Risk Perception, Risk Attitude and Decision : a Rank-Dependent Approach

Michèle Cohen

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Risk Perception, Risk Attitude and Decision : a Rank-Dependent Approach

Michèle COHEN

2008.84

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://ces.univ-paris1.fr/cesdp/CES-docs.htm
ISSN : 1955-611X

Risk Perception, Risk Attitude and Decision : a Rank-Dependent Approach

Michèle Cohen

Paris School of Economics and
CES -University Paris I, Panthéon-Sorbonne

December 3, 2008

Résumé

Le modèle classique d'espérance d'utilité dans le risque a été critiqué aussi bien d'un point de vue expérimental (Paradoxe d'Allais) que pour son manque de pouvoir explicatif. The modèle Rank-Dependent Expected Utility (RDU) (Quiggin 1982) tente de répondre à certaines de ces critiques. Le décideur est caractérisé par deux fonctions : une fonction d'utilité qui mesure la satisfaction des résultats dans le certain et une fonction de transformation des probabilités qui mesure le poids subjectif des probabilités. Nous montrons, dans ce papier, que le modèle RDU permet des comportements plus diversifiés : il est cohérent avec le paradoxe d'Allais ; le décideur peut ne pas aimer le risque (préférer à toute décision son espérance) sans nécessairement éviter tout accroissement de risque (à espérance constante) ; un décideur peut avoir une utilité marginale décroissante tout en ayant du goût « faible » pour le risque ; deux décideurs ayant la même fonction d'utilité peuvent avoir des comportements différents s'ils ont des fonctions de transformation des probabilités différentes ; de plus, le même décideur peut avoir des croyances subjectives sur les événements qui dépendent du contexte.

Mots clé: décision dans le risque, perception des risques, aversion pour le risque, paradoxe d'Allais, modèle Rank-Dependent Expected Utility.

Abstract

The classical expected utility model of decision under risk (von Neumann-Morgenstern, 1944) has been criticized from an experimental point of view (Allais' paradox) as well as for its restrictive lack of explanatory power. The Rank-Dependent Expected Utility model (RDU) model (Quiggin, 1982) attempts to answer some of these criticisms. The decision maker is characterized by two functions: a utility function on consequences measuring preferences over sure outcomes and a probability weighting function measuring the subjective weighting of probabilities. As we show and illustrate in this paper, this model allows for more diversified types of behavior: it is consistent with the behavior revealed by the Allais paradox; the decision maker could dislike risk (prefer to any lottery its expectation) without necessarily avoiding any increase in risk; diminishing marginal utility may coexists with "weak" risk seeking attitudes; decision makers with the same utility function may differ in their choices between lotteries when they have different probability weighting functions; furthermore, the same decision maker may have different, context-dependent, subjective beliefs on events.

Key words: Decision under risk, risk perception, risk aversion, Allais paradox, Rank-Dependent Expected Utility model.

*Classification-JEL : D81

Acknowledgement

I am most grateful to André de Palma, Johanna Etner, Meglena Jeleva, Isaac Meilijson, Benoit Menoni, Nicolas Roux and two anonymous referees for very helpful suggestions and comments.

1 Introduction

The Expected Utility (EU) model, introduced in the seminal work of von Neumann and Morgenstern (1944) is the classical model of decision under risk. The well known Allais paradox (Allais 1953) provides experimental evidence that the observed behavior of a majority of DMs is in contradiction with the EU model.

Further criticisms of the EU model concerns its lack of explanatory power:

(1) The same function u characterizes two distinct attitudes: attitude towards risk and attitude towards wealth (shape of the marginal utility) that have no a-priori reason to be identified with each other ; so, if a DM, adhering to the EU model, has a diminishing marginal utility (i.e., u is concave), then he must be risk averse.

(2) Theoretically, it could be possible that a DM avoids risk whenever possible, but faced with two risky situations, chooses the riskier one with the hope of a higher gain: Weak Risk Aversion (preferring to any random variable its expected value) needs not imply Strong Risk Aversion (aversion to mean preserving increases in risk). Under EU, Weak Risk Aversion forces u to be concave, and this, in turn, implies Strong Risk Aversion.

(3) Experimental findings (Slovic 1987) show that the probability of the same event can be perceived differently, depending on subjective properties of the DM and, for the same DM, on subjective characteristics of the decision problem encountered. These different ways of weighting probabilities cannot be taken into account in the EU framework.

(4) There are observed economic behaviors that can not be explained in the framework of the EU model as, for instance, the behavior of a policy holder, asking for complete insurance even though the premium is loaded.

The Rank-Dependent Expected Utility (RDU) model (Quiggin 1982, Yaari 1987, first version 1982) has been built, in part, as an attempt to answer to some of these criticisms.

The purpose of this paper is to illustrate how does the RDU model, which is more flexible than the EU model, accommodate the diversity of behaviors mentioned above. The paper is organized as follows: Section 2 introduces the general framework of a decision making problem under uncertainty and establishes some convenient notations. Section 3 introduces different concepts of increase in risk and risk aversion (risk seeking being the dual concept), as independently as possible of the exact model of decision under risk adopted. The treatment of these concepts under EU presented in Section 4, unduly restrictive, is contrasted in Section 5 by the flexibility of the RDU model in differentiating between the various concepts of risk aversion.

2 The decision problem under uncertainty: general presentation

Most of our decisions concerning, for instance, employment, investment, insurance, portfolio choice, health, transportation, quality of life or quality of our environment... are taken in an uncertain situation. We have to choose a decision without knowing its consequences with certainty because they depend on events that may or may not occur. Thus, the study of decision making needs, first of all, an appropriate formalization of decision problems under uncertainty.

We have first to define formally the set of decisions. We take as primitive a set of states of nature, S, representing the set of all elementary events that can occur, and a set $\mathcal{C}$ of all possible consequences for this problem. A decision is then a mapping from the set of states of nature S to the set of consequences $\mathcal{C}$. The consequence set is specific to a given decision problem: it can contain only monetary outcomes, but its elements can also correspond to consumption levels, employment levels, qualities of health or qualities of life, transportation times....The set of states of nature has to be described sufficiently precisely for the given decision problem and finally, the available information on events has to be identified in order to construct a "belief" on the events.

2.1 Typology of situations of uncertainty

In decision theory, we make a distinction between different types of information on events, subsets of S, set of states of nature.

