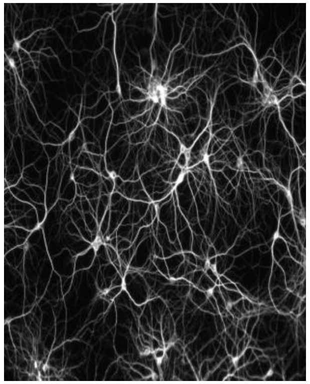
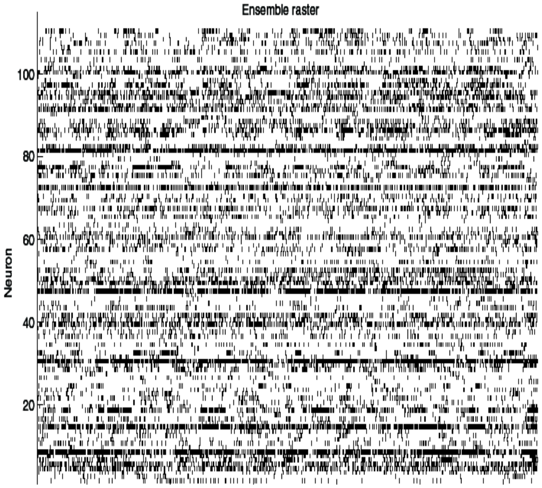
Mathematics of Multilevel Anticipatory Complex Systems

