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Lack of Preemption Under Irreversible Investment

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Abstract

This article considers the classic model of irreversible investment under imperfect competition and stochastic demand, and characterizes the markov perfect equilibrium. To do so, I introduce a new way to define strategies, permitting the players to create endogenous jumps in the state variable. The markov equilibrium is then similar to the open-loop equilibrium, meaning that the irreversibility of investment does not create a preemption effect in this model. This is due to the form of investment's cost, which creates an incentive to invest as soon as possible, reducing the strategic interaction to the one of a static problem.

Keywords: Capacity investment. Cournot competition. Markov-perfect equilibrium. Real option games. Differential games.

JEL classification: D43 L13 L25

Abstract

Cet article considère le modèle classique d'investissement irréversible en environnement Brownien lorsque la concurrence est imparfaite, et caractérise l'équilibre Markovien. Pour ce faire, j'introduis une nouvelle façon de définir des stratégies, permettant aux joueurs de faire sauter la variable d'état. L'équilibre Markovien est alors similaire à l'équilibre en Open-loop, ce qui signifie que, dans ce modèle, l'irréversibilité de l'investissement ne crée pas un effet de préemption dans ce modèle. Cela est dû à la forme du coût de l'investissement, qui crée une incitation à investir dès que possible, en réduisant l'interaction stratégique entre les entreprises à celui d'un problème statique.

Mots-clefs: Investissement en capacité. Concurrence à la Cournot. Equilibre markovien. Option réelle. Jeux différentielle.

1 Introduction

Investment in capacity of production has usually three main features. It is irreversible, as a firm cannot recover its installation fees if it decides to dismantle some unit of capacity. It is done in a long run perspective, with an uncertain evolution of the environment. It is also often done in an imperfect competition framework. In that case, the commitment due to the irreversibility of investment has an impact not only on the firm which invests, but also on its opponents. Indeed, the competitors of the investing firm will have to deal with its new capacity and should reduce or delay their own capacity adoption. This usually creates a preemption effect, as the investing firm may invest not just for the project value, but also for its impact on opponents' future decisions.

This article considers the classic real option model of irreversible investment under uncertainty and imperfect competition. It shows that, in this model, there is no preemption effect at the equilibrium. This is due to the form of investment's cost, which creates an incentive to invest as soon as possible and thus reduces the strategic interaction between firms to the one of a static problem. This result shows that the irreversibility of investment alone is not sufficient to create preemption, and provides a better understanding of the reason driving preemption. This paper also introduces a refinement of the theory of differential games, allowing strategic jumps in the state variable.

More precisely, I consider a duopoly producing an homogenous good, and competing *à la Cournot* in a continuous time setting. The quantity produced by a firm depends of its capacity. At each time, firms can decide to increase their capacity at a linear cost. Demand evolves according to a continuous stochastic process, and capacities are smooth.

When there is a single firm in the market, the optimal strategy is well known. (See for example Abel et al (1996) or Dixit and Pindyck (1994)). There exists a trigger, a level of demand depending of the capacity of the firm, such that the monopoly invest only when the demand exceeds the trigger, and then install the smaller level of capacity which releases the trigger. As long as the demand is inferior to the trigger, the monopoly does nothing.

Under imperfect competition, this model has already been studied by several authors. Baldursson (1998) and Grenadier (2002) assume that firms have the same behavior as the monopoly, i.e. invest when demand reaches a trigger. They characterize the Nash equilibrium in triggers' strategy and show that competition makes the firms more reactive to demand. Back and Paulsen (2009) and Steg (2012) show that the behavior assumed by Baldursson (1998) and Grenadier (2002) is consistent with the concept of open-loop equilibrium. One of the disadvantages of open loop equilibria is that they usually fail sub-game perfection. Indeed, in open loop equilibrium, firms commit to their path of investment along time, and the open loop equilibrium corresponds to the Nash equilibrium in these paths.

In markov perfect equilibrium[1], firms decide of their investment in function of the capacity repartition at the investment time. This equilibrium concept is a refinement of sub-game perfect equilibrium, and its comparison with open loop equilibria permits to identify the strategic effects in a game (see for example Figuières (2009) or Genc and Zaccour (2014)). However, in our case of irreversible investment, there exists a difficulty to define a markovian strategy. Indeed, the linearity of the investment's cost creates a incentives to invest as soon as possible and the optimal path of capacity can presents some jumps[2]. Such jumps are not allow in the usual theory of differential games.

Back and Paulsen (2009) and Chevalier-Roignant, Huchzermeier and Trigeorgis (2011) focus on markov perfect equilibrium. Back and Paulsen (2009) define the strategy of a firm in function of its opponent decisions[3]. Under that condition, they show that firms can play strategies which ensure them the same profit as the open-loop equilibrium. In that case, a firm can increase its profit by committing to an open loop strategy (according to the fact that its opponent will reacts to its new strategy). This has often be interpreted as the fact that the open loop equilibrium differs from markov perfect equilibrium, as a firm can preempt its opponent. This is misleading, as the possibility of preemption highly depends on the assumption that the opponent will change its path of capacity. If the opponent does not change its path of capacity, the preemption is not necessary profitable. In fact,

[1] In this case, the notions of markov perfect equilibrium and of closed loop equilibrium coincide, as the closed loop information structure is the state variable of markov perfect strategies.

[2] At the equilibrium, these jumps are made at the very first moment of the game. However there exist sub-games, which are not on the equilibrium path, in which firms make several jumps. It is therefore important, for the determination of the markov perfect equilibrium, to permits the firm to jump at any time, and not just on the first moment of the game.

[3] In Back and Paulsen (2009), page 4538, the capacity choosed by firm i at time t depends of the capacity of its opponents at the same time t.

preemption is not profitable, as open-loop and markov perfect equilibrium coincide. The right interpretation of this result of Back and Paulsen (2009) should be that Stackelberg equilibrium differs from open-loop equilibrium, and that a firm which has the possibility to commit to its open-loop strategy will have a higher profit than its markovian follower[4]. Chevalier-Roignant, Huchzermeier and Trigeorgis (2011) use the usual theory of differential games to show that firms invest when the marginal revenue of capacity is higher than the price of investment, where the competitive externality and the effect of uncertainty are taken into account in the marginal revenue of capacity. In that way, Tobin's q theory is still valid. The point of the present work will be to determine the marginal value of capacity and therefore to fully characterizes the dynamic equilibrium.[5]

When there is no demand uncertainty, this model is similar to some classic papers on dynamic oligopoly, as Fudenberg and Tirole (1983), Reynolds (1991), or Figuières (2009), except in the form of investment's cost. Fudenberg and Tirole (1983) assume an exogenous boundary on firms' investments, and show the existence of strategic effects. Reynolds (1991) and Figuières (2009) assume quadratic cost of investment and characterize the impact of strategic effects in function of the form of the competition. When there is complementarity (i.e. in price competition), commitment to open-loop strategies decreases the profit of the firms, whereas when there is substitutability (i.e. in quantity competition), commitment to open-loop strategies decreases firms' profit.

In both papers, the strategic effect comes from the fact that each firm has an unilateral incentive to delay investment, due to an exogenous bound in the case of Fudenberg and Tirole (1983), and to an investment's cost increasing in the velocity of the investment in Figuières (2009) or Reynolds (1991). In order to get the intuition when firms face a linear cost I start focusing on a two period model without uncertainty. In that framework, each firm knows that its opponent's incentive to invest during the second period decreases with the amount of capacity invested at the first period. The strategic effect should thus increases the incentive to invest in the first period. However, as firms can make money on

[4]Such interpretation should be taken carefully, as the definition of a Stackelberg equilibrium in such a continuous setting is not straightforward.

