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Abstract. We study a 2D quasi-static discrete *crack* anti-plane model of a tectonic plate with long range elastic forces and quenched disorder. The plate is driven at its border and the load is transfered to all elements through elastic forces. This model can be considered as belonging to the class of self-organized models which may exhibit spontaneous criticality, with four additional ingredients compared to sandpile models, namely quenched disorder, boundary driving, long range forces and fast time crack rules. In this "crack" model, as in the "dislocation" version previously studied, we find that the occurrence of repeated earthquakes organizes the activity on well-defined fault-like structures. In contrast with the "dislocation" model, after a transient, the time evolution becomes periodic with run-aways ending each cycle. This stems from the "crack" stress transfer rule preventing criticality to organize in favor of cyclic behavior. For sufficiently large disorder and weak stress drop, these large events are preceded by a complex space-time history of foreshock activity, characterized by a Gutenberg-Richter power law distribution with universal exponent $B = 1 \pm 0.05$. This is similar to a power law distribution of small nucleating droplets before the nucleation of the macroscopic phase in a first-order phase transition. For large disorder and large stress drop, and for certain specific initial disorder configurations, the stress field becomes frustrated in fast time : out-of-plane deformations (thrust and normal faulting) and/or a genuine dynamics must be introduced to resolve this frustration.

1 Introduction

In this paper, we continue our exploration of the hypothesis according to which earthquake and fault characteristics can be understood, at time scales of years and above, only by using a global perspective, treating on the same level the growth of faults by repeated earthquakes on one hand and the localization of earthquakes on faults on the other hand. A lot of studies have documented the self-similar structure of fault patterns (King (1983); Wu and Aki (1986); Davy et al. (1990); Kagan (1991); King and Sammis (1992); Barton and Zoback (1992); Hanks (1992); Scholz et al. (1993)). At short time scales (typically less than or of the order of a century), earthquakes occur on these pre-existing set of faults and one can neglect the evolution of the fault network to focus on the question of the role of the fault structure in the observed earthquake phenomenology. On the other hand, faults are evolving, nucleating, growing, branching, healing, dying eventually, being screened by others faults (Andrews (1989)). This evolution occurs as a result of the accumulation of deformations, accounted for by earthquakes for a significant fraction (varying upon the location on earth). Therefore, the fault geometry, which must be introduced to get a correct description of earthquake occurrence, is not an arbitrary fractal, but results from the accumulation of earthquakes and other more ductile modes of deformations which are themselves determined by the geometry. Our purpose here is to explore further the implications of this "hen and egg" problem, within a simple model.

To tackle this question, we have previously introduced a 2-D quasi-static "dislocation" model for the generation and organization of faults by repeated earthquakes in an heterogeneous elastic plate driven at its border (Cowie et al. (1993,1995); Miltenberger et al. (1993); Sornette et al. (1994a)). The main results are the spontaneous generation of fractal fault structures and the existence of a well-defined Gutenberg-Richter earthquake energy distribution

$$N(E)dE \sim E^{-(1+B)}dE,$$

with $B = 0.3 \pm 0.05$ in 2D describing the earthquake population. The faults have been found to be globally optimal structures in the sense that they can be mapped onto a minimal interface problem, which in 2D corresponds to the random directed polymer problem (Halpin-Healy and Zhang (1995)). More precisely, for small stress drops, we have shown that a fault minimizes the sum of the random thresholds of the elements along it. This global minimization problem is achieved by the spontaneous organization of the medium, in which after a long "learning" transient regime, the deformation becomes localized on the optimal fault structure. In a sense, the elastic plate, endowed with its rules of rupture and stress redistribution, can be viewed as an analog computer which solves an optimization problem. Concretely, the outcome of this optimization correspondence is that the faults are self-affine with a roughness exponent which is known exactly and equal to 2/3.

In the present work, we study the "crack" version of the model : a constant "dynamical" stress is assumed to characterize the "fast time" rupture, i.e. the stress on all broken elements is fixed during a given earthquake event. In contrast, the previous "dislocation" model (Cowie et al. (1993,1995); Miltenberger et al. (1993); Sornette et al. (1994a)) corresponds to imposing a slip to the ruptured element, allowing it to be reloaded in fast time in the succession of ruptures producing a complete earthquake. In the present "crack" model, an element thus fails only once in a given earthquake and stress enhancement occurs at the crack tips, growing, as usual, as the square root of the length of the evolving earthquake. The motivation to study this variant of the initial model is twofolds: 1) there is some debate in the litterature on the correct model (dislocation or crack) to use for large earthquakes (Scholz (1982,1994); Romanowicz and Rundle (1993,1994); Sornette and Sornette (1994)); 2) in the context of self-organization, it is important to assess the role of local rules in the resulting large-scale organization (Gabrielov et al. (1994)).

We explore the various regimes as a function of initial disorder (on the stress thresholds and elastic coefficients) and dynamical stress drop amplitude. As in the "dislocation" model, we find that the occurrence of repeated earthquakes organizes the activity on well-defined fault-like structures. The main difference with the "dislocation" model stems from the tendency for the model to synchronize, i.e. to generate large periodic events, albeit of a complex internal spatio-temporal structure. We interpret this behavior as resulting from the physics of coupled relaxation oscillators with threshold (Christensen (1992); Sornette (1994); Sornette et al. (1995); Corral et al. (1995); Middleton and Tang (1995); Bottani (1995); Gil and Sornette (1995)). For sufficiently large disorder and weak stress drop, these large events are preceded by a complex space-time history of foreshock activity, characterized by a Gutenberg-Richter distribution with exponent $B = 1 \pm 0.1$ which is universal in the sense that the exponent is found essentially the same for all systems explored. For large disorder and large stress drop, for certain specific initial disorder configuration, and after a long time, the model does not have a solution anymore: this corresponds to a situation where the stress field becomes "frustrated" and the antiplane quasi-static crack modelling is no more defined. This shows that the quasi-static "crack" version of the model is not self-consistent and additional types of deformations must be allowed to get rid of this frustration. For instance, out-of-plane deformations (thrust and inverse faulting) and/or the introduction of a genuine dynamics can resolve this frustration. This breaking of self-consistency is reminiscent of the breaking of unicity accompanying the appearence of mechanical instabilities for instance in elastic-plastic transition (Lubliner (1990)).

Ref.(Lomnitz-Adler et al. (1992)) has explored a variety of avalanches and epidemic models which have the same type of stress enhancement transfer at the crack tips. In noiseless systems, they find, in agreement with us, periodic behavior and in general large events of the size of the system.

