Gibbs point field models for extraction problems in image analysis

Xavier Descombes, Elena Zhizhina

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1 Probabilistic approach in image analysis.

The basic idea of probabilistic approach in image analysis, see e.g. [1, 2], was to rewrite an image processing procedure in the language of statistical physics using concepts of statistical ensembles, equilibrium and non-equilibrium dynamics. Under this view, images are considered as configurations of a Gibbs field. The implicit assumption behind the probabilistic approach in image analysis is that, for a given problem, there exists a Gibbs field such that its ground states represent regularized solutions of the problem. Thus, the crucial step in the probabilistic approach is the choice of a proper configuration space and the choice of a distribution, or equivalently, in the case of the Gibbs random fields approach, the choice of an energy function $H(X)$. The energy function contains usually few types of terms. One of them arises from the observable image (a data driven term) and has the form of an external field term. Others are due to generic or prior knowledge on the structure of images. Prior terms in the energy function are specified by potentials associated with local interactions of neighboring variables. Thus, each variable directly depends only on its neighborhood, although from a global point of view, all variables are mutually dependent through the combination of successive local interactions.

Recently, there has been again growing interest in the applications of Gibbs point fields and Markov point processes to inverse problems of image processing such as feature extraction, object detection, surface reconstruction, stereo matching. All these problems related with consideration of strong geometrical constraints in a priori potential. In this paper we present a new multiple birth and death algorithm constructed as approximation of a stochastic Glauber type dynamics. We discuss results of its implementation on the example of two extraction problems.

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2 Gibbs fields models for feature extraction problems. The general setting.

We discuss here stochastic algorithms in the framework of the Gibbs fields approach for feature extraction problems. These problems become critical in remote sensing with the development of high resolution sensors for which the object geometry is well defined. Marked point processes framework is found very proper for extraction problems, since it is difficult to incorporate strong non-local geometrical constraints in the potential in lattice based models. Random sets of objects are represented in the models by marked point configurations in continuous space. Features of the objects, such as shape and/or size are described by a mark, and locations of the objects by a point configuration.

If we denote by $\Gamma := \Gamma(V)$ the set of all point configurations from a finite volume $V \subset R^2$, by S a space of marks (a spin space) and by π_z the Poisson measures with activity z, $z > 0$, then the marked configuration space $\hat{\Gamma}$ of the model is:

$$\hat{\Gamma} := \big\{ \hat{\gamma} = (\gamma, \sigma_\gamma), \gamma \in \Gamma,\ \sigma_\gamma = \{\sigma_x(\gamma)\}_{x\in\gamma} = \{\sigma_x\}_{x\in\gamma},\ \sigma_x \in S \big\}.$$

A reference measure μ_0 on $\hat{\Gamma}$ can be written as $d\mu_0(\hat{\gamma}) = d\omega(\sigma_\gamma)\, d\pi_z(\gamma)$, where $d\omega(\sigma_\gamma) = \prod_{x\in\gamma} d\omega(\sigma_x)$ is the conditional (under given configuration γ for positions of marks) free marks measure equals to the product of the free mark measures ω over all points from the configuration γ. The probability distribution on the configuration space $\hat{\Gamma}$ is defined then as a Gibbs reconstruction μ_β of the reference measure μ_0 with the energy function $H(\hat{\gamma})$ involving both objects positions and their marks. To find global minimizers of the energy function, one can consider various stochastic dynamics with a given stationary Gibbs measure under the annealing procedure.

Here we will discuss two models for extraction problems (a random disc model and a random point model), both of them can be described as pure point models without marks, in this case $\hat{\Gamma} = \Gamma$. We consider an equilibrium birth-and-death dynamics with the stationary Gibbs measure μ_β given by the following generator

$$(L_\beta\ f)(\gamma) \ = \ \sum_{x\in\gamma} e^{\beta E(x,\gamma\backslash x)}(f(\gamma\backslash x) - f(\gamma)) \ + \ z \int_V (f(\gamma \cup y) - f(\gamma))\, dy, \tag{1}$$

defined in the functional space $f(\gamma) \in L^2(\Gamma, \mu_{\beta,z})$, where $E(x, \gamma\backslash x) \ = \ H(\gamma) - H(\gamma\backslash x)$.