[5]The results of Chevalier-Roignant, Huchzermeier and Trigeorgis (2011) is established in a more general setting than the model of this work, as they consider a general investment's cost function,with a disinvestment possibility.

both periods, there is no incentive to delay investment. The game is thus reduced to a one shot game, and the strategic effect of investment disappears. This highlights the fact that both commitment and an incentive to delay investment are important to the creation of strategic effect in a simultaneous game.

When time is continuous, the optimal capacity path of the firms is not continuous in time but presents some jumps. This is due to the linearity of investment cost, which creates an incentive to invest as soon as possible. In order to introduce the possibility for the firm to jump, I introduce a new control variable, the desired capacity. When the capacity of the firm (which is the state variable) differs from the desired capacity, the firm jumps to the desired level of capacity. With this definition of firms' strategy, I characterize the markov perfect equilibrium. Both firms invest to a Cournot level of capacity if both have capacity smaller than the Cournot level. When this is not the case, only the smallest of both firms invests, except when it already has a capacity too large. Such investments take place at the first time of the game, and the markov perfect equilibrium coincides with the open-loop equilibrium.

When demand evolves stochastically, the equilibrium strategies are the same. The optimal levels of capacity depend on the solution of a differential equation with smooth pasting and value matching conditions, due the real option effect. However, the equivalence between open-loop and markov perfect equilibrium remains valid.

Section 2 describes the two period model. Section 3 presents the differential game without uncertainty, and section 4 the differential game with uncertainty. Section 5 concludes. All non-included proofs are given in appendix.

2 Discrete game

2.1 The model

In order to understand why there is no strategic interaction under irreversible investment, let's focus on the simplest game of irreversible investment, with two periods[6] and without uncertainty. Consider a duopoly competing *à la Cournot* in capacities. Each firm i, for $i \in \{A, B\}$, starts with some amount of capacity k_0^i ($k_0^i \geq 0$), which can be extended through buying some assets. Purchases are made at a linear price p^+ ($p^+ > 0$). Let k_1^i be the capacity installed by firm i in the first period, and k_2^i the capacity installed in period 2. Each firm's production depends of its capacity, according to the technology $q^i = k^i$. Such technology is classic in dynamic investment models, and has been used by Fudenberg and Tirole (1983), Grenadier (2002), Merhi and Zervos (2007) among others[7].

At each periods, firms produce and sell an homogenous good, at a price $P\left(k_t^i + k_t^j\right)$ depending of the total quantity sold on the market. The profit of firm i is thus;

$$\Pi^i = P\left(k_t^i + k_t^j\right) k_1^i - p^+\left(k_1^i - k_0^i\right) + \delta\left[P\left(k_t^i + k_t^j\right) K_2^i - p^+\left(k_2^i - k_1^i\right)\right], \tag{1}$$

where $\delta \in [0,1]$ is the discount rate. As investments are irreversible, firm's i capacities should verify:

$$k_2^i > k_1^i > k_0^i. \tag{2}$$

In this game, I compare two kinds of equilibria concepts, open-loop equilibria and sub-game perfect equilibria. In open-loop equilibria, at the beginning of the game, firms commit on their action for the two periods, and then played a classic Nash equilibrium where the strategy space is $\mathbb{R}_+^2$. Formally, this gives:

[6] Two periods are needed in order to have some possibility of dynamic interaction (the action of a player impacts the action of the other on the next period).

[7] As it was shown in Reynolds (1987), this technology assumption is also the result of a dynamic games with limited Cournot competition, without uncertainty. Therefore we can relay this hypothesis in section 2 and 3 (for $q^i \leq k^i$). However when there is uncertainty, firms have an incentive to keep their unused capacity for a possible further use in case of demand increases. Assuming that quantities are equal to capacities permits to avoid such adaptability effects and focus on the direct effect of uncertainty on capacity choice.

Open Loop Equilibrium: The group of strategies $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$ is an open loop equilibrium if, for all $i \in \{A, B\}$,

$$\Pi^i\left[k_1^*, k_2^*, k_1^{*j}, k_2^{*j}\right] = \max_{K_1^i, K_2^i} \Pi^i\left[k_1^i, k_2^i, k_1^{*j}, k_2^{*j}\right]. \tag{3}$$

□

In sub-game perfect equilibria, at each time firms choose their action knowing the history of all the previous actions. Here, the choice of capacity in the second period depends of the capacities installed in the first period. The formal definition follows.

Sub-game Perfect Equilibrium: The group of strategies $(k_1^{*i}, \tilde{k}_2^{*i})^{i\in\{A,B\}}$, where $\tilde{k}_2^{*i}$ is a function from $\mathbb{R}_+^2$ to $\mathbb{R}_+$ and k_1^{*i} a real, is a sub-game perfect equilibrium if, for all $i \in \{A, B\}$, the two following conditions are verified:

· for all vectors of capacity $k_1 \in \mathbb{R}_+^2$ installed in the first period, $\tilde{k}_2^{*i}$ verifies:

$$\Pi^i\left[k_1^i, \tilde{k}_2^{*i}(k_1), k_1^{*j}, \tilde{k}_2^{*j}(k_1)\right] = \max_{\tilde{k}_2^i(.)} \Pi^i\left[k_1^i, \tilde{k}_2^i(K_1), K_1^{*j}K_2^{*j}(K_1)\right] \tag{4}$$

· the vector of first period capacity, k_1^{*i}, verifies:

$$\Pi^i\left[k_1^{*i}, \tilde{k}_2^{*i}(k_1^*), k_1^{*j}, \tilde{k}_2^{*j}(k_1^*)\right] = \max_{k_1^i} \Pi^i\left[k_1^i, \tilde{k}_2^{i*}(k_1), k_1^{*j}, \tilde{k}_2^{*j}(k_1^*)\right]. \tag{5}$$

□

Equation (4) represents the fact that the strategies should be optimal in all sub-game, i.e. whatever happens at the first period, and not just on the equilibrium path.

Open-loop equilibria usually fails sub-game perfection, but are used as a benchmark for discussing the effects of dynamic strategic incentives, i.e. the incentives to change current play so as to influence the future play of opponents[8]. In the game considered here, the open-loop equilibrium coincides with the sub-game perfect equilibrium, and there is no strategic interaction.

[8] See for example Figuières (2009) or Genc and Zaccour (2014).

2.2 An example

This sub-section focus on what happens with a linear price function $P\left(k_t^A+k_t^B\right)=1-k_t^A-k_t^B$. It solves the model by backward induction, in order to give intuition on the lack of strategic effect under irreversible investment, and on the proof of proposition 1.

In the last period, given the capacity of period one, the best response of firm A is given by

$$k_2^A(k_2^B)=\max\left\{\frac{1-p^+-k_2^B}{2},k_1^A\right\}. \tag{6}$$

Therefore, there exist three possibilities, in function of the capacities installed at the first period. Both firms are constrained (In the meaning that they wish to have a capacity in period 2 inferior to their capacity in period 1). One firm is constrained and the other is not. No firms are constrained.

When no firms are constrained, the equilibrium is straightforward, both firms investing until the usual Cournot level. This happens when k_1^A and k_1^B are smaller than the Cournot equilibrium, $\frac{1-p^+}{3}$. When firm A is constrained and firm B is not, firm B installed a capacity $k_2^B=\frac{1-p^+-K_1^A}{2}$. If $k_1^B>\frac{1-p^+-K_1^A}{2}$, then firm B has no interest to invest, and both firms are constrained. Graphic 1 presents the area where firms are constraint, in function of the capacities chosen in the first period, and Graphic 2 the equilibrium of second period.