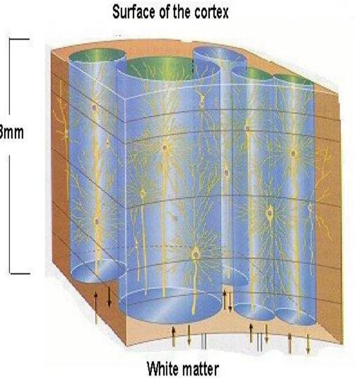
informatiques mathématiques

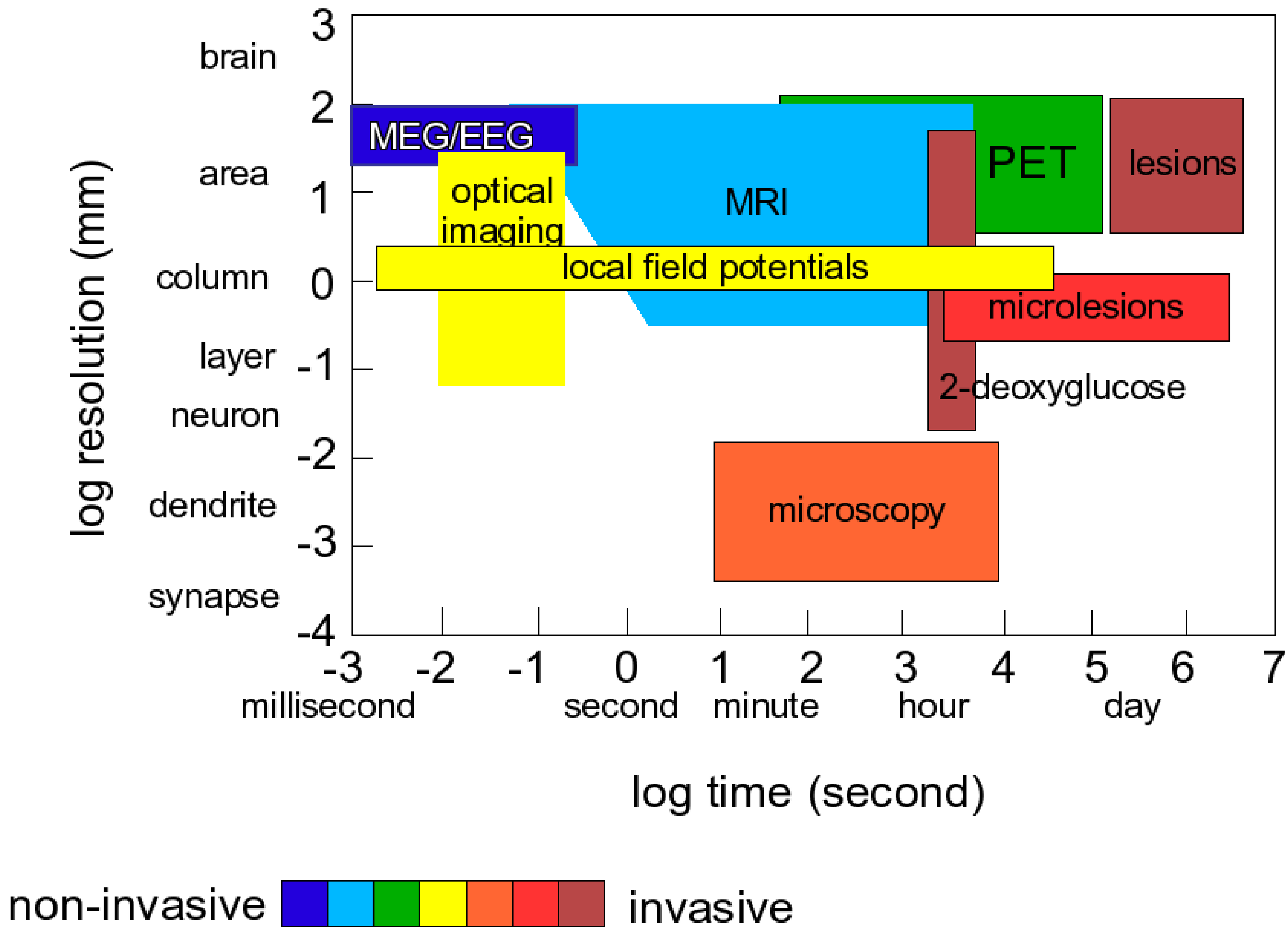
Mean Field Methods in Neuroscience

B. Cessac,
Neuromathcomp,
INRIA



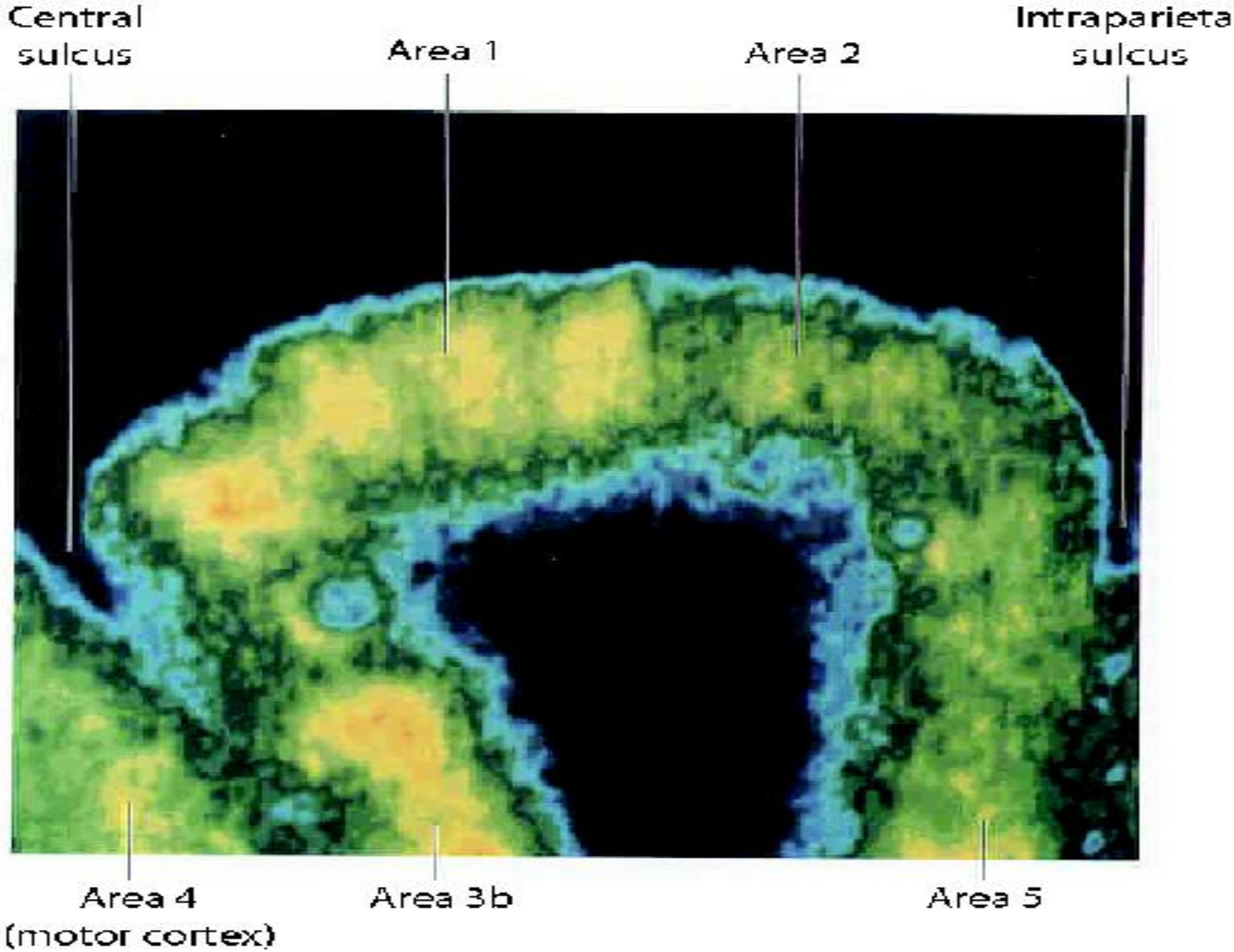

Ensemble raster
Neuron
100
80
60
40
20


Surface of the cortex
3mm
White matter

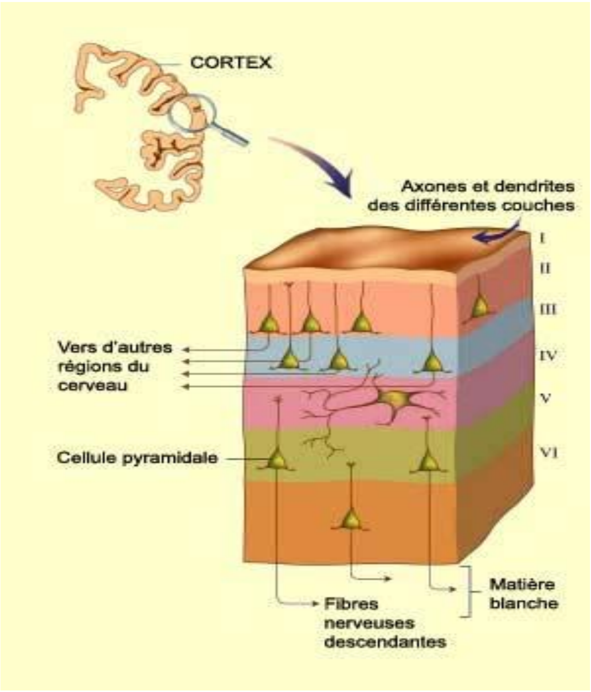


brain
area
column
layer
neuron
dendrite
synapse
3
2
1
0
-1
-2
-3
-4
log resolution (mm)
MEG/EEG
optical imaging
MRI
PET
lesions
local field potentials
microlesions
2-deoxyglucose
microscopy
-3
-2
-1
0
1
2
3
4
5
6
7
millisecond
second
minute
hour
day
log time (second)
non-invasive
invasive

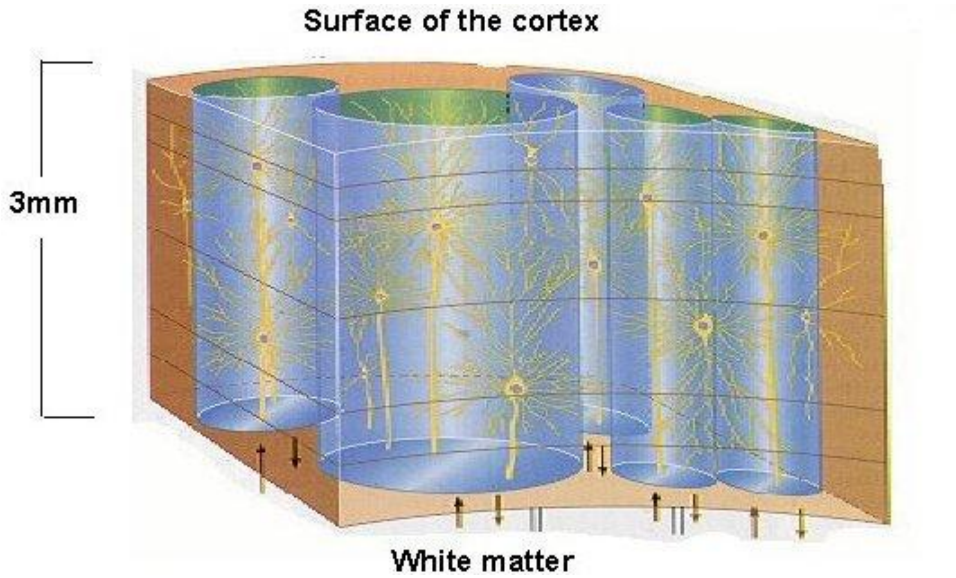
Cortical columns.


Central
sulcus
Area 1
Area 2
Intraparietal
sulcus
Area 4
(motor cortex)
Area 3b
Area 5

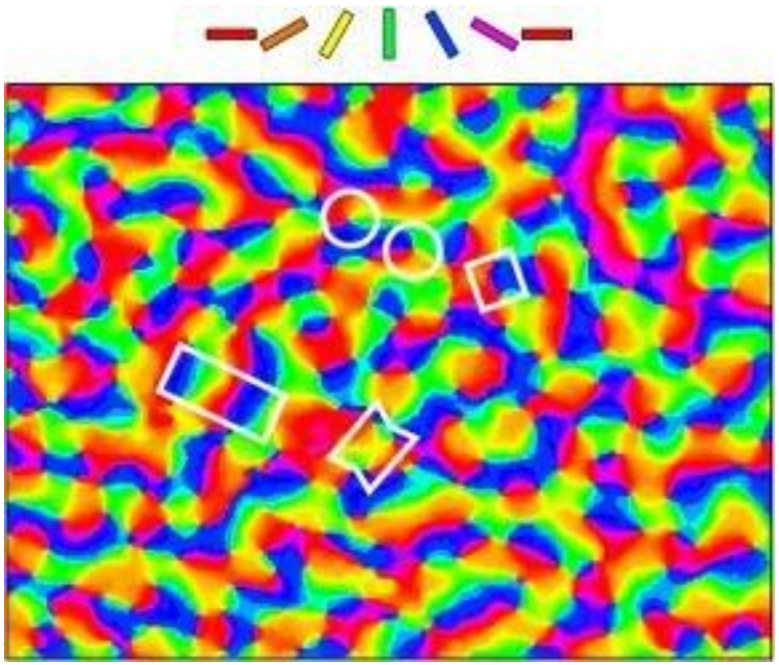
Cortical columns.