2 Description of the "crack" model

The model is a direct extension to the crack case of the dislocation model developed in (Cowie et al. (1993,1995); Miltenberger et al. (1993); Sornette et al. (1994a)). We consider an elastic plate embedded in the (0x, 0y) plane and composed of plaquettes of unit sizes paving the plane. The boundaries between the plaquettes constitute the elementary fault segments. They are tilted at 45 degrees with respect to the $0x$ axis, ensuring a symmetric role for all plaquette borders. A constant velocity boundary condition in the z direction is applied along the upper edge while the bottom edge is kept fixed (both the upper and bottom edges are parallel to Ox). Due to this externally imposed deformation and the stress transfer due to elasticity, each plaquette will deform. Discretizing the mechanical problem, we attribute a single vertical displacement $w(x, y)$ along the direction $0z$ perpendicular to the plate, at the center or node (x, y) of a plaquette. Each plaquette border is characterized by an elastic constant g which may vary from element to element (quenched disorder on the elastic coupling coefficients). Only two components of stress are non-zero in this antiplane model, namely the stress $\sigma_{yz}(x, y)$ along z applied on the border/fault between the plaquette centered on (x, y) and the plaquette centered on $(x, y-1)$ given by

$$\sigma_{yz}(x, y) = g[w(x, y) - w(x, y-1)]$$

and the stress $\sigma_{xz}(x, y)$ along z applied on the border between the plaquette centered on (x, y) and the plaquette centered on $(x-1, y)$ given by

$$\sigma_{xz}(x, y) = g[w(x, y) - w(x-1, y)].$$

Note that these expressions are just the discretized version of Hooke's law for elasticity, expressed for principal axis along Ox and Oy. For the present 45 degrees tilted lattice, the formulas are deduced from those above by the standard rule of transformation under a rigid rotation. The elastic displacement $w(x, y)$ in the direction z normal to the lattice plane is solution of the discretized version of the equilibrium elasticity equation

$$div\left(g(x,y)\mathbf{grad}w(x,y)\right) = 0.$$

Rupture occurs on a boundary between two plaquettes when the stress applied on it reaches a threshold σ_c which may depend on the position (quenched disorder on the rupture thresholds). The stress threshold rupture criterion used in our model can be interpreted as a standard Mohr-Coulomb criterion as follows. Recall that the Mohr-Coulomb criterion is believed to apply for rupture in the brittle crust at depth. It states that slip occurs when the shear stress on a fault plane reaches a value equal to the normal stress applied on this fault plane times a so-called friction coefficient which is a function of roughness and material properties of the fault. We notice that the non-vanishing stress components σ_{xz} and σ_{yz} of our antiplane model correspond to pure shear stresses along Oz applied on the vertical plaquette borders, interpreted as elementary fault segments. Let us introduce a constant hydrostatic pressure in the plane (Ox, Oy) of the tectonic plate. This pressure creates a stress normal to the fault segment. According to the Mohr-Coulomb criterion, slip occurs on a given fault segment if its shear stress (namely either σ_{xz} or σ_{yz}) reaches a constant equal to the pressure times the coefficient of friction of this fault. This justifies our choice of the stress threshold rupture criterion used in our model. The heterogeneity of the thresholds σ_c then reflects that of the friction coefficients on different fault segments.

When an element breaks, the elastic strain in the element is relaxed but the broken element suffers no change in its material properties and it can support stress again in the future. The stress field is assumed to obey the equation of mechanical equilibrium immediately after the rupture of an element. The redistribution of elastic stresses can bring other elements to rupture in a domino effect, creating model earthquakes. What we denominate as "fast time" is thus the succession of element ruptures within an event, during which the macroscopic load at the plate border does not increase ("slow time" is quenched during "fast time"). This separation into these two time scales is intended to represent the difference between the fast dynamical rupture which lasts minutes at most compared to the tectonic loading which does not change over this time scale.

In our previous dislocation model, the nature of the rupture on an element was simply characterized by the amplitude of the slip, chosen to be proportional to elastic deformation with a constant of proportionality β. This amounts to model a ruptured element as equivalent to a dipole (antiplane is scalar) whose strength is fixed until the element breaks again. The total slip occurring on a fault corresponds to the cumulative dipole amplitude on that fault. A large earthquake in the dislocation model can be viewed as a nonlinear rupture pulse propagating in and being multiply scattered by the heterogeneous medium, with the possibility for an element to rupture several times in fast time. In the present "crack" version, the strength of the dipole is not fixed in "fast time", but must be reajusted at each rupture event in fast time during a given earthquake such that the dynamical stress σ_{dyn}, defined as

$$\sigma_{dyn} = (1-\beta)\sigma_c$$

where $\beta\sigma_c$ is the average stress drop, remains constant and equal to a preassigned value on all rupture elements in this earthquake. If n elements have ruptured and a new element is brought to rupture in fast time due to the stress redistribution induced by these n previous ruptures, the dipole strengths of the $n+1$ elements are determined from a set of $n+1$ equations as follows. Each dipole exerts a contribution to the stress on all other elements. The stress on any element is therefore the sum of the background stress prior to the earthquake plus the contribution of all the dipoles created in the event. These dipoles are then self-consistently determined such that the stress on all the ruptured elements in fast time is fixed and equal to the preassigned value. Physically, this models a situation where the faults remains "open" during the whole duration of the earthquake, at the opposite of the dislocation model which corresponds to an instantaneous healing (closing) of the fault after each rupture.

In the crack model, there is another subtlety that was not present in the dislocation model. Suppose two or more elements are brought above their threshold in fast time due to the stress redistribution. The correct physics would be to solve the elastodynamics equations which direct the evolution of these unstable elements. This is however too difficult to implement practically for an heterogeneous system with many interacting faults. Our quasi-static approach circumvents this difficulty at the price that one has to choose, rather arbitrarily, a rule for the evolution of the unstability. A priori, two rules can be introduced: 1) one breaks them all simultaneously or 2) one ruptures only the element with the largest ratio (larger than one) of its stress to its threshold. We have checked that these two rules do not make any significant difference in the dislocation model. In the crack model, only the second rule has been explored in details in irreversible models of rupture, whereas the first rule may lead to un-ending ruptures. Our simulations have thus been carried out with this second rule. The elastic equations have been solved using a conjugate gradient technique with stopping criterion 10^{-20}.