$[Insert\ Graphic\ 1]$

$[Insert\ Graphic\ 2]$

In the first period, a firm has two possibilities. Choosing a capacity which will constrained him in the second period, or choosing a capacity which will not. If firm A chooses a capacity which does not constraint her, its best response is:

$$k_1^A(k_1^B)=\max\left\{\frac{1-p^+-k_1^B}{2},k_0^A\right\}, \tag{7}$$

so firm A wishes to be constrained in the second period. This is due to the fact that a unit of capacity brings the same level of profit in the first and in the second period, and there is no interest to delay investment. Therefore, at the closed loop equilibrium, firms have the same level of capacity in both periods. Such level maximizes

$$\text{For } i\in\{A,B\},\ \Pi^i=(1+\delta)\left(1-k_1^i-k_1^j\right)k_1^i-p^+\left(k_1^i-k_0^i\right). \tag{8}$$

Therefore, the equilibrium strategy of firm A is to install the Cournot level if firm B starts with a capacity inferior to the Cournot:

$$k_2^{*A} = k_1^{*A} = \max\left\{\frac{1-\frac{p^+}{(1+\delta)}}{3}, k_0^A\right\} \text{ if } k_0^B \leq \frac{1-\frac{p^+}{(1+\delta)}}{3}, \tag{9}$$

and to install the best response to firm B's initial level when firm B's capacity exceed the Cournot level:

$$k_2^{*A} = k_1^{*A} = \max\left\{\frac{1-\frac{p^+}{(1+\delta)} - k_0^B}{2}, k_0^A\right\} \text{ if } k_0^B > \frac{1-\frac{p^+}{(1+\delta)}}{3}. \tag{10}$$

Firm B has the same equilibrium strategy. In some way, this game is equivalent to a one-shot game, where firms invest for both periods. This is due to the linearity of investment cost, which overthrow the incentives to delay investment. The next section will show that the same effect happens in a differential game. Before that, the following sub-section presents this result for general demand and cost function.

2.3 The proposition

In order to present proposition 1, let $k^i_{Irr}(K^j)$ be the best response of firm i for $i \in \{A, B\}$, when firm i has no initial capacity and firm j has the same capacity in both periods, i.e. k^j. k^i_{Irr} is defined as the implicit solution to the following equation:

$$P\left(k^i_{Irr} + k^j\right) + P'\left(k^i_{Irr} + k^j\right) k^i_{Irr} = \frac{p^+}{1+\delta}. \tag{11}$$

Let also k_C be the Cournot level of capacity when both firms has no initial value, i.e. the solution to

$$k^A_{Irr}(k^B_C) = k^A_C \text{ and } k^B_{Irr}(k^A_C) = k^B_C. \tag{12}$$

In order to ensure the existence of k^i_{Irr} and k_C, I make the following assumption:

H1: P is a concave, twice differentiable positive function.

This assumption permits to state proposition 1.

Proposition 1: Assume $H1$. Let $(k_t^{*i})^{i\in\{A,B\}}_{t\in\{1,2\}}$ be the strategies defined by:

$$\text{For } i \in \{A, B\}\,,\; k_2^{*i} = k_1^{*i} = \begin{cases} \max\left\{k_{Irr}^i(k_0^j), k_0^i\right\} \text{ if } k_0^j > k_C^j \\ \max\left\{k_C^j, k_0^i\right\} \text{ if } k_0^j \leq k_C^j \end{cases}. \tag{13}$$

Then, $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$ is the unique open loop equilibrium of the game. It is also the only sub-game perfect equilibrium.

As seen in the example, the linearity of the investment price overthrows the incentives to delay investment. This reduces the strategic interaction to the first period game, and explains the equivalence of the notion of open-loop and equilibrium in that game.

3 Dynamic game without uncertainty

3.1 Model and definition

This section presents a dynamic model of investment in capacity under imperfect competition. It shows that, under some smoothness condition, the markovian equilibria are similar to the one shot game previously considered, and there is no preemption effect.

Let k_t^i be the capital of firm i at time t, time is continuous and capacity partially reversible. Let $\pi^i(k_t)$ be the instantaneous payoff of firm i, k_t being the vector of firms capacities. I assume Cournot competition. Let $P(.)$ be the inverse demand function. The instantaneous revenue of firm i is

$$\pi^i(k_t) = P\left(\bar{k}_t\right) k_t^i, \tag{14}$$

where $\bar{k}_t = \sum_{i=1}^n k_t^i$, as previously. The interest rate is r and the purchase price of capacity is p^+. As investment is irreversible, the path of investment, k_t^i, is assumed to be an increasing function of the time. The total profit of firm i at time 0 is thus:

$$\Pi^i = \int_0^{+\infty} e^{-rt}\pi^i(k_t)dt - p^+ \int_0^{+\infty} e^{-rt} dk_t^i. \tag{15}$$

The objective of firm i is to choose the process k_t^i which maximizes its own expected profit, given the initial levels of capital and demand. Obviously, the optimal process will depend of the processes chosen by the other players. To properly define the game, I must define the strategic variable used by the players.

In this framework, the usual theory of differential games assume that the path of investment of the firm is described by the flow of investment I_t^i:

$$\frac{\partial k_t^i}{\partial t} = I_t^i, \tag{16}$$

and that the strategic variable of the players is the flow of investment. Coming from an initial value, the flow of investment describe a unique capacity path. Equation (16) implicitly assume that this capacity path is a continuous function of the time (as it is differentiable). In most of the application of differential game, such assumption is natural. However, in case of irreversible investment, the linearity of investment cost might create an incentive to install the desired capacity as soon as possible. Indeed, the Bellman formula gives:

$$r\Pi^i = \sup_{I^i} \left\{ P\left(\bar{k}\right) k^i - p^+ I^i + I^i \frac{\partial \Pi^i}{\partial K^i} + I^{-i} \frac{\partial \Pi^i}{\partial K^{-i}} \right\}. \tag{17}$$

The optimal investment policy maximizes[9] $I^i \frac{\partial \Pi^i}{\partial K^i} - p^+ I^i$, and when $\frac{\partial \Pi^i}{\partial K^i} < p^+$ the firm has no incentives to invest. When $\frac{\partial \Pi^i}{\partial K^i} > p^+$, the optimal flow of investment I_t^i is infinite: the firm wishes to install its new capacity as soon as possible. This creates a jump in the optimal capacity process of the firms, which is not allowed by (16).

To address this difficulty, I introduce a new control variable, K_t^i, which is the capacity desired by firm i at time t. If this desired capacity is equal to the installed capacity, k_t^i, the firm continue to invest in a continuous way. If the desired capacity is different from the installed capacity, the firm installs the desired capacity. Formally, this is expressed in the following definition.

Definition: The investment game previously considered is in its jump-control form if:
(i) a strategy of player i is a pair $(K_t^i, I_t^i)_{t \in \mathbb{R}_+}$, where for each t, $(K_t^i, I_t^i) \in \mathbb{R}_+^2$.
(ii) the state variable at time t, $k_t = (k_t^1, k_t^2)$, is defined by the two equations:

$$\frac{\partial^+ k_t^i}{\partial t} = I_t^i, \tag{18}$$

[9] This is the case under markovian or open-loop equilibrium. The choice of the equilibrium concept impacts only the differential equation determining firms' profit.

and

$$\lim_{\substack{s \to t \\ s > t}} k_s^i = K_t^i, \tag{19}$$

where k_0 is the given initial level of capital, and $\frac{\partial^+}{\partial t}$ denotes the right-hand derivatives.[10]

(iii) the strategic variables of player i is its level of investment, I_t^i, and the desired capacity K_t^i. $\square$

Equation (19) states than the installed capacity k_t^i is continuous if and only if $K_t^i = k_t^i$. When the strategies are functions of the state variable, firm i jumps as long as $k_t^i \neq \tilde{K}^i(k_t)$. Graphic 3 introduces two examples of strategies than can be follow, and their implication in term of capacity installation.