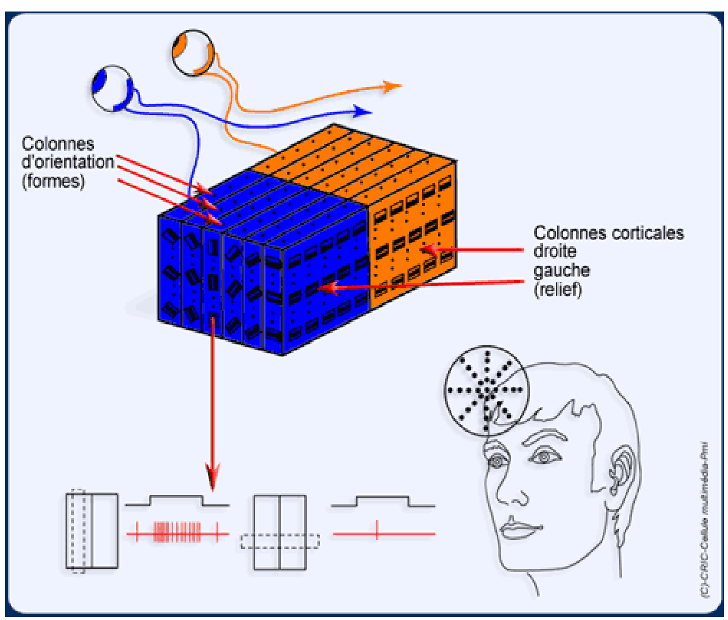
Small cylinders, of diameter 0.1~1mm, crossing cortex layers, with about 10^3-10^4 neurons, from different types, strongly connected.

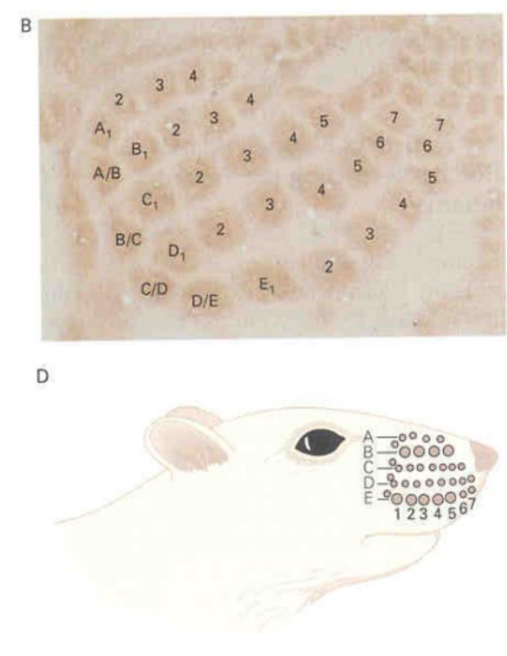


Cortical columns.



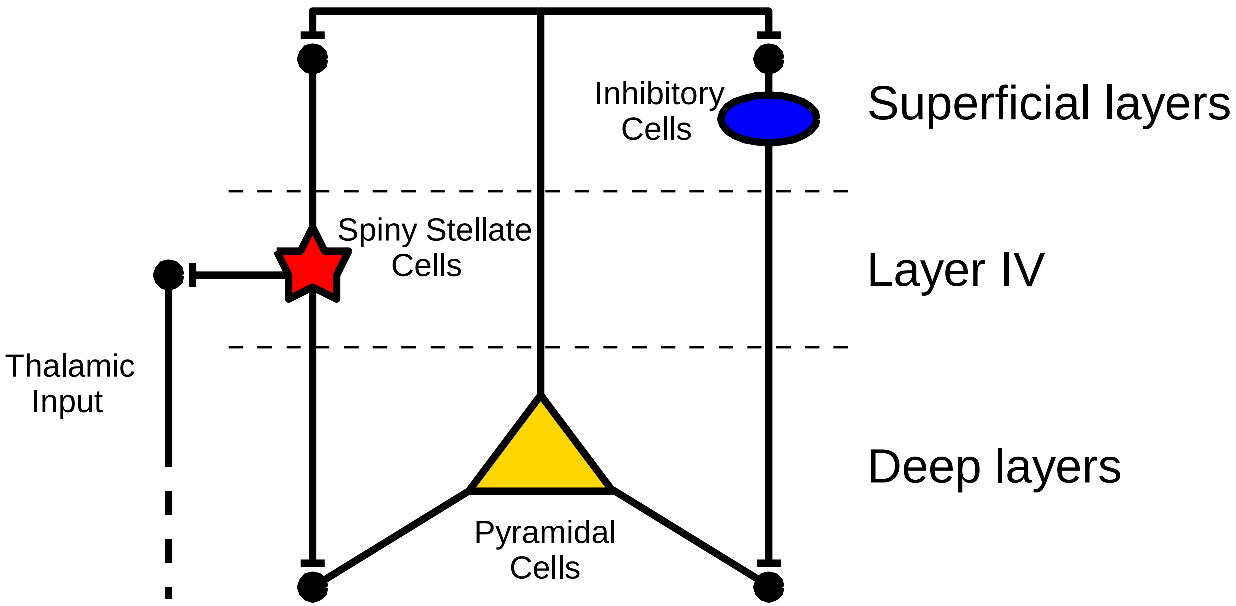
Cortical columns are involved in elementary sensori-motor like vision.





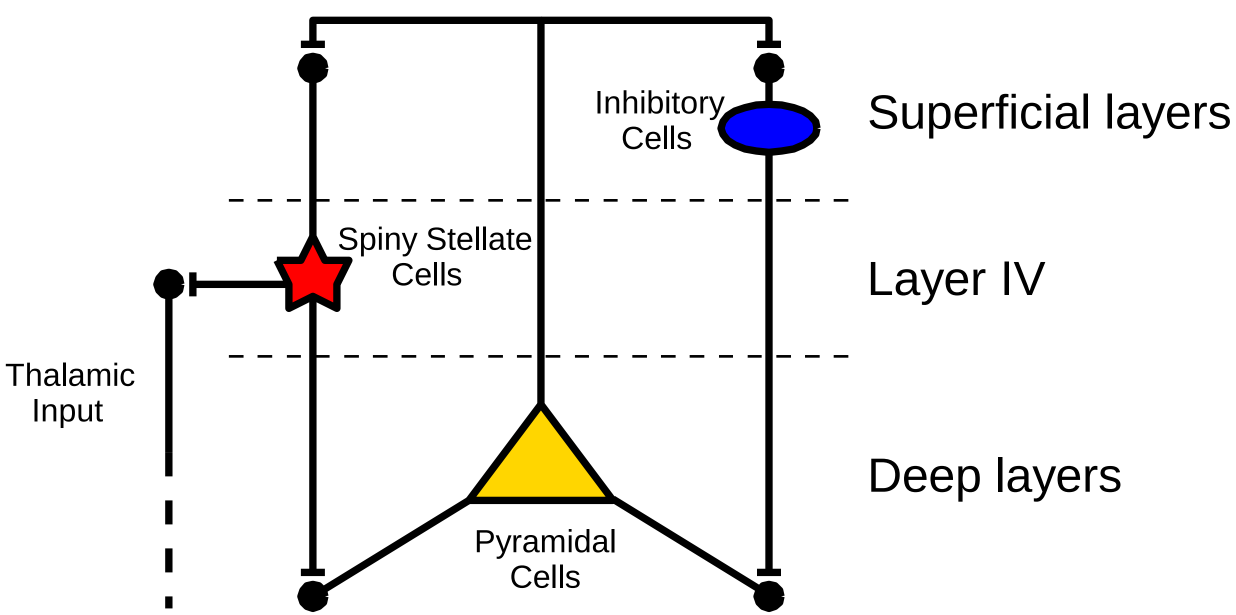
Cortical columns.

