Carbon tax and OPEC's rents under a ceiling constraint

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Abstract: We study the MPNE of a game between oil importing countries seeking to maintain atmospheric carbon concentration under a given ceiling and exporting countries, when the former sets a carbon tax and the latter controls the producer price. We obtain implicit feedback rules and explicit non-linear time paths of extraction, carbon tax and producer price. Consumers are always able to reap some part of the scarcity and monopoly rents, whereas producers partially preempt the carbon tax only if the marginal damage under the ceiling is small. We compare the MPNE to the efficient, open loop and cartel without tax equilibria.

Keywords: global warming, non-renewable resources, differential games, non-linear strategies.

JEL classification: C73, Q30, Q4.

I Introduction[1]

Three key lessons stand out from the Copenhagen Summit in December 2009. First, the agreement recognizes the need for the temperature rise to stay below 2 degrees Celsius, which is usually associated with a greenhouse gas concentration ceiling of 450 ppm (IPCC (2007)). This shared objective is a significant progress obtained in Copenhagen. The two other lessons are less constructive: the inability of the largest emitting countries to reach even a basic effective agreement on an international policy architecture designed to aim towards this common objective; and OPEC's hostility to any international agreement which would finally result in a sensible contraction of world oil demand. OPEC repeatedly argues that climate policy is still another excuse to steal the oil rent to finance public spending in oil importing countries, and even asks for compensation for the reduction of its income.

That OPEC does not contemplate introducing a climate policy, and reacts negatively when oil importing countries do, is not entirely surprising. As Wirl and Dockner (1995) mentioned, an example of such strategic OPEC's reaction is the oil price increase of $4 per barrel in the 1992's first-half, matching the first step of the EEC proposal of a hybrid energy-carbon tax. The crude oil price increase allows OPEC to make climate policy to a certain extent useless since it may be sufficient to trigger the demand decrease desired by consuming countries. By doing so, OPEC captures a part of what we can call the carbon or climate rent.

However, oil consuming countries can also behave as a coalition and adopt a carbon

[1]The authors acknowledge the support of the French National Research Agency (ANR) under the CLEANER project (ANR_NT09_505778). We are grateful to two anonymous referees for helpful suggestions and comments.

tax which allows them to reap some part of the oil rent[2]. A global climate agreement of oil importing countries coordinating their carbon taxation could be interpreted as a consumers' cartel, would it happen to exist[3]. The implementation of a high carbon tax could prompt OPEC to lower its producer price in order to limit the decrease of oil demand.

Since the increases in oil price between 1998 and 2008, and especially the very strong increase of summer 2008, many OECD countries have led the idea that the government must control the consumer price of oil and that, to do so, taxes on energy should be "floating", or "additional", to use two terms coined in France: they should decrease when the producer price increases, and *vice versa*. This proposal is clearly a bad one, as OECD (2006) points out, for strategic reasons: "If oil importers start to reduce taxes in order to stabilise tax-inclusive fuel prices, oil exporters will know that they can at no risk increase their resource rents by restraining their production and thus increase crude prices further. Normally such actions would trigger reductions in demand that could reduce the incomes of the oil producers, but the demand reductions will be absent if tax-inclusive user prices are kept stable by tax reductions."

What is the result of the previous two-sided strategic behaviour? Does the producers' cartel capture the climate rent or does the consumers' cartel capture the scarcity and monopoly rents? Is the time path of extraction more or less conservative than what would

[2]Besides objectives of public spending financing and energy savings, the high levels of fuel taxes in the European countries can be viewed also as an attempt to capture a part of the scarcity and monopoly rents. On average, between one-third and one-half of the total price of unleaded gasoline is excise tax.

[3]Some oil consuming countries have already implemented carbon taxes: Finland, Denmark, Norway and Sweden since 1991, Switzerland in 2008, but these policies remain isolated. Cap and trade schemes have the same consequences but even the largest existing system, the EU-ETS, cannot be interpreted as a global agreement on climate policy.

be optimal? What are the consequences of strategic climate policy in terms of welfare and in terms of distribution among the two coalitions? These are the main questions we address in this paper.

They have to some extent already been addressed from a theoretical point of view by Wirl (1994), (1995), Tahvonen (1995), (1996), Rubio and Escriche (2001) and Liski and Tahvonen (2004). These papers solve a similar differential game between the producers' and the consumers' cartels, in a framework where global warming creates damages to consumers' welfare. They study the Markov Perfect Nash Equilibrium (MPNE) of the game, and compare it to some benchmarks.

Liski and Tahvonen (2004) "consolidate, clarify, and extend" the results obtained in these papers. They show that the optimal design of the carbon tax in the presence of two-sided strategic interactions generally deviates from a Pigouvian tax that internalizes only the environmental damage. Moreover, considering linear strategies, they solve explicitly the MPNE in the linear-quadratic case, and prove that when the damage is not too severe, the carbon tax shifts more rents than what is necessary to internalize the environmental externality whereas it is the contrary when the damage is large. They also study the time profile of the carbon tax and the producer price. If the damage is small, the carbon tax is decreasing with time and the producer price is increasing; if the damage is intermediate, both are increasing; if the damage is large, the carbon tax is increasing and the producer price decreasing. Finally, they conjecture that the sellers' payoff is always – whatever the damage is – lower and the buyers' payoff higher in the MPNE than in the pre-tax situation.

The aim of the present paper is to delve further into this question. We revisit the differential game between oil consuming and producing countries when the former shape their climate policy to take into account the existence of a physical upper limit on atmospheric carbon concentration, which overtaking would lead to catastrophic consequences. The level of this ceiling is based on scientific evidence. It has been set at 450 parts per million carbon dioxide equivalent by the Intergovernmental Panel on Climate Change in 2007, in its Fourth Assessment Report (IPCC (2007)), target which would provide a "reasonable chance" of averting warming beyond 2°C above pre-industrial temperature. Following the work of Allen *et al.* (2009) and Meinshausen *et al.* (2009), it has been recently argued that the correct way of thinking about global warming is in terms of a global "CO_2 emission budget", which overtaking would trigger temperature rises above 2°C, because of all the uncertainties in the carbon cycle, the climate response to an increase in the atmospheric carbon concentration, and the natural decay. This global carbon budget is exactly equivalent to a cap on carbon atmospheric concentration, provided that natural carbon absorption by sinks is negligible. The precise level of the ceiling is of course dependent on current scientific knowledge and likely to evolve. We shall nevertheless consider it as fixed. Finally, imposing a ceiling constraint beyond which damages become infinite does not mean that there exists no damage before the ceiling. Consistently with the existence of the ceiling, we assume that these damages are relatively small. We then combine in this paper a ceiling on atmospheric carbon concentration and a "small" damage function before (and at) the ceiling.

This ceiling framework was introduced in the theoretical litterature on optimal fossil resources extraction and global warming by Chakravorty, Magné and Moreaux (2006a), (2006b). Its introduction in a differential game raises a number of technical problems. Whereas in general linear-quadratic specifications make it possible to obtain explicitly a solution of the game, the linear one, the ceiling constraint introduces here an intrinsic non-linearity in the problem and we cannot expect to obtain a linear solution. We are nevertheless able to obtain a solution of the MPNE, which is non-linear .

