,16H PERSON OF AGE X/10X,55(1H*))
ISKIP=3
DO 85 I=1,NR
IF (1.NE.ISKIP) GO TO 82
PRINT 9999
ISKIP=ISKIP+2
82 CONTINUE
PRINT 83, REC(IO)
83 FORMAT //1X,3HACE,bX,28HREGION 
of residence at age X, 
12X,A8/1X,3(1H*),6X,38(1H*)/)
PRINT 9100, (REG(J),J=1,NR)
PRINT 66
DO 81 X=1,NA
CMT=0.
DO 84 J=1,NR
IF (I.EQ.J) CM(J)=ZFNY2*(1.+P(X,J,I))
IF (I.NE.J) CM(J)=ZFNY2*P(X,J,I)
IF (X.EQ.NA) CM(J)=CC(J,I)
CMT=CMT+CM(J)
84 CONTINUE
PRINT 9103, NAGE(X),CMT,(CM(J),J=1,NR)
81 CONTINUE
85 CONTINUE
C *---------------------------------------------------------------------
C LIFE TABLE MORTALITY AND MIGRATION RATES (NO MULTIPLE TRANSITION)
C *---------------------------------------------------------------------
IF (IPROB.EQ.3) GO TO 159
PRINT 9999
PRINT 4833
4833 FORMAT (20X,30HDEATH RATE AND MIGRATION RATES/20X,30(1H*))
ISKIP=3
DO 59 I=1,NA
IF (ISKIP .NE. I) GO TO 122
PRINT 9999
ISKIP = ISKIP + 2
122 CONTINUE
PRINT 9001, REG(I)
PRINT 9011, REG(I)
PRINT 9020, (REG(J), J = 1, NR)
PRINT 66
DO 59 X = 1, NA
   21 = ZFNY2 * (1. - P(X, I, I))
   PMIGT = 0.
   DO 35 J = 1, NR
      PMIGT = PMIGT + P(X, J, I)
   35 ZQ = 1. - PMIGT
      RATDV = ZQ / Z1
   DO 36 J = 1, NR
      CM(J) = P(X, J, I) / Z1
   36 IF (X .NE. NA) GO TO 37
      RATDV = RATDV(NA, I)
   DO 38 J = 1, NR
      CM(J) = 0.
   37 CONTINUE
59 CONTINUE
159 CONTINUE

C SURVIVORSHIP PROPORTIONS
C

DO 61 X = 1, NAA
   XX = X + 1
   DO 74 IO = 1, NR
      DO 74 J = 1, NR
         CC(IO, J) = L(X, IO, J)
      CALL INVERT (NR)
      DO 75 IO = 1, NR
         DO 75 J = 1, NR
            SU(X, IO, J) = 0.
            DO 75 JJ = 1, NR
               SU(X, IO, J) = SU(X, IO, J) + CC(IO, JJ) * L(XX, JJ, J)
         75 CONTINUE
   61 CONTINUE

C SURVIVORSHIP PROPORTIONS
C

DO 64 I = 1, NR
   IF (ISKIP .NE. I) GO TO 125
   PRINT 9999
   ISKIP = ISKIP + 2
125 CONTINUE
PRINT 9001, REG(I)
PRINT 9100, (REG(J), J = 1, NR)
NAA = NA - 1
PRINT 66
DO 63 X = 1, NAA
SSU=0.
   DO 62 J=1,NR
62   SSU=SSU+SU(X,I,J)
   PRINT 9103, NAGE(X),SSU,(SU(X,I,J),J=1,NR)
   CONTINUE
C------------------------------------------------------------------
C NUMBER OF YEARS LIVED BEYOND AGE X AND LIFE EXPECTANCY BY PLACE OF BIRTH
C------------------------------------------------------------------
   PRINT 9999
   PRINT 4835
4835 FORMAT (10X,40HTOTAL NUMBER OF YEARS LIVED BEYOND AGE X/110X,40(1H*)
   ISKIP=3
   DO 51 IO=1,NR
      IF (ISKIP.NE.IO) GO TO 126
      PRINT 9999
      ISKIP=ISKIP+2
126   CONTINUE
      PRINT 9502, REG(IO)
      PRINT 9100, (REG(J),J=1,NR)
      PRINT 66
   DO 14 X=1,NA
   TT=0.
   DO 17 I=1,NR
      T(I)=0.
   DO 12 XY=X,NA
      T(I)=T(I)+L(XY,IO,I)
17    TT=TT+T(I)
   PRINT 9103, NAGE(X),TT,(T(J),J=1,NR)
   CLT=0.
   DO 333 J=1,NR
      CLT=CLT+CL(X,IO,J)
   DO 13 J=1,NR
      E(X,IO,J)=0.
      IF (CLT.EQ.0.) GO TO 13
         E(X,IO,J)=T(J)/CLT
13    CONTINUE
   CONTINUE
51   CONTINUE
C PRINT LIFE EXPECTANCY
   PRINT 9999
   PRINT 4830
4830 FORMAT (30X,38HEXPECTATIONS OF LIFE BY PLACE OF BIRTH/130X,38(1H*)
   ISKIP=3
   DO 65 IO=1,NR
      IF (ISKIP.NE.IO) GO TO 127
      PRINT 9999
      ISKIP=ISKIP+2
127   CONTINUE
      IF (IPREX.EQ.1) PRINT 9502, REG(IO)
      IF (IPREX.EQ.25) PRINT 83, REG(IO)
   PRINT 9100, (REG(J),J=1,NR)
   PRINT 66
   DO 65 X=1,NA
EE = 0.
DO 15 J = 1, NR
15 EE = EE + E(X, IO, J)
PRINT 9103, NAGE(X), EE, (E(X, IO, J), J = 1, NR)
65 CONTINUE
IF (IPREX.EQ.25) GO TO 877
C ----------------------------------------------------------------------
C LIFE EXPECTANCY BY PLACE OF RESIDENCE
C ----------------------------------------------------------------------
PRINT 9999
PRINT 56
56 FORMAT (30X, 42HEXPECTATIONS OF LIFE BY PLACE OF RESIDENCE/
130X, 42(1H*))
DO 49 I = 1, NR
DO 49 J = 1, NR
49 A(I,J) = 0.
NA = NA + 1
DO 57 IX = 1, NA
X = NA - IX
DO 52 I = 1, NR
DO 52 J = 1, NR
52 CC(I,J) = CL(X, I, J)
CALL INVERT (NR)
DO 54 I = 1, NR
DO 54 J = 1, NR
A(I,J) = A(I,J) - L(X, I, J)
54 B(I,J) = CC(J, I)
CALL MULTIP (NR, NR, NR)
DO 55 I = 1, NR
DO 55 J = 1, NR
55 E(X, I, J) = C(J, I)
57 CONTINUE
IPREX = 25
GO TO 876
877 CONTINUE
RETURN
END
SUBROUTINE PRLF2 (NA, NY, IPROB)

DIMENSION ZEX(2)
COMMON /CL/ L(18,7,7)
COMMON /CNAG/ NAGE(18)
COMMON /CPQ/ P(18,7,7)
COMMON /CRAD/ RADIX(7), RADIXT
COMMON /CRATE/
RATD(18,7), RATH(18,7,7), RATF(18,7)
COMMON /CTIT/ TIT(20)
COMMON /CSU/ SU(18,7,7)

REAL L
INTEGER X, XY

PRINT 1, (TIT(J), J=1,20)
1 FORMAT (1H1, 50X, 20A4//)
PRINT 5054, IPROB
5054 FORMAT (7X, 5H TABLE, 4X, 39H - MULTIREGIONAL (TWO-REGION) LIFE TABLE, 1X, 6H OPTION, 12/7X, 5(1H-), 4X, 48(1H-)/)

I = 1
J = 2
PRINT 5059, I, ((I, I, J, J), K=1,3), J, I, (I, I, J, J), K=1, 2
DO 5057 X=1, NA
5057 CONTINUE
ZCE=0.
DO 13 J=1, 2
CLT=CLT+CL(X, 1, J)
13 CLT=CLT+CL(X, 1, J)
DO 15 J=1, 2
ZEX(J)=0.
15 TT=TT-L(X, 1, J)
ZEX(J)=TT/CLT
14 XY=XY, NA
DO 14 XY=XY, NA
1 CONTINUE
PRINT 5056, NAGE(X), ZZ, P(X, 1, 1), P(X, 2, 1), ZCL1, ZCL2, 1L(X, 1, 1), L(X, 1, 2), RATH(X, 2, 1), RATF(X, 1), SU(X, 1, 1), SU(X, 1, 2), 1(ZEX(J), J=1, 2)
5057 CONTINUE
5056 FORMAT (1X, 3H X, 3F10.6, 2F9.0, 2F10.6, 2F10.5, 4F10.6, 2F9.2)
PRINT 66
66 FORMAT (1X)
I = 1
J = 2
PRINT 5059, J, ((J, J, I, J), K=1,3), I, J, (J, J, I, J), K=1, 2
DO 5058 X=1, NA
ZCE=0.
1 CONTINUE
ZCE=0.
2ZCL1=CL(X, 2, 1) * RADIX(1)
ZCL2=CL(X, 2, 2) * RADIX(1)
CLT=0.
DO 16 J=1,2
  CLT=CLT+CL(X,2,J)
16 DO 18 J=1,2
  ZEX(J)=0.
  IF (CLT.EQ.0.) GO TO 18
  TT=0.
  DO 17 XX=X,NA
    TT=TT+L(XX,2,J)
  ZEX(J)=TT/CLT
17 TT=TT
18 CONTINUE
PRINT 5056, NAGE(X),ZI,P(X,2,2),P(X,1,2),ZCL1,ZCL2
  L(X,2,2),L(X,2,1),RATH(X,1,2),RATD(X,2),SU(X,2,2),SU(X,2,1),
  ZEX(2),ZEX(1)
RETURN
END
SUBROUTINE WHOLE (NA,ZFNY,NR)
DIMENSION CLLTOT(18),CLTOT(18)
COMMON /CCL/ CL(18,7,7)
COMMON /CL/ L(18,7,7)
COMMON /CRAD/ RADIX(7),RADIXT
COMMON /CREG/ REG(13)
COMMON /CTOTRAT/ POPC(18),RATDT(18),RATFT(18),RATMT(18)
DOUBLE PRECISION REG,REGL
REAL L
INTEGER X,XX
DATA REGL/8H AGGREG./
DO 10 X=1,NA
CLTOT(X)=0.
CLTOT(X)=0.
DO 10 I=1,NR
ZCLT=0.
ZCLLT=0.
DO 11 J=1,NR
ZCLT=ZCLT+CL(X,I,J)
ZCLLT=ZCLLT+L(X,I,J)
11 CONTINUE
CLTOT(X)=CLTOT(X)-ZCLT*RADIX(I)/RADIXT
CLTOT(X)=CLTOT(X)+ZCLLT*RADIX(I)/RADIXT
10 CONTINUE
C -------------------------------------
C COMPUTE LIFE TABLE DEATH RATES ASSOCIATED WITH AGGREGATED SYSTEM
C -------------------------------------
NAA=NA-1
DO 12 X=1,NAA
XX=X+1
12 RATDT(X)=(CLTOT(X)-CLTOT(XX))/CLTOT(X)
RATDT(NA)=CLTOT(NA)/CLTOT(NA)
PRINT 13
13 FORMAT (1H1,10X,31HLIFE TABLE OF AGGREGATED SYSTEM/
111X,31(1H*))
PRINT 14, (REG(J),J=1,NR)
14 FORMAT (6X,12(2X,A8))
PRINT 15
15 FORMAT (1X)
PRINT 16, (RADIX(J),J=1,NR)
16 FORMAT (6X,12F10.0)
PRINT 15
PRINT 15
CALL TOTSY (NA,ZFNY,1.,1.,1.,0,REGL)
RETURN
END
SUBROUTINE LMAT (NA, NR, ZFNY, ILIF)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)
COMMON /CMUL/ A1(7,7), B(7,7), C(7,7)
COMMON /CPQ/ P(18,7,7)
COMMON /CRATE/ RAI18(7), RATM(18,7,7), RATP(18,7)
COMMON /CMMLA/ RMLA(7,7)
REAL L
INTEGER X