3 Approximation process

In this section we present the mathematical background of our algorithm, which consists of two main steps:
1) the construction of the approximation process and the proof of the convergence of the approximation to the continuous time process as the discretization step tends to zero, and
2) the proof of the convergence of the corresponding evolution of measures under the annealing regime to a measure concentrated on the global minima of the energy function with a minimal number of points in the configuration.

We define a discrete time approximation $T_{\beta,\delta}(n), n = 0,1,2,\ldots$ of the continuous time birth-and-death process generated by (1). It is a Markov chain on the same space $\Gamma(V)$ with the transition operator $P_{\beta,\delta}$ ($T_{\beta,\delta}(n) = P^n_{\beta,\delta}$) of the form:

$$(P_{\beta,\delta}f)(\gamma) = \sum_{\gamma_1\subseteq\gamma} \prod_{x\in\gamma_1} \frac{1}{1+a_x\delta} \prod_{x\in\gamma\setminus\gamma_1} \frac{a_x\delta}{1+a_x\delta} \tag{2}$$

$$\Xi_\delta^{-1} \sum_{k=0}^{\infty} \int\limits_{V^k} \frac{(z\delta)^k}{k!} f(\gamma_1 \cup y_1 \cup \ldots \cup y_k)\, dy_1 \ldots dy_k,$$

where $\Xi_\delta = \Xi_\delta(V,z,\delta)$ is the normalizing factor, $a_x = a_x(\gamma) = e^{\beta E(x,\gamma\setminus x)}$.

Let $\mathcal{L} = B(\Gamma(V))$ be a Banach space of bounded functions on $\Gamma(V)$ with a norm

$$\|F\| = \sup_{\gamma\in\Gamma(V)} |F(\gamma)|,$$

and by $\mathcal{B}$ we denote a family of measures with a bounded density w.r.t. the Poisson measure π_z (and hence also w.r.t. the Gibbs measure μ_β).

Theorem 3 (Convergence of the approximations) [3]. *For each $F \in \mathcal{L}$*

$$\|T_{\beta,\delta}(\left[\frac{t}{\delta}\right])F - T_\beta(t)F\|_{\mathcal{L}} = \sup_\gamma |(T_{\beta,\delta}(\left[\frac{t}{\delta}\right])F)(\gamma) - (T_\beta(t)F)(\gamma)| \to 0, \tag{3}$$

as $\delta \to 0$ for all $t \geq 0$ uniformly on bounded intervals of time.

Let $S_{\beta,\delta}(n)$ be an adjoint to $T_{\beta,\delta}(n)$ semigroup acting on measures, such that for any $\nu \in \mathcal{B}$:

$$\langle S_{\beta,\delta}(n)\nu, F\rangle = (p_\nu, T_{\beta,\delta}(n)F)_{\mu_\beta} \text{ with } p_\nu = \frac{d\nu}{d\mu_\beta}.$$

Let $n_0 \in N \cup \{0\}$ be the minimal number of points in configurations $\bar\gamma$ minimizing the energy function $H(\gamma)$. Then the Gibbs distributions μ_β converge weakly as $\beta \to \infty$ to a distribution μ_∞ on $\Gamma(V)$ of the form

$$\mu_\infty = \sum_{\bar\gamma:|\bar\gamma|=n_0} C_{\bar\gamma}\delta_{\bar\gamma} \text{ if } n_0 > 0, \quad \text{and} \quad \mu_\infty = \delta_{\{\emptyset\}} \text{ if } n_0 = 0. \tag{4}$$

Here $\delta_{\bar\gamma}$ is the unit measure concentrated on the configuration $\bar\gamma$, and $\sum\limits_{\bar\gamma:|\bar\gamma|=n_0} C_{\bar\gamma} = 1$.