Most of our study will be carried out in the presence of quenched disorder in the stress thresholds σ_c, which are drawn once for all from a probability distribution $P_\sigma(\sigma_c)$ chosen uniform in the interval $[1-\frac{\Delta\sigma}{2}, 1+\frac{\Delta\sigma}{2}]$ with the value of $\Delta\sigma$ between 0.1 and 1.9.

The basic difference between the dislocation and crack model is that stress enhancement at the fault tip is much stronger in the latter, with a stress growing as the square root of the fault length. As a consequence, the nature of fault and slip organization depends on the system size L. An earthquake of size L will generate a stress enhancement at its tip of magnitude equal to $\beta\sigma_c\sqrt{L}$. Two cases appear: if $\beta\sigma_c\sqrt{L} < \Delta\sigma$, the amplitude of stress enhancement generated by the dynamical evolution of the model is smaller than the quenched heterogeneity. The latter thus dominates and we expect an organization similar to that observed in the dislocation model where stress enhancement is small. On the contrary, for $\beta\sigma_c\sqrt{L} > \Delta\sigma$, sufficiently large earthquakes will always create stress enhancements larger than the pre-existing barriers. Beyond a characteristic nucleation size L^* given by

$$\beta\sigma_c\sqrt{L^*} \simeq \Delta\sigma,$$

earthquakes will not be stopped and will always break through the system (so-called "run-away" events). In addition, these large earthquakes will tend to smooth out the stress heterogeneity along the fault due to the condition of equal dynamical stress drop on the ruptured elements. These ingredients favor an approximate periodic state characterized by a repetition of large similar earthquakes (Christensen (1992); Sornette (1994); Sornette et al. (1995); Corral et al. (1995); Middleton and Tang (1995); Bottani (1995); Gil and Sornette (1995)). Take for instance $\beta = 0.1$. Then, the run-away will be absent for systems sizes smaller than of the order of 100. For such small stress drops, we have verified in particular that the fault patterns selected in the dislocation and the crack models have similar statistical properties in this regime. For larger stress drops or larger system sizes, there are two populations of events : 1) the small ones similar to that occurring in the dislocation model albeit with a different distribution; 2) large earthquakes of size comparable to the system size.

3 Threshold disorder

Here, we wish to explore the crack regime. Our simulations have thus been carried out on large systems 130 by 130. The influence of the dynamical stress σ_{dyn} has been explored in the range $0 \leq \sigma_{dyn} < 1 - \frac{\Delta\sigma}{2}$, since otherwise unending ruptures occur (the second inequality expresses that the dynamical stress must obviously be smaller than all static stress thresholds).

• $\Delta\sigma = 0.1$ (leading to $\sigma_{dyn} \leq 0.95$).

For $\sigma_{dyn} = 0$ (large stress drop), after a short transient, the dynamics is that of a perfect "characteristic earthquake" (Schwartz and Coopersmith (1984)): because of our tilted lattice structure, a well-defined regular fault, made of two linear strands oriented at 45 degrees with respect to the $0x$ axis and forming a V, is activated regularly in a perfect periodic fashion by a single great earthquake in which all elements on the fault break once in fast time with exactly the same slip. This regime corresponds to a perfect synchronization of all the threshold elements constituting the fault. The Gutenberg-Richter distribution is a Dirac function. The same is found for intermediate stress drop ($\sigma_{dyn} = 0.4$). For small stress drop ($\sigma_{dyn} = 0.9$), we find again a perfect synchronization corresponding to the repetition of a single large identical event. However, the fault on which this event occurs is now rough, characterized by linear strands separated by rough portions. Its specific structure is however dependent upon the specific realization of the disorder on the thresholds. In summary, a small disorder favors a very regular organization.

• $\Delta\sigma = 1$ (leading to $\sigma_{dyn} \leq 0.5$). For $\sigma_{dyn} = 0$, the behavior is very similar to the previous case $\Delta\sigma = 0.1$, except that the fault is now rough all along its length. For larger dynamical stress $\sigma_{dyn} \leq 0.2$ and 0.4, the dynamics is still periodic. However, a period contains a much more complex history than just the succession, as documented until now, of a quiescent phase followed by a single great earthquake. In contrast, after a great earthquake, there is long quiescent phase, followed by the appearence of a diffuse foreshock activity spread over the plate. This diffuse activity is made of many small earthquakes. It accelerates up to the time where the great earthquake occurs on a fault. This fault is again well-defined and is finally selected after a long transient. The distribution of earthquake energies contains two parts: a nice powerlaw distribution

$$P(E)dE \sim E^{-(1+B)}$$

for small earthquakes $0.2 \leq E \leq 20$ (in the units where the elastic coefficients are all equal to 1) with $B = 1 \pm 0.05$ and a Dirac peak at the energy of the great event (around $E \simeq 4000$). Notice the huge separation of energy scales. This can be rationalized using the nucleation argument outlined above. From the expression $\beta\sigma_c\sqrt{L^*} \simeq \Delta\sigma$, we get a nucleation scale L^* of the order of 4 elements. The energy being proportional to the square of the length in crack elasticity, this yields a characteristic maximum energy scale of 16 not far from the maximum energy observed in the power law distribution. In contrast, the great earthquake has a size of the order of one hundred and its release energy is thus of order 10^4.

• $\Delta\sigma = 1.9$ (leading to $\sigma_{dyn} \leq 0.05$). The progressive organization of the earthquake activity on a localized fault network is shown in fig.1. This case is similar to the previous one with the larger σ_{dyn}. However, quantitatively, the rupture history during a period separating