C NAA=NA-1
ILIF=1
DO 23 I=1, NR
DO 23 J=1, NR
IF (I.EQ.J) C(J,I)=1.
IF (I.NE.J) C(J,I)=0.

23 CONTINUE
C ............................................................
C NUMBER OF YEARS LIVED BETWEEN X AND X+H
C ............................................................
DO 333 X=1, NAA
DO 24 I=1, NR
DO 24 J=1, NR
IF (I.EQ.J) A1(J, I)=(P(X, J, I)+1.)*ZFNY*0.5
IF (I.NE.J) A1(J, I)=P(X, J, I)*ZFNY*0.5

24 B(J, I)=C(J, I)
CALL MULTIP (NR, NR, NR)
DO 25 I=1, NR
DO 25 J=1, NR
L(X, I, J)=C(J, I)

25 A1(J, I)=P(X, J, I)

CALL MULTIP (NR, NR, NR)

333 CONTINUE
C ............................................................
C NUMBER OF YEARS LIVED IN LAST AGE GROUP
C ............................................................
DO 2 I=1, NR
DO 2 J=1, NR
IF (I.EQ.J) C(J, I)=1.
IF (I.NE.J) C(J, I)=0.

2 DO 4 X=1, NAA
DO 3 I=1, NR
DO 3 J=1, NR
A1(J, I)=P(X, J, I)

3 B(J, I)=C(J, I)
CALL MULTIP (NR, NR, NR)

4 CONTINUE
DO 5 I=1, NR
DO 5 J=1, NR

5 CC(J, I)=RMLA(J, I)

CALL INVERT (NR)
DO 6 I = 1, NR
DO 6 J = 1, NR
A(J,I) = CC(J,I)
6 B(J,I) = C(J,I)
CALL MULTIP (NR, NR, NR)
DO 27 I = 1, NR
DO 27 J = 1, NR
27 L(NA, I, J) = C(J, I)
RETURN
END

SUBROUTINE RELAM (ZFNY, ZLAMDK, RSTAB)
READ (5, 3) ZLAMDK
3 FORMAT (F12.8)
RSTAB = ALOG(ZLAMDK) / ZFNY
RETURN
END
SUBROUTINE GROWTH (NA, ZFNY, NR, ILIF)

COMMON /CNAG/ NAGE(18)
COMMON /CGROU/ BR(l8,7,7),POPR(18,7)
COMMON /CINV/ CC(7,7)
COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
COMMON /CPQ/ P(18,7,7)
COMMON /CRATE/
    RATD(18,7),RATM(18,7,7),RATF(18,7)
COMMON /CREG/
    REG(13)
COMMON /CRMLA/ RMLA(7,7)
COMMON /CTIT/ TIT(20)
COMMON /CSU/ SU(18,7,7)

DOUBLE PRECISION REG
INTEGER X, XX
REAL L
NAA = NA - 1
ZZ = ZFNY * 0.25
ZFNY2 = ZFNY * 0.5

C --------------------------------------------------------
C COMPUTE SURVIVORSHIP PROPORTIONS IF ILIF=0
C --------------------------------------------------------
IF (ILIF.NE.0) GO TO 50
DO 30 X = 1, NAA
    XX = X + 1
    DO 21 I = 1, NR
        CC(I, I) = 1.0 + P(X, I, I)
        DO 21 J = 1, NR
            IF (I .NE. J) CC(J, I) = P(X, J, I)
    21 CONTINUE
    CALL INVERT (NR)
    DO 22 I = 1, NR
        DO 22 J = 1, NR
            A1(J, I) = P(X, J, I)
    22 B(J, I) = CC(J, I)
        CALL MULTIP (NR, NR, NR)
        IF (X .EQ. NAA) GO TO 44
        DO 23 I = 1, NR
            A1(I, I) = 1.0 + P(XX, I, I)
            DO 23 J = 1, NR
                IF (I .NE. J) A1(J, I) = P(XX, J, I)
        23 CONTINUE
        CALL INVERT (NR)
        DO 27 I = 1, NR
            DO 27 J = 1, NR
                CC(J, I) = RMLA(J, I)
        27 CONTINUE
        DO 28 I = 1, NR
            DO 28 J = 1, NR
                SU(X, I, J) = CC(J, I) / ZFNY2
        28 CONTINUE
    30 CONTINUE
CONTINUE

C ------------------------------------------
C COMPUTE FIRST ROW OF GENERALIZED LESLIE MATRIX
C ------------------------------------------

DO 4 X=1,NAA
   XX=XX+1
   DO 3 I=1,NR
      DO 3 J=1,NR
         IF(I.EQ.J) A1(J,I)=RATF(XX,I)
         IF(I.NE.J) A1(J,I)=0.
      3 B(J,I)=SU(X,I,J)
      CALL MULTIP (NR,NR,NR)
   DO 5 I=1,NR
      DO 5 J=1,NR
         IF(I.EQ.J) B(J,I)=RATF(X,I)*C(J,I)
         IF(I.NE.J) B(J,I)=C(J,I)
   5 IF (I.EQ.J) A1(J,I)=ZZF(P(1,J,I)+1.)
      IF (I.NE.J) A1(J,I)=ZZF(P(1,J,I))
      CONTINUE
      CALL MULTIP (NR,NR,NR)
   DO 8 I=1,NR
      DO 8 J=1,NR
         BR(X,J,I)=C(J,I)
   8 IF (I.EQ.J) B(J,I)=RATF(X,I)+C(J,I)
      IF (I.NE.J) B(J,I)=C(J,I)
   4 CONTINUE

C ------------------------------------------
C PRINT GROWTH MATRIX (FIRST ROW AND SUBDIAGONAL ELEMENTS)
C ------------------------------------------

