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Asymptotic Analysis of Regularized Zero-Forcing Precoding in MISO Broadcast Channels with Limited Feedback

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Abstract—In this paper we analyse the asymptotic sum-rate of regularized zero-forcing (RZF) precoding in MISO broadcast channels with limited feedback, transmit correlation and path-loss. Our analysis assumes that the ratio of the number of transmit antennas \( M \) to the number of users \( K \) is bounded as \(( K, M) \) grow large. By applying recent results from random matrix theory we derive a deterministic equivalent of the SINR and compute the sum-rate maximizing regularization term as well as sum-rate bounds for high SNR. Numerical simulations show that the asymptotic results extend well into finite regimes.

I. INTRODUCTION

It has been shown in [1] that the capacity achieving precoding strategy of the Gaussian MIMO broadcast channel is based on the non-linear dirty-paper coding (DPC) technique. But so far no efficient practical algorithm implementing the optimal DPC scheme has been proposed. Therefore, low complexity linear precoding strategies have gained a lot of attention since they achieve a large portion of the channel capacity [2], [3].

The RZF filters have first been analysed in the MU-MIMO context in [3]. It has been observed that the RZF precoding matrix has a similar structure as the transmit Wiener filter context in [3]. It has been observed that the RZF precoding strategy of the Gaussian MIMO broadcast channel is based

Definition 1: Let \( F \) be a probability distribution function. For \( z \in \mathbb{C} \) outside the support of \( F \), we define the Stieltjes transform of \( F \) as the function

\[
m_F(z) = \int \frac{1}{\lambda - z} dF(\lambda)
\]

In this paper we are interested in the Stieltjes transform \( m_{B_K} \) of random matrices \( B_K \in \mathbb{C}^{M \times M} \) of the type

\[
B_K = R_K^{1/2} X_K^H L_K X_K R_K^{1/2} + A_K
\]

where \( R_K \in \mathbb{C}^{M \times M} \) and \( L_K \in \mathbb{C}^{K \times K} \) are positive definite Hermitian matrices, \( X_K \in \mathbb{C}^{K \times M} \) is random with independent and identically distributed (i.i.d.) entries of zero mean and variance \( 1/M \), and \( A_K \) is a nonnegative Hermitian matrix, with same eigenspace as \( R_K \); denote then \( a(\lambda) \) a function mapping the eigenvalues of \( R_K \) to those of \( A_K \). In our derivations, we will require the following result.

Theorem 1: [7] Under the above model for \( B_K \) where \( L_K \) and \( R_K \) have uniformly bounded spectral norm (w.r.t. \( M \)), as \(( K, M) \) grow large with ratio \( \beta = \beta(M) \Delta M/K \) such that \( 0 < \lim \inf M \beta(M) \leq \lim \sup M \beta(M) < \infty \), for \( z \in \mathbb{C}^+ \),

\[
m_{B_K}(z) - \tilde{m}_{B_K}(z) \rightarrow \frac{M}{z^2} 0
\]

almost surely, where \( \tilde{m}_{B_K}(z) \) is defined as

\[
m_{\tilde{B}_K}(z) = \int \frac{1}{a(\lambda) + \lambda c(z) - z} dF(R_K)(\lambda)
\]

with

\[
c(z) = \frac{1}{\beta} \int \frac{\nu}{1 + \nu e_K(z)} dF(R_K)(\nu)
\]

where \( e_K(z) \in \mathbb{C}^+ \) is the unique solution of

\[
e_K(z) = \int \frac{\lambda}{a(\lambda) + \lambda c(z) - z} dF(R_K)(\lambda)
\]

where \( \tilde{m}_{B_K}(z) \) is referred to as a deterministic equivalent of \( m_{B_K}(z) \).

II. MATHEMATICAL PRELIMINARIES

Definition 1: Let \( F \) be a probability distribution function. For \( z \in \mathbb{C} \) outside the support of \( F \), we define the Stieltjes transform of \( F \) as the function

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where \( \tilde{m}_{B_K}(z) \) is referred to as a deterministic equivalent of \( m_{B_K}(z) \).

