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An application of wage bargaining to price negotiation with discount factors varying in time

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Abstract. We consider a non-cooperative price bargaining model between a monopolistic producer and a monopsonic consumer. The innovative element that our model brings to the existing literature on price negotiation concerns the parties' preferences which are not expressed by constant discount rates, but by sequences of discount factors varying in time. We assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent. Under certain parameters, the price negotiation model coincides with wage bargaining with the exogenous always strike decision. We determine the unique subgame perfect equilibrium in this model for no-delay strategies independent of the former history of the game. Then we relax the no-delay assumption and determine the highest equilibrium payoff of the seller and the lowest equilibrium payoff of the buyer for the general case. We show that the no-delay equilibrium strategy profiles support these extreme payoffs.

JEL Classification: C78, J52, E31

Keywords: price bargaining, alternating offers, varying discount rates, subgame perfect equilibrium

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1 Introduction

1.1 Brief literature overview

The paper concerns price bargaining – undoubtedly an important issue in most economic and market negotiations. In such a bargaining, a seller wants to sell his product at a highest price to maximize his profit whereas a buyer wants to buy it at a lowest price to maximize his surplus. If the seller and buyer do not agree on a price, then there will be no transaction.

Numerous works are devoted to price bargaining between sellers and buyers. Non-cooperative two-person sequential bargaining models are used to examine the bargaining behavior in different kinds of markets. Frequently the analysis takes notice of reference points – the concept introduced in prospect theory (Kahneman and Tversky (1979); Tversky and Kahneman (1991, 1992)). Some reference points are external such as previous paid prices or market values (Kahneman (1992); Kristensen and Gaerling (1997b);

Northcraft and Neale (1987)), and others are internal such as reservation price or aspiration price (Kristensen and Gaerling (1997a)). In the price bargaining literature, it is still unclear what are the internal reference points. Kristensen and Gaerling (1997a) use an experimental study for determining the reference points of bargaining price and show the importance of reservation prices of both sellers and buyers in a competitive market. A reservation price is the point at which the bargainers are indifferent to accept or reject the offer of the other party. In other words, in a seller-buyer bargaining, it is the maximum (minimum) price at which the buyer (seller) is willing to buy (sell) the product. Kristensen and Gaerling (1997a) find in their experiment that if the expected market price is lower and the first offer is higher than the reservation price, then using it as a reference point will not be significant. However, White et al. (1994) find that a buyer's reservation price is the most important reference point for the buyers. Kwon et al. (2009) create a reservation price reporting mechanism by using an experimental study. Poucke and Buelens (2002) introduce the notion of an offer zone, which is the difference between aspiration price and initial offer, and study its influence on the negotiated outcome, by running simulated seller-buyer negotiations between managers.

Many works on non-cooperative two-person bargaining models are based on Rubinstein (1982b) formulation of sequential bargaining process in discrete time with alternating offers and counteroffers and on the determination of subgame perfect equilibria (abbreviated here as SPE). Time and information are important elements in these models. Some authors consider one-sided or two-sided asymmetric information and present models of sequential bargaining under incomplete information. Price bargaining between manufacturer and distributor under asymmetric and incomplete information of distributor's knowledge about buyers' reservation price is tested in an experimental study of sequential bargaining by Srivastava et al. (2000). Feri and Gantner (2011) modify Rubinstein's sequential bargaining model by two-sided incomplete information and study experimentally price bargaining. Cramton (1991) adds transaction cost to Rubinstein's sequential bargaining model with asymmetric information. Gul and Sonnenschein (1988) identify the delay to agreement with a screening process of a price bargaining model between a buyer and a seller where there exists an uncertainty about the valuation of one party.

An important issue in non-cooperative bargaining models concerns preferences of bargainers, in particular, non-stationarity of preferences. Although several works emphasize that stationary bargaining models are rare in real-life situations (e.g., Cramton and Tracy (1994)), models with discount factors varying in time do not receive enough attention so far. Non-stationarity of parties' preferences in the original Rubinstein model is discussed, e.g., in Binmore (1987), Coles and Muthoo (2003). Also Rusinowska (2001, 2002, 2004) generalizes the original model of Rubinstein to bargaining models with preferences described by sequences of discount rates or/and bargaining costs varying in time. Ozkardas and Rusinowska (2012) propose a union-firm wage bargaining model in which preferences are described by discount rates varying in time and strike decision is given exogenously. Trefler (1999) modifies Rubinstein and Wolinsky (1985) bargaining framework by adding the Markov process of pairwise matching to analyze the impact of market supply and demand on bilateral bargaining outcomes. Dickinson (2003) introduces the importance of risk preferences on the bargaining outcomes in price negotiation.

Price bargaining models are frequently tested by laboratory experiments (Roth and Kagel (1995)). For example, price bargaining on perishable goods market is studied experimentally by Moulet and Rouchier (2008) to determine the effects of time on sequential bargaining model. Cason et al. (2003) compare posted price versus bilateral bargaining price by using laboratory experiments and find that the bargaining price is higher and sticker than posted prices. Other studies use field experiments for reference points of price bargaining (Abdul-Muhmin (2001)).

Although price negotiation between a seller and a buyer can be seen as a microeconomic problem, several authors apply price negotiation models to macroeconomic issues. Application of price bargaining to international trade between two countries over two non-storable goods is analyzed by Fernández-Blanco (2012). Oczkowski (1999) applies Nash bargaining framework to an econometric analysis of price and quantity bargaining model.