PRINT 1, (TIT(J),J=1,20)
1 FORMAT (1H1,50X,20A4)
PRINT 10
10 FORMAT (1H0,5X,48HTHE DISCRETE MODEL OF MULTIREGIONAL DEMOGRAPHIC
1,6HGROWTH/6X,54(1H*)/6X,54(1HV/)
PRINT 11
11 FORMAT (/5X,31HMULTIREGIONAL PROJECTION MATRIX/5X,31(1H*))
DO 20 I=1,NR
   IF (I.NE.1) PRINT 120
120 FORHAT (1H1,1X)
   PRINT 12, REG(I)
12 FORMAT (1H1,1X)
   PRINT 13, REG(I)
13 FORMAT (/5X,31HMULTIREGIONAL PROJECTION MATRIX/5X,31(1H*))
   PRINT 14, (REG(J),J=1,NR)
14 FORMAT (11X,12(2X,A8))
   PRINT 15
15 FORMAT (1X)
   DO 16 X=1,NAA
16 PRINT 17, NAGE(X),(BR(X,J,I),J=1,NR)
17 FORMAT (5X,13,3X,12F10.6)
   PRINT 18
18 FORMAT (/5X,31HMULTIREGIONAL PROJECTION MATRIX/5X,31(1H*))
   PRINT 19, (REG(J),J=1,NR)
19 PRINT 15
   DO 19 X=1,NAA
   PRINT 17, NAGE(X),(SU(X,J,I),J=1,NR)
20 CONTINUE
RETURN
SUBROUTINE PROJEC(NA,NY,ZMIN, NR, ZLAM1, IPROJ)
DIMENSION ZMIN(7), HUP(7), ZLAM1(7), AGEM(7), ZR(7)
DIMENSION PREC(7), HU(7)
DIMENSION POPTOT(7)
COMMON /CPAR/ INIT, NHORIZ, INTV, ITOLX, NTOLL
COMMON /CNAG/ NAGE(18)
COMMON /CGROW/ BR(18,7,7), POPR(18,7)
COMMON /CMUL/ A1(7,7), B(7,7), C(7,7)
COMMON /CRA/ RATD(18,7), RATM(18,7), RATF(18,7)
COMMON /CRE/ REG(13)
COMMON /CTIT/ TIT(20)
COMMON /CSU/ SU(18,7,7)
COMMON /CRO/ SU(13)
DOUBLE PRECISION REG
INTEGER X, XI, X2
DATA ZDAT1/5HM.AGE/, ZDAT2/5HSHA/, ZDAT3/5HLM/,
IPROJ=1
JGO-0
Z1 I=(-1)*NTOLL
TOLX=10.*TOLX
NYEAR1=INIT
NYEAR1=NYEAR1+INTV
PRINT 1876, (TIT(J), J=1, 20)
1876 FORMAT (1H1, 50X, 20A4)
PRINT 1
1 FORMAT (1H0, 5X, 35HMULTIREGIONAL POPULATION PROJECTION/6X,
135(1H*))
GO TO 509
C--------------------------------------------------
C PROJECT POPULATION NY YEARS
C--------------------------------------------------
500 CONTINUE
C IPROJ = ITERATION NUMBER
C NYEAR = PROJECTION YEAR (INIT + IPROJ * NY)
C ZMIN = POPULATION OF REGION I AT TIME T-1
C ZMINT = POPULATION OF TOTAL SYSTEM AT TIME T-1
IPROJ=IPROJ+1
NYEAR1=NYEAR1+NY
DO 3 I=1, NR
HUP(I)=0.
3 ZMIN(I)=POPTOT(I)
ZMINT=PTOTA
C FIRST AGE GROUP
DO 2 X=1, NA
DO 4 J=1, NR
B(J, X)=POPR(X, J)
4 A(J, I)=BR(X, J, I)
CALL MULTIP (NR, NR, 1)
DO 5 J=1, NR
5 HUP(J)=HUP(J)+C(J, 1)
2 CONTINUE
C OTHER AGE GROUPS
    DO 6 X=1,NAA
       X1=NA-X
       X2=X1+1
    DO 7 J=1,NR
       B(J,1)=POPR(X1,J)
       DO 7 I=1,NR
          A1(J,I)=SU(X1,I,J)
    CALL MULTIP (NR,NR,1)
7 CONTINUE
6 CONTINUE
C COMPUTE TOTAL POPULATION
    DO 11 X=1,NA
       PCT(X)=0.
       DO 11 J=1,NR
          PCT(X)=PCT(X)+POPR(X,J)
    DO 13 J=1,NR
       POPTOT(J)=0.
       DO 13 X=1,NA
          POPTOT(J)=POPTOT(J)+POPR(X,J)
       PTOTA=0.
       DO 17 J=1,NR
    17 PTOTA=PTOTA+POPTOT(J)
C CHECK WHETHER OUTPUT MUST BE PRINTED
C IF ((NYEAR1.GT.NHORIZ).AND.(NYEAR1.NE.NYEAPR)) GO TO 501
C PRINT PROJECTED POPULATION
C IF (IPROJ.GT.0) PRINT 51
51 FORMAT (1H1,1X)
   PRINT 52, NYEAR1
   PRINT 253
52 FORMAT (5X,4HYEAR,1X,15/5X,1O(lH-/)
   PRINT 54
253 FORMAT (10X,10HPOPULATION/10X,5(2H-))/
   IF (NR.LE.10) PRINT 53, (REG(J),J=1,NR)
53 FORMAT (1X,3HAGE,2X,5X,5HTOTAL,10(3X,A8))
   IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
54 FORMAT (1X,3HAGE,2X,6X,5HTOTAL,12(1X,A8))
   IF (NR.LE.10) PRINT 56, (REG(J),J=1,NR)
56 FORMAT (1X,3HAGE,2X,11F11.0)
   IF (NR.GT.10) PRINT 81, (REG(J),J=1,NR)
55 FORMAT (1X,3HAGE,2X,F11.0,12F9.0)
   IF (NULL) PRINT 57, PTOTA,(POPTOT(J),J=1,NR)
57 FORMAT (1X,5HTOTAL,11F11.0)
IF (NR.GT.10) PRINT 82, PTOTA,(POPTOT(J),J=1,NR)
82 FORMAT (1X,5HTOTAL,F11.0,12F9.0)
C PERCENTAGE DISTRIBUTION
PRINT 58
58 FORMAT (/10X,23HPERCENTAGE DISTRIBUTION/10X,12(2H-)/)
IF (NR.LE.10) PRINT 53, (REG(J),J=1,NR)
IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
PRINT 54
ZHU=0.
DO 23 J=1,NR
23 HU(J)=0.
DO 59 X=1,NA
PRCT=100.*PCT(X)/PTOTA
ZHU=ZHU+PRCT
DO 14 J=1,NR
PERC(J)=100.*POPR(X,J)/POPTOT(J)
14 HU(J)=HU(J)+PERC(J)
IF (NR.LE.10) PRINT 60, NAGE(X),PRCT,(PERC(J),J=1,NR)
60 FORMAT (1X,13,2X,11F11.4)
IF (NR.GT.10) PRINT 84, NAGE(X),PRCT,(PERC(J),J=1,NR)
84 FORMAT (1X,13,2X,F11.2,12F9.2)
IF (NR.LE.10) PRINT 761, ZHU, (HU(J),J=1,NR)
761 FORMAT (/1X,5HTOTAL,11F11.4)
IF (NR.GT.10) PRINT 85, ZHU,(HU(J),J=1,NR)
85 FORMAT (/1X,5HTOTAL,F11.2,12F9.2)
C MEAN AGE
ACEMT=0.
DO 21 J=1,NR
21 AGEM(J)=0.
DO 20 X=1,NA
N9:NAGE(X)
A9:FLOAT(N9)+ZFNY*0.5
AGEM=AGEM-A9*PCT(X)/PTOTA
20 J=1,NR
DO 20
21 AGEM(J)=AGEM(J)+A9*POPR(X,J)/POPTOT(J)
IF (NR.LE.10) PRINT 22, ZDAT1,AGEMT,(AGEM(J),J=1,NR)
22 FORMAT (1X,A5,11F11.4)
IF (NR.GT.10) PRINT 86, ZDAT1,AGEMT,(AGEM(J),J=1,NR)
86 FORMAT (1X,A5,F11.4,12F9.4)
C REGIONAL SHARE
Z=0.
DO 16 J=1,NR
16 HUP(J)=(POPTOT(J)/PTOTA)*100.
Z=Z+HUP(J)
IF (NR.LE.10) PRINT 22, ZDAT2,Z,(HUP(J),J=1,NR)
IF (NR.GT.10) PRINT 86, ZDAT2,Z,(HUP(J),J=1,NR)
501 CONTINUE
C GROWTH RATIO (LAM)
IF (IPROJ.EQ.0) GO TO 500
IF (JGO.GE.1) GO TO 505
DO 62 J=1,NR
62 ZLAMB(J)=POPTOT(J)/ZMIN(J)
Z=ZLAMB(J)
IF (NYEAR.NE.THORIZ) AND (NYEAR1.NE.NYEAPR) GO TO 502
IF (NYEAR1.GT.THORIZ) NYEAPR=NYEAPR+INTV
CONTINUE
IF (NR.LE.10) PRINT 64, ZDAT3, ZZ, (ZLAMB(J), J=1, NR)
FORMAT (1X, A5, 11F11.6)
IF (NR.GT.10) PRINT 88, ZDAT3, ZZ, (ZLAMB(J), J=1, NR)
FORMAT (1X, A5, F11.6, 12F9.6)
C ANNUAL GROWTH RATE
RSTAB = ALOG(ZZ)/ZFNY
DO 27 J=1, NR
27 HUP(J) = ALOG(ZLAMB(J))/ZFNY
IF (NR.LE.10) PRINT 64, ZDAT4, RSTAB, (HUP(J), J=1, NR)
IF (NR.GT.10) PRINT 88, ZDAT4, RSTAB, (HUP(J), J=1, NR)
CONTINUE
IF (JGO.GE.1) GO TO 504
C COMPARE GROWTH RATIO WITH TOLERANCE LEVEL
C
IF (ITOLX.EQ.3) ZTOLX = ZLAMB(1) - ZLAM1
IF (ITOLX.EQ.3) ZLAM1 = ZLAMB(1)
IF (ITOLX.EQ.2) ZTOLX = ZLAMB(NR) - ZLAMB(1)
TTOLX = TOLX
IF ((ZTOLX.GT.TOLX).OR.(ZTOLX.LT.TTOLX)) GO TO 500
JGO = JGO + 1
C ZLAMDA = STABLE GROWTH RATIO
ZLAMDA = ZZ
18 FORMAT (1HO,1X,3OH TOLERANCE LEVEL FOR EIGENVALUE, E14.4)
PRINT 65, IPROJ
65 FORMAT (2X, 39H NUMBER OF ITERATIONS TO REACH STABILITY, I7)
C STABLE EQUIVALENT
C
ZS = ZLAMDA**IPROJ
DO 66 J=1, NR
POPTOT(J) = POPTOT(J)/ZS
DO 66 X=1, NA
66 POPR(X,J) = POPR(X,J)/ZS
DO 68 X=1, NA
66 PCT(X) = PCT(X)/ZS
PTOTA = PTOTA/ZS
PRINT 69
69 FORMAT (1H1, 1X, 4OH STABLE EQUIVALENT TO ORIGINAL POPULATION/EX,
14O(1B*)/)
GO TO 578
CONTINUE
RETURN
END
SUBROUTINE AGEDIS (NA,ZFNY,NR,R)
DIMENSION HULP(7),HU1(7)
COMMON /C1/ POP(18,7)
COMMON /CNAG/ NAGE(18)
COMMON /CEX/ EX(18)
COMMON /CL/ L(18,7,7)
COMMON /CREG/ REG(13)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X
REAL L
DO 3 X=1,NA
Z=FLOAT(NAGE(X))+ZFNY*0.5
Zn-Z*R
EX(X)=EXP(Z)
3 CONTINUE
PRINT 1, (TIT(J),J=1,20)
1 FORMAT (1H1,50X;20A4)
PRINT 5
5 FORMAT (1HO,9X,41HPOPULATION DISTRIBUTION BY AGE AND REGION/
110X,41(1H*)/)
C PRINT OBSERVED POPULATION DISTRIBUTION
C-----------------------------------------------
PRINT 6
6 FORMAT (/10X,43H OBSERVED POPULATION (BY PLACE OF RESIDENCE)/
110X,43(1H=))
PRINT 7, (REG(J),J=1,NR)
7 FORMAT (/6~,12(2x,~ej)
PRINT 64
64 FORMAT (1X)
DO 25 J=1,NR
HULP(J)=0.
DO 25 X=1,NA
25 HULP(J)=HULP(J)+POP(X,J)
DO 8 X=1,NA
PRINT 9, NAGE(X),(POP(X,J),J=1,NR)
9 FORMAT (1X,I3,2X,1ZF10.0)
PRINT 15, (HULP(J),J=1,NR)
15 FORMAT (/1X,5HTOTAL,10F10.0)
C COMPUTE AND PRINT LIFE TABLE AND STABLE POPULATION DISTRIBUTION
C ITER = 1 : LIFE TABLE
C ITER = 2 : STABLE POPULATION
C-----------------------------------------------
DO 21 ITER=1,2
10 FORMAT (1H1,9X,21HLIFE TABLE POPULATION/10X,21(1H=))
11 FORMAT (//10X,24HINITIAL REGION OF COHORT,2X,A8/
110X,34(1H=//)
20 FORMAT (1H1,9X,33HSTABLE POPULATION (GROWTH RATE =.F10.6,
12H/)10X,45(1H=))
