Modelling SAR with a Generalisation of the Rayleigh Distribution

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Abstract: Synthetic aperture radar (SAR) imagery has found important applications since its introduction, due to its clear advantage over optical satellite imagery, being operable in various weather conditions. However, due to the physics of radar imaging process, SAR images contain unwanted artefacts in the form of a granular look which is called speckle. The assumptions of the classical SAR image generation model lead to the convention that the real and imaginary parts of the received wave follow a Gaussian law, which in turn means that the amplitude of the wave has a Rayleigh distribution. However, some experimental data show impulsive characteristics which correspond to underlying heavy-tailed distributions, clearly non-Rayleigh. Some alternative distributions have been suggested such as Weibull and log-normal distributions, however, in most of the cases these models are empirical, not derived with the consideration of underlying physical conditions and therefore are case specific. In this report, relaxing some of the assumptions leading to the classical Rayleigh model and using the recent results in the literature on α-stable distributions, we develop a generalised (heavy-tailed) version of the Rayleigh model based on the assumption that the real and the imaginary parts of the received signal follows an isotropic α-stable law which is suggested by a generalised form of the central limit theorem. We also derive novel methods for the estimation of the heavy-tailed Rayleigh distribution parameters based on negative fractional-order statistics for model fitting. Our experimental results show that the heavy-tailed Rayleigh model can describe a wide range of data which could not be described by the classical Rayleigh model.

Key-words: α-stable distribution, generalised Rayleigh distribution, heavy-tailed Rayleigh distribution, synthetic aperture radar (SAR) imagery, negative order statistics.

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Modélisation d'Image RSO par Généralisation d'une Distribution de Rayleigh

Résumé : L'imagerie Radar à Synthése d'Ouverture (RSO) a conduit à d'importantes applications, du fait de son avantage certain sur l'imagerie satellitaire optique (utilisation tout temps). Cependant, du fait de la physique du capteur RSO, les images produites présentent des artefacts non désirables, connus sous le nom de bruit de chatoiement. L'hypothèse que les parties réelles et imaginaires de l'onde reçue suivent une loi Gaussienne (ce qui revient à dire que l'amplitude de l'onde suit une distribution de Rayleigh) découle des hypothèses classiquement faites sur le modèle de génération de l'image RSO.

Cependant, des données expérimentales présentent des charactéristiques impulsionnelles correspondant à des distributions à queue lourde sous-jascentes, qui ne sont pas de type Rayleigh. D'autres distributions telles que les lois de Weibull ou log-normale ont été proposées. Cependant, dans la plupart des cas, ces modèles sont empiriques ne prenant pas, en compte la physique du capteur, et sont trop spécifiques.

Dans ce rapport, en relachant quelques hypothèses qui conduisent au modèle de Rayleigh et en utilisant des résultats récents publiés dans la littérature sur les distributions α-stables, nous proposons une version généralisée (à queue lourde) du modèle de Rayleigh. Ceci est fondé sur l'hypothèse que les parties reélle et imaginaire du signal reçu suivent une loi α-stable isotrope, suggérée par une généralisation du théorème central limite. Nous présentons également de nouvelles mèthodes d'estimation des paramètres d'une distribution de Rayleigh à queue lourde fondées sur des statistiques d'ordre fractionnaire négatif. Les tests expérimentaux montrent que le modèle de Rayleigh à queue lourde permet de décrire une grande variété de données qui ne pourraient pas être décrites de façon satisfaisante par un modèle de Rayleigh classique.

Mots-clés : distribution α-stable, distribution de Rayleigh généralisée, distribution de Rayleigh à queue lourde, imagerie radar à synthèse d'ouverture (RSO), statistiques d'ordre négatif.

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Contents

1 Introduction

Synthetic aperture radar (SAR) imagery has become increasingly popular over the last couple of decades over optical satellite imagery due to its operability regardless of weather conditions and the independence of resolution from sensor highth. However, there are problems associated with the nature of radar imaging process: due to comparability of the wavelength to surface roughness in some cases the wave reflections occur in various directions dictated by the incidence angle, surface structure, dielectric constant of the surface, factors too many to be successfully accounted by a simple deterministic model. The coherent addition of these reflected waves in the receiver out-of phase results in the granular look of the images which is referred to as speckle noise.

Presence of speckle noise degrade SAR images significantly and it may hide important details on the images and therefore may lead to the loss of crucial information. The first step towards removing speckle noise is to understand its statistical properties.

Traditionally, several assumptions have been made for developing a statistical model for the speckle noise. These can be summarised as [21]: 1) the scatterers are statistically independent, 2) the number of scatterers is a large number, 3) the scattering amplitude and the instantaneous phase are independent random variables, 4) the phase is uniformly distributed over the range $[0, 2\pi]$, 5) no individual scatterer dominates the whole scene, 6) the reflection surface is large when compared to the size of individual reflectors.