1.2 The present paper

We consider a monopolistic seller that sells a unique and indivisible good in a market with only one buyer. They bargain over the price of the product by making alternating offers. Initial offer is made by the seller and the buyer is free to either accept or reject it. If he rejects the offer, then it is his turn to make a new offer. We use therefore Rubinstein's bargaining procedure (Rubinstein (1982a)), but similarly as in Rusinowska (2001) we generalize the model by assuming that preferences of each party are expressed by discount factors varying in time. There are several differences between the present model and the model analyzed in Rusinowska (2001). In the latter, two players bargain over a division of one unit of infinitely divisible good and the utility of a player is given by the discounted agreement (i.e., the discounted part of the good received by the given player). In our model, the seller and the buyer bargain over the price of a good, the payoffs are different from the ones defined in Rusinowska (2001), and the utility of a bargainer is given by the discounted sums of the payoffs from period 0 to infinity. We assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent. Ozkardas and Rusinowska (2012) consider a wage bargaining in which a union and a firm bargain over a wage contract and the union may go on strike if an offer is rejected. They analyze subgame perfect equilibria under exogenous strike decisions and for history independent strategies with no delay. Under some assumptions on the parameters in the model, the utilities of the seller and the buyer coincide with the utilities of the union and the firm in the wage bargaining in which the union commits to go on strike whenever there is a disagreement (Ozkardas and Rusinowska (2012)). Consequently, the particular case of wage bargaining can be applied to the price negotiation model.

In the present paper, first we restrict our analysis to history independent strategies with no delay which means that an offer of a player is independent of the previous offers of the players and when a player has to make an offer, his equilibrium offer is accepted by the other party. Similarly as in Ozkardas and Rusinowska (2012), we determine the unique subgame perfect equilibrium for no-delay strategies independent of the former history of the game. Then we relax the no-delay assumption and determine the highest equilibrium payoff of the seller and the lowest equilibrium payoff of the buyer for the general case. We show that the no-delay equilibrium strategy profiles support these extreme payoffs.

Our approach to the analysis of equilibrium payoffs in the price bargaining is similar to the one used in Houba and Wen (2008) who apply the method by Shaked and Sutton (1984) to derive the exact bounds of equilibrium payoffs in wage bargaining introduced in Fernandez and Glazer (1991). However, while preferences of the union and firm in the model of Fernandez and Glazer (1991) are constant in time, in our model the seller and the buyer have preferences varying in time.

Section 2 describes the price bargaining model with discount rates varying in time. In Section 3 we determine the unique subgame perfect equilibrium of the model, when we restrict the analysis to history independent strategies with no delay. Then we we analyze equilibrium payoffs for the general model without the restriction to no-delay strategies. In Section 4 we conclude and mention our future research agenda. To make the paper self-contained, in the Appendix we present proofs of all the results stated in the present paper, although some of the proofs are very similar to the ones concerning wage bargaining with the exogenous 'always strike' decision and presented in Ozkardas and Rusinowska (2012).

2 The Model

We introduce a model of price negotiation between a seller and a buyer on a unique indivisible product. We suppose that the seller is in a monopolistic situation and the buyer is monopsone which means that the market is constituted by two players.

Buyer has a reservation price of R for the unique product and he buys it for personal satisfaction. His reservation price is an indicator of the buyer's willingness to buy. If the buyer cannot obtain the product, he pays a dissatisfaction cost of D. On the other hand, if he gets the product, he has a positive satisfaction gain of S, where $R \geq S \geq D \geq 0$. The seller desires to sell the product and to make a positive and maximum profit. If the seller cannot sell it, he pays a cost of $0 < C \leq S + D$ of producing the product. The bargaining procedure between the seller and the buyer is the following. Two parties (the seller and the buyer) bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of price that the other party is free either to accept or to reject.

Let P_s^{2t} denote the offer of the seller made in an even-numbered period $2t$, where $t \in \mathbb{N}$, and let P_b^{2t+1} denote the offer of the buyer made in an odd numbered period $2t+1$. The range of the proposed price is $[0, S+D]$, i.e., neither the seller nor the buyer can propose a price above the sum of the satisfaction value and the dissatisfaction cost. In period 0 the seller proposes P_s^0, and if the buyer accepts this price, than the agreement is reached and the payoffs in period 0 are $(P_s^0 - C, R - P_s^0 + S)$. If the buyer rejects it, then the payoffs in period 0 are $(-C, R - D)$, and it is the buyer's turn to make a counter-offer P_b^1 in period 1. If the seller accepts this offer, then the payoffs in period 1 are $(P_b^1 - C, R - P_b^1 + S)$. Otherwise, the payoffs in period 1 are $(-C, R - D)$, and the seller makes a new offer in the next period. This procedure goes on until an agreement is reached.

In the price negotiation, preferences of the seller and the buyer are described by sequences of discount factors varying in time, $(\delta_{s,t})_{t\in\mathbb{N}}$ and $(\delta_{b,t})_{t\in\mathbb{N}}$, respectively, where $\delta_{s,t}$ is the discount factor of the seller in period $t \in \mathbb{N}$, $\delta_{s,0} = 1$, $0 < \delta_{s,t} < 1$ for $t \geq 1$ and $\delta_{b,t}$ is the discount factor of the buyer in period $t \in \mathbb{N}$, $\delta_{b,0} = 1$, $0 < \delta_{b,t} < 1$ for $t \geq 1$.

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The result of the price negotiation is either a pair (P,T), where $P \in [0, S+D]$ is the agreed price of the product and $T \in \mathbb{N}$ is the number of periods before reaching the agreement, or a disagreement denoted by (d,∞) and meaning the situation in which the parties never reach an agreement.

