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Dominique Guegan, Bertrand Hassani

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The authors are experts in Operational Risk modelling. They published several papers on the topic. Their biography is detailed below:

- Pr. Dr. Dominique Guégan is the Director of the Paris 1 Doctoral School of Economics. She is the Head of the Financial Axis at the Center of Economic Sciences of the Sorbonne.

  Université Paris 1 Panthéon-Sorbonne, CES UMR 8174, 106 boulevard de l'Hopital 75647 Paris Cedex 13, France, phone: +33144078298, e-mail: dguegan@univ-paris1.fr.

- Dr. Bertrand K. Hassani was the Head of the AMA project at BPCE. He is an associate researcher at the University Paris 1. He was leading the Operational Risk Modelling at Aon Limited - AGRC. He has recently been appointed to the position of Head of Major Risk Management at Santander UK.

  Université Paris 1 Panthéon-Sorbonne CES UMR 8174, 106 boulevard de l'Hopital 75647 Paris Cedex 13, France, phone: +44 (0)2070860973, e-mail: bertrand.hassani@malix.univ-paris1.fr.

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Abstract

Following the Banking Committee on Banking Supervision, operational risk quantification is based on the Basel matrix which enables sorting incidents. In this paper we analyze these incidents in depth and propose strategies for carrying out the supervisory guidelines proposed by the regulators. The objectives are as follows:

- On the first hand, banks need to provide a univariate capital charge for each cell of the Basel matrix. That requires constructing Loss Distribution Functions (LDFs), which implies estimating a frequency and a severity distribution. We show that the choice of the theoretical distributions to build the LDFs has a tremendous impact on the capital charges, especially if we do not take into account extreme losses.

- On the other hand, banks also need to provide a global capital charge corresponding to the whole matrix. We highlight that a lack of consideration or a poor appreciation of the dependence structure may lead to incorrect capital charges.

- Finally, we draw the attention of regulators and managers to two crucial points:

  1. The necessity of splitting information sets in two parts while adjusting the severity distribution. The first covering small and medium losses, and the latter containing extreme losses (this point implies problems of granularity mentioned in the last Basel II guidelines),

  2. The choice of the risk measure which provides the capital amount. We emphasize that the expected shortfall measure enables a better anticipation of large incidents pertaining to operational risks.

Keywords: Operational risks - Loss Distribution Function - Risk Measures - EVT - Vine Copula

1 Introduction

In 2001, the Basel Committee on Banking Supervision provided a set of principles [4] for the effective management and supervision of operational risks, designed for banks and domestic authorities. According to these principles, banks could either use a basic, standard or advanced approach to calculate their capital charges. The last approach known as advanced measurement approach (AMA) requires banks to develop internal models.

However, at that time, operational risks management was a very young topic and due to its absence in the Basel I accords [3], the level of maturity was not the same as for credit risk. Therefore, Basel II initial requirements were very brief. They bound banks and financial institutions to use internal and external data, scenarios and qualitative criteria. They required banks to compute capital charges on a yearly basis and at a "99.9% confidence level". Therefore, at this time domestic authorities had no real experience of what would be a good approach to model the operational risks.

It was only after banks initial attempt to comply with the regulatory requirements that the BCBS started to be more precise on the kind of models they where expecting ([5, 6, 7]). New proposals have been done, they correspond to what we could refer to as Basel II++ principles which have to be monitored by the banks for the next few years. The purpose of this article is to analyse the effects of these Basel II principles, to discuss the strategy before the deployment of the next Basel system and to anticipate the applications of these new principles with respect to the data on which these principles have been applied. The results presented in this paper have been obtained using Caisse d'Epargne data sets (2006-2010), except the results presented in Table 2, which have been obtained on a Banques Populaires data set (2008-2010)[1].

1.1 Initial principles in Basel II proposals

In the second part of the 80's when the regulators decided to establish rules to strengthen the banking system there was nothing concerning operational risks. It was only in 2001 that supervisors and the banking industry recognized the importance of operational risk in shaping

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[1]The data sets used in this paper were sampled from Bertrand HASSANI's PhD thesis financed by BPCE.

the risk profile of financial institutions. Thus, the Basel Committee on Banking Supervision in their first Pillar about minimum regulatory capital charge for risks, required banks to take into account operational risks. As a result the Risk Management Group of the Basel Committee introduced a new regulatory capital approach based on banks' internal risk estimates called the "Advanced Measurement Approach" (AMA). The principle of this regulatory capital charge for operational risks was based on the following definition of operational risk: "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events". This definition includes legal risks, but not the strategic, reputational or systemic risks. At the time, the biggest challenge in applying the recommended guidelines stood in collecting and analysing loss data sets, because this assumed efficient risk management devices and materials. Collection system have been deployed in the past ten years. In this paper we will further discuss data collection (in particular the granularity issue) and organisation as part of calculating the minimum regulatory capital charge required from banks and insurance companies, in particular the problem of granularity.

The first Pillar proposal was established in close collaboration with the banking industry. It categorised operational risk exposures and losses into a series of standardised business lines and events types. Table 1 provides an example of the matrix on which the supervisor worked to provide the notorious capital requirement. This matrix was likely responsible for errors in the calculation of the capital requirement due to the excessive aggregation of the data collected. This was reflected in the guidelines for future Basel accords, and we will show in this paper the impact of the level of granularity of matrix and the importance of taking heed of this point.