IF (ITER.EQ.1) PRINT 10
IF (ITER.EQ.2) PRINT 20,R
ISKIP=3
DO 21 I=1,NR
  IF (ISKIP.LE.1) GO TO 40
  PRINT 41
  ISKIP=ISKIP+2
  CONTINUE
  PRINT 11, REG(I)
  IF (NR.LE.10) PRINT 14, (REG(J),J=1,NR)
  14 FORMAT (12X,5HTOTAL,10(3X,A8))
  IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
  80 FORMAT (11X,5HTOTAL, 12(1X,A8))
  PRINT 64
  ZH=0.
  DO 24 J=1,NR
  24 HULP(J)=0.
  DO 22 I=1,NA
    Z=0.
    DO 23 J=1,NR
      IF (ITER.EQ.1) HU1(J)=L(X,I,J)
      IF (ITER.EQ.2) HU1(J)=EX(X)*L(X,I,J)
      Z=Z+HU1(J)
    23 HULP(J)=HULP(J)+HU1(J)
    IF (NR.LE.10) PRINT 13, NAGE(X), Z,(HU1(J),J=1,NR)
    13 FORMAT (1X,I3,2X,11F11.6)
    IF (NR.GT.10) PRINT 81, NAGE(X), Z,(HU1(J),J=1,NR)
    81 FORMAT (1X,I3,2X,13F9.5)
  DO 26 J=1,NR
  26 ZH=ZH+HULP(J)
  IF (NR.LE.10) PRINT 18, ZH,(HULP(J),J=1,NR)
  18 FORMAT (/1X,5HTOTAL,11F11.6)
  IF (NR.GT.10) PRINT 82, ZH,(HULP(J),J=1,NR)
  82 FORMAT (/1X,5HTOTAL,13F9.5)
  CONTINUE
  RETURN
END
SUBROUTINE FERMOB (NA,ZFNY,NR,NOPMOB,NEIG,R)
DIMENSION ZMOMT(7)
DIMENSION HULP(7,7),HULP2(7,7),ZGRR(7),ZMOM(3,7,7)
DIMENSION HULP7(7),HU(18)
DIMENSION ZMON(~,~)
COMMON /CNAG/ NAGE(18)
COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)
COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
COMMON /CREG/ REG
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X
REAL L
64 FORMAT (1X)
C NOPMOB = 1 LIFE TABLE ANALYSIS
C NOPMOB = 2 STATIONARY (ZPG) POPULATION ANALYSIS
C NOPMOB = 3 STABLE POPULATION ANALYSIS
C -----------------------------------------------------------------
PRINT 222
222 FORMAT (1H1,1X)
PRINT 1, (TIT(J),J=1,20)
1 FORMAT (50X,20A4//)
PRINT 223
223 FORMAT (10X,41(1H*)/10X,41(1H*)/10X,3H* * ,35X,3H* *)
IF (NOPMOB.EQ.1) PRINT 224
224 FORMAT (10X,3H* *,1X,33HANALYSIS OF LIFE TABLE POPULATION,
11X,3H* *)
IF (NOPMOB.EQ.2) PRINT 229
229 FORMAT (10X,3H* *,1X,33HANALYSIS OF STATIONARY POPULATION,
11X,3H* *)
IF (NOPMOB.EQ.3) PRINT 226
226 FORMAT (10X,3H* *,1X,29HANALYSIS OF STABLE POPULATION,5X,
13H* *)
PRINT 227
227 FORMAT (10X,3H* *,35X,3H* */10X,41(1H*)/10X,41(1H*)//)
DO 228 X=1,NA
HU(X)=1.
C COMPUTE WEIGHTS FOR STABLE AGE COMPOSITION
IF (NOPMOB.NE.3) GO TO 228
DO 2 X=1,NA
Z=FLOAT(NAGE(X))+ZFNY*0.5
Z1=Z*R
HU(X)=EXP(Z1)
2 CONTINUE
228 CONTINUE
C -----------------------------------------------------------------
C FERTILITY ANALYSIS (INTEGR=1) AND MOBILITY ANALYSIS (INTEGR=2)
C DO 580 INTEGR=1,2
IF (((INTEGR.EQ.2).AND.(NOPMOB.EQ.2)) GO TO 580
DO 121 I=1,NR
DO 121 X=1,NA
IF (INTEGR.NE.2) GO TO 24
Z=0.
DO 23 J=1,NR
Z=Z+RATM(X,J,I)
ZGRAL(X,I)=Z
23 CONTINUE
IF (INTEGR.EQ.1) ZGRAL(X,I)=RATF(X,I)
121 CONTINUE
IF (INTEGR.EQ.1) PRINT 36
36 FORMAT (1HO,9X,18H FERTILITY ANALYSIS/10X,18(H*))
IF (INTEGR.NE.2) GO TO 853
PRINT 222
PRINT 
1.
(TIT(J),J=1,20)
.. 
.. 
.. 
PRINT 37
37 FORMAT (1H1,9X,18H MIGRATION ANALYSIS/10X,18(H*))
853 CONTINUE
C
..............................................................
C PRINT RATES
C
..............................................................
PRINT 5
5 FORMAT (/10X,18H AGE-SPECIFIC RATES/10X,18(H*))
PRINT 7, (REG(J),J=1,MR)
7 FORMAT (/1X,3H AGE,2X,12(2X,A8))
PRINT 64
DO 687 X=1,NA
687 PRINT 686, NAGE(X), (ZGRAL(X,J),J=1,NR)
686 FORMAT (1X,J3,2X,12F10.6)
DO 684 J=1,NR
ZGRR(J)=0.
DO 684 X=1,NA
684 ZGRR(J)=ZGRR(J)+ZGRAL(X,J)*ZPMT 
DO 110 I=1,NR
110 HULF(J,I)=0.
DO 120 I=1,NR
120 FORMAT (1H1,9X,47H INTEGRALS OF GENERALIZED NET MATERNITY FUNCTION/10X,47(H*))
IF ((INTEGR.EQ.1).AND.(NOPMOB.LE.2)) PRINT 120
124 FORMAT (1H1,9X,36H INTEGRALS OF WEIGHTED GENERALIZED NET MATERNITY FUNCTION/10X,56(H*))
IF ((INTEGR.EQ.2).AND.(NOPMOB.LE.2)) PRINT 122
122 FORMAT (1H1,9X,46H INTEGRALS OF GENERALIZED NET MOBILITY FUNCTION/10X,46(H*))
IF ((INTEGR.EQ.2).AND.(NOPMOB.LE.2)) PRINT 123
123 FORMAT (1H1,9X,36H INTEGRALS OF WEIGHTED GENERALIZED NET MOBILITY FUNCTION/10X,55(H*))
DO 110 I=1,NR
DO 110 J=1,NR
110 HULF(J,I)=0.
ISKIP=3
DO 3 I=1,NR
IF (ISKIP.NE.1) GO TO 40
PRINT 222
ISKIP=ISKIP+2
CONTINUE
40 CONTINUE
PRINT 4, REG(I)
FORMAT (/10X,2HINITIAL REGION OF COHORT,2X,A8/110X,34(1H-)/)
PRINT 7, (REG(J),J=1,NR)
DO 8 X=1,NA
DO 109 J=1,NR
HULP4(J)=HU(X)*ZGRAL(X,J)*L(X,I,J)
109 HULP(J,I)=HULP(J,I)+HULP4(J)
8 PRINT 9, NAGE(X),(HULP4(J),J=1,NR)
9 FORMAT (1X,I3,2X,12F10.6)
PRINT 108, (HULP(J,I),J=1,NR)
108 FORMAT (/1X,JXTOTAL,12F10.6)
3 CONTINUE
C------------------------------------------
C MOMENTS OF INTEGRAL FUNCTIONS
C------------------------------------------
PRINT 33
33 FORMAT (/1H1,9X,28HMOMENTS OF INTEGRAL FUNCTION/10X,28(1H=))
NMOMEN=2
NMOM=NMOMEN-1
DO 13 IMOM=1,NMOM
12 IN8=IMOM-1
DO 12 I=1,NR
DO 12 J=1,NR
ZMOM(IMOM,J,I)=0.
PhTFU=HU(X)*ZGRAL(X,J)*L(X,I,J)
12 IF (IN8.EQ.0) GO TO 12
Z=FLOAT(NAGE(X))+ZFNY*0.5
Z=Z*IN8
ZMOM(IMOM,J,I)=ZMOM(IMOM,J,I)+Z*PhTFU
PRINT 61, IN8
61 FORMAT (/9X,I2,1X,6HMOMENT/9X,9(1H-))
DO 90 J=1,NR
90 CE(J,I)=ZMOM(IMOM,J,I)
CALL EIGEN(NR,1,NEIG)
13 CONTINUE
C------------------------------------------
C MATRICES OF MEAN AGES
C------------------------------------------
PRINT 167
167 FORMAT (/1H1,9X,35HMATRICES OF MEAN AGES AND VARIANCES/10X,35(1H=))
PRINT 125
125 FORMAT (/1X)
IX=1
PRINT 723, IX
723 FORMAT (3X,2H***,1X,11HALTERNATIVE,I2,1X,2H***/3X,19(1H*))
PRINT 67
67 FORMAT (/G9X,5HMEANS/G9X,5(1H-))
DO 19 I=1,NR
DO 19 J=1,NR
19 CE(J,I)=ZMOM(2,J,I)/ZMOM(1,J,I)
CALL EIGEN(NR,2,NEIG)
PRINT 68
68 FORMAT (/G9X,9HVARIANCES/G9X,9(1H-))
DO 21 I=1,NR
DO 21 J=1,NR
Z=CE(J,I)*CE(J,I)
21 CE(J,I)=ZMOM(3,J,I)/ZMOM(1,J,I)-Z
CALL EIGEN (NR,2,NEIG)
PRINT 125
IK=2
PRINT 723, IK
PRINT 67
DO 14 I= 1, NR
DO 14 J=1,NR
14 CC(J,I)=ZMOM(1,J,I)
CALL INVERT (NR)
DO 17 I=1,NR
DO 17 Jr1,NR
A1(J,I)=ZMOM(2,J,I)
17 B(J,I)=CC(J,I)
CALL MULTIP (NR, NR,NR)
DO 91 I=1,NR
DO 91 Jr1,NR
91 CE(J,I)=HULP(J,I)-C(J,I)
CALL EIGEN (NR,1,0)
C EXPECTANCIES AND ALLOCATIONS
C---------------------------------------------------------------------
C EXPECTANCIES AND ALLOCATIONS
C---------------------------------------------------------------------
IF (INTEGR.EQ.1) GO TO 579
PRINT 222
PRINT 1, (TIT(J), J=1,20)
PRINT 777
777 FORMAT (1HO,6X,30HSPATIAL MIGRATION EXPECTANCIES/10X,30(1H*))
DO 771 I = 1, NR
DO 771 J = 1, NR
CE(J, I) = 0.
DO 771 X = 1, NA
CE(J, I) = CE(J, I) + HU(X) * L(X, I, J)
PRINT 772
772 FORMAT (/10X, 20HEXPECTATIONS OF LIFE/10X, 20(1H-)/)
CALL EIGEN (NR, 1, 1)
PRINT 774
774 FORMAT (/10X, 16HMIGRATION LEVELS/10X, 16(1H-)/)
DO 775 I = 1, NR
Z = 0.
DO 776 J = 1, NR
Z = Z + CE(J, I)
DO 775 J = 1, NR
CE(J, I) = CE(J, I) / Z
CALL EIGEN (NR, 1, 0)
CONTINUE
PRINT 579
PRINT 222
PRINT 1, (TIT(J), J = 1, PO)
IF (INTEGR.EQ.2) PRINT 777
IF (INTEGR.EQ.1) PRINT 882
882 FORMAT (/10X, 9X, 30HSpatial Fertility Expectancies/10X, 30(1H-)/)
IF (INTEGR.EQ.2) PRINT 886
886 FORMAT (/10X, 24HNET MIGRATION RATE /10X, 24(1H-)/)
IF (INTEGR.EQ.1) PRINT 887
887 FORMAT (/10X, 21HNET REPRODUCTION RATE/10X, 21(1H-)/)
DO 888 I = 1, NR
DO 886 J = 1, NR
CE(J, I) = ZMOM(1, J, I)
CALL EIGEN (NR, 1, 1)
PRINT 64
IF (INTEGR.EQ.2) PRINT 891
IF (INTEGR.EQ.1) PRINT 892
891 FORMAT (/10X, 31HNET MIGRATION ALLOCATIONS/10X, 31(1H-)/)
IF (INTEGR.EQ.1) PRINT 892
892 FORMAT (/10X, 28HNET REPRODUCTION ALLOCATIONS/10X, 28(1H-)/)
DO 889 I = 1, NR
Z = 0.
DO 890 J = 1, NR
Z = Z + CE(J, I)
DO 889 J = 1, NR
CE(J, I) = CE(J, I) / Z
CALL EIGEN (NR, 1, 0)
CONTINUE
RETURN
END
SUBROUTINE RVALUE (NA,ZFNY,NR,R,ZVT)