For each $t \in \mathbb{N}$, we introduce the following notation:

$$\delta_s(t) := \prod_{k=0}^{t} \delta_{s,k},\ \delta_b(t) := \prod_{k=0}^{t} \delta_{b,k},\ \text{for } 0 < t' \leq t,\ \delta_s(t',t) = \prod_{k=t'}^{t} \delta_{s,k},\ \delta_b(t',t) = \prod_{k=t'}^{t} \delta_{b,k}$$

The utility of the result (P,T) for the seller, where $S+D \geq P \geq 0$ and $T \in \mathbb{N}$, is equal to

$$U_s(P,T) = \sum_{t=0}^{\infty} \delta_s(t)\, u_s(t) \tag{1}$$

where $u_s(t) = P - C$ for each $t \geq T$, and if $T > 0$ then $u_s(t) = -C$ for each $0 \leq t < T$.

The utility of the result (P,T) for the buyer is equal to

$$U_b(P,T) = \sum_{t=0}^{\infty} \delta_b(t)\, u_b(t) \tag{2}$$

where $u_b(t) = R - P + S$ for each $t \geq T$, and if $T > 0$ then $u_b(t) = R - D$ for each $0 \leq t < T$, where $R \geq S \geq D \geq 0$ and $S + D \geq P \geq 0$.

The utilities of the disagreement for the seller and the buyer are equal to

$$U_s(d,\infty) = -C \sum_{t=0}^{\infty} \delta_s(t),\ U_b(d,\infty) = (R-D) \sum_{t=0}^{\infty} \delta_b(t)$$

At the seller's side, when the agreement (P,T) is reached, his payoff in every period $t \geq T$ will be equal to $u_s(t) = P - C$, i.e., to the difference between the price and the production cost. If $P \geq C$, the seller will make profit from this agreement. On the other hand, if the agreement is not reached in period T, then the seller's payoff at period T will be $u_s(T) = -C$, i.e., the production cost which is equal to the lost of the seller. We therefore assume that the product can be used only within one period and must be produced each time when a new period starts.

For the buyer, the agreement (P,T) gives to the buyer in every period $t \geq T$ the payoff equal to $u_b(t) = R - P + S$, i.e., to the difference between his reservation price for that product and the agreement price, plus the satisfaction value for obtaining the product. Hence, the buyer's payoff in the agreement has two components: the surplus of the buyer which is the amount of money that stays in his pocket and the satisfaction value that comes from obtaining the product. In case of a disagreement, the payoff level of the buyer in period T is equal to $u_b(T) = R - D$, i.e., to the difference between the reservation price and the cost of the disagreement. This means that the buyer suffers from not obtaining the product, but he still has some money in his pocket.

Remark 1 Note that if $R = D = 1 - S$ and $C = 0$, then we recover the wage bargaining with discount rates varying in time, where the union commits to strike whenever there is a disagreement; see Ozkardas and Rusinowska (2012).

The utilities for both parties depend on the infinite series, so we need to well define the sequences of discount rates. What are the conditions for convergence of these infinite series?

Remark 2 The necessary conditions for the convergence of the infinite series which define $U_s(P,T)$ and $U_b(P,T)$ in (1) and (2) are

$$\delta_s(t) \to_{t\to+\infty} 0 \quad \text{and} \quad \delta_b(t) \to_{t\to+\infty} 0 \tag{3}$$

but these are not sufficient conditions. The necessary conditions come immediately from the necessary condition of the convergence of the infinite series. To see that these are not sufficient conditions, consider $\delta_{b,k} = \frac{k}{k+1}$ for each $k \geq 1$, $\delta_{b,0} = 1$. Then

$$\delta_b(t) = \frac{1}{2} \cdot \frac{2}{3} \cdots \frac{t}{t+1} = \frac{1}{t+1} \to_{t\to+\infty} 0$$

If the agreement P is reached immediately, then $U_b(P,0) = (R - P + S) \sum_{t=0}^{\infty} \frac{1}{t+1}$ which is a divergent series. Similarly, if P is reached in a certain period $T > 0$, then $U_b(P,T) = \sum_{t=0}^{T-1} \delta_b(t) u_t + (R - P + S) \sum_{t=T}^{\infty} \frac{1}{t+1}$.

Remark 3 If $(\delta_{s,t})_{t\in\mathbb{N}}$ and $(\delta_{b,t})_{t\in\mathbb{N}}$ are bounded by a certain number smaller than 1, i.e., if

$$\text{there exist } \Phi_s < 1 \text{ and } \Phi_b < 1 \text{ such that } \delta_{s,t} \leq \Phi_s \text{ and } \delta_{b,t} \leq \Phi_b \text{ for each } t \in \mathbb{N} \tag{4}$$

then the series which define $U_s(P,T)$ and $U_b(P,T)$ in (1) and (2) are convergent. We have for each $t \in \mathbb{N}$

$$0 \leq \delta_b(t)(R - P + S) \leq (\Phi_b)^t (R - P + S)$$

Let the agreement P be reached immediately. Since $\sum_{t=0}^{\infty} (\Phi_b)^t$ is the convergent geometric series, by virtue of the comparison test, $U_b(P,0)$ is also convergent. The proof is similar, if P is reached in a certain period $T > 0$ and it is analogous for the seller. The sufficient conditions given in (4) are not necessary conditions. To see that, consider $\delta_{b,k} = \frac{k}{k+2}$ for each $k \geq 1$, $\delta_{b,0} = 1$. The sequence does not satisfy the condition (4). However, we have

$$\delta_b(t) = \frac{1}{3} \cdot \frac{2}{4} \cdots \frac{t}{t+2} = \frac{2}{(t+1)(t+2)} \to_{t\to+\infty} 0$$

If the agreement P is reached immediately, then $U_b(P,0) = (R - P + S) \sum_{t=1}^{\infty} \frac{2}{(t+1)(t+2)}$ which is convergent by virtue of the comparison test: $\frac{1}{t^2} \geq \frac{1}{(t+1)(t+2)}$ and we know that $\sum_{t=1}^{\infty} \frac{1}{t^2}$ is convergent. The proof is similar, if P is reached in a certain period $T > 0$.