We mentioned earlier data collection specific to AMA to evaluate capital charges, which lead to inquest another problem concerning the regulatory capital. The Committees's preliminary assessments of the possible level of operational risk regulatory capital was 20% of the current regulatory capital (CRM). Practitioners thought that a 20% figure should be lowered, and that 12% of minimum regulatory capital would provide a more reasonable cushion and produce required capital amounts more in line with the operational risks faced by large, complex banking organisations. However, it seems in reality that these first assesments (20% to 12%) were calibrated at a too low level. The first evaluation of operational risk capital requirements was used

BUSINESS UNITS	BUSINESS LINES LEVEL 1	(1) Internal Fraud	(2) External Fraud	(3) Employment Practices & Workplace Safety	(4) Clients, Products & Business Practices	(5) Damage to Physical Assets	(6) Business Disruption & System Failures	(7) Execution, Delivery & Process Management
INVESTMENT BANKING	(1) Corporate Finance (including Municipal/Gov.t Finance & merchant banking)				F_1			
	(2) Trading & Sales		F_8		F_2			
BANKING	(3) Retail Banking					F_5	F_9	
	(4) Commercial Banking							
	(5) Payment & Settlement		F_{10}		F_3		F_6	F_7
	(6) Agency Services & Custody							
OTHERS	(7) Asset Management							
	(8) Retail Brokerage				F_4			

Table 1: Basel Matrix for operational risks: each cell represents Business line/event types classification. F_i, $i \in [1, 10]$ denotes the distribution associated to each cell for which data is available.

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to define the Basic approach, and induced the weights of the standard approach. Since the banks' incentive to adopt the AMA was a 25% capital allocation reduction compared to the standard approach disregarding their risk profile, it produced a modelling bias. It could be argued that the capital reduction was driven by better risks management within the banks, which is not entirely false, nevertheless the inital lack of experience led to too low forfaitary capital charges which were used as a benchmark for the AMA. Besides, and we would like to emphasize this point, loss data sets (internal or external) are not used in the basic and the standard approach to compute capital requirements. The regulatory capital in those cases is evaluated as a fraction of the net banking income. Therefore, we cannot compare these approaches with the AMA. We show with empirical studies the difficulty to assess a correct level of charges. Therefore, after suggesting higher risk levels for the basic and the standard approach, we provide conservative and accurate solutions to model operational risks. The choice of methodologies is crucial and we detail those evidences at different steps of the calibration process.

1.2 The basic tools: LDA and VaR

A traditional way to deal with operational risks is the Loss Distribution Approach (LDA) [12]. This aims at building a Loss Distribution Function (LDF) ([13] for instance) G which is the mixture of a frequency distribution $p(k)$ and a severity distribution $F(x)$ (whose density is given by f), thus

$$G = \sum_{\gamma=1}^{\infty} p(k)F^{*\gamma}(x), \, x > 0, \, G_{b,e} = 0, \, x = 0, \tag{1.1}$$

where $*$ denotes the convolution operator and $F^{*\gamma}$ the γ-fold convolution of F with itself. We denote g the density of G, and

$$g = \sum_{k=\gamma}^{\infty} p(k)f^{*\gamma}(x), \, x > 0. \tag{1.2}$$

Experts working on operational risks agree on the fact that the frequency distribution has a negligible impact on the LDF's quantile which provides *in fine* the regulatory capital. However, the choice of the severity distribution has a tremendous impact on this because it is the most pertinent information in terms of level of loss. The capital charge pertaining to G is obtained, at the request of the regulators, through a 99.9% Value-at-Risk (VaR) - a quantile computed from this LDF. We recall now the definition of the VaR measure,

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Definition 1.1. *Given a confidence level $\alpha \in [0, 1]$, the VaR associated to a random variable X is given by the smallest number x such that the probability that X exceeds x is not larger than $(1 - \alpha)$*

$$VaR_{(1-\alpha)\%} = \inf(x \in \mathbb{R} : P(X > x) \leq (1 - \alpha)). \tag{1.3}$$

Here, the random variable X is associated to a cell of the Basel matrix (Table 1). To compute $P(X > x)$ we need to determine the LDF associated with this random variable, thus the choice of the severity distribution which constitutes this LDF will have a tremendous impact on the final capital allocation.

Severity distribution	K-S test	VaR (95%)	ES (95%)	VaR (99.9%)	ES (99.9%)
Lognormal	<2.2 e-16	6 448 883	7 531 208	13 354 724	17 863 146
Weibull	<2.2 e-16	4 081 999	4 204 917	4 530 362	4 631 586
GPD	<2.2 e-16	33 512 000	192 902 521	1 009 472 708	4 865 819 191
Gumbel	<2.2 e-16	4 247 871	4 336 857	4 597 061	4 669 047
GB2	<2.2 e-16	9 321 772	19 497 566	76 326 374	253 422 675
g-and-h	<2.2 e-16	6 030 146	6 289 353	6 940 614	7 231 981

Table 2: Several theoretical distributions have been fitted from a data set representing the External Fraud events of the Retail Banking business line of the Banques Populaire perimeter. The table presents very poor goodness-of-fit tests results. In this case it is not possible to state which distribution is better than the others. The fifth column presents the capital charges we would have using these distributions, and the sixth the Expected Shortfall. These results are compared to 95% confidence level risk measures.

