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Walking analysis of young and elderly people by using an intelligent walker ANG

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Abstract

This paper proposes a new method to analyze human walking by using a 3-wheels rollator walker instrumented with encoders and a 3D accelerometer/gyrometer. In order to develop walking quality index and monitor the health state of elderly people at home, the walking of 23 young adults and 25 elderly people (> 69 years) with the help of the walker, are compared. The results show that many general walking indicators such as walking speed, stride length do not present obvious difference between the two groups, but that new indicators obtained by using the walker measurements and not

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available otherwise are very discriminating, e.g., the lateral motion of elderly people is larger, their walking accuracy is lower, but their effort distributed on the handles are more symmetrical. We also show that this walker may have other purposes such as updating collaborative maps with sideway slopes and location of lowered kerbs.

1. Introduction

The elderly population is growing fast all over the world. Population ageing will cause significant challenges of care giving. One of the problems that affect the most of the elderly population is the reduction of mobility. Therefore, personal assistance mobility devices are strongly desired to keep the elderly independent. Among the possible assistance devices, the walkers have large number of users because of its simplicity while using the person's remaining locomotion capability in order to move. Besides of the physical benefits of maintaining the standing position, there are also other important psychological benefits, such as maintaining self-esteem and social relationship.

There are many studies and projects regarding advanced versions of walkers. According to the user's needs, the functions of the proposed walkers are not restricted to their primary task, i.e. physical support and mobility assistance. There are other functions such as sensorial assistance, cognitive assistance and health monitoring [1]. These walkers focus on mobility assistance [2], [3], [4], sit-to-stand transfer [5], [6], navigation help [7], [8], obstacle

avoidance and fall detection [9]. Besides of these, there are other multifunctional walkers such as PAMM SmartWalker [10], which was designed to offer extra support for walking, guidance, scheduling (reminding the time of medicines, for an example) and health monitoring for elderly users. The Medical Automation Research Center (MARC) smart walker [11], which was installed a pair of tridimensional force/torque sensors on it's handles, can be used to determine gait characteristics such as the heel strike, toe-off, double support, and single support [12], [13].

To study the extension of the functions of walkers we have developed our own family of walking aids, the ANG family [14], i.e., ANG-light and ANG-2. It has been tested that the walker ANG-2 can perform multifunctions such as navigation, street mapping, fall detection/prevention and autonomous object recovery [14]. Our walker is also a communication device that can receive and emit information (e.g., fall detection for emission, change of user profile for reception). In addition, the walker is able to download and execute new services according to the end-user's trajectory of life (e.g., specific rehabilitation program). It is therefore much more flexible in its application than other walkers. In this work we use the simplest version, ANG-light (Fig. 1) which is based on a commercially available 3-wheels Rollator. It has been instrumented with encoders at the two fixed rear wheels and a 3D accelerometer/gyrometer at the front with the purpose of determining the walker's trajectory on a 24/24 basis. A small, low energy consumption fit-pc computer manages the measurements and records all the data. Additionally the walker has GPS, GSM and infrared network connection that are not used in this paper. Compared with the walkers proposed above, our walker is low

cost, simple to be used at home and its functions can be easily extended. This paper will present how it can be used for medical monitoring of walking patterns and what kind of medical information may be obtained.

Figure 1: The walking aid ANG-light

Many studies have examined the effect of age on the walking by comparing younger with older adults [15], [16], [17], [18], [19], [20], [21], [22]. These studies calculated some gait parameters, such as step length, gait cycle, step width, cadence and gait speed [22], [23], [24], [25]. Usually gait speed or walking velocity is regarded as a very important indicator of health. Some studies claimed that older subjects exhibited significantly reduced gait speed compared to younger adults [15], [16], [22]. Other studies showed that there were little or no differences in the gait speed between the healthy younger and elderly people [20], [19]. In fact, [25] has showed that the measured gait

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speed is based on age, sex [23], use of mobility aids, chronic conditions, smoking history, blood pressure, body mass index, and hospitalization. Therefore, the classical gait parameters are not sufficient to monitor the user's health. Some studies have considered also gait variability [18], [26], [19], [27]. The variability of gait parameters can be characterized by the coefficient of variation (CV) of kinematic gait parameters [28], [15], which is an index of gait stability and complexity. The increased variability of gait parameters corresponds to decreased gait stability, complexity and increased risk of falling. However, gait instability is multifactorial and the results of previous studies are often inconsistent according to the conditions of experiment. Therefore, we need to do more tests and find more pertinent indicators of walking. As proposed in [29], at least the following components of a person's gait should be considered: initiation of gait, step length, height, and symmetry, step continuity, step path, trunk motion, walking stance, turning, and heel-to-toe walking. Presently, although some studies began to analyze other gait characteristics such as medial-lateral displacement, center of mass [30] and foot placement [21], [31], they are still not sufficient to describe comprehensively walking motion.

This paper proposes a new method of walking analysis by using our instrumented smart walker. A 10 meters straight line walking test has been performed by two groups of younger and elderly people. Thanks to encoders and a 3D accelerometer/gyrometer, we can calculate accurately the classical gait parameters such as gait cadence, walking speed and stride cycle, stride length and their variability without the subjectivity of an human examination. We also can obtain the trajectory of the walker and therefore

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compare it with the reference trajectory. This comparison will allow to establish several original gait parameters which have not been considered in previous studies and the walking accuracy of two groups of people can be compared. We define a mobile frame attached to the walker with X_1 being the forward direction and Y_1 being the lateral direction (see Fig. 3). Because of the elasticity of the wheels, when leaning forward to push the walker there will be a clock-wise rotation around the Y_1 axis and when leaning on the right (left) handle a rotation around the X_1 axis should be observed. Therefore, by using the measured angular velocity of the walker, the proportion of forward/backward support force and left/right support force on the handles during the whole walking can be estimated. Overall, using our walker the gait characteristics can be described more comprehensively. Secondly, a drawback of most studies is that these measures are obtained on a reduced space with specialized laboratory equipment such as motion capture systems and instrumented walkways, which may not be available in many clinics and certainly not during daily activities. In contrast, our walker can be easily used at home and outdoors, so it is possible to develop it for individual medical monitoring of walking patterns at any time of the day and in any context. Moreover, motion capture systems don't have large workspace and are not very accurate. Conversely, we have measured the error on the final pose of the walker after a 10 meters walk: the positioning error was less than 1 cm and 0.1 $degree$ in orientation, which is much better than a motion capture system on such large workspace. Besides of this, a very small abnormal limb motion during one step, which characterizes an emerging pathology, cannot be detected by motion capture but can be detected on the walking trajectory.

2. Description of experiments

Physical functioning tests have showed significant aged-related differences for older adults [32]. Several classical tests used to assess the mobility of elderly people are the $10m$ walk test (`10mWT`) (measure: time duration) [33], Timed Up and Go test (`TUG`) (measure: time duration) [34], Tinetti Test (`TT`) (analysis of gait parameters through a video) [35]. Such tests are easy to implement but are basically global (the time for the `10mWT` and the `TUG` may be identical for two subjects which have however very different walking patterns) or are subjective (for the `TT` [36]). Furthermore these tests are performed only during medical visits and consequently are not appropriate to detect abnormal events in the walking patterns. Hence we have decided to examine if the measurements of our walking aid allow one to refine the output of the above walking tests.