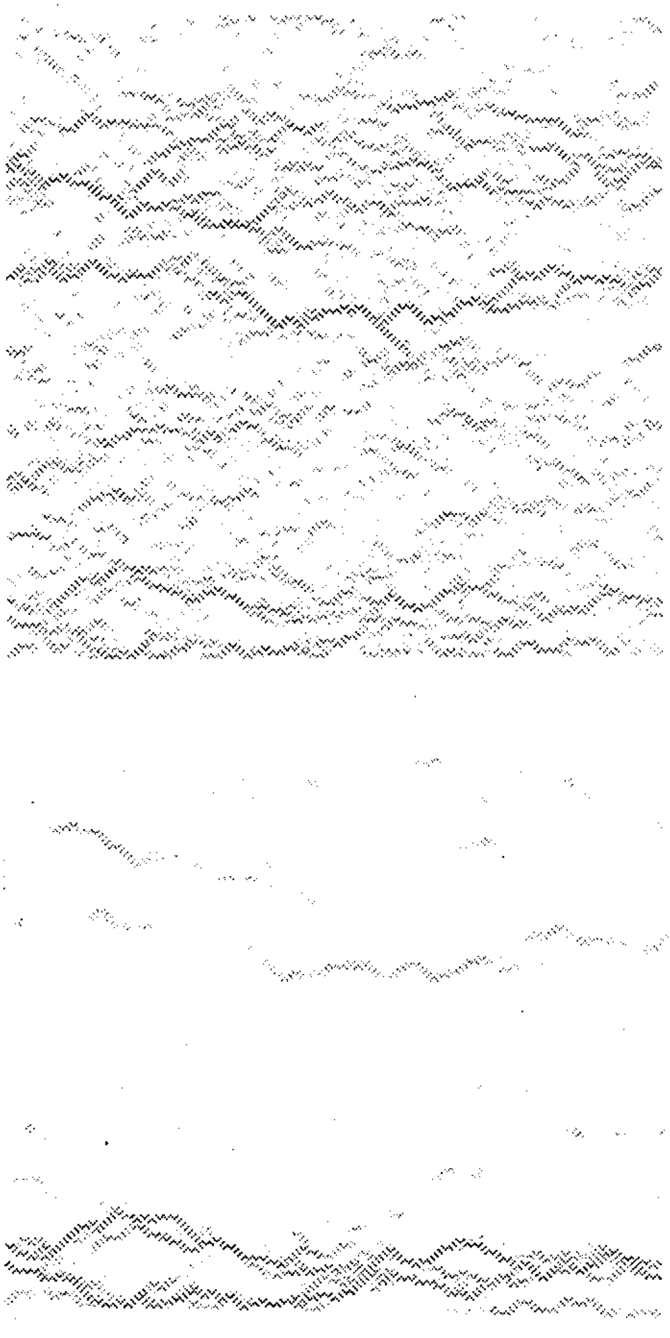


Fig.1 : The parameters are $\Delta\sigma = 1.9$, $\sigma_{dyn} = 0.04$, and the constant velocity imposed at the boundary is $V = 10^{-3}$. The figure shows two maps of the accumulated slip in fault segment at two different times in a square system of size $L = 130$ in the transient regime. We represent those elements which have slipped at least once, the light grey to black scale corresponding to increasing cumulative slips. The two times of observations correspond respectively to the first 1500 events (top) and 2000 events (bottom). One observes a rather diffuse "damage" at early times and localization of the deformation at long times.

the recurrence of two great earthquakes is even more characterized by the appearence of a multitude of small earthquakes (see fig.2c). The activity is always localized on a well-defined fault structure (see fig.2a) which becomes fixed at large times. However, the fault is no more a linear object, but contains loops defining internal "micro-plates". For the largest allowed dynamical stress drops (say $\sigma_{dyn} = 0.04$), we observe additional properties. After a great earthquake, there is, as usual, a quiescent time, followed by a progressing activation of the main fault by small earthquakes (fig.2c). The main fault is defined as the locus of the great earthquakes (see fig.2a). The activity of small earthquakes then shifts progressively away from the main fault to become delocalized in the bulk of the plate. This activity accelerates until the great earthquake occurs on the main fault. Again, we observe a huge separation of energy scales between the small events distributed according to a power law distribution for energies between 10^{-2} and a few 10^{-1} and the great characteristic earthquake of energy of the order of 1500. This is again correctly explained by the nucleation argument. The exponent of the power law distribution for small earthquakes is again $B = 1 \pm 0.05$ (see fig.2d)

Before the periodic regime organizes itself (see fig.2b) with its complex spatio-temporal structure of small earthquakes preceding the run-away, we witness a long transient aperiodic regime. The time duration of this transient is all the longer, the larger is the dynamical stress σ_{dyn}. For $\sigma_{dyn} = 0.04$ for instance, the transient is so long that one can measure with good accuracy the distribution of earthquake energies in time windows sufficiently large so that the statistics is good, but sufficiently small so that the evolution of the organization in this transient regime is negligible. (Note however that the transient is nevertheless quite small compared to that observed in the dislocation model: the crack model introduces a much stronger coupling via the crack stress redistribution rule, favoring a faster and stronger organization). We again obtain a nice power law for small earthquakes in the transient regime, with the same exponent B. However, the sizes of these "small" earthquakes in the transient regime are typically one order of magnitude larger than in the asymptotic periodic regime.

In summary, these numerical explorations show that the asymptotic dynamics is always periodic, with however a more and more complex sequence of ruptures within a period, the larger are the disorder $\Delta\sigma$ and the dynamical stress σ_{dyn} (i.e. the smaller the stress drop). This can be qualitatively understood as an intermediate regime between the fully periodic regime, which is characteristic of low disorder and large stress drop, and the self-organized critical behavior observed for the dislocation model, which is controlled essentially by large disorder and small stress drop (Cowie et al. (1993,1995); Miltenberger et al. (1993); Sornette et al. (1994a)). We have already underlined the analogy between this problem and that of a set of interacting relaxation oscillators with threshold. In this analogy, the stress drop parameter measures the coupling strength between elements, whereas the amplitude of the disorder $\Delta\sigma$ quantifies the disorder in the natural frequencies of the individual elements. The analogy predicts that synchronization, hence regular periodic behavior, will be the stronger the stronger the coupling and the weaker the disorder (Christensen (1992); Sornette (1994); Sornette et al. (1995); Corral et al. (1995); Middleton and Tang (1995); Bottani (1995); Gil and Sornette (1995)).

Let us present a mean field toy version of the model, in the spirit of (Mirollo and Strogatz (1990)), which allows one to understand the mechanism underlying the synchronization process and the appearence of periodic states. For the purpose of clarity, consider an homogeneous system and the situation where the earthquake cycle is constituted of two earthquakes, involving respectively N_1 and $N_2 < N_1$ fault elements. This can be the situation at the end of the transient regime, after many small events have disappeared. Our reasoning carried out below on these two earthquakes can be easily generalized to a more general situation with more earthquakes. We should stress that the origin of periodicity found here does not rely on the assumption of two faults but rather on the fact that the large earthquake is an "absorbing" state (see below). The mean field character of the argument is to assume that, when an element reaches its threshold σ_c, it redistributes its stress to all the other $N = N_1 + N_2$ active elements, each thus getting a stress increment equal to $\alpha \frac{\sigma_c}{N}$ where we allow for an arbitrary *positive* coupling strength α. For simplicity of the argument, suppose also that the stress of the ruptured element is put to zero. Suppose that we start reasoning at the time where the stress on the elements of fault 1 is $\sigma_1 > \sigma_2$, where σ_2 is the stress on the elements of fault 2. Earthquake 1 will occur first, when all N_1 elements are at their σ_c in fast time. At that time, the stress on the elements of fault 2 is

$$\sigma_2 + \sigma_c - \sigma_1,$$

due to the uniform tectonic loading. Just after event 1, the stress on the elements of 1 is zero by our rules while the stress on the fault 2 is