DIMENSION V(18,7,7),HU(18)
COMMON /C1/ POP(18,7)
COMMON /CMAG/ NAGE(18)
COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)
COMMON /CMUL/ A(7,7),B(7,7),C(7,7)
COMMON /CPQ/ P(18,7,7)
COMMON /CPHI/ PSI(~,~),VRPSI(~),VLPSI(~),ROPSI
COMMON /CPSX/ REG(13)
COMMON /CTIT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X ,X1 ,XX
REAL L

NAA=NA-1

64 FORMAT (1X)
78 FORMAT (1H1,lX)

C ...........................................................
C COMPUTE LEFT EIGENVECTOR OF CHARACTERISTIC MATRIX : VLPSI(J)
C ...........................................................

67 FORMAT (1H1/1X)
PRINT 1. (TIT(J),J=1,20)
1 FORMAT (5X,2OA4)
PRINT 64
DO 10 X=1,NA
HU(X)=1.
IF (R.EQ.0.) GO TO 10
Z=FLOAT(NAGE(X))+ZFNY*0.5
Z=Z*R
EX(X) = EXP(Z)
HU(X)=EX(X)
10 CONTINUE
DO 11 I=1,NR
DO 11 J=1,NR
CE(J,I)=0.
DO 11 X=1,NA
11 CE(J,I)=CE(J,I)+HU(X)*RATF(X,J)*L(X,1,J)
CALL EIGEN (NR,1) ROOT
ROPSI=ROOT
DO 12 I=1,NR
VLPSI(I)=VECTL(I)
VRPSI(I)=VECT(I)
12 DO 12 J=1,NR
DO 251 IRES=1,2
PRINT 67
PRINT 1, (TIT(J),J=1,20)
IF (IRES.EQ.1) PRINT 250
250 FORMAT (110,1X,32HTHE SPATIAL REPRODUCTIVE VALUE ;, 1X, 133HRESULTS FOR PEOPLE AT EXACT AGE X /2X,30(1H*),3X, 133(1H*)/2X,30(1H*),3X,33(1H*))
IF (IRES.EQ.2) PRINT 258
258 FORMAT (110,34X,33HRESULTS FOR PEOPLE IN AGE GROUP X / 135X,33(1H*)/35X,33(1H*))
PRINT 51
51 FORMAT (//10X,41HDISCOUNTED NUMBER OF OFFSPRING PER PERSON, 1/10X,41(1H*)/)
C ---------------------------------
C COMPUTE DISCOUNTED NUMBER OF OFFSPRING PER PERSON OF EXACT AGE X
C ---------------------------------
IF (IRES.EQ.2) GO TO 257
Z=-ZFNY'O.5*R
ZZ=EXP(Z)
ZZZ=ZFNY'O.5*ZZ
DO 252 I= 1, NR
DO 252 J=1 , NR
252 V(NA,J,I)=RATF(NA,J)*ZZZ
DO 255 XI 1,NAA
XX=NA-X
X1=XX+1
DO 254 I=1, NR
DO 254 J=1, NR
IF (I.EQ.J) A1(J,I)=ZZZ*ZFNY'O.5*ZZ+V(X1,J,I)
IF (I.NE.J) A1(J,I)=ZZZ*ZZ*V(X1,J,I)
254 B(J,I)=P(XX,J,I)
CALL MULTIP (NR,NR,NR)
DO 255 I=1, NR
DO 255 J=1, NR
IF (I.EQ.J) V(XX,J,I)=ZZZ*ZFNY'O.5*ZZ+V(X1,J,I)
IF (I.NE.J) V(XX,J,I)=C(J,I)
255 CONTINUE
DO 268 I=1, NR
DO 268 J=1, NR
268 V(1,J,I)=PSI(J,I)
GO TO 730
C ---------------------------------
C COMPUTE MATRIX OF SURVIVORSHIP PROPORTIONS
DO 53 I=1, NR
DO 53 J=1, NR
53 V(NA,J,I)=RATF(NA,J)*ZFNY'O.5
Z=-ZFNY'O.5*R
ZZ=EXP(Z)
DO 52 X=1, NAA
XX=NA-X
X1=XX+1
DO 21 I=1, NR
DO 21 J=1, NR
21 C(J,I)=L(XX,I,J)
CALL INVERT (NR)
DO 22 I=1,NR
DO 22 J=1,NR
A(I,J)=L(X1,I,J)
22 B(J,I)=CC(J,I)
CALL MULTIP (NR,NR,NR)
C COMPUTE DISCOUNTED NUMBER OF OFFSPRING
DO 54 I=1,NR
DO 54 J=1,NR
IF (I.EQ.J) A(I,J)=ZFNY*0.5*RATF(X1,J)*V(X1,J)
IF (I.NE.J) A(I,J)=V(X1,J)
54 B(J,I)=C(J,I)*ZZ
CALL MULTIP (NR,NR,NR)
DO 55 I=1,NR
DO 55 J=1,NR
IF (I.EQ.J) V(JX,J,I)=ZFNY*0.5*RATF(JX,J)*C(J,I)
IF (I.NE.J) V(JX,J,I)=C(J,I)
52 CONTINUE
730 CONTINUE
C .............................................................
C PRINT RESULTS
C .............................................................
ISKIP=3
DO 58 I=1,NR
IF (ISKIP.NE.1) GO TO 888
PRINT 67
ISKIP=ISKIP+2
888 CONTINUE
PRINT 56, REG(I)
56 FORMAT (/10X,19HREGION OF RESIDENCE,2X,A8/10X,29(1H-)/)
PRINT 57
57 FORMAT (10X,28HREGION OF BIRTH OF OFFSPRING)
IF (NR.LE.10) PRINT 80, (REG(J),J=1,NR)
80 FORMAT (10X,5HTOTAL,10(2X,A8))
IF (NR.GT.10) PRINT 81, (REG(J),J=1,NR)
81 FORMAT (9X,5HTOTAL,12(1X,A8))
PRINT 64
DO 58 X=1,NAA
Z=0.
DO 60 J=1,NR
Z=Z+V(X,J)
60 IF (NR.LE.10) PRINT 59, NAGE(X),Z,(V(X,J),J=1,NR)
59 FORMAT (1X,I3,1X,F10.6,10F10.6)
IF (NR.GT.10) PRINT 83, NAGE(X),Z,(V(X,J),J=1,NR)
83 FORMAT (1X,I3,1X,13F9.6)
58 CONTINUE
C .............................................................
C COMPUTE AND PRINT SPATIAL REPRODUCTIVE VALUES
C .............................................................
PRINT 61
61 FORMAT (1H1,10X,37HSpatial reproductive value per person /
111X,37(1H*)/)
PRINT 65, (REG(J),J=1,NR)
65 FORMAT (5X,12(2X,A8))
PRINT 64
DO 62 X=1,NAA
DO 63 J=1, NR
A(1,J)=VLPSI(J)
DO 63 I=1, NR
63 B(J,I)=V(X,J,I)
CALL MULTIP(1, NR, NR)
IF (IRES.NE.1) GO TO 264
IF (X.NE.1) GO TO 264
DO 263 J=1, NR
263 C(1,J)=VLPSI(J)
264 CONTINUE
PRINT 66, NAGE(X),(C(J,J),J=1,NR)
66 FORMAT (1X,I3,1X,12F10.6)
62 CONTINUE
251 CONTINUE
C ............................................................
C RESULTS FOR TOTAL POPULATION
C ............................................................
PRINT 67
PRINT 1, (TIT(J),J=1,20)
PRINT 71
71 FORMAT (1HO,36HTOTAL DISCOUNTED NUMBER OF OFFSPRING, 123H OF OBSERVED POPULATION /1X,58(1H*/)
DO 72 I=1, NR
DO 72 J=1, NR
CE(J,I)=0.
DO 72 X=1, NAA
72 CE(J,I)=CE(J,I)+V(X,J,I)*POP(X,I)
ICHE=0.
ZZTOT=0.
DO 89 J=1, NR
HU(J)=0.
DO 88 I=1, NR
88 HU(J)=HU(J)+CE(I,J)
89 ZZTOT=ZZTOT+HU(J)
IF (ZZTOT.LT.10000000.) GO TO 23
ICHE=1
PRINT 24
24 FORMAT (25X,8HIN1,000/25X,8(1H*/)
ZZTOT=ZZTOT*0.001
DO 25 I=1, NR
HU(I)=HU(I)*0.001
DO 25 J=1, NR
25 CE(I,J)=CE(I,J)*0.001
23 CONTINUE
IF (NR.LE.10) PRINT 84, (REG(J),J=1,NR)
84 FORMAT (16X,5HTOTAL,10(2X,A8))
IF (NR.GT.10) PRINT 259, (REG(J),J=1,NR)
259 FORMAT (15X,5HTOTAL,12(1X,A8))
64 PRINT 64
DO 87 I=1, NR
ZZT=0.
DO 86 J=1, NR
86 ZZT=ZZT+CE(I,J)
IF (NR.LE.10) PRINT 85, REG(I),ZZT,(CE(I,J),J=1,NR)
85 FORMAT (1X,A8,2X,11F10.0)
IF (NR .GT. 10) PRINT 26, REC(I), ZZZT, (CE(I, J), J=1, NR)
26 FORMAT (1X, A8, 2X, 13F9.0)
87 CONTINUE
IF (NR .LT. 10) PRINT 28, ZZZTOT, (HU(J), J=1, NR)
28 FORMAT (/4X, 5HTOTAL, 2X, 11F10.0)
IF (NR .GT. 10) PRINT 29, ZZZTOT, (HU(J), J=1, NR)
29 FORMAT (/4X, 5HTOTAL, 2X, 13F9.0)
PRINT 265
265 FORMAT (/1X, 42HREPRODUCTIVE VALUE OF THE TOTAL POPULATION
1/1X, 42(1H*)/
IF (ICHE .EQ. 1) PRINT 27
27 FORMAT (15X, 8HIN 1,000/15X, 8(1H*)/
PRINT 887
887 FORMAT (/17X, 5HTOTAL, 1X, 10HPERCENTAGE/)
DO 260 J=1, NR
A(A, J) = VLPSI(J)
DO 260 I=1, NR
260 E(J, I) = CE(J, I)
CALL MULTIP (1, NR, NR)
ZVT=0.
DO 92 I=1, NR
92 ZVT=ZVT+C(I, I)
ZZ=0.
DO 261 I=1, NR
Z=100. * C(I, I)/ZVT
ZZ=ZZ+Z
261 PRINT 262, REC(I), C(I, I), Z
262 FORMAT (1X, A8, 2X, F11.0, F11.2)
PRINT 93, ZVT, ZZ
93 FORMAT (/4X, 5HTOTAL, 2X, F11.0, F11.2)
IF (ICHE .EQ. 1) ZVT=ZVT*1000.
RETURN
END
SUBROUTINE RINTR (NA,ZFNY,NR,R,ZVT)