For that purpose we have led a large scale experiment that was approved by the regional ethical committee (Comité de Protection des Personnes). In this paper only the results of a `10mWT` will be presented. Each subject was asked to walk along a $10m$ straight line trajectory with the help of the walker. The experiment takes place at INRIA and at Nice hospital. The subjects were 23 INRIA members (with age between 25 and 65 years, mean value 32) and 25 elderly people (age over 65 years) recruited by Nice hospital (see Fig. 2). No subject has pathological walking deceases. All the subjects were asked to perform twice the trajectory with the walking aid.

Figure 2: The walking aid ANG-light at Nice Hospital

3. Methods

This section will explain how the walker can obtain the walking trajectory and determine the stride. During all the measurements, the calculation of walking trajectory and the detection of stride are the two most important issues as the measurement of all the gait parameters and their variability are based on the detection of stride.

3.1. Calculation of the trajectory

As shown in Fig. 3, the origin of the walker frame $O_1X_1Y_1$ is supposed to be the position of the middle point between two rear wheels. The position of the walker in a reference frame OXY is described by $[x, y, \theta]$, where θ describes the walking direction of the rollator and represents the angle between the horizontal axis of two rear wheels and the X axis. In our experiment
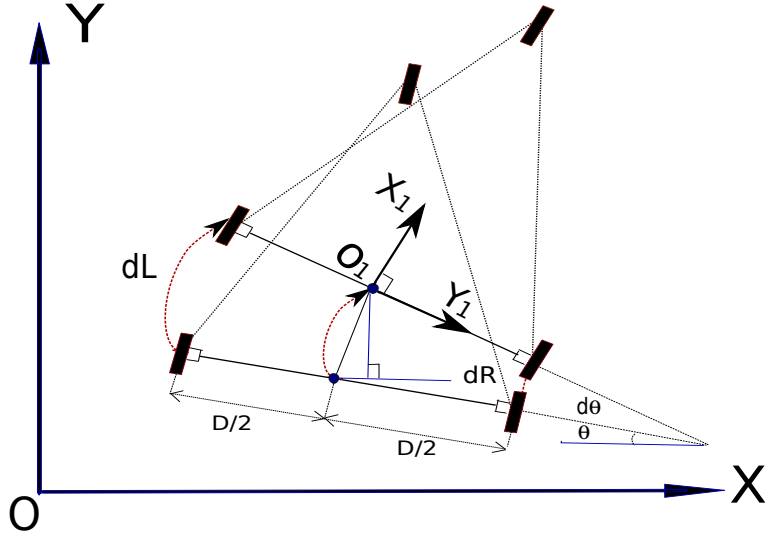
Figure 3: Simple kinematic model of the walker

of `10mWT`, as the reference trajectory was directed along the Y axis we have $\theta = 0$ at the beginning of the walker's trajectory. The trajectory of the walker is determined by using the encoders. Assuming that at the $(k+1)_{th}$ time sample moment the measurement of the encoders of two rear wheels are Δ_L and Δ_R, the displacement of the left and right wheel are obtained respectively by using (1) and (2):

$$dL = \frac{2\pi r}{4C \cdot 360}\Delta_L \tag{1}$$

and

$$dR = \frac{2\pi r}{4C \cdot 360}\Delta_R \tag{2}$$

where r is the radius of the rear wheel and C is a constant parameter of the transformation between the value of encoder and the wheel radius. The change of the direction angle θ during the $(k+1)_{th}$ sampling time can be

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estimated as:

$$dθ = \frac{dL - dR}{D},$$ (3)

where D is the distance between the two rear wheels.

According to the kinematic model shown in Fig. 3, the changes of the walker's position can be obtained as follows [37]:

$$dx = \frac{dL + dR}{2} sin(θ_k + \frac{dθ}{2})$$ (4)

$$dy = \frac{dL + dR}{2} cos(θ_k + \frac{dθ}{2})$$ (5)

Finally, the new position of the walker can be calculated by using:

$$\begin{cases} x_{k+1} = x_k + dx \\ y_{k+1} = y_k + dy \\ θ_{k+1} = θ_k + dθ \end{cases}$$ (6)

Using the above equations, the trajectory of the walker can be determined by using the encoders. The experiments have shown that after a straight line walking of 10 meters the estimated positioning has an absolute accuracy better than $1cm$.

3.2. Detection of the stride

The instruments generally used to evaluate human's gait are pedometers, accelerometers or gyrometers. To be appropriate for long-term measurements in everyday environments, these devices should be practical and not interfere with normal movement behaviour. Pedometers are small, easy to use and count the number of steps. The Yamax Digi-Walker SW-200 is considered the most accurate electronic pedometer, but its precision decreases at slower

walking speeds, making it less suitable for seniors with low physical fitness or gait abnormalities [38]. We have also used a pedometer (Omron) during the experiment and we have noticed large errors on the number of steps. Accelerometers are utilized to detect the walking stride in many studies [39], [40]. Most of methods use the peak value of forward acceleration to detect the walking cycle. However, some steps often does not lead to a high-peak forward acceleration, and hence they are not counted although there is displacement during these periods. A recent study [31] used thresholds on the magnitude of the gyroscope and accelerometer signals to identify the zero velocity instant and regarded it as the end of a step.

Our walker ANG also uses the gyrometer data to detect the walking stride. An interesting contribution of ANG is that it allows one to differentiate the right and left steps. Indeed when the subject is on the left (right) support phase the walking aid rotates on the left (right). Hence the rotational velocity of the walker around the vertical axis, which can be easily obtained by the gyrometer, is used to detect the walking stride. Its zero value instant is regarded as the end of a step. An example of rotational velocity for an elderly people is shown in Fig. 4.

Since the position of the walker at every moment has been calculated by using the method presented in Subsection 3.1, the displacement of the walker during every step, which is regarded as the step length of the subject, can be easily calculated as soon as all the steps are detected, as shown in Fig. 5. Accordingly, all gait speed characteristics (such as mean value, minimum and maximum value) can be obtained for each step. Moreover, other spatial-temporal gait parameters such as minimum and maximum acceleration can

be analyzed. With a sampling time of $1ms$ for the encoders and $4.8ms$ for the gyrometer, we may obtain a quiet reasonable accuracy on these parameters.
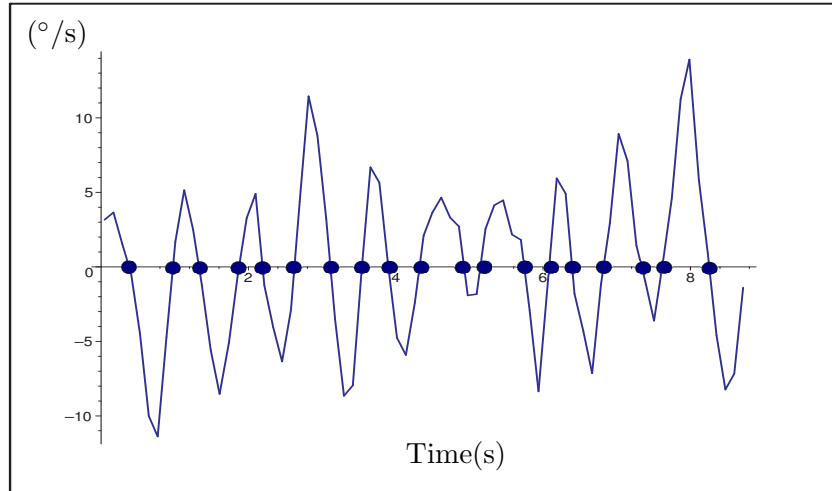
Figure 4: The rotational velocity of the walker around the vertical axis as function of time (°/s). One step is finished when it passes zero.

4. Results

In order to analyze the result comprehensively, the walking accuracy, ability and stability of the younger and elderly people are compared respectively in the following subsections.