For instance to model the severities of a sample representing the External Fraud event on the Retail Banking business line, we have fitted by maximum likelihood estimation and tested the following distributions (for which we provide the densities in appendix B): the lognormal distribution, the Weibull distribution, the Gumbel distribution, the Generalized Pareto distribution (GPD) [23, 11], the GB2 distribution ([17], [24]), and the g-and-h distribution ([20], [9]). For the choosen sample, we provide, for each distribution the corresponding capital requirement computed using the 95% and 99.9% Value-at-Risk (VaR) introduced in (1.3). The second column of Table 2 provides the result of an adequacy test based on the Kolmogorov-Smirnov distance

to check the quality of the fit of these statistical distributions to the data sets. They all failed, thus statistically we cannot accept any of these distributions. Nevertheless if we continue the exercise[2] columms 3 and 5 contain the capital computed using the VaR for two values of α. Colums 4 and 6 exhibit the capital requirements that the banks need if we use another measure named Expected Shortfall given by (3.1). Looking at these four columns we observe that the choice of the distribution has a tremendous impact on the capital allocation. Thus at this point, there is a trade-off between the choice of a distribution that minimises the capital allocation (here the Gumbel one), or a distribution which provides a more conservative capital (here the GB2). We also observe large differences with respect to the choice of the so-called risk measure. Thus, besides the calibration problem, the choice of risk measure is crucial. We will now expand these points more in details.

In the first part of section 2, we will discuss estimation of the LDF for each cell of the matrix, and provide the corresponding regulatory capital with respect to the corresponding LDF and the chosen risk measure. In section 3 we introduce a dependence structure between the cells, and for the different risk measures we will produce several capital charges. Finally the last section concludes with some important remarks focusing on the impact of the distribution families, the parameter estimation procedures and the risk measures on the regulatory capital. To complete our analysis, Appendixes provide statistics of the data sets used (Appendix A), exact densities of the distribution considered in this paper (Appendix B) and a comparison of estimation methods for the GPD parameters (Appendix C).

2 Estimation of the severities in each cell: the importance of the tail distribution

Before providing a capital associated with the global matrix, we need to analyse the behavior of each cell. We need to know the correct distribution that characterizes each cell. It is now recognised that for a proper statistical adjustment of the severity distribution, we must take into account the existence of extreme values in the analysis of the loss data sets (Table 2). There are

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[2]To continue this exercise is not unrealistic as soon as severity distributions like the lognormal distribution is commonly used in banks to compute the LDF

several approaches to consider which are all part of the extreme value theory [21]. Following this theory in a recent paper, we have proposed a flexible approach [15] to model the severity distributions. Our approach is a mixture of two distributions: one characterizing the most important losses (the tail) using the Generalized Pareto Distribution (GPD), and the latter modelling the remaining data (the central part of the distribution or corpus). We used a GPD on the right tail for which we provided innovative theoretical and practical solutions, and fit a lognormal[3] distribution on the remaining data via an EM algorithm. Figure 1 illustrates the approach. Then, to build the final LDF, we applied an adapted Monte Carlo algorithm. Once the threshold of the GPD has been found, the method chosen to estimate GPD's parameters tremendously impacts the VaR: it seems that the influence of the estimation procedure on the computation of the capital allocation has never been discussed before and we provide some details in Appendix C. The estimation procedure for the GPD we selected in this paper is based on the maximization of the Anderson-Darling statistic [22].

Thus, we define the severity distribution as a mixture of a lognormal distribution on the corpus, and a GPD on the right tail (cauda,) whose density $f(x; u, \beta, \xi)$ is :

$$f(x; u, \beta, \xi, \mu, \sigma) = \begin{cases} f^{(corpus)}(x; \mu, \sigma), \text{ if } x < u \\ f^{(cauda)} = \frac{1}{1 - \int_0^u f^{(center)}(x; \mu, \sigma)dx} \times f^{GPD}(x; u, \beta, \xi)), \text{ if } x \geq u \end{cases}, \quad (2.1)$$

where, μ and σ are the lognormal distribution parameters, and f^{GPD} the density of the GPD given in Appendix B.

In Table 3, we provide the capital requirements computed using different distributions for the cell associated with the business line "Payment & Settlement" and the event type "Delivery, Execution and Process Management" for the year 2006. We observed that by using the lognormal distribution only provides a very less conservative regulatory capital than using the POT method we introduced above. Note that when we adjusted only a GPD on the data set, we did not obtain workable results because either the parameters could not be properly estimated, or we faced an infinite mean model ($\xi > 1$).

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[3]The lognormal assumption was the best on our data sets, but one can fit any other distributions.