DIMENSION DISV(7),HULP(13,6),HZ(18,7,7)

COMMON /CAGEM/ AGEM(7,7)
COMMON /CNAG/ NAGE(18)
COMMON /CEIGEN/ CE(7,7),ROOT(7),VECT(7),VECTL(7)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)

DOUBLE PRECISION HU
INTEGER X,X1,XX

REAL L
NAA=NA-1
DO 16 X=1,NA
HU(X)=1.
IF (R.EQ.0.) GO TO 16
Z=FLOAT(NAGE(X))+ZFNY*0.5
ZZ=-Z*R
HU(X)=EXP(ZZ)
16 CONTINUE

C COMPUTE NORMALIZING FACTOR

DO 10 I=1,NR
DO 10 J=1,NR
CC(J,I)=0.
A1(J,I)=0.
DO 10 X=1,NA
Z=FLOAT(NAGE(X))+ZFNY*0.5
PATFU=HU(X)*RATF(X,J)*L(X,I,J)
A1(J,I)=A1(J,I)+Z*PATFU
10 CC(J,I)=CC(J,I)+PATFU
CALL INVERT (NR)
DO 11 I=1,NR
DO 11 J=1,NR
11 B(J,I)=CC(J,I)
CALL MULTIP (NR,NR,NR)
DO 12 I=1,NR
DO 12 J=1,NR
12 AGEM(J,I)=C(J,I)
DO 85 I=1,NR
B(I,1)=VRPSI(I)
DO 85 J=1,NR
85 A1(J,I)=AGEM(J,I)
CALL MULTIP (NR,NR,1)
ZNORM=0.
DO 86 I=1,NR
86 ZNORM=ZNORM+C(I,1)*VLPSI(I)
VKNORM=ZVT/ZNORM

C STABLE EQUIVALENTS OF BIRTHS

DO 94 I=1,NR
   QQ(I)=VRPSI(I)*VKNORM
94 CONTINUE

C STABLE EQUIVALENT OF TOTAL POPULATION

PRINT 1, (TIT(J),J=1,20)
1 FORMAT ('H1,50X,20A4)
PRINT 96
96 FORMAT ('H0,10X,37HSTABLE EQUIVALENT OF TOTAL POPULATION/
  11X,37(',1H*')/)
DO 121 X=1,NA
   DO 121 J=1,NR
      HULP(J,1)=0.
   DO 120 I=1,NR
      HULP(J,1)=HULP(J,1)+L(X,I,J)*QQ(I)
   POPST(X,J)=HU(X)*HULP(J,1)
   YT=0.
DO 122 I=1,NR
   YY(I)=0.
   DO 134 X:1 ,NA
      YY(I)=YY(I)+POPST(X,I)
   YT=YT+YY(I)
IF (NR.LE.10) PRINT 133, (REC(J),J=1,NR)
133 FORMAT (11X,5HTOTAL,10(2X,A8))
IF (NR.GT.10) PRINT 33, (REC(J),J=1,NR)
33 FORMAT (11X,5HTOTAL,12(1X,A8))
DO 123 X=1,NA
   HZ(X)=0.
   DO 132 J=1,NR
      HZ(X)=HZ(X)+POPST(X,J)
   IF (NR.LE.10) PRINT 124, NAGE(X),HZ(X),POPST(X,J),J=1,NR
124 FORMAT (1X,I3,2X,F10.0)
   IF (NR.GT.10) PRINT 34, NAGE(X),HZ(X),POPST(X,J),J=1,NR
34 FORMAT (1X,I3,2X,F10.0,12F9.0)
123 CONTINUE
   IF (NR.LE.10) PRINT 125, YT, (YY(J),J=1,NR)
125 FORMAT (1X,5HTOTAL,11F10.0)
   IF (NR.GT.10) PRINT 35, YT,(YY(J),J=1,NR)
35 FORMAT (1X,5HTOTAL,F10.0,12F9.0)
PRINT 31
31 FORMAT ('/10X,23HPERCENTAGE DISTRIBUTION ',/10X,23(',1H-'))
   IF (NR.LE.10) PRINT 133, (REC(J),J=1,NR)
   IF (NR.GT.10) PRINT 33, (REC(J),J=1,NR)
PRINT 64
   DO 126 X=1,NA
      HZ(X)=100.*HZ(X)/YT
   DO 127 J=1,NR
      DISV(J)=100.*POPST(X,J)/YY(J)
IF (NR.LE.10) PRINT 135, NAGE(X),HZ(X),(DISV(J),J=1,NR)
135 FORMAT (1X,I3,2X,1F10.3)
128 FORMAT (1X,I3,2X,F10.3,12F9.3)
126 IF (NR.GT.10) PRINT 128, NAGE(X),HZ(X),(DISV(J),J=1,NR)
PRINT 64
Z=100.
IF (NR.LE.10) PRINT 141, Z,(Z,J=1,NR)
141 FORMAT (1X,5HTOTAL,11F10.3)
126 IF (NR.GT.10) PRINT 142, Z,(Z,J=1,NR)
142 FORMAT (1X,5HTOTAL,F10.3,12F9.3)
DO 138 J=1,NR
138 HZ(J)=100.*YY(J)/YT
IF (NR.LE.10) PRINT 139, Z,(HZ(J),J=1,NR)
139 FORMAT (1X,5HSHARE,11F10.3)
140 FORMAT (1X,5HSHARE,F10.3,12F9.3)
138 IF (NR.GT.10) PRINT 140, Z,(HZ(J),J=1,NR)