We obtain implicit feedback rules and explicit time paths of extraction, carbon tax and producer price. We show that the producer and consumer prices are monotonically increasing in time, whereas the carbon tax may be first decreasing and is always increasing near the ceiling. As far as rent capture is concerned, we show that the carbon tax always shifts more rent than necessary for an environmental motive, meaning that consumers are able to reap some part of the scarcity and monopoly rents, and that producers are able to preempt to some extent the carbon tax only if the marginal damage under the ceiling is small.

We then compare the MPNE and three benchmarks. The comparison with the efficient equilibrium allows us to assess to what extent the carbon tax of the MPNE departs from the tax of the efficient equilibrium, designed to correct the environmental problem only. The comparison with the open loop equilibrium highlights the impact of the producers' and consumers' feedback strategies. The comparison with the cartel without carbon tax equilibrium which, as argued by Liski and Tahvonen (2004) is the proper benchmark since it is the pre-tax situation, allows us to see whether consumers gain in terms of welfare when they adopt a common environmental policy.

The paper is organized as follows. Section 2 presents the assumptions of the model, solves the Markov-perfect Nash equilibrium and studies its properties. Section 3 studies the three benchmarks against which the properties of the MPNE will be assessed. The equilibria are compared in Section 4. Section 5 concludes.

II Markov-perfect Nash Equilibrium

We study the strategic interactions between a cartel of fossil fuel producers and a coalition of consumers coordinating their carbon emission taxation to fight global warming, in a differential games framework of analysis. Whereas producers set the fossil fuel price, consumers set the carbon tax in order to meet an atmospheric carbon concentration constraint based on scientific knowledge and taken as given, and to internalize the damage caused by the rise in temperature.

Consumers' area

In the consumers' area, the utility of the representative consumer is derived from the use of the fossil resource. The utility function is denoted $u(x_t)$, where x is the consumed resource flow (let's say oil). It is assumed to be quadratic and concave:

$$u(x) = ax - \frac{b}{2}x^2, \quad a > 0, \quad b > 0. \tag{1}$$

$u'(0) = a$ is the choke price, for which demand becomes nil.

The initial resource stock is X_0, the stock still in the ground at date t is X_t. The additional (to the pre-industrial level) atmospheric carbon concentration at date t is Z_t. We assume that natural carbon absorption is nil, so that the additional atmospheric carbon concentration at date t is strictly equal to the stock yet extracted and burnt at this date: $Z_t = X_0 - X_t$. This assumption has two justifications: firstly, natural absorption by sinks (oceans, forests) is uncertain and likely to decrease while carbon concentration increases; secondly and more technically, it allows us to consider only one stock in the problem and obtain tractable solutions.

Climate policy takes the form of a ceiling constraint: $X_0 - X_t \leq \overline{Z}$ where $\overline{Z}$ is the ceiling on the additional carbon stock. $\overline{Z}$ is a physical constraint, and can be seen as the global carbon budget allowing humanity to contain the rise of temperature under 2°C. But this does not mean of course that the rise of temperature does not cause any damage before the ceiling is reached. To be consistent with the choice of the value of the ceiling, it can be assumed that these damages are small – otherwise the ceiling would have been ill-chosen. We then introduce the damage that indeed appears before the ceiling, and, following Amigues *et al.* (2011), we suppose this damage linear:

$$D(X_t) = d(X_0 - X_t), \quad d > 0. \tag{2}$$

The buyers' regulator sets a unit carbon tax for the area to control the pollution accumulation due to the resource use, taking into account the demand function of the representative consumer, given by:

$$u'(x) = p + \theta, \tag{3}$$

where p is the producer price and θ the carbon tax. The regulator maximizes on the whole horizon the discounted net surplus, difference between the consumers' utility and the amount paid to producers, subject to the law of carbon accumulation in the atmosphere and the ceiling constraint. Tax revenues are reimbursed as lump-sum transfers to consumers, so they do not appear into the net surplus.

The two players act simultaneously, and the game is played with feedback (Markovian) strategies. The buyers' regulator, when choosing the carbon tax θ, takes into account the fact that the producer price depends on the resource stock, that is $p(X)$.

The buyers' regulator problem is[4]:

$$V_c^m(X_0) = \max_{\theta_t} \int_0^\infty e^{-\rho t} \left[u\left(x\left(p\left(X_t\right)+\theta_t\right)\right) - p_t x\left(p\left(X_t\right)+\theta_t\right) - D\left(X_t\right)\right] dt$$

$$\text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x\left(p\left(X_t\right)+\theta_t\right) \\ X_0 - X_t \leq \overline{Z} \\ X_0,\ \overline{Z} \text{ given.} \end{array} \right. \tag{4}$$

Let λ_c be the shadow price of the resource and ω the Lagrange multiplier associated to the ceiling constraint. First order optimality conditions and the complementarity slackness condition read[5]:

$$\theta_t = \lambda_{ct} \tag{5}$$

$$\dot{\lambda}_{ct} = \rho\lambda_{ct} + p'(X_t)x_t + D'(X_t) - \omega_t \tag{6}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ct}X_t = 0 \tag{7}$$

$$\omega_t \geq 0,\ X_t - X_0 + \overline{Z} \geq 0,\ \omega_t(X_t - X_0 + \overline{Z}) = 0. \tag{8}$$

Let T_m be the date at which the ceiling becomes binding. Given the structure of the problem, if the initial atmospheric carbon concentration is lower than the ceiling, which

[4]Superscript m for Markov. We will use in what follows the superscripts o for optimum/efficient equilibrium, ol for open loop equilibrium and c for cartel without tax equilibrium.

[5]Conventionally, Hamilton–Jacobi–Bellman equations are used to solve the MPNE because they yield directly strategies depending on the state variable, here X. We have prefered to use here the maximum principle, because the value function is potentially non-differentiable at the ceiling, and consequently discontinuities of the feedback rule at the ceiling are possible. The use of the maximum principle allows us to overcome this difficulty. Notice that the non-differentiability of the value function at the ceiling is equivalent to the discontinuity of the stock shadow price at the ceiling. We study this potential discontinuity in the on-line Appendix B and show that indeed it exists. We solve the same problem using Hamilton–Jacobi–Bellman equations in the on-line Appendix E, to convince the reader that we actually obtain the same strategies.

we assume, the ceiling will not be binding before T_m ($\omega_t = 0 \ \forall t < T_m$), and will remain binding forever after T_m ($\omega_t > 0 \ \forall t \geq T_m$). After the ceiling is reached, the resource consumer price, sum of the producer price and the carbon tax, remains equal to $u'(0)$, the price at which demand is choked, and $x = 0$.

Before the ceiling i.e. $\forall t < T_m$, equations (5) and (6) yield the following evolution of the carbon tax:

$$\dot{\theta}_t = \rho\theta_t + p'(X_t)x_t + D'(X_t). \tag{9}$$

(9) integrates into:

$$\theta_t = \left(e^{-\rho T_m}\theta_{T_m-}\right)e^{\rho t} - \int_t^{T_m} e^{-\rho(s-t)}D'(X_s)ds - \int_t^{T_m} e^{-\rho(s-t)}p'(X_s)x_s ds. \tag{10}$$

The carbon tax is the sum of three terms.