The term "*risk*" is reserved to situations in which *all events of S have "objective" probabilities* with which the DM agrees (Knight, 1921). This is typically the case in games of chance, such as card games, roulette...; risk also encompasses all situations in which reliable statistical data are available: risk of car accident according to the type of driver and vehicle, risk of heart attack for a 55 year old person with high blood pressure, risk of flood....

All the other situations without such (probabilistic) information are called situations of *uncertainty*. Let us just mention different kinds of uncertainty :

(i) *Imprecise risk*: the probability of each event is only known to belong to an interval ; (ii) *Complete uncertainty*: we know all the elements of the set S but no further information on the events ; (iii) *Radical uncertainty*: the entire set of states of nature is not completely identified and the unknown states are denoted "unforeseen contingencies"[1].

In Bayesian decision theory, the Decision Maker acts under any type of uncertainty as if he could always summarize the uncertainty by a subjective probability and thus uses a model of decision under risk. However, it has been observed experimentally (Ellsberg, 1961) that individuals *dislike ambiguity*, meaning that

[1] Even though we will not study here these situations, let us just notice some examples that occured these last decades : Aids, Contamined blood, mad cow, 9/11, GMO, risks of cancer linked with the cellular phones ...., cases where the possibility of these events has not even been anticitaped.

they prefer a situation of risk to an uncertain one, and such a behavior is incompatible with Bayesian theory.

Nevertheless, in order to improve our understanding of behavior in uncertain situations, we will focus, in this paper, to a situation of risk.

From now on, we will assume that the probability distribution on the set S is given; we are thus dealing with a problem of decision under risk.

2.2 Given probability and perceived risk

It is well established now, since Slovic (1987), that there exists a difference between "given" probabilities (issues from frequencies or probabilistic calculus) and "perceived" probabilities that a DM uses to evaluate his decisions.

Risk perception appears to depend not only on the DM's characteristics but also, for the same DM, on the decision problem he is facing and on the context in which the decisions are taken.

In the following, we give some examples on DM's and decision problem's characteristics that have been pointed out as relevant for risk perception (see for example Slovic, 1987 ; Weber, 1997 ; Weber, Blais and Betz, 2002). Moreover the DM' characteristics like gender, culture, level of education, level of knowledge, differences between experts and non-experts, risk perception may depend on decision problem's or context's characteristics. For the same probability (Slovic, 1987), the "perceived" risk (considered here as danger) can depends on:

(i) whether the danger is controlled or not (chosen *vs* undergone danger like, for example, in a car's travel, driver *vs* passenger); (ii) whether there is equity or not about the consequences (nuclear wastes in a neighborhood where inhabitants have or not taken advantage of the nuclear power); (iii) whether there is catastrophic potential or not (car accident *vs* aircraft accident); (iv) whether there is reversibility of the danger or not (ski accident *vs* nuclear accident); (v) whether there is knowledge (or familiarity) of the danger or not (household accident *vs* nuclear accident);

Another factor, that may be more important than all those mentioned before, can influence the perceived probability: the link between beliefs and consequences. A DM does not treat a given probability the same way when corresponding to a "good" consequence or corresponding to a "bad" consequence ; the perception may also depend on the reference point (see for instance Kahneman and Tversky, 1979).

How to deal with the differences between given risk and perceived risk ?

Knowing that different decision makers with the same given information construct different subjective beliefs and different preferences, it is important, from an individual point of view as well as from a public policy (or an insurance company) point of view, to know how each DM constructs his preferences, using his risk perception and his attitude towards risk.

Knowing that, in the classical Expected Utility model, it is not possible to discriminate, for the same given probability, between different risk perceptions, how can we fill the gap between all these experimental findings and the classical model? How can we take into account such differences in risk perception?

We will try to prove that RDU model is a partial answer to these questions and we will show that, for a RDU Decision Maker, the probability weighting function φ can be a way of linking the DM perceived probability to consequences, and that all the other factors influencing the perception of risk can be modelled as additional parameters to the φ function.

2.3 Formalization and Notations

A general problem of decision making under uncertainty is formalized by introducing a set S called set of states of nature, identifying events to subsets of S, forming a σ-algebra $\mathcal{A}$. If we denote by $\mathcal{C}$ a set of consequences, a decision is then a mapping from $(S, \mathcal{A})$ to $\mathcal{C}$. We denote by $\mathfrak{X}$ the set of such mappings (set of decisions).

For a given decision problem, each DM has a preference relation denoted $\succsim$ on $\mathfrak{X}$ which is assumed to be a weak order (i.e. a reflexive, transitive and complete relation); the strict preference is denoted $\succ$ and the indifference $\sim$. This relation on $\mathfrak{X}$ *induces* (by the way of constant mappings) a preference relation on $\mathcal{C}$, also denoted by $\succsim$.

The purpose is to represent the DM's preferences $(\mathfrak{X}, \succsim)$ by a real-valued utility function, (i.e. a mapping V from $\mathfrak{X}$ to $\mathbb{R}$ such that: $X \succsim Y \iff V(X) \geq V(Y)$).

Since we are under *risk*, the set $(S, \mathcal{A})$ is equipped with a probability measure P. Any decision X of $\mathfrak{X}$ is then a random variable[2] and has then a probability distribution denoted P_X; Let F_X denote the cumulative distribution function of $P_X : F_X(x) = P\{X \leq x\}$ and $E(X)$ its expected value.

Under the classical assumption that two random variables with the same probability distribution are indifferent, the preference relation $\succsim$ on $\mathfrak{X}$ induces, on the set of probability distributions on $\mathcal{C}$, denoted $\mathcal{L}$, a preference relation also denoted $\succsim$. This assumption means, in fact, that the preference relation depends only on the consequences and their probabilities.

We also identify the consequence $c \in \mathcal{C}$ with the Dirac function δ_c at c $(\delta_c \in \mathcal{L})$ and we equivalently denote: $\delta_c \succsim P_X$ or $c \succsim P_X$ or $c \succsim X$.

Here, for the sake of simplicity, we assume that $\mathcal{C}$ is a *closed and bounded interval* of $\mathbb{R}$. In some sections, we limit our study to the set $\mathcal{L}_0$ of *finite* probability distributions on $\mathcal{C}$; generic element of $\mathcal{L}_0$ will be denoted $P_X = (x_1, p_1; ...; x_n, p_n)$ where the consequences are always *ranked with increasing order*: $x_1 \leq ... \leq x_n$, with $p_i \geq 0$ and $\sum p_i = 1$.