Under the first two assumptions, invoking the central limit theorem, it has been assumed that the real and the imaginary parts of the received complex signal are independent, zero mean, identically distributed Gaussian random variables [4]. That is, if the total scattered signal is

$$X_{re} + jX_{im} = \Sigma_{i=1}^{N} X_i \exp(j\phi_i) \tag{1}$$

$$= \Sigma_{i=1}^{N} X_i \cos(\phi_i) + j\Sigma_{i=1}^{N} X_i \sin(\phi_i), \tag{2}$$

where X_{re}, X_{im} are the real and imaginary parts of the total signal, and X_i and ϕ_i are the amplitude and phase of the ith scattered complex wave, then the real and the imaginary parts have the following joint probability density function:

$$p(X_{re}, X_{im}) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{X_{re}^2 + X_{im}^2}{2\sigma^2}\right) \tag{3}$$

Converting this expression into polar coordinates, with $r = (X_{re}^2 + X_{im}^2)^{1/2}$, $\phi = \arctan(\frac{X_{im}}{X_{re}})$, we obtain

$$p(r, \phi) = \frac{r}{\pi\sigma^2} \exp(-\frac{r^2}{2\sigma^2}) \tag{4}$$

Following from the third assumption, the amplitude and the phase are independent and the phase, which does not even appear in the formula, can be integrated out to obtain the following expression for the amplitude distribution which is the Rayleigh probability density function (pdf):

$$p(r) = \frac{r}{\sigma^2} \exp(-\frac{r^2}{2\sigma^2}) \tag{5}$$

Despite the theoretical appeal and the analytical simplicity of this model, frequently, real-life SAR data deviate from the Rayleigh distribution [22] in the form of impulsive behaviour which indicates underlying heavy-tailed distributions. To accommodate for such behaviour various models such as Weibull [22], and log-normal distributions [21] and the K-distribution [7] were suggested.

Among these, the log-normal distribution provides a convenient choice when one is using the multiplicative noise model since homomorphic filtering converts the problem into that of an additive Gaussian noise model.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), \qquad x > 0 \tag{6}$$

it fails modelling the lower half of the SAR histograms and overestimates the range of variation [10]. Another popular model is the Weibull distribution which showed some degree of success in modelling urban scenes and sea clutter [20]. The Weibull distribution the pdf of which is given as follows,

$$f(x) = \frac{\alpha}{\gamma^\alpha} x^{\alpha-1} \exp\left(-\left(\frac{x}{\gamma}\right)^\alpha\right) \tag{7}$$

unfortunately is an empirical model with no theoretical justification. The K-distribution is represented with its pdf

$$f(x) = \frac{4}{\gamma\Gamma(\alpha)} \left(\frac{x}{\gamma}\right)^\alpha K_{\alpha-1}\left(2\frac{x}{\gamma}\right) \tag{8}$$

where $K_{.}(.)$ is the modified Bessel function of the first type [3]. K-distribution is a successful model for sea clutter, and as the Weibull distribution, has the Rayleigh distribution as its special case. However, the parameter estimation of K-distribution is not straight forward [10]. These models are generally empirical, were not derived considering underlying physical generation mechanisms and are, therefore, case-specific. We argue that a theoretically more sound and more general model needs be developed which accounts both for cases the Rayleigh distribution can model and the heavy tailed cases in which Rayleigh is unsuccessful.

As the first step, we suggest that the assumptions of the classical Rayleigh model [4] needs to be reconsidered for a more realistic model. In Section 2, we will discuss an important physical fact not taken into account by the Rayleigh model. This will lead us to the discussion of an alternative model for the real and imaginery parts of the received signal. The properties of this alternative model namely the α-stable distribution is presented in Section 3. In Section 4, we will proceed to develop our new model for SAR images which we call *heavy-tailed Rayleigh distribution.* In Section 5, we will present novel estimation techniques based on negative order statistics for the estimation of parameters of the heavy-tailed Rayleigh distribution, which will be used in Section 6 for model fitting to SAR images. Conclusions will be given in Section 7.

2 Considerations of the Physical Environment

We argue that a theoretically more sound model can be developed by reconsidering the assumptions of the Rayleigh model [4]. The assumptions of the Rayleigh model were given in Section 1. What is missing in this model is the consideration of the propagation conditions of the reflected waves and the spatial and temporal distribution of the reflectors on the surface. Since the waves are propagating in a physical medium, one should also include in the model the way the waves are effected from the medium. A widely accepted model [13] is attenuation by a factor which varies with a power of the distance:

$$a(r) = \frac{K}{r^m} \tag{9}$$

where r is the distance travelled by the wave, K is related with the transmitted power and m depends on the electrical properties of the medium. Then, we can describe the received wave as [6]:

$$\mathbf{Y} = \sum_{i=1}^{\infty} \frac{K}{r^m} \mathbf{X}_i. \tag{10}$$

Further we assume that the reflectors form a Poisson point process in the plane, that is, the probability of k scatterers in the unit region is