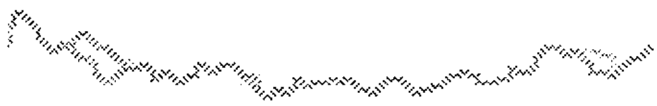
$$\sigma_2 + \sigma_c - \sigma_1 + \frac{N_1 \alpha \sigma_c}{N}.$$

Fault 2 will then reach itself σ_c at which time the stress on fault 1 is

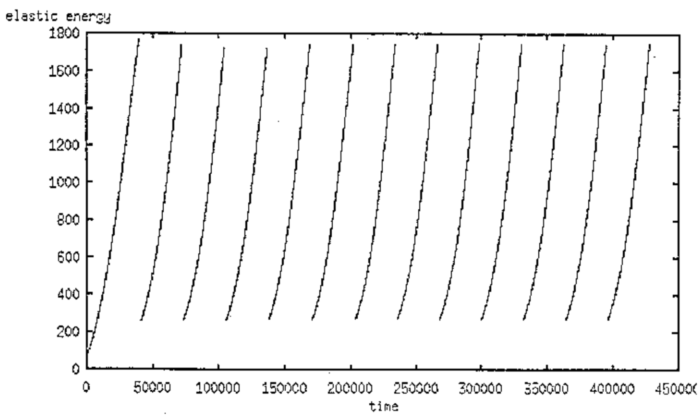
$$\sigma_1 - \sigma_2 - \frac{N_1 \alpha \sigma_c}{N}.$$

After earthquake 2, the stress on fault 2 is zero by definition while the stress transfer loads fault 1 to the level

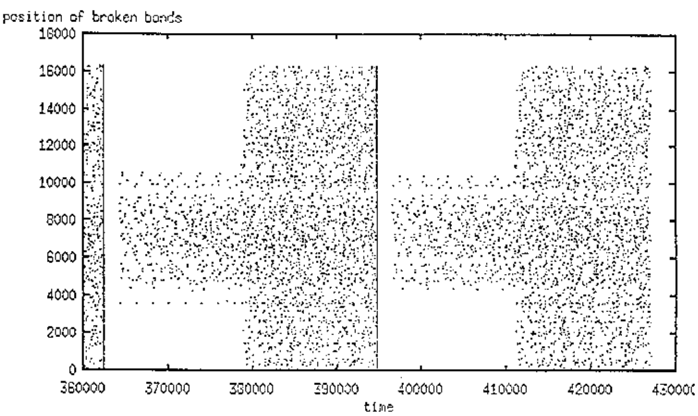
$$\sigma_1 - \sigma_2 - \frac{(N_1 - N_2) \alpha \sigma_c}{N}.$$



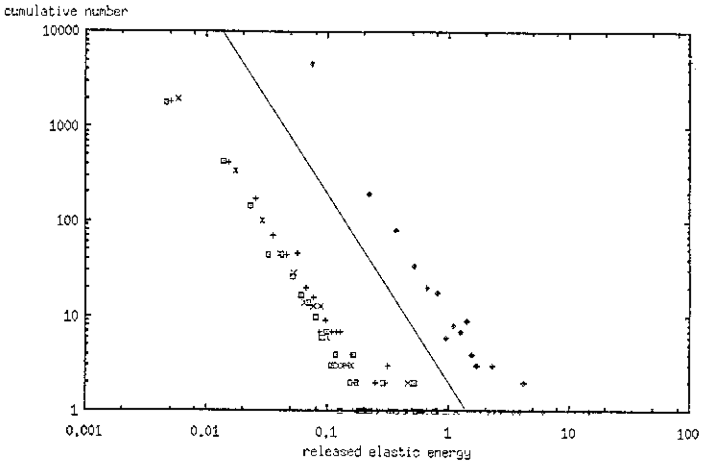
(a)



(b)



(c)



(d)

Fig.2 : The parameters are $\Delta\sigma = 1.9$, $\sigma_{dyn} = 0.04$, and the constant velocity imposed at the boundary is $V = 10^{-3}$.

a) Long-time accumulated slip after localization for the same parameters as for fig.1 but with another disorder realization. All the earthquakes occur on this 1D fault.

b) Time dependence of the total elastic energy stored in the plate. The periodic behavior established after the transient up to time 50000 is clearly visible.

c) Coding of the active fault elements as a function of time. The position of a given fault element is coded by a single index (denoted "position of broken bonds" in the figure) increasing from 1 to $L^2 = 16900$. The index spans $x = 1$ to L at fixed y for each $y = 1$ to L. Note the periodic cycle with three phases: i) quiescence after the great event, ii) reactivation of the seismic activity mainly on the main fault and iii) increase of the foreshocks frequency in the system.

d) Gutenberg-Richter distribution of the number of events having a given energy. The energy of an event is defined as the difference between the total elastic energy stored in the system before and after the event. The crosses, plus and square symbols corresponds to different times in the dynamics at long times, showing the stability of the distribution. The diamonds correspond to the distribution of small events in a time window in the third regime close to the run-away in the periodic regime. In all cases, we observe powerlaw distribution $P(E)dE \sim E^{-(1+B)}$ with a constant exponent $B = 1 \pm 0.05$. The straight line has slope -2 for comparison. It is interesting to note that the foreshocks represented by the diamonds have the same B-value but are larger on average, signaling the nucleation of the run-away. The great earthquake is not represented on the figure, being out of scale.

This last result shows that the stress difference, which was initially $\sigma_1 - \sigma_2$ has decreased during an earthquake cycle. The largest earthquake (1 in this example) is an absorbing state. This argument summarized here for an homogeneous system works also for heterogeneous systems, if the disorder is not too large (Bottani (1995)).

4 Elasticity and threshold disorder

We explore in this section the possibility to destroy the periodic behavior and kill the great earthquake, by introducing disorder also in the elastic coefficients of the elements and by varying the nature of the disorder, for instance by allowing for the existence of very strong and rigid elements (described by a powerlaw distribution of thresholds and/or elastic coefficients). This comes about because, the stronger the disorder, the stronger will be the barriers to stop the run-away.

