Topologies for Optical Interconnection Networks Based on the Optical Transpose Interconnection System

David Coudert, Afonso Ferreira, Xavier Munoz

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David Coudert
Project SLOOP
I3S-INRIA
BP 93
F-06902 Sophia-Antipolis
France

Afonso Ferreira
Project SLOOP
CNRS-I3S-INRIA
BP 93
F-06902 Sophia-Antipolis
France

Xavier Muñoz
DMAT
UPC
08034 Barcelona
Spain

Abstract

Many results exist in the literature describing technological and theoretical advances in optical network topologies and design. However, an essential effort has yet to be done in linking those results together. In this paper, we propose a step in this direction, by giving optical layouts for several graph-theoretical topologies studied in the literature, using the Optical Transpose Interconnection System (OTIS) architecture. These topologies include the family of Partitioned Optical Passive Star (POPS) and stack-Kautz networks as well as a generalization of the Kautz and de Bruijn digraphs.

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1 Introduction

Many results exist in the literature describing technological and theoretical advances in optical network topologies and design. However, an essential effort has yet to be done in linking those results together. In this paper, we propose a step in this direction, by giving the optical layout of several graph-theoretical topologies studied in the literature, using existing optical technologies.

Our motivation stems from the fact that it has been shown that the break-even line length where optical communication lines become more effective than their electrical counterparts is less than 1cm, in terms of speed and power consumption [1]. Therefore, the use of optical interconnections on-board is nowadays justified, and some studies even suggest that on-chip optical interconnects will be soon cost-effective [2]. Moreover the emergence of cutting-edge technologies as Vertical Cavity Surface-Emitting Lasers (VCSELs) [3, 4], high sensibility optical transimpedance receivers [5], beam splitters (based on diffractive optical elements which can split the optical beam of a VCSEL in 16 beams with a 16th of the incoming power each, without signal amplification) [6, 7], micro-lenses, and holograms, makes possible the fabrication of complex optical communication networks.

An interesting optical passive interconnection system is the Optical Transpose Interconnection System (OTIS) architecture, from the Optoelectronic Computing Group of UCSD

[8, 9]. The OTIS architecture is a simple scalable means of implementing very dense interconnections. It provides a transpose interconnection utilizing only a pair of lenslet arrays and making use of free-space optical interconnections. It has been shown in [10] that the OTIS architecture can be used to build 4-D meshes, hypercubes and complete networks. Also many algorithms on OTIS-Mesh were studied in [11–13]. The OTIS architecture is used by the 3-D OptoElectronic Stacked Processors (3-D OESP) consortium to connect a 16-by-16 switch (For more details, see http://soliton.ucsd.edu/3doesp/public/).

From a logical topology viewpoint, our work focuses on topologies allowing one-to-many communications in a single communication step. The building block is the Optical Passive Star (OPS) coupler [14] which can be used as a *single-OPS broadcast-and-select* network in which an incoming optical signal is broadcast to all output ports [15]. Although a great deal of research efforts have been concentrated on single-OPS topologies [16–19], the OPS coupler presents a severe drawback since its size is technologically limited by its splitting capabilities. That is the reason why we concentrate our study on *multi-OPS* topologies which seem more viable and scalable.

One such multi-OPS network can be built using the stack-Kautz topology, modeled and described in [20]. The stack-Kautz is based on the Kautz digraph (i.e. directed graph) [21] and on stack-graphs (a useful model to manipulate multi-OPS networks) [15]. The Kautz digraphs have a large number of nodes, $n = d^D + d^{D-1}$, for a given constant degree d and a low diameter D. As an example, $KG(5, 3)$ has $N = 150$ nodes, degree 5 and diameter 3. Figure 6 shows three Kautz digraphs: $KG(2, 1) = K_3^+$, $KG(2, 2)$ and $KG(2, 3)$. It is both Eulerian and Hamiltonian and optimal with respect to the number of nodes if $d > 2$ [21]. Notice that routing on the Kautz digraph is very simple, since a shortest path routing algorithm is induced by the label of the nodes. The mean eccentricity (i.e. average distance between each pair of nodes) of Kautz networks, for various routing protocols, was compared to different network topologies, namely Shufflenet, GEMNET and de Bruijn, in [22]. This study showed that Kautz networks are very attractive and more efficient than the others in case it would be chosen as the logical topology of a communication network. Moreover, Kautz digraphs were used as the logical topology of the Rattlesnake ATM switch presented in [23, 24], which is a cost effective switching system designed to build a local area ATM network that supports multimedia applications. The Kautz digraphs were chosen because of their simple routing mechanism, efficient broadcast and multicast protocols [25], and the possibility to generate node-disjoint routes, which makes it node (or link) fault tolerant [26, 27].

The stack-Kautz network is obtained from a Kautz digraph by replacing each node by a group of s nodes (where s is called *stacking-factor*) and each arc by an OPS coupler connecting two groups of s nodes. Hence, this network is regular with the small constant degree and the low diameter of the underlying Kautz digraph and has a large number of nodes, equal to sn, where n is the number of nodes of the Kautz digraph and s is the size of the groups replacing the nodes. The stack-Kautz network has also a simple routing protocol and an efficient and optimal broadcast algorithm [20]. We remark that a stack-Kautz of stacking-factor $s = 1$ is the original Kautz digraph and that the Kautz digraph of diameter 1 is the complete digraph. Furthermore, the singlehop multi-OPS Partitioned Optical Passive Star (POPS) network, introduced in [28], was modeled in [29] as a stack-graph built from a complete digraph, implying that a POPS network is in fact a stack-Kautz network of diameter 1.

In this paper, we give the design of the family of POPS and stack-Kautz networks, using the OTIS architecture and OPS couplers. Our constructions are based on Proposition 3.2.1 which states a more general result, showing that the OTIS architecture allows to implement every Imase and Itoh digraph, a broader family of digraphs, which contains both Kautz digraphs and de Bruijn digraphs [30–32].