Moreover, we define F^{-1} from $(0,1]$ into $\mathbb{R}$ by $F^{-1}(p) = \inf\{z \in \mathcal{R} | F(z) \geq p\}$, the highest gain among the least favorable $p\%$ of the outcomes.

[2] We assume here all random variables possess properties of measurability.

3 Attitude towards risk

Risk aversion has two aspects: preference for certainty when it is possible to avoid any risk, and preference for reduction of risk when it is impossible to completely eliminate risk.

Thus, the first notion of risk aversion corresponds to a propensity to choose, when possible, to avoid risk, or more precisely, to always prefer to any random variable X the certainty of its expected value $E(X)$. However, when it is not possible to completely eliminate the risk, this notion is not sufficient to compare the riskiness of two random variables.

Aversion to increase in risk is more delicate to define because many different concepts of increase in risk that can be proposed. To any type of increase in risk will be associated a specific definition of risk aversion (corresponding to aversion to this type of increase in risk).

Let us first give two notions of increase in risk.

3.1 Definition and properties of different notions of increase in risk

In this section, to define a partial order on random variables (namely, Y is an increase in risk of X), and focus our attention to the riskiness of these random variables, we will only compare random variables with the *same mean*.

For X and Y with the same mean, Y is a general mean preserving increase in risk or *Mean Preserving Spread (MPS)* of X if $\int_{-\infty}^{t} F_Y(x)dx \geq \int_{-\infty}^{t} F_X(x)dx$ for all $t \in \mathbb{R}$.

This classical notion, special case, for equal means, of second order stochastic dominance, was introduced in economics by Rothschild and Stiglitz (1970). A more intuitive characterization is that the more risky Y is obtained by adding a noise (random variable independent of X and with 0 mean) to X.

Rothschild and Stiglitz themselves (1971) showed that there exist many economic situations where their notion of increase in risk does not seem to fit well to the problem. For example, let us assume $Y\ MPS\ X$ and two EU decision makers such that D2 is more risk averse than D1 and D1 is ready to pay c to exchange Y for the less risky X : in a very counter-intuitive way, it can be the case that D2, more risk averse than D1, is ready to pay *less than* c for the same exchange.

Some of these counter-intuitive examples gave rise to other notions of increase in risk (see Jewitt 1989 ; Lansberger and Meilijson, 1994 ; Chateauneuf and alii, 2004). In particular, Quiggin (1992) defined a more demanding notion of increase in risk, more fitted with many economic problems[3], where, for instance, the above counter-intuitive result can no more be possible :

[3] See for instance Landsberger and Meilijson (1994), Chateauneuf and alii (2005).

For X and Y with the same mean, Y is a *Monotone Mean Preserving Spread (MMPS)* with respect to X if $F_Y^{-1}(q) - F_Y^{-1}(p) \geq F_X^{-1}(q) - F_X^{-1}(p)$ for all $0 < p < q < 1$.

The interpretation of this formula is that all the *interquantile intervals are shorter for* X than for Y[4]. With this interpretation, no one could deny that Y is more risky than X (being more dispersed everywhere). The following table shows examples of these different increases (reductions) of risk.

Table 1 : Examples of different concepts of increase (reduction) in risk

Proba	1/5	1/5	1/5	1/5	1/5	
Y	-2000	-1000	0	1000	2000	*Initial risk*
$X1$	-2000	0	0	0	2000	*Rothschild − Stiglitz reduction of risk*
$X2$	-1250	-500	0	500	1250	*Monotone reduction of risk*
$X3$	0	0	0	0	0	*Complete reduction of risk*

It can easily be proved that $Y\ MPS\ X_1$ and $Y\ MPS\ X_2$ but only $Y\ MMPS\ X_2$.

Let us try to add more justification of this new concept of monotone increase in risk: looking at the probability distribution in Table 1 as an income distribution, then, exchanging Y for X_2 can always be viewed as a reduction of inequalities, whereas exchanging Y for X_1, can no more be viewed as a reduction of inequalities and at least for the poorest one, could even be viewed as an increase in inequalities.

Remark 1 *Let us just mention that there exist also other definitions of increasing risk (reducing risk) that do not treat symmetrically good and bad results like the concepts of downside risk (see C Menezes and alii, 1980) or left monotone increase in risk (Jewitt, 1989 ; Landsberger and Meilijson, 1994 ; Chateauneuf and alii, 2004).*

3.2 Model-free concepts of risk aversion, risk seeking

For a DM with a preference relation $\succsim$ on $\mathfrak{X}$, we can now give three model-free concepts of risk aversion and their precise definitions.

- A DM has *Weak Risk Aversion (WRA)* if, for any random variable X of $\mathfrak{X}$, he prefers to the random variable X, its expected value $E(X)$ with

[4] More details on this notion can be found in Chateauneuf and alii (2005).

certainty: $\forall X \in \mathfrak{X},\ E(X) \succsim X$; has *Weak Risk-Seeking(WRS)* if $\forall X \in \mathfrak{X},\ X \succsim E(X)$; is risk-neutral if $\forall X \in \mathfrak{X}, X \sim E(X)$.

Each other concept of risk aversion will now be defined as aversion to a particular concept of increase in risk.

- A DM has *Strong Risk Aversion (SRA)*, if, for any pair of random variables X, Y in $\mathfrak{X}$ with Y being a Mean Preserving Spread of X, he always prefers X to Y: $\forall X, Y \in \mathfrak{X},\ Y\ MPS\ X \implies X \succsim Y$; has *Strong Risk-Seeking(SRS)* if $\forall X, Y \in \mathfrak{X},\ Y\ MPS\ X \implies Y \succsim X$; is risk-neutral if $X \sim Y$.

Intuitively, these two notions capture distinct behaviors: a DM may want to avoid completely risk when possible, but when he cannot do so and has to choose between two situations where he cannot avoid risk, he could choose the riskier one, hoping to get the best consequences.

The third concept is based on aversion to monotone increases in risk :

- A DM has *Monotone Risk Aversion (MRA)*, if, for any pair of random variables X, Y of $\mathfrak{X}$ with Y Monotone Mean Preserving Spread (MPS) of X, he always prefers X to Y: $\forall X, Y \in \mathfrak{X},\ Y\ MMPS\ X \implies X \succsim Y$; has *Monotone Risk-Seeking (MRS)* if $\forall X, Y \in \mathfrak{X},\ Y\ MMPS\ X \implies Y \succsim X$; is risk-neutral if $X \sim Y$.