III. SYSTEM MODEL

Consider the MISO broadcast channel composed of one central transmitter equipped with \( M \) antennas and of \( K \) single-antenna receivers. Assume narrow-band communication. Denoting \( y_k \) the signal received by user \( k \), the concatenated
received signal vector $y = [y_1, \ldots, y_K]^T \in \mathbb{C}^K$ at a given time interval reads

$$y = \sqrt{M}Hx + n$$  \hspace{1cm} (7)

with transmit vector $x \in \mathbb{C}^M$, channel matrix $H \in \mathbb{C}^{K \times M}$ containing i.i.d. elements of zero mean and variance $1/M$ and noise vector $n \sim C\mathcal{N}(0,\sigma^2I_K)$. The transmit vector $x$ is obtained by linear precoding

$$x = Gs$$  \hspace{1cm} (8)

where $s \sim C\mathcal{N}(0,I_K)$ is the symbol vector and $G = [g_1, \ldots, g_K] \in \mathbb{C}^{M \times K}$ is the precoding matrix. The transmitter has a limited amount of transmit power $P > 0$, thus we have the constraint

$$\text{tr}(E[xx^H]) = \text{tr}(GG^H) \leq P$$  \hspace{1cm} (9)

In this paper we consider regularized channel inversion precoding

$$G = \sqrt{M} \xi \left( M\hat{H}^H\hat{H} + \alpha I_M \right)^{-1} \hat{H}^H$$  \hspace{1cm} (10)

$$= \frac{\xi}{\sqrt{M}} \left( \hat{H}^H\hat{H} + \alpha I_M \right)^{-1} \hat{H}^H \Delta \frac{\xi}{\sqrt{M}} W\hat{H}^H$$  \hspace{1cm} (11)

where $\hat{H}$ is the estimated channel matrix available at the transmitter and the scaling factor $\xi$ is set to fulfill the power constraint (9). The regularization scalar $\alpha$ in (10) is scaled by $M$ to ensure that, as $(K, M)$ grow large, both $\text{tr}\hat{H}^H\hat{H}$ and $\text{tr}M\alpha I_M$ grow with the same order of magnitude. From (9) we obtain

$$\xi^2 = \frac{P}{\text{tr}(\hat{H}^H\hat{H})} \frac{P}{\text{tr}(\hat{H}^H\hat{H} + \alpha I_M)^{-2}} \frac{P}{\Psi(\alpha)}$$  \hspace{1cm} (12)

where (a) follows from (1) and (·)' denotes the derivative w.r.t. $z$ in $z = -\alpha$. The received symbol $y_k$ of user $k$ is given by

$$y_k = \xi h_k^H\hat{W}_k s_k + \xi \sum_{i=1, i \neq k}^K h_k^H\hat{W}_i s_i + n_k$$  \hspace{1cm} (13)

where $h_k^H$ and $\hat{H}_k^H$ denote the $k^{th}$ row of $H$ and $\hat{H}$, respectively. The SINR $\gamma_k$ of user $k$ can be written in the form

$$\gamma_k = \frac{|h_k^H\hat{W}_k|^2}{h_k^H\hat{W}_k U_k W_k + \rho \Psi(\alpha)}$$  \hspace{1cm} (14)

where $U_k^H = [\hat{h}_1, \ldots, \hat{h}_{k-1}, \hat{h}_{k+1}, \ldots, \hat{h}_K] \in \mathbb{C}^{M \times (K-1)}$ and $\rho = P/\sigma^2$ denotes the SNR. The sum-rate $R_{\text{sum}}$ is defined as

$$R_{\text{sum}} = \sum_{k=1}^K \log (1 + \gamma_k) \quad \text{[nats/s/Hz]}$$  \hspace{1cm} (15)

Under the assumption of a rich scattering environment the correlated channel can be modeled as [8]–[10]

$$H = L^{1/2}H_w \Theta_T^{1/2}$$  \hspace{1cm} (16)

where $H_w \in \mathbb{C}^{K \times M}$ has i.i.d. zero-mean entries of variance $1/M$, $\Theta_T \in \mathbb{C}^{M \times M}$ is the nonnegative definite correlation matrix at the transmitter and $L = \text{diag}([l_1, \ldots, l_K])$ is a diagonal matrix containing the user’s channel gain. We assume $||\Theta_T||^{-1}$ to be uniformly bounded with respect to $M$.