Theorem 4 (Convergence in the annealing regim) [3]. *Let $F \in B(\Gamma(V))$ and an initial measure $\nu \in \mathcal{B}$. Then under relation $\delta\, e^{\beta b} < const$ with $b = \sup_{\gamma\in\Gamma_d(V)} \sup_{x\in\gamma} E(x,\gamma\setminus x)$ we have*

$$\lim_{\beta\to\infty,\ t\to\infty,\ \delta\to 0} \langle F\rangle_{S_{\beta,\delta}([\frac{t}{\delta}])\nu} = \langle F\rangle_{\mu_\infty}, \tag{5}$$

where $\langle F\rangle_{S_{\beta,\delta}([\frac{t}{\delta}])\nu} = \langle S_{\beta,\delta}([\frac{t}{\delta}])\nu, F\rangle$.

4 A new Multiple Birth and Death algorithm.

The main idea behind our algorithm is to use the continuous time stochastic dynamics generated by (1) and then to take the transition operator of the discrete time approximation process (2) as a base of stochastic iterative steps of the algorithm. The algorithm simulating the process is defined as follows:

- **Computation of the birth map:** To speed up the process, we consider instead of z a non homogeneous birth rate $B(s),\ s \in V$ to favor birth where the data term is strong. This non homogeneous birth rate refers to a non homogeneous reference Poisson measure.

- **Main program:** initialise the inverse temperature parameter $\beta = \beta_0$ and the discretization step $\delta = \delta_0$ and alternate birth and death steps. Birth step is taken with density $\delta B(s)$ w.r.t. the Lebesgue measure on V. Death step: for each point from the configuration, the death probability is defined as follows:

  $$D(x) = \frac{\delta a_x}{1 + \delta a_x}.$$

  Decrease the temperature and the discretization step by a given factor and go back to the birth step.

5 Results

5.1 Application to birds detection

We consider a model of partially overlapping discs $\{d_{x_1}, \ldots, d_{x_k}\}$ of the same radius r with a hard core distance $\epsilon_0 < r$ between any two elements, lying in a bounded domain $V \subset R^2$. Then Γ is the configuration space of the centers of the discs. The energy function is a sum of data and a priori terms

$$H(\gamma) = \alpha \sum_{x \in \gamma} H_1(x) + \sum_{\{x,y\} \subset \gamma} H_2(x, y),$$

where α is a weighting parameter. The second term represents prior knowledge on the discs configuration and it is defined by pair interactions (repulsion on small distances) between neighboring discs. A data term is added for each object to fit the disc configuration onto the data, it is a sum of local energy functions associated with each object. For a given object, the local energy depends on a statistical test between the pixel distribution inside of the projection of the disc on the lattice and the pixel distribution in the neighborhood of the disc. The higher the contrast between the interior of the object and its neighboring ring, the lower the energy.

The fragment of the initial image of flamingo colony and the result image of detected birds are given on figure 1.

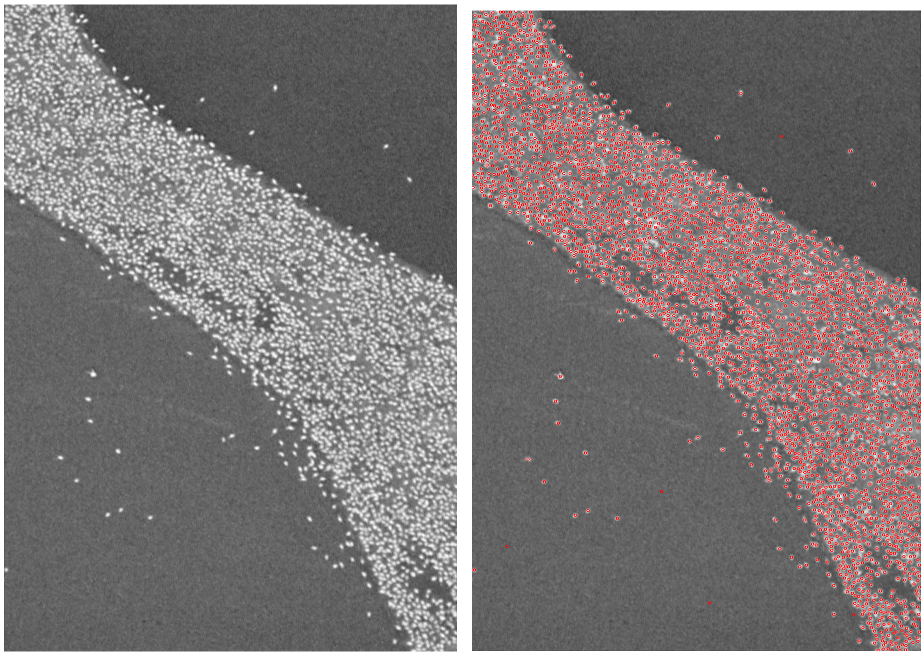

Figure 1: Fragment of the image of the bird population, Station Biologique Tour du Valat (left); image of detected birds (right)