$$\frac{e^{-\lambda}\lambda^k}{k!}. \tag{11}$$

Eq. (10) is in the form of Le Page series [19] and has interesting convergence properties as discussed in [6]. In [6], Ilow and Hatzinakos consider modelling the interference in multiple access communication systems and showed that under these assumptions (independence of interferers, large number of interferers, attenuation according to Eq. (9) and Poisson distribution of interferers), the characteristic function of Y follows an α-stable distribution. This is in accordance with Nikias and Shao's earlier result [13]. Nikias and Shao, in [13], considered the problem of modelling the limiting distribution of superposition of a large number of noise pulses generated by noise sources randomly located in space. They considered a filtered-impulse mechanism of the noise process under appropriate assumptions on the spatial and temporal distributions of noise sources and their propagation conditions. They arrived at the result that limit distribution is a symmetric α-stable law, the parameters of which is related to the physical environment parameters.

One of the main justifications of the classical Rayleigh model was that due the assumption of a large number of reflectors one can invoke the central limit theorem according to which the distribution of the real and complex parts of the received wave approaches the Gaussian distribution. However, a generalised version of the central limit theorem says that the sum of a large number of i.i.d. processes approach the α-stable law [11] which contains the Gaussian distribution as a special case but can also describe impulsive and skewed behaviour. Therefore, the α-stable model is an equally well-justified and more general model. Motivated by the *generalised central limit theorem* [11], and the results of Nikias and Shao [13] and Ilow and Hatzinakos [6], we suggest adopting the symmetric α-stable model for the real and the imaginary parts of the signal in the receiver of a synthetic aperture radar.

3 The α-Stable Distribution

The α-stable distribution family which found important applications in financial time series analysis [12] and signal processing [13], was first developed by Levy [11] during his study of the generalisation of the central limit theorem as the only distribution family suggested by the generalised central limit theorem. For detailed accounts of the properties of this distribution family, the reader is referred to [19] and [13]. A recent account of diverse applications can also be found in [14]. Due to the lack of a compact analytical expression for its probability density function (pdf), the α-stable distribution is represented most conveniently by its characteristic function which is the Fourier transform of its pdf given below:

$$\varphi(t) = \begin{cases} \exp\{j\delta t - \gamma|t|^\alpha[1 + j\beta \mathrm{sign}(t)\tan(\frac{\alpha\pi}{2})]\}, & \text{if } \alpha \neq 1 \\ \exp\{j\delta t - \gamma|t|^\alpha[1 + j\beta \mathrm{sign}(t)\frac{2}{\pi}\log|t|]\}, & \text{if } \alpha = 1 \end{cases} \tag{12}$$

where $-\infty < \delta < \infty$ is the location parameter and simply corresponds to a shift in the x-axis in the pdf domain, $\gamma > 0$ is the scale parameter and corresponds to the dispersion of the random variable around the mean in analogy to variance in the Gaussian case, $0 < \alpha \leq 2$ is the characteristic exponent which sets the thickness of the tails of the distribution and $-1 \leq \beta \leq 1$ is the symmetry parameter which sets the skewness of the distribution.

In this paper, we will restrict our attention to a special case, namely the (zero-mean) symmetric α-stable ($S\alpha S$) distribution, which has the characteristic function

$$\psi(t) = \exp(-\gamma|t|^\alpha) \tag{13}$$

following the suggestions of [6, 13] and due to its simplicity.

Symmetric α-stable distributions have found successful applications in various branches of signal processing and a recent account of diverse applications can be found in [14]. In particular, in [2], Banerjee *et al.* used α-stable distributions in modelling the clutter in ultra-wideband (UWB) SAR images. The problem we study is different in that we consider narrow band SAR images and we aim at modelling the whole image with speckle rather than only the clutter arising from non-target objects in the image. Other work in image processing with α-stable distributions include [15] where Pesquet-Popescu and Pesquet study α-stable textures with long-range dependence and [9] where Kuruoglu and Zerubia study autoregressive α-stable textures and suggest model estimation techniques. Another related work in image processing is [17] where Poliannikov al. and Krim study Levy random field models for infrared images.

4 Heavy-tailed Rayleigh Model

Following the arguments of [6, 13] and invoking the generalised central limit theorem, we assume that the real and the imaginary parts of the received signal (Eq. 2) are jointly $S\alpha S$. Therefore we need to consider the bivariate isotropic α-stable distribution [13] which is described by its characteristic function as:

$$\psi(t_1, t_2) = \exp(-\gamma|\mathbf{t}|^\alpha) \tag{14}$$

where $|\mathbf{t}| = \sqrt{t_1^2 + t_2^2}$. The marginal distributions of t_1 and t_2 are as given in Eq. (13). The probability density function can be evaluated by taking the Fourier transform of Eq. (14):

$$f_{\alpha,\gamma}(x_{re}, x_{im}) = \frac{1}{(2\pi)^2} \int_{t_1} \int_{t_2} \exp(-\gamma|\mathbf{t}|^\alpha) \ \exp(-(x_{re}t_1 + x_{im}t_2)) \ dt_1 dt_2 \tag{15}$$