C STABLE EQUIVALENTS AND INTRINSIC RATES OF DEATH AND MIGRATION
C
DO 43 I=1,NR
HULP(I,1)=QQ(I)
43 CONTINUE
PRINT 1, (TIT(J),J=1,20)
PRINT 131
131 FORMAT (1HO,1OX,38HSTABLE EQUIVALENTS AND INTRINSIC RATES/
111X,38(1H*)/)
PRINT 49
49 FORMAT (13X,4(3X,6X,6HNUMBER,8X,4HRATE)/)
DO 25 X=1,NAA
DO 21 I=1,NR
DO 21 J=1,NR
IF (I.EQ.J) CC(J,I)=P(X,J,I)+1.
21 IF (I.NE.J) CC(J,I)=P(X,J,I)
CALL INVERT (NR)
Z2=0.5*ZFNY
Z1=Z2*R
Z=ZFNY*R
Z6=EXP(Z)
Z9=EXP(Z1)/Z2
DO 22 I=1,NR
DO 22 J=1,NR
B(J,I)=CC(J,I)
22 IF (I.EQ.J) A1(J,I)=1.-P(X,J,I)*Z8
22 IF (I.NE.J) A1(J,I)=-P(X,J,I)*Z8
CALL MULTIP (NR,NR,NR)
DO 23 I=1,NR
DO 23 J=1,NR
IF (I.EQ.J) HM(J,I)=Z9*C(J,I)-R
23 IF (I.NE.J) HM(J,I)=Z9*C(J,I)
DO 25 I=1,NR
Z7=0.
DO 24 J=1,NR
IF (I.EQ.J) GO TO 24
Z7=Z7+HM(J,I)
CONTINUE
RATD(X,I)=HM(I,I)+Z7
DO 25 J=1,NR
RATM(X,J,I)=-HM(J,I)
CONTINUE
DO 27 I=1,NR
DO 26 J=1,NR
RATM(NA,J,I)=0.
DO 118 I=1,NR
HULP(I,J)=0.
DO 111 X=1,NA
HULP(I,3)=HULP(I,3)+POPST(X,I)*RATD(X,I)
DO 119 J=1,NR
IF (I.EQ.J) GO TO 119
HULP(I,5)=HULP(I,5)+POPST(X,I)*RATM(X,J,I)
HULP(I,7)=HULP(I,7)+POPST(X,J)*RATM(X,J,I)
CONTINUE
CONTINUE
NR1=NR+1
DO 20 J=1,8
HULP(NR1,J)=0.
DO 113 I=1,NR
DO 113 J=1,4
JK=J+2
JJ=JK+1
HULP(I,JJ)=HULP(I,JK)/YY(I)
HULP(NR1,JK)=HULP(NR1,JK)+HULP(I,JK)
CONTINUE
DO 177 I=1,NR
PRINT 114, REG(I),(HULP(I,J),J=1,8)
FORMAT (5X,A8,4((X,F12.0,F12.6))
DO 130 J=1,4
JJ=J+2
HULP(NR1,JJ)=HULP(NR1,JJ-1)/YT
PRINT 116, (HULP(NR1,J),J=1,8)
FORMAT (/8X,5HTOTAL,4((X,F12.0,F12.6))
PRINT 178, R
FORMAT (/10X,18HSTABLE GROWTH RATE,4X,F10.6)
PRINT 179, ZNORM
FORMAT (/10X,18HNORMALIZING FACTOR,2X,F12.4)
RETURN
END
SUBROUTINE MOMENT (NA,ZFNY,NR,R)
DIMENSION RO(7,7),R1(7,7),HU(7,7),QQZF(7),EO(7,7),
YYZF(7)
COMMON /CNAG/ NAGE(18)
COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
COMMON /CINV/ L(18,7,7)
COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
COMMON /CPSI/ PSI(7,7),VRPSI(7),VLPSI(7),ROPSI
COMMON /CQQ/ QQ(7),POPST(18,7),YY(7)
COMMON /CREG/ REG(13)
COMMON /CTIT/ TIT(20)
COMMON /CQQ/ QQ(7),POPST(18,7),YY(7)
COMMON /CRATE/ RATD(18,7),RATF(18,7)
COMMON /CRAT/ TIT(20)
DOUBLE PRECISION REG
INTEGER X
REAL L
PRINT 1, (TIT(J),J=1,20)
1 FORMAT (1H1,50X,ZOA4//)
PRINT 50
50 FORMAT (1HO,1OX,42HSPATIAL MOMENTUM OF ZERO POPULATION GROWTH/
111X,42(lH')/llX,42(lH')/
C.........................
C COMPUTE AND PRINT MATRIX CONVERTINGS STABLE TO STATIONARY BIRTHS
C.........................
DO 3 I=1,NR
DO 3 J=1,NR
EO(J,I)=0.
RO(J,I)=0.
R1(J,I)=0.
DO 3 X=1,NA
EO(J,I)=EO(J,I)+L(X,I,J)
IZ=NAGE(X)
ZI=FLOAT(IZ)+ZFNY'0.5
Z=RATF(X,J)*L(X,I,J)
RO(J,I)=RO(J,I)+Z
3 R1(J,I)=R1(J,I)+Z1*Z
DO 4 I=1,NR
DO 4 J=1,NR
HU(J,I)=RO(J,I)-PSI(J,I)
4 CC(J,I)=R1(J,I)
CALL INVERT (NR)
DO 5 I=1,NR
DO 5 J=1,NR
A1(J,I)=CC(J,I)
5 CALL MULTIP (NR,NR,NR)
DO 6 I=1,NR
B(I,1)=QQ(I)
6 CALL MULTIP (NR,NR,1)
DO 7 I=1,NR
QQZF(I)=C(I,1)
7 PRINT 11
11 FORMAT (//10X,45HMATRIX CONVERTING STABLE TO STATIONARY BIRTHS/
    110X,45(1H-))
   DO 10 I=1,NR
   DO 10 J=1,NR
10  CE(J,I)=AI(J,I)
   CALL EICEN (NR,1,0)
C ---------------------------------------------------------------------
C COMPUTE AND PRINT STABLE AND STATIONARY EQUIVALENTS AND MOMENTUM
C ---------------------------------------------------------------------
   DO 8 I=1,NR
     B(I,1)=QQZP(I)
     DO 8 J=1,NR
8  Al(J,I)=EO(J,I)
   CALL MULTIP (NR,NR,1)
   DO 9 I=1,NR
9  YYZP(I)=C(I,1)
   PRINT 12
12 FORMAT (//10X,33HSTABLE AND STATIONARY EQUIVALENTS/
    110X,33(1H-))
   PRINT 13
13 FORMAT (/21X,6HBIRTHS,16X,10HPopulation,9X,10HPopulation)
   PRINT 14
14 FORMAT (/13X,2(5X,6HSTABLE,1X,10HSTATIONARY,2X),3X,8HMOMENTUM)
   PRINT 64
64 FORMAT (//1X)
   DO 15 I=1,NR
15 IF (YY(I) .NE. 0.) THEN
       Z=YYZP(I)/YY(I)
   PRINT 16, REC(I),QQ(I),QQZP(I),YY(I),YYZP(I),Z
16 FORMAT (3X,A8,2X,2(2F11.0,2X),F11.4)
   DO 17 I=1,NR
17 HU(I,1)=0.
   DO 18 I=1,NR
18 HU(1,1)=HU(1,1)+QQ(I)
     HU(2,1)=HU(2,1)+QQZP(I)
     HU(3,1)=HU(3,1)+YY(I)
     HU(4,1)=HU(4,1)+YYZP(I)
     Z=HU(4,1)/HU(3,1)
   PRINT 19, (HU(I,1),I=1,4),Z
19 FORMAT (//6X,5HTOTAL,2X,Z(2F11.0,2X),F11.4)
   RETURN
END
SUBROUTINE ZERO (NA, NR, NZERO, RONRR)
C RONRR = DOMINANT EIGENVALUE OF NET REPRODUCTION RATE MATRIX
C NZERO = 1 COHORT REPLACEMENT ALTERNATIVE
C NZERO = 2 PROPORTIONAL REDUCTION ALTERNATIVE

DIMENSION VI(7)
COMMON /CEIGEN/ CE(7,7), ROOT, VECT(7), VECTL(7)
COMMON /CINV/ CC(7,7)
COMMON /CL/ L(18,7,7)
COMMON /CRATE/
RATD(18,7), RATH(18,7,7), RhTF(18,7)
COMMON /CTIT/ TIT(20)
REAL L
INTEGER X
DATA BLAN/4H/, ZPG1/4HZPG1/, ZPG2/4HZPG2/

64 FORMAT (1X)
PRINT 29
DO 432 I = 1, 20
IF (TIT(I).NE.BLAN) GO TO 432
IF (TIT(I+1).NE.BLAN) GO TO 432
IF (NZERO.EQ.1) TIT(I+1) = ZPG1
IF (NZERO.EQ.2) TIT(I+1) = ZPG2
GO TO 433

432 CONTINUE
433 CONTINUE
PRINT 1, (TIT(J), J = 1, 20)
1 FORMAT (1H1, 50X, 2OA4//)
29 FORMAT (1HO, 5X, 70(1H*))/6X, 70(1H*))/
PRINT 30, NZERO
30 FORMAT (1HO, 5X, 22HZERO POPULATION GROWTH, 5X, 11HALTERNATIVE
1, 12/1X, 22(1H*), 5X, 13(1H*)/11X, 22(1H*), 5X, 13(1H*)/)

C COMPUTE NET RATE OF REPRODUCTION MATRIX
C-------------------------------------------------------------
IF (NZERO.NE.1) GO TO 41
PRINT 8
8 FORMAT (1HO, 10X, 32HNET REPRODUCTION RATE MATRIX NRR/11X,
132(1H*)/)
DO 6 I = 1, NR
DO 6 J = 1, NR
CE(J, I) = 0.
DO 6 X = 1, NA
CE(J, X) = CE(J, I) + RATF(X, J) * L(X, I, J)
6 CONTINUE
CALL EIGEN (NR, 1, 1)
RONRR = ROOT
C-------------------------------------------------------------
C COMPUTE FERTILITY ADJUSTMENT FACTORS (ALTERNATIVE 1)
C-------------------------------------------------------------
DO 32 I = 1, NR
DO 32 J = 1, NR
CC(J, I) = CE(I, J)
32 CONTINUE
PRINT 64
PRINT 64
PRINT 64
C COMPILE FERTILITY ADJUSTMENT FACTORS (ALTERNATIVE 2)

C

DO 38 I = 1, NR
   VI(I) = .1/90NR

CONTINUE

C PRINT MATRIX OF FERTILITY ADJUSTMENT FACTORS

C

DO 36 I = 1, NR
   DO 36 J = 1, NR
      CE(J, I) = VI(I)

CONTINUE

C REPLACE FERTILITY RATES BY RATES AT REPLACEMENT LEVEL

C

DO 40 I = 1, NR
   DO 40 X = 1, NA
      RATF(X, I) = RATF(X, I) * VI(I)

RETURN

END
11.3 Main Program
MAIN PROGRAM FOR SPATIAL POPULATION ANALYSIS

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS (IIASA)

ATTENTION ZERO = 0

DIMENSION RATFZE(18,7)
COMMON /C1/ POP(18,7)
COMMON /CNAG/ NAGE(18)
COMMON /CGROW/ BR(18,7,7),POPR(18,7)
COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
INTEGER X
NPR=1
IHIST=0
ILIF=0
IPROJ=0
CALL DATAS (NPR,NA,NY,ZFNY,NR,XZB,XZD,XZO,IPROB,NEIG)

--- PRELIMINARY ANALYSIS ---
CALL PRELIM (NA,NY,ZFNY,NR,XZB,XZD,XZO)

--- COMPUTE PROBABILITIES ---
CALL PROBSC (NA,ZFNY,NR,IPROB)

--- MULTIREGIONAL LIFE TABLE ---
CALL HIST (NA,NR, IHIST)
CALL LIFE (NA,ZFNY,NR,IPROB,IHIST,ILIF)
CALL WHOLE (NA,ZFNY,NR)
IF (NR.EQ.2) CALL PRLIFP (NA,NY,IPROB)

--- MULTIREGIONAL DEMOGRAPHIC PROJECTION ---
CALL GROWTH (NA,ZFNY,NR,ILIF)
DO 10 I=1,NR
DO 10 X=1,NA
10 POPR(X,I)=POP(X,I)
CALL PROJEC (NA,NY,ZFNY,NR,ZLAMDK,IPROJ)

--- FERTILITY AND MOBILITY ANALYSIS ---
IF (ILIF.NE.1) CALL LMAT (NA,NR,ZFNY,ILIF)
CALL FERMOB (NA,ZFNY,NR,1,NEIG,0.)