$[Insert\ Graphic\ 3]$

In such framework, the strategy of player i is markovian if its strategy is only function of the state variable (firms capacity and level of demand), i.e. if there exists $\tilde{K}^i(.)$ and $\tilde{I}^i(.)$ such that

$$K_t^i = \tilde{K}^i(k_t) \text{ and } I_t^i = \tilde{I}^i(k_t). \tag{20}$$

A strategy is an open-loop strategy if K_t and I_t only depends on the time, and not on the installed capacity k_t.

The definition of the jump-control form permits to extend the class of capacity path from the class of continuous functions implicitly assumed by (16) to the class of the left-hand continuous function with right-hand derivative. However, this definition allows multiple jumps: a jump of firm i can imply a jump of its opponents in reaction, which can bring a new jump of firm i, and so on... In order to prevent such multiple jumps, I introduce the following assumption.

H2: The strategies of the firms i verifies assumption H2 if, for all $k \in \mathbb{R}_+^n$,

$$\tilde{K}\left(\tilde{K}(k)\right) = \tilde{K}(k). \tag{21}$$

[10] As process k_t can be discontinuous in t, the right-hand derivative is defined by: $\frac{\partial^+ k_t^i}{\partial t} = \lim_{n \to +\infty} \frac{l - k_{t+h}^i}{h}$, where $l = lim_{\substack{s>t \\ s \to t}} k_s^i$.

A markov perfect equilibrium in the markovian state-control form is defined as a vector of functions $\left(\tilde{K}^{*}(.), \tilde{I}^{*}(.)\right) = \left(\tilde{K}^{*1}(.), \tilde{I}^{*1}(.), .., \tilde{K}^{*n}(.), \tilde{I}^{*n}(.)\right)$ such that, for all firm $i \in \{1, .., n\}$ and for all other markovian strategy of firm i, $\left(\tilde{K}'^{i}(.), \tilde{I}'^{i}(.)\right)$,

$$\forall k \in \mathbb{R}^n_+, \; \Pi^i(k^*_t) \geq \Pi^i(k'_t), \tag{22}$$

where k^*_t is the path defined by (18) and (19) and the strategies $\left(\tilde{K}^{*}(.), \tilde{I}^{*}(.)\right)$, and k'_t is the path created when firm i uses the strategy $\left(\tilde{K}'^{i}(.), \tilde{I}'^{i}(.)\right)$ instead of $\left(\tilde{K}^{*i}(.), \tilde{I}^{*i}(.)\right)$, and the other firm does not change their strategies. Furthermore, a markovian strategy for firm i is said continuous if the function $\tilde{K}^i$ is continuous, and a continuous markov perfect equilibrium is a markov perfect equilibrium with continuous strategy.

To my knowledge, this is a new way of modeling differential games. In this context, it permits to properly define the possibility to jump.

3.2 Characterization of the continuous markov equilibrium

As in the case of the discrete game, let $k^i_{Irr}(k^j)$ be the best response of firm i when firm i has no initial capacity and its opponent install the level k^j at the beginning of the game and maintain the same level afterwards. k^i_{Irr} is defined by:

$$P(k^i_{Irr} + k^j) + P'(k^i_{Irr} + k^j)k^i_{Irr} = rp^+, \tag{23}$$

and the Cournot level, k_C, is given by:

$$k^A_{Irr}(k^B_C) = k^A_C \text{ and } k^B_{Irr}(k^A_C) = k^B_C. \tag{24}$$

Assumption $H1$ is required in order to be sure that equations (23) and (24) are well defined.

Proposition 2 shows that firms have interest to invest at the beginning of the game, and to keep their capacity forever after that. This is the situation in both the continuous markov perfect equilibrium and the open-loop equilibrium. Indeed, as the linearity of investment cost overthrows the incentives to delay investment, everything happens like in a one-shot game, and there is no preemption effect.

Proposition 2: Assume **H1** and **H2**. Let $(\tilde{K}^{*i}, \tilde{I}^{*})^{i\in\{A,B\}}$ be the markovian strategies define by:

$$\text{For } i \in \{A, B\}\,,\ K^{*i}(k_t) = \begin{cases} \max\left\{k^i_{Irr}(k^j_0), k^i_0\right\} \text{ if } k^j_0 > k^j_C \\ \max\left\{k^i_C, k^i_0\right\} \text{ if } k^j_0 \leq k^j_C \end{cases} \text{ and } \tilde{I}^{*i}(A_t, k_t) = 0. \tag{25}$$

Then, $(\tilde{K}^{*i}, \tilde{I}^{*})^{i\in\{A,B\}}$ is the unique continuous markov perfect equilibrium of the game. It is also the unique open loop equilibrium.

4 Dynamic game with uncertainty

In this subsection, I characterize the continuous markovian equilibrium when demand evolves randomly. A_t is the parameter of demand, following a diffusion process:

$$dA_t = \beta(A_t)dt + \sigma(A_t)dW_t, \tag{26}$$

where W_t is a standard Wiener process. The price at time t is now a function of the parameter of demand at time t and of the capacities of the firm at time t. The revenue of firm i at time t is thus:

$$\pi^i(k_t) = P\left(A_t, k^1_t + k^2_t\right) k^i_t. \tag{27}$$

In this real option game model, similar to the one of Grenadier (2002), Back and Paulsen (2009) or Chevalier-Roignant, Huchzermeier and Trigeorgis (2011) there is no strategic effects. As previously, this is due to the linearity of investment's cost, which overthrows the incentives to delay investment. The introduction of uncertainty does not change the strategic interaction. In order to prove this result, I make the following assumptions:

H3: $P(.)$ is a positive function, four times differentiable, such that $P' \leq 0$, $P'' \leq 0$.

H4: $\beta(A)$ and $\sigma(A)$ are continuous functions, and verify the Lipschitz conditions.

H5: There exists a function $G : \mathbb{R}_+ \to \mathbb{R}_+$, such that, $\forall(A, x) \in \mathbb{R} \times \mathbb{R}_+$, $xP(A, x) < G(A)$, and $\int_0^{+\infty} e^{-rt}G(A_t)dt < +\infty$.

The high order of differentiability in assumption $H3$ is needed in order to use Ito's Lemma and to inverse Ito's Lemma results. H4 is classic to ensure the existence of a strong solution to (26). H5 ensures the existence of the stochastic integral determining the profit of the firms.