Not only every decreasing sequence $(\delta_{s,t})_{t\in\mathbb{N}}$ ($(\delta_{b,t})_{t\in\mathbb{N}}$, respectively) satisfies (4) and gives the convergent series defined in (1) ((2), respectively) but also some increasing sequences do that; see, e.g., $\delta_{b,k} = \frac{1}{3} - \frac{1}{3k+3}$ for each $k \geq 1$.

Remark 4 We restrict our analysis to the case in which the discount rates satisfy condition (4). Hence, in particular, for each $t \in \mathbb{N}$,

$$\sum_{k=2t+1}^{\infty} \delta_s(2t+1,k) \leq \frac{\Phi_s}{1-\Phi_s}, \quad \sum_{k=2t+2}^{\infty} \delta_b(2t+2,k) \leq \frac{\Phi_b}{1-\Phi_b} \tag{5}$$

3 The Results

3.1 Subgame perfect equilibrium of the model

First, we find the unique SPE if we restrict our analysis to strategies independent of the former history of the game and with no-delay. We introduce the following notation for every $t \in \mathbb{N}_+$

$$\Delta_s(t) = \frac{\sum_{k=t}^{\infty} \delta_s(t,k)}{1+\sum_{k=t}^{\infty} \delta_s(t,k)},\ \Delta_b(t) = \frac{\sum_{k=t}^{\infty} \delta_b(t,k)}{1+\sum_{k=t}^{\infty} \delta_b(t,k)} \tag{6}$$

Hence, we have also

$$1-\Delta_s(t) = \frac{1}{1+\sum_{k=t}^{\infty} \delta_s(t,k)},\ 1-\Delta_b(t) = \frac{1}{1+\sum_{k=t}^{\infty} \delta_b(t,k)} \tag{7}$$

and for every $t \in \mathbb{N}_+$

$$\Delta_s(t) \leq \Phi_s \text{ and } \Delta_b(t) \leq \Phi_b \tag{8}$$

Proposition 1 *Consider the price bargaining model in which preferences of the seller and the buyer are described by the sequences of discount factors $(\delta_{i,t})_{t\in\mathbb{N}}$, where $\delta_{i,0}=1$, $0<\delta_{i,t}<1$ for $t \geq 1$, $i=s,b$. Consider the following family of strategies (s_s, s_b):*
in each period $2t+1$ the seller accepts an offer y of the buyer if and only if $y \geq P_b^{2t+1}$, and in each period $2t$ the buyer accepts an offer x of the seller if and only if $x \leq P_s^{2t}$, where P_s^{2t} is an offer of the seller in $2t$ and P_b^{2t+1} is an offer of the buyer in $2t+1$.
Then (s_s, s_b) is a SPE of this game if and only if the offers satisfy the following infinite system of equations for each $t \in \mathbb{N}$:

$$R - P_s^{2t} + S = (R-D)\left(1-\Delta_b(2t+1)\right) + \left(R - P_b^{2t+1} + S\right)\Delta_b\left(2t+1\right) \tag{9}$$

$$P_b^{2t+1} - C = -C(1-\Delta_s(2t+2)) + \left(P_s^{2t+2} - C\right)\Delta_s\left(2t+2\right) \tag{10}$$

For the proof of Proposition 1, see the Appendix.

Proposition 1 presents necessary and sufficient conditions for the profile (s_s, s_b) to be a SPE. The first equation means that the buyer is indifferent between accepting the equilibrium offer of the seller and rejecting that offer. Similarly, the second equation expresses indifference of the seller between accepting and rejecting the equilibrium offer of the buyer. By solving the infinite system (9) and (10), we determine the equilibrium offers proposed under the strategies (s_s, s_b) and show that this SPE is unique.

Proposition 2 *Consider the price bargaining model with preferences of the seller and the buyer described by the sequences of discount factors $(\delta_{i,t})_{t\in\mathbb{N}}$, where $\delta_{i,0}=1$, $0<\delta_{i,t}<1$ for $t \geq 1$, $i=s,b$. Then there is the unique SPE of the form (s_s, s_b) stated in Proposition 1, in which the offers of the parties, for every $t \in \mathbb{N}$, are given by*

$$P_s^{2t} = (S+D)\left(1-\Delta_b(2t+1) + \sum_{m=t}^{\infty}(1-\Delta_b(2m+3))\prod_{j=t}^{m}\Delta_s(2j+2)\Delta_b(2j+1)\right) \tag{11}$$

$$P_b^{2t+1} = P_s^{2t+2}\Delta_s\left(2t+2\right) \tag{12}$$

For the proof of Proposition 2, see the Appendix.

We could expect that in the price negotiation model the agreed prices (P_s^{2t}) and $\left(P_b^{2t+1}\right)$ would depend on the reservation price R, the dissatisfaction cost D, the satisfaction value S, the production cost C and the discount factors $(\delta_{s,t})$ and $(\delta_{b,t})$, since in the literature they are usually supposed to be the reference points of the price determination. However, the results obtained in our model show that there is no dependence of the agreement price level on some of these determinants. More precisely, the offered prices at the equilibrium depend only on the sum of the dissatisfaction cost and satisfaction value of the buyer, and on the discount rates of both parties. In particular, this means that when proposing a price the seller does not care about his production cost but he does care about the (dis)satisfaction values of the buyer. The higher these values are, the higher the prices offered by the seller and the buyer are, i.e., if the buyer is highly attached to the product and the seller knows that, the seller will offer higher prices and the buyer will accept it. Moreover, the more patient the seller will be in the future, the higher the prices offered by both parties are.