4.1. Comparison of the walking accuracy

Using the calculation method proposed in section 3.1, the trajectories of all the subjects for the 10mWT are presented in Fig. 6. Here and in the following figures the younger adults' trajectories are presented in red while the trajectories of the elderly are presented in blue. The reference trajectory is the horizontal axis and the vertical axis is scaled to illustrate the lateral

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Figure 5: Displacement (cm) of the walker during every step as function of time (s). The results of left steps and right steps were put together and they appeared alternately.
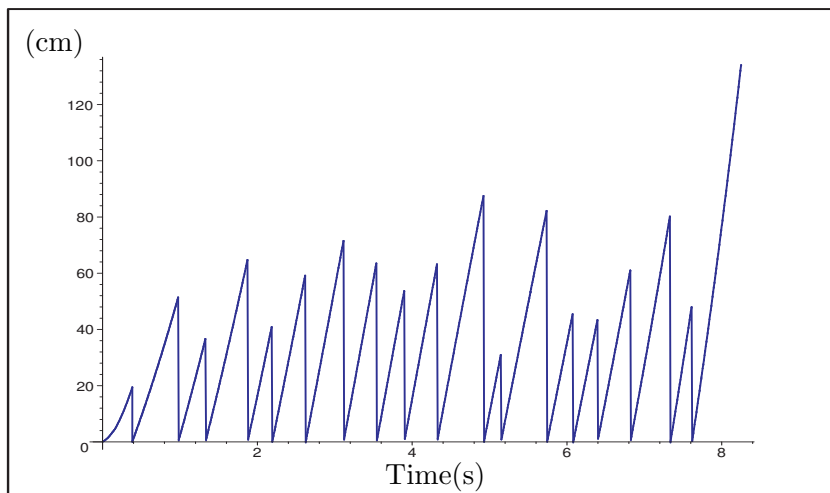
deviations between the real and reference trajectories. Fig. 6 clearly shows that the elderly subjects have larger deviations than the younger.

Several indicators about the gait can be calculated from a trajectory, such as the maximum and mean value of the lateral deviations between the real and reference trajectory, the domain of the later deviation, the area between the real and reference trajectory, and the relative Standard Deviation (SD) values. Detailed results are given in the Appendix. Fig. 7 and Fig. 8 show the maximum lateral deviation and the area between the real and reference trajectory respectively, where the results of every group of subjects are sorted into ascending order. These figures illustrate that the results of the elderly subjects have a significantly larger deviation from the reference trajectory than that of the young subjects. For example, the mean value (with a standard deviation) of the maximum lateral deviation for the elderly people is 11.048 ± 5.99 cm while that for the young people is only 3.963 ± 2.301 cm.

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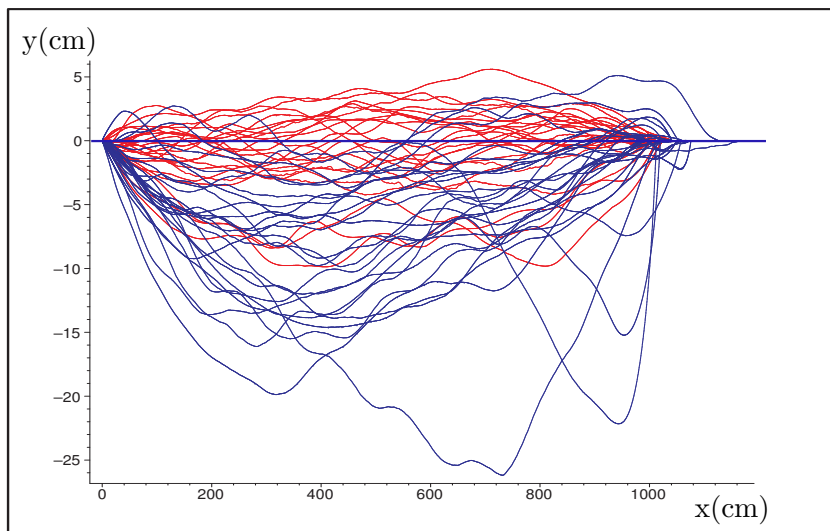
Figure 6: Trajectory of the subjects in the xy plane, where the blue color denotes the elderly subjects and the red one denotes the young subjects. The reference trajectory is the horizontal line: $y = 0$.

Similarly, the mean value of the traveled area relative to the reference trajectory for the elderly people is $5930.639 \pm 3218.4\ cm^2$ while that for the young people is $2085.702 \pm 1708.313\ cm^2$. [30] has proposed that there exists significant group difference in the medio-lateral displacement of center of mass between healthy elderly adults and elderly patients. It is consistent with our result and they reveal that walking accuracy can be regarded as a pertinent walking quality index.

In addition, several other indicators such as the traveled Manhattan distance and the absolute mean orientation angle presented in Appendix also can be used to measure the walking accuracy of the subjects, and we found that their values for the elderly people are significantly larger than that of the younger people. For the elderly people the mean value of the Manhattan
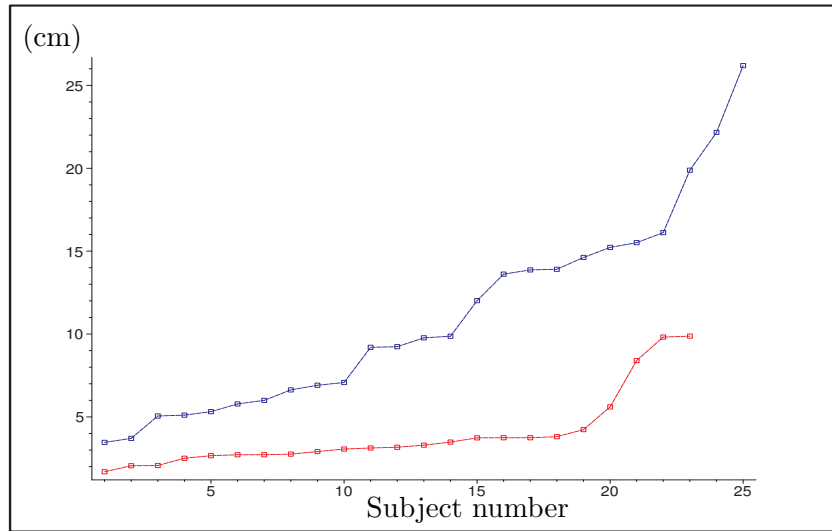
Figure 7: The maximum lateral deviation (cm) between the real and reference trajectory, where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

Figure 8: The area (cm^2) between the real and reference trajectory of the elderly and young adults. Blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.
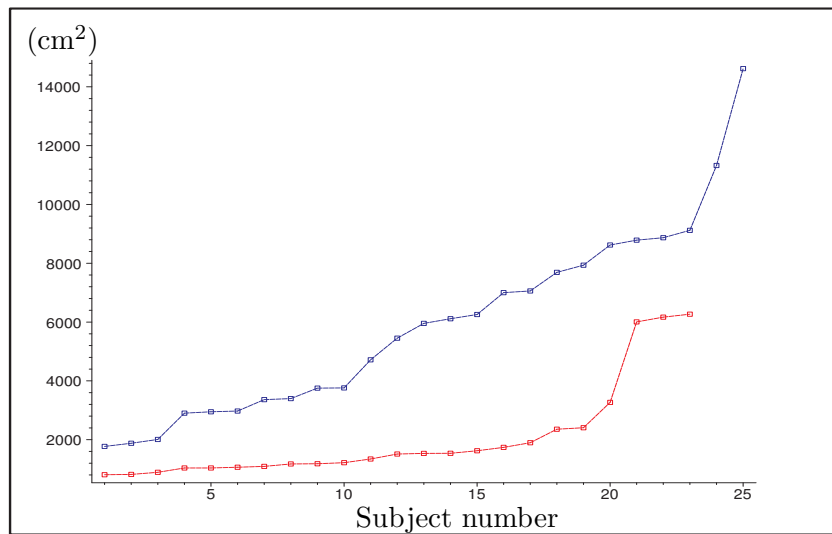
distance is 1070.003 ± 37.077 cm while for the young people this value is 1020.863 ± 14.885 cm. Similarly, the mean value of the absolute mean orientation angle for the elderly people is $1.538 \pm 0.553°$ while that for the young people is $0.951 \pm 0.311°$. In summary, the lateral motion of the elderly is larger than the younger, and the indicators that can be used to characterize this deviation are:

- the relative values of the lateral deviations between the real and reference trajectory,

- the area between the real and reference trajectory,

- the travelled Manhattan distance,

- the relative values of the orientation angle of the walking aid.