Moreover, we suppose that only $\hat{H}$, an imperfect estimate of the true channel matrix $H$, is available at the transmitter. The channel-gain matrix $L$ as well as the transmit correlation $\Theta_T$ can be estimated accurately and are assumed to be perfectly known. We therefore model $\hat{H}$ as

$$\hat{H} = L^{1/2}H_w \Theta_T^{1/2}$$  \hspace{1cm} (17)

with $\hat{H}_w = \sqrt{1 - \tau^2} H_w + \tau Q$  \hspace{1cm} (18)

where $Q \in \mathbb{C}^{K \times M}$ has i.i.d. zero-mean entries of variance $1/M$ which are not necessarily Gaussian distributed. Furthermore we suppose that $H_w$ and $Q$ are mutually independent as well as independent of the symbol vector $s$ and noise $n$. A similar model for imperfect CSIT has been used in [11]–[13].

IV. DETERMINISTIC EQUIVALENT OF THE SINR

In the following we will derive a deterministic equivalent $\gamma^*_k$ of the SINR of user $k$, i.e. $\gamma^*_k$ is such that, almost surely,

$$\gamma_k - \gamma^*_k \xrightarrow{M \rightarrow \infty} 0$$  \hspace{1cm} (19)

That is, $\gamma^*_k$ is an approximation of $\gamma_k$ independent of the particular realizations of $H_w$ and $Q$. We will proceed by calculating deterministic equivalent expressions of $\Psi(\alpha)$, the signal power $|h_k^H\hat{W}_k|^2$ and the power of the interference $h_k^H U_k W_k^H$. Consider $\Psi(\alpha)$ in (12), from Theorem 1, with $R_K = \Theta_T$ and $A_K = 0$, we know that, for $K$ large, $m_{H^H\hat{H}}(-\alpha)$ is close to $m^{\xi}_{H^H\hat{H}}(-\alpha)$ given by Equation (3). Therefore we have

$$\Psi(\alpha) \approx \left[ m_{H^H\hat{H}}(-\alpha) - \alpha m^{\xi}_{H^H\hat{H}}(-\alpha) \right] \xrightarrow{M \rightarrow \infty} 0$$  \hspace{1cm} (20)

The Stieltjes transform is differentiable and $m^{\xi}_{H^H\hat{H}}$ is well defined.

At this point we need the following results.

Corollary 1: [14] Let $A$ be a deterministic $N \times N$ complex matrix with uniformly bounded norm. Let $x \in \mathbb{C}^N$ have i.i.d. complex entries of zero mean and variance $1/N$. Then, almost surely,

$$x^HA - \frac{1}{N} \text{tr}A \xrightarrow{N \rightarrow \infty} 0$$  \hspace{1cm} (21)

and for vector $y \in \mathbb{C}^N$ with standard i.i.d. entries, independent of $x$,

$$y^H A y \xrightarrow{N \rightarrow \infty} 0$$  \hspace{1cm} (22)

almost surely.

In addition we will make use of the following identity

Lemma 1: [15, Lemma 2.2] Let $A$ be an $N \times N$ invertible matrix and $x \in \mathbb{C}^N$, $c \in \mathbb{C}$ for which $A + cxx^H$ is invertible. Then

$$x^H (A + cxx^H)^{-1} = \frac{x^HA^{-1}}{1 + cxx^H A^{-1} x}$$  \hspace{1cm} (23)
A. Signal Power

Applying Lemma 1, we have

\[ \hat{h}_k^H W_k h_k = \frac{\tilde{h}_k^H (U_k^H U_k + \alpha I_M)^{-1} h_k}{1 + \tilde{h}_k^H (U_k^H U_k + \alpha I_M)^{-1} h_k} \]  
(24)

Together with \( \hat{h}_k = \sqrt{1 - \tau^2} h_k + \tau \hat{q}_k \) and \( \hat{q}_k = \sqrt{\tau} q_k \Theta_{T/2}^1 \), we obtain

\[ \hat{h}_k^H W_k h_k = \frac{\sqrt{1 - \tau^2} l_k h_w k A_w^{-1} h_w k + \tau l_k q_k^H A_w^{-1} h_w k}{1 + l_k h_w k A_w^{-1} h_w k} \]

with \( V_k = [\hat{h}_w(1), \ldots, \hat{h}_w(k), \ldots, \hat{h}_w(K)] \) and \( A_k = V_k^H L K V_k + \alpha \Theta_{T/2}^{-1} \). Since \( h_w k \) and \( q_k \) have i.i.d. entries of variance 1/M and are independent of \( A_k \) we invoke Corollary 1 and obtain

\[ h_{w k}^H A_k^{-1} h_w k = \frac{1}{M} \text{tr} A_k^{-1} \]

\[ \hat{h}_{w k}^H A_k^{-1} \hat{h}_{w k} = \frac{1}{M} \text{tr} A_k^{-1} \]

\[ q_k^H A_k^{-1} q_{w k} = \frac{1}{M} \text{tr} A_k^{-1} \]