The first term on the right-hand side of equation (10) is what we may name the "pure Hotellinian tax". It comes from the existence of the ceiling constraint and grows at the discount rate. The second term represent the “pure Pigouvian tax” of Liski and Tahvonen (2004). It is due to the existence of damages before the ceiling and is the usual discounted sum of future marginal damages. Without ceiling and small damage, T_m would tend to infinity and both terms would be nil[6]. There would then exist no environmental motive for taxation.

The third term on the right-hand side of (10) is the strategic component of the carbon tax. It represents the “pure import tariff” of Liski and Tahvonen (2004) if $p'(.) < 0$, and their “pure import subsidy” if $p'(.) > 0$. In the first case, the carbon tax is positive even absent any damage. Consumers tax oil more heavily than the environmental motive would require, and are thus able to reap to some extent the scarcity and monopoly rents

[6]This comes from the transversality condition (7), since X_t would tend in the long run to a strictly positive value because of assumption (15) on the extraction cost function.

of producers. In the second case, consumers subsidize oil consumption to correct the monopoly distorsion.

After the ceiling i.e. $\forall t \geq T_m$, equations (5) and (6) yield the following evolution of the carbon tax:

$$\dot{\theta}_t = \rho\theta_t + D'(X_t) - \omega_t. \tag{11}$$

Using the transversality condition (7), (11) integrates into:

$$\theta_t = \int_t^{\infty} e^{-\rho(t-s)}\omega_s ds - \int_t^{\infty} e^{-\rho(s-t)} D'(X_s) ds. \tag{12}$$

Producers' area

Producers face a unit extraction cost depending on the resource stock still in the ground. The smaller this stock the higher the marginal extraction cost: the last drop of oil is very costly to extract. More precisely, the unit extraction cost is $c(X_t)$, with $c(X) > 0$, $c'(X) < 0$. We use the following linear specification:

$$c\left(X\right) = c_1 - c_2 X, \quad c_1 > 0, \quad c_2 > 0. \tag{13}$$

We make the assumption that initial extraction is profitable:

$$c(X_0) < u'(0) \Leftrightarrow X_0 > \frac{c_1 - a}{c_2}. \tag{14}$$

With a constant marginal extraction cost, scarcity is purely physical. Here, scarcity can be economic, in the sense that the marginal cost of extraction of the last drop of oil can be higher than the choke price. Following Liski and Tahvonen (2004), we therefore assume that economic scarcity is binding:

$$c(0) > u'(0) \Leftrightarrow c_1 > a. \tag{15}$$

Then the last drop will never be extracted; producers will stop extraction before and leave some oil in the ground. Without any environmental constraint – neither ceiling nor damage in our framework, they will leave in the ground a stock X_∞ defined by:

$$c(X_\infty) = u'(0) \Leftrightarrow X_\infty = \frac{c_1 - a}{c_2}. \tag{16}$$

We assume that producers do not intend to adopt any climate policy, but are perfectly aware that consumers do.

The sellers' regulator, when choosing the producer price, maximizes on the whole horizon its discounted profits, subject to the law of evolution of the resource stock:

$$V_p^m(X_0) = \max_{p_t} \int_0^\infty e^{-\rho t}\left(p_t - c(X_t)\right) x\left(p_t + \theta(X_t)\right) dt$$

$$\text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x\left(p_t + \theta(X_t)\right) \\ X_0 \text{ given.} \end{array} \right. \tag{17}$$

Producers are assumed to be aware of the reaction function of buyers to the state variable, $\theta(X)$.

The first order conditions give the producers' price strategy and the evolution of the shadow price of the resource λ_p, together with the transversality condition:

$$p_t = c(X_t) - \frac{x\left(p_t + \theta(X_t)\right)}{x'\left(p_t + \theta(X_t)\right)} + \lambda_{pt} \tag{18}$$

$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + (c'(X_t) + \theta'(X_t))x\left(p_t + \theta(X_t)\right) \tag{19}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt}X_t = 0. \tag{20}$$

In equation (18), $c(X) - x(.)/x'(.)$ is the static component of the monopoly price. The scarcity rent must be added to this static component.

Integrating equation (19) forward before the ceiling ($t < T_m$) yields:

$$\lambda_{pt} = \left(e^{-\rho T_m}\lambda_{pT_m-}\right)e^{\rho t} - \int_t^{T_m} e^{-\rho(s-t)}(c'(X_s) + \theta'(X_s))x_s ds. \tag{21}$$

The scarcity rent exhibits a Hotelling component, and a second component expressing the fact that a marginal amount of stock extracted at a given date s affects future profits by $c'(X_s)+\theta'(X_s)$, because of increased extraction costs and the variation of the consumers tax rate, which must be reflected in the current price. While the extraction cost effect clearly reduces future profits and increases the scarcity rent and today's price, the tax effect is at this stage ambiguous.

With the specification (1) adopted for the utility function, the demand function is:

$$x(p+\theta) = \frac{a-(p+\theta)}{b}, \tag{22}$$

and (18) reads before the ceiling:

$$p = \frac{1}{2}\left[c(X) + a - \theta(X) + \lambda^p\right]. \tag{23}$$

This formulation highlights the two effects of the carbon tax on the producer price: a negative static effect through the monopoly price (when θ increases, the cartel decreases p to support demand, and *vice versa*), and a dynamic effect through the scarcity rent. The sign of this last effect is at this stage indeterminate. If it is positive, it can be interpreted as a preemptive behaviour of the cartel: when the cartel extracts one unit of oil it knows that it will increase the atmospheric carbon concentration and that consequently the consumers' coalition will increase the carbon tax, and it reacts by increasing the producer price.

Consider now what happens after the ceiling is reached. Integrating (19) forward after T_m and using the transversality condition (20) shows that the scarcity rent is nil after the ceiling, which makes sense since the resource won't be extracted anymore. Then (18) yields $p_t = c(X_0 - \overline{Z}) = \overline{p}$, $t \geq T_m$. Then, since $p_t + \theta_t = u'(0)$ $t \geq T_m$, $\theta_t = u'(0) - \overline{p} = \overline{\theta}$, $t \geq T_m$.

Notice that nothing insures neither that the scarcity rent is continuous at the ceiling, i.e. that $\lambda_{pT_m-} = 0$, nor that the producer price and the carbon tax are.

The non-linear solution

In general, the Markov Perfect Nash Equilibria of differential games are studied under the assumptions of quadratic objective functions (utility, profit) and linear cost functions. Linear solutions are then the only ones that can be computed throughout analytically, even if other solutions may exist. But here, the ceiling constraint introduces an intrinsic non-linearity in the problem. Hence it is natural that the linear solution holds only if this constraint is not binding, that is if the ceiling is sufficiently high, so that the increase in unit extraction cost triggers the stop of extraction for economic reasons before the ceiling is reached. We make the opposite assumption that the ceiling is sufficiently low, so that it requires to stop extraction before the date at which it would have stopped spontaneously, and to leave more fossil fuel in the ground. The study of the linear solution, valid under the assumption of a high ceiling, is relegated to the on-line Appendix A.