They are composed of neurons belonging to a small number of populations interacting together. These populations belong to different cortex layers.



Cortical columns.

It is possible and useful to propose phenomenological models characterizing the mesoscopic activity of cortical columns, predicting the behaviour of local field potential generated by the electric activity of neurons and to relate this behavior with measures and clinical observations (epilepsy).



Courtesy. S. Chemla.

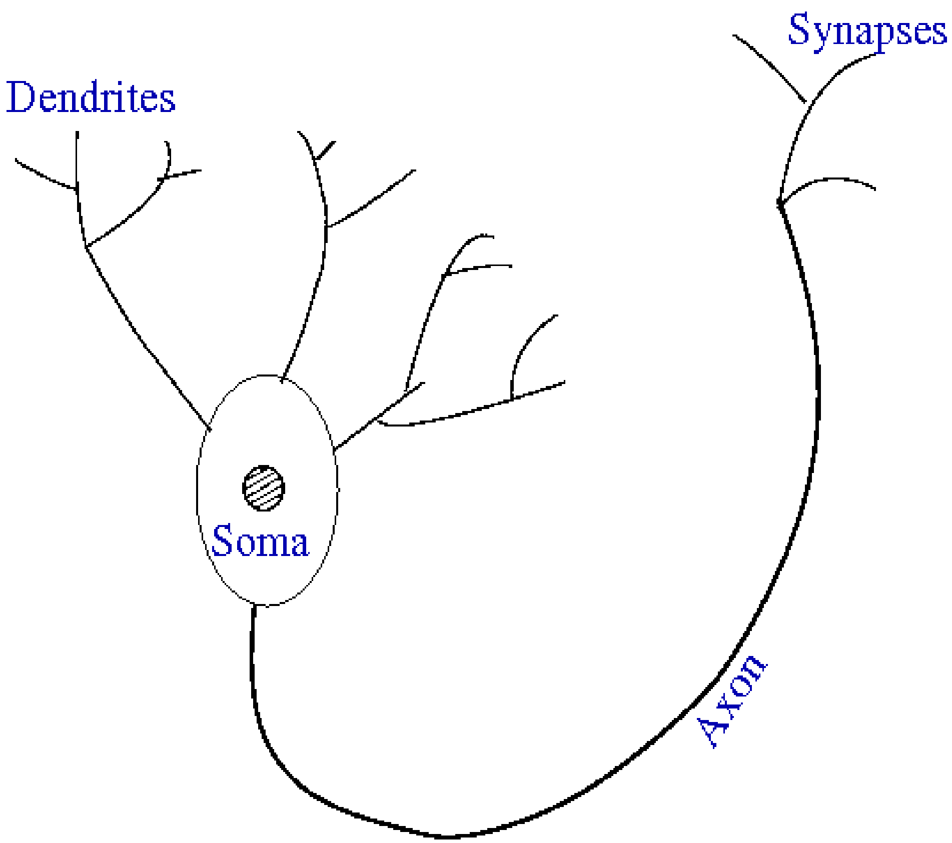
Cortical column paradigm

Type of cortical column	Anatomical	OI pixel	Functional	Physico-functional	Cortical Area
Definition	Micro-column Mini-column	**Our Column**	Orientation column	Macro-column or Hyper-column (V1)	Neural Mass
Spatial scale	40-50 µm	**50-100 µm**	200-300 µm	600 µm (and more)	10 mm
Number of neurons	80-100 neurons	**150-200 neurons**	Several mini-columns	60-100 mini-columns 10000 neurons	100XThousand neurons of the same type (pyr, stellate,…)

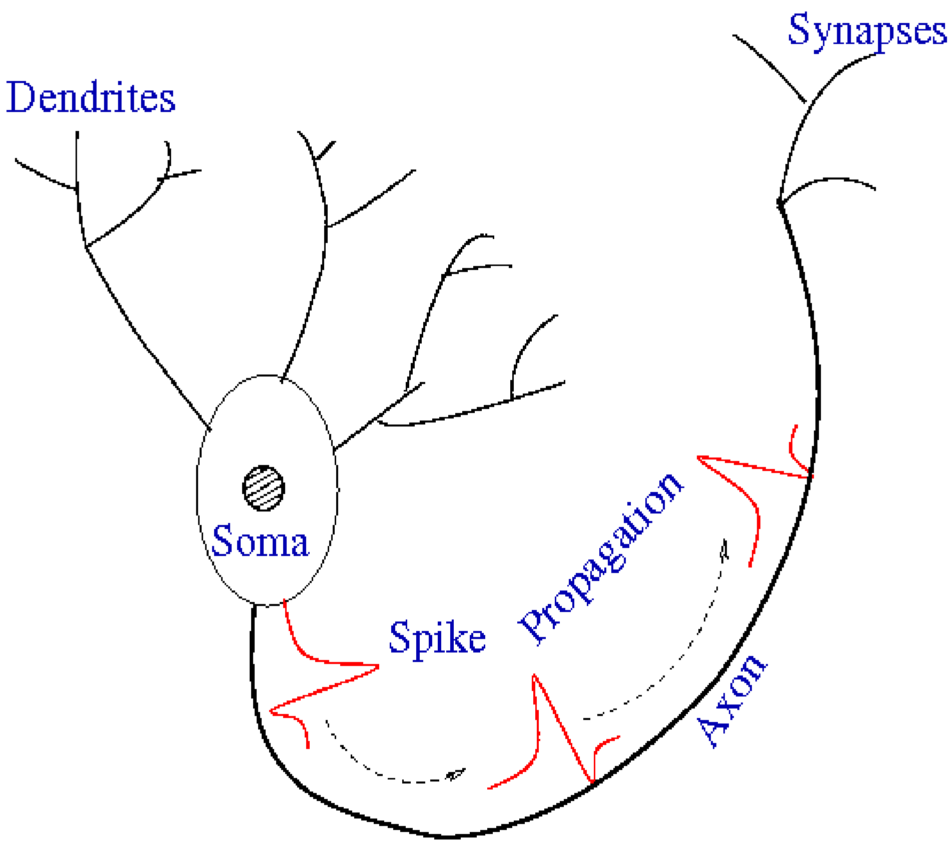
Mean field models.

Neurons and synapses.

