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Theoretical Aspects of the Optical Transpose Interconnecting System Architecture
(extended abstract)

D. Coudert†‡ A. Ferreira† S. Perennes†

1 Introduction

An attractive way of implementing efficient local interconnection networks is to use the Optical Transpose Interconnecting System (OTIS) architecture proposed in [8]. This system allows to optically interconnect some set of processors in a Free Space of Optical Interconnections (FSOI). Briefly, it consists of two lenslet arrays allowing a large number of optical interconnections from a set of transmitters to a set of receivers. Note that the OTIS architecture is indeed a three dimension (3D) one but it can always be modelized in a 2D-space.

Two approaches exist, in the first one (All-Optical) the OTIS architecture provides all the interconnections of the network in one-hop, while in the second (Hybrid), some electronic connections are necessary and moreover some connections are directly implemented but may require several hops.

The hybrid approach has been motivated by the results of [5], where it was shown that as soon as an electronic wire is more than 1 cm long, it has more power consumption than its optical counterpart which connects an optical transmitter to an optical receiver through a FSOI. Consequently, the OTIS architecture was used in [9] to realize parts of interconnection networks such as hypercubes, 4-D meshes, mesh-of-trees and butterflies.

The All-Optical approach is enhanced by the opportunity, offered with the OTIS architecture, to easily build a real one-to-one symmetric complete digraph with loops (\(K^*_{64}\)). In fact, using this architecture, it is practically possible to connect 64 processors in a complete graph [7], each processor having 64 transceivers (corresponding to the 64 arcs of one vertex). It has also been shown in [4] how to realize the single-hop multi-OPS POPS network [2], and the multi-hop multi-OPS stack-Kautz network [3] with the OTIS architecture.

In this work, we focus on All-Optical networks.

The number of transceivers per processor is technologically limited (the number of transceivers per cm\(^2\) cannot exceed 64 at the moment). Also, whereas a network having the topology of a complete graph with 64 processors is actually feasible and is an important advance for network design, it is not enough as one can wish layouts for large networks having hence bounded degree (\(d \ll 64\)). Moreover, a low number of transceivers per processor means a reduced price per processor. Consequently, it is important to study the set of network topologies for which it exists an efficient layout with the OTIS architecture and to find which “good” networks admit such a layout, we will call it an OTIS\(_{2D}\) (OTIS\(_{3D}\)).

So we will first try to provide ways of determining if a given general network admits an OTIS layout or not. Then we will study particular case of regular and symmetric networks. Finally we will show that classical topologies like de Bruijn, Kautz and complete digraphs admit an OTIS\(_{2D}\)-layout. At the end, the results obtained for the OTIS\(_{2D}\) model are applied to the OTIS\(_{3D}\) case.

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2 Preliminaries

2.1 OTIS

The Optical Transpose Interconnection System (OTIS) architecture, was first proposed in [8]. OTIS\((p, q)\) is an optical system which allows point-to-point (1-to-1) communications from \(p\) groups of size \(q\) onto \(q\) groups of size \(p\). This architecture connects the transmitter \((i, j)\), \(0 \leq i \leq p - 1, 0 \leq j \leq q - 1\), to the receiver \((q - 1 - j, p - 1 - i)\).

Optical interconnections in the OTIS architecture are realized with two planes of lenses [6] in a free optical space as shown in the figure opposite. The transmitter \(t = (i, j)\) and the receiver \(r = (i', j')\) are also numbered as integer modulo \(m = pq, t = ip + j\) and \(r = qj' + j'\). Transceivers are associated to Processors in the following way: the transmitters of processor \(p_i\) are \(d_i, d_i + 1, \ldots, d(i + 1) - 1\) and the receivers of \(p_i\) are \(d_i, d_i + 1, \ldots, d(i + 1) - 1\). Hence there is a connection from \(p_i\) to \(p_j\) if one transmitter of \(p_i\) is connected to one receiver of \(p_j\).

Actually, the OTIS architecture which is implemented has three dimensions. OTIS\(_{3D}(p, q, p', q')\) connects \(pp'\) groups of size \(qq'\), to \(qq'\) groups of size \(pp'\). The transmitter \((i, j, i', j')\), \(0 \leq i < p, 0 \leq i' < p', 0 \leq j < q\) and \(0 \leq j' < q'\), is connected to the receiver \((q - j - 1, p - i - 1, q' - j' - 1, p' - i' - 1)\), but the OTIS\(_{3D}\) architecture can be modeled by two OTIS\(_{2D}\) (See Section 3.5).