This paper is organized as follows. We start by recalling, in Section 2, the results from the literature upon which we construct ours. In particular, we present the Optical Transpose Interconnecting System (OTIS) architecture from [8], the Optical Passive Star, and the Partitioned Optical Passive Star (POPS) network from [28]. We also recall the stack-graphs from [15], the definition of the Kautz graph from [21], generalized with the graph of Imase and Itoh from [31], and close this section by presenting the multihop multi-OPS stack-Kautz network from [20]. In Section 3, we propose the implementation of some building blocks using the OTIS architecture: first, the optical interconnections between a group of nodes and its corresponding OPS couplers, and then an optical interconnection network having the topology of a graph of Imase and Itoh. Finally, Section 4 presents the application of these building blocks to the optical design of the families of POPS networks and stack-Kautz networks. We close the paper with some discussion on system implementation.

2 Preliminaries

In this section, we recall the basic features of the OTIS architecture and OPS couplers. We also show the model proposed in [15] for studying multi-OPS networks with stack-graphs.

Then, we exemplify the use of this model on the POPS network. Finally, we recall the definition of the Kautz digraph, the Imase and Itoh's digraph and the stack-Kautz.

2.1 OTIS

The **Optical Transpose Interconnection System** (OTIS) architecture, was first proposed in [8]. $OTIS(p, q)$ is an optical system which allows point-to-point (1-to-1) communications from p groups of q transmitters onto q groups of p receivers. This architecture connects the transmitter (i, j), $0 \leq i \leq p-1$, $0 \leq j \leq q-1$, to the receiver $(q-j-1, p-i-1)$.

Optical interconnections in the OTIS architecture are realized with a pair of lenslet arrays [9] in a free space of optical interconnection as shown in Figure 1.

The OTIS architecture was used in [10] to realize interconnection networks such as hypercubes, 4-D meshes, mesh-of-trees and butterfly for multi-processors systems. It was shown that, in such electronic interconnection networks (in which connections are realized with electronic wires), a set of wires can be replaced with pairs of transmitters and receivers connected using the OTIS architecture. This is interesting in terms of speed, power consumption [1] and space reduction.

2.2 Optical passive stars

An **optical passive star coupler** is a singlehop one-to-many optical transmission device. An OPS(s, z) has s inputs and z outputs. In the case where s equals z, the OPS is said to be of degree s (see Figure 2, left). When one of the input nodes sends a message through an OPS coupler, the s output nodes have access to it. An OPS coupler is a **passive** optical system, i.e. it requires no external power source. It is composed of an optical multiplexer followed by an optical fiber or a free optical space and a beam-splitter [7] that divides the incoming light signal into s equal signals of a s-th of the incoming optical power. The interested reader can find in [33] a practical realization of an OPS coupler using a hologram [6] at the outputs. Another realization using optical fibers is described in [34]. Throughout this paper, we will deal with **single-wavelength** OPS couplers of degree s, implying that only one optical beam has to be guided through the network (see Figure 2, right). Consequently, only one node can send an optical signal through it per time step.

2.3 Stack-graphs

Hypergraphs are a generalization of graphs where edges may connect more than two nodes [35–37]. Let us define a special kind of directed hypergraphs, called *stack-graphs*, which have the property of being very easy to deal with. Informally, they can be obtained by piling up s copies of a digraph G and subsequently viewing each stack of s nodes as a hypergraph node and each stack of s arcs as a hyperarc. The value s is called the *stacking-factor* of the corresponding stack-graph. Let $\varsigma(s, G)$ denote the stack-graph of stacking-factor s, obtained from the digraph G. A formal definition can be found in [15].

Therefore, an OPS coupler of degree s can be modeled by a hyperarc linking two hypergraph nodes composed of s nodes each, meaning that the processors of one set (the OPS sources) can only send messages through the hyperarc while the other set (the OPS destinations) can only receive messages through the same hyperarc. Figure 3 shows an OPS coupler modeled by a hyperarc.

2.4 POPS

The **Partitioned Optical Passive Star network** $POPS(t, g)$, introduced in [28], is composed of $N = tg$ nodes and g^2 OPS couplers of degree t. The nodes are divided into g groups of size t (see Figure 4). Each OPS coupler is labeled by a pair of integers (i, j), $0 \leq i, j < g$. The input of the OPS (i, j) is connected to the i-th group of nodes, and the output to the j-th group of nodes. The POPS is a singlehop multi-OPS network.

Since an OPS coupler is modeled as a hyperarc, the POPS network $POPS(t, g)$ can be modeled as a stack-K_g^* (or $\varsigma(t, K_g^*)$, for short) of stacking-factor t, where K_g^* is the complete digraph with loops (a loop is an arc from a node to itself) with g nodes and g^2 arcs (see Figure 5), as proposed in [29].

2.5 Kautz digraphs and Imase and Itoh digraphs

The Kautz digraph was first defined in [21] as follows.

Definition 2.5.1 *[21] The* **Kautz digraph** $KG(d, k)$ *of degree d and diameter k is the digraph defined as follows (see Figure 6).*

1. *A vertex is labeled with a word of length k, $(x_1, \cdots, x_k)$, on the alphabet $\Sigma = \{0, \cdots, d\}$, $|\Sigma| = d + 1$, in which $x_i \neq x_{i+1}$, for $1 \leq i \leq k - 1$.*

2. *There is an arc from a vertex $x = (x_1, \cdots, x_k)$ to all vertices y such that $y = (x_2, \cdots, x_k, z)$, $z \in \Sigma$, $z \neq x_k$.*

Two properties are worth mentioning here: the Kautz digraph $KG(d, k)$ has $N = d^{k-1}(d+1)$ nodes, constant degree d and diameter k ($k = \lfloor \log_d N \rfloor$). It is shown in [38] that $KG(d, 1)$

is the complete digraph without loops K^+_{d+1}.

Although very interesting in terms of number of nodes for fixed degrees and diameters, Kautz digraphs do not yield digraphs of any size. Digraphs of Imase and Itoh have thus been introduced as a Kautz digraph generalization in order to obtain digraphs of every size [31] .