Consequently we have

\[ \hat{h}_k^H W_k h_k - \sqrt{1 - \tau^2} l_k m_{A_k}^0 \frac{1}{M} \text{tr} A_k^{-1} \]

\[ \frac{1}{1 + l_k m_{A_k}^0 \frac{1}{M} \text{tr} A_k^{-1}} \]

\[ \frac{M \rightarrow \infty}{0} \]

\[ \frac{M \rightarrow \infty}{0} \]

\[ \frac{M \rightarrow \infty}{0} \]

(28)

In [16] we prove that a rank-1 perturbation has no impact on the trace of \( A_k^{-1} \) for asymptotically large \( K \). Therefore, almost surely,

\[ \frac{1}{M} \text{tr} A_k^{-1} - \frac{1}{M} \text{tr} A_k^{-1} \]

\[ \frac{M \rightarrow \infty}{0} \]

(29)

where \( A = \tilde{h}_{w k}^H L \tilde{h}_{w k} + \alpha \Theta_{T/2}^{-1} \).

Furthermore we can write \( \frac{1}{M} \text{tr} A_k^{-1} \) in terms of the Stieltjes transform \( m_A(z) \) in \( z = 0 \) (which is valid since \( \| \Theta_{T/2}^{-1} \| \) is bounded away from zero) which, for large \( K \) is close to

\[ m_A^0(0) = \int \frac{\lambda}{\lambda - \alpha} dF_{\Theta_{T/2}}(\lambda) \]

with \( c(0) = \frac{1}{\beta} \int \frac{\nu}{1 + \nu m_A^0(0)} dF_L(\nu) \)

(31)

(32)

Finally, Equation (28) implies

\[ \hat{h}_k^H W_k h_k - \sqrt{1 - \tau^2} l_k m_{A_k}^0 \frac{1}{1 + l_k m_{A_k}^0} \frac{M \rightarrow \infty}{0} \]

(33)

B. Interference Power

After applying Lemma 1 twice, we obtain

\[ h_k^H W_k U_k^H U_k W_k h_k = \frac{l_k h_{w k}^H B_k^{-1} V_k^H L_k V_k B_k^{-1} h_{w k}}{1 + l_k \tau^2 (h_{w k}^H B_k^{-1} h_{w k})^2} \]

(34)

where \( B_k = A_k + l_k \tau^2 q_k q_k^H + l_k \tau \sqrt{1 - \tau^2} h_{w k} q_k^H + l_k \tau \sqrt{1 - \tau^2} h_{w k} q_k \).

Lemma 2: Let \( A \in \mathbb{C}^{N \times N} \) of bounded norm. Let \( x, y \in \mathbb{C}^N \) have i.i.d. complex entries of zero mean and variance 1/N. Then, for \( c_1 \in \mathbb{C} \), \( i = 0, 1, 2 \) and \( u = \frac{1}{\text{tr} A^{-1}} \)

\[ x^H (A + c_0 xx^H + c_1 yy^H + c_2 xy^H + c_2 yx^H)^{-1} x \]

\[ u(1 + c_1 u)^{N - 2} u \]

\[ \frac{1}{(c_0 c_1 - c_0^2) u^2} + (c_0 + c_1) u + 1 \]

almost surely. The proof can be found in [16].

Applying Lemma 2, after some algebraic manipulations we obtain

\[ l_k m_{A_k}^0(0) - c m_{A_k}^0 \Theta_{T/2}^1(0)[1 + l_k \tau^2 (l_k m_{A_k}^0(0) + 2)m_{A_k}^0(0)] \]

\[ \frac{1}{(1 + l_k m_{A_k}^0(0))^2} \]

\[ \hat{h}_k^H W_k U_k^H U_k W_k \]

\[ \frac{M \rightarrow \infty}{0} \]

(36)

where \( m_{A_k}^0 \Theta_{T/2}^1(0) = \frac{1}{M} \text{tr} \left( \Theta_{T/2}^{1/4} \tilde{h}_{w k}^H L_k^H \tilde{h}_{w k} + \Theta_{T/2}^{1/4} + \alpha \Theta_{T/2}^{-1/2} \right)^{-2} \).