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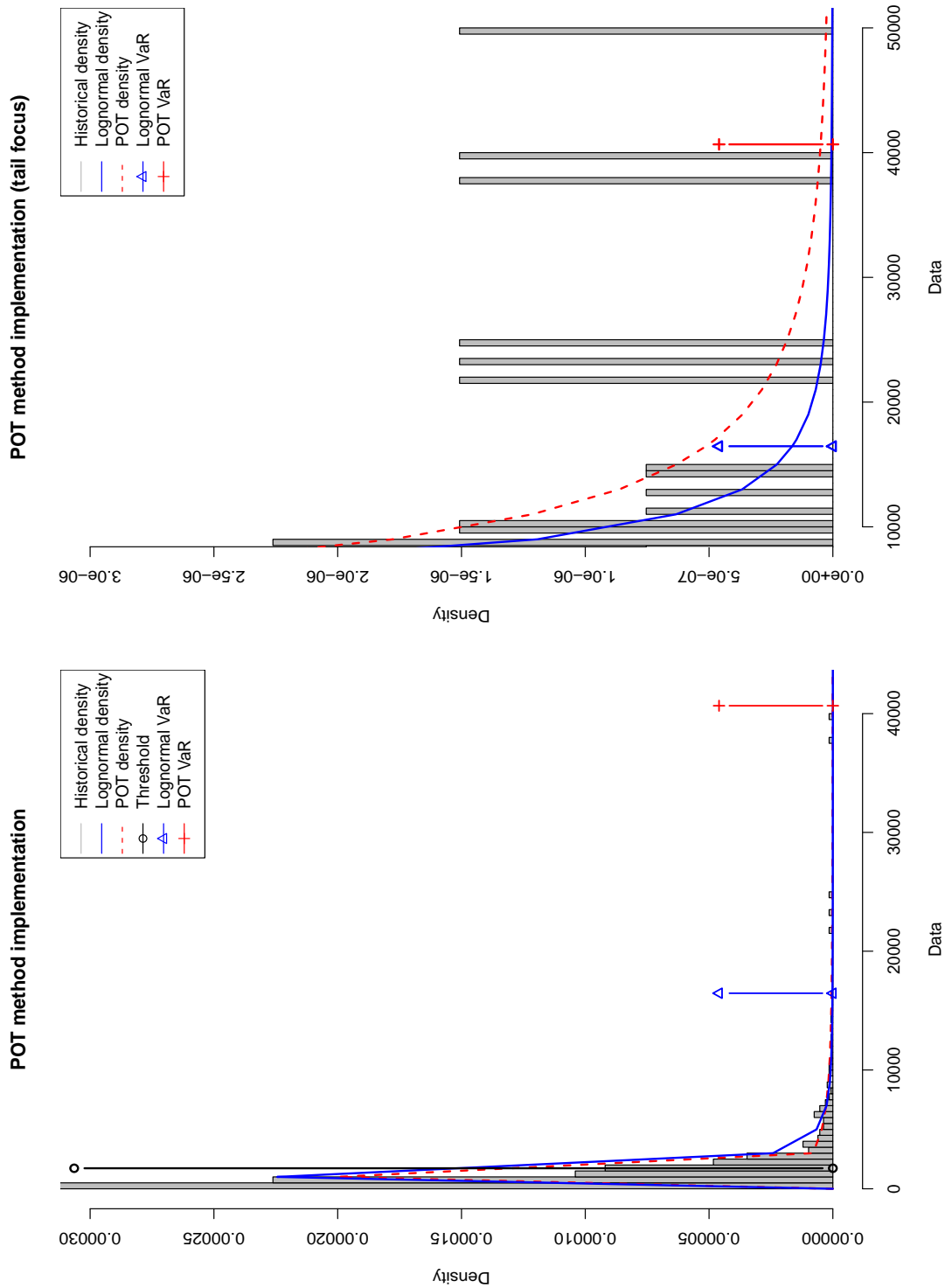
Figure 1: Method Illustration: This figure presents the histogram (in grey) of the Historical LDF. The black line corresponds to a LDF mixing a Poisson and a lognormal distribution. The 99.9% quantile representing the VaR is pointed out by Δ. The dash line represents a LDF mixing a Poisson distribution and a multiple pattern severity (lognormal-GPD). + indicates the location of the VaR. The right graph focuses on the right tail of the LDFs described above. This figure highlights the fact that this method enables thickening up the right tail of the LDF.

Severity distribution	VaR (95%)	ES (95%)	VaR (99.9%)	ES (99.9%)
Lognormal	305 303	336 455	463 192	533 160
GPD	∞	∞	∞	∞
POT	1 388 655	3 070 808	15 627 945	30 166 707

Table 3: We provide risk measures on the data set which represents the severity of the business line "Payment & Settlement" and the event type "Delivery, Execution and Process Management" for the year 2006, comparing different hypothesis for the severity distribution. We cannot provide a GPD estimation on the whole data set as the shape parameter is superior to 1, and therefore we face an infinite mean model.

We also observed large amount differences with respect to the choice of the risk measure: the VaR or the ES. This is discussed in more detail below.

3   Capital charge associated with a coherent risk measure

The regulator imposed a confidence level of 99.9% for the capital charge, therefore if we use the VaR measure, we have $\alpha = 0.1\%$ in (1.3). Nevertheless, using this risk measure, we face several problems, especially while computing a global capital charge. Indeed, the sum of the VaRs may be lower than the VaR of the sum i.e. the VaR is known not to be subadditive, and the result is questionnable as soon as several loss distributions are aggregated. On the other hand, this measure -by definition- does not take into account the large losses. However, the Conditional VAR (CVaR) or Expected Shortfall (ES) is a coherent measure [2] and therefore it is more appropriate to work with it. We recall its definition:

Definition 3.1. *For a given α in $[0,1]$, η the $VaR_{(1-\alpha)\%}$, and X a random variable which represents losses during a prespecified period (such as a day, a week, or some other chosen time period) then,*

$$ES_{(1-\alpha)\%} = E(X|X > \eta) \tag{3.1}$$

The ES measure which is subadditive takes into account the information contained in the distribution tail contrary to the VaR measure. Therefore, some extremal event exposures are captured by the ES measure. However in practice the ES value may be much higher than the corresponding VaR value, therefore, a lower confidence level could be used when considering the ES measure

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compared to the VaR measure.

As illustrated in Table 2, it might be judicious to use a GB2 distribution to model the severity and to compute a 95% ES than to use a lognormal distribution at a 99.9% VaR. Thus, we see that, to take into account very large losses, it could be more appropriate to associate at the same time the use of extreme distributions and the ES measure. In this case the result is closer to reality, and more effective to help banks understanding extremal events and by the way preventing them. In Tables 2 and 3, we illustrate these facts. We observed a large difference between the capital computed with the VaR and the ES measures. Thus, a trade-off between a VaR at 99.9% and an ES at 95% should be considered .