Converting this integral into the polar coordinates, by $s = |\mathbf{t}| = \sqrt{t_1^2 + t_2^2}$ and $\theta = \arctan(\frac{t_1}{t_2})$, one obtains:

$$f_{\alpha,\gamma}(x_{re}, x_{im}) = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^\infty s \exp(-\gamma s^\alpha) \ J_0(s|\mathbf{x}|) \ ds \ d\theta \tag{16}$$

where we used the definition of the Bessel functions (J_0 is the zeroth order Bessel function of the first kind [3]) and $|\mathbf{x}| = \sqrt{x_{re}^2 + x_{im}^2}$. Since, θ does not explicitly appear in the integral, the outer integral can be evaluated simply which only provides a 2π factor:

$$f_{\alpha,\gamma}(x_{re}, x_{im}) = \frac{1}{2\pi} \int_0^\infty s \exp(-\gamma s^\alpha) \ J_0(s|\mathbf{x}|) \ ds \tag{17}$$

Since in most schemes one is interested in amplitude detection, we make the following transformation (into polar coordinates)

$$f(r,\phi) = r \ f_{\alpha,\gamma}(r\cos\phi, r\sin\phi), \quad r \geq 0, \ 0 \leq \phi \leq 2\pi \tag{18}$$

substituting from Eq. (17), we reach to

$$f_{\alpha,\gamma}(r,\phi) = \frac{r}{2\pi} \int_0^\infty s \exp(-\gamma s^\alpha) \ J_0(sr) \ ds \tag{19}$$

Once again, integrating out ϕ which is uniformly distributed the final expression is obtained for the amplitude.

$$\eta_{\alpha,\gamma}(r) = r \int_0^\infty s \exp(-\gamma s^\alpha) \ J_0(sr) \ ds \tag{20}$$

We prefer to call this distribution *heavy-tailed Rayleigh distribution.* The reason for this choise of terminology is that this new generalised form of the Rayleigh distribution can describe impulsive data and its p.d.f. has thicker tails when compared to the classical Rayleigh distribution.

Considering the special case, $\alpha = 2$, we use the following identity from [5]

$$\int_0^\infty z \ \exp(-az^2)\ J_0(bz)\, dz = \frac{1}{2a} \exp\left(-\frac{b^2}{4a}\right), \quad a > 0,\ b > 0 \tag{21}$$

in Eq. (20) to obtain

$$\eta_{2,\gamma}(r) = \frac{r}{2\gamma} \exp\left(-\frac{r^2}{4\gamma}\right). \tag{22}$$

which is basically the classical Rayleigh distribution as expected since for $\alpha = 2$, the member of the α-stable family is Gaussian. Next, we consider the special case $\alpha = 1$ (which corresponds to the Cauchy distribution), using the following identity from [5]

$$\int_0^\infty z^{n+1} \ \exp(-az)\ J_n(bz)\, dz = \frac{a 2^{n+1} b^n \Gamma(n+3/2)}{\sqrt(\pi)(a^2+b^2)^{n+3/2}}, \quad a > 0, b > 0, n > -1 \tag{23}$$

one can obtain

$$\eta_{1,\gamma}(r) = \frac{r\gamma}{(r^2+\gamma^2)^{3/2}}, \tag{24}$$

For other values of α, we cannot evaluate the integral in Eq. (20). Asymptotic series expansions exist [13]; however, these expansions are numerically very unstable and need a large number of components for convergence. Therefore, we do not suggest using them; instead, one can use numerical integration to evaluate Eq. (20).

To give a feeling about the characteristics of this class of distributions, they are plotted for various values of the characteristic exponent α in Figure (1) ($\gamma = 1$).

5 Estimation of the Heavy-Tailed Rayleigh Model

For the heavy-tailed Rayleigh model to be of any practical use, one should be able to estimate the parameters α and γ from the observed data. Eq. (20) is not in a compact form and it does not seem to be possible to invert it to obtain parameter values.

Pierce in [16], where he uses the heavy-tailed Rayleigh distribution to model sea clutter, claims that for $1.3 < \alpha < 2$, α is approximately 1.5 times the reciprocal of the first negative order moment of the distribution. He suggests that one can then calculate the p.d.f.s for a few α values around this value and choose the one which gives the best fit to data. He does not provide any schemes to estimate γ. This is a tedius and, we believe, unreliable way of estimating α. In this paper, we suggest an estimation method belonging the class of *method-of-moments* where we exploit the negative order moments of the heavy-tailed Rayleigh distribution to estimate both α and γ.