The notion of Weak Risk Aversion was defined by Arrow and Pratt (1961). If we compare *Weak Risk Aversion* and *Strong Risk Aversion*, we can notice that, since for any X, X is a mean preserving spread of $E(X)$, *Strong Risk Aversion* implies *Weak Risk Aversion*. The reciprocal implication is not true in general.

More generally, it can be proved that[5]:

$$Strong\ Risk\ Aversion \Longrightarrow Monotone\ Risk\ Aversion \Longrightarrow Weak\ Risk\ Aversion$$

while the reciprocal assertions are not necessarily true.

4 The Expected Utility Model

4.1 General presentation

The classical normative model (due to von Neumann and Morgenstern, 1944), the Expected Utility (EU) model, is based on simple and appealing axioms and

[5] See for example Lansdberger and Meilijson (1994) or Chateauneuf and alii (2004).

has important properties[6]. Let us recall briefly the value function representing the preferences in this axiomatic model.

For a decision $P_X = (x_1, p_1; ...; x_n, p_n)$, preferences can be represented by a *functional* U, such that :

$U(P_X) = E_P(u(X)) = \sum_{i=1}^{n} p_i u(x_i)$ where u is a mapping from $\mathcal{C}$ to $\mathbb{R}$ defined by $u(x) = U(\delta_x)$ strictly increasing and unique up to an affine increasing weighting.

In this formula, the functional U is *linear in probabilities*. This formula can also be written as:

$$U(P_X) = u(x_1)+..+(\sum_{j=i+1}^{j=n} p_j)[u(x_{i+1})-u(x_i)]+\cdots+(p_n)[u(x_n)-u(x_{n-1})] \quad (1)$$

The best decision is the one maximizing this Expected Utility. Let us notice that a DM, who behaves in accordance to the EU model, is completely characterized by this unique function u ; thus, EU is a very parsimonious model.

4.2 Characterization of different notions of risk aversion in the EU model

In this model, it has been proved, since 1970, by Rothschild and Stiglitz that:

(i) Weak Risk Aversion is equivalent to the concavity of u;

(ii) Strong Risk Aversion is equivalent to the concavity of u.

Thus, these two results imply that, in the *framework of EU model*,

$$\textit{Strong Risk Av.} \Longleftrightarrow \textit{Monotone Risk Av.} \Longleftrightarrow \textit{Weak Risk Av.} \Longleftrightarrow \text{Concavity of } u$$

This means that, in the EU model, as soon as the DM has Weak Risk Aversion, he has necessarily Monotone Risk Aversion and Strong Risk Aversion. Thus, the EU model cannot take into account some diversified behaviors like having Weak but not Strong Risk Aversion.

4.3 Discussion on the EU model

4.3.1 Advantages

This classical model has many nice properties. The EU model has axiomatic foundations; the axioms are simple and appealing. Moreover, Bayes rule provides an intuitive updating rule and the linearity in probabilities guarantees dynamic consistency which allows the treatment of decision trees by dynamic programming.

Despite its numerous advantages, the EU model has been questioned for not taking into account, in many situations, the DM's observed behavior.

[6] For an axiomatic presentation of the model, and all its properties, see for instance Kreps (1988).

4.3.2 Drawbacks

Allais' paradox From an experimental point of view, the well-known Allais' paradox has pointed out, since 1953, that real behavior of a majority of DMs is in contradiction with the EU model. Let us first recall this Allais paradox (Allais, 1953) that gave rise to the new models of behavior under risk. In fact, we expose an example from Kahneman and Tversky (1979) similar to Allais paradox. Subjects are requested to express their preferences first between these two prospects:

- A: winning $3,000 with probability 1 *or* B : winning $4,000 with probability 0.8

and then, *independently*, between the two prospects :

- C : winning $3,000 with probability 0.25 *or* D : winning $4,000 with probability 0.20.

Typical preferences are $A \succ B$ *and* $D \succ C$. Such preferences are in contradiction with the EU model: indeed, setting, without lack of generality, $u(4000) = 1$ and $u(0) = 0$ and $u(3000) = \alpha$, $0 < \alpha < 1$, $A \succ B$ implies, in the EU model, $\alpha > 0.80$ whereas $D \succ C$ implies $0.25\alpha < 0.20$, a contradiction.

This type of experiment has been conducted many times with heterogeneous samples, in different countries and each time, about 2/3 of the subjects gave the choices A and D.

Other drawbacks There are more criticisms concerning the lack of explanatory power of the EU model.

(1) As we already mention in the introduction, the EU model also raised a theoretical difficulty, namely, the interpretation of the function u (Bernoulli's utility function) characterizing the decision-maker's behavior. Indeed, as pointed out by Allais, the function u has a double role of expressing the DM's attitude with respect to risk (concavity of u implying risk aversion) and the DM's valuation differences under certainty (concavity of u implying then diminishing marginal utility of wealth), so that, for instance, it is impossible to explain, in this model, the behavior of a DM having at the same time a diminishing marginal utility for certain wealth and a risk seeking attitude.

(2) In the same way, the attitude of a DM having Weak Risk Aversion without having automatically Strong Risk Aversion is not allowed in the framework of EU model.

(3) As mentioned in the introduction, the probability of the same event can be perceived differently, depending on many subjective characteristics of the decision problem and also many subjective characteristics of the DM. These different ways of "weighting" probabilities, again, cannot be taken into account in the EU framework where the DM is always supposed to use the same "given" probability distribution.

There are many economic illustrations of the rigidity of the model as explained, for instance, in Rothschild and Stiglitz (1971). As we already mention in the introduction, since, in the EU model, it is never optimal to take a complete insurance coverage when the premium is loaded, this EU model cannot explain why many policy holders buy complete insurance coverage, knowing that, in insurance companies, the premium is always loaded[7].

For these reasons, even if the EU model has the advantage to be parsimonious (with only one function u to assess), since many observed economic behaviors cannot be explained in the framework of this model, we will present, in the next section, the Rank-Dependent Expected Utility (RDU) model (Quiggin, 1982), a more general model, less parsimonious but more explanatory, built, in part, as an attempt to answer to some of the criticisms of the EU model.

5 The Rank Dependent Expected Utility model

5.1 General presentation

The Rank Dependent Expected Utility (RDU) model is due to Quiggin (1982) under the denomination of Anticipated Utility. Variants of this model are due to Yaari (1987, first version, 1982), and Allais (1988). More general axiomatizations can be found in Wakker (1994), Chateauneuf (1997).