--- SPATIAL STABLE POPULATION ANALYSIS ---
IF (ILIF.NE.1) CALL LMAT(NA,NR,ZFNY,ILIF)
IF (IPROJ.NE.1) CALL RELAM(ZFNY,ZLAMDK,RSTAB)
CALL AGEDIS (NA,ZFNY,NR,RSTAB)
CALL FERMOB (NA,ZFNY,NR,3,NEIG,RSTAB)
CALL RVALUE (NA,ZFNY,NR,RSTAB,ZVT)
CALL RINTR (NA,ZFNY,NR,RSTAB,ZVT)
C SPATIAL ZPG ANALYSIS : ANALYTICAL APPROACH (MOMENTUM)
CALL MOMENT (NA,ZFNY,NR,RSTAB)
C SPATIAL ZERO POPULATION GROWTH ANALYSIS
C

C NUMERICAL APPROACH : TWO ALTERNATIVE FERTILITY REDUCTION SCHEMES
NNZERO=2
DO 33 NZERO=1,NNZERO
CALL ZERO (NA,NR,NZERO,RONRR)
CALL FERMOB (NA,ZFNY,NR,2,NEIG,O.)
CALL RVALUE (NA,ZFNY,NR,0.,ZVT)
CALL RINTR (NA,ZFNY,NR,0.,ZVT)
DO 31 I=1,NR
DO 31 X=1,NA
30 RATFZE(X,I)=RATF(X,I)
31 RATF(X,I)=RATFZE(X,I)
33 CONTINUE
C
STOP
END
REFERENCES


Feeney, G.M. (1970), Stable Age by Region Distributions, Demography, 6, 341-348.


Schoen, R. (1975), Constructing Increment-Decrement Life Tables, Demography, 12, 313-324.


Appendix A

Glossary of Mathematical Symbols and FORTRAN Names of Demographic Variables
Glossary of mathematical symbols and FORTRAN names of demographic variables.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page in Rogers (1975a)</th>
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**Subscripts**

- **X**: age group (1,2,3,...)
- **I0 (or I)**: region of birth
- **I**: region of residence
- **J**: region of destination (in case of migration)

**Data and Preliminary Analysis**

- **Ki(x)**: POP(X,I) population by age and region
- **Bi(x)**: BIRTH(X,I) births
- **Di(x)**: DEATH(X,I) deaths
- **Kij(x)**: OMIG(X,J,I) migrants from I to J by age and region
- **Mi0(x)**: RADIX(I) radix of region I
- **REG(I)**: name of region I
- **Mij(x)**: RATD(X,I) age-specific death rates of region I
- **RATF(X,I)**: age-specific fertility rates of region I
- **RATM(X,J,I)**: age-specific migration rates from I to J
<table>
<thead>
<tr>
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<tr>
<td>$RATDT(X)$</td>
<td>age-specific death rates for whole system</td>
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<tr>
<td>$RATFT(X)$</td>
<td>age-specific fertility rates for whole system</td>
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<tr>
<td>$RATMT(X)$</td>
<td>age-specific migration rates for whole system</td>
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<tr>
<td>$POPC(X)$</td>
<td>age distribution of population for total system</td>
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**Life Table**

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<td>$q_i(x)$</td>
<td>probability of dying in I between ages $X$ and $X + h$.</td>
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<tr>
<td>$p_{ij}(x)$</td>
<td>probability of being in J at age $X + h$, while in I at $X$</td>
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<tr>
<td>$i^0j(x)$</td>
<td>probability that an individual born in I will be in J at age $X$</td>
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<tr>
<td>$i^0L_j(x)$</td>
<td>number of years lived in J between ages $X$ and $X + h$ by an individual born in I</td>
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<tr>
<td>$i^0e_j(x)$</td>
<td>expected number of years remained to be lived in region J by an individual born in I, and now of age $X$</td>
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<tr>
<td>$s_{ij}(x)$</td>
<td>proportion of people aged $X$ to $X + h$ in region I, surviving to be in region J and $X + h$ to $X + 2h$ years old $h$ years later</td>
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**Population Projection**

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<td>$s_{ij}(x)$</td>
<td>see life table</td>
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<tr>
<td>$b_{ij}(x)$</td>
<td>average number of babies born during the unit time interval and alive in region J at end of that interval, per X to $(X + h)$ year old resident of region I at beginning of that interval</td>
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### Fertility and Mobility Analysis

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<td>ZMOM(P,J,I)</td>
<td>$P$-th moment of the integral function</td>
</tr>
<tr>
<td>HU(X)</td>
<td>weighting factor for generalized net maternity and mobility functions</td>
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<tr>
<td>$\text{HU}(X) = 1$ for stationary populations</td>
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<tr>
<td>$\text{HU}(X) = e^{-r(x+N_Y*0.5)}$ for stable populations</td>
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<tr>
<td>ZGRAL(X,J)</td>
<td>the age- and region-specific rates entering the integral function</td>
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### Spatial Reproductive Value and Further Stable Population Analysis

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<td>$\bar{n}_x$</td>
<td>$V(X,J,I)$ discounted number of offspring in region $J$ per person residing in region $I$ at age $X$, respectively in age group $X$ to $X+4$</td>
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<tr>
<td>$\bar{\Psi}(z)$</td>
<td>$\text{PSI}(J,I)$ zero-th moment of the weighted generalized net maternity function</td>
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<tr>
<td>${Q^0}$</td>
<td>$\text{ROPSI}$ dominant eigenvalue of $\bar{\Psi}(r)$</td>
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<tr>
<td>${Q}$</td>
<td>$\text{VRPSI}(J)$ right eigenvector of $\bar{\Psi}(r)$, associated with RORPSI</td>
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<td>${\Psi(0)}$</td>
<td>$\text{VLPSI}(I)$ left eigenvector of $\bar{\Psi}(r)$, associated with RORPSI</td>
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<tr>
<td>${q}$</td>
<td>$\text{QQ}(I)$ stable equivalent of regional births</td>
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<tr>
<td>${\gamma}$</td>
<td>$\text{YY}(I)$ stable equivalent of the regional total population</td>
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<tr>
<td>${\kappa(x)}$</td>
<td>$\text{POPST}(X,I)$ stable equivalent of the population by age group and region (discrete case)</td>
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<td>${k(x)}$</td>
<td>stable equivalent of population by age and region (continuous case)</td>
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### Momentum of Spatial Zero Population Growth

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<td>matrix of life expectancies at birth: average lifetime an I-born person may expect to live in region J.</td>
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<td>NRR - matrix: the average number of children an I-born person may expect to have in region J.</td>
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<td>{Q}</td>
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<td>{Y}</td>
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<td>regional allocation of the total population in ZPG-population</td>
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<tr>
<td>{γ}</td>
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<td>vector containing the diagonal elements of the matrix of fertility adjustment factors.</td>
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### Zero Population Growth Analysis

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<td>VI(I)</td>
<td>vector containing the diagonal elements of the matrix of fertility adjustment factors.</td>
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Appendix B

Multiregional Life Table: Slovenia - Rest of Yugoslavia

Option 1
| age | q(x,1) | p(x,1) | p(x,2,1) | l(x,1) | l(x,2,1) | l(x,1|x,2) | l(x,2|x,1) | m(x,2,1) | s(x,1) | s(x,2) | s(x,1|x,2) | s(x,2|x,1) |
|-----|--------|--------|----------|--------|----------|------------|------------|----------|--------|--------|------------|------------|
| 0   | 0.010077 | 0.956075 | 0.015938 | 99.9999 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |
| 2   | 0.000207 | 0.999803 | 0.000197 | 95.0497 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |
| 4   | 0.000037 | 0.999963 | 0.000037 | 94.9639 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |
| 6   | 0.000006 | 0.999994 | 0.000006 | 94.9594 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |
| 8   | 0.000001 | 0.999999 | 0.000001 | 94.9593 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |
| 10  | 0.000000 | 0.999999 | 0.000000 | 94.9593 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 | 100.0000 |

This table corresponds to life table statistics given in Rogers, 1975a, Chapter 3. For an explanation of column variables, see Table 2.10.
Appendix C

Sample Data and Outputs for Multiregional Demographic Analysis†

C1. Great Britain, 1970‡‡

C2. Hungary, 1974+++ 

†As part of the Comparative Migration and Settlement Study, IIASA is applying the computer programs presented in this report to analyze patterns of multiregional demographic change in the IIASA National Member Organization countries. This analysis is carried out jointly with national scholars. The data given in this Appendix are sample input data.

‡‡The data for Great Britain have been provided by Prof. P. Rees of the University of Leeds. The outputs are analyzed in his report, Migration and Settlement in the United Kingdom, IIASA Research Report (forthcoming).

+++The data for Hungary have been provided by Dr. K. Tekse and Dr. K. Bies of the Demographic Research Institute, Budapest. The outputs are analyzed in their study, Migration and Settlement in Hungary, IIASA Research Report (forthcoming).
Observed population characteristics: Great Britain

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**EIGENVALUES**

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### Eigenvalues and Eigenvectors

**Eigenvalue:** 68.94656

**Eigenvector**

- **Right:** 1.000000 0.458241 0.556257 0.398676 0.407601 0.448172
- **Left:** 1.000000 1.014909 1.024267 1.011877 1.007252 1.013979

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### Spatial Migration Expectancies

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- **Central**: 4.024927
- **N. Hung.**: 4.171292
- **N. Plain**: 4.272367
- **S. Plain**: 4.444483
- **N. T. Danu**: 5.045033
- **S. T. Danu**: 4.350386

#### Eigenvalue

- **Eigenvalue**: 4.09273

#### Eigenvector

- **Right**
  - 1.000000
  - 0.504220
  - 0.612827
  - 0.530904
  - 0.944555
  - 0.531261

- **Left**
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**TOTAL**

- **Central**: 1.000000
- **N. Hung.**: 1.000000
- **N. Plain**: 1.000000
- **S. Plain**: 1.000000
- **N. T. Danu**: 1.000000
- **S. T. Danu**: 1.000000
### Spatial Fertility Expectancies

#### Net Reproduction Rate

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**Eigenvalue** 1.083136

**Eigenvector**
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- Left: 1.000000 1.167921 1.053948 1.059429 1.055553 1.057445

#### Net Reproduction Allocations

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