Imase and Itoh's digraphs are defined using a congruence relation, as follows.

Definition 2.5.2 *[31] The* **digraph of Imase and Itoh** $II(d, n)$*, of degree* d *with* n *nodes, is the directed graph in which:*

1. *Nodes are integers modulo* n.
2. *There is an arc from the node* u *to all nodes* v *such that* $v \equiv (-du - \alpha) \mod n$, $1 \leq \alpha \leq d$.

Figure 7 shows an example of Imase and Itoh's digraph with $II(3, 12)$. It has $n = 12$ nodes, degree 3 and diameter $\lfloor \log_3 12 \rfloor = 2$, vertex set $V = \{0, 1, \ldots, 11\}$ and arc set $\{(u, v) \mid u \in V,\ v \equiv (-3u - \alpha) \mod 12,\ \alpha = 1, 2, 3\}$ (e.g., arcs from 5 to 6,7,8). The figure is drawn in such a way that the left nodes and the right nodes represent the same nodes, and hence, arcs should be understood outgoing from the left set and incoming to the right one. Furthermore, we note that a digraph $II(3, 12)$ is also a Kautz digraph $KG(3, 2)$ [32]. Therefore, in Figure 7 we also give the labels that the nodes would have in $KG(3, 2)$. In such an example, for a given node u of $II(3, 12)$, its label (x_1, x_2) in $KG(3, 2)$ can be computed as $x_1 = 3 - u/3$ and $x_2 = u \mod 4$ (e.g. $u = 7$, $(x_1, x_2) = (1, 3)$). For more information on this subject, we refer the interested reader to [39, p. 69].

2.6 Stack-Kautz

We now dispose of a good model for multi-OPS networks (the stack-graphs) and also of a digraph having good properties to build a multihop network (the Kautz digraph). Hence, we can recall the multihop multi-OPS architecture based on the *stack-Kautz*.

In order to define the optical interconnection network called stack-Kautz, we use the Kautz digraph with loops $KG^*(d,k)$, where every node has a loop and hence degree $d+1$. Thus, we can define the stack-Kautz graph as follows.

Definition 2.6.1 *The* **stack-Kautz graph** $SK(s,d,k)$ *is the stack-graph* $\varsigma(s, KG^*(d,k))$ *of stacking-factor* s*, degree* $d+1$ *and diameter* k *(see Figure 8).*

The *stack-Kautz* network has the topology of the stack-Kautz graph $SK(s,d,k)$ and $N = sd^{k-1}(d+1)$ nodes. Each node is labeled by a pair (x,y) where x is the label of the stack in $KG(d,k)$ and y is an integer $0 \leq y < s$, i.e., x is the label of a node group and y is the label of a node in this group. Since the stack-Kautz network inherits most of the properties of the Kautz digraph, like shortest path routing, fault tolerance and others, we showed in a previous work that it is a good candidate for the topology of an OPS-based lightwave network [20].

We note that the definition of stack-Kautz network can be trivially extended to the stack-Imase-Itoh network.

3 Implementing some building blocks with OTIS

In this section, we explain how the OTIS architecture can be used both to connect a group of nodes to the inputs of OPS couplers, and to connect the outputs of OPS couplers to a group of nodes, and we show that the optical interconnections of an Imase and Itoh's based network can be simply realized using the OTIS architecture.

3.1 Creating groups of nodes

We can realize the optical interconnections between the t nodes of a group and the inputs of g OPS couplers (each node being connected to the g OPS couplers), in a simple way using one $OTIS(t, g)$ architecture plus g optical multiplexers (input part of an OPS coupler). Figure 9 shows how to connect the transmitters of a group of 6 nodes to 4 optical multiplexers, using $OTIS(6, 4)$.

Analogously, we can also realize the optical interconnections between the outputs of g OPS couplers and the t nodes of a group, using one $OTIS(g, t)$ architecture plus g beam-splitters. Figure 10 shows how to connect 3 beam-splitters to the receivers of a group of 5 nodes, using $OTIS(3, 5)$.

To build specific topologies, the output optical multiplexers are to be connected to its corresponding input beam-splitters, through the topology of the Optical Interconnection Network.

3.2 Optical implementation of the digraph of Imase and Itoh

The main result of this paper, proved below, shows how the $OTIS(d,n)$ architecture can be used to optically realize the interconnections of the Imase and Itoh digraph $II(d,n)$, of n nodes of degree d each (see Definition 2.5.2). In order to prove this result, we first consider each node as a group of d transmitters plus d receivers, thus taking the degree of $II(d,n)$ into account. Then, we show two functions which associate the inputs and outputs of $OTIS(d,n)$ with the nodes of $II(d,n)$. Finally, the last step is to prove that the neighbourhood obtained with $OTIS(d,n)$ is the same neighbourhood of $II(d,n)$. Formal arguments are developed in the following.

Proposition 3.2.1 *The optical interconnections of the digraph of Imase and Itoh $II(d,n)$ of degree d with n nodes can be perfectly realized with the OTIS architecture $OTIS(d,n)$.*

Proof: The OTIS architecture $OTIS(d,n)$ has dn inputs (d groups of size n) and dn outputs (n groups of size d). The inputs $e = (i,j)$, $0 \leq i < d$, $0 \leq j < n$, are connected to the outputs $s = (n-j-1, d-i-1)$.

The digraph of Imase and Itoh $II(d,n)$ of degree d with n nodes connect a node u to the nodes $v \equiv (-du - \alpha) \mod n$, $1 \leq \alpha \leq d$.

We associate d inputs of $OTIS(d,n)$ to each node of $II(d,n)$, such that the input $e = (i,j)$ is associated to the node $u = \lfloor (ni+j)/d \rfloor$. In other words, a node u of $II(d,n)$ is associated to inputs $e_{du+\alpha-1} = (\lfloor (du+\alpha-1)/n \rfloor, du+\alpha-1-\lfloor (du+\alpha-1)/n \rfloor n)$, $1 \leq \alpha \leq d$.