In these two tables we have seen the impact of the choice of risk measure on the capital computation. Now this impact can also be illustrated if we study the composition of the matrix and its granularity. Indeed, until now we used collected data organised into the Basel Matrix [4]. The first level of granularity is made up of 56 cases - 8 business lines ("b") $\times$ 7 event types ("e")[4]. Nevertheless, each event type might be further broken down into several elements. For example, the "external fraud" event may be shared in two items - "Theft and Fraud" and "Systems Security" (second level of granularity). In a third level, the element "Theft and Fraud" may be split in several components: "Theft/Robbery", "Forgery" and "Check kiting". After a deep analysis, we observe that the kind of losses expected from a fraud with a credit card does not correspond to losses caused by someone hacking the system for instance. Nevertheless these two different kinds of losses are in the same cell. Therefore, considering the largest level of granularity, we could face multimodal empirical distributions. Consequently, the methods used to model the losses depend on the choice of granularity of the Basel matrix. This choice might have a tremendous impact on capital requirement computations. Besides, there is a trade-off between the quantity of data and the robustness of the estimations. Indeed, if the quantity of data is not sufficient, we cannot go to a more granular level; on the other hand the empirical distribution is therefore an aggregate of various types of data and the estimation of this last empirical distribution can

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[4]The business lines are corporate finance, trading & sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage. The event types are internal fraud, external fraud, employment practices & workplace safety, clients, products & business practices, damage to physical assets, business disruption & system failures and execution, delivery & process management.

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be source of unusable results.

For the moment, we cannot empirically illustrate these remarks because we do not have the appropriate information set. Nevertheless it seems reasonable, in order to be close to reality, to introduce the computation of the distributions associated to the second or even third level of data granularity in order not to bias the LDF, as long as this information exists.

4   Influence of the dependence structure between the cells on the regulatory capital computation

Banks need to have *in fine* an amount calculated using the global matrix. This means that, as soon as the LDFs have been determined for each cell, the next question is the best way to combine these distributions to provide a global regulatory capital i.e. corresponding to the whole matrix.

Traditionally the experts computed the global capital requirement summing the capital calculated in each cell. This procedure does not take into account the true dependence which exists within the cells. One way to bypass this problem is to use a copula distribution which is a multivariate distribution permitting to link a huge number of distributions. During recent years some experts used the Gaussian copula to take these dependences into account, claiming inability to perform calculations using other copulas in high dimensions. The well-known Gaussian structure is not adapted to loss data sets (indeed, the Gaussian structure is elliptical, and does not capture tail dependence), and since 2005 it is now possible to work with copulas in high dimensions using nested copulas or vines [1]. Recently, we used this last methodology and adapted it to compute the capital requirement associated with operational risks in high dimensions [14]. Extending this work the main improvements for practitioners are the following:

- Firstly, this methodology enables the use of numerous classes of copulas without restricting to the elliptic domain. One can consider copulas which focus on information contained in the tails, where we find the large losses.

- Secondly, this approach allows several combinations of margins (corresponding to the dis-

tributions computed for each cell) to derive robust adjustments in the statistical sense.

- Thirdly, even working in the highest dimension, the procedure is easy to implement and is not too time consuming.

- Finally, this method complies with the lastest Basel Committee [7] requirements.

Below, the results obtained considering several cells of the Basel matrix[5] are provided. In Table 4 we introduce some notations corresponding to the losses we have studied.

BUSINESS UNITS	BUSINESS LINES LEVEL 1	Loss Distributions
INVESTMENT BANKING	(2) Trading & Sales	B_1
BANKING	(3) Retail Banking	B_2
	(5) Payment & Settlement	B_3
OTHERS	(8) Retail Brokerage	B_4

Table 4: Restricted Basel Matrix used to compute operational risk global capital allocation. B_1, B_2, B_3, B_4 are the four loss distributions used in the vine methodology. B_2 and B_3 are built considering respectively (F_2, F_5, F_8, F_9) and (F_3, F_6, F_7, F_{10}) as single data sets. In this case, our aggregation is "Business Line" oriented (Table 1).

In a first exercise, we show how to compute the amount corresponding to these four cells. The diagram is provided in Figure 2. We begin by estimating the margins associated with the cells B_1, B_2, B_3 and B_4. Thus, in a first step, we link the couples (B_1, B_2), (B_1, B_4) and (B_3, B_4) with a copula whose parameter is estimated by maximum likelihood (and sense checked using the Kendall τ). In a second step we link the copulas previously obtained by other copulas and so on. The choice of the cells to find the copula results from a sharp analysis of the data sets (cells). Nevertheless, some statistical studies enabling the decision of which links are appropriate can be found in ([16] and [14]).

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[5]Nevertheless the complete Basel matrix could contain more than 250 cells, and thus more research will be necessary to work with such a large matrix, mainly to limit the time of computation. Recent improvements using parallel computing, seem to provide interesting solutions to achieve this purpose [8].
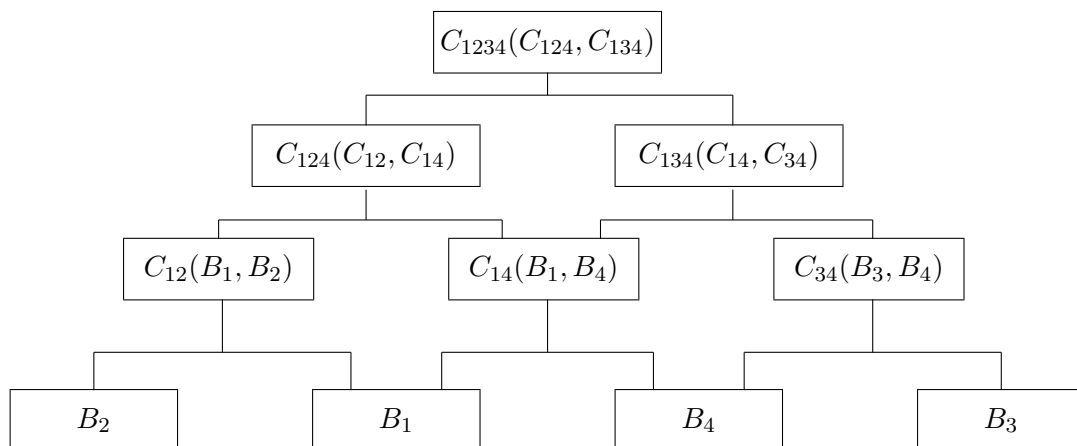
Figure 2: 4-dimensional vine estimation to obtain the dependence structure for the whole Basel Matrix.