2.2 A graph-theoretic model for OTIS\(_{2D}\) networks

The OTIS\(_{2D}(p, q)\) architecture connects \(m = pq\) transmitters to \(m\) receivers. Let \(m = dn\) and let \(p_i, 0 \leq i < n\) be the processor corresponding to the transmitters \(d_i, d_i + 1, \ldots, d(i + 1) - 1\) and the receivers \(d_i, d_i + 1, \ldots, d(i + 1) - 1\). The OTIS\(_{2D}(p, q)\) architecture connects \(n\) processors in a network of constant
Let $H(p, q, d)$ be the directed graph (digraph) corresponding to this network topology. $H(p, q, d)$ has $n = \frac{pq}{d}$ nodes of constant degree $d$ and $m = pq$ arcs. There is an arc from the node $u$, $0 \leq u < n$, to the nodes $v_\alpha$, $0 \leq \alpha < d$, such that

$$v_\alpha = \left\lfloor \frac{(pq - 1) \left( \left\lceil \frac{du + \alpha}{n} \right\rceil + 1 \right) - p(du + \alpha)}{d} \right\rfloor, \quad 0 \leq \alpha < d$$

Let $G = (V, E)$ be a digraph, with $|V| = n$ nodes of constant degree $d$ and $|E| = m = dn$ arcs. $G$ has an OTIS$_{2D}$-layout if there exist $p$ and $q$ with $pq = m$ and an isomorphism $\sigma$ from $G$ onto $H(p, q, d)$. This notion can be extended to OTIS$_{3D}$.

**Remark 1** Let us consider the case $p = d$ and $q = n$. We obtain:

$$v_\alpha = \left\lfloor n \left( \left\lceil \frac{du + \alpha}{n} \right\rceil + 1 \right) - (du + \alpha) - \left\lceil \frac{du + \alpha}{n} \right\rceil + 1 \right\rfloor, \quad 0 \leq \alpha < d$$

$$\equiv -(du + \alpha) + \left\lfloor \left\lceil \frac{du + \alpha}{n} \right\rceil + 1 \right\rfloor \mod n, \quad 0 \leq \alpha < d$$

As far as $0 \leq du + \alpha \leq d(n - 1) + d - 1 = dn - 1$, it follows that $\left\lceil \frac{du + \alpha}{n} \right\rceil = -1$.

Consequently,

$$v_\alpha \equiv -du - \alpha \mod n, \quad 1 \leq \alpha \leq d$$

### 3 Results

Characterizing OTIS-layout graphs appears to be difficult, so we generally restrict ourselves to the case of regular digraphs. Two very simple and useful results are the following ones.

**Remark 2** If there exists an OTIS-layout for a digraph $G$, then $G$ contains a cycle of length 2.

**Remark 3** Let $G^-$ be the digraph obtain by reversing all the arcs of the digraph $G$. If $G$ admit an OTIS$_{2D}(p, q)$-layout then $G^-$ has an OTIS$_{2D}(q, p)$-layout.

Hence the optical interconnections of the OTIS$_{2D}(p, q)$ architecture are clearly “equivalent” to those of the OTIS$_{2D}(q, p)$ architecture (up to reversing the arcs).

#### 3.1 OTIS$_{2D}(1, m)$

The OTIS$_{2D}(1, m)$ architecture is used to reverse the order of the incoming optical beams, as the transmitter $t$, $0 \leq t < m$ is connected to the receiver $m - t - 1$.

**Proposition 1** Let $G = (V, E)$ be a simple digraph with $|V| = n$ nodes and $|E| = m$ arcs. If there exists an OTIS$_{2D}(1, m)$-layout for $G$, then:

1. $\forall u \in V$, $|\{v \in \Gamma^+(u) | d^-(v) > 1\}| \leq 2$.
2. $\forall u \in V$, $|\{w \in \Gamma^-(u) | d^+(w) > 1\}| \leq 2$.

Using Proposition 1, it is possible to obtain a polynomial algorithm which compute an OTIS$_{2D}(1, m)$-layout for a digraph $G$ when it exists and reject $G$ otherwise.
3.2 Symmetric digraphs

Symmetric digraphs play an important role as interconnection network topologies. We show in this section that only very few of them admit an OTIS\textit{2D}-layout.

**Proposition 2** Let \( G = (V, E) \) be a strongly connected symmetric digraph with \(|V| = n\) nodes, \(|E| = m\) arcs and a constant degree \( d \). Then an OTIS-layout for \( G \) can only be realized with an OTIS\textit{2D}(p, q) architecture in which \( p - 1 \leq q \leq p + 1 \).

**Proof:** Considering such a symmetric digraph, a quick study of the adjacencies of the node number 0 leads to \( m - \alpha q - 1 - (d - 1) \leq m - \alpha p - 1 \leq m - \alpha q - 1 + (d - 1) \), with \( 0 \leq \alpha < d \). The result is given by \( \alpha = d - 1 \).

**Proposition 3** Let \( G = (V, E) \) be a strongly connected symmetric digraph with \(|V| = n\) nodes, \(|E| = m\) arcs and a constant degree \( d \), which has an OTIS\textit{2D}-layout. Then,

1. If \( G \) is without loop, then \( G = K_n^+ \) and \( d = q = p - 1 \) or \( d = q - 1 = p \).
2. If each node of \( G \) has a loop, then \( G = K_n^* \) and \( d = p = q \).

Consequently, Propositions 2 and 3 imply that there exists no OTIS\textit{2D}-layout for hypercubes, grids and torus of any sizes and more generally for all symmetric digraphs with constant degree in which either all nodes have a loop or there is no loop (in particular for transitive digraphs), except for complete digraphs \( K_n^+ \) and \( K_n^* \).