In the market with only one seller and one buyer, both parties do not have any other alternatives and they want to reach an agreement quickly. If there were other buyers in the market that desired to buy the product, the monopolistic seller could make higher profits. On the other hand, if there were many sellers that wanted to sell their products, the buyer could find lower prices. The market with many sellers and buyers gives the perfect competition situation. In our model with one seller and one buyer, it seems natural that the price does not depend on the production cost or the reservation price. However, the reservation price which indicates the buyer's willingness to buy and the production cost of the seller will determine the payoffs of the parties in every period as defined in (1) and (2). Indeed, note that in a single period the sum of the agreement payoffs is equal to $(R + S - C)$ and the sum of the disagreement payoffs is equal to $(R - D - C)$.

3.2 On equilibrium payoffs for the general model

Next we relax the no-delay assumption that strategies are of the form (s_s, s_b), and we determine the highest SPE payoff of the seller and the lowest SPE payoff of the buyer for the general case when making an unacceptable offer is allowed.

Houba and Wen (2008) apply the method of Shaked and Sutton (1984) to the wage bargaining model of Fernandez and Glazer (1991) to derive the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs. We generalize this method to the price negotiation model with sequences of discount rates varying in time.

Let M_s^{2t} denote the supremum of the seller's SPE payoff in any even period $2t$, where the seller makes an offer. Let m_b^{2t+1} denote the infimum of the buyer's SPE payoff in any odd period $(2t + 1)$, where the buyer makes an offer.

First we will derive necessary conditions for M_s^{2t} and m_b^{2t+1}. We can notice that for every $t \in \mathbb{N}$

$$-C \leq M_s^{2t} \leq S + D - C, \quad R - D \leq m_b^{2t+1} \leq R + S$$

We have the following necessary conditions.

Proposition 3 *For all* $(\delta_{s,t})_{t\in\mathbb{N}}$, $(\delta_{b,t})_{t\in\mathbb{N}}$, $R \geq S \geq D \geq 0$, $0 < C \leq S + D$, *and* $t \in \mathbb{N}$,

$$M_s^{2t} \leq S + D - C + \left(R - D - m_b^{2t+1}\right) \Delta_b(2t + 1) \tag{13}$$

and

$$m_b^{2t+1} \geq R + S - \left(C + M_s^{2t+2}\right) \Delta_s(2t+2) \tag{14}$$

For the proof of Proposition 3, see the Appendix. It appears that under SPE neither the seller nor the buyer makes an unacceptable offer, as making the least irresistible offer gives always a higher payoff than proposing an unacceptable offer.

Next, from Proposition 3 we will calculate M_s^{2t} and m_b^{2t+1} for $t \in \mathbb{N}$.

Proposition 4 *For all* $(\delta_{s,t})_{t\in\mathbb{N}}$, $(\delta_{b,t})_{t\in\mathbb{N}}$, $R \geq S \geq D \geq 0$, $0 < C \leq S + D$, *and* $t \in \mathbb{N}$,

$$M_s^{2t} = (S+D)\left(1 - \Delta_b(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_b(2m+3)) \prod_{j=t}^{m} \Delta_s(2j+2)\Delta_b(2j+1)\right) - C \tag{15}$$

$$m_b^{2t+1} = R + S - (C + M_s^{2t+2})\Delta_s\,(2t+2) \tag{16}$$

For the proof of Proposition 4, see the Appendix.

Remark 5 Note that M_s^{2t} and m_b^{2t+1} calculated in Proposition 4 coincide with the results presented in Proposition 2 on the prices offered under the SPE with no-delay. Indeed, by combining Propositions 2 and 4 we get for each $t \in \mathbb{N}$,

$$M_s^{2t} = P_s^{2t} - C \text{ and } m_b^{2t+1} = R + S - P_b^{2t+1}$$

Consequently, the no-delay equilibrium strategies (s_s, s_b) presented in Proposition 2 support the extreme payoffs M_s^{2t} and m_b^{2t+1}.

4 Concluding remarks

Many of the previous studies in the literature focus on determining the reference points and did not reveal the optimal price between sellers and buyers. Although we make some restrictions in our model, we determine both the price level and the reference points that have impact on the price negotiation. We use complete information and sequential bargaining procedure where the preferences of the seller and the buyer vary in time. Using varying discount factors gives more possibilities for the characteristics of the parties and makes the model more realistic. Although preferences of the individuals may be constant while buying many consumption goods, for rare and/or privileged goods the parties' patience levels and preferences may vary during negotiations. Also some economic and social changes caused, for instance, by climate changes, epidemic increase, varying fashion requirements, make the preferences vary in time. Our generalized framework is therefore more suitable to model real-life situations.

Our results concern determining the unique SPE for no-delay strategies independent of the former history of the game and determining the equilibrium extreme payoffs of the seller and the buyer for the general case, i.e., without the restriction to no-delay strategies. It appears that the no-delay equilibrium strategy profiles support these extreme payoffs. Under equilibrium, neither the seller nor the buyer makes an unacceptable offers.

For the future agenda, we would like to apply this model to one of the important economic issues – pharmaceutical product price determination; see e.g. Jelovac (2005);

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Garcia-Marinoso et al. (2011). Although the drug market is quite complex, applying our monopolistic and monopsonistic model to pharmaceutical price negotiations would help to get a deeper insight into such negotiations. In the pharmaceutical product market, there are two main parties that negotiate for the price: state or an agency that represents the state and a firm that produces the drug. Although the marginal cost of drug production is very low, R&D expenses are relatively high in comparison with the other markets. Most of the patented drugs are produced only by one firm that creates a monopole in the market. Especially using discount rates varying in time has more importance in the drug market, where the consumers' patience levels vary according to the urgency of their illnesses and the producers' patience levels vary according to the risk of losing the market despite the high R&D expenses.