In order to states proposition 2, let v be the expected price when no firms change their capacity. v is a solution to the differential equation:

$$\forall x > 0,\ rv(A,x) = P(A,x) + \beta(A)\frac{\partial v}{\partial A}(A,x) + \frac{\sigma^2(A)}{2}\frac{\partial^2 v}{(\partial A)^2}(A,x), \tag{28}$$

and should verify that, for all (A,k) such that

$$\sup_{i\in\{A,B\}}\left\{v\left(A,\bar{k}\right) + \frac{\partial v}{\partial k}\left(A,\bar{k}\right)k^i\right\} = p^+, \tag{29}$$

then

$$\frac{\partial v}{\partial A}\left(A,\bar{k}\right) + k^i\frac{\partial^2 v}{\partial A\partial k}\left(A,\bar{k}\right) = 0. \tag{30}$$

The conditions (29) and (30) are the value matching and smooth pasting conditions. As previously, I define $k^i_{Irr}(A,k^j)$, the optimal level of investment of firm i for a level of demand A, by

$$v\left(A, k^i_{Irr} + k^j\right) + \frac{\partial v}{\partial k}\left(A, k^i_{Irr} + k^j\right)k^i_{Irr} = p^+ \tag{31}$$

and k_C, the Cournot equilibrium, by

$$k^A_{Irr}(k^B_C) = k^A_C \text{ and } k^B_{Irr}(k^A_C) = k^B_C. \tag{32}$$

Remark that the Cournot level, k_C, depends on the level of demand, A. With these definition, firms jump at the beginning of the game and make their capacity evolves continuously with the continuous evolution of demand. This continuity of firms' decision overthrow the possibility of strategic effect, as it is seen in theorem 2.

Proposition 3: Assume **H2**, **H3, H4** and **H5**. Let $(\tilde{K}^{*i}, \tilde{I}^*)^{i\in\{A,B\}}$ be the markovian strategies define by:

$$\text{For } i \in \{A,B\},\ K^{*i}(A_t,k_t) = \begin{cases} \max\left\{k^i_{Irr}(A_t,k^j_t), k^i_t\right\} \text{ if } k^j_t > k^j_C \\ \max\left\{k^i_C(A_t), k^i_t\right\} \text{ if } k^j_t \le k^j_C \end{cases} \text{ and } \tilde{I}^{*i}(A_t,k_t) = 0. \tag{33}$$

Then, $(\tilde{K}^{*i}, \tilde{I}^*)^{i\in\{A,B\}}$ is the unique continuous markov perfect equilibrium of the game. It is also the unique open loop equilibrium.

An important point concerning proposition 2 and 3 is that it focus only on continuous markov perfect equilibrium. However there exists a range of other markov perfect equilibrium, as the state variable contains enough information on the past of the game to implement punishment strategies. The continuous markov perfect equilibrium has some interesting properties. It assumes some regularity on the decisions of the firms: a small change in the state variable should not imply an important change in firms' actions. It is unique. When the firms are initially symmetric, it is equivalent to the Cournot equilibrium. For these reasons, I fill confident enough to claim that the continuous markov perfect equilibrium describes the regular equilibrium of the market. It plays the same role as the stage game in a repeated game framework.

5 Conclusion

This work shows that, in the classic model of irreversible investment in capacity, there is no preemption effect. This is due to the linearity of investment price which destroys any incentive to delay investment, and thus the possibility to influence future decisions. The introduction of uncertainty does not change the nature of strategic interactions between players (at least as long as the uncertainty is a generalized Brownian motion and evolves continuously). These results contradict the intuition of the real option game literature. The possibility of preemption, found in models of entry (models of investment in a single project), may not be valid for investment in capacity, as long as the capacity is sufficiently smooth.

These results are highly dependent on the nature of the investment's cost. In the industrial organization literature, there exists two other assumption which permits to recover the strategic effects. The first one is to assume quadratic cost of investment, as in Figuières (2009). Quadratic investment cost creates incentives to always postpone a part of the remaining investment. A monopoly with quadratic cost will never achieve its stationary capacity, even in a static environment. The second possibility is to assume linear investment's cost but also an exogenous boundary on the flow of investment of the firms, as in Fudenberg and Tirole (1983). In that case both firms invest as much as they can, until the marginal value of their investment is inferior to the investment cost. The path of capacities is thus highly dependent on the exogenous boundary, representing the financial constraint of the firm. It is this financial constraint which creates the strategic effects. Recent empirical papers, as Angelini and Generale (2008) and Bottazzi et al. (2014) shows that small firms are more restricted than large firms on the capital market. It suggests that strategic effects should be more accurate one market composed of small firms than on market composed of large competitors.

On a game theory perspective, this work contributes to extend the theory of differential games, introducing a way to define strategies which enables players to create endogenous jumps in the state variable. In this framework, there exists a multiplicity of markovian equilibrium, as in discrete dynamic games with infinite time-horizon, but only one of them is describe by a strategy continuous in the state variable.

6 Appendix

Proof of proposition 1:

In step 1 and 2, I show that in any sub-game perfect equilibrium $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$, firms have the same capacity in period 1 and 2. Existence, unicity and characterization of this equilibrium follow. Step 3 focuses on open-loop equilibrium.

Step 1: In this step, I show that for all choices of capacity made in the first period, the equilibrium is:

$$\text{For } i \in \{A,B\}\,,\ k_2^{*i} = \begin{cases} \max\left\{\hat{k}_{Irr}^i(k_1^{*j}), k_1^{*i}\right\} \text{ if } k_1^{*j} > \hat{k}_C^j \\ \max\left\{\hat{k}_C^j, k_1^{*i}\right\} \text{ if } k_1^{*j} \leq \hat{k}_C^j \end{cases}, \tag{34}$$

where $\hat{k}_{Irr}^i(k_1^{*j})$ is implicitly defined by

$$P\left(\hat{k}_{Irr}^i + k^j\right) + P'\left(\hat{k}_{Irr}^i + k^j\right) k_{Irr}^i = p^+, \tag{35}$$

and $\hat{k}_C$ by

$$\hat{k}_{Irr}^A(\hat{k}_C^B) = \hat{k}_C^A \text{ and } \hat{k}_{Irr}^B(\hat{k}_C^A) = \hat{k}_C^B. \tag{36}$$

The existence of $\hat{k}_{Irr}$ is due to assumption 2. The existence and unicity of $\hat{k}_C$ comes from Novshek (1985). Novshek (1985) also implies that the equilibrium of the last stage game exists and is unique (the profit function of the last stage verifies the assumption of theorem 3).

The best response of firm i is

$$k_2^i = \max\left\{\hat{k}_{Irr}^i(k_1^{*j}), k_1^{*i}\right\}. \tag{37}$$

Therefore, if opponent of firm i have a capacity inferior to the Cournot level, the equilibrium strategy of firm i will be to install the Cournot level, except if its capacity already exceed this level. If the capacity of its opponents exceed the Cournot level, firm i will invest until its best response level, knowing that the other firm will not invest (if firm j invest until its best response level, both firms install the Cournot level).

Step 2: This step shows that, at the equilibrium, firms are constraint in the second period. Assume that $k_2^{*i} > k_1^{*i}$, for one i. Then, there is two possibility: first $k_1^{*j} > \hat{k}_C^j$ and $k_2^{*i} = \hat{k}_{Irr}^i(k_1^{*j}) > k_1^{*i}$. In that case, the profit of firm i in the first period is

$$\Pi^i|_{\text{first period}} = P\left(k_1^i + k_1^j\right)k_1^i - p^+\left(k_1^i - k_0^i\right) \tag{38}$$

which is a concave function of k_1^i, with a maximum in $\hat{k}_{Irr}^i(k_1^{*j})$. However, $k_1^{*i} < \hat{k}_{Irr}^i(k_1^{*j})$, and firm 1 does not invest until its best response level, which contradict the fact that $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$ is an equilibrium.

The second possibility is that $k_1^{*j} < \hat{k}_C^j$ and $k_2^{*i} = \hat{k}_C^j > k_1^{*i}$. In that case, the profit of firm i is the same as (38), and, as $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$ is an equilibrium, $k_1^{*i} = \hat{k}_{Irr}^i(k_1^{*j})$. Using the theorem of implicit function to (35) implies that $\hat{k}_{Irr}^i(.)$ is a decreasing function of the capacity of firm i's opponent. Therefore, as $k_1^{*j} < \hat{k}_C^j$, $k_1^{*i} > \hat{k}_{Irr}^i(\hat{k}_C^j) = \hat{k}_C^j$, which contradict the fact that firm i is not constrained.