With respect to the fault structure and the time history, the addition of disorder on the elastic coefficient, when not too large, is tantamount to increasing the threshold disorder with no elastic disorder: we observe a periodic behavior, after a transient which is significantly larger than previously, all parameters being the same otherwise. When measurable, the distribution of small earthquakes is a power law with an exponent B always equal to 1 ± 0.05. Fig.3 presents such a simulation with $\Delta\sigma = 1.9$, a disorder on the elastic coefficients defined as for the threshold with a flat distribution of elastic coefficient and a width equal to $\Delta g = 1$ and $\sigma_{dyn} = 0.04$.

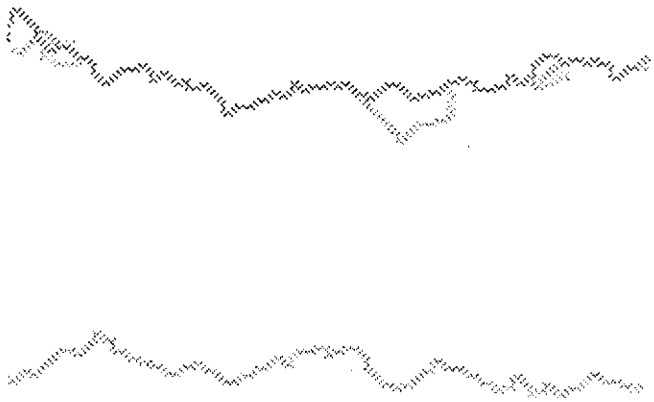
For large threshold disorder $\Delta\sigma = 1.9$, we find the novel feature that, in some system realizations, the time history does not seem to become periodic, however long we wait. A global stationary regime seems to emerge nevertheless, with an elastic energy stored in the plate which fluctuates around a well-defined average. The fault structure becomes very rough with some overhangs (this is allowed in the scalar anti-plane elastic model used here but would be unphysical in in-plane stress or strain models since compressive and extensive stress would accumulate without limit in the region of the overhang, according to a "hook" effect (Sornette (1988)). In figure 3a, one can see such a configuration in the form of a fault segment parallel to the Oy axis inside the largest loop on the upper fault. Furthermore, the run-away does not exist anymore and is replaced by a continuous distribution of earthquake of all sizes.

We have also explored different disorders, for instance a powerlaw distribution of rupture thresholds

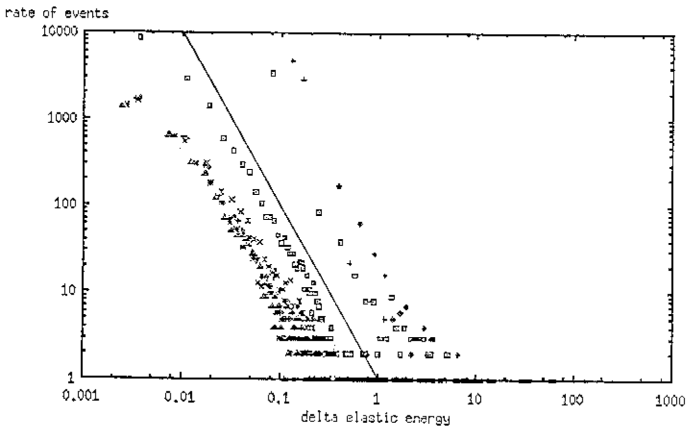
$$P_\sigma(\sigma_c) \sim \sigma_c^{-(1+\mu_\sigma)}$$

for $\sigma_c \geq 1$, with $\mu_\sigma = 0.5$ and 3. The first case corresponds to a very broad distribution (the average is mathematically divergent) with a significant fraction of the bonds having a very large threshold. Depending up the specific realization, we find a similar phenomenology as before. For instance, for $\sigma_{dyn} = 0$, a single great run-away punctuated the periodic dynamics. However, the fault becomes very complex, with many branches and loops. Correlatively, the number of elements rupturing in the great earthquake is about three times the system width.

For other disorder realizations with the powerlaw distribution of rupture thresholds, in presence or absence of elastic disorder, we have found a completely new effect, that we identify as a "frustration" which destroys the self-consistency of the quasi-static crack model. This also occurs for the previous bounded distribution in the presence of quenched disorder in the elastic coefficients. The phenomenon is best illustration by examination of figure 4. It shows a small part of the stress field during an earthquake in fast time in a network with a power law distribution of thresholds with $\mu_\sigma = 0.5$ and $\sigma_{dyn} = 0.5$. The fault segments which have ruptured are indicated by an arrow with a thick line. These ruptured elements all carry a stress whose absolute value is $\sigma_{dyn} = 0.5$. The arrows indicate the sign of the stress: downward (resp. upward) arrows correspond to positive (resp. negative) stresses (recall that all the stresses are along the Oz axis). Alternatively, one can interpret the map as a electric current field, within the usual mechanical-electrical analogy (stress $\rightarrow$ current, displacement $\rightarrow$ voltage, elastic coefficient $\rightarrow$ conductance) which is exact for antiplane elasticity. The stress carried by each fault element is indicated on the arrow. The rupture threshold for each element is written on the other side of the bond. The mechanical equilibrium translates into the condition that the sum of stresses carried by the fault segments attached to the same node be zero, easily verified in this example. The self-consistency of the model is obeyed when the mechanical equilibrium is such that all stresses are smaller than the corresponding threshold. When this is not the case, a rupture occurs relaxing the stress on the rupture element to σ_{dyn}. However, there are "frustrated" situations in which this is not possible. To see this, examine the element carrying a stress equal to 1.5 whose threshold is $\sigma_c = 1.495$. By the definition of the model, since its stress is larger than its threshold, it must rupture and its stress must thus be lowered to $\sigma_{dyn} = 0.5$. When this occurs, the mechanical equilibrium is broken, because the three other fault elements connected to the same node have all their stresses imposed to be equal to the dynamical stress. The model looses its self-consistency due to an over-determination. In general, it is straightforward to realize that this "frustrated" state occurs when three fault elements connected to the same node have ruptured. As a consequence, their stresses is imposed to be equal to σ_{dyn} and from the rule of quasi-static equilibrium, the stress on the remaining element connected to the same node is completely determined. If it happens that its threshold be less than this stress,



(a)



(b)

Fig.3 : a) Fault localization obtained at long times for $\Delta\sigma = 1.9$, disorder on the elastic coefficients $\Delta g = 1$, $\sigma_{dyn} = 0.04$, and the constant velocity imposed at the boundary is $V = 10^{-3}$. Note that the lower fault in grey becomes inactive after the transient and its contribution to the cumulative slip vanishes at long times.

b) Gutenberg-Richter distribution of the number of events having a given energy for different time windows at increasing times when going from right to left. In all cases, we observe powerlaw distribution $P(E)dE \sim E^{-(1+B)}$ with a constant exponent $B = 1 \pm 0.05$. The straight line has slope -2 for comparison. For early times, in the power law presents the same exponent, the events have a larger size which finally settle to a stationary distribution.