The equilibrium of the game is characterized by equations (5) and (23) giving the carbon tax and the producer price depending on the shadow price of the resource respectively for consumers and producers, and equations (6) and (19) giving the evolution of these shadow prices.

Considering the shadow prices λ_c and λ_p as functions of the resource stock, notice that $\dot{\lambda}_c = \lambda_c'(X)\dot{X} = -\lambda_c'(X)x$, and that the same holds for $\dot{\lambda}_p$. Then the sum of equations (9) giving the evolution of the carbon tax before the ceiling and (19) giving the evolution of the scarcity rent yields, at the equilibrium:

$$\rho\left(\theta(X) + \lambda_p(X)\right) + (p'(X) + c'(X) + 2\theta'(X) + \lambda_p'(X))x + D'(X) = 0. \qquad (24)$$

Differentiating (23) w.r.t. X and replacing in (24) yields:

$$\rho\left(\theta(X)+\lambda_p(X)+\frac{D'(X)}{\rho}\right)+\frac{3}{2}(c'(X)+\theta'(X)+\lambda_p'(X))x=0. \tag{25}$$

Wirl and Dockner (1995) suggests (in a somewhat different framework) to introduce a proper sum of the relevant variables in order to help solving the problem. One possibility here turns out to be:

$$\Phi\left(X\right)=c(X)+\theta(X)+\lambda_p(X)+\frac{D'(X)}{\rho}. \tag{26}$$

Equation (25) now reads:

$$\rho\left(\Phi(X)-c(X)\right)+\frac{3}{2}\Phi'(X)x=0, \tag{27}$$

while, from (23) and using the assumption of a constant marginal damage (2),

$$p(X)+\theta(X)=\frac{1}{2}\left[a+\frac{d}{\rho}+\Phi(X)\right] \tag{28}$$

which can also be written:

$$a-bx=\frac{1}{2}\left[a+\frac{d}{\rho}+\Phi(X)\right]. \tag{29}$$

By elimination of x, (27) and (29) yield a non-linear differential equation in X:

$$\Phi'\left(X\right)=\frac{4\rho b}{3}\left(\frac{\Phi(X)-c\left(X\right)}{\Phi(X)-\left(a-\frac{d}{\rho}\right)}\right). \tag{30}$$

We assume that $a>d/\rho$ which means that at date 0 the choke price is higher than the discounted value of the marginal damage or, to put it differently, that the marginal damage before the ceiling is small.

We show in the on-line Appendix A that a linear solution to this equation exists, but that it is valid only in the case where the ceiling constraint is not binding. We want to find a solution in the case of a sufficiently stringent ceiling, in the sense that the oil stock that is left in the ground with the ceiling constraint is larger than the one that would be left without it (but taking into account the existence of the damage):

$$X_0 - \overline{Z} > \frac{c_1 - \left(a - \frac{d}{\rho}\right)}{c_2} = \widetilde{X}. \tag{31}$$

This solution has to be non-linear. Moreover, it is unique. Indeed, as shown by Tsutsui and Mino (1990), Dockner and Long (1993), Rowat (2007) and Wirl (2007), when the ceiling is not binding, (uncountably many) non-linear strategies coexist with the linear strategy. They all admit as a steady state an atmospheric carbon concentration lower than the one reached asymptotically by the Markov linear strategy, which amounts to say that the oil stock ultimately left in the ground is higher than $\widetilde{X}$. These non-linear strategies are Pareto-inferior to the linear one (Wirl and Dockner (1995)). But in our game the boundary condition (the ceiling) allows us to pin down the unique solution, among the non-linear ones, which admits as a steady state an atmospheric carbon concentration equal to $\overline{Z}$. The boundary condition rules multiple equilibria out.

It is not possible to find $\Phi(X)$ analytically, so we resort to a phase diagram (Figure 1).

< Figure 1 about here >

On the phase diagram, $\Phi'(X) = 0 \Leftrightarrow \Phi(X) = c(X)$; the admissible zone is under the line $\Phi(X) = a - d/\rho$, since $p + \theta = (a + d/\rho + \Phi(X))/2 < a \Rightarrow \Phi(X) < a - d/\rho$; $\Phi'(X) < 0$ when $\Phi(X) > c(X)$, and $\Phi'(X) > 0$ when $\Phi(X) < c(X)$; finally, $\Phi'(X) \to -\infty \Leftrightarrow \Phi(X) = a - d/\rho$. Starting from $\Phi(X_0) > c(X_0)$, the unique stable arm is travelled along from the right to the left, towards the equilibrium $\Phi(X_0 - \overline{Z}) = a - d/\rho$. X decreases, and as $\Phi'(X) < 0$, $\Phi(X)$ increases.

We can now express the feedback rules for extraction, the scarcity rent, the carbon tax and the producer price. Equation (29) is equivalent to

$$x = \frac{1}{2b}\left[a - \frac{d}{\rho} - \Phi(X)\right]. \tag{32}$$

From (19),

$$\rho\lambda_p(X) + \Phi'(X)x = 0. \tag{33}$$

i.e., with (32) and (30),

$$\lambda_p(X) = \frac{1}{2\rho b}\left[\Phi(X) - a + \frac{d}{\rho}\right]\Phi'(X) = \frac{2}{3}\left[\Phi(X) - c(X)\right]. \tag{34}$$

From (25), using (32), (34) and (30),

$$\theta(X) = \frac{1}{3}\left[\Phi(X) - c(X)\right] + \frac{d}{\rho}. \tag{35}$$

Finally, from (28), using (35),

$$p(X) = \frac{1}{6}\Phi(X) + \frac{1}{3}c(X) + \frac{1}{2}\left(a - \frac{d}{\rho}\right). \tag{36}$$

Notice that absent any damage before the ceiling ($d = 0$), the carbon tax is equal to half the scarcity rent ($\theta(X) = \lambda_p(X)/2 \quad \forall X$). This implies that the dynamic strategic effect of the carbon tax on the producer price is stronger than the static monopoly effect (see equation (23)), and that consequently the producers' cartel is able to preempt to some extent the carbon tax, as explained by Wirl (1995). This is not true anymore with damage: we now have $\theta(X) = \lambda_p(X)/2 + d/\rho$, and preemption is all the less likely since d is large. On the phase diagram, the carbon tax can be read as d/ρ plus one third of the vertical distance between the curve $\Phi(X)$ and the line $\Phi'(X) = 0$ (see equation (35)).

At the ceiling, x is nil, X is equal to $X_0 - \overline{Z}$, and we have shown that, $\forall t \geq T_m$:

$$p_t = \overline{p} = c\left(X_0 - \overline{Z}\right) \tag{37}$$

$$\theta_t = \overline{\theta} = a - c\left(X_0 - \overline{Z}\right) \tag{38}$$

$$\omega_t = \overline{\omega} = \rho\overline{\theta} - d. \tag{39}$$

The producer price at the ceiling is equal to the unit extraction cost. The level of the carbon tax is then such that demand is totally choked, since the consumer price is equal to the choke price a. The scarcity rent is nil.

Proposition 1 ***MPNE***

(i) Before the ceiling, the consumer and producer prices are monotonically increasing; the carbon tax may be first decreasing and then increasing, and is always increasing near the ceiling. The carbon tax includes an import tariff element.