A DM behaves in accordance with the Rank-Dependent Expected Utility (RDU) model if the DM's choices between decisions are characterized by two functions u and φ:

(i) a continuous, increasing utility function u: $\mathcal{C} \rightarrow \mathbb{R}$ (that plays the role of *utility on certainty*) and

(ii) an increasing *probability weighting* function $\varphi : [0,1] \rightarrow [0,1]$ that satisfies $\varphi(0) = 0, \varphi(1) = 1$.

Such a DM prefers the decision X to the decision Y if and only if

$V(X) \geq V(Y)$, where the *functional* V is given by :

$$V(X) = u(x_1) + .. + \varphi(\sum_{j=i+1}^{j=n} p_j)[u(x_{i+1}) - u(x_i)] + \cdots + \varphi(p_n)[u(x_n) - u(x_{n-1})] \quad (2)$$

$V(X)$ happens to be a Choquet integral (Choquet 1953).

We can interpret u as the satisfaction of the consequences and φ as the way the DM transforms the decumulative probabilities.

Let us give an interpretation of this valuation. The DM takes for sure the utility of the worst outcome $u(x_1)$ and weights the additional possible increases

[7] Let us mention another example: In the theory of portfolio choice, the observed behavior of a DM, putting all his wealth in the riskless asset, even though the expected value of the risky asset's rate is greater than the safe rate, is incompatible with the EU model.

of utility $[u(x_i)-u(x_{i-1})]$ by his personal weighting $\varphi(\sum_{j=i+1}^{j=n} p_j)$ of the probability $\sum_{j=i+1}^{j=n} p_j$ of getting *at least* x_i.

According to this interpretation of the formula, if the DM is such that $\varphi\ (p) \leq p$ for all p in $[0,1]$, comparing the formula giving respectively EU and RDU functionals, it means that this DM underweights all the additional utilities of gains. Thus, we give the following definitions.

- A RDU DM with a function φ such that $\forall p$ in $[0,1]$, $\varphi\ (p) \leq p$, is called a *pessimist under risk*.
- A RDU DM with a function φ such that $\forall p$ in $[0,1]$, $\varphi\ (p) \geq p$, is called an *optimist under risk*.

Let us finally notice that, if the weighting function φ is the identity function $\varphi(p) \equiv p$, formula 1 and 2 are then identical, $V(X)$ reduces to the expected utility of the random variable.

5.2 Evaluating a simple decision (EU *vs* RDU)

A very simple example can be used to show the role of a probability weighting function φ. Let X be a simple decision giving consequence x_1 with probability $1-p$ and x_2 with probability p.

Valuation of X in a EU model: $U(X) = (1-p)u(x_1) + pu(x_2)$

Valuation of X in a RDU model:

$$\begin{cases} RDU(X) = [1-\varphi(p)]u(x_1) + \varphi(p)u(x_2) & if\ x_1 \leq x_2 \\ RDU(X) = \varphi(1-p)u(x_1) + [1-\varphi(1-p)]u(x_2), & if\ x_1 \geq x_2 \end{cases}$$

If φ is different from identity, the DM will not evaluate the same event with the same weight depending on whether the event is favorable or unfavorable. If the DM is pessimistic under risk, he always underweights the probability of the best outcome and overweights the probability of the worse outcome.

Numerical example: $S = \{s_1, s_2\}$. S has two elements with respective probabilities $2/10$ and $8/10$.

The following table gives, in the two first columns, directly *the utilities of the outcomes* for the two decisions X and Y according to which state occur, and in the three next columns, the valuation of these two decisions in *EU* and in two types of *RDU* models.

	$2/10$	$8/10$	EU	$RDU(pess)$ $\varphi(p)=p^2$	$RDU(opt)$ $\varphi(p)=\sqrt{p}$
X	-1000	0	-200	-360	-105.6
Y	1000	0	200	40	447

- Valuation in EU: $U(X) = -200$; $U(Y) = 200$
- Valuation in RDU with $\varphi(p) = p^2$,for p in$[0, 1]$, revealing *pessimism* under risk since $0 \leq p^2 \leq p \leq 1$.

We evaluate $\varphi(0.20) = 0.04; 1 - \varphi(0.20) = 0.96; \varphi(0.80) = 0.64; 1 - \varphi(0.80) = 0.36$; For such a pessimist DM, when 0 corresponds to an unfavorable event, its probability is over-estimated whereas the same probability is under-estimated when 0 corresponds to a favorable event.

Since $-1000 < 0$, $RDU(X) = -1000 * 0.36 + 0 * 0.96 = -360$; since $0 < 1000$, $RDU(Y) = 0 * 0.96 + 1000 * 0.04 = 40$.

- The last column of the table corresponds to the values of a RDU DM with $\varphi(p) = \sqrt{p}$, for p in$[0, 1]$, revealing *optimism* under risk, since $0 \leq p \leq \sqrt{p} \leq 1$.

The probability weighting function φ depends thus on the DM. Moreover, if we want to model the fact that risk perception depends also on the context in which the decisions are taken, for a DM having a utility function u, his probability weighting function φ could depend on a parameter $\varphi_s(.)$ where s characterizes the context (section 5.5.2.).

5.3 Characterization of different notions of risk aversion in the RDU model

The different concepts of risk aversion, while equivalent in EU theory, have different characterizations in the RDU model. However, there is a common necessary condition for the three different concepts of Risk Aversion:

$$Pessimism\ under\ risk: \ \varphi(p) \leq p, \text{ for } p \text{ in}[0, 1].$$

5.3.1 Strong Risk Aversion/Risk-Seeking

Chew, Karni and Safra (1987) found an appealing characterization of Strong Risk Aversion.

A RDU decision maker, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ, has Strong Risk Aversion (SRA) if and only if his weighting function φ is convex and his utility function u is concave.

This result shows that Strong Risk Aversion cannot be disentangled from diminishing marginal utility, even in the framework of RDU model, more flexible than EU model.

We have exactly the same kind of characterization for the dual notion of Strong Risk Seeking.

A RDU decision maker, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ, has Strong Risk-Seeking (SRS) if and only if his weighting function φ is concave and his utility function u is convex (Chew, Karni and Safra, 1987).