In Table 5 we provide the amount computed using VaR and ES measures. The results given in the first line correspond to the global amounts obtained summing the VaR provided by the four margins $B_1, ..., B_4$, considering different distributions to model the severities: 1 corresponds to a non parametric estimation procedure, 2 corresponds to the lognormal distribution estimated and 3 corresponds to the Gumbel adjustment. In the second line we provide the results obtained linking the marginal distributions with the Gumbel copula with a parameter $\theta = 5.34$ obtained applying the previous vine methodology presented above; 1, 2 and 3 correspond to the same distributions for the margins as before. Looking at the third column we observe that we do not have the same amount if we sum the four cells (methodology based on a univariate approach) or if we use a copula methodology. The capital charges are very similar, but bigger in the first case. As a result, not taking into account the dependencies at all may lead to over conservative capital charges, even more conservative than using an extreme value copula (for instance a Gumbel one). The similar results between the two aggregation methodologies is a coincidence due to the fact that a simple sum of high quantiles (VaR) intuitively implies a strong dependence of large losses and the Gumbel copula we obtained mimics this behaviour. If we compare the third and the fourth column we observe that the capital charges are always bigger using the ES measure: this confirms comments made previously. When a Gumbel distribution is used on the margins, the two methods are competitive, indeed, the results are nearly similar. The use of lognormal distribution provides more conservative results. The use of nonparametric modelling provides a

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value between the two previous situations. It seems to us that obtaining a very good fit on the margins is conducive with providing a realistic amount of capital. The modelling of the margins can be more influential than the fit of the copula, nevertheless, using upper tail dependence copula structures allows a more conservative capital charge for banks.

$C_{1234}(B_1,B_2,B_3,B_4)$	Margins	VaR	ES
Univariate	1	68 468 561	71 030 541
	2	76 817 362	94 196 483
	3	48 113 918	48 244 680
Gumbel Copula	1	68 517 234	71 028 526
	2	76 564 982	93 187 912
	3	48 112 494	48 241 814

Table 5: This table provides the capital allocation (VaR) and the ES for the whole data set, considering three classes of severities (1 denotes the non parametric approach of the LDF, 2 the lognormal approach and 3 the Gumbel one.) and two classes of dependence. Univariate corresponds to the VaRs sum of each LDF. The alternative corresponds to an aggregation using a Gumbel copula.

We now propose another exercise. In Table 1 we consider the cell F_9 corresponding to Business Disruption & System Failure events in the Retail Banking business unit and the distribution associated to the cell F_6 characterizing the same events in the Payment & Settlement business unit. For the distribution F_9 we estimate a Gumbel distribution or a lognormal distribution, for the distribution F_6 we estimate a Generalized Pareto distribution (GPD) or a lognormal distribution. Tables 6 and 7 provide the capital values when we link these two distributions with a Gumbel copula on one hand and with a Clayton copula on another hand. In columns 4 and 5 we provide the capital computed using the VaR and the ES measures respectively. Columns 2 and 3 gives the amount corresponding to each cell obtained by projection from the fourth column.

The results of these tables show that depending on the way we model the margins, we have tremendous differences between the VaRs. For example, we would have a VaR equal to 117 207

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402 euros if F_9 is modeled with a lognormal distribution and F_6 with a GPD distribution versus a VaR equal to 2 037 655 euros if F_9 is modeled with a lognormal distribution and F_6 with a Gumbel one. Depending on the way we model the LDFs, the aggregated VaR may be multiplied by 57.52. The same behavior is observable when we project the corresponding values on the cells. For example, the multivariate VaR projection on F_9 is € 2 655 055 if F_6 is modeled using a lognormal distribution, and is equal to € 15 405 192 if F_6 is modeled using a GPD distribution. The peak for the VaR observed in that latter case is due to the capture of extreme events through the choice of the margins: the Gumbel one. Finally, using at the same time copula and severity distributions, which take into account information in the tail, provides very accurate results. Indeed, when we model F_6 using a GPD associated with a Gumbel copula, we provide a larger VaR than with the Clayton one. The differences are observed comparing the amounts € 105 422 356 with € 103 249 260 on the one hand, and € 117 207 402 with € 107 807 238 on the other.

In Table 6 we see that the capital requirements obtained using a Gumbel copula are bigger than those obtained with a Clayton one, thus the choice of the dependence structure also has an impact on the computation of the capital charges. Now, if we use the VaR measure the difference is not significant for a bank (columns 4 in Tables 6 and 7), but if we compare the results obtained with the ES measure (columns 5 in Tables 6 and 7) the difference is tremendous (up to almost a billion euros). Nevertheless, choosing the ES measure at a 99.9% confidence level induces a much larger capital amount considering an upper tail depencence structure (Gumbel copula) than an lower tail dependence (Clayton copula).