At equilibrium, firms do not invest in the second period. The sub-game perfect equilibrium can thus be seen as a one shot game, and the characterization of the equilibrium, as presented in proposition 1, comes from the same reasoning as step 1.

Step 3: Assume that $(k_t^{*i})_{t\in\{1,2\}}^{i\in\{A,B\}}$ is an open loop equilibrium, and that firm A is not constrained in the second period ($k_2^{*A} > k_1^{*A}$). Then, by definition,

$$k_2^{*A} = \arg\max\left[P\left(k_2^i + k_2^{*j}\right)k_2^i - p^+\left(k_2^i - k_0^i\right)\right] \tag{39}$$

and

$$k_1^{*A} = \arg\max \left[P\left(k_1^i + k_1^{*j}\right) k_1^i - (1-\delta)\, p^+ k_1^i \right]. \tag{40}$$

Let $X(k^j, y)$ be the solution to the implicit equation

$$P\left(X + k^j\right) + P'\left(X + k^j\right) X = y. \tag{41}$$

By assumption $H1$, (41) is well defined, and, by application of the theorem of implicit function, decreasing in k^j and y. Furthermore, $k_2^{*A} = X(k_2^{*j}, p^+)$ and $k_1^{*A} = X(k_1^{*j}, (1-\delta)\, p^+)$. As by assumption $k_2^{*j} \geq k_1^{*j}$,

$$k_2^{*A} = X(k_2^{*j}, p^+) \leq X(k_1^{*j}, p^+) < X(k_1^{*j}, (1-\delta)\, p^+) = k_1^{*A}. \tag{42}$$

■

In the following proof, I put the time in index when I consider a variable as a stochastic process and nothing in index when I consider the variable as a constant. Thereby, $k^i = \tilde{K}^i(A, k)$ is the capacity implemented by firm i for a level of demand A and a vector of installed capacity k, whereas $K_t^i = \tilde{K}^i(A_t, k_t)$ is the stochastic process implemented during a period of time.

Theorem 1 is a particular case of theorem 2. In order to prove theorem 2, I define H, the set of state variables such that no firms jump:

$$H = \left\{ (A, k) \ \mid \tilde{K}^*(A, k) = k \right\}. \tag{43}$$

Proof of theorem 2:

The proof is devised in four steps. The first step characterizes the optimal strategy and the profit function when both firms belong to H. The second step ensures that the implicit functions expressed in (31) are well defined. To do so, I show that the profit functions are concave (in H). Third step described the equilibrium, and the last step fully characterizes the profit functions using the smooth pasting and value-matching conditions.

Step 1: Characterization of the profit functions when (A, k) belongs to H^i.

If the initial point (A, k) belongs to $\mathring{H}$, Bellman theorem applies and:

$$r\Pi^i(A,k) = \max_{I^i}\left\{\pi^i(A,k) - p^+\left(I^i\right)_+ - p^-\left(I^i\right)_- + I^i\frac{\partial \Pi^i}{\partial K^i} + D\Pi^i\right\}, \tag{44}$$

where $D\Pi^i$ does not depend of the control I^i. If $\frac{\partial \Pi^i}{\partial K^i} \leq p^+$, then the optimal control is $I^i = 0$. If $\frac{\partial \Pi^i}{\partial K^i} > p^+$, then there is no optimal control available in $\mathbb{R}$, and firm i wishes to jump. The same works for firm j, and, H verifies

$$H = \left\{k \in \mathbb{R}_+^2 \mid \text{for } i \in \{A,B\}, \frac{\partial \Pi^i}{\partial K^i} \leq p^+\right\}. \tag{45}$$

Inside H, the best strategy of each firm is to maintain its capacity constant and thus the firms profit depends only on the evolution of the uncertainty. Therefore, for all $i \in \{A,B\}$,

$$r\Pi^i = \pi^i(A,k) + \beta(A)\frac{\partial \Pi^i}{\partial A} + \frac{\sigma^2(A)}{2}\frac{\partial^2 \Pi^i}{\partial A^2}. \tag{46}$$

The solutions to this differential equation have the form

$$\Pi^i(A,k) = v\left(A,\bar{k}\right)k^i, \tag{47}$$

where v is a solution to:

$$rv(A,x) = P(A,x) + \beta(A)\frac{\partial v}{\partial A}(A,x) + \frac{\sigma^2(A)}{2}\frac{\partial^2 v}{(\partial A)^2}(A,x). \tag{48}$$

This permits to pass from two differential equations in (46) to a unique one in (48). However, this differential equation can still have many solutions.

Step 2: Concavity of the profit functions

The principal difficulty of that step is that (48) defines v only on H. So I will use a general solution to (48), w, and show that assumption $H3$ implies the concavity of v.

Let $w(A,x)$ be a solution to (48) for all $(A,x) \in \mathbb{R}_+^2$. Using Itô Lemma on the function $e^{-rt}w(A_t,\bar{k})$ gives:

$$d\left(e^{-rt}w(A_t,\bar{k})\right) = \left(\beta(A_t)\frac{\partial w}{\partial A}(A_t,\bar{k}) + \frac{\sigma^2(A_t)}{2}\frac{\partial^2 w}{(\partial A)^2}(A_t,\bar{k}) - rw(A_t,\bar{k})\right)e^{-rt}dt$$
$$+ \sigma(A_t)e^{-rt}\frac{\partial w}{\partial A}(A_t,\bar{k})dW_t.$$

By (48), this equation becomes

$$d\left(e^{-rt}w(A_t,\bar{k})\right) = -P\left(A_t,\bar{k}\right)e^{-rt}dt + \sigma(A_t)e^{-rt}\frac{\partial w}{\partial A}(A_t,\bar{k})dW_t, \tag{49}$$

which can be rewritten, for $h > 0$,

$$\forall A_0 \in \mathbb{R}_+,\ w(A_0, \bar{k}) - e^{-rh} w(A_h, \bar{k}) = \int_0^h P\left(A_t, \bar{k}\right) e^{-rt} dt - \int_0^h \sigma(A_t) e^{-rt} \frac{\partial w}{\partial A}(A_t, \bar{k}) dW_t \tag{50}$$

and thus

$$w(A_0, \bar{k}) - e^{-rh} E\left[w(A_h, \bar{k}) | A_0\right] = E\left[\int_0^h P\left(A_t, \bar{k}\right) e^{-rt} dt \mid A_0\right]. \tag{51}$$

When $h \to +\infty$, (51) becomes,

$$w(A_0, \bar{k}) = E\left[\int_0^{+\infty} P\left(A_t, \bar{k}\right) e^{-rt} dt \mid A_0\right]. \tag{52}$$

Assumption $H5$ implies that this integral is well defined. Assumption $H3$ implies that $v' < 0$ and $v'' < 0$, so w verifies assumption $H1$. Furthermore, on H, $w(A, \bar{k})$ is a particular solution to the differential equation (48), and, as v is also a solution to (48), there exists a function $g(.)$ such that, on H:

$$v(A, \bar{k}) = g(A) + w(A, \bar{k}), \tag{53}$$

thus v also verifies assumption $H1$.

Step 3: Equilibrium outside H.

