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Proposition structure in framed decision problems: A formal representation

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Proposition Structure in Framed Decision Problems: A Formal Representation

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Abstract

Framing effects, which may induce decision-makers to demonstrate preference description invariance violation for logically equivalent options varying in semantic emphasis, are an economically significant decision bias and an active area of research. Framing is an issue *inter alia* for the way in which options are presented in stated-choice studies where (often inadvertent) semantic emphasis may impact on preference responses. While research into both espoused preference effects and its cognitive substrate is highly active, interpretation and explanation of preference anomalies is beset by variation in the underlying structure of problems and latitude for decision-maker elaboration. A formal, general scheme for making transparent the parameter and proposition structure of framed decision stimuli is described. Interpretive and cognitive explanations for framing effects are reviewed. The formalism's potential for describing extant, generating new stimulus tasks, detailing decision-maker task elaboration. The approach also provides a means of formalising stated-choice response stimuli and provides a metric of decision stimuli complexity. An immediate application is in the structuring of stated-choice test instruments.

Keywords

Decision-making / risk / prospect theory / framing effects / task logical description

Introduction

Risk taking behaviour of individuals, small groups and organisations is subject to two avenues of research. A normative tradition that informs management science and operations analysis deduces optimal decisions from assumption of instrumental rationality, utility maximisation and Bayesian information efficiency. A complementary descriptive tradition takes a positive approach to the reality of decision-making. Contrasting findings between the two reveals that real decision-making is cognitively 'rich' with actual choice and preference demonstrating a series of systematic deviations – referred to as decision biases or anomalies - from normative precepts. One anomaly that has been extensively researched under both laboratory and ecological conditions is the 'framing effect' (Tversky & Kahneman 1981; 1986) where rating of a single option or preference between available options (usually, but not inevitably a pair) is reversed depending on the semantic emphasis of the outcome description *even though those presentations are, at face, logically equivalent*. This represents an ostensible violation of expected utility description invariance.

This paper addresses a lacuna by developing a formal, rigorous and quite general logical representation of decision problems proposed to be subject to the framing effect in earlier literature. It complements Levin, et al's (1998) tubular representation of framing problem structure. Such formalism is practical and has value in the analysis of extant decision problems and creation of well-defined novel problem stimulus, including those in stated-choice instruments. The formalism makes clear that the scope for stimulus problem construction is far greater than those currently subjected to experimental investigation. The formalism is also of use in revealing decision construal in ecological settings and can inform the development of prescriptive technologies to debias and enhance professional decision-making. It also makes explicit logical and outcome / risk facets of problem structure that have been suggested to be of import to framing effects and are called upon as a basis for their explanation. The distinction between risky choice, attribute and goal framing highlighted by Levin, et al (1998) is emphasised. Examples of framed decision stimuli from a transport context are developed.

The paper is structured as follows. First an outline of Prospect Theory, which provides the primary conceptual platform for explaining framing effect, is presented. Its predictions are contrasted with those of normative expected utility theory (EUT). This is followed by a brief review of framing effects in which investigation of business arena decisions, framing typology (a distinction between risky choice, attribute and goal framing), proposed explanations and outstanding research issues are discussed. This discussion makes a strong case for the relevance and value of the proposed formalism. The formalism is then expiated. Elements common to all three types of framing problem are discussed first with elements specific to the formalisation of risk choice, attribute and goal framing following. The paper concludes with comments on how the formalism may be extended to accommodate more general and ecologically relevant features of stated-choice decision challenges.

Prospect Theory: An overview

Most decisions are taken in the face of uncertainty. Outcomes vary in their degree of desirability and likelihood. Actions must be taken in light of their potential to deliver preferred outcomes and to ameliorate, if not eliminate, risk. The normative approach to decision under risk (established in the early post-war years by *inter alios* von-Neumann & Morgenstern (1947) and Savage (1954)) dictates optimal preferences under objective and subjective risk respectively on the basis of a set of consistency axioms and a target of expected utility maximisation. Decision-makers are presumed be computationally efficient, logically omniscient and properly Bayesian in deriving beliefs from available information. This tradition informs much management science, finance theory and operations analysis. It provides a platform for stated-choice methodologies.

The empirical predictions of normative theory have stimulated a tradition of experimental and ecological evaluation of human decision making under a variety of situations to assess the extent to which it is truly 'rational' on an aggregated and (increasingly) individual level. This descriptive tradition has generated extensive evidence that actual decision-making rarely meets with normative precepts. Further, preference deviations are not random but tend to be distorted in a systematic and consistent way. This has led to the development of descriptive models of decision-making under risk that generalise expected utility theory in an attempt to better map the reality of choice and preference (Reviews are offered by Schoemaker (1982), Sugden (1986), Weber & Camerer (1987), Machina (1987), Fishburn (1988a ; 1988b), Camerer (1989), Munier (1989), Camerer & Weber (1992), Starmer (2000).