(ii) The carbon tax and the producer price are not continuous at the ceiling, whereas the consumer price is. When reaching the ceiling, the carbon tax jumps upwards and the producer price jumps downwards to the marginal extraction cost.

Proof. Equation (28) shows that the consumer price is monotonically increasing in time, since $\Phi'(X) < 0$ and X is monotonically decreasing in time. Moreover, (28) and (37)–(38) show that the consumer price is continuous at the juncture, and equal to the choke price $u'(0) = a$.

Equation (35) yields $\theta'(X) = \left(\Phi'(X) - c'(X)\right)/3$. As $\Phi'(X) < 0$ and $c'(X) < 0$, the sign of $\theta'(X)$ is indeterminate. However, $\lim_{X \to X_0 - \overline{Z}} \theta'(X) = -\infty$, and, by continuity, $\theta'(X) < 0$ and the carbon tax is increasing in time near the ceiling.

Equation (36) yields $p'(X) = \Phi'(X)/6 + c'(X)/3 < 0$. Hence the producer price is increasing with time.

This proves part (i) of the proposition.

When $X \to X_0 - \overline{Z}$, (35) and (36) show that $\theta(X) \to \left[(a - d/\rho) - c\left(X_0 - \overline{Z}\right)\right]/3 + d/\rho$ and $p(X) \to \left[2\,(a - d/\rho) + c\left(X_0 - \overline{Z}\right)\right]/3$. However at the ceiling, according to (38) and (37), $\theta_t = \overline{\theta} = a - c\left(X_0 - \overline{Z}\right)$ and $p_t = \overline{p} = c\left(X_0 - \overline{Z}\right)$. This proves part (ii) of the proposition. ■

The important result lies in the first part of the Proposition. It concerns the time profile of the carbon tax and the producer price. Whereas the price increases along the whole trajectory, the carbon tax may be first decreasing, but always ends up increasing. Moreover, the carbon tax always includes an import tariff, allowing consuming countries to reap a part of the oil and monopoly rents. This result challenges the robustness of Liski and Tahvonen (2004) findings, namely that the carbon tax includes an import tariff when the damage is not too severe, whereas it includes an import subsidy when the damage is large.

Though the feedback rules cannot be expressed as closed-form functions of X, it is possible to obtain the time paths of extraction, carbon tax and producer price implied by the MPNE.

Differentiating (32) w.r.t. time and using (30) yields:

$$\dot{x} = -\frac{2\rho}{3}\left(\frac{\Phi(X) - c\,(X)}{\Phi(X) - \left(a - \frac{d}{\rho}\right)}\right)\dot{X}. \tag{40}$$

Notice that $\dot{x} = -\ddot{X}$ and that, from (32), $\Phi(X) - (a - d/\rho) = -2bx = 2b\dot{X}$. Equation (40) then reads:

$$\ddot{X} = \frac{\rho}{3b}\left(\Phi(X) - c\,(X)\right) = \frac{\rho}{3b}\left(2b\dot{X} + a - \frac{d}{\rho} - c\,(X)\right).$$

Replacing $c(X)$ by its linear specification (13), we obtain a second order differential equation in X :

$$3b\ddot{X} - 2\rho b\dot{X} - \rho c_2 X + \rho\left(c_1 - \left(a - \frac{d}{\rho}\right)\right) = 0. \tag{41}$$

The solution of this differential equation is:

$$X_t = \alpha_1 e^{v_1 t} + \alpha_2 e^{v_2 t} + \frac{c_1 - \left(a - \frac{d}{\rho}\right)}{c_2}, \tag{42}$$

with:

$$v_1 = \frac{\rho}{3}\left(1 + \sqrt{1 + \frac{3c_2}{\rho b}}\right) > 0 \text{ and } v_2 = \frac{\rho}{3}\left(1 - \sqrt{1 + \frac{3c_2}{\rho b}}\right) < 0. \tag{43}$$

If the ceiling constraint never binds, that is if $X_0 - \overline{Z} \leq \widetilde{X}$, the v_1 root can be ruled out as X_t can neither become negative (if $\alpha_1 < 0$) nor increase (if $\alpha_1 > 0$). In such a case, the solution is the linear one, where X converges asymptotically to $\widetilde{X}$. We have excluded this case. Under the opposite assumption, the two roots must be conserved and we obtain the non-linear solution.

Three equations are needed to determine implicitly the three unknown α_1, α_2, and T_m: the initial condition X_0, the condition at the ceiling $X_{T_m} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling[7], $x_{T_m} = 0$. These conditions read:

$$X_0 = \alpha_1 + \alpha_2 + \frac{c_1 - \left(a - \frac{d}{\rho}\right)}{c_2} \tag{44}$$

$$X_0 - \overline{Z} = \alpha_1 e^{v_1 T_m} + \alpha_2 e^{v_2 T_m} + \frac{c_1 - \left(a - \frac{d}{\rho}\right)}{c_2} \tag{45}$$

$$0 = v_1 \alpha_1 e^{v_1 T_m} + v_2 \alpha_2 e^{v_2 T_m}. \tag{46}$$

Equations (45) and (46) yield:

$$\alpha_1 = -\frac{v_2}{v_1 - v_2}\left(X_0 - \overline{Z} - \widetilde{X}\right) e^{-v_1 T_m} > 0 \tag{47}$$

$$\alpha_2 = \frac{v_1}{v_1 - v_2}\left(X_0 - \overline{Z} - \widetilde{X}\right) e^{-v_2 T_m} > 0. \tag{48}$$

[7]The proof is relegated to the on-line Appendix B.

Then (44) yields

$$\frac{v_1 e^{-v_2 T_m} - v_2 e^{-v_1 T_m}}{v_1 - v_2} = \frac{X_0 - \widetilde{X}}{X_0 - \overline{Z} - \widetilde{X}}, \tag{49}$$

which gives T_m implicitly. It is an increasing function of $\overline{Z}$. It is also an increasing function of d. The existence of a small damage puts forward the juncture date: indeed, because of this damage, at each date oil consumption is lowered and the ceiling is reached later.

We can now obtain the time paths of the extraction, consumer price (from (3)), scarcity rent (from (33) and (40)), carbon tax and producer price:

$$x_t = -\dot{X}_t = -v_1\alpha_1 e^{v_1 t} - v_2\alpha_2 e^{v_2 t} \tag{50}$$

$$p_t + \theta_t = a - bx_t = a + b(v_1\alpha_1 e^{v_1 t} + v_2\alpha_2 e^{v_2 t}) \tag{51}$$

$$\lambda_{pt} = -\frac{2b}{\rho}\dot{x}_t = \frac{2b}{\rho}\left(v_1^2\alpha_1 e^{v_1 t} + v_2^2\alpha_2 e^{v_2 t}\right) \tag{52}$$

$$\theta_t = \frac{1}{2}\lambda_{pt} + \frac{d}{\rho} = \frac{b}{\rho}\left(v_1^2\alpha_1 e^{v_1 t} + v_2^2\alpha_2 e^{v_2 t}\right) + \frac{d}{\rho} \tag{53}$$

$$p_t = a - \frac{d}{\rho} + \frac{b}{\rho}\left[v_1\alpha_1\left(\rho - v_1\right)e^{v_1 t} + v_2\alpha_2\left(\rho - v_2\right)e^{v_2 t}\right]. \tag{54}$$

III Benchmarks

Three possible benchmarks are studied, against which the properties of the MPNE will be assessed: the efficient equilibrium, the open loop equilibrium of the game, and the cartel without carbon tax equilibrium. The first one allows us to assess whether the monopoly power of the producers' cartel and the strategic behaviour of the two players lead to too much or too little extraction, compared to what is optimal. The second one allows us to assess the effect of the feedback strategies on the producer price and the carbon tax. The last one is the proper benchmark, as argued by Liski and Tahvonen (2004), since it is the pre-tax situation. The comparison of the pre-tax and the MPNE outcomes for consumers shows whether consumers benefit in terms of welfare from the implementation of the carbon tax.