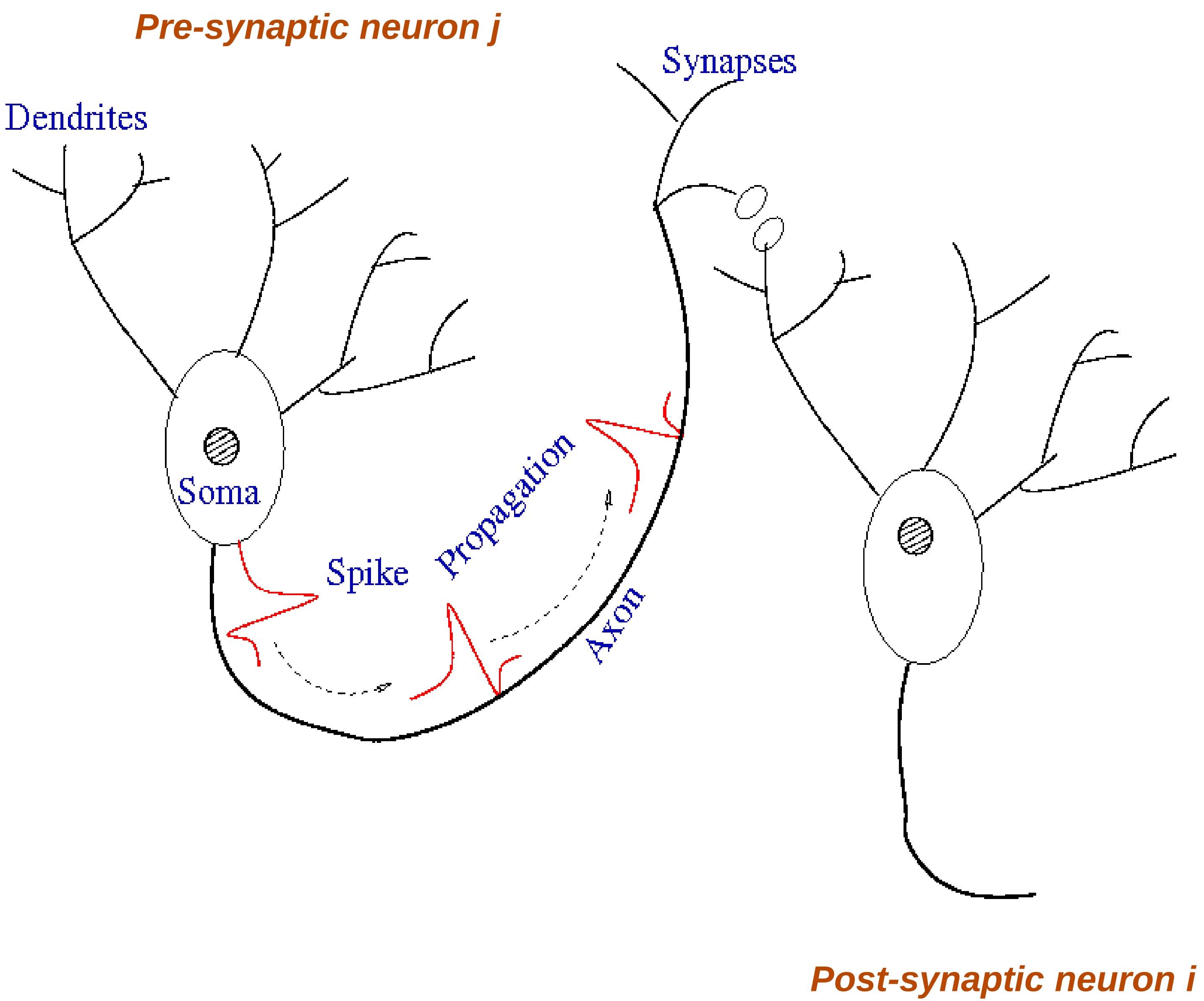
Neurons and synapses.



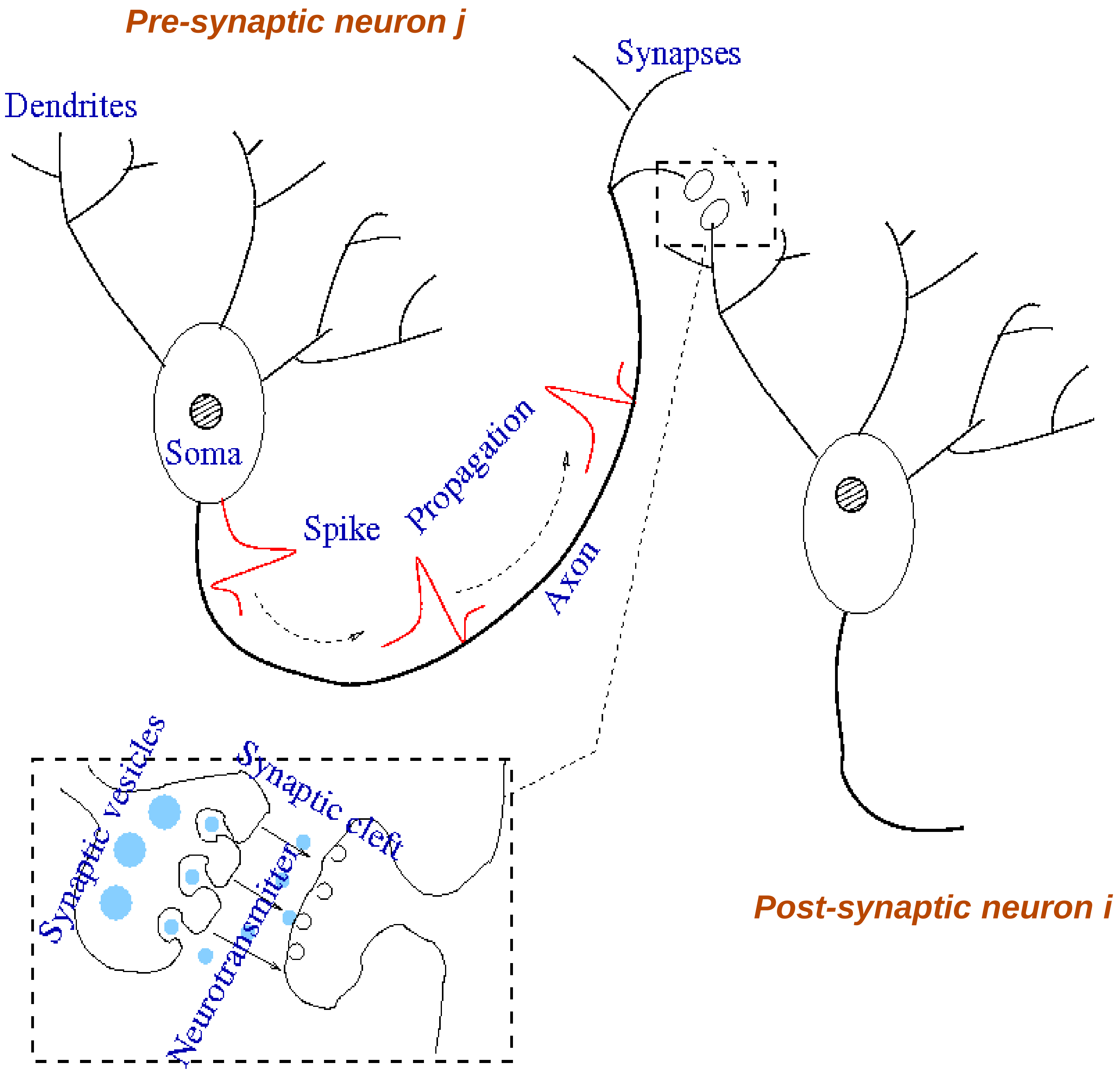
Neurons and synapses.