Of these generalisations, Prospect Theory (Kahneman & Tversky 1979; Tversky & Kahneman 1992) has proved to be the most successful in stimulating research and providing a basis of descriptive accounts of practical decision-making. Prospect theory has a number of postulates that distinguish it from expected utility theory. Consider a decision-maker facing a prospect, □ , that is a set of discrete outcomes $\mathbf{x} \subset X$, $\mathbf{x} = (x_1 ,\dots, x_n)$, to which the decision-maker assigns a utility $U : X \to$ □ over a probability distribution $\mathbf{p} = (p_1 ,\dots, p_n)$, $p \in [0 , 1]$, $\Sigma_i\, p_i = 1$. Prospect Theory suggests that decision-makers evaluate potential changes in asset position, $\Delta x = (x - r)$ relative to an initial reference point, r , rather than final asset position. Such changes are psychologically coded as gains ($\Delta x > 0$) or losses ($\Delta x \leq 0$). It further proposes that the utility function has a specific shape (S-shaped: concave in gains, convex in losses discontinuous at the reference point) such that gains tend to induce preference for risk averse options while losses induce preference for risk seeking options. Further, probabilities are not entertained unmodified, but are weighted as a function $w(p)$ with $w(0) = 0$ and $w(1) = 1$ but with a typical pattern of low probabilities, $p_{<<}$ being overweighted and moderate to high probabilities, $p_{>}$ being underweighted $w(p_{<<}) > p_{<<}$, $w(p_{>}) < p_{>}$.

In total, whereas expected utility theory suggests a prospect should be (rationally) valued as V_{EUT}(□) $= u(y , 1) = \Sigma_i\, u(x_i)\, pi$, Prospect Theory proposes that the prospect is valued as V_{PT}(□) $= u(y , 1) = \Sigma_i\, v(\Delta x_i) w(p_i)$, where v is the value function that has the properties of a utility function over asset changes.

The switch from risk aversion in gains to risk seeking in losses is manifest as two distinct (though often confused, see Fagley 1993 and Li 1998) decision phenomena. The *reflection effect* occurs when a decision-maker prefers one (usually the less risky) of two prospects □$^{+}_{s}$ □ □$^{+}_{r}$ when gains are involved ($\forall x > 0$) , but refers the other (usually the riskier) option when gains are reflected to losses ($\forall x \to -x$) $\Rightarrow$ □$^{-}_{s}$ □ □$^{-}_{r}$. As the gain and loss prospects are logically distinct, this reversal in preference might reflect consistent preferences and so is not necessarily irrational. The *framing effect*, on the other hand, relates to an inversion of preference between options differing in risk when *logically identical* pairs of prospects are presented with descriptions that differ only in semantic emphasis: if the description is in terms of affectively positive aspects or outcomes (gains, opportunities, improvement or attractive features) then risk aversion usually results; if the description is in terms of affectively negative (and antonymic to the positive) aspects or outcomes (losses, threats, depreciations or unattractive features) then risk seeking ensues. The reversal may be observed intra-participant when the two versions are presented to the same individual on different (cognitively isolated) occasions and inter-participant when the reversal is manifest as a shift in modal preference between comparable groups.

The two descriptions are logically equivalent assuming that the negative description is antonymic to the positive and reference to set partitions switched. Hence the preference reversal is *ostensibly* irrational (the need for this qualification will become apparent later). Prospect Theory accounts for the framing effect as a consequence of the decision-maker psychologically coding positive descriptions as gains and negative as losses so inducing risk averse and risk seeking preferences respectively.

Framing effects: Typology, examples and explanations

Explicit attention to framing effects was first drawn by Tversky & Kahneman (1981) with the now canonical ‘Asian Disease’ problem. An initial semantically common context was set with:

> “Imagine the US is preparing for the outbreak of an usual Asian disease, which is expected to kill 600 people. Two alternative programs have been prepared to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows:”

Two versions of the choice set (two options, forced choice) then follow, one with a ‘positive’ frame emphasising ‘lives saved’:

> “If Program A is adopted, 200 people will be saved.
>
> If Program B is adopted, there is 1/3 probability that 600 people will be saved and 2/3 probability that no people will be saved”

The other with a ‘negative’ frame emphasising ‘deaths’:

> “If Program C is adopted, 400 people will die.
>
> If Program D is adopted, there is 1/3 probability that nobody will die and 2/3 probability that 600 people will die.”

Assuming that ‘die’ and ‘saved’ are complete and mutually exclusive, and that ‘die’ ≡ ‘(not) saved’ and ‘saved’ ≡ ‘(not) die’ then the consequences of Program A ≡ Program C and Program B ≡ Program D. Under expected utility theory it is presumed that such invariance should not influence preference. Yet Tversky & Kahneman (1981) found that when presented to different participant groups 72% (n = 152) preferred the ‘safe’ (certain outcome) Program A (coded as gains) and 78% (n = 155) preferred the ‘risky’ (probabilistic outcome) Program D (coded as losses). Given the assumptions above, this inter-participant preference reversal is an ostensible violation of strategic invariance.

Follow up studies by Tversky and Kahneman (1986) demonstrated the robustness of the effect with a series of alternative problems adopting the same two-choice: safe-risky template. Further studies into the effect (largely for business-context decisions) have been undertaken by Neale & Bazerman (1985) and Schurr (1987) for bargaining, Qualls & Puto (1989) for industrial buying, Roszkowski & Snelbecker (1990) for financial planning, Schneider & Eble

(1994) for employment, Schoorman, et al (1994) for company acquisition and disposal and Chang, et al (2002) for accounting decisions. These studies largely confirmed the framing effect as found by Tversky & Kahneman (1981; 1986) and demonstrated that professional decision makers are as subject to it as naive. Real financial incentives do not eliminate the effect and may even exacerbate it (Camerer, 1995:635; Kühberger, et al, 2002). Highhouse, et al (1996) demonstrated contrast effects in the framing of business issues as opportunities or threats. Decision-makers exposed to unequivocal opportunities framed equivocal issues as threats and vice versa.