5.1 Moments of the Heavy-Tailed Rayleigh Distribution

The pth order moment of heavy-tailed Rayleigh distribution can be written as

$$E(r^p) = \int_0^\infty r^{p+1} \int_0^\infty s \exp(-\gamma s^\alpha)\ J_0(sr)\ ds\ dr \tag{25}$$

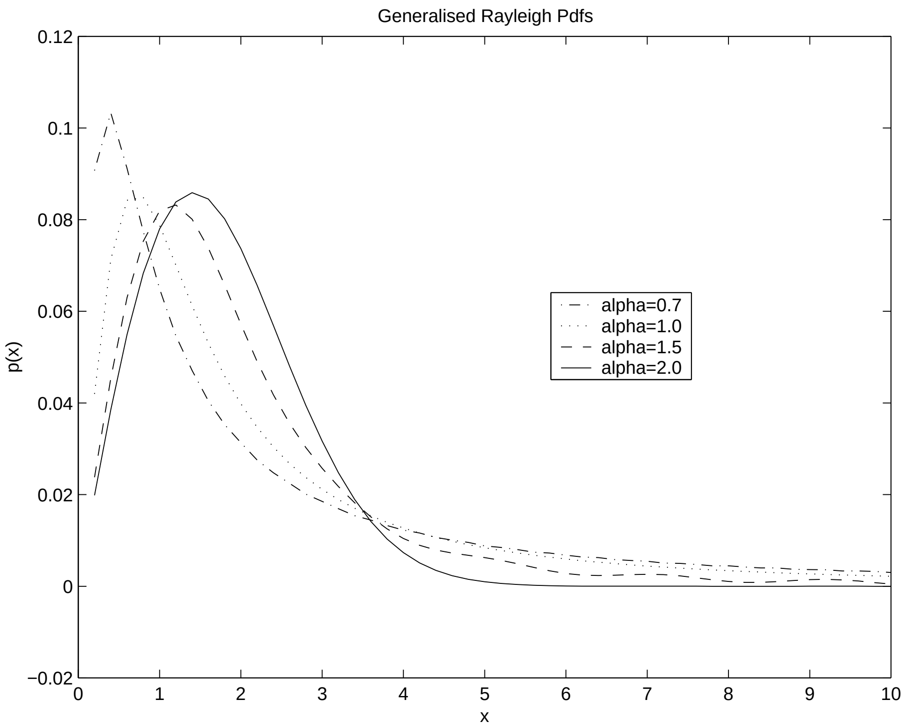

Figure 1: Generalised Rayleigh pdfs.

We, firstly, change the order of the integration and then make a change of variables $t = sr$.

$$\int_0^\infty s^{-(p+1)} \exp(-\gamma s^\alpha) \int_0^\infty t^{p+1} J_0(t)\ dt\ ds \tag{26}$$

Next we use the following identity from Abramowitz and Stegun (p.489, eq. 11.4.16) [1]

$$\int_0^\infty t^\mu J_\nu(t)\mathrm{dt} = \frac{2^\mu \Gamma\left(\frac{\nu+\mu+1}{2}\right)}{\Gamma\left(\frac{\nu-\mu+1}{2}\right)},\ Re(\mu+\nu) > -1,\ Re(\mu) < 1/2 \tag{27}$$

to obtain

$$E(r^p) = \frac{2^{p+1}\Gamma\left(\frac{p}{2}+1\right)}{\Gamma\left(-\frac{p}{2}\right)} \int_0^\infty s^{-(p+1)} \exp(-\gamma s^\alpha)\ ds. \tag{28}$$

Using the following identity from Gradhsteyn and Ryzhik (p. 342, eq. 3.478.1) [5]:

$$\int_0^\infty x^{\nu-1} \exp(-\mu x^p) dx = \frac{1}{p}\mu^{-\frac{\nu}{p}} \Gamma\left(\frac{\nu}{p}\right),\quad Re(\mu) > 0, Re(\nu) > 0, p > 0 \tag{29}$$

one arrives at the following formula:

$$E(\lambda^p) = \frac{2^{p+1}\Gamma(\frac{p}{2}+1)}{\Gamma(-\frac{p}{2})} \frac{\gamma^p \Gamma(-\frac{p}{\alpha})}{\alpha},\quad -2 < p < -\frac{1}{2}. \tag{30}$$

5.2 Parameter Estimation: Method of Moments

To obtain the parameter values we solve Eq. (30) for α and γ against the empirical moments $\hat{E}(\lambda^p)$ calculated from data.