Optimum and efficient equilibrium

The world central planner's problem reads[8]:

$$V^*(X_0) = \max_{x_t} \int_0^\infty e^{-\rho t} \left[u(x_t) - c(X_t)x_t - D(X_t) \right] dt$$

$$\text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x_t \\ X_0 - X_t \leq \overline{Z} \\ X_0 \text{ given.} \end{array} \right. \tag{55}$$

Denoting by ν_t the shadow price of the resource stock and ω_t the Lagrange multiplier associated to the ceiling constraint, first order optimality conditions and the complementarity slackness condition are:

$$u'(x_t) = c(X_t) + \nu_t \tag{56}$$

$$\dot{\nu}_t = \rho\nu_t + c'(X_t)x_t + D'(X_t) - \omega_t \tag{57}$$

$$\lim_{t\to\infty} e^{-\rho t}\nu_t X_t = 0 \tag{58}$$

$$\omega_t \geq 0,\ X_t - X_0 + \overline{Z} \geq 0,\ \omega_t(X_t - X_0 + \overline{Z}) = 0. \tag{59}$$

Before the ceiling, the marginal utility on the optimal path is the sum of the marginal extraction cost and of a rent ν_t, encompassing the scarcity rent and the carbon shadow value, in this simplified framework where the same stock characterizes the fossil resource stock in the ground and the atmospheric carbon concentration.

It is possible to show that extraction x is continuous at the juncture, in the same line as for the MPNE. Then, from (56), the costate ν is also continuous at the juncture.

[8] Notice that it is exactly equivalent to the problem without ceiling but with an initial resource stock $X_0 - \overline{Z}$.

Differentiating (56) w.r.t. time and using (57) yields:

$$u''(x)\dot{x} = c'(X)\dot{X} + (\rho\nu + c'(X)x + D'(X) - \omega) = \rho\nu + D'(X) - \omega.$$

Before the ceiling, $\omega = 0$ and we get, using (56) again, the optimal extraction path:

$$u''(x)\,\dot{x} = \rho\,(u'(x) - c(X)) + D'(X). \tag{60}$$

With the specifications adopted for the utility, damage and extraction cost functions, (60) reads:

$$b\ddot{X} - \rho b\dot{X} - \rho c_2 X + \rho\left(c_1 - \left(a - \frac{d}{\rho}\right)\right) = 0, \tag{61}$$

a linear differential equation of the second order, as in the MPNE, but with different coefficients.

The solution is:

$$X_t = \beta_1 e^{u_1 t} + \beta_2 e^{u_2 t} + c_1 - \left(a - \frac{d}{\rho}\right), \tag{62}$$

with

$$u_1 = \frac{\rho}{2}\left(1 + \sqrt{1 + \frac{4c_2}{\rho b}}\right) > 0 \text{ and } u_2 = \frac{\rho}{2}\left(1 - \sqrt{1 + \frac{4c_2}{\rho b}}\right) < 0. \tag{63}$$

As in the case of the MPNE, three boundary conditions allow us to obtain the unknown β_1, β_2 and T_o : the initial condition X_0, the condition at the ceiling $X_{T_o} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling, $x_{T_o} = 0$. With the same argument as in the MPNE, it is possible to show that $\beta_1, \beta_2 > 0$.

At date T_o, the ceiling is reached, and we have, $\forall t \geq T_o$:

$$\begin{cases} X_t = X_0 - \overline{Z} \\ x_t = 0 \\ \nu_t = u'(0) - c(X_0 - \overline{Z}) \\ \omega_t = \rho\nu_t - d. \end{cases} \tag{64}$$

The decentralization of the optimum leads to an efficient competitive equilibrium, provided that the right environmental tax –redistributed by lump-sum transfers to consumers– is implemented in the consumers' area.

The demand function of the representative consumer is given by $u'(x) = p + \theta$.

On the producer side:

$$V_p^o(X_0) = \max_{x_t} \int_0^\infty e^{-\rho t} [p_t - c(X_t)] x_t dt \tag{65}$$

$$\text{s.t.} \quad \dot{X}_t = -x_t, \quad X_0 \text{ given.}$$

Denoting by λ_p the scarcity rent, the first order conditions read:

$$p_t = c(X_t) + \lambda_{pt} \tag{66}$$

$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + c'(X_t)x_t \tag{67}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt}X_t = 0. \tag{68}$$

The equilibrium is then defined by:

$$u'(x_t) = c(X_t) + \lambda_{pt} + \theta_t. \tag{69}$$

Differentiating equation (69) w.r.t. time and using (67) yields:

$$b\ddot{X} - \rho b\dot{X} - \rho c_2 X + \rho(c_1 - a) = \dot{\theta} - \rho\theta. \tag{70}$$

Comparing (61) and (70) shows that for the equilibrium to be an optimum the carbon tax before the ceiling must be such that $\dot{\theta}_t - \rho\theta_t = -d$, which integrates into

$$\theta_t = \left(\theta_{T_o} e^{-\rho T_o}\right) e^{\rho t} + \frac{d}{\rho}\left(1 - e^{-\rho(T_o - t)}\right), \quad \forall t \geq 0. \tag{71}$$

The optimal carbon tax is the sum of the Hotellinian tax and the pure Pigouvian tax.

Extraction x and the marginal extraction cost $c'(X)$ being continuous at the ceiling, (67) implies that $\dot{\lambda}_p - \rho\lambda_p$ is continuous, and yields, integrating forward and using the transversality condition (68)[9],

$$\lambda_{pt} = -\int_t^{\infty} e^{-\rho(s-t)} c'(X_s) x_s ds, \quad \forall t \geq 0. \tag{72}$$

Then, as $x_s = 0$, $s \geq T_o$, $\lambda_{pT_o} = 0$. The scarcity rent is continuous at the juncture, and equal to 0. Then the carbon tax is also continuous, and it is equal to $\overline{\theta}$ given by (38). This allows us to obtain the initial level of the carbon tax: $\theta_0 = \overline{\theta} e^{-\rho T_o} + d/\rho \left(1 - e^{-\rho T_o}\right)$. Moreover, it is straightforward to show that the carbon tax is monotonically increasing iff $\overline{\theta} > d/\rho$ which, according to the definition (38) of $\overline{\theta}$, is true when the ceiling is binding.