Appendix - Proofs

To make the paper self-contained, we present proofs of all the results stated in the present paper. Some of the proofs are very similar to the ones concerning wage bargaining with the exogenous 'always strike' decision and presented in Ozkardas and Rusinowska (2012).

Proof of Proposition 1

($\Leftarrow$) Let (s_p, s_c) be defined by (9) and (10), which can be equivalently written as

$$\left(R - P_s^{2t} + S\right) + \left(R - P_s^{2t} + S\right) \sum_{k=2t+1}^{\infty} \delta_b(2t+1, k) =$$

$$(R - D) + \left(R - P_b^{2t+1} + S\right) \sum_{k=2t+1}^{\infty} \delta_b(2t+1, k) \tag{17}$$

$$\left(P_b^{2t+1} - C\right) + \left(P_b^{2t+1} - C\right) \sum_{k=2t+2}^{\infty} \delta_s(2t+2, k) = -C + \left(P_s^{2t+2} - C\right) \sum_{k=2t+2}^{\infty} \delta_s(2t+2, k) \tag{18}$$

We show that (s_s, s_b) is a SPE.

Consider an arbitrary subgame starting in period $2t$ with the seller making an offer. Under (s_s, s_b), the seller gets $(P_s^{2t} - C) + (P_s^{2t} - C) \sum_{k=2t+1}^{\infty} \delta_s (2t+1, k)$ and the buyer gets $(R - P_s^{2t} + S) + (R - P_s^{2t} + S) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$. If the seller deviates from s_s and proposes a certain $x > P_s^{2t}$, then the seller gets $-C + \left(P_b^{2t+1} - C\right) \sum_{k=2t+1}^{\infty} \delta_s (2t+1, k)$. From (17), $0 \leq (D + S - P_s^{2t}) = \left(P_s^{2t} - P_b^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_b (2t+1, k)$, and hence $P_s^{2t} \geq P_b^{2t+1}$. The seller is then not better off by this deviation, because we have $(P_s^{2t} - C) + (P_s^{2t} - C) \sum_{k=2t+1}^{\infty} \delta_s (2t+1, k) \geq -C + \left(P_b^{2t+1} - C\right) \sum_{k=2t+1}^{\infty} \delta_s (2t+1, k)$.

Suppose that the seller deviates from s_s and proposes a certain $x < P_s^{2t}$. Then the seller gets $(x-C)+(x-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$, but he is worse off since $(x-C)+(x-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k) < (P_s^{2t}-C)+(P_s^{2t}-C)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$.

Suppose that the buyer deviates from s_b and rejects P_s^{2t}. Then he gets at most $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, which from (17) is equal to $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, so the buyer is not better off by this deviation.

The analysis of an arbitrary subgame starting in $2t+1$ with the buyer making an offer is analogous to the study of the subgame starting in $2t$, except that we use (18) instead of (17).

Consider an arbitrary subgame starting in period $2t$ with the buyer replying to an offer $x \leq P_s^{2t}$. Under (s_s, s_b) he accepts it and gets $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. A deviation from s_s does not change the result for the seller. Suppose that the buyer deviates from s_b and rejects such x. We know that it is optimal for the buyer to propose P_b^{2t+1} in $2t+1$, so the buyer gets $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. By virtue of (17), we have $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) \geq (R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) = (R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, and hence the buyer is not better off by this deviation.

Consider an arbitrary subgame starting in period $2t$ with the buyer replying to an offer $x > P_s^{2t}$. Under (s_s, s_b) the buyer rejects it and proposes P_b^{2t+1} which is accepted. The seller gets then $-C+\left(P_b^{2t+1}-C\right)\sum_{k=2t+1}^{\infty}\delta_s(2t+1,k)$ and the buyer gets $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1)$. If the buyer deviates from s_b and accepts such x, then it gets $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$. But from (17) we have $(R-x+S)+(R-x+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) < (R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k) = (R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$, so the buyer is worse off.

The analysis of subgame starting in $2t+1$ by the seller replying to an offer $y \geq P_b^{2t+1}$ and to an offer $y < P_b^{2t+1}$ is analogous to the analysis of the corresponding subgames starting in period $2t$ by the buyer replying to x.

($\Rightarrow$) Let (s_s, s_b) be a SPE. We will show that it must be defined by (17) and (18) which are equivalent to (9) and (10). Consider an arbitrary subgame starting in period $2t$ with the seller making an offer. Under (s_s, s_b) the seller proposes P_s^{2t} which is accepted and gives $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b(2t+1,k)$ to the buyer. By rejecting

P_s^{2t}, the buyer would get $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)$. Since (s_s,s_b) is a SPE, it must be $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)\geq(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)$. Suppose that the following holds $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)>(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)$. Then there exists $\widetilde{x}>P_s^{2t}$ with $(R-P_s^{2t}+S)+(R-P_s^{2t}+S)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)>(R-\tilde{x}+S)+(R-\tilde{x}+S)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)>(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)$. Since $\tilde{x}>P_s^{2t}$, the buyer rejects it and gets $(R-D)+\left(R-P_b^{2t+1}+S\right)\sum_{k=2t+1}^{\infty}\delta_b\left(2t+1,k\right)$, but he would be better off if he accepted this offer. Hence we get a contradiction and prove (17). Proving (18) is analogous by considering an arbitrary subgame starting in period $2t+1$ with the buyer making an offer. ■