We can express \( m_{A_k}^0 \Theta_{T/2}^1(0) \) in terms of the derivative of \( m_{A_k}^0 \Theta_{T/2}^1(0) \)

\[ m_{A_k}^0 \Theta_{T/2}^1(0) = \frac{\partial m_{A_k}^0 \Theta_{T/2}^1(0)}{\partial z} \]

\[ \mid z = 0 = m_{A_k}^0 \Theta_{T/2}^1(0) \]

(37)

Applying Theorem 1 to matrix \( A \Theta_{T/2}^{-1} \) leads to

\[ m_{A_k}^0 \Theta_{T/2}^1(0) = \frac{1}{1 + \nu m_{A_k}^0(0)} dF_L(\nu) \]

(38)

(39)

For the derivatives we obtain

\[ m_{A_k}^0 \Theta_{T/2}^1(0) = \frac{1}{1 + \nu m_{A_k}^0(0)} dF_L(\nu) \]

(41)

(42)

Finally, the deterministic equivalent \( \gamma_k^0 \) is given by (44). Note that the computation of (44) requires the solution of only one fixed-point equation [16].

V. Sum-Rate Maximizing Regularization

To optimize the achievable sum-rate, \( \alpha \) in (44) should be chosen to maximize (15). We then define \( \alpha^* \) as

\[ \alpha^* = \arg \max_{\alpha > 0} \sum_{k=1}^{K} \log (1 + \gamma_k^0) \]

(45)

For the general channel model (17) the optimization in (45) is very tedious and no closed-form solution for \( \alpha^* \) exists. However, in case of a homogeneous network (\( L = I_M \)) without transmit correlation (\( \Theta_{T} = I_M \)) \( \alpha^* \) has a closed-form. In this case \( m_{A_k}^0(0) = m_{A_k}^0 \Theta_{T/2}^1 = m_{A_k}^0 H_k^H H_w (\alpha) \) is the Stieltjes
\[
\gamma_k^\circ = \frac{r_k^2 (1 - \tau^2) (m_\Lambda^\circ(0))^2}{l_k \left[ m_\Lambda^\circ(0) - \text{c.m.} \Theta_{\nu_2} \right] \left[ 1 + l_k \tau^2 (l_k m_\Lambda^\circ(0) + 2) m_\Lambda^\circ(0) \right] + \frac{1}{\beta} (1 + l_k m_\Lambda^\circ(0))^2 \Psi(\alpha)}
\] (44)

The transform of the Marčenko-Pastur law and has the unique solution given by [17]

\[
m_\Lambda^\circ(\nu_2) = \frac{\beta (1 - \alpha) - 1 + d(\alpha, \beta)}{2 \alpha \beta}
\]

with \(d(\alpha, \beta) = \sqrt{\beta^2 \alpha^2 + 2 \alpha \beta (1 + \beta) + (1 - \beta)^2}\) (46)

Substituting (46) into (44) and setting the derivative w.r.t. \(\alpha\) to zero, we obtain

\[
\alpha^* = \frac{\left( 1 + \frac{\tau^2}{\beta} \right)}{\beta} \frac{1}{\beta}
\] (47)

For this \(\alpha^*, \gamma_k^\circ\) in (44) takes the surprisingly simple form

\[
\gamma_k = m_\Lambda^\circ(\nu_2) = \frac{\beta (1 - \alpha^*) - 1 + d(\alpha^*, \beta)}{2 \alpha^* \beta}
\] (48)

The sum-rate saturation level at high SNR is

\[
R_{\text{sum}}^{\text{lim}} = \lim_{\rho \to \infty} K \log(1 + \gamma^\circ) = K \log \left( 1 - \frac{1 + \beta (\tau^2 - 1) + d(\beta, \tau)}{2 \tau^2} \right)
\] (49)

with \(d(\beta, \tau) = \sqrt{\beta^2 \tau^2 + \beta \tau^2 (1 - \tau^2) (4 - \beta) + (1 - \beta)^2}\)

Let’s assume that the parameters involved in (47) are mutually independent and look at their asymptotic values.

First, notice that for perfect CSIT \((\tau = 0)\) we have \(\alpha^* = 1/(\beta \rho)\) which corresponds to the result derived in [3]. As mentioned in [3], for large \((K, M)\) the RZF precoder is equal to the MMSE precoder in [4], [11].