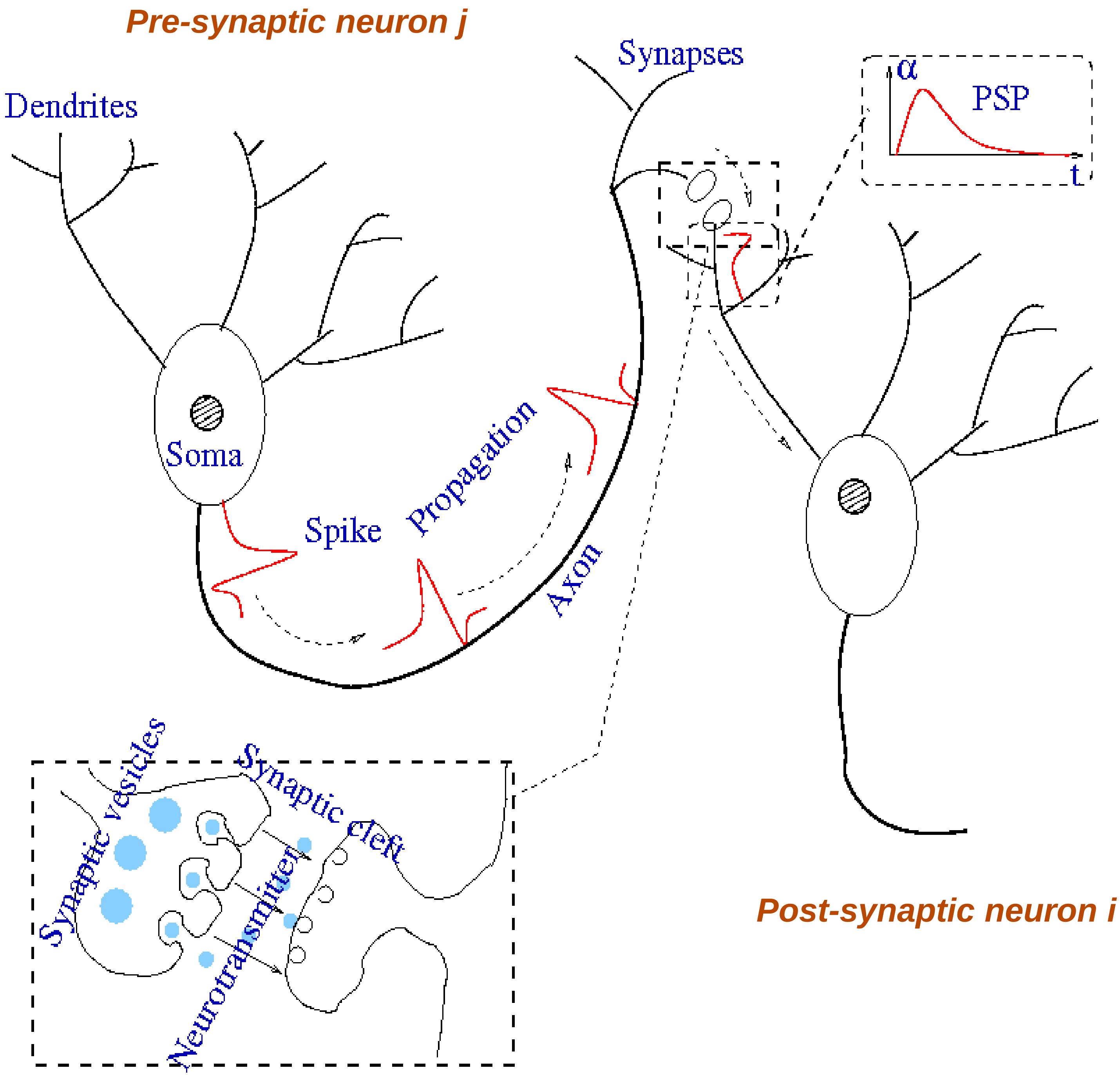
Neurons and synapses.



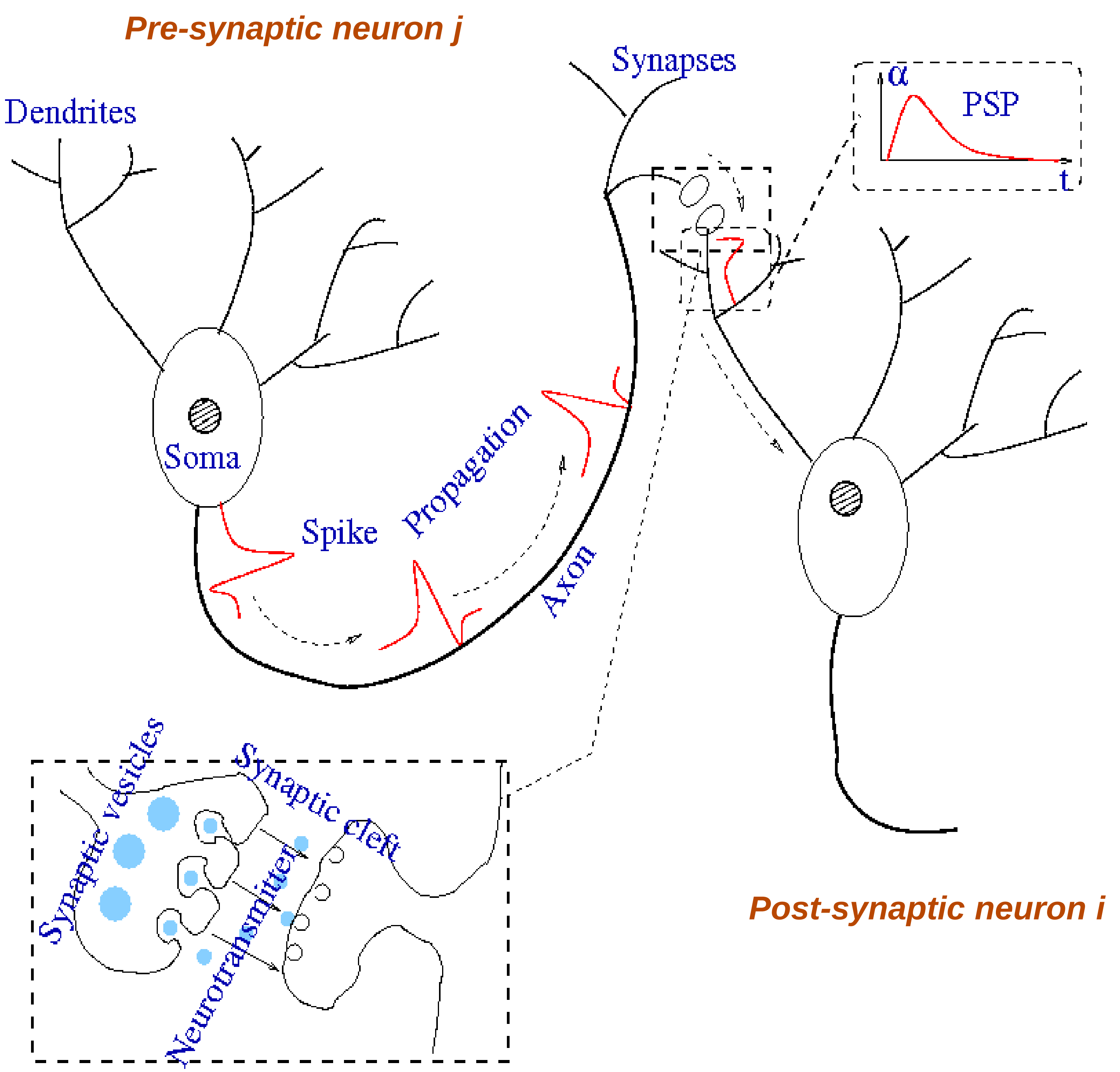
Neurons and synapses.



Neurons and synapses.