We also associate d outputs of $OTIS(d,n)$ to each node v of $II(d,n)$, an output $s = (n-j-1, d-i-1)$ being associated to node $v = n-j-1$, and a node v being associated to outputs $s = (v, d-\alpha)$, $1 \leq \alpha \leq d$.

Due to the OTIS architecture, a node u of $II(d,n)$ is then connected to nodes

$$\begin{aligned} v_\alpha &= n - \left(du + \alpha - 1 - \left\lfloor \tfrac{du+\alpha-1}{n} \right\rfloor n\right) - 1,\ 1 \le \alpha \le d, \\ v_\alpha &= n\left(1 + \left\lfloor \tfrac{du+\alpha-1}{n} \right\rfloor\right) - du - \alpha,\ 1 \le \alpha \le d, \\ v_\alpha &\equiv (-du - \alpha) \mod n,\ 1 \le \alpha \le d. \end{aligned}$$

Therefore, the neighbourhood of u, obtained by using the OTIS architecture, is exactly the neighbourhood of u in the digraph of Imase and Itoh, proving the proposition. □

To illustrate this proposition, Figure 11 shows how the optical interconnections of the digraph of Imase and Itoh are realized with the OTIS architecture. Notice that each node of $II(3,12)$ is represented as a group of 3 consecutive transmitters (in the left side) and a group of 3 consecutive receivers (in the right side) in $OTIS(3,12)$. This stems from the fact that $II(3,12)$ has degree 3.

Corollary 3.2.1 *The Kautz digraph $KG(d,k)$ is the digraph of Imase and Itoh $II(d,d^{k-1}(d+1))$. Hence, we can realize the optical interconnections of $KG(d,k)$ using $OTIS(d,d^{k-1}(d+1))$.*

Corollary 3.2.2 *The de Bruijn $B(d,D)$ is the digraph of Imase and Itoh $II(d,d^D)$. Hence, we can realize the optical interconnections of $B(d,D)$ using $OTIS(d,d^D)$.*

4 Implementing multi-OPS networks with OTIS

We saw in the previous section that the OTIS architecture can be used to realize the optical interconnections of the digraph of Imase and Itoh and consequently of the Kautz digraph. It was also shown how to connect the nodes of a group with its corresponding OPS couplers.

In this section, we show how to build the stack-Kautz and the POPS networks using these building blocks.

4.1 Stack-Kautz with OTIS

The stack-Kautz network $SK(s,d,k)$ has $d^{k-1}(d+1)$ groups of s nodes and $d^{k-1}(d+1)^2$ OPS couplers of degree s. Each node has degree $d+1$.

The groups: We have explained how to connect a group of nodes to its corresponding OPS couplers. Hence, using $d^{k-1}(d+1)$ $OTIS(s,d+1)$ plus $d^{k-1}(d+1)$ $OTIS(d+1,s)$ architectures, we can connect all the groups of s nodes to their corresponding $d+1$ optical multiplexers and $d+1$ beam-splitters.

The Optical Interconnection Network: Now, we have to realize the optical interconnection network which connects the optical multiplexers to their corresponding beam-splitters. The stack-Kautz network $SK(s,d,k)$ is based on $KG^*(d,k)$ which is a Kautz digraph with loops. By Corollary 3.2.1, the optical interconnections of a Kautz network $KG(d,k)$ can be realized using one $OTIS(d,d^{k-1}(d+1))$. It remains to be shown how to optically interconnect the loops.

The loops: A loop is a local link for a group of nodes. Hence, the optical interconnection between the optical multiplexer and the beam-splitter corresponding to a loop, can be locally realized using an appropriate technique (e.g., optical fiber).

Synthesis: As it will be shown in the example below, the optical interconnections of the stack-Kautz network $SK(s,d,k)$ are realized using $d^{k-1}(d+1)$ $OTIS(s,d+1)$ plus $d^{k-1}(d+1)$ $OTIS(d+1,s)$, $d^{k-1}(d+1)^2$ optical multiplexers plus $d^{k-1}(d+1)^2$ beam-splitters and one $OTIS(d,d^{k-1}(d+1))$.

The groups are connected to the optical multiplexers using $OTIS(s,d+1)$ and to the beam-splitters using $OTIS(d+1,s)$. The first optical multiplexer and the first beam-splitter of a group correspond to its loop. Hence, for each group of nodes, the remaining d optical multiplexers are connected to their corresponding beam-splitters through the $OTIS(d,d^{k-1}(d+1))$ architecture, the first group being connected to the d first inputs and to the d first outputs of $OTIS(d,d^{k-1}(d+1))$, the second group to the d next inputs and outputs and so on.

An example: Figure 12 shows how those interconnections are realized for $SK(6,3,2)$ (modeled in Figure 8) using 12 $OTIS(6,4)$, 12 $OTIS(4,6)$, 48 optical multiplexers, 48 beam-splitters and one $OTIS(3,12)$. $SK(6,3,2)$ has 72 nodes (12 groups of 6 nodes) of degree 4, connected in a network of diameter 2.

4.2 POPS with OTIS

$POPS(t,g)$ has been modeled as a stack-K_g^* of stacking factor t and, due to the definition of the Kautz digraph, we know that $KG(g-1,1)$ is isomorphic to K_g^+ and that $KG^*(g-1,1)$ is isomorphic to K_g^*. Hence, the POPS network $POPS(t,g)$ is isomorphic to the stack-Kautz network $SK(t,g-1,1)$. Thus, we can realize the optical interconnections of $POPS(t,g)$ using g $OTIS(t,g)$ plus g $OTIS(g,t)$, g^2 optical multiplexers and beam-splitters and one

$OTIS(g-1,g)$. This implementation does not include the loops.

On the other hand, $OTIS(g,g)$ realizes the optical interconnections of K_g^*, including the loops. Thus, we can also realize the optical interconnections of $POPS(t,g)$ using g $OTIS(t,g)$ plus g $OTIS(g,t)$, g^2 optical multiplexers and beam-splitters and one $OTIS(g,g)$, as shown in Figure 13. Indeed, optical interconnections of $SK(s,d,1)$ can be better realized using $OTIS(d+1,d+1)$ which includes the loops.