In contrast, for \(\tau > 0\) the RZF transmit filter and the MMSE transmit filter are not identical anymore, even in the large \(K\) limit. Moreover, for asymptotically high SNR, (47) becomes

\[
\lim_{\rho \to \infty} \alpha^* = \frac{\tau^2}{1 - \tau^2} \frac{1}{\beta}
\] (50)

Thus, for \(\rho \to \infty\), RZF has a finite sum-rate limit. That means, as soon as there are errors in the CSIT a sum-rate saturation effect occurs and the system becomes interference-limited. An open question is to determine how the distortion \(\tau^2\) has to scale in order to assure that the system sum-rate does not saturate at high SNR.

VI. NUMERICAL RESULTS

In our simulations all results are averaged over 10,000 independent channel realizations. Additional results can be found in [16].

Figure 1 shows the ergodic sum-rate two different precoder, RZF-1 using the sum-rate maximizing regularization term \(\alpha^*\) in (47) and RZF-2 with \(\alpha = 1/(\beta \rho)\) i.e. designed based on perfect CSIT. For comparison we also plot the performance of the MMSE filter proposed in [11] and the ZF precoder.

We observe that as soon as the error variance \(\tau^2\) dominates over the noise power \(\sigma^2\), the ergodic sum-rate of the RZF-2 filter decreases and approaches ZF precoding for high SNR. We further notice that the RZF-1 and MMSE filters achieve similar performance since they are almost identical for small values of \(\tau^2\). But, since \(\alpha^*\) is derived for asymptotically large \((K, M)\), the performance advantage of the RZF-1 over the MMSE filter increases with increasing \((K, M)\).

Figure 2 illustrates the ergodic sum-rate of RZF with the asymptotic optimal regularization \(\alpha^*\) and the true optimal regularization \(\alpha_{\text{Opt}}\) found by exhaustive search for \(M = \{2, 4, 8\}\). As expected, we observe that for an increasing number of transmit antennas \(\gamma_k\) approaches \(\gamma_k^\circ\).

Figure 3 compares our deterministic results to Monte-Carlo simulations for a correlated channels with unequal user path loss. We indicate the standard deviation of the simulations by error bars.

The transmit correlation is assumed to depend only on distance \(d_{ij}, i,j = 1, 2, \ldots, M\) between antennas \(i\) and \(j\) placed on a uniform circular array (UCA). Thus, \((\Theta_r)_{ij} = \frac{\pi d_{ij}}{\lambda}\) [18], where \(\lambda\) is the zero-order Bessel function of the first kind and \(\lambda\) is the signal wavelength. To assure that \(\|\Theta_r\|^{-1}\) is bounded we suppose that the distance between adjacent antennas \(d = d_{k, i+1}\) is independent of \(M\), i.e. as \(M\) grows the radius of the UCA increases.

Furthermore, we consider a uniform density of users in a circular cell. We take \(K\) samples of the user distribution which remain constant over all channel realizations. According to [19] (“Suburban Macro”) \(l_k = -31.5 + 35 \log_{10}(d_k)\) dB, where \(d_k\) is the distance of user \(k\) to the transmitter.

From Figure 3 we observe, that the expressions derived for large \((K, M)\) lie approximately within one standard deviation of the simulation results even for finite \((K, M)\). To avoid the small divergence of the asymptotic results from the simulation results for high SNR, \(M\) has to be increased.

VII. CONCLUSION

In this paper we derived a deterministic equivalent of the SINR of RZF precoding in MISO broadcast channels by applying recent results from random matrix theory. We use the deterministic equivalent expression of the SINR to compute the sum-rate maximizing RZF precoder for large \((K, M)\). Simulations show that the asymptotic results extend well into finite regimes.

REFERENCES


Fig. 1. Ergodic sum-rate vs. average SNR with $\Theta_T = I_M, L = I_K, M = 10, \beta = 1, \tau^2 = 0.1$.

Fig. 2. Ergodic sum-rate vs. average SNR with $\Theta_T = I_M, L = I_K, \beta = 1, \tau^2 = 0.1$.

Fig. 3. Ergodic sum-rate vs. average SNR with $M = 32, \beta = 1$, simulation results are indicated by circle marks with error bars indicating the standard deviation.