4.2. Comparison of the walking ability

By using the walker, most of gait parameters presented in the usual walking test can be calculated or estimated, such as gait cycle, gait or walking speed, step length, cadence and forward acceleration. Detailed results are given in Table A.19 in the Appendix. Although the step width cannot be calculated precisely, the analysis of the walker' lateral motion in the previous section can reflect the characteristic of the subjects' step width.

In addition, the instantaneous walking velocity can be derived from the encoder measurements. The instantaneous walking velocity is given in Fig. 9 which shows that there is no obvious difference between the elderly and young subjects. [19] and [20] also mentioned that the age-related differences in walking velocity were not significant. Fig. 10 gives their maximum values

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and it illustrates that the maximum walking velocity of the younger subjects is a little larger than that of the elderly. For the elderly people the mean value of the maximum walking velocity is 117.969 ± 15.851 cm/s and for the young people this value is 119.967 ± 16.019 cm/s.

Figure 9: Instantaneous walking velocity (cm/s) of elderly and young adults. Blue color denotes the elderly subjects and the red one denotes the young subjects. In order to estimate it more precisely, only the middle part of the trajectory is used to do the derivation.

It has been estimated that a comfortable walking speed for young adult lies in the range 130 cm/s–160 cm/s while for elderly people this speed is given by the formula $117 - 0.4 \times age$ [33]. The mean speed value for elderly people is coherent with this formula while the result of the younger adults is lower than the normal walking speed. Experiences without the walking aid have shown that the younger subjects were presenting a mean velocity that was close to the normal walking speed. Our interpretation is that elderly people are more familiar with walking aids and have walking patterns that

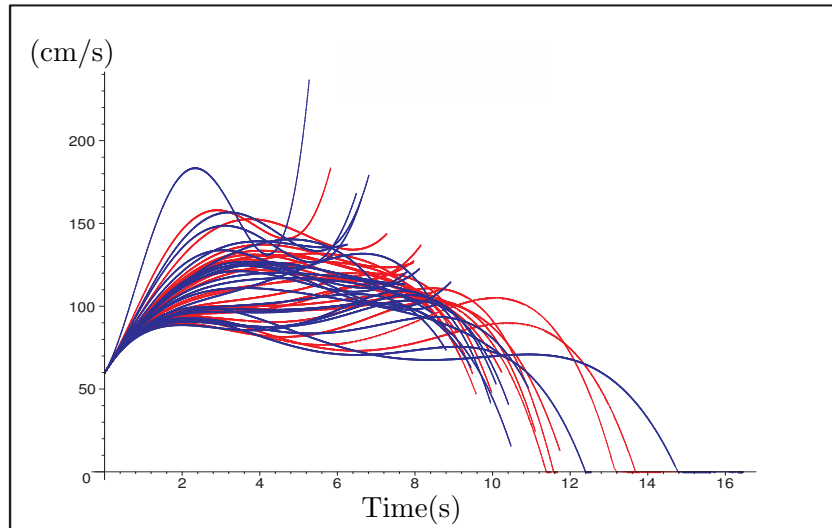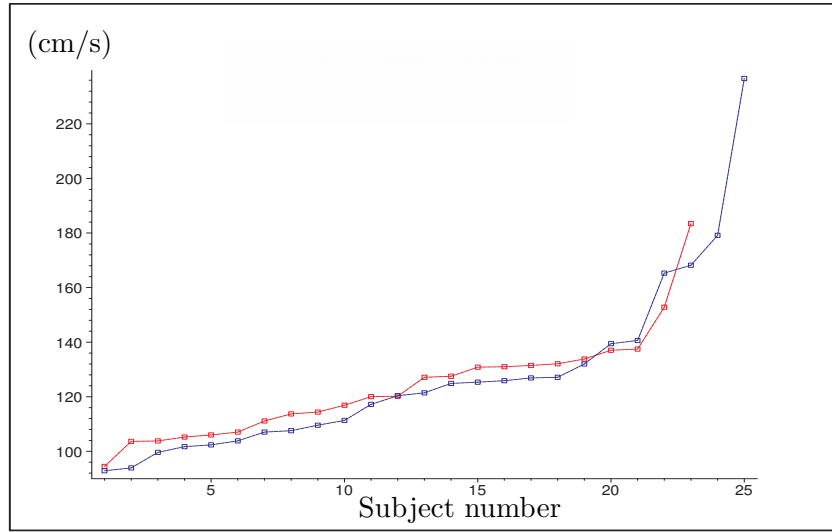Figure 10: Maximum instantaneous walking velocity (cm/s) of elderly and young adults. Blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

benefit from such an aid, while younger people have a more dynamic pattern that is jeopardized by the aid. This can explain why the maximum velocities of the younger are higher, as shown in Fig. 10.

The results of step period and step length for the two groups are given in Fig. 11 and Fig. 12. Our previous work [41] has shown that there is no difference between the left steps and the right steps so here and in the following contents the results of two steps are put together. The results illustrate that there is almost no difference between the two groups which explains why the two groups have similar walking speed. The mean values of step period and step length of the elderly people are 0.526 ± 0.1 s and 54.862 ± 11.643 cm receptively, and these values of the young people are 0.537 ± 0.095 s and 55.050 ± 8.605 cm. Exactly, about 78% of the subjects

(37 of 48) have a step period between 0.4 s and 0.6s, and 75% of the subjects (36 of 48) have a step length between 40 cm and 60 cm,

Figure 11: Mean value of step period (s), where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

Next we have considered the mean value of forward acceleration shown in Fig. 13. It illustrates that the forward accelerations of the elderly are larger than that of the younger. In addition, almost 70% of the younger's (16 of 23) mean forward accelerations are less than zero while for the elderly this number is only 40% (10 of 25). Therefore, we can deduce that the minimum velocity of the younger is less than that of the elderly although their mean speed is almost the same. As a result, the elderly subjects can use less time to arrive to the goal, as shown in Fig. 14. This contradicts the usual assumption (which is based on the `10mWT`) that the walking time may be used to evaluate the walking ability.