A meta-study by Kühberger (1998) of 136 published experimental studies found the gain – risk averse to loss – risk seeking violation of invariance to be robust. However, subtle variations in problem structure such as problem arena, framing by reference point shift or outcome salience, response mode (forced choice or rating), qualitative or quantitative risk description and single or multiple initiator events were influential on the extent of the effect. All of these issues are relevant to stated-choice survey instruments. So too was whether the effect was measured on an inter- or intra-participant basis (see also Stanovich (1999:107) for discussion of divergence between the two). Zickar & Highhouse (1998) summarise outstanding issues in decision framing research as including individual differences in susceptibility, frame response consistency and framing arena effects. The formalism proposed in this paper makes transparent subtle variation in problem structure, parameters and logic and can support investigation into these issues.

The Asian Disease problem (and those using it as a logical template) represents choice over alternative options varying in risk. However, as Levin, et al (1998) point out in a major review of framing effects, the issue of framing has been extended (often without overt recognition) to two other types of problem that are of quite different structure: *attribute framing* in which (logically, but not semantically equivalent) alternative descriptions of salient characteristics of an item (item set) impact on object selection or rating and *goal framing* in which the consequences of action or inaction are framed with the persuasiveness of the message to undertake action being influenced.

The simplest form of attribute framing will be familiar to marketers who prefer to label a food product as '90% fat free' rather than (the logically equivalent) 'contains 10% fat'. As might be expected, description in terms of the positive attribute description is usually rated more highly than that with the negative description. Levin & Gaeth (1988) found that for a beef product not only was the positively labelled product rated more highly, but also that in gustatory tests consumers actually reported that it tasted better.

A number of studies have been conducted to ascertain the impact of attribute framing for problems with this template have been conducted. These include decisions in resource allocation (Duchon, et al 1989; Dunegan 1993; 1995; Dunegan, et al 1995), penalties for deceptive advertising (Dunegan 1996), financial control systems (Emby 1994) and joint business ventures (Highhouse & Yüce 1996).

A more complex example of attribute framing (one that involves pairwise choice rather than rating, multiple attributes and probabilistic rather than certain attributes) might be as follows:

> "You are making a business trip and need to be at your destination for a particular time. The train journey is 50 minutes. You have a choice between three trains.

In the positive frame:

08:30 train: There is a 95% chance that you will arrive on time, and a 60% chance that you will get a seat for the entire journey.

09:00 train: There is an 85% chance that you will arrive on time, and an 80% chance that you will not get a seat for the entire journey.

09:30 train: There is an 80% chance that you will arrive on time, and a 90% chance that you will get a seat for the entire journey.

Which train would you prefer?

In the negative frame:

08:30 train: There is a 5% chance that you will be late, and a 40% chance that you will not get a seat and have to stand for the entire journey.

09:00 train: There is a 15% chance that you will be late, and a 20% chance that you will not get a seat and have to stand for the entire journey.

09:30 train: There is a 20% chance that you will be late, and a 10% chance that you will not get a seat and have to stand for the entire journey.

Which train would you prefer?
Such a stimulus might be used to investigate how individuals were willing to play off arrival time risk against journey comfort. These descriptions are equivalent given that the attributes 'arrive on time and 'be late' and 'get a seat' and 'have to stand' both constitute exclusive and complete sets. The proposed formalism captures more general problems of this type with the 'single item rating – certain attribute' template a sub-type.

Levin, et al (1998) define goal framing as 'a positive emphasis on the goal of obtaining a positive consequence as opposed to the negative emphasis on avoiding a negative consequence'. In both cases the same action (referred to as a *project* in this formalism) and consequence is being promoted. However the persuasiveness of the message is frame sensitive. It is usually the case that the action presents some short-term cost or inconvenience to the decision-maker. A simple example of goal framing in the travel choice arena is, in a positive frame:

> "Leaving earlier increases your chance of arriving on time";

Versus the negative frame:

> "Not leaving earlier increases your chance of being late".

This template of 'for a single project: do and achieve good – not do and face bad, (with certainty)' is dominant in goal framing studies. This does however represent only a small subspace of possible goal framing problem types. The generalisation within the proposed formalism occurs along three dimensions. First the possibility of selecting between alternative projects rather than just accepting (rejecting) or rating a single project is entertained; secondly, the projects are considered to offer desired consequences at a probability rather than for certain (this introduces the notion of the project's instrumentality); thirdly, projects are regarded as symmetric in that they can prevent the delivery of (negative) consequences as well as help ensure those with a positive (projects might have a negative instrumentality as well as positive).

For example, in the arena of congestion charging:

In a positive frame:

Would you support a congestion charge of £5 between the hours of 07:00-09:00 in your city centre if it had a 90% chance of reducing your travel time of at least 15 minutes and a 60% chance of reducing your travel time by at least 30 minutes?

Yes, no?

And in a negative frame:

Would you support a congestion charge of £5 between the hours of 07:00-09:00 in your city centre if it had a 10% chance of not reducing your travel time by up to 15 minutes and a 40% chance of not reducing your travel time by up to 30 minutes?

Yes, no?