To isolate α in eq. (30), we take the ratio of

$$\frac{E(\lambda^{2p})}{(E(\lambda^p))^2} = \frac{\Gamma(p+1)\Gamma^2(-\frac{p}{2})}{2\Gamma(-p)\Gamma^2(\frac{p}{2}+1)} \frac{\alpha\Gamma(-\frac{2p}{\alpha})}{\Gamma(-\frac{p}{\alpha})},\quad -1 < p < -\frac{1}{2}. \tag{31}$$

γ can be expressed in terms of α (from eq. (30)):

$$\gamma = \left(\frac{\Gamma(\frac{p}{2})}{2^{p+1}\Gamma(\frac{p}{2}+1)} \frac{\alpha}{\Gamma(-\frac{p}{\alpha})}\right)^{\alpha/p}. \tag{32}$$

We would like to solve Eq. (31) for the parameter α using empirical moment values calculated from data for the left hand side of the equation. Unfortunately, Eq. (31) is a highly nonlinear equation and does not have a compact solution; however, for the range of values of p, the Gamma function is well-behaved and we can always solve Eq. (31) with any of the numerical optimisation techniques such as bisection [18]. Once α is obtained, solution for γ is straight-forward from Eq. (32).

6 Simulation Results

We tested the heavy-tailed Rayleigh model on various images provided by CNES (Centre National d'Etudes Spatiales) which were obtained by the satellite ERS-1 of the European Space Agency. Our simulations showed us that the Generalised Rayleigh Model developed can model all the cases the classical Rayleigh model can, as expected. In addition, it can model various other cases where the tails of the distribution is heavier. We give an example for its performance: a C-Band SAR image of an urban area in Figure (2) and the comparison of its histogram with the classical and generalised Rayleigh model fits in Figure (3). The parameters were estimated to be $\alpha = 1.37$ and $\gamma = 331$.

We further tested the algorithm on X-band SAR images obtained by the DH-6 aircraft of Sandia National labs. An example image is given in Figure (4). We provide a comparison of the performances of Rayleigh, heavy-tailed Rayleigh and Weibull models. The Figure (5) is in logarithmic scale so that the performance difference between Weibull and heavy-tailed Rayleigh distribution in fitting the tails can be clearly seen. Despite heavy-tailed Rayleigh model's algebraic tails, Weibull model has exponential tails which cannot describe the data well enough. The estimated values of α and γ were 1.50 and 113 respectively in this case.

Experimentally, it has been observed that most urban-area SAR images have α's in the range $[1.2, 1.6]$. It has also been observed that in some SAR images that contain mixed scenes and for some UHF data, the heavy-tailed Rayleigh model was not successful [8]. We give an example of this observation in Figure (6) and Figure (7).

This may be due to the fact that we considered only symmetric α-stable distributions in our derivation while a more general model can be obtained starting from general (possibly skewed) α-stable density function. Unfortunately, in this case, the heavy-tailed Rayleigh model gets much more complicated, which is also reflected in the parameter estimation problem. However, it is a future research direction we are considering. One solution is to directly work on the I and Q components of the SAR image, in which case one use the α-stable parameter estimation techniques presented in [8].

The assumption of zero location parameter is also questionable since we observed SAR histograms which are shifted from origin. In this case, one can simply estimate the shift and pull the histogram back to the origin and then apply the techniques suggested in this paper. However, care must be taken since the parameter estimation techniques have proved to be very sensitive to the shift. This is due to the use of negative order moments which give emphasis to samples near origin.

Finally, for mixed scenes greater success can be obtained by developing a mixture model which provides flexibility to model multimodal data.

7 Conclusions

In this report, we have developed a generalisation of the classical Rayleigh model for SAR images and provided a novel technique based on negative order moments for parameter estimation. The model provides us more flexibility in SAR image modelling and showed