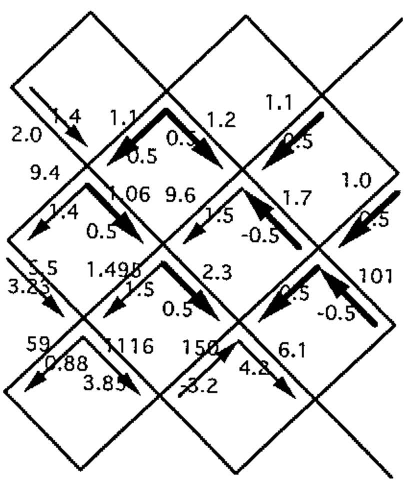


Fig.4 : Map of a small part of the stress field during an earthquake in fast time in a network with a power law distribution of thresholds with $\mu_\sigma = 0.5$ and $\sigma_{dyn} = 0.5$. The fault segments which have ruptured are indicated by an arrow with a thick line. These ruptured elements all carry a stress whose absolute value is $\sigma_{dyn} = 0.5$. The arrows indicate the sign of the stress: downward (resp. upward) arrows correspond to positive (resp. negative) stresses (recall that all the stresses are along the Oz axis). The stress carried by each fault element is indicated on the arrow. The rupture threshold for each element is written on the other side of the bond. A "frustrated" element is seen, with a stress equal to 1.5 whose threshold is $\sigma_c = 1.495$.

the frustration appears and the model collapses.

This effect is reminiscent of the concept of "frustration" introduced in the physics of general disordered systems (Toulouse (1977)). In general, frustration arises in a system whose interactions compete or conflict in such a way that not all constraints on the system can be simultaneously satisfied. This is what happens in our model with the conflict between the stress conservation and the dynamical stress which may produce frustration. The usual outcome of frustation in the presence of disorder is the existence of, not a single well-defined stable equilibrium state, but rather to a very large number of disordered equilibrium states of equivalent energies (Mézard et al. (1987)). The number of these minimum states usually increases exponentially with the number of degrees of freedom (spins). The energy landscape is extremely complicated with a hierarchy of barriers of increasing sizes separating the minimum states. In other words, in order to go from one minimum state to another, an energy barrier must be passed. This hierarchical structure produces novel behaviors (long time relaxations, breakdown of ergodicity, etc.) and important fluctuations. In our model, mechanical equilibrium is not more possible in the presence of frustration, as we have shown. It is tempting to extrapole from this analogy and suggest that the state of stress chosen by the plate, in an extended model allowing for a genuine dynamics, could have a multi-valley structure similar to that obtained in spinglasses for instance. This could be an important underlying ingredient at the origin of the observed spatio-temporal complexity. In this spirit, we have recently proposed that the mechanics of coupled blocks and especially the coupling between rotations exhibit the frustration property (Sornette et al. (1994b)). We believe that this is an ubiquitous and key property at the basis of the complexity observed in fault and earthquake organization.

The present crack model must be enriched to get rid of the overdetermination. The conceptually simplest solution is to introduce a genuine dynamics allowing a transient breakdown of static equilibrium. For instance, elastodynamics allow for an unbalance of the stress, corresponding to the generation and radiation of elastic waves. Physically, this describes the fact that the stored elastic energy is converted into two types of dissipation: 1) frictional heating, that we take into account by our rupture criterion and 2) elastic wave radiation that we neglect. A model in which the dissipation is not completely converted into friction, i.e. the stress is not immediately put to its asymptotic dynamical value, would cure the observed frustration and over-determination. Another alternative, within quasi-static mechanics, is to introduce other types of deformations, such as thrust or normal faulting. Algorithmically, this will lead to allow a fraction of the stress to be removed by going out of plane, thereby removing the constraint of zero antiplane stress at all nodes. However, we still expect frustation to occur for certain realizations due to the competition between static mechanical equilibrium and constant dynamical stress.

It is interesting to note that there is a particular value of the dynamical stress drop for which the frustration never occurs, namely for $\sigma_{dyn} = 0$. In this case, the stress on the fourth element attached to a node, for which the three other fault segments have ruptured, is zero by the law of local static mechanical equilibrium. This is consistent both with mechanical equilibrium by definition and also with the dynamical stress drop condition. Notice that this is the only situation for which the condition of dynamical stress drop is always compatible with static mechanical equilibrium. However, the crack model presents a curious and probably rather artificial behavior in this case. Remember that a rupture cycle is characterized usually by a progressive acceleration of the rate of small earthquakes prior to the occurrence of the run-away. Since the run-away spans the whole width of the lattice, and since the dynamical stress is imposed on its ruptured elements, this amounts to impose the total stress within the plate equal to zero. Previous ruptures on small faults within a cycle have produced localized slip and stress sources. They cannot remain in the presence of this global vanishing of the stress within the plate. As a consequence, we observe in fast time a cloud of small earthquakes accompanying the run-away, which are the "ghosts" of all the previous small earthquakes. These "ghosts" present exactly a slip which is the opposite of the slip that they have develop in the foreshock phase. In other words, the aftershocks occurring in fast time are the exact symmetric of all the foreshocks. The difference is that the foreshocks are spread in time over the period of the cycle while the aftershocks "ghosts" are occurring in fast time, just after the run-away. While the details of this behavior is clearly model specific, this phenomenon is not without recalling field observations that foreshocks occur usually years or even decades before a great event, while the huge majority of aftershocks are clustered over a few months after the main event. The present model does not contain however the necessary ingredients to describe the time delays associated with the coupling with other modes of creep or ductile deformations, the ductile crust and the fluid in the crust.