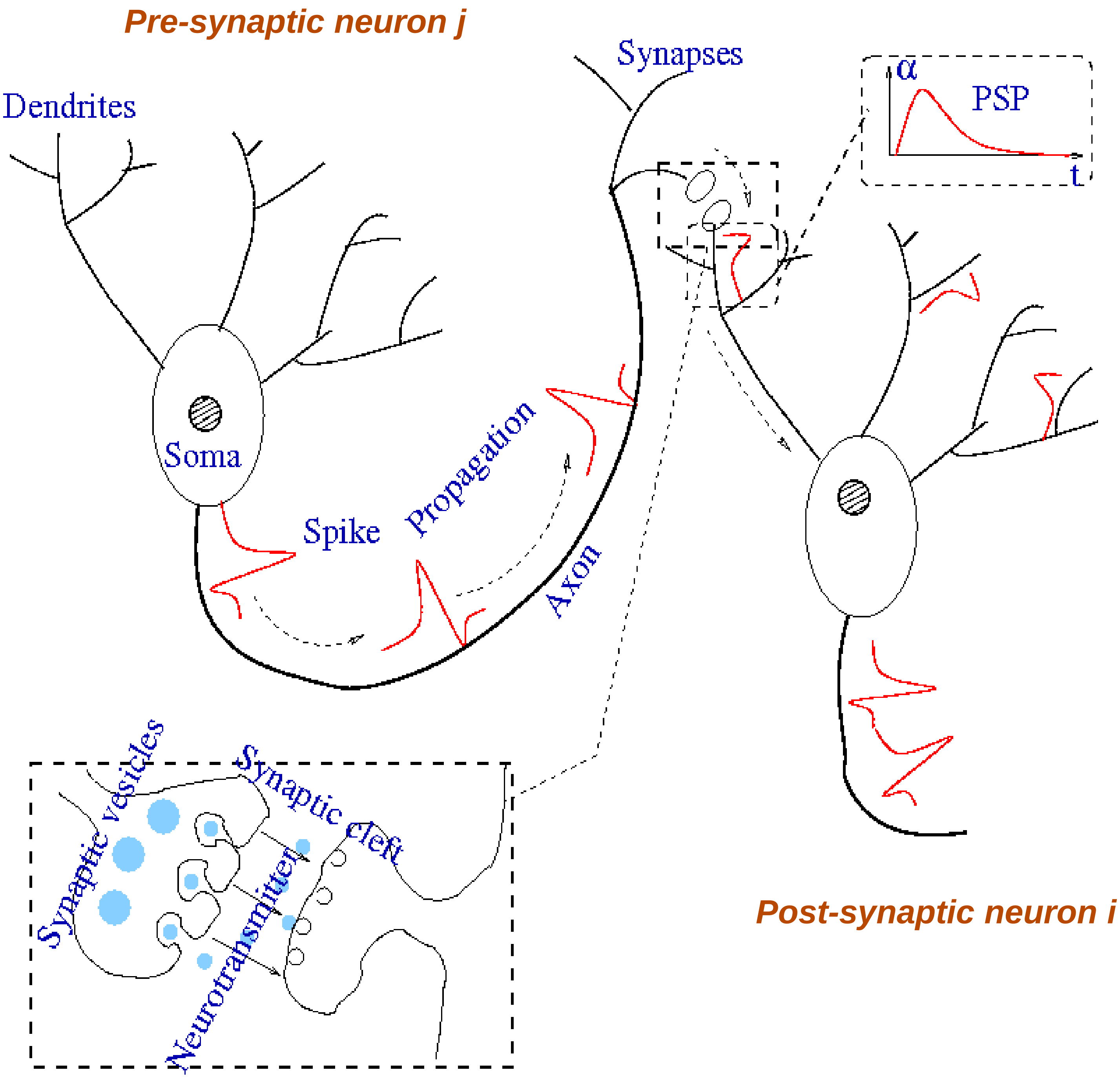


Neurons and synapses.



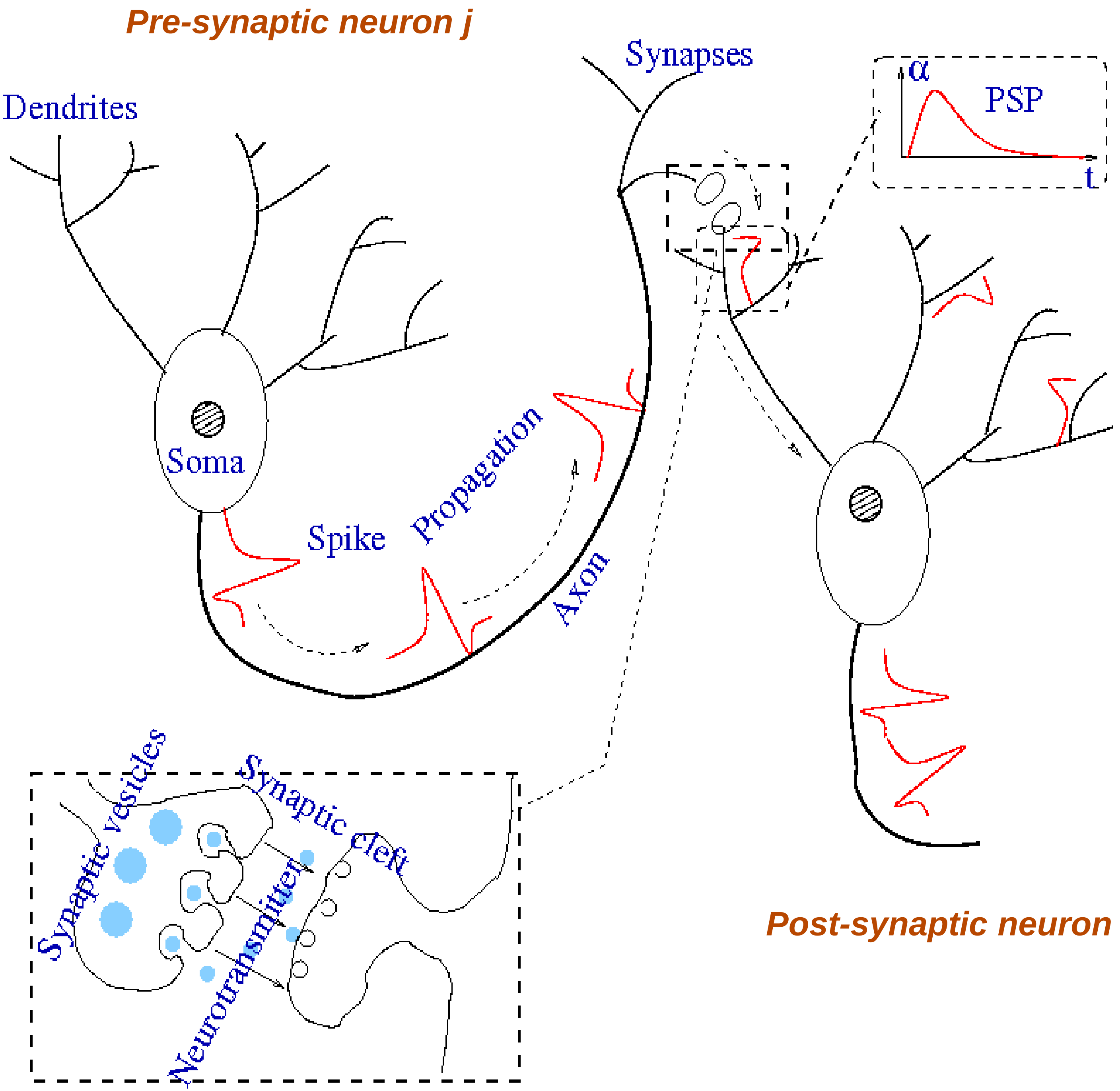
$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

Neurons and synapses.