Goal framing has not received the same level of research attention as risky choice framing in the most decision arenas. Health choice is perhaps an exception where the impact of health improvement messages is important. Goal framing effects have not always been found, but where they have, the negative frame usually has the greater impact on espoused behavioural intention. Two management arena studies are McCusker & Carnevale (1995) for resource deployment and Newberry et al (1993) for tax practitioner behaviour. While risky choice framing has capitalised the attention of theorists, the latter two types of framing are likely to be of increasing interest in empirical investigation of the practical impact of descriptive valence variation on travel choice.

Levin, et al (1998) make it clear that interpretation of causes into framing effects will be hindered if these three types of stimulus problem are not clearly delineated. As revealed by the proposed formalism, the logical structure of these three types of framing problem is quite distinct.

Explaining framing effects

A number of explanations have been proposed for framing effects that do not call upon Prospect Theory's postulation of value function convexity-concavity. These largely centre on problem interpretation, elaboration or comprehension.

An immediate possibility is that individuals do not integrate outcome and probability data in a way consistent with expected utility theory, but focus on one particular dimension. If the outcome dimension is prioritised in the Asian Disease problem, then the '600 saved' of B might be attended in the positive frame while the 'nobody dies' of D is attended to in the negative frame, thus not inducing (at a perceptual level) invariance violation. If the risk dimension is attended to however then preference would be frame sensitive as it is the probability of both 'good' and 'bad' outcomes that increase leading to preference for B in the positive frame and C in the negative. This pattern of preferences is however counter to that of the Prospect Theory prediction usually observed. Zickar & Highhouse (1998) develop an Item Response Theory (IRT) approach to analysing framing effects. This establishes the probability of an individual affirming or rejecting a particular item. Applied to four problems with the Asian Disease template they found consistent inter-participant preference for the risk averse option in the positive frame and risk seeking in the negative (albeit with some statistical anomalies for the original problem). 40% of variance could be accounted for by a single factor the authors refer to as preference for risk.

The suggestion of invariance violation is presupposed on the decision-maker sharing the problem constructor's logical intentions of the completeness and exclusivity of descriptive antonyms and negations. The constructor also supposes that the decision-maker will regard the described outcome conditions as final and not available for elaboration. If the decision-maker does not share these assumptions then a violation of invariance may not be occurring *within her interpretation of the problem*.

For example, Gigerenzer (1991) suggests that decision-makers do not take the problem information at face (Bayesian) value. Rather they construct a Local Mental Model (LMM) of the described scenario based on the information provided. Within the LMM of the Asian Disease problem, for example, the decision-maker may (reasonably) include the idea that 'die' is a final and unambiguous condition whereas the 'not saved' condition (which is not made explicit in the problem, it being presumed the decision-maker deduces this and so increasing its available for elaboration) is ambiguous and includes the possibility that given time and new developments some of these people might be saved *later*. Further, the decision-maker may (intuitively) doubt the veracity of the number dying and supposing that it is a lower estimate with more people dying *later*. Both presumptions can remove the burden of invariance violation. In a naturalistic context, a 'buy more time' strategy rather than (immediate) utility maximisation might be considered reasonable.

Reyna & Brainerd (1991) and Reyna & Ellis (1994) apply Fuzzy-Trace Theory (FTT) to the framing issue. FTT proposes that rather than processing quantitative data, decision-makers develop a qualitative representation of the problem. For the Asian Disease problem this might take the form: Program A – 'Some people will be saved'; Program B – 'Some people will be saved or no one will be saved'; Program C – 'Some people will die' and Program D – 'Nobody will die or some people will die'. Given that 'Some people will be saved is common to A and B, the unique '…no one will be saved' in B shifts preference to A, while with C and D, the '(…) some people will die' is common while the unique 'Nobody will die' shifts preference to of D.

These competing explanations for the framing effect make specific predictions in relation to the influence of changing problem structure, parameters (relative outcome value and probabilities), and explicitness of logical assumptions. The proposed formalism is of value in that it makes these aspects of the decision problem transparent. Further, it clearly distinguishes between risky choice, attribute and goal type problems. The formalism can be used to construct decision stimulus problems with specific structural, parametric and logical properties.

In addition, the formalism can be used to rigorously analyse decisions as practically construed by decision-makers in ecological settings. Application here includes recognition of option possibilities and intuitive option framing. Variation in such between different members of decision-making groups and resulting impact on group dynamics and decision quality is a major research opportunity.

Further, consultants and researchers developing prescriptive technologies that enhance managerial decision-making are concerned with 'de-biasing' technologies that reduce or eliminate susceptibility to framing effects. Proposed techniques include development of causal cognitive maps of the problem (e.g. Hodkinson, et al 1999), imposing justification for the decision made (e.g. Takemura 1994) or evaluation the credibility of the evidence that underpins the decision (e.g. Emby & Finley 1997). The formalism provides a tool for developing debiasing techniques and empirical evaluation of their performance.

Formal description of framed decision problems

After a common initial generalisation, the three types of framing are subject to different formalisations. This reflects the divergence in their logical structures.

Framed decision problem common structure

A decision problem, $\square$ can be regarded as consisting of three information sets: a context setting introduction, K , a set of explicit decision options, A and a set of instructions, T . Thus: $\square = K \wedge A \wedge T$. The context is a set of propositions that instigates the nature and terms of the problem. The amount of information in K will vary. A distinction might be made between propositions including information that is relevant to the normative interpretation favoured by the problem constructor K_N and normatively irrelevant 'filler' information K_D , which may be intended to colour interpretation, distract or provide complexity load, so $K = K_N \wedge K_D$.