Equations (66) and (67) show that the producer price is monotonically increasing before the ceiling. Moreover, by (66), the producer price is continuous at the juncture and equal to $\overline{p}$ given by (37).

These results are summarized in the following proposition.

Proposition 2 ***Efficient equilibrium***

(i) Before the ceiling, the carbon tax is the sum of a pure Hotellinian tax and a pure Pigouvian tax, monotonically increasing, and the producer price is also monotonically increasing.

(ii) The carbon tax and the producer price are continuous at the ceiling.

Open loop equilibrium

In this case, the players base their strategies on time alone.

The consumers' regulator problem is similar to problem (4), but for the fact that he takes the producer price as given. Equally, the producers' regulator problem is similar to (17), but for the fact that he takes the carbon tax as given.

[9] Notice that the same reasoning cannot be made in the case of the MPNE, since we are not sure that $\theta'(X)$ is continuous.

The first order optimality conditions are, on the consumers' side:

$$\theta_t = \lambda_{ct} \tag{73}$$

$$\dot{\lambda}_{ct} = \rho\lambda_{ct} + D'(X_t) - \omega_t \tag{74}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ct}X_t = 0, \tag{75}$$

which shows that before the ceiling the carbon tax is the sum of the Hotellinian tax and the Pigouvian tax.

On the producer side, the first order optimality conditions read:

$$p_t = \frac{1}{2}\left[c(X_t) + a + \lambda_{pt} - \theta_t\right] \tag{76}$$

$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + c'(X_t)x_t \tag{77}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt}X_t = 0. \tag{78}$$

The equilibrium before the ceiling is characterized by:

$$u'(x) = p + \theta \Longleftrightarrow a - bx = \frac{1}{2}\left[c(X) + a + \theta + \lambda_p\right]. \tag{79}$$

Differentiating (79) w.r.t. time and using (74) and (77) yields:

$$2b\ddot{X} - 2\rho b\dot{X} - \rho c_2 X + \rho\left(c_1 - \left(a - \frac{d}{\rho}\right)\right) = 0, \tag{80}$$

again a linear differential equation of the second order, but with different coefficients.

The solution has the same form as in the MPNE and the efficient equilibrium:

$$X_t = \gamma_1 e^{w_1 t} + \gamma_2 e^{w_2 t} + \frac{c_1 - \left(a - \frac{d}{\rho}\right)}{c_2}, \tag{81}$$

with

$$w_1 = \frac{\rho}{2}\left(1 + \sqrt{1 + \frac{2c_2}{\rho b}}\right) > 0 \text{ and } w_2 = \frac{\rho}{2}\left(1 - \sqrt{1 + \frac{2c_2}{\rho b}}\right) < 0. \tag{82}$$

As in the cases of the MPNE and the efficient equilibrium, three boundary conditions allow us to obtain the unknown γ_1, γ_2 and T_{ol}, the date at which the ceiling is reached:

the initial condition X_0, the condition at the ceiling $X_{T_{ol}} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling, $x_{T_{ol}} = 0$. It is possible to show that $\gamma_1, \gamma_2 > 0$.

Proposition 3 ***Open loop equilibrium***

(i) Before the ceiling, the carbon tax is the sum of a Hotellinian tax and a Pigouvian tax and is always increasing, whereas the producer price may be first increasing and then decreasing, and is decreasing when approaching the ceiling.

(ii) The carbon tax and the producer price are continuous at the ceiling.

Proof. To prove part (ii), remark that λ_p is continuous at the juncture by the same argument as in the efficient equilibrium case. Its continuity implies that of the carbon tax (from (79)) and the producer price.

To prove part (i) note first that the consumer price is increasing as $p + \theta = a + b\dot{X} \Rightarrow d(p+\theta)/dt = b\ddot{X} > 0$. As far as the carbon tax before the ceiling is concerned, (73) and (74) show that it has the same expression than in the efficient equilibrium (equation (71)), hence has the same properties. Finally, the sign of $\dot{p}$ is indeterminate. However at the juncture at date T_{ol}, using (76) and (77), $\dot{p}_{T_{ol}-} = -\dot{\theta}_{T_{ol}-}/2 < 0$. This proves part (i) of the proposition. ■

The discontinuity of the carbon tax and the producer price at the juncture at the MPNE is then the consequence of the feedback strategies of the players, since this discontinuity does not exist at the open loop equilibrium.

Cartel equilibrium without carbon tax

This equilibrium can be seen as the present situation, where everybody is aware of the existence of the physical limit $\overline{Z}$ to atmospheric carbon concentration but ignores it.

The buyers' demand for oil is simply given by $u'(x) = p$. The sellers' regulator solves the same problem as in the open loop game, but for the fact that now the carbon tax is nil. The equilibrium is characterized by equation (80), as in the open loop game; but now producers behave as if no constraint could prevent them from extracting all what is economically profitable. They intend to leave asymptotically X_∞ in the ground, and choose the extraction path accordingly. In the solution of the linear differential equation (80), the positive exponential has to be ruled out. Therefore the time path of oil stock and extraction are:

$$X_t = (X_0 - X_\infty) e^{w_2 t} + X_\infty \tag{83}$$

$$x_t = -w_2 (X_t - X_\infty) \tag{84}$$

with w_2 given in (82).

But ignoring the ceiling does not make it disappear. Once the ceiling is reached, the damage from consuming more oil becomes infinite whereas the marginal utility of consumption remains finite, and therefore consuming countries are not willing to buy oil any more, even under a zero tax. Extraction drops to zero in finite time (at date T_c), while a stock of oil $X_{T_c} = X_0 - \overline{Z}$ is left forever in the ground. From (83) we get:

$$T_c = \frac{1}{w_2} \ln \frac{X_0 - \overline{Z} - X_\infty}{X_0 - X_\infty}. \tag{85}$$

Finally, the producer price is given by:

$$p_t = a + b w_2 (X_0 - X_\infty) e^{w_2 t}, \quad t \leq T_c, \tag{86}$$

from which we deduce $p_{T_c} = a + b w_2 \left(X_0 - \overline{Z} - X_\infty \right) < a$.

These results are summarized in the following proposition:

Proposition 4 ***Cartel without tax equilibrium***

Producers and consumers behave as if the ceiling did not exist. The resource price is an increasing and concave function of time. It is lower than the choke price at the ceiling, where it jumps upwards to the choke price while extraction jumps downwards to zero.

IV Comparison of equilibria

The comparison of the MPNE and the efficient equilibrium allows us to assess to what extent the carbon tax of the MPNE departs from the Pigouvian tax of the efficient equilibrium, designed to correct the environmental problem only, and to see if the game is more or less conservative than what is optimal. The comparison of the MPNE and the open loop equilibrium highlights the impact of the producers' and consumers' feedback strategies. Finally, the comparison of the MPNE and the cartel without carbon tax equilibrium allows us to see whether consumers gain in terms of welfare when they adopt a common environmental policy.

Technically, the comparison is made easier by the fact that all equilibria reach in finite time the same state, where a stock of oil $X_0 - \overline{Z}$ is left in the ground, atmospheric carbon concentration is at the ceiling and oil consumption is nil. What changes in the different equilibria is the intertemporal allocation of extraction, driven by different producer prices and carbon taxes, and thus producers and consumers' payoffs.