5 Discussion

Clearly, the implementation of such system is limited by technological constraints stemming from several factors, like components design (beam-splitters, power budget, ...), packaging, alignment, etc. In the case of the Stack-Kautz, if the size of the beam-splitters – and consequently, the stacking factor – is limited around 16, then the power budget remains reasonable [7]. Another constraint is the number of transceivers a processor can hold. It suffices to keep the degree smaller than the stacking factor to overcome this obstacle, since technology allows for processors to hold 8×8 VCSEL matrices [40]. Finally, the connection of the multiplexers' outputs to the input of the central OTIS, as well as of its output to the inputs of the beam-splitters, can be done with a precise alignment if fiber optics are used. On the other hand, the alignment of the micro-lenses matrices forming the different OTIS used in the proposed networks seems delicate to implement and could be the subject of further specific studies. In this way, we can reasonably think that near-future technology will allow for the construction of stack-Kautz networks with up to 69.632 processors and diameter 3 (i.e., $s=d=16$).

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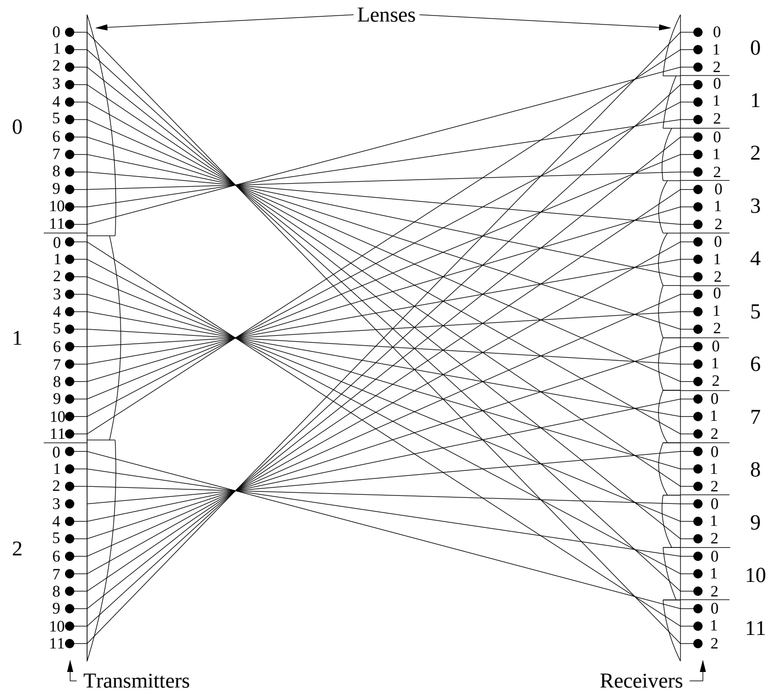
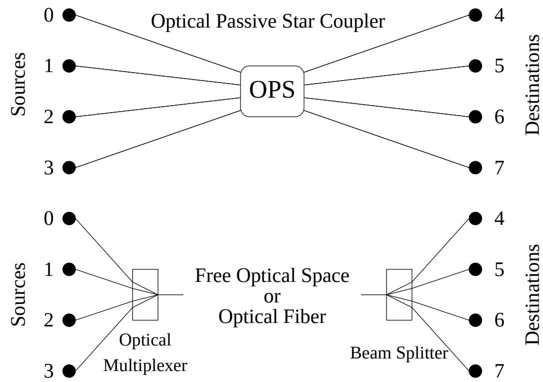


Figure 1: $OTIS(3, 12)$.



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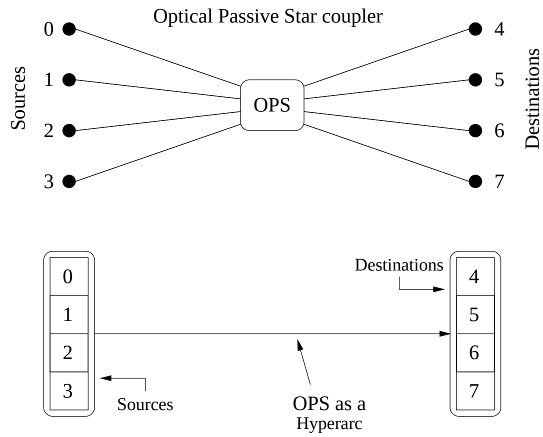

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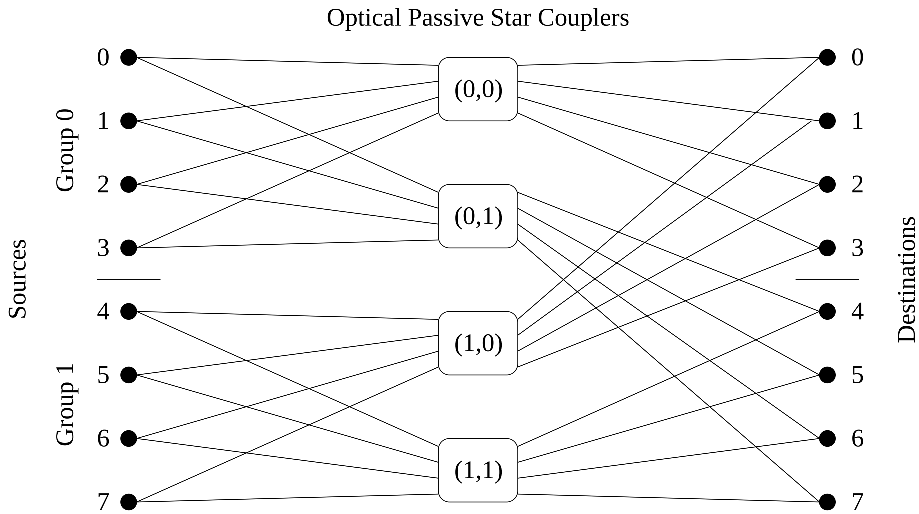