Figure 12: Mean value of step length (cm), where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.
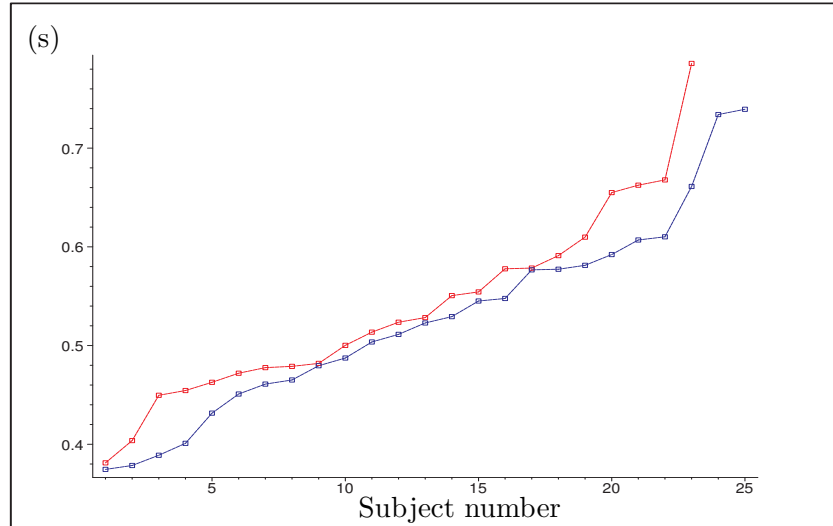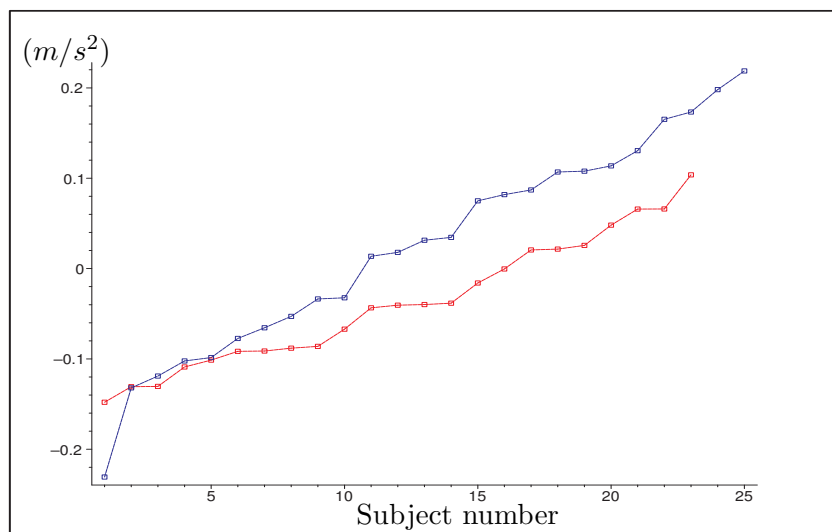

Figure 13: Mean value of forward acceleration (m/s^2), where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.
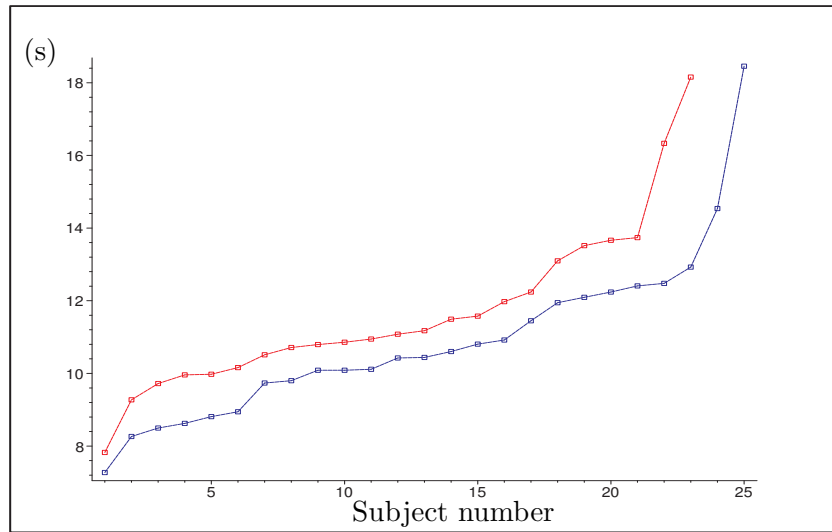
Figure 14: Time used for `10mWT` (s) of elderly and young adults. Blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

In summary, the elderly people have similar walking speed, step length, step period as the younger people, but there are three indicators for which the difference exists between the younger and elderly adults:

- maximum instantaneous walking velocity,

- mean value of the forward acceleration,

- time used for the total test.

4.3. Comparison of the walking stability

Gait variability is an index of gait stability and complexity. The increased variability of gait parameters corresponds to decreased gait stability, complexity and increased risk of falling. Gait variability is defined as changes

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in gait parameters from one stride to the next. It can be characterized by the coefficient of variation (CV) of kinematic gait parameters [15], [28]. The coefficient of variation (CV) is defined as the ratio of the standard deviation (SD) to the mean, i.e., for a set of gait parameter A, it's CV is:

$$CV(A) = \frac{SD(A)}{mean(A)}. \tag{7}$$

Hence CV shows the extent of variability in relation to mean of the population. Here we have compared the CV of step length, step period and walking speed between the two groups. The results are given in Fig. 15 and detailed information are given in Table A.19 in the Appendix. Fig. 15 shows that the CV of step length for the two groups are similar. The comparison of the CV of stride time illustrates the same characteristic thus it was not presented here. Our results are similar to those of some previous studies [19], [20], [42], which presented that there were no significant differences between young and elderly healthy people in CV of step length and stride time. For the elderly people the mean value of CV of step length is $0.545 \pm 0.194 \; cm$ and for the young people this value is $0.513 \pm 0.143 \; cm$. Next, the CV of walking speed is shown in Fig. 15, as expected the walking speed of elderly exhibits less variability than that of younger adults but the difference is small. Exactly, the mean values of CV of walking speed of the elderly and young people are respectively $0.206 \pm 0.09 \; cm/s$ and $0.238 \pm 0.07 \; cm/s$.

As indicated in the introduction, other information about the pressure on the handles can be used to analyze the walking stability by measuring the angular velocity around the forward direction X_1 and the lateral direction Y_1 (see Fig. 3). We were surprised that the angular velocity measurements were sensitive enough to estimate changes in the forward/backward support force
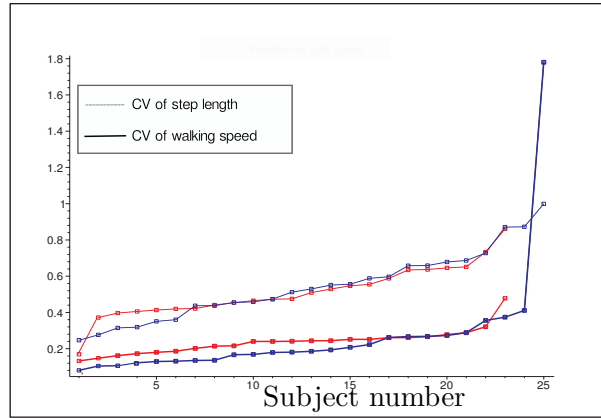
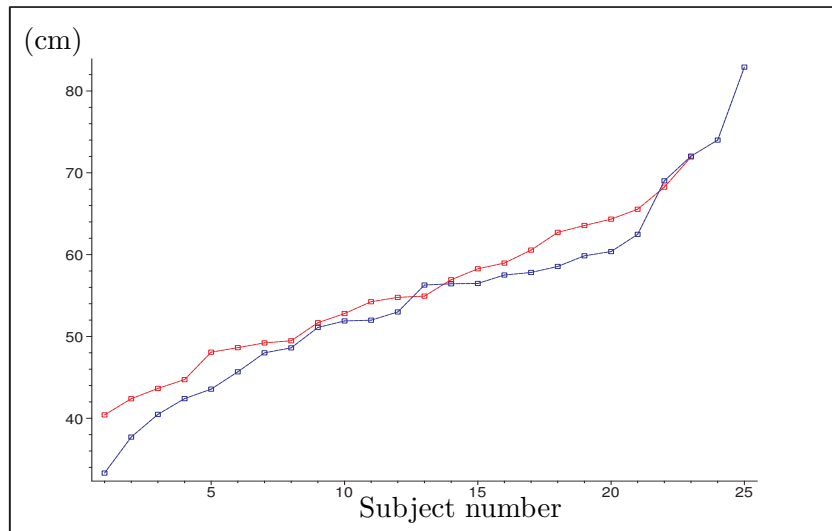Figure 15: Coefficient of variation for the step length and walking speed, where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

(change on the angular velocity around Y_1) and on the left/right support force (change on the angular velocity around X_1).