Proof of Proposition 2

By virtue of Proposition 1, we need to solve the infinite system of equations (9) and (10), which can be equivalently written for each $t\in\mathbb{N}$, as

$$P_s^{2t}-P_b^{2t+1}\Delta_b\left(2t+1\right)=(S+D)(1-\Delta_b(2t+1)) \tag{19}$$

and

$$P_b^{2t+1}-P_s^{2t+2}\Delta_s\left(2t+2\right)=0 \tag{20}$$

From (20) we get immediately (12). In order to calculate P_s^{2t}, we use a similar matrix method as the one applied in Ozkardas and Rusinowska (2012) for the union-firm wage bargaining. The infinite system of (19) and (20) is a regular triangular system $AX=Y$, where $A=[a_{ij}]_{i,j\in\mathbb{N}^+}$, $X=\left[(x_i)_{i\in\mathbb{N}^+}\right]^T$, $Y=\left[(y_i)_{i\in\mathbb{N}^+}\right]^T$, for each $t,j\geq1$

$$a_{t,t}=1,\quad a_{t,j}=0 \text{ for } j<t \text{ or } j>t+1 \tag{21}$$

for each $t\in\mathbb{N}$

$$a_{2t+1,2t+2}=-\Delta_b\left(2t+1\right),\quad a_{2t+2,2t+3}=-\Delta_s\left(2t+2\right) \tag{22}$$

$$x_{2t+1}=P_s^{2t},\ x_{2t+2}=P_b^{2t+1},\ y_{2t+1}=(S+D)(1-\Delta_b(2t+1)),\quad y_{2t+2}=0 \tag{23}$$

Any regular triangular matrix A possesses the (unique) inverse matrix B, which is also regular triangular. In other words, there exists $B=[b_{ij}]_{i,j\in\mathbb{N}^+}$ such that $BA=I$, where I is the infinite identity matrix, and

$$b_{t,t}=1,\ b_{t,j}=0 \text{ for each } t,j\geq1 \text{ such that } j<t \tag{24}$$

for each $t\in\mathbb{N}$

$$b_{2t+1,2t+2}=\Delta_b\left(2t+1\right),\quad b_{2t+2,2t+3}=\Delta_s\left(2t+2\right) \tag{25}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_s(2j+2)\,\Delta_b(2j+3) \tag{26}$$

$$b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_s(2j+2)\,\Delta_b(2j+3)\,\Delta_s(2m+2) \tag{27}$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_s(2j+2)\,\Delta_b(2j+1) \tag{28}$$

$$b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_s(2j+2)\,\Delta_b(2j+1)\,\Delta_b(2m+1) \tag{29}$$

We have then

$$\begin{bmatrix} 1 & -\Delta_b(1) & 0 & 0 & \cdots \\ 0 & 1 & -\Delta_s(2) & 0 & \cdots \\ 0 & 0 & 1 & -\Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} P_s^0 \\ P_b^1 \\ P_s^2 \\ P_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} (S+D)(1-\Delta_b(1)) \\ 0 \\ (S+D)(1-\Delta_b(3)) \\ 0 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} P_s^0 \\ P_b^1 \\ P_s^2 \\ P_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & \Delta_b(1) & \Delta_b(1)\Delta_s(2) & \Delta_b(1)\Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 1 & \Delta_s(2) & \Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 0 & 1 & \Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} (S+D)(1-\Delta_b(1)) \\ 0 \\ (S+D)(1-\Delta_b(3)) \\ 0 \\ \vdots \end{bmatrix}$$

and hence we get P_s^{2t} as given by (11).

Note that $P_s^{2t}, P_b^{2t+1} \in [0, S+D]$ for each $t \in \mathbb{N}$. Obviously $P_s^{2t} \geq 0$. Let us consider the sequence of partial sums for $k > t$:

$$S_k = (S+D)\left(1 - \Delta_b(2t+1) + \sum_{m=t}^{k-1}(1-\Delta_b(2m+3))\prod_{j=t}^{m}\Delta_s(2j+2)\Delta_b(2j+1)\right)$$

The sequence is obviously increasing, and also $S_k \leq S+D$ for each $k > t$. Hence, $P_s^{2t} = \lim_{k\to+\infty} S_k \leq S+D$. ■

Proof of Proposition 3

Necessary condition for M_s^{2t}

Consider an arbitrary even period $2t$. The seller makes either an unacceptable offer or an irresistible offer. If the buyer rejects the seller's offer, then he will get at least $(R-D)(1-\Delta_b(2t+1))+m_b^{2t+1}\Delta_b(2t+1)$. Hence, the seller gets at most $R+S-C-(R-D)(1-\Delta_b(2t+1))-m_b^{2t+1}\Delta_b(2t+1)$ from making the least acceptable offer. Alternatively, the seller gets at most $-C(1-\Delta_s(2t+1))+(R+S-C-m_b^{2t+1})\Delta_s(2t+1)$ from making an unacceptable offer. Hence, we get

$$M_s^{2t} \leq \max \begin{cases} R+S-C-(R-D)(1-\Delta_b(2t+1)-m_b^{2t+1}\Delta_b(2t+1) \\ -C(1-\Delta_s(2t+1))+(R+S-C-m_b^{2t+1})\Delta_s(2t+1) \end{cases} \tag{30}$$

which can be equivalently written as

$$M_s^{2t} \leq \max \begin{cases} S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1) \\ -C+\left(R+S-m_b^{2t+1}\right)\Delta_s(2t+1) \end{cases} \tag{31}$$

which leads to

$$M_s^{2t} \leq \begin{cases} S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1) & \text{if (33)} \\ -C+\left(R+S-m_b^{2t+1}\right)\Delta_s(2t+1) & \text{otherwise} \end{cases} \tag{32}$$

where

$$S(1-\Delta_s(2t+1))+D(1-\Delta_b(2t+1)) \geq \left(R-m_b^{2t+1}\right)(\Delta_s(2t+1)-\Delta_b(2t+1)) \tag{33}$$

However, we can show that (33) always holds.