The instruction set T is concerned with directing the decision-maker's response, for example: to select one from a set of ≥ 2 options (forced choice), to accept or reject a single option, to rank options in order of preference or to rate option attractiveness on a numerical or verbal scale. A number of studies indicate that such alternative approaches to revealing preference may impact on stated preferences in decision in general (Hershey, et al 1982; Farquar 1984) and framing in particular (Kühberger, 1998).

Framing is introduced by generating two versions of the decision problem:

$$\square = K \wedge \tilde{A} \wedge T , \square = K \wedge \tilde{A} \wedge T$$

reflecting articulation of the decision-problem in terms of positive and negative valence descriptors of the option set respectively. Note that the change in framing usually only impacts on the description of options, not the context set K (though potential for manipulating context remains). Under positive framing a positive valence lexical term, g and (potentially) its negation $\neg$g (where $\neg$ indicates logical negation) are used in the outcome description propositions; under negative framing a negative valence lexical term b and (potentially) its negation$\neg$b are used in the outcome description propositions. Positive valence terms imply gain, opportunity, improvement and/or attractiveness. Negative valence lexical terms imply loss, threat, depreciation and/or unattractiveness.

g is presumed to be strictly antonymic to b . Syntactically, it is assumed that $g \equiv \neg b$, $\neg g \equiv b$. Further it is assumed that the positive and negative pair are complete and mutually exclusive: $\square = g \cup b$, $g \cap b = \varnothing$, hence $\square^+$ and $\square^-$ represent logically (but not semantically) equivalent representations of the problem. The framing effect is said to occur if preference orderings over a preference structure $\langle A , \square \rangle$ are inter- or intra-participant inconsistent given a framing shift between $\square^+$ and $\square^-$ where $\square$ is a complete, transitive weak ordering

Risky option framing

Under a risky option problem, the decision-maker is presented with a set of N items (which may be individuals or a divisible sum of money or other commodity). These items are in an initial condition, $S_0(N)$. The item set faces the possibility of an event, E (which may be certain or occur with a probability p_E). This event will distribute the items to a partition of new conditions, $\Box = \{S_1 ,\ldots, S_m\}$, $S_i \cap S_j = \varnothing$

$$E(p_E) : S_0(N) \rightarrow \Box$$

It is supposed that the decision-maker has a preference structure $\langle \Box , \Box \rangle$ over these conditions, ordering such that: $S_1 \Box S_2 \Box ,\ldots, \Box S_m$, where $\Box$ indicates a complete, transitive weak ordering. In general, the partitioning of items occurs over a discrete probability distribution, $\mathbf{p} = \{p_1 ,\ldots, p_z\}$, $\forall p \in [0 , 1]$, $\Box_i p_i = 1$.

The decision maker is presented with a set of options (acts) $A = \{A^1 ,\ldots, A^y\}$ that might ameliorate the partitioning of items into these conditions. It is normally assumed that the options are discrete and only one may be taken.

Thus:

$$\begin{aligned}(E(p_E) \mid A^j) : S_0(N) \rightarrow\ & \{(n^j_{11} \oplus n^j_{21} \oplus \ldots \oplus n^j_{m1}); p_1\} + \\ & \{(n^j_{12} \oplus n^j_{22} \oplus \ldots \oplus n^j_{m2}) ; p_2\} + \ldots.. + \\ & \{(n^j_{1z} \oplus n^j_{2z} \oplus \ldots \oplus n^j_{mz} ; p_z\}\end{aligned}$$

Where, following Luce's (2000:23) convention the symbol '$\oplus$' represents joint receipt and '+' probabilistic combination. The term n^j_{ki} should be read as the number of items out of the set $S_0(N)$ apportioned to condition S_k , at probability p_i given that action A^j has been taken in response to, or anticipation of event E with $\Box_k n^j_{ki} = N$ and $\Box_i p_i = 1$. This partitioning is dependent on E occurring. It is assumed that if E does not occur, then the action taken has no impact and all items remain in $S_o(N)$.

Each option $A^j \in A$ might then be regarded as being described in terms of a linear combination of atomic propositions:

$$[A^{j+}] \equiv \wedge_{ki} [E(p_e) \mid A^j \Rightarrow (n^j_{ki} , S^+_k , p_i)]$$
$$[A^{j-}] \equiv \wedge_{ki} [E(p_e) \mid A^j \Rightarrow (n^j_{ki} , S^-_k , p_i)]$$

Which can be read as "Given that event E occurs (and its probability is p_e) and action A^j is taken then there is a probability p_i that n^j_{ki} items will be transferred to condition $S^{+/-}_k$". The $^{+/-}$ superscripts indicate that a positive or negative valence term may be used to describe the condition to create the positive and negatively framed problem respectively. The full description of act $[A^j]$ is a linear combination of propositions over all conditions and probabilities [1].

The numeric distribution to conditions consequential on E given $A^j \in A$ can be depicted by a matrix representation (figure 1).

[1] Note that in this and all following, A^j , etc. refers to an option while $[A^j]$, etc. refers to a *proposition* describing that option.