Proposition 5 *(i) The MPNE is more conservative than the open loop equilibrium, in the sense that initial extraction is lower. Both are excessively conservative, compared to what is efficient, and are also more conservative than the cartel without tax equilibrium. The ceiling is reached later in the MPNE than in the open loop equilibrium, and later in the open loop equilibrium than in the efficient equilibrium and than in the cartel without tax equilibrium. The cartel without tax equilibrium is less conservative than the efficient* *equilibrium and the ceiling is reached sooner if it is low; it is the contrary if the ceiling is high.*

(ii) The ranking of the payoffs for the producers' and the consumers' area respectively are: $V_p^{ol}(X_0) > V_p^m(X_0) > V_p^o(X_0) = 0$ and $V_c^o(X_0) > V_c^{ol}(X_0) > V_c^m(X_0)$. When the marginal damage is small enough, the consumers' area gets a higher payoff in the MPNE than in the cartel without tax case if the ceiling is high; it is the contrary if it is low.

Proof. (i) We prove in the on-line Appendix C that $T_m > T_{ol} > T_o$, $T_{ol} > T_c$, and that $x_0^o > x_0^{ol} > x_0^m$, $x_0^c > x_0^{ol}$. Notice that we cannot deduce the ranking of initial extractions from the ranking of the dates at which the ceiling is reached and the fact that total extraction is the same in the three equilibria, because in some of the equilibria extraction can be convex-concave. It may be the case for instance in the MPNE since θ_t can be first decreasing and then increasing and $\ddot{x}_t = -\rho/b.\dot{\theta}_t$.

The comparison of T_c and T_o on the one hand, x_0^c and x_0^o on the other hand is also relegated to the on-line Appendix C.

(ii) The ranking of payoffs is deduced from the ranking of initial extractions (except for the producers' payoff at the efficient equilibrium and both payoffs at the cartel without tax equilibrium), since $V_p^m(X_0) = \frac{b}{\rho}(x_0^m)^2$, $V_p^{ol}(X_0) = b/\rho.\left(x_0^{ol}\right)^2$, $V_c^o(X_0) = b/(2\rho).(x_0^o)^2$, $V_c^m(X_0) = b/(2\rho).(x_0^m)^2$, $V_c^{ol}(X_0) = b/(2\rho).\left(x_0^{ol}\right)^2$, see the on-line Appendix E. The producers' payoff at the efficient equilibrium is nil.

For the cartel without tax equilibrium the Hamilton–Jacobi–Bellman equation cannot be used since consumers do not take into account the evolution of the stock in their problem (the damage is an externality). The consumers' payoff must be computed directly. We get

$$V_c^c(X_0) = \frac{b}{2}\int_0^\infty e^{-\rho t}x_t^2 dt - d\int_0^\infty e^{-\rho t}(X_0 - X_t)dt, \tag{87}$$

x_t and X_t being given by (83) and (84). We compare in the on-line Appendix D the consumers' payoffs $V_c^m(X_0)$ and $V_c^c(X_0)$ in the case $d = 0$ and prove the result stated in the Proposition. This result can be extended by continuity to d small. As $V_c^m(X_0)$ and $V_c^c(X_0)$ both are decreasing functions of d, it is not possible to obtain analytically a ranking of the payoffs for any value of d. ■

This Proposition contains at least three strong results.

First of all, when the two players act strategically, the sellers win. They get a higher payoff than at the efficient equilibrium, whereas the buyers' payoff is reduced.

Secondly, consumers and producers are both better off in the open loop equilibrium than in the MPNE. In this sense, playing feedback strategies is a lose-lose situation, both parties ending up being worse off. There exists in this game a commitment value.

Lastly, when the ceiling is not too stringent and the marginal damage small enough, consumers gain in the MPNE with respect to the pre-tax case. To put it differently, consumers are better off with the carbon tax than without it if the global warming problem is not too severe. In this case indeed, they do not suffer from a too drastic reduction of their oil consumption whereas they benefit from the reduction of damages. Conversely, consumers may lose, and we are sure that this is the case if the marginal damage is small enough and the ceiling very stringent.

V Conclusion

Studying the MPNE of a game between two coalitions of oil producing and oil consuming countries, Liski and Tahvonen (2004) show, within the damage function approach, that the carbon tax is not purely Pigouvian. If the damage is not too severe, it includes an import tariff element and exceeds the present value of marginal damages, allowing oil consuming countries to reap resource rents from the cartel of oil producers, whereas for a serious damage this element is an import subsidy and the strategic tax falls short of the Pigouvian one. The optimal design of the strategic tax (import subsidy or import tariff, tax increasing or decreasing in time) depends on the value of the parameter of the quadratic damage function, featuring the severity of the damage. This severity also determines the temporal profile of the strategic producer price: increasing when the damage is not too severe, decreasing otherwise. In terms of payoffs, Liski and Tahvonen conjecture that the strategic tax reduces the producers' payoff and enhances the consumers' payoff, compared to the pre-tax case, whatever the severity of the damage.

We revisit this game within the ceiling approach. We obtain a monotonically increasing strategic producer price before the ceiling, and a carbon tax which may be decreasing or increasing at the beginning of the planning horizon, but is always increasing near the ceiling, and this independently on the stringency of the ceiling and the severity of the "small" linear damage before the ceiling. Moreover, in this framework, the strategic tax includes an import tariff element whatever the stringency of the ceiling. These results challenge the robustness of the conclusions of the existing literature.

Compared to the open loop solution, behaving strategically is a lose-lose situation, both parties ending up being worse off. Compared to the pre-tax situation (the cartel without tax equilibrium), we prove that when the ceiling is tight and the "small" marginal damage small enough the consumers are worse off in the MPNE, whereas when the ceiling is relatively high they are better off. We do not confirm here the conjecture of Liski and Tahvonen (2004), which is that consumers always gain from introducing the carbon tax.

The practical discussions about the introduction of a carbon tax very often concentrate on the distributive consequences of the tax within each country and between countries adopting the environmental policy and countries refusing to do so, without considering a central actor in the climate change game, namely fossil fuel producers. We have in this paper contributed to fill this gap, in a two-zones framework. But a lot remains to be done.

Some very recent papers open new directions of research in this area. For instance Fujiwara and Long (2010) consider a game with Stackelberg leadership, where the leader can be the oil producing area or the oil consuming area, and wonder whether being the Stackelberg leader is better than being the follower or not. Compared to Rubio and Escriche (2001) they study two varieties of Stackelberg leadership, the global and the stagewise ones, but do not introduce the climate motive for oil taxation. Another path is opened by Wei *et al.* (2010), which consider that oil producing countries also consume oil, and can counteract climate policy by using a strategy of price discrimination, subsidizing the oil they consume. They only study the open loop equilibrium of the game.

In the same spirit, it would be very useful to distinguish between two different zones of oil consuming countries, a "Kyoto zone", setting a common carbon tax, and a "non-Kyoto zone" refusing to do so. In this three players game, the oil cartel's power should be enhanced, but the consequences of the unilateral climate policy on Kyoto and non-Kyoto countries is not trivial and deserves further research.

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