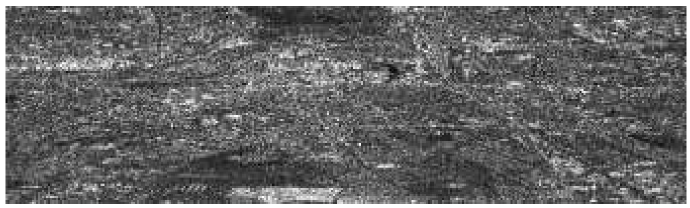

Figure 2: C-Band SAR image of an urban area

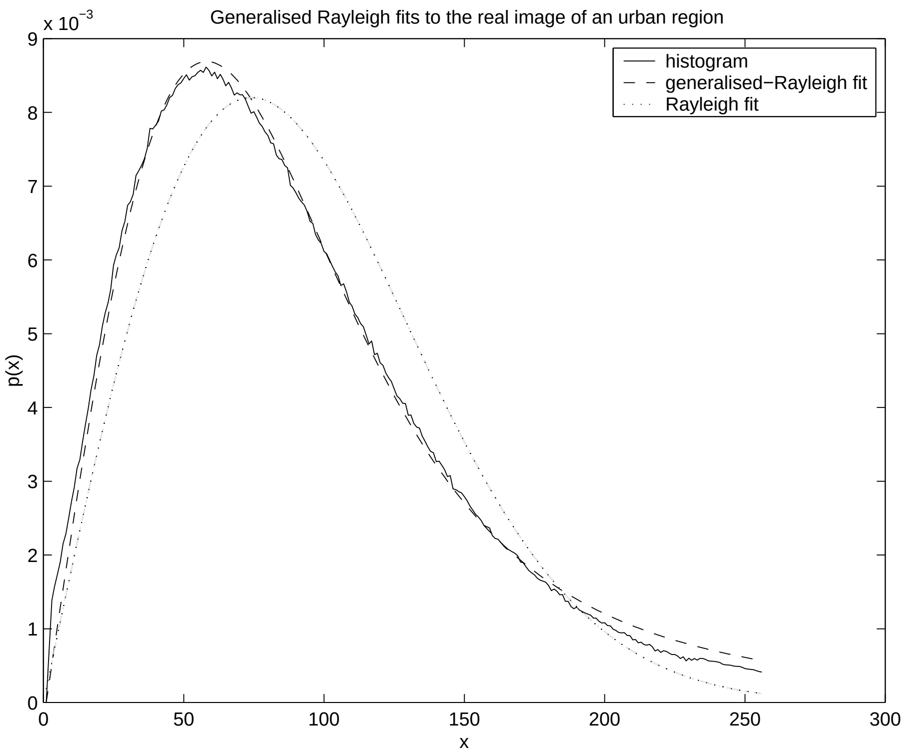

Figure 3: Histogram of Fig. 2, and classical and generalised Rayleigh model fits to it.

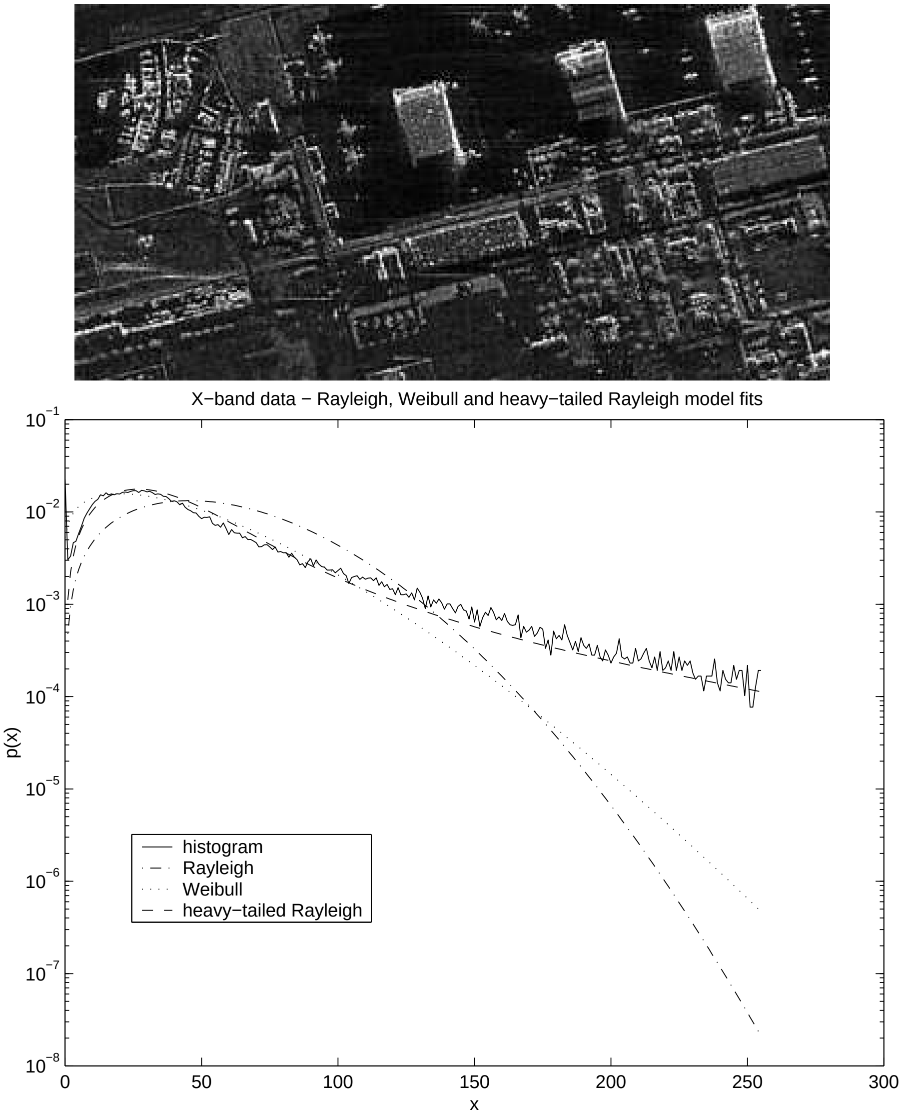

Figure 5: Histogram of Fig.4, and classical, generalised Rayleigh and Weibull model fits in logarithmic scales.