5.3.2 Monotone Risk Aversion/Risk seeking

For the case of Monotone Risk Aversion, assuming differentiability of u, the characterization is based on two indexes:

Index of *non-concavity* (or *greediness*) : $G_u = \sup_{y<x} u'(x)/u'(y)$

This index satisfies $G_u \geq 1$ but the value 1 corresponds exclusively to concavity of u, meaning that $G_u > 1$ for any non-concave function u.

Index of *pessimism* under risk *:* $P\varphi = \inf_{0<p<1} [\frac{1-\varphi(p)}{\varphi(p)} / \frac{1-p}{p}]$

This index satisfies ≥ 1 as soon as $\varphi(p) \leq p$; moreover, the more pessimistic the DM, the greater P_φ : $\varphi(p) \leq \phi(p$, implies $P_\varphi \geq P_\phi)$. This index is irrelevant if there exists p such that $\varphi(p) > p$ (relevant only for a pessimist). Let us give an possible interpretation of this index : For a probability p of winning, $\frac{1-p}{p}$ is the odds-ratio against winning. Pessimists exaggerate this odds ratio by amplifying it to $\frac{1-\varphi(p)}{\varphi(p)}$. The index of pessimism can be intuitively understood as the minimal such amplification factor.

On the basis of these 2 indexes, we have the following characterization :

A RDU DM, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ, is Monotone Risk Averse (MRA) if and only if his index of pessimism under risk exceeds his index of non-concavity (Chateauneuf and alii, 2005).

$$MRA \iff P_\varphi \geq G_u$$

The idea of this theorem is the following : it is possible to have monotone risk aversion with a non-concave u provided that the more non-concave u is, the more the DM is pessimist under risk.

For Monotone Risk Seeking, we also need two dual indexes:

Index of *non-convexity :* $T_u = \sup_{y<x} \frac{u'(y)}{u'(x)}$. This index always satisfies $T_u \geq 1$, and the value 1 corresponds exclusively to convexity.

Index of *optimism* under risk : $O_\varphi = \inf_{0<v<1} [\frac{\varphi(p)}{1-\varphi(p)} / \frac{p}{1-p}]$. This index satisfies $O_\varphi \geq 1$ as soon as $\varphi(p) \geq p$.

This index is irrelevant if there exists p such that. $\varphi(p) < p$ (relevant only for an optimist).

On the basis of these 2 indexes, we have the following characterization :

A RDU DM, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ, is Monotone Risk Seeking (MRS) if and only if his DM's index of optimism exceeds his DM's index of non-convexity: $O_\varphi \geq T_u$ (Chateauneuf and alii, 2005).

$$MRS \iff O_\varphi \geq T_u$$

With the same dual interpretation : it is possible to have monotone risk seeking with a non-convex u provided that the more non-convex u is (may be concave), the more the DM is optimist under risk.

We will see in next section, examples of such attitudes.

5.3.3 Weak Risk Aversion/Risk-Seeking

As we already mentioned, a necessary condition for Weak Risk Aversion is $\varphi(p) \leq p$, $\forall p \in [0,1]$. Here, we have only sufficient conditions *(Chateauneuf and Cohen, 1994).*

Let a RDU DM, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ*. If* $\exists k \geq 1$ *satisfying the 2 following conditions :*

(i) $u\prime(x) \leq k\frac{u(x)-u(y)}{x-y}$, $y < x$ *and*

(ii) φ *is such that* $\varphi(p) \leq p^k$, $\forall p \in [0,1]$

then, this DM has Weak Risk Aversion (WRA).

The interpretation can be the following : (1) If $k = 1$, the 2 conditions reduce to u concave and $\varphi(p) \leq p$ (pessimism) ; (2) if $k > 1$, u is not concave (can be convex) and again, the larger k (meaning the more non-concave u), the more pessimist the DM has to be.

We have similar results for Weak Risk Seeking. A necessary condition for Weak Risk-Seeking is $\varphi(p) \geq p$, $\forall p \in [0,1]$. Then, we have sufficient conditions.

Let a RDU DM, characterized by a differentiable and increasing utility u and an increasing probability weighting function φ*. If* $\exists h \geq 1$ satisfying the 2 following conditions :

(i) $u\prime(y) \leq h\frac{u(x)-u(y)}{x-y}$, $y < x$ *and*

(ii) φ *is such that* $\varphi(p) \geq 1-(1-p)^h$, $\forall p \in [0,1]$,

then, this DM has Weak Risk-Seeking (WRS) (Chateauneuf and Cohen, 1994).

Although there are only sufficient conditions, the key point is that a DM can have Weak Risk Aversion without having a concave u (for instance, can have a convex u) and the DM can have *Weak Risk Seeking with a concave u interpreted as a diminishing marginal utility.*

Moreover, for two DMs with the same utility function u, with convex and concave parts, one can have Weak Risk Aversion, if sufficiently pessimist and the other can have Weak Risk Seeking, if sufficiently optimist (see examples in the next section).

5.4 Some examples

Let us exhibit some examples of diversified attitudes towards risk in the RDU model, where, in particular, different DMs with the same function u, can exhibit different behaviors according to their different φ functions.

5.4.1 Examples of functions u and φ and their indexes

For reasons of simplicity, we assume that $\mathcal{C} = [0,1]$. We propose first some functions u in Table 2 and φ in Table 3 and their respective indexes.

Table 2 : Some examples of u functions and their indexes

u		*Shape of u*	G_u	T_u
u_1	$1-e^{-ax}$	*concave*	1	e^a
u_2	e^{ax}	*convex*	e^a	1
u_3	x^3-x^2+x	$S-shape$	3	3/2
u_4	x^2	*convex*	∞	1
u_5	$1-(1-x)^2$	*concave*	1	∞

Table 3 : Some examples of φ functions and their indexes

φ		*shape of* φ	$P\varphi$	$O\varphi$
φ_1	p^3	$\varphi_1(p) \leq p$ *and convex*	3	*irrel.*
φ_2	$\frac{p}{p+(1-p)\theta}$ $\theta \geq 1$	$\varphi_2(p) \leq p$ *and convex*	θ	*irrel.*
φ_3	$1-(1-p)^3$	$\varphi_3(p) \geq p$ *and concave*	*irrel.*	3

Examples of some diversified attitudes towards risk in RDU In the following table, we show how different combinations of the previous utilities and probability weighting functions illustrate different types of risk aversion/risk seeking in the RDU model.