Ref.(Bhagavatula et al. (1994)) have studied the same crack model and it is instructive to discuss how their results differs from ours in many aspects. They have only studied the case $\sigma_{dyn} = 0$. They have thus not found the frustration effect discussed above. In addition, they consider an annealed disorder, i.e. all the threshold of the fractures elements are re-set to new random numbers after each event. As a consequence, they cannot obtain earthquake localization on well-defined faults but only observe diffuse earthquakes. Also, this reshuffling of the disorder prevents the synchronization to a periodic cycle and the appearence of a run-away. Nevertheless, they observe that the distribution of small earthquakes is a powerlaw with $B = 0.8 \pm 0.1$, not far from our estimate and that the distribution presents a peak at large earthquakes whose energy scales with the square of the system size. While they interpret this as a finite size effect, we rather conclude that these large events are the shadows of the great run-aways in the presence of annealed noise, which sizes are given by that of the system. In a sense, the annealed disorder makes their system function permanently in our transient regime. They have only studied disorder on the thresholds by the method of Green functions, which is not useful practically in the presence of elastic disorder. The gradient conjugate method that we have used is slower but more general to tackle this second case. Finally, they have used infinite system Green functions, and have not addressed the question of the effect of boundaries in finite systems. While in the statistical physics of critical phenomena, one would like to get results which are independent of boundaries, in the present mechanical problem as well as in the general mechanical case, the existence of well-defined boundary conditions on the stress or strain fields at the border of the system is know to control drastically the localization of the mechanical deformation, as we have been able to observe.

5 Concluding remarks

Except for special realizations with the strongest disorder on rupture thresholds and elastic coefficients, we have found that the quasi-static crack model of earthquake recurrence leads to periodic cycles, characterized by small foreshocks distributed according to a universal Gutenberg-Richter law with exponent $B = 1$ up to a maximum nucleation size and a large run-away ending the cycle. This periodic behavior results from the strong synchronization brought by imposing a constant dynamical stress, corresponding to an attractive or absorptive state. The other main result is the discovery of a fundamental frustration resulting from an overdetermination of the stress field in the presence of large disorder and imposed dynamical stress drop. The general solution to this breakdown of self-consistency is to re-introduce a genuine dynamics allowing the local breakdown of static mechanical equilibrium, associated to the radiation of elastic waves. Our study pinpoints the fundamental role played by elastodynamics in repetitive crack ruptures. We thus believe that, in crack models (and not in dislocation models), there is no other way than incorporate the full elastodynamic equations to get a self-consistent solution in all situations.

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References

King G.C.P., *Pageoph,124*, 567-585, 1983; *Pageoph,123*, 806, 1985.

Wu R.-S. and K. Aki, *Pageoph, 121*, 761-815, 1986.

Davy P., Sornette A. and Sornette D., *Nature, 348*, 56-58, 1990.

Kagan Y.Y., *J. Nonlinear Sci., 1*, 1-16, 1991.

King G.C.P. and C.G. Sammis, *Pageoph, 138*, 611-640, 1992.

Barton C.A. and M.D. Zoback, *J. Geophys. Res., 97*, 5181-5200, 1992.

Hanks T.C., *Science, 256*, 1430-1432, 1992.

Scholz C.H., N.H. Dawers, J.-Z. Yu and M.H. Anders, *J. Geophys. Res., 98*, 21951-21961, 1993.

Andrews D.J., *J. Geophys. Res., B7*, 9389-9397, 1989; *Bull. Seism. Soc. Am., 84*, 1184-1198, 1994.

Cowie P., D. Sornette and C. Vanneste, *J.Geophys.Res., 98*, 21809-21821, 1993; *Geophys. J. Int., 122*, 457-469, 1995.

Miltenberger, P., Sornette, D. and Vanneste, C., *Phys.Rev.Lett., 71*, 3604-3607, 1993.

D.Sornette, P. Miltenberger and C. Vanneste, *Pageoph, 142*, 491-527, 1994.

Halpin-Healy T. and Y.-C. Zhang, *Phys. Rep., 254*, 215-415, 1995.

Scholz C.H., *Bull. Seism. Soc. Am., 72*, 1-14, 1982; *Bull. Seism. Soc. Am., 84*, 215-218, 1994; *Bull. Seism. Soc. Am., 84*, 1677-1678, 1994.

Romanowicz B. and Rundle J.B., *Bull. Seism. Soc. Am., 83*, 1294-1297, 1993; *Bull. Seism. Soc. Am., 84*, 1684, 1994.

Sornette D. and A. Sornette, *Bull.Seism.Soc.Am., 84*, 1679-1683, 1994.

Gabrielov A., W. Newman and L. Knopoff, *Phys.Rev., E 50*, 188, 1994.

Christensen K., Self-organization in models of sandpiles, earthquakes, flashing fireflies, *PhD Thesis* Oslo, Nov. 1992.

Sornette D., Les phénomènes critiques auto-organisés, *Images de la Physique 1993*, édition du CNRS, january 1994.

Sornette D., P. Miltenberger and C. Vanneste, Statistical physics of fault patterns self-organized by repeated earthquakes : synchronization versus self-organized criticality, in Recent Progresses in Statistical Mechanics and Quantum Field Theory, *Proceedings of the conference 'Statistical Mechanics and Quantum Field Theory'*, USC, Los Angeles, May 16-21, 1994, eds. P. Bouwknegt, P. Fendley, J. Minahan, D. Nemeschansky, K. Pilch, H. Saleur and N. Warner, (World Scientific, Singapore, 1995), p.313-332.

Corral A., C.J. Pérez, A. Díaz-Guilera and A. Arenas, *Phys.Rev.Lett., 74*, 118-121, 1995.

Middleton A.A. and C. Tang, *Phys.Rev.Lett.* 74, 742-745, 1995.

Bottani S., *Phys.Rev.Lett., 74*, 4189-4192, 1995.

Gil L. and Sornette D., Landau-Ginzburg theory of self-organized criticality, *Phys.Rev.Lett.* submitted, 1995.

Lubliner J., *Plasticity theory*, Macmillan Publishing company, London, 1990.

Lomnitz-Adler J., L. Knopoff nd G. Martínez-Mekler, *Phys.Rev.*, *A 45*, 2211-2221, 1992.

Schwartz and K.J. Coopersmith, *J.Geophys.Res.*,*89*, 5681-5698, 1984.

Mirollo R. and Strogatz S., *SIAM J. Appl. Math.*, *50*, 1645, 1990.

Sornette D., *J.Physique (Paris)*, *49*, 1365, 1988.

Toulouse G., *Commun.Phys.*, *2*, 115-119, 1977.

Mézard M., Parisi G. and Virasoro M.A., *Spinglass Theory and Beyond*, World Scientist Lecture Notes in Physics, *Vol. 9*, 1987.

Sornette A., D. Sornette and P. Evesque, *Nonlinear Processes in Geophysics*, *1*, 209-218, 1994.

Bhagavatula R., K. Chen and C. Jayaprakash, *J.Phys.A*, *27*, L155-L162, 1994.