Assume now that, for one firm, the optimal strategy is to jump at the initial point (A, k). Let $\hat{k}^i$ be the vector of capacity just after the jump:

$$\hat{k} = \tilde{K}(k).$$

If $\hat{k}^i = k^i$, firm i does not jump. By assumption $H2$, firms do not jump just after a jump, and $\hat{k} \in J$. Then, the profit of firm i is equal to:

$$\Pi^i(A, k) = \Pi^i\left(A, \hat{k}\right) - p^+ \max\left(\hat{k}^i - k_0^i, 0\right). \tag{54}$$

As v verifies assumption $H1$, the implicit theorem ensures the existence of (31). Then, there is two possibilities:

· If both firms have capacity inferior to the Cournot level, as defined by (32), firms invest until the Cournot level.

· If firm i have a capacity higher than the Cournot level, $k^i > k_C^i$, then its opponents will jump until $\hat{k}^j = k_{Irr}^j(k^i)$ whereas firm i keep its capacity constant. If firm j has a capacity higher than its best response to firm i's capacity, $k^j > k^j_{Irr}(k^i)$ then no firms invest, and $k \in H$.

This gives the unicity and the form of equilibrium for any solution of (48). However, v should also verify the smooth-pasting and value matching conditions.

Step 4: Smooth pasting and value-matching condition.

Let $k \in H(A)$. Let A^+ be the level of demand such that one firm invests if the demand increases over this level. Then, if firm i is a firm which invests just after A^+, the profit function of firm i should verify,

$$\frac{\partial \Pi^i}{\partial k^i}\left(A^+, k\right) = p^+. \tag{55}$$

If firm i does not invest just after A^+,

$$\frac{\partial \Pi^i}{\partial k^i}\left(A^+, k\right) < p^+. \tag{56}$$

The smooth pasting condition is thus

$$\max\left\{\frac{\partial \Pi^A}{\partial k^A}\left(A^+, k\right), \frac{\partial \Pi^B}{\partial k^B}\left(A^+, k\right)\right\} = p^+, \tag{57}$$

which can be written as:

$$\sup_{i\in\{A,B\}}\left\{v\left(A^+, \bar{k}\right) + \frac{\partial v}{\partial k}\left(A^+, \bar{k}\right) k^i\right\} = p^+. \tag{58}$$

The differentiation of (55) gives the optimal matching condition:

$$\frac{\partial^2 \Pi^i}{\partial A \partial k^i}\left(A^+, k\right) = 0, \tag{59}$$

for all i which maximizes (57), which can be written

$$\frac{\partial v}{\partial A}\left(A^+, \bar{k}\right) + \frac{\partial^2 v}{\partial A \partial k^i}\left(A^+, \bar{k}\right) k^i = 0. \tag{60}$$

Equations (58), (60), characterizes the solution of the differential equation (48).

At this point of the proof the profit function of firm i can be rewritten:

$$\Pi^i(A, k) = \max_{K^i | K \in H(A)} \left\{v\left(A, \bar{K}\right) K^i - \max\left(K^i - k^i, 0\right)\right\}, \tag{61}$$

where v is a solution to the differential equation (48), which verifies (58) and (60). As v verifies assumption $H1$, the same reasoning as in the last step of the discrete game gives us theorem 2.