Table 4 : Examples of DM' attitudes

		SRA	MRA	WRA	SRS	MRS	WRS
$DM1$	u_1, φ_1	yes	yes	yes			
$DM2$	u_3, φ_1	no	yes	yes			
$DM3$	u_2, φ_2	no	if $e^a \leq \theta$	if $e^a \leq \theta$			
$DM4$	u_4, φ_1	no	no	yes			
$DM5$	u_4, φ_3				yes	yes	yes
$DM6$	u_3, φ_3				no	yes	yes
$DM7$	u_1, φ_3				no	if $e^a \leq 3$	if $e^a \leq 3$
$DM8$	u_5, φ_3				no	no	yes

These examples show that:

(i)A RDU DM, with a unique marginal utility with diminishing parts and increasing parts (like u_3, in the examples), can have Monotone Risk Aversion (thus Weak Risk Aversion), if sufficiently pessimistic (like for DM2 with (u_3,φ_1)) and Monotone Risk-Seeking (thus Weak Risk-Seeking), if sufficiently optimistic (like for DM6 with (u_3, φ_3)).

(ii) Weak risk Aversion does not imply necessarily Monotone Risk Aversion like for DM4 and similarly, Weak Risk-Seeking does not imply necessarily Monotone Risk-Seeking like for DM8. The properties of the two last examples are proved in Chateauneuf, Cohen and Meilijson (2006). However, in the same paper, they show that, in fact, there is few room for Weak Risk Averse DMs that are not Monotone Risk Averse, so that Monotone Risk Aversion can capture most of the Weak Risk Averse attitudes.

5.5 Adequation of observed behavior to the RDU model

This RDU model under risk, will not only prove to be compatible with observed behaviors in Allais' paradox, but also, in some cases, compatible with many observed attitudes, already mentioned in the section 4.3.2. that are not explainable in the EU model.

5.5.1 Explanation of Allais' paradox

It can be easily proved that Allais' paradox is explainable by RDU theory.

In the example given in section 4, any DM choosing simultaneously A and D conform with the RDU model, as soon as his φ satisfies:

$$\varphi(0,8) = \varphi(\frac{0,20}{0,25}) < \frac{\varphi(0,20)}{\varphi(0,25)}$$

Indeed, with the normalization $u(4000) = 1, u(0) = 0$ and denoting $\alpha = u(3000)$, we have $V(A) = \alpha; V(B) = \varphi(0,8); V(C) = \alpha\varphi(0,25); V(D) = \varphi(0,20)$ and thus, $A \succ B$ and $D \succ C$, as soon as $\alpha > \varphi(0,8)$ and $\alpha\varphi(0,25) < \varphi(0,20)$, which gives the result above.

5.5.2 Adequation of other observed behaviors

RDU model can thus answer to several criticisms raised by the EU model :

(1) In the examples, we have seen that the RDU model can explain the behavior of a Monotone Risk-Seeking (or Weak Risk-Seeking) DM with a diminishing marginal utility: see, for instance the behavior of DM7 where u is concave (with $e^a \leq 3$) and φ reveals optimism.

(2) We have seen, in the examples, that a DM can have Weak Risk Aversion without having Strong Risk Aversion: see, for instance the behavior of DM2 or DM4.

(3) RDU model is constructed to take into account the link between perceived probabilities and consequences. The intensity of variation between given probabilities and perceived probabilities are well taken into account by the probability weighting function φ. Moreover, to take into account that, for the same probability, perception of risk can depend on the decision problem the DM is facing, the function φ can depend on a parameter s, indicating the influence of the context, past experience, framing, feelings of the DM.... For example, if s indicates the level of the danger, φ_s can be more (less) optimist if the danger is more (less) controlled ; if s indicates the level of knowledge or familiarity of the danger, again, φ_s can be more (less) optimist if the danger is more (less) familiar. A formalization of attitudes characterized by (u, φ_s), where s indicates the past-experience, can be found in Cohen, Etner, Jeleva (2006).

(4) Finally, let us just mention, among others, an observed economic behavior explained in the framework of RDU: it can be optimal for a policy holder to buy a complete insurance coverage, even when the premium is loaded, if he is sufficiently *pessimistic under risk*.

5.6 Assessment of the different parameters

In a perspective of Decision Aiding, the next step to help each Decision Maker to take an optimal decision is to assess his different parameters in the RDU model.

For each DM and each decision problem, we have to assess the two functions u and φ. Luckily, an EU DM is a RDU DM whose probability weighting function

φ is the identity. We thus can use the same experimental method for all DMs. Unfortunately, the assessment of the two parameters u and φ are linked and we need a sequential experiment to assess simultaneously the two functions. There exists sophisticated methods to achieve this goal (Wakker and Deneffe, 1996).

From an empirical point of view, the perception function φ has been assessed, first at a global level, by, among others, Abdellaoui (2000), Tversky and Wakker (1995), Wu and Gonzalez (1996), and axiomatically justified by Prelec (1998).

Then, experimental studies have been conducted, *at an individual level*, to assess the two functions of the DM in different types of consequences and context: monetary gains in Abdellaoui (2000), risks of monetary losses with different levels of losses in Etchard-Vincent (2004), risk of bad consequences of medical decisions in Bleichrodt and Pinto (2000), probabilities based on QALY (Quality Adjusted Life Year) in Bleichrodt and Quiggin ((1997), risk of nuclear accidents in nuclear power plant maintenance in Beaudouin, Munier and Serquin (1999). They show not only heterogeneity in individual preferences,but also heterogeneity according to the different contexts of the experiment.

The last study (De Lapparent, 2006) concerns the risk of time loss in travel activity (for air route choices) and it is an empirical one, based on a large data base. De Lapparent (2006) prove that (i) travellers choices are not compatible with EU theory but compatible with RDU theory (ii) that travellers are optimistic under risk ($\varphi(p) \geq p$) which is interesting because this optimism can be interpreted as a signal of trust that passengers have, concerning their airline company.

6 Concluding remarks

We have seen that RDU model accounts for many observed behaviors under risk. However, this model has a drawback: it is not dynamically consistent.

A solution to deal with a dynamic model with RDU static preferences is to use a recursive model a la Kreps-Porteus. Moreover, in such a recursive model, we can take into account the possible evolution, during time, of the risk perception, more particularly the influence that history of each DM can have on this perception. Cohen, Etner, and Jeleva [2006] have studied such a model and their illustration by an insurance problem shows that this dynamic model can explain changes in behavior about insurance coverage, observed in real behaviors, that are not explainable in the other classical models.