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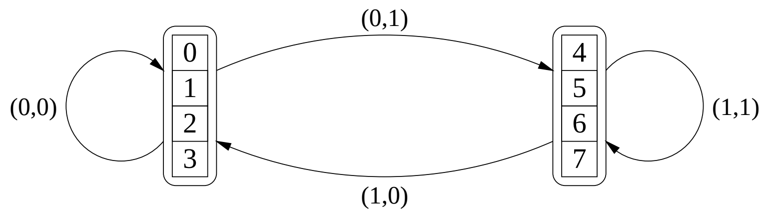

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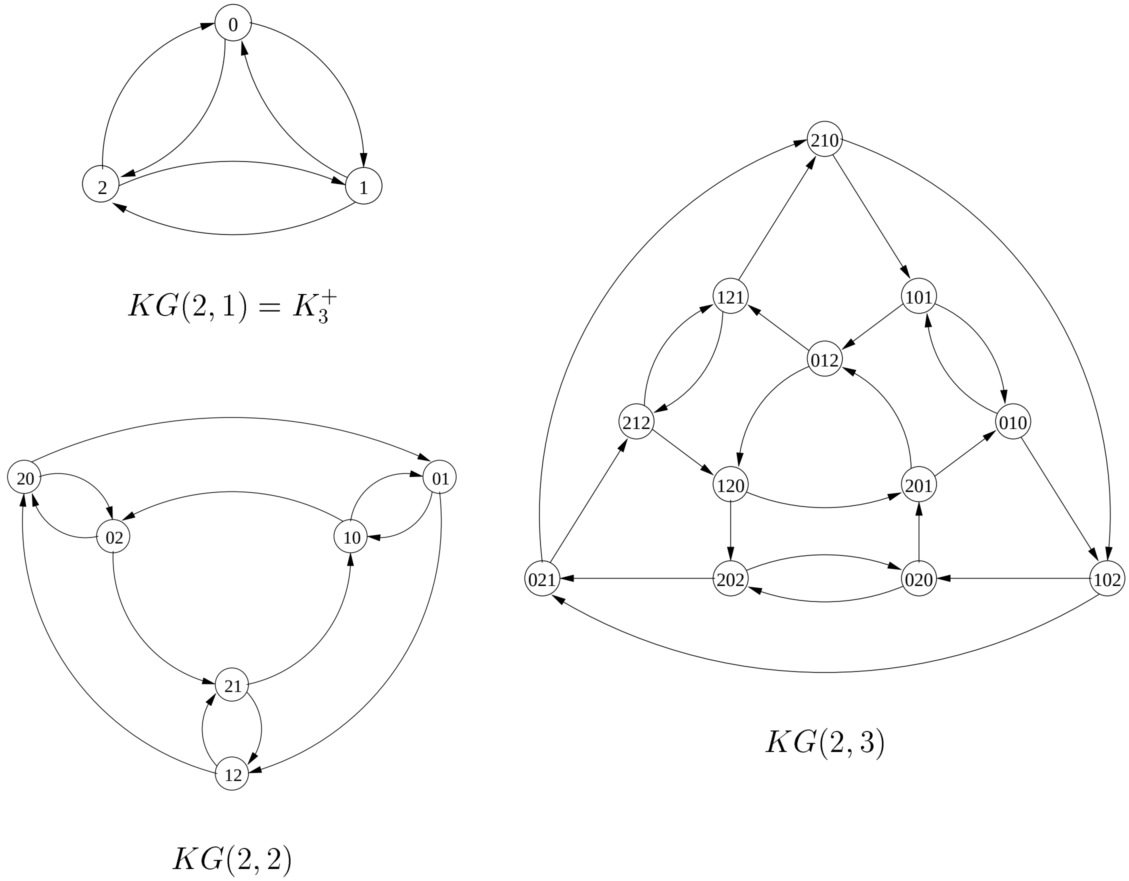

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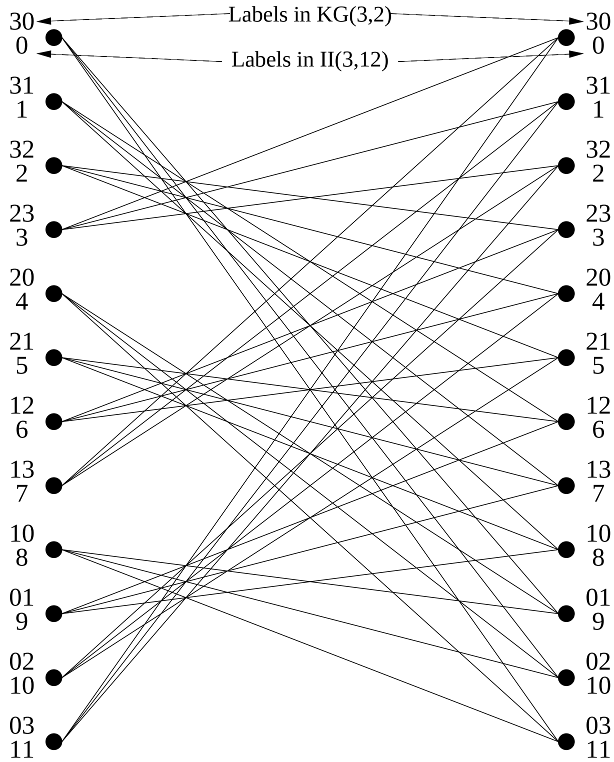