Model	Gumbel Copula			
	LDF_9	LDF_6	VaR	ES
Gumbel-GPD	2 322 782	103 099 574	105 422 356	1 603 169 459
Gumbel-lognormal	1 471 343	566 312	2 037 655	3 091 139
lognormal-GPD	15 405 192	101 802 210	117 207 402	1 904 323 684

Table 6: For the LDF corresponding to F_9 and F_6 we provide the VaRs and the ES computed from a Gumbel copula for the year 2006. They are given respectively for three classes of severities. For instance, "Gumbel-GPD" means that we have chosen a Gumbel distribution to model F_9 and a mix of a lognormal and a GPD to model F_6.

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Model	Clayton Copula			
	LDF_9	LDF_6	VaR	ES
Gumbel-GPD	1 154 681	102 094 579	103 249 260	739 977 372
Gumbel-lognormal	1 455 693	649 164	2 104 857	2 637 980
lognormal-GPD	5 631 004	102 176 234	107 807 238	1 092 923 925

Table 7: For the LDF corresponding to F_9 and F_6 we provide the VaRs and the ES computed from a Clayton copula for the year 2006. They are given respectively for three classes of severities. For instance, "Gumbel-GPD" means that we have chosen a Gumbel distribution to model F_9 and a mix of a lognormal and a GPD to model F_6.

Finally, another exercise allows us to see the influence of the dependence structure on the cells for which we need to know the capital allocation. Applying a vine approach, we obtained a Gumbel copula to model the dependence between several LDFs. Computing the corresponding multivariate VaR, we derived a global capital charge. In Table 11, we projected from the multivariate VaR given in column 7, the amount corresponding to each cell (axis). For example, for the second line corresponding to the "Retail Banking", we can provide the amounts pertaining independently to the "External Fraud", the "Clients, Products & Business Practices", the "Damage to Physical Assets" and the "Business Disruption & System Failures" event types (Table 1 line 3, column 2, 5, 8, 9). Our approach is interesting because it provides the capital for each cell through the dependence structure between the cells. This approach is totally different from the approach mainly used by practitioners who directly compute the capital associated with each cell without taking into account the information given by any other cell.

Table 11 highlights the fact that an upper tail dependence structure (line 3) always provides larger capital charges than the sum of the univariate VaRs. We can also say that the larger the tail of the theoretical distribution, the larger the gap between the ES and the VaR.

5   Conclusion: New Proposals

In this paper, we discussed a range of options for assessing Basel Pillar 1's capital charges for operational risk. We have expanded them and increased both reliability and precision in our

Approach	LDF	LDF_2	LDF_5	LDF_8	LDF_9	VaR	ES
	1	19 650 986	3 182 731	14 212 411	2 241 011	39 287 139	40 899 526
Univariate	2	6 240 984	2 151 627	11 676 534	4 049 831	24 118 976	32 957 873
	3	13 599 313	2 478 087	8 599 313	1 087 410	25 764 123	25 887 436
	1	20 578 056	3 300 471	15 191 828	2 386 106	41 456 461	43 024 128
Gumbel Copula	2	10 732 933	2 310 337	29 525 155	13 608 000	56 176 425	79 500 929
	3	13 617 419	2 486 453	8 603 916	1 095 587	25 803 375	25 927 487

Table 8: This table provides the VaRs and ES associated with each LDF of the set LDF_2, LDF_5, LDF_8 and LDF_9 when we decompose the dependence structure of the 4-dimensional set C_{2589}, considering three classes of severities (1 denotes the non parametric approach of the LDF, 2 the lognormal approach and 3 the Gumbel one.).

measurement, management and control of operational risks. As we had the opportunity to experiment the methods suggested in literature using real data sets, we found various drawbacks and pitfalls, and we proposed solutions to bypass them.

We suggested the use of a Peak-over-Threshold method to thicken the right tail of the loss distribution function. Presenting this solution, we suggested an efficient way to obtain generalized Pareto distribution parameters which is accurate regarding goodness-of-fit tests, and therefore compliant regarding the regulator. Furthermore, our methods have shown conservative results and quantitatively supported the idea that some data sets might be badly built.

We provided an innovative solution to compute aggregated risk measures (VaR and ES), dealing with dependences between Basel categories. This solution is based on nested structures and vine architectures. Carrying out this methodology, we were able to take into account specific dependences (upper tail etc.) between many margins with Archimedean and extreme value copulas. We also studied the sensitivity of multivariate VaRs to modeled LDFs (margins), to dependence architectures and to copulas parameters.

In addition of these results, we observed that estimating dynamically the parameter of the de-

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pendence structure creates important variations in the values of the Gumbel copula parameter. We illustrated this fact in Table 9. We computed the parameter of the Gumbel copula linking the LDFs of the cells F_9 and F_6. This parameter θ varied with respect to the information set used for its estimation. The parameters' value obtained using the year 2006, was different when we used the year 2007 or the year 2008, or the whole sample. We noticed that the upper tail dependence was larger when we used this last data set. This will have an impact on the computation of capital requirements. Thus, with the information set we use, the capital requirement appeared to be more or less conservative, and notion of dynamics inside the data needs to be taken into account.

Year	θ	θ
2006	4.9202 (0.94)	
2007	3.7206 (0.75)	10.6610 (0.88)
2008	5.8490 (0.51)	

Table 9: Parameter estimation of Gumbel copulas estimated on F_9 and F_6 for each year 2006, 2007 and 2008 (second column). These parameters are compared to a Gumbel copula parameter estimated on the entire time series (third column). The corresponding standard deviation are provided in brackets.

Therefore, we suggest to work dynamically and therefore measure the impact of the time passing on the distribution shapes. This idea led us to challenge the 5 years data sets required by the authorities. Indeed, these data sets may include outdated data - for example, a incident occurred in a department that does not exist anymore - or not old enough and some long memory process should be involved.