References

[1] Abdellaoui, M.(2000). Parameter-free elicitation of utilities and probabilities weighting functions, *Management Science*, 46, 11, 1497-1512.

[2] Allais, M.(1953). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine, *Econometrica*, **21**, 503–546.

[3] Allais, M.(1988)The general theory of random choices in relation to the invariant cardinal utility function and the specific probability function, in *Risk, Decision and Rationality*, B. R. Munier (Ed.), Reidel: Dordrech, 233–289.

[4] Beaudouin, F., B. Munier et Y. Serquin, (1999). Multi-Attribute Decision Making and Generalized Expected Utility in Nuclear Power Plant Maintenance , in: M. Machina, B. Munier, eds., *Beliefs, Interactions and Preferences in Decision Making*, Dordrecht/Boston, KAP.

[5] Bleichrodt, H. and J. Quiggin (1997). Characterizing QALYs under a General Rank Dependent Utility Model*Journal of Risk and Uncertainty* Volume 15, 2.

[6] Bleichrodt, H. and JL. Pinto (2000). Parameter-Free Elicitation of Utility and Probability Weighting Functions*Management Science*, Volume 46, 11.

[7] Chateauneuf, A. (1999). On the use of comonotony in the axiomatization of EURDP theory for arbitrary consequences*Journal of Mathematical Economics*, 32.

[8] Chateauneuf, A. and M. Cohen (1994). Risk seeking with diminishing marginal utility in a non-expected utility model, *Journal of Risk and Uncertainty*, **9**, 77–91.

[9] Chateauneuf, A., M. Cohen and I. Meilijson, (2004). Four notions of mean preserving increase in risk, risk attitudes and applications to the Rank-Dependent Expected Utility model , *Journal of Mathematical Economics*, 40, p 547-571.

[10] Chateauneuf, A., M. Cohen and I. Meilijson, (2005).More pessimism than greediness: A characterization of monotone risk aversion in the Rank Dependent Expected Utility model , *Economic Theory*, 25, 649-667.

[11] Chateauneuf, A., M. Cohen and I. Meilijson, (2006).Weak risk aversion and the quest for simplicity: A characterisation of the preference for safety in the RDU model. WorkingPaper.

[12] Chew, S., E. Karni and Z., Safra (1987). Risk aversion in the theory of expected utility with Rank Dependent preferences, *Journal of Economic Theory*, **42**, 370–381.

[13] Choquet, G, (1953). Theorie des capacités, *Ann. Institut Fourier* (Grenoble), **V**, 131-295.

[14] Cohen, M. , J. Etner and M. Jeleva , (2007). Dynamic Decision Making when Risk Perception Depends on Past Experience, to appear in *Theory and Decision*.

[15] de Lapparent, M. (2006) Optimism, pessimism towards risk of time losses in business air travel demand, to appear in *Intelligent Transportation Systems*.

[16] Ellsberg, D., (1961). " Risk, Ambiguity, and the Savage Axioms", *The Quarterly Journal of Economics*, p643-669.

[17] Etchart-Vincent, N. (2004).Is Probability Weighting Sensitive to the Magnitude of Consequences ? An Experimental Investigation on Losses,*Journal of Risk and Uncertainty*, vol. 28(3).

[18] Jewitt, I., (1989). Choosing between risky prospects: the characterisation of comparative static results and location independent risk. *Management Science*, **35**, 60–70.

[19] Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decisions under risk. *Econometrica*, 47, 263-291.

[20] Knight, F (1921). Risk, Uncertainty and Profit., University of Chicago Press.

[21] Kreps, D. (1988). *Notes on the theory of choice*. Boulder and London, Westview Press.

[22] Landsberger, M. and I. Meilijson.(1994). Comonotone allocations, Bickel-Lehmann dispersion and the Arrow-Pratt measure of risk aversion, *Annals of Operations Research*, **52**, 97–106.

[23] Landsberger, M. and I. Meilijson, (1994). The generating process and an extension of Jewitt's location independent risk concept. *Management Science*, **40**, 662–669.

[24] C Menezes, C Geiss and J. Tressler (1980). Increasing Downside Risk, *American Economic Review*, vol. 70, n° 5, p 921-932.

[25] Prelec, D. (1998) The probability weighting function, *Econometrica*, 66, 3, p497-527.

[26] Quiggin, J.(1982) A theory of anticipated utility,*Journal of Economic Behavior and Organisation*, **3**, 323–343.

[27] Quiggin, J.(1992) Increasing risk: another definition, In *Progress in Decision, Utility and Risk Theory*, A. Chikan (Ed.), Dordrecht: Kluwer.

[28] Rothschild, M. and J. Stiglitz (1970). Increasing Risk: I. A definition, *Journal of Economic Theory*, **2**, 225–243.

[29] Rothschild, M. and J. Stiglitz (1971).Increasing Risk II: Its economic consequences, with J.E. Stiglitz *Journal of Economic Theory*, 3,1.

[30] Slovic, P.(1987) Perception of Risk. *Science,* 236, p 280-285.

[31] Tversky, A. and P. Wakker.(1995) Risk attitudes and decision weights, *Econometrica*, 63, 6, p1255-1280.

[32] von Neumann J. and O. Morgenstern (1944). Theory of Games and Economic Behavior. New York: John Wiley and Sons.

[33] Wakker, P.(1994) Separating marginal utility and risk aversion, *Theory and Decision*, **36**, 1–44.

[34] Wakker, P. P., D. Deneffe. (1996). Eliciting von Neumann-Morgenstern utilities when probabilities are distorted or unknown. *Management Science*, 42, 1131–1150.

[35] Weber, E. U. (1997). The utility of measuring and modeling perceived risk. In A. A. J. Marley (Ed.), Choice, Decision, and Measurement: Essays in Honor of R. Duncan Luce (p45-57). Mahwah, NJ: Lawrence Erlbaum Associates.

[36] Weber, E. U., Blais, A.-R., & Betz, N. (2002). A domain-specific risk-attitude scale: Measuring risk perceptions and risk behaviors. Journal of Behavioral Decision Making, 15, 1-28.

[37] Wu, G. and R. Gonzalez. (, 1996). Curvature of the probability weighting function, *Management Science*, 42, 1676-1690.

[38] Yaari, M.(1987). The dual theory of choice under risk, *Econometrica*, **55**, 95–115.