Event E **occurs and action** A^j **is taken**	**Condition**				
Probability		S_1	S_2	…	S_m
	p_1	n^j_{11}	n^j_{21}	…	n^j_{m1}
	p_2	n^j_{12}	n^j_{12}	…	n^j_{m2}
	:	:	:		:
	p_z	n^j_{1z}	n^j_{2z1}	…	n^j_{mz}

Figure 1
Matrix representation of the probabilistic apportioning of items in $S_0(N)$ to conditions on event E given amelioration by act A^j

Some problem presentations neglect to explicitly present an apportioning to a consequence state on the grounds that this is logically implied by the consequence states that are presented (this is so with the original Kahneman-Tversky 'Asian Disease' problem). It is presumed that the decision-maker will recognise this, but it may impact on the latitude for decision-maker to elaborate a Local Mental Model of the problem (Gigerenzer, et al, 1991). In this case the modal operator '□' ('is necessarily the case') is used to indicate this condition within the formalism. Thus:

$$\square\ [E(p_e) \mid A^j \Rightarrow (n^j_{ki}\ ,\ S^{+/-}_{k}\ ,\ p_i)]$$

Implies the decision-maker is left to deduce this proposition.

The framing effect occurs if there is a violation of invariance with a reversal in preferences:

$$A^{s+}\ ,\ A^{r+}\ ,\ A^{s-}\ ,\ A^{r-} \in \mathit{A}$$
$$A^{s+}\ \square\ A^{r+} \text{ and } A^{r-}\ \square\ A^{s-}$$

With □ indicating strict preference.

Attribute framing

Attribute framing relates to choice over an item or set of items in which the decision-maker is called upon to either accept or reject and option, to select a preferred option from a set or rate an option given a description of the option (or option set) on the basis of a description of the attributes of the option set.

The decision concerns a set, G of $N \geq 2$ of items $G^j \in G$, where G^j may itself be a set of objects. G^j may be described in terms of g_i , $b_i \in F$ with F a set of decision-salient attributes where g has a positive and b a negative valence

The decision-maker is called upon to accept or reject an option $G^j \in G$, rate its (un)attractiveness on a numeric or verbal scale or express preference for $\exists G^j \in G$. Again. it is presumed that $\forall i\ g_i \equiv \neg b_i$, $b_i \equiv \neg g_i$ and $g_i \cap b_i = \varnothing$. g_i and b_i are quantified for $\forall G^j \in G$ at a level $q^j(g_i) \in [0\ ,\ 1]$ and $q^j(b_i) = 1 - q^j(g_i)$, g and b being antonymic. The level may of course be represented more meaningfully to the decision-maker as a percentage or numerical ratio. For $\forall G^j \in G$ the attribute g at level $q^j(g)$ may be definite or, more generally, it may occur over a probability distribution $(q^j(g_i)\ , p^j(g_i))$. $\forall g\ , b \in F\ , p(g_i) = p(b_i)$. By implication, the distribution of the negative valence attribute for G^j is $(g^j(b_i)\ , p^j(b_i)) = ((1 - g^j(g_i))\ , p^j(g_i))$. It is presumed that the decision-maker has monotonic preferences over $\forall G^j \in G\ , q^j(g) \geq q^k(g)\ ,$ $p^j(g) \geq p^k(g) \Rightarrow G^j \square G^k$. Each option $G^j \in G$ is then described by a proposition of the form:

$$[G^{j+}] = \wedge_i\ [G^j\ , q^j_i(g)\ , p^j_i(g)]$$
$$[G^{j-}] = \wedge_i\ [G^j\ , (1 - q^j_i(b))\ , p^j_i(b))]$$

This may be read as, for the positive frame "[Option G^{j+} has a (positive valence) attribute g at a level $q^j_i(g)$ with probability $p^j_i(g)$]" or for the negative frame "[Option G^{j-} has a (negative valence) attribute b at a level $(1 - q^j_i(b))$ with probability $p^j_i(b)$]". The full description of the choice option combines linearly over probabilities. The expectation of the positive attribute for G^{j+} is then $E(G_j(g)) = \square_i\ g^j(g_i))\ .\ p^j(g_i))$. Normatively, a rational decision-maker might be expected to maximise this.

The framing effect occurs if G^{j+} is accepted, while G^{j-} is rejected (or vice versa), if the relative (un)attractiveness rating of G^{j+}, G^{j-} is consistently different or that over pairwise preference $G^{j+} \square G^{j-}$ reverses to $G^{j+} \square G^{j-}$.

A graphical representation of the options in *G* is illustrated in figure 2. In this the horizontal bars normalised to a length of unity are divided into lengths representing the level of positive and negative attribute, $q^j_i(g)$ and $q^j_i\ (b) = (1 - q^j_i\ (g))$. The height of the bar represents $p^j_i\ (g) = p^j_i\ (b)$. The area $\square_i\ g^j(g_i))\ .\ p^j(g_i))$ is proportional to the expectation of the positive attribute for G_j .