Hence we have been able to determine the respective percentage of forward/backward and left/right support with a reasonable accuracy without any force sensors in the handles [43]. Fig. 17 shows the percentage of forward support. It is interesting to note that the results of the younger people are much farther away from 50% than that of the elderly people. Exactly, the mean value of the respective percentage of forward support of the young people is $61.902\% \pm 5.093\%$ while that of the elderly people is $58.160\% \pm 4.378\%$. That means for younger people the difference between forward and backward support is larger. Surprisingly the younger adults are leaning significantly more on the aid than the elderly people.

Based on the above analysis, it appears that the following three indicators should be investigated in the future:
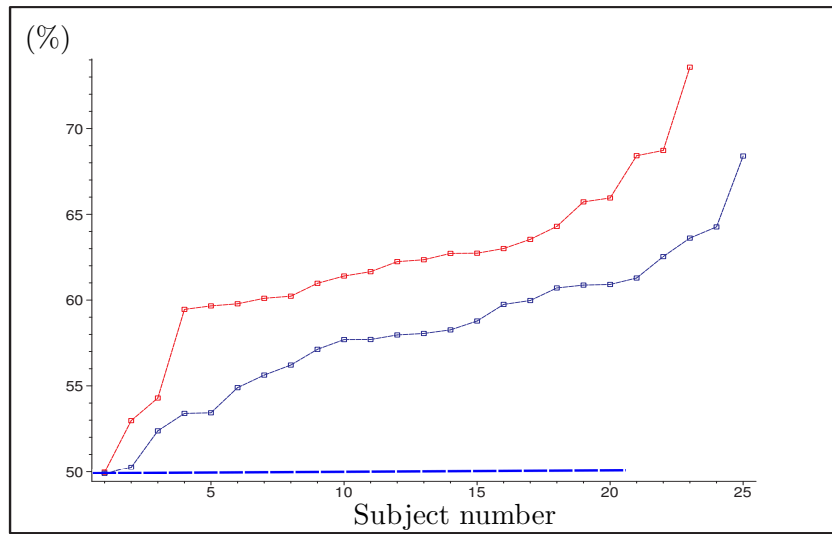
Figure 16: Percentage of the forward support, where the blue color denotes the elderly subjects and the red one denotes the young subjects. The results of every group of subjects are sorted into ascending order.

- variability of walking speed,

- percentage of forward/backward support,

- percentage of right/left support.

5. Other applications of the walker ANG

Besides of gait analysis, our walker ANG [14] can perform multifunctions such as navigation, fall detection/prevention and street mapping. Here we will give an example of its application for updating collaborative maps with sideway slopes and location of lowered kerbs.

The walker is instrumented with an accelerometer that measures both the gravity and the walker's acceleration, which is usually much lower than
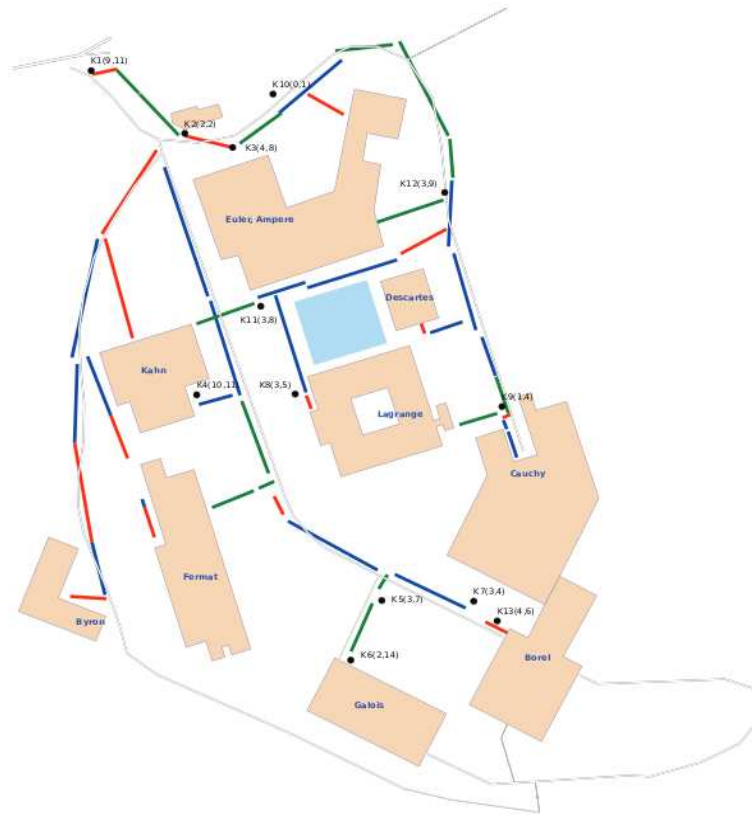
Figure 17: Qualified map of INRIA Sophia Antipolis by using the walker ANG, where the red lines denote the roads with high slope and the green lines denote the almost horizontal roads that we walked on with the walker.

the gravity. Hence an appropriate processing of the measurement allows to determine the direction of gravity in the walker's frame. In turn this directly allows to determine the roll and tilt of the walker. According to these information, we can measure the slope of sideways, detect lowered kerbs and measure the quality of the sidewalk surfaces. For example, Fig. 17 shows a qualified map of INRIA Sophia Antipolis, where the red lines denote the roads with high slope and the green lines denote the almost horizontal roads

that we walked on with the walker. Post-processing of this information allows to report the results in OpenStreetMap to produce an ad-hoc map. Hence it is significant for the end-users of walker or wheelchair because they can share more information of a city while planners can take into account sideway slopes and location of lowered kerbs to determine an optimal itinerary for them.

6. Conclusions

This paper has proposed a gait analysis method based on the use of an instrumented walker. The results of a $10\ m$ straight line test for 23 younger people and 25 elderly people were compared comprehensively. The comparison includes the relative information about walking accuracy, ability and stability. Several important indicators that exhibit significant difference between the two groups were obtained, such as the maximum lateral deviations between the real and reference trajectory, the area and the Manhattan distance between the real and reference trajectory. For the elderly people these indicators are much larger than that of the young people, and exhibit also significant difference within the same group.

As for the gait parameters describing the walking ability, it appears that there is no obvious difference in step length, step period and walking speed between two groups. The instantaneous walking velocity has been obtained and we found out the maximum instantaneous walking velocity of the younger people is a little larger than that of the elderly people. On the other hand, surprisingly, the time required to perform the trajectory is usually lower for the elderly than for young people. Our explanation is that the normal gait of younger adults was more affected by the presence of the walker than for

elderly that are more accustomed with the use of the walker. In addition, when we tried to use the variability of gait parameters to analyze the walking stability, there are similar results for the two groups in the variability of step length and step period while for the younger subjects the variability of the walking speed is larger.