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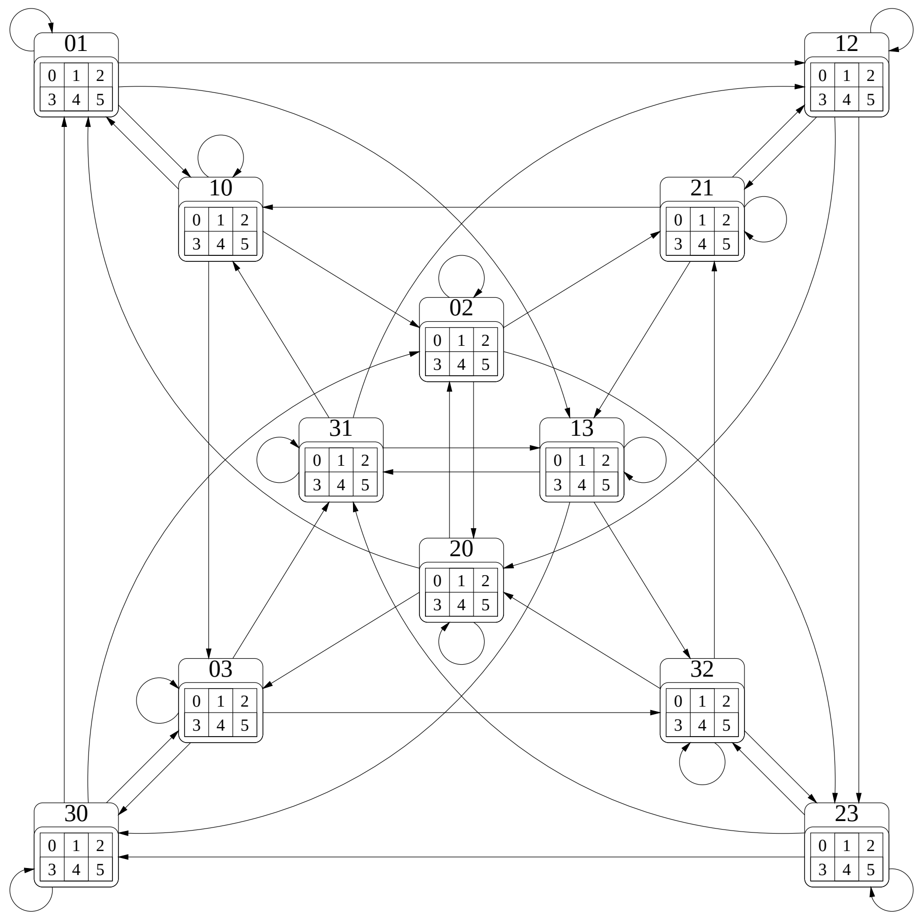

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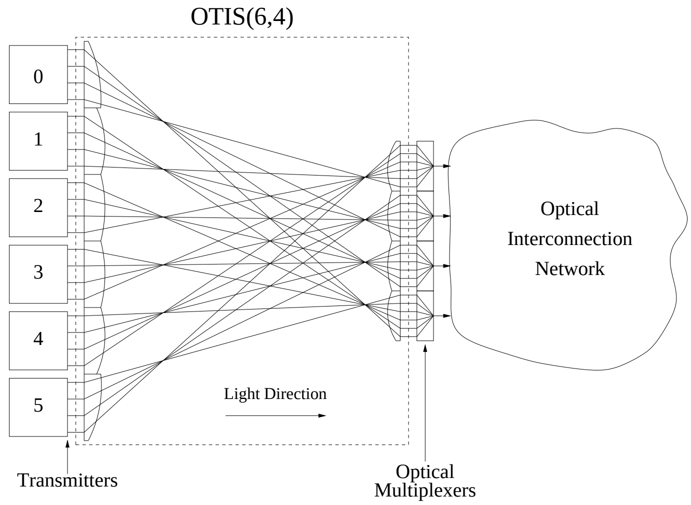

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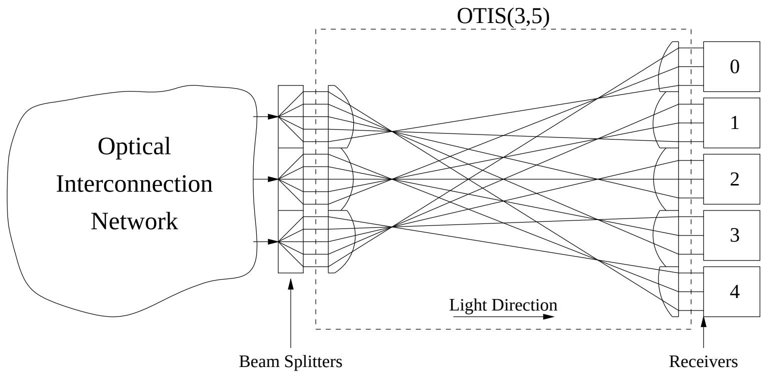

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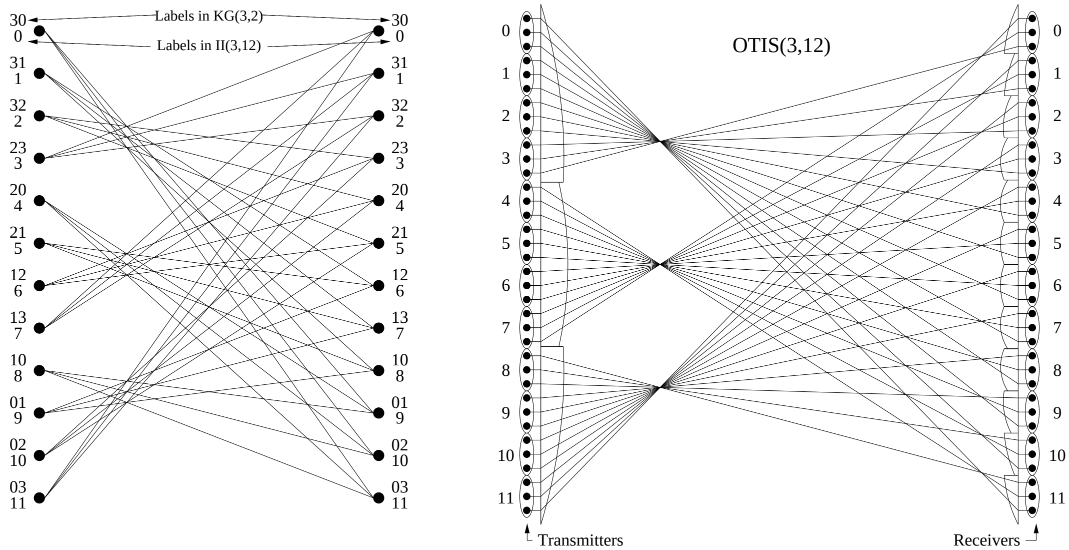