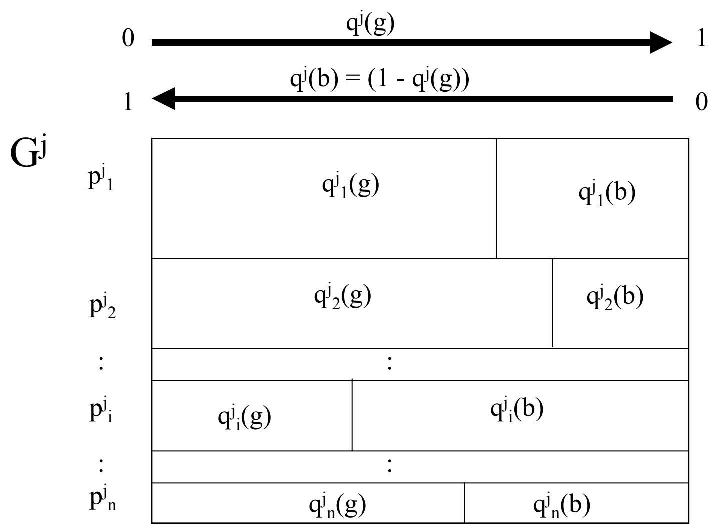


Figure 2
Graphical representation of the parameter space for an attribute framing problem

Goal framing

A goal framing decision problem is one in which the terms are an individual or item, Q (which may be the decision-maker herself, or may invoke he decision-maker as agent on behalf of another individual or organisation) is in an initial condition S_0 . The individual or item will transit to a new condition out of a partitioned set $\square = \{S_1, \ldots, S_m\}$, $S_i \cap S_j = \varnothing$ over a probability distribution $\mathbf{p} = \{p_1, \ldots, p_z\}$, $\forall p \in [0, 1]$. It is supposed that the decision-maker has a preference ordering over these conditions, ranked such that: $S_1 \square S_2 \square, \ldots, \square S_m$, where $\square$ indicates a complete, transitive weak ordering.

The decision-maker has available a set of projects $P = \{P^1, \ldots, P^n\}$ that ameliorates the probability of the individual or item transiting to the conditions in $\square$. I.e.:

$$P : S_0 \rightarrow \square$$

It is supposed that a null project ('do nothing') P^0 is available (though this may not be presented as an option to the decision-maker), thus $S_0 \in \square$, $P^0 : S_0 \rightarrow S_0$.
The instrumentality of a project P^i in achieving a condition S_j for the individual or item is defined as:

$$I(P^i, S_j) = p(S_j \mid P^i) + p(\square \backslash S_j \mid \neg P^i) - 1$$

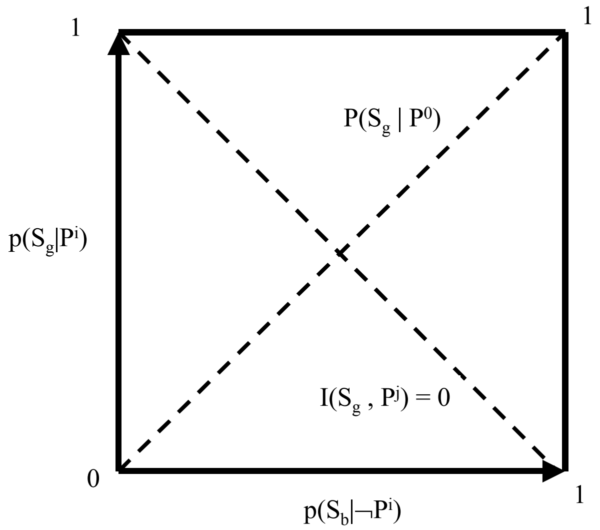
Which might be read as '[If project P_i is undertaken, then there is a probability $p(S_j \mid P^i)$ that state S_j will occur and if project P_i is not undertaken then there is a probability $p(\square \backslash S_j \mid \neg P^i)$ that state S_j will not occur]'. The 'not undertake P^i action, $\neg P^i$ may be defined in terms of do nothing, P^0 , undertake an alternative project, P_j (thus indicating opportunity cost of P^i versus P^j) or, potentially, some weighted sum of alternative projects $\square(P^{k \neq i})$. The instrumentality metric takes a value of +1 if instigating P^i *guarantees* the delivery of Q to condition S_j , if P^i is not undertaken then it is definite that $\neg S_j$ will occur; at I = -1 the project guarantees that Q will *not* be in condition S_j (otherwise it will) and at 0 the project has no influence on whether Q will be in condition S_j or not (the project has no instrumentality). By definition $\forall S_j \in \square$, $I(P^0, S_j) = 0$. The 'inherent' probability of S_j is specified as $p(S_j \mid P^0)$. The overall structure of the problem may be depicted schematically as in figure 3.

Each project $P^j \in P$ might then be regarded as being described in terms of a combination of atomic propositions of the form:

$$[P^{j+}] \equiv \wedge_j [p(S_j^+ \mid P^i) \wedge p(\square \backslash S_j^+ \mid \neg P^i)]$$
$$[P^{j-}] \equiv \wedge_j [p(S_j^- \mid P^i) \wedge p(\square \backslash S_j^- \mid \neg P^i)]$$

Which might be read as a linear combination over all potential states of propositions of the form "[Given that project P^i is undertaken then the item will transit to condition S_j with a probability $p(S_j^{+/-} \mid P^i)$; if the project is not undertaken, then the item will not transit to condition S_j (that is, will go to some other condition) with a probability $p(\square \backslash S_j^{+/-} \mid \neg P^i)$ "