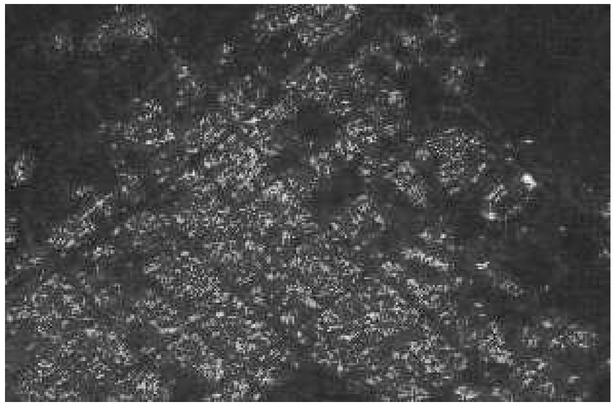

Figure 6: UHF-Band ($>$ 125 GHz) SAR, 2m resolution, obtained by Sandia National labs DH-6 Aircraft

success in examples that the classical Rayleigh distribution was unsuccessful. The future research directions are: 1) running a thorough survey on SAR images recorded in various conditions to gain an understanding of the relation between the model parameters and the medium properties, 2) development of a more general heavy-tailed Rayleigh distribution considering the skewed case as well, 3) devising techniques based on this new model for eliminating speckle noise.

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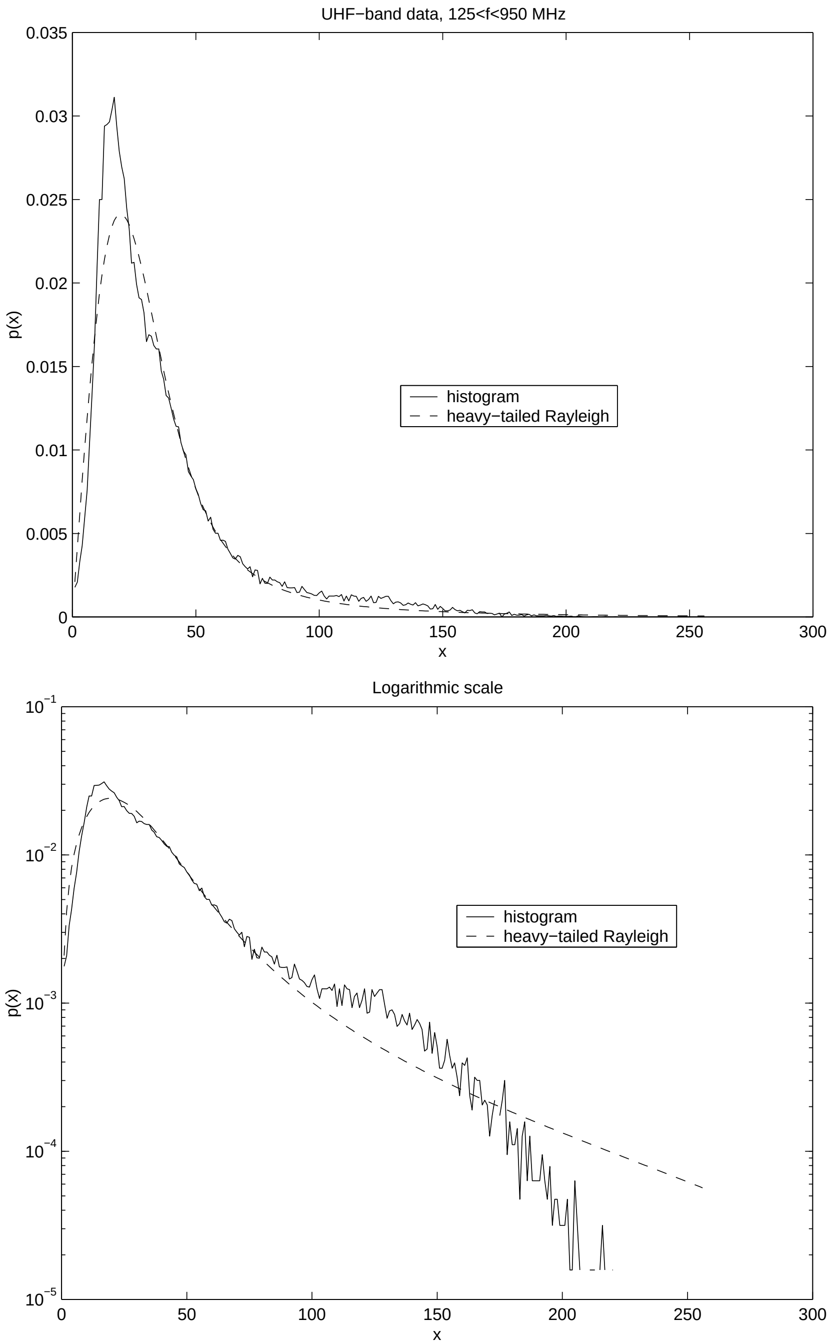

Figure 7: Histogram and the log-histogram of the UHF data and the generalised Rayleigh model fit

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