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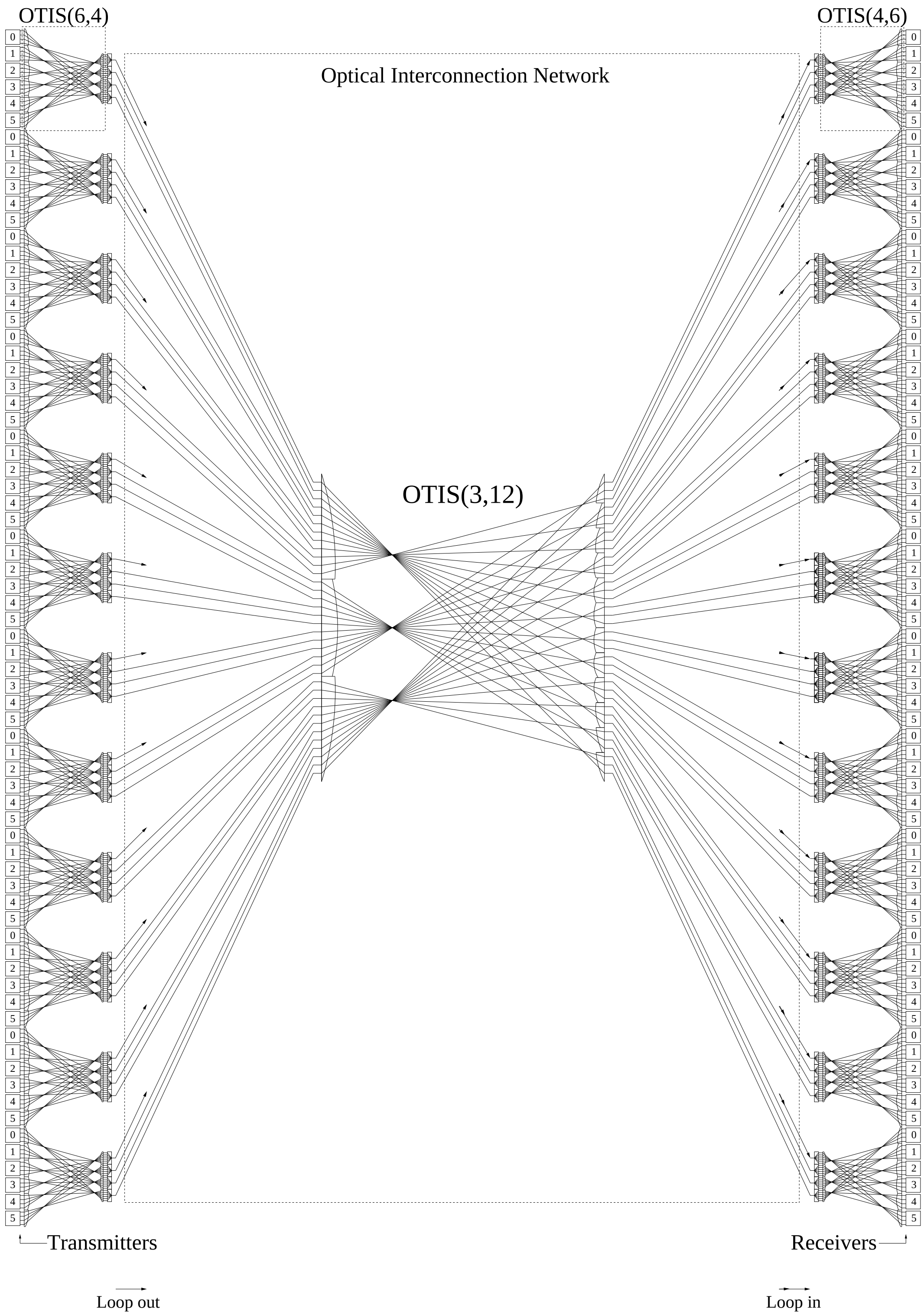

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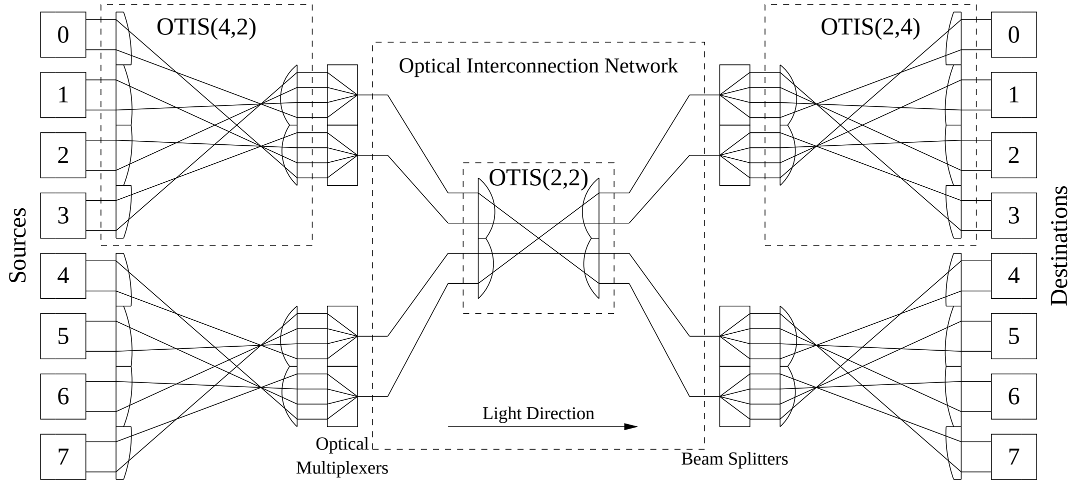

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