5.2 Application to road network extraction problem

Different approaches for a fully automatic road network extraction from satellite images have been proposed recently, and Gibbs fields models among them. We present here a new model with simple objects (points) but with a complicated energy including the interaction energy of a special form. We propose here a new preprocessing procedure to extract from the input image a significant information in the form of another pixel-wise marked image $\mathcal{P}$, where each pixel p has two marks: an angle and a contrast. The value of contrast n_p describes how much pixel p is likely to belong to the road, and then angle θ_p defines a local direction of the road for relevant pixels. The results $\mathcal{P}$ of this preprocessing procedure is used in the data driven energy and in the interaction energy terms.

The energy function is a sum of three terms: the data term, a priori term and an interaction term. The prior knowledge models the high connectivity and the low curvature of a road network, the priori potential is used for reconstruction of hidden parts of the roads as well as for junction detection. The interaction energy is generated by pair potential depending on preprocessing image $\mathcal{P}$. It is a repulsive energy on short distances to prevent accumulation of points in the configuration and an attractive energy on fitting distances. In exceptional situations (junctions or parallel roads), that can be seen from the preprocessing image, the repulsive energy is vanishing. The data term contributes in the energy through a sum of local energy functions at each point of the configuration using preprocessing data. Then we exploit algorithms based on approximations of stochastic birth and death dynamics embeded into a simulated annealing regime, see Sect. 4 above.

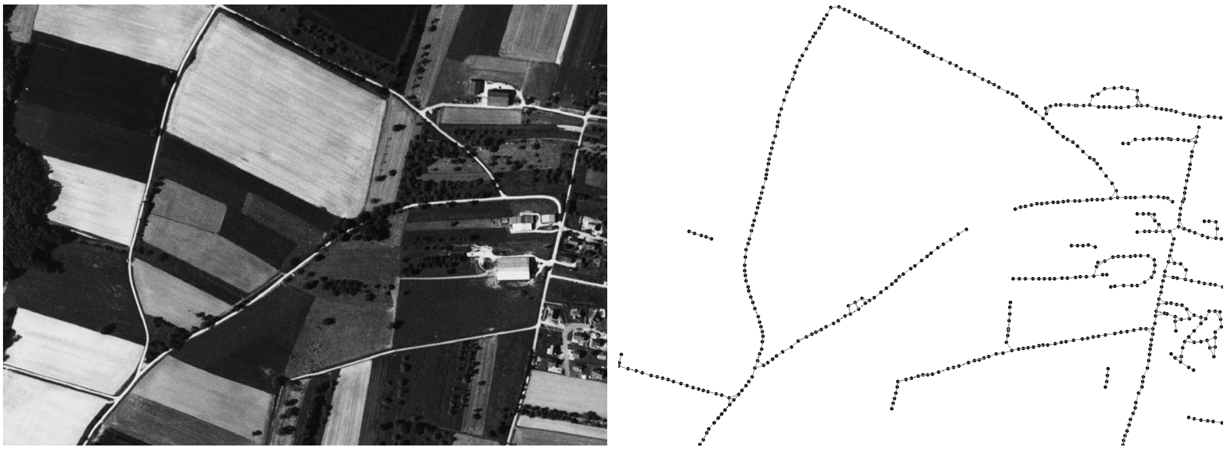

Figure 2: Aerial image and obtained result (the point configuration)

6 Conclusion

Thus, the main advantages of marked point process in image analysis are in their geometrical adaptativity and generality. Any geometrical properties can easily be introduced into the model through the object geometry. Different types of objects (trees, roads, buildings, etc.) can be considered within the same model but with appropriate interactions. Moreover, interactions between points allows to model some prior information on the object configuration, and the data are taken into account at the object level, thus improving robustness of the algorithms.

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