Moreover, we also found that the younger adults are leaning significantly more on the aid than the elderly people. Hence the influence of the walker on the walk pattern of the younger people should be investigated. Besides of this, another walking test with a returning trajectory for two groups people will be studied. We also want to examine if a learning process may be implemented in order to characterize the walking pattern at a given time and customize the walking analysis software in order to better determine future trends. Finally, it is worth mentioning that the present analysis method depends on the measurement of sensors with noise that do not satisfy usual statistical hypothesis (e.g., they are not Gaussian). In the future we plan to take measurement uncertainties into account by using interval analysis for obtaining the indicators as interval values that will be guaranteed to include the real values, with the advantage that the width of the interval will be a measure of the quality of the result.

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Appendix A. Original results

Results of the elderly (1--25) and young subjects (26--48)

Subject	Total time used(s)	Time used for 10m	Traveled euclidean distance(cm)	Traveled manhattan distance(cm)	Error domaine(cm)	Maximum absolute error(cm)	Mean error (cm)	Standard deviation of error (cm)	Average absolute deviation (cm)	Area between real traj and reference traj(cm^2)	Mean orientation (degree)	Standard deviation of orientation (degree)	Average absolute deviation of orientation (degre)
1	10.113	9.718	1040.686	1069.070	13.863	12.016	-7.620	3.222	2.650	7057.905	-0.274	1.837	1.562
2	10.423	8.934	1166.656	1179.347	5.055	5.055	-2.877	1.483	1.325	3398.169	-0.176	1.024	0.815
3	8.496	8.308	1022.638	1034.198	4.699	3.464	-1.749	1.020	0.908	1768.897	-0.015	0.924	0.743
4	14.537	14.760	984.854	1001.812	7.839	6.907	-2.555	2.292	1.968	2900.350	0.080	1.240	0.989
5	10.088	9.563	1054.914	1082.618	13.866	13.866	-6.384	4.647	4.033	7000.278	-0.154	1.940	1.670
6	11.946	11.898	1004.027	1031.728	13.903	13.903	-8.286	4.213	3.613	8786.815	-0.235	1.973	1.690
7	8.624	8.156	1057.320	1085.517	9.772	9.772	-5.710	2.186	1.767	6258.550	-0.604	1.702	1.351
8	10.601	10.236	1035.678	1068.704	15.968	15.510	-7.343	5.280	4.688	8619.661	0.045	2.287	1.903
9	8.945	8.787	1018.028	1066.260	23.568	22.163	-4.163	6.331	4.844	5955.932	-1.202	2.194	1.831
10	12.239	11.383	1075.235	1092.079	5.783	5.783	-2.944	1.632	1.452	3362.376	-0.252	1.337	1.003
11	9.800	9.314	1052.148	1068.561	6.686	3.702	0.004	2.030	1.788	1880.020	0.120	1.090	0.875
12	18.456	17.242	1070.415	1088.041	6.959	6.628	-3.456	2.606	2.446	4716.062	-0.274	1.373	1.130
13	12.479	12.591	991.120	1031.188	19.880	19.880	-10.213	6.084	5.172	11326.516	-0.079	3.152	2.520
14	12.408	11.700	1060.484	1077.227	7.498	7.072	-3.600	2.477	2.179	3752.420	-0.142	1.379	1.122
15	10.440	10.209	1022.716	1066.890	15.224	15.224	-7.994	3.430	2.701	8868.822	-1.005	2.229	1.866
16	8.264	8.102	1019.958	1047.481	13.614	13.614	-7.761	4.260	3.813	7688.381	-0.506	2.656	2.119
17	12.095	11.587	1043.780	1068.686	11.825	9.238	-2.644	3.933	3.607	3761.081	-0.266	2.169	1.700
18	9.736	9.377	1038.248	1072.650	16.314	16.116	-7.630	5.086	4.674	7935.143	-0.203	2.672	2.186
19	12.927	12.325	1048.851	1067.417	8.315	6.000	-2.460	2.228	1.853	2972.125	0.038	1.558	1.131
20	10.087	9.843	1024.833	1076.353	26.192	26.192	-13.479	8.265	7.312	14619.055	0.079	3.628	3.076
21	7.273	6.997	1039.459	1063.419	8.052	5.319	-1.042	2.249	1.888	2006.372	0.138	1.684	1.389
22	8.810	7.827	1125.638	1147.934	9.124	5.108	0.649	2.930	2.707	2947.763	-0.111	1.688	1.304
23	11.448	11.565	989.839	1010.958	9.206	9.197	-5.439	2.309	1.823	5450.687	-0.126	1.539	1.314
24	10.920	10.189	1071.751	1093.331	9.893	9.863	-5.399	2.747	2.443	6111.605	0.086	1.583	1.306
25	10.806	10.492	1029.881	1058.610	14.617	14.617	-9.007	4.354	3.896	9120.989	-0.237	2.153	1.796
26	16.332	16.107	1013.973	1029.073	4.445	4.236	-2.104	1.503	1.413	2406.272	-0.106	1.188	0.955
27	10.793	10.735	1005.426	1019.371	4.469	2.715	0.337	1.174	1.051	1512.369	-0.008	1.051	0.848
28	13.666	13.636	1002.144	1013.737	2.549	2.056	0.559	0.793	0.692	809.319	-0.048	0.910	0.730
29	10.857	10.713	1013.364	1021.897	2.227	2.067	0.826	0.680	0.607	1036.115	-0.082	0.747	0.614
30	11.576	11.624	995.917	1011.723	3.744	3.734	-1.519	1.157	1.054	1896.943	-0.009	1.184	0.905
31	10.511	10.499	1001.144	1013.170	4.569	3.739	1.223	1.413	1.243	1737.640	-0.020	1.003	0.820
32	11.976	11.964	1001.035	1010.207	3.073	2.908	1.141	0.922	0.844	1344.696	0.002	0.738	0.614
33	13.737	13.190	1041.459	1055.006	5.603	5.603	2.913	1.497	1.253	3268.150	0.049	1.024	0.859
34	12.241	11.782	1038.936	1051.297	3.492	2.661	0.896	0.932	0.795	1180.521	0.019	1.016	0.816
35	9.977	9.861	1011.707	1035.818	8.397	8.397	-5.465	2.668	2.304	6003.658	-0.190	1.946	1.582
36	10.944	10.811	1012.325	1033.995	9.874	9.874	-5.492	2.848	2.389	6266.182	0.107	2.171	1.678
37	10.160	9.938	1022.427	1049.399	9.817	9.817	-5.563	2.678	2.180	6165.286	0.102	2.009	1.701
38	11.493	11.558	994.394	1007.535	4.321	3.806	-1.346	1.163	0.967	1538.759	-0.036	1.004	0.787
39	13.517	13.509	1000.582	1014.853	4.727	3.298	-0.956	1.447	1.215	1622.401	0.020	1.154	0.916
40	13.097	13.161	995.136	1007.984	4.731	3.123	0.153	1.276	1.024	1092.119	-0.092	1.000	0.824
41	11.176	11.145	1002.730	1010.248	2.271	1.693	0.726	0.623	0.540	820.150	0.074	0.745	0.599
42	11.079	11.087	999.285	1012.005	3.504	3.168	-1.183	0.974	0.846	1218.985	-0.144	1.116	0.910
43	9.960	9.896	1006.504	1022.513	4.570	2.717	0.252	1.291	1.094	1172.839	-0.132	1.255	1.037
44	9.720	9.738	998.128	1011.245	4.028	3.482	-1.282	1.159	0.973	1534.383	-0.170	1.089	0.887
45	10.711	10.702	1000.886	1012.559	3.662	3.058	-0.790	1.028	0.907	1036.045	-0.023	0.882	0.703
46	9.272	9.191	1008.871	1023.952	4.312	2.755	0.528	0.980	0.775	889.333	0.218	1.563	1.162
47	7.824	7.941	985.252	1003.301	4.927	2.509	-0.210	1.263	1.013	1061.180	-0.059	1.353	1.144
48	18.157	18.204	997.402	1008.961	3.856	3.742	2.071	1.227	1.077	2357.790	-0.126	1.021	0.816

Figure A.18: Result of trajectory for the elderly $(1-25)$ and younger subjects $(26-48)$.