Let $\Delta_s(2t+1) \leq \Delta_b(2t+1)$. We know that $-S \leq R-m_b^{2t+1} \leq D$. If $0 \leq R-m_b^{2t+1} \leq D$, then the right hand side of (33) is not positive. Hence, since the left hand side of (33) is not negative, (33) holds. If $-S \leq R-m_b^{2t+1} < 0$, then we have
$0 \leq \left(R-m_b^{2t+1}\right)(\Delta_s(2t+1)-\Delta_b(2t+1)) \leq -S(\Delta_s(2t+1)-\Delta_b(2t+1)) = S(\Delta_b(2t+1)-\Delta_s(2t+1)) \leq S(1-\Delta_s(2t+1)) \leq S(1-\Delta_s(2t+1))+D(1-\Delta_b(2t+1))$, and therefore (33) also holds.

Let $\Delta_s(2t+1) > \Delta_b(2t+1)$. If $-S \leq R-m_b^{2t+1} < 0$, then the right hand side of (33) is negative, and therefore (33) holds, since the left hand side of (33) is not negative. If $0 \leq R-m_b^{2t+1} \leq D$, then we have
$0 \leq \left(R-m_b^{2t+1}\right)(\Delta_s(2t+1)-\Delta_b(2t+1)) \leq D(\Delta_s(2t+1)-\Delta_b(2t+1)) \leq D(1-\Delta_b(2t+1)) \leq S(1-\Delta_s(2t+1))+D(1-\Delta_b(2t+1))$, and therefore (33) also holds.

Necessary condition for m_b^{2t+1}

Consider an arbitrary odd period $2t+1$. The buyer makes either an unacceptable offer or an irresistible offer. If the seller rejects the buyer's offer, then he will get at most $-C(1-\Delta_s(2t+2))+M_s^{2t+2}\Delta_s(2t+2)$. Hence, the buyer gets at least $R+S-C+C(1-\Delta_s(2t+2))-M_s^{2t+2}\Delta_s(2t+2)$ from making the least irresistible offer. Alternatively, the buyer gets at least $(R-D)(1-\Delta_b(2t+2))+(R+S-C-M_s^{2t+2})\Delta_b(2t+2)$ from making an unacceptable offer. Hence, we get

$$m_b^{2t+1} \geq \max \begin{cases} R+S-(C+M_s^{2t+2})\Delta_s(2t+2) \\ R-D+(S+D-C-M_s^{2t+2})\Delta_b(2t+2) \end{cases} \tag{34}$$

which leads to

$$m_b^{2t+1} \geq \begin{cases} R+S-\left(C+M_s^{2t+2}\right)\Delta_s(2t+2) & \text{if (36)} \\ R-D+\left(S+D-C-M_s^{2t+2}\right)\Delta_b(2t+2) & \text{otherwise} \end{cases} \tag{35}$$

where

$$(S+D)\left(1-\Delta_b(2t+2)\right) \geq \left(C+M_s^{2t+2}\right)\left(\Delta_s(2t+2)-\Delta_b(2t+2)\right) \tag{36}$$

However, note that (36) is always satisfied, since $S+D \geq C+M_s^{2t+2}$ and $1-\Delta_b(2t+2) \geq \Delta_s(2t+2)-\Delta_b(2t+2)$. This completes the proof. ■

Proof of Proposition 4

When looking for the upper bound of M_s^{2t} and the lower bound of m_b^{2t+1}, we need to solve the following infinite system: for each $t \in \mathbb{N}$

$$M_s^{2t} = S+D-C+\left(R-D-m_b^{2t+1}\right)\Delta_b(2t+1)$$

and

$$m_b^{2t+1} = R+S-\left(C+M_s^{2t+2}\right)\Delta_s(2t+2)$$

Hence, we get immediately (16), and if $-C \leq M_s^{2t} \leq S+D-C$, then $R-D \leq m_b^{2t+1} \leq R+S$. Furthermore, we have

$$\begin{bmatrix} 1 & \Delta_b(1) & 0 & 0 & \cdots \\ 0 & 1 & \Delta_s(2) & 0 & \cdots \\ 0 & 0 & 1 & \Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} M_s^0 \\ m_b^1 \\ M_s^2 \\ m_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} S+D-C+(R-D)\Delta_b(1) \\ R+S-C\Delta_s(2) \\ S+D-C+(R-D)\Delta_b(3) \\ R+S-C\Delta_s(4) \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} M_s^0 \\ m_b^1 \\ M_s^2 \\ m_b^3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & -\Delta_b(1) & \Delta_b(1)\Delta_s(2) & -\Delta_b(1)\Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 1 & -\Delta_s(2) & \Delta_s(2)\Delta_b(3) & \cdots \\ 0 & 0 & 1 & -\Delta_b(3) & \cdots \\ 0 & 0 & 0 & 1 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} S+D-C+(R-D)\Delta_b(1) \\ R+S-C\Delta_s(2) \\ S+D-C+(R-D)\Delta_b(3) \\ R+S-C\Delta_s(4) \\ \vdots \end{bmatrix}$$

which gives us (15). Obviously, $M_s^{2t} \geq -C$, and similarly to the proof of Proposition 2, one can show that $M_s^{2t} \leq S+D-C$. ■

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