Note that there exist symmetric digraphs with \( l \) loops, \( 0 < l < n \), admitting an OTIS-layout.

3.3 De Bruijn, Kautz and their generalizations

Two important classes of digraphs with large number of vertices and small diameter are the de Bruijn and the Kautz digraphs which are both particular cases of the generalized Kautz digraphs (also called Imase and Itoh digraphs). The de Bruijn digraph is also a particular case of the generalized de Bruijn digraphs.

**Theorem 1** The Imase and Itoh digraph, \( II(d, n) \), with \( n \) nodes of degree \( d \), has an OTIS\textit{2D}(d, n)-layout.

**Proof:** Inside the Imase and Itoh digraph, \( II(d, n) \), the nodes are integer modulo \( n \) and there is an arc from a node \( u \) to the nodes \( v_\alpha \), \( 0 \leq \alpha < d \), such that \( v_\alpha \equiv -du - \alpha - 1 \mod n \). Using remark 1, the proof follows.

**Corollary 1** The de Bruijn and the Kautz digraphs have an OTIS\textit{2D}-layout.

For both \( B(d, k) \) and \( K(d, k) \), we have a simple linear algorithm to label the nodes. Notice that the generalized de Bruijn digraph, \( GB(d, n) \), with \( n \) nodes of degree \( d \), has no OTIS\textit{2D}(d, n)-layout, and we proposed an OTIS-layout using one OTIS\textit{2D}(d, n) and one OTIS\textit{2D}(1, dn).

3.4 De Bruijn and Kautz bus networks

The de Bruijn and Kautz bus networks were defined in [1] as hypergraphs built from the generalized de Bruijn and the generalized Kautz digraphs. As an Optical Passive Star coupler is equivalent to a directed bus, we propose for both networks an optical implementation using two OTIS\textit{2D} and one OTIS\textit{2D}(1, m).
3.5 Extension of the Results to OTIS$_{3D}$

The OTIS$_{3D}$ architecture realizes optical interconnections from a matrix of transmitters to a matrix of receivers. From a topological aspect, those interconnections can be realized by applying an OTIS$_{2D}$ on the matrix columns followed by an OTIS$_{2D}$ on the rows.

**Definition 1** The conjunction $G_1 \otimes G_2$ of two digraphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the digraph with vertex-set $V_1 \times V_2$ and an arc joining $(u_1, u_2)$ to $(v_1, v_2)$ if and only if there is an arc joining $u_1$ to $v_1$ in $G_1$ and an arc joining $u_2$ to $v_2$ in $G_2$.

**Theorem 2** A digraph $H$ admits an OTIS$_{3D}$-layout if and only if it exist two digraphs $G$ and $K$, admitting both an OTIS$_{2D}$-layout, and such that $H = G \otimes K$.

Consequently, OTIS$_{2D} \otimes$ OTIS$_{2D} = $ OTIS$_{3D}$ and questions related to 3D case can be reduced to the 2D model. The OTIS$_{3D}$ architecture allows more digraphs layout than the OTIS$_{2D}$ architecture. As an example, the digraph $K_{n_1}^+ \otimes K_{n_2}^*$ has an OTIS$_{3D}$-layout whereas Proposition 3 forbid its OTIS$_{2D}$-layouts. Otherwise, some digraphs having an OTIS$_{2D}$-layout have also an OTIS$_{3D}$-layout like de Bruijn digraphs, as $B(d, k) \otimes B(d', k) = B(dd', k)$, and $K_{n}^*$.

4 Conclusion

We have obtained several results on OTIS networks but the one which is still open is the question to know if finding an OTIS-layout for a given digraph is a polynomial problem or not. Also as OTIS contains several good networks, it would be interesting to study the properties (degree, diameter, routing) of digraphs induced by the OTIS$_{2D}(p, q)$ architecture when processors have degree $d$.

As the problem of computing the diameter of an OTIS-network seems difficult, we are interested in the problem of finding the largest OTIS-network for given degree $d$ and diameter $D$. Experimentaly, it appears that Imase and Itoh’s digraphs are always the largest. Futhermore, we obtain the same results when we design the OTIS-network of lowest diameter for given number of node and degree. On the other hand, an Imase and Itoh’s digraph, $II(d, n)$, has an OTIS$_{d}(d, n)$ - layout. When $n$ is large compare to $d$, the OTIS architecture has a large number of small lenses which may be difficult to built. Hence, we are also interested in finding goods OTIS-networks, for given degrees and diameters, in which the values of $p$ and $q$ are close. Table 1 gives the number of nodes and the size of the OTIS($p, q$) architecture for networks of degree $d = 4$ and diameter $D = 5$ with $n \geq 400$.

Another interesting issue is to consider combining several OTIS and other optical devices, in order to construct a wide variety of networks, as shown in [4] for the POPS and the stack-Kautz networks.
Table 1: Number of nodes, \( n \), and size of the \( OTIS(p, q) \) architecture for networks of degree \( d = 4 \), diameter \( D = 5 \) and \( n \geq 400 \) nodes. All of these networks are isomorphic to Imase and Itoh’s digraphs except the ones which are indicated.

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References