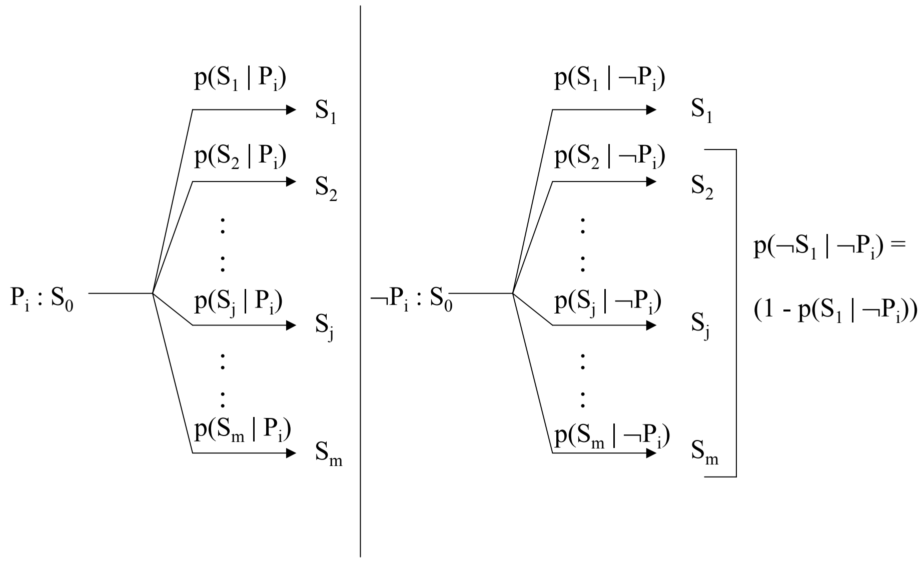
Figure 3
Graphical representation of the parameter space for a goal framing problem



The $^{+/-}$ superscript indicates that an positive or negative valence term may be used to describe the condition to create the positive and negatively framed problem respectively.

The framing effect is said to occur if P^{j+} is indicated as acceptable and P^{j-} is indicated as unacceptable (or vice versa) or if P^{j+} □ P^{k+} and P^{j-} □ P^{k-} .

Figure 4
Map of the parameter space for a two-state goal framing problem



It is worthwhile to consider the restricted case of Q transiting to one of three conditions, retention in S_0 or transit to a 'good' state S_g or a 'bad' state S_b with
S_g □ S_0 □S_b . The 'do nothing' option is not available. This represents the majority of goal framing decision problems and has an interesting graphical representation. Consider figure 4. This represents a graph of $p(S_g \mid P^i)$ against $p(S_b \mid \neg P^i)$. This implies that the top-left – bottom-

left diagonal represents the instrumentality of project P^i on a scale of [-1 , +1] and the bottom-left – top-right diagonal represents the inherent probability of S_g , $p(S_g \mid P^0)$ on a scale of [0 , 1] .

The full set of project options, *P* is then represented by a map of points within the graph. Points above the top-left – bottom-left instrumentality diagonal reflect projects with a positive instrumentality that contribute to the delivery of S_g while points below the diagonal have a negative instrumentality and hinder delivery of S_g .

Potential extensions of the formalism

The formalism developed here has potential for extension into problems where probabilities are vague or where receipt is complicated by factors other than or in addition to risk.

Ambiguity represents a decision condition in which probabilities are available but (in contrast to the condition of strict risk) these are not precisely known or there is reason to doubt their veracity. Ambiguity is a more common condition than risk in many decision contexts and is known to influence decision-making in a number of ways that violate expected utility theory independently of risk (Ellsberg, 1961; 2001; Einhorn & Hogarth, 1986). Hogarth & Einhorn (1990) develop a model that integrates ambiguous probabilities into prospect theory probability weighting. A series of studies of framing under ambiguous probabilities (indicated by either probability ranges or adjectival qualifiers to definite probabilities) conducted by Kuhn (1997) suggests that (with some order effect qualifiers) under negative frames, preference for vague probability options increased. As probabilities (with clear distinction between probabilities of event occurrence, outcome likelihood or option character) are made explicit within the formalism it is straightforward to integrate vague probabilities be they specified by probability range, adjectival qualifier, second order probability distribution or fuzzy set representation.

Many project decisions do not consider specific projects in isolation, but rather as tactical contributions to an overall strategic goal. This means that projects must be considered in the way in which they support or hinder each other, as well as the way in which they reflect upon each other as opportunity costs. The formal description of goal framing can accommodate this by invoking the instrumentality of two (or more) concurrent or sequential projects in relation to delivering specific preferred states.

In addition to risk decision-makers are often called upon to make choices under conditions where receipt is delayed (intertemporal choice) [2], options must be evaluated in terms of a number of desirable or undesirable attributes (multiattribute choice) or the decision-maker holds concern for the distribution of outcomes to others as well as self (social choice). In all cases, receipt may be under conditions of risk or certainty.

Reflecting the tradition of distinct research streams into these different decision dilemmas, framing research has been largely concerned with immediate, single-attribute non-social choice under risk. Progression of research into the impact of framing on decision with complexity other than, or extra to, risk remains an opportunity. The potential for extending the formalism into describing decision problems in these areas is recognised.

[2] A number of anomalies in Intertemporal choice parallel those of choice under uncertainty. See Prelec & Loewenstein (1991).

Conclusions

Framing effects offer a clear indication of (ostensibly irrational) psychological effects in economic and personal decision-making. They have been demonstrated in a number or decision arenas including the managerial, health and political (though travel behaviour is quite neglected). They are influential for both espoused and practical decision-making and have potential to impact on personal economic well-being. The formalism described here has the potential to both provide a rigorous description of extant decision stimuli and in clarifying their full logical scope the creation of new stimuli. It also provides a metric of decision stimuli complexity. One immediate application is the recognition of potential for and investigation into the effect of framing biases in stated-choice test instruments.

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