Last but not least, we suggested to compute capital charges considering another risk measure than the VaR measure, say the expected shortfall. This measure is coherent and also take into account the whole information contained in the tail. Computing a Capital allocation from a 99.9% ES is not realistic as it appears to be too conservative, but a lower critical threshold could be considered. For example one could consider a 95% ES instead of a 99.9% VaR (Table 2) as we illustrate in the previous exercises.

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A   Appendix: Distributions statistics

Next table provides the four first moments of the empirical severities corresponding to the cells of the Basel matrix (Table 1)used in this paper.

Distributions	Mean	Variance	Skewness	Kurtosis
F_1	195.37	292732.86	7.31	71.69
F_2	1522.83	372183311.54	27.57	910.13
F_3	175.42	3804557.63	30.03	956.75
F_4	1805.81	93274002.03	18.74	457.58
F_5	1824.95	189175093.33	17.79	354.00
F_6	1200.08	438224165.80	23.69	563.48
F_7	800.14	24268504.39	10.88	139.39
F_8	1779	1602373386	19.27	435.88
F_9	1824.95	189175093.3	17.79	354.00
F_{10}	12104	519962084.2	108.03	11806.23

Table 10: Statistics of the data sets used. The distributions are right skewed and present large kurtosis.

B   Appendix: Distributions for the severities

We provide the densities of the main severity distributions we used along this paper.

- lognormal distribution:

$$f_{b,e}(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}}, \tag{B.1}$$

for $x > 0$, $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^{+*}$.

- Weibull distribution:

$$f_{b,e}(x; \beta, \xi) = \frac{\xi}{\beta} \frac{x}{\beta}^{\xi-1} e^{-(\frac{x}{\beta})^\xi}, \tag{B.2}$$

for $x > 0$, $\beta > 0$, $\xi > 0$.

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- Gumbel distribution:

$$f_{b,e}(x; u, \beta) = \frac{1}{\beta} e^{\frac{x-u}{\beta} - e^{\frac{x-u}{\beta}}}, \tag{B.3}$$

with $u \in \mathbb{R}$ and $\beta > 0$.

- Generalized Pareto distribution (GPD) ([23, 11]):

$$f_{b,e}(x; u, \beta, \xi) = \frac{1}{\beta} \Big(1 + u \frac{(x - \xi)}{\beta} \Big)^{\left(-1 - \frac{1}{u}\right)}, \tag{B.4}$$

with $1 + u\frac{(x-\xi)}{\beta} > 0$, $\beta > 0$ and $\xi \neq 0$ (or $f_{b,e}(x; u, \beta, \xi) = \frac{1}{\beta}\Big(- \frac{(x-\xi)}{\beta} \Big)$ if $u = 0$).

- GB2 distribution ([17], [24]):

$$f_{b,e}(x; \alpha, \beta, p, q) = \frac{\alpha x^{\alpha p - 1}}{\beta^{\alpha p} B(p, q) \left[1 + \left(\frac{x}{\beta} \right)^{p+q} \right]} \tag{B.5}$$

where $\alpha, \beta, p, q, x >, B(u, v) = \Gamma(u)\Gamma(v)\Gamma(u + v)$ is the Beta function, and $\Gamma(.)$ is the Gamma function.

- g-and-h distribution ([20], [9]):

$$f_{b,e}(x; g, h) = \exp^{((g*x)-1)} * \exp^{\frac{\left(\frac{(h*p^2)}{2} \right)}{g}} \tag{B.6}$$

when $g = 0$ and $h = 0$, the g-and-h distribution reduces to a standard normal distribution.

C Appendix: Influence of estimation methods on the amount of regulatory capital

We show the influence of the estimation procedures of GPD's parameters used in section 2 on capital requirements. Assuming a bootstrap method to estimate the threshold ([18], [10]), we estimate the remaining ξ and β parameters of the GPDs defined in (2.1) using the method introduced by [22], denoting this method M1. We also consider three other alternative estimation methods to estimate these parameters in order to check their impact on VaR computations. These ones are respectively the Pickands method (M2) [23], the Hill method (M3) [19], and the

Maximum Likelihood method (M4). We provide in Table 11 the estimations of GDPs parameters (with their standard deviation in brackets) obtained for both sets, using these four methods and the corresponding capital charges.

Note that a shape parameter $\xi > 1$ in (2.1) induce an infinite mean model which naturally provides very high VaRs. Nevertheless, non-parametric estimator such as Pickands might provide this kind of value and by the way unusable models.

Method	β	ξ	VaR	ES
M1	932.854	0.767	15 700 112	34 215 896
	(83.71)	(0.101)		
M2	682.615	1.144	538 480 990	∞
	(160.70)	(0.266)		
M3	1007	0.66	5 725 341	14 780 214
	(214.36)	(0.228)		
M4	904.087	0.827	27 944 558	125 019 034
	(92.31)	(0.097)		

Table 11: We provide risk measures on the data set which represents the severity of the business line "Payment & Settlement" and the event type "Delivery, Execution and Process Management" for the year 2006, given estimations of the GPD's parameters ξ and β (Appendix B) using four methods for $\hat{u} = 179$. M1 is the method introduced by Luceno [22], M2 is the Pickands method, M3 is the Hill method, and M4 is the Maximum Likelihood method. We provide in brackets the standard deviations computed by bootstrapping. The third column gives the 99.9% VaR (regulatory capital allocation) pertaining to these estimates ($\mu = 3.593098$, $\sigma = 1.510882$). The fourth column presents the corresponding expected shortfall capital value.