Subject	Step number	Mean step length (cm)	Minimum step length (cm)	Maximum step length (cm)	Variation of step length	Mean walking cycle (s)	Minimun walking cycle(s)	Maximum walking cycle(s)	Variation of walking cycle	Mean walking speed(cm/s)	Variation of walking speed	Mean foward acceleration (m/s^2)	Standard deviation of foward acceleration (m/s^2)
1	12	82.921	52.920	202.720	0.551	0.734	0.482	1.692	0.501	112.608	0.129	-0.102	0.208
2	25	42.404	7.022	136.153	0.658	0.389	0.078	1.073	0.553	109.091	0.268	0.031	0.233
3	18	56.293	1.427	101.454	0.555	0.461	0.001	0.818	0.507	217.939	1.780	-0.053	0.385
4	23	45.676	6.320	69.053	0.319	0.610	0.081	1.045	0.349	75.833	0.106	0.219	0.144
5	17	72.040	1.126	235.058	0.872	0.661	0.012	1.974	0.800	107.812	0.135	0.107	0.213
6	20	51.100	20.202	99.100	0.459	0.577	0.200	0.960	0.406	89.776	0.193	0.075	0.120
7	14	74.004	17.926	130.485	0.360	0.581	0.328	1.233	0.400	127.207	0.207	-0.231	0.261
8	19	58.561	27.199	86.262	0.276	0.523	0.339	0.689	0.201	110.614	0.131	0.035	0.198
9	17	59.859	3.074	207.145	0.871	0.504	0.016	1.513	0.740	126.354	0.288	-0.034	0.259
10	21	47.998	2.916	152.318	0.728	0.480	0.028	1.577	0.721	100.605	0.168	0.014	0.189
11	20	51.994	1.416	100.955	0.473	0.465	0.005	0.866	0.447	121.855	0.411	-0.132	0.494
12	31	37.701	0.669	110.098	0.687	0.607	0.049	2.673	0.833	68.111	0.374	0.165	0.199
13	27	40.483	1.137	99.439	0.678	0.451	0.006	1.016	0.658	94.144	0.272	0.108	0.240
14	29	33.312	5.858	97.523	0.588	0.379	0.055	1.006	0.568	90.361	0.167	0.131	0.266
15	18	56.441	9.332	91.475	0.455	0.529	0.079	0.915	0.476	109.992	0.136	0.087	0.224
16	19	48.610	8.972	97.786	0.438	0.401	0.255	0.706	0.311	118.447	0.267	-0.119	0.459
17	20	52.996	1.223	106.687	0.529	0.592	0.006	1.748	0.679	99.928	0.262	0.173	0.173
18	17	62.476	6.570	89.165	0.351	0.545	0.107	0.789	0.336	112.329	0.179	-0.077	0.291
19	24	43.558	15.888	87.297	0.437	0.511	0.174	1.443	0.551	87.792	0.080	0.198	0.191
20	18	60.378	18.027	75.697	0.247	0.548	0.438	0.657	0.117	110.144	0.223	-0.099	0.240
21	17	57.511	6.455	85.754	0.512	0.375	0.041	0.625	0.502	151.234	0.104	0.018	0.198
22	20	57.823	19.110	89.430	0.315	0.432	0.220	0.606	0.261	134.145	0.186	-0.066	0.408
23	17	69.029	20.903	198.043	0.659	0.739	0.234	2.075	0.640	93.700	0.122	0.114	0.195
24	22	51.904	15.072	145.279	0.597	0.487	0.130	1.211	0.491	103.673	0.181	0.082	0.482
25	18	56.484	1.473	221.339	0.999	0.577	0.007	2.124	0.946	103.840	0.356	-0.032	0.197
26	17	63.562	4.135	128.368	0.454	0.786	0.315	2.275	0.630	83.235	0.288	0.022	0.099
27	23	42.397	2.444	128.813	0.861	0.404	0.017	1.104	0.798	108.786	0.277	-0.101	0.285
28	21	44.727	13.856	79.080	0.441	0.500	0.242	0.806	0.326	88.666	0.252	-0.086	0.193
29	24	48.638	6.452	99.169	0.465	0.450	0.045	0.852	0.488	116.145	0.240	0.104	0.317
30	19	65.546	17.305	169.675	0.646	0.668	0.211	1.453	0.592	99.821	0.261	0.026	0.293
31	19	60.559	18.135	110.274	0.421	0.528	0.137	0.947	0.425	117.764	0.173	-0.040	0.269
32	22	48.086	9.139	93.776	0.473	0.472	0.083	1.084	0.559	106.607	0.147	-0.000	0.159
33	21	54.244	7.390	85.969	0.420	0.578	0.073	0.853	0.365	93.539	0.214	-0.038	0.295
34	21	52.800	1.019	78.531	0.405	0.554	0.007	0.764	0.334	97.361	0.244	-0.016	0.244
35	16	71.941	32.556	184.565	0.509	0.655	0.280	1.779	0.540	112.527	0.186	-0.043	0.265
36	16	68.243	46.257	90.528	0.169	0.591	0.375	1.289	0.374	120.280	0.133	-0.092	0.201
37	19	62.723	3.583	109.991	0.413	0.551	0.026	0.935	0.350	114.700	0.201	-0.109	0.336
38	17	56.930	9.549	137.246	0.588	0.610	0.248	1.428	0.486	91.555	0.321	-0.088	0.314
39	20	58.976	13.937	106.636	0.475	0.662	0.173	1.430	0.476	90.444	0.244	0.021	0.271
40	25	40.416	1.155	79.878	0.637	0.482	0.004	1.038	0.588	94.237	0.478	0.048	0.207
41	21	49.478	5.605	126.980	0.733	0.514	0.083	1.248	0.646	95.306	0.266	-0.041	0.227
42	18	58.275	2.376	129.605	0.635	0.524	0.038	1.143	0.622	109.730	0.261	-0.131	0.331
43	19	54.777	7.646	111.694	0.555	0.463	0.056	1.253	0.609	123.488	0.252	-0.067	0.276
44	22	51.677	7.011	79.897	0.397	0.454	0.135	0.747	0.349	112.852	0.241	-0.091	0.363
45	20	54.936	2.875	79.032	0.372	0.478	0.105	0.662	0.309	110.902	0.240	-0.148	0.395
46	22	49.220	2.809	101.165	0.651	0.381	0.017	0.895	0.702	137.518	0.180	0.066	0.384
47	16	64.334	11.461	116.511	0.547	0.479	0.075	1.001	0.595	141.711	0.162	-0.130	0.359
48	27	43.658	9.531	89.937	0.529	0.578	0.141	1.135	0.488	76.389	0.216	0.066	0.208

Figure A.19: Result of gait parameters for the elderly $(1-25)$ and younger subjects$(26-48)$.