■

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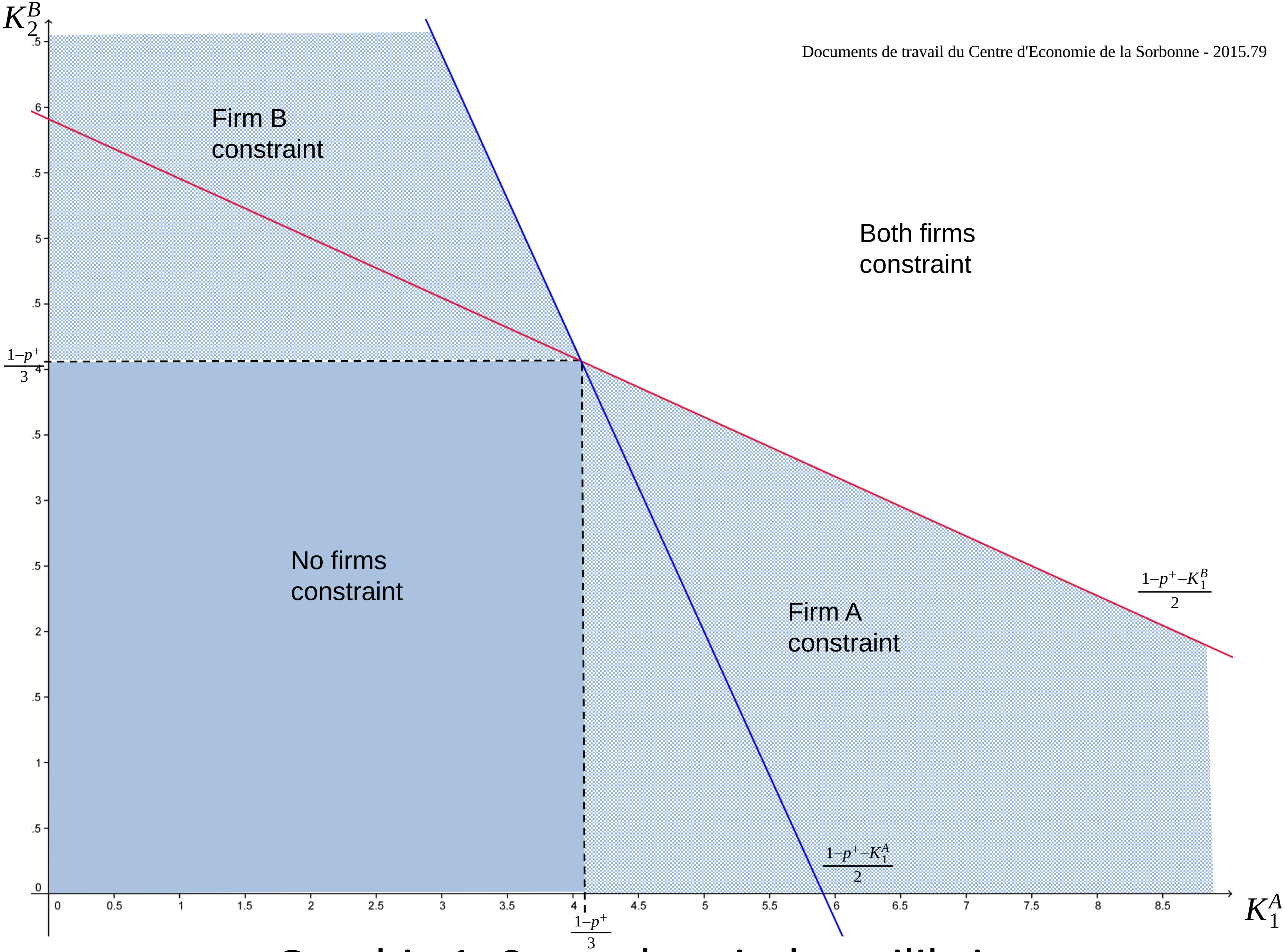
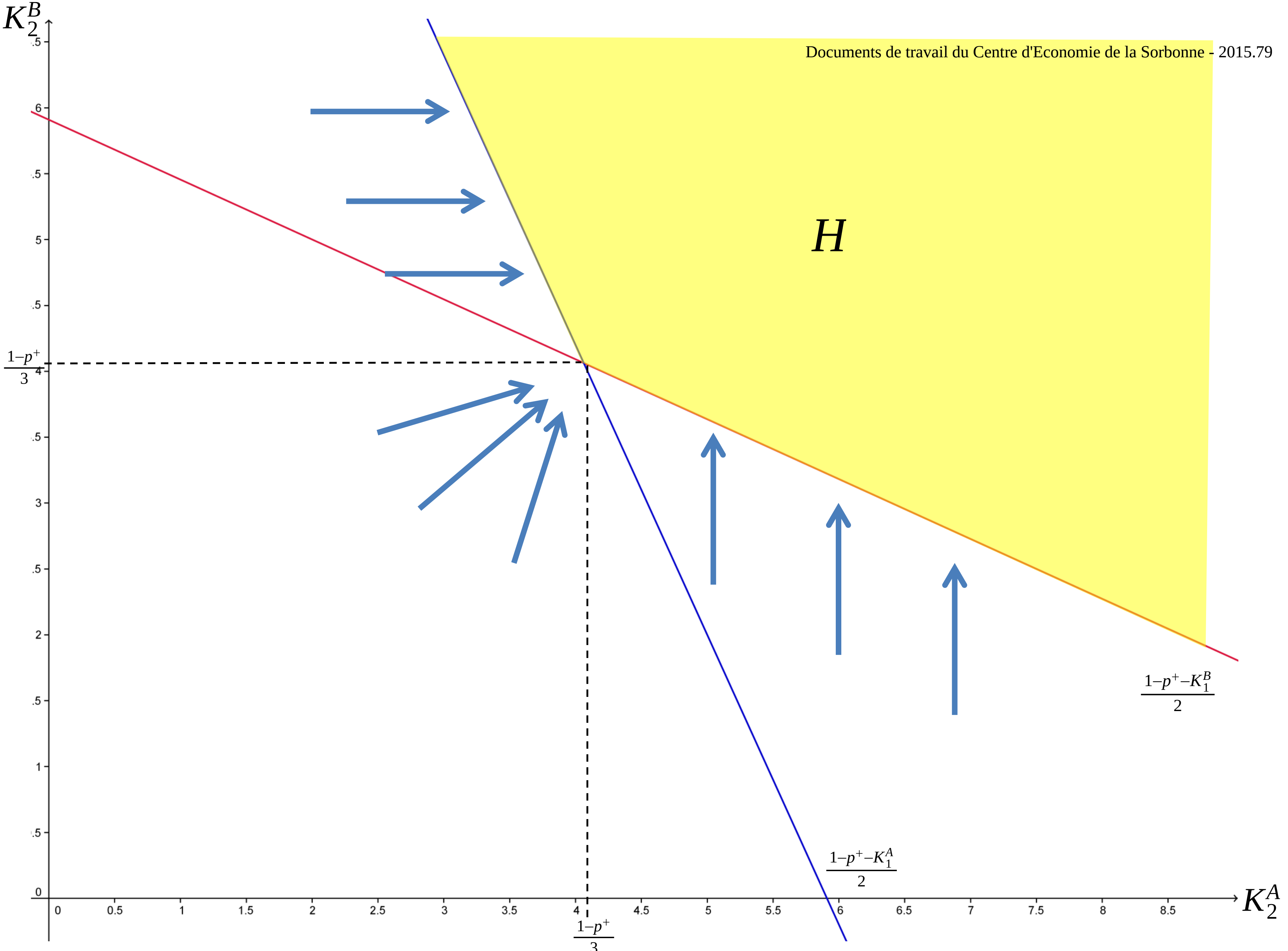
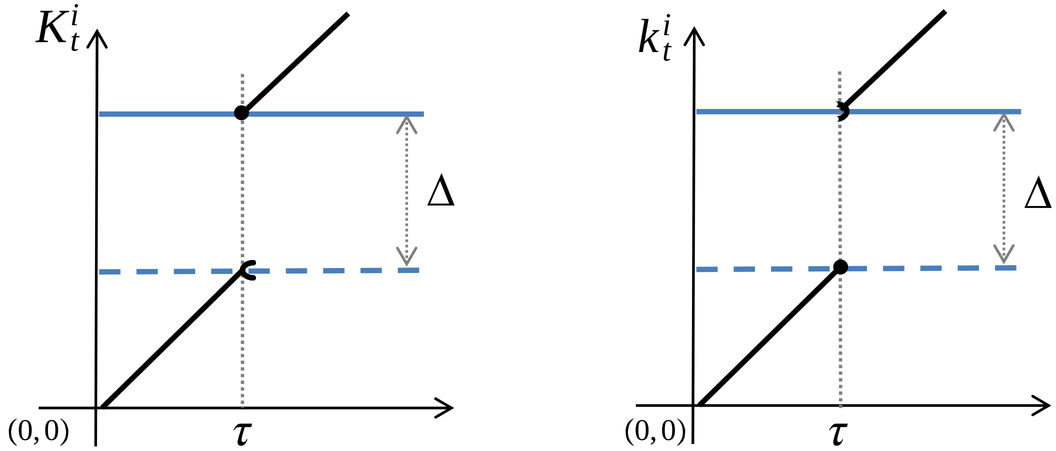


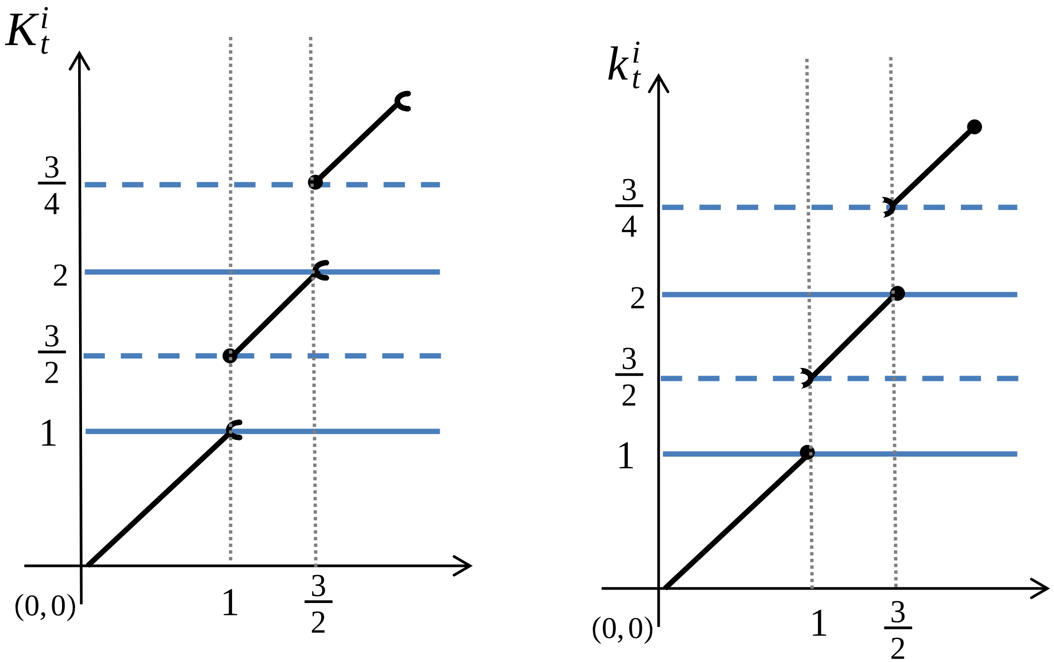
Graphic 1: Second period equilibrium



Graphic 2: Second period equilibrium

$\tilde{I}(A_t, k_t) = 1$, and
$\tilde{K}(A_t, k_t) = k_t^i + \Delta 1_{\{k_t^i = \tau\}}$.





$\tilde{I}(A_t, k_t) = 1$, and
$\tilde{K}(A_t, k_t) = k_t^i + \frac{1}{2} 1_{\{|k_t^i| = k_t^i\}}$.

Graphic 3: Examples