$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

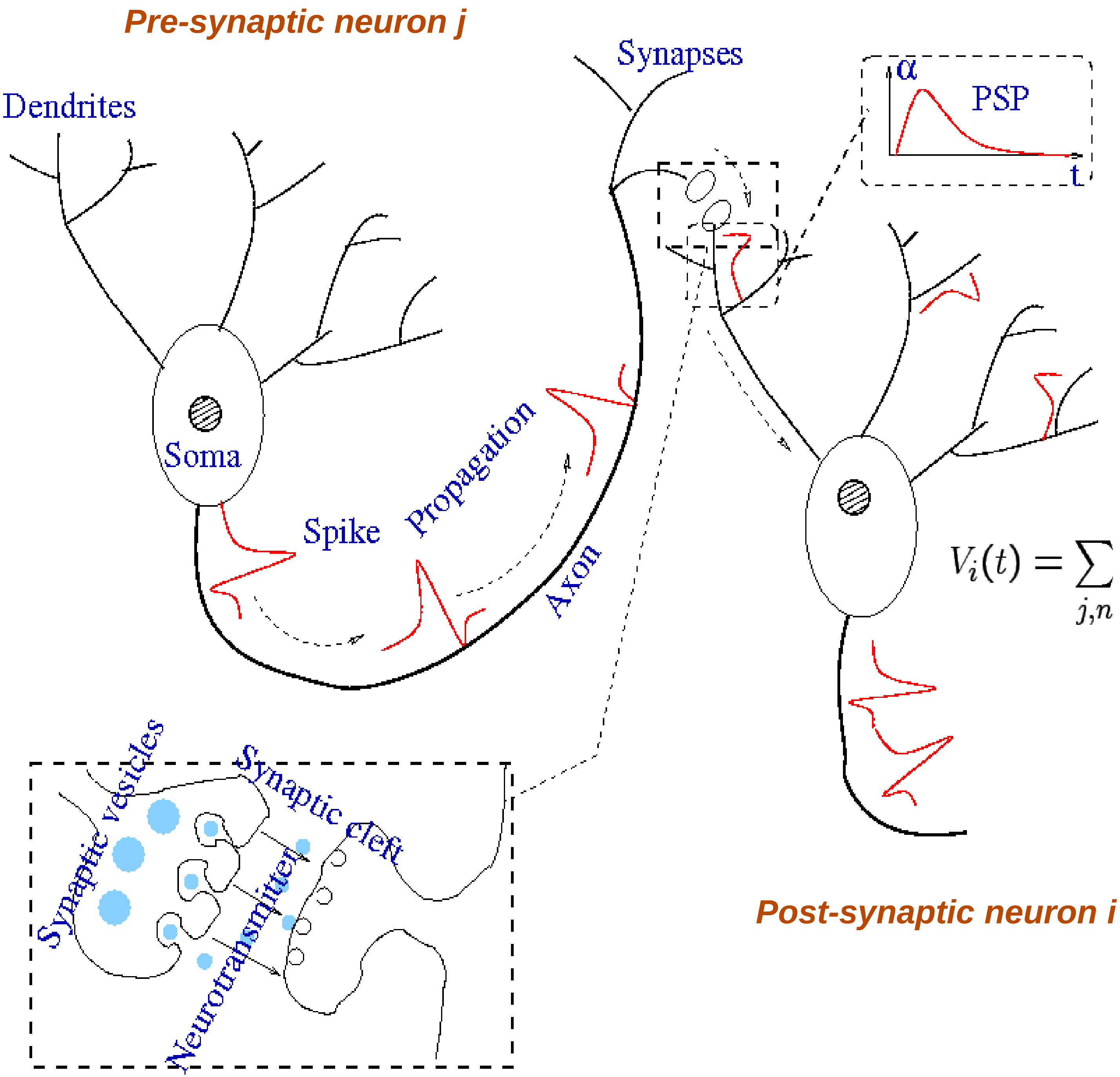
Neurons and synapses.



$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

$$\alpha_{ij}(t) = W_{ij}\alpha_i(t)$$

Neurons and synapses.

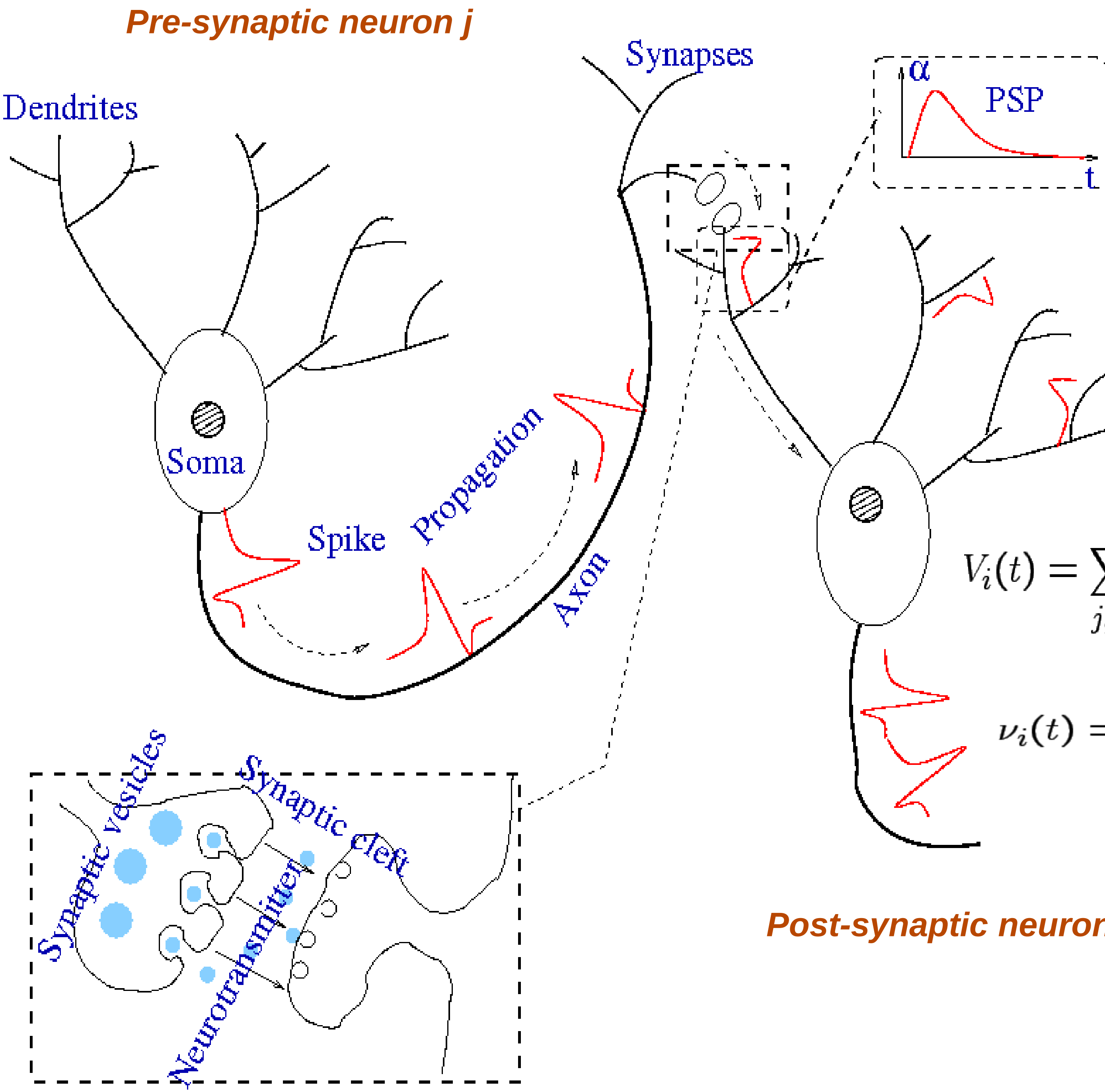


$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

$$\alpha_{ij}(t) = W_{ij}\alpha_i(t)$$

$$V_i(t) = \sum_{j,n} \alpha_{ij}(t - t_j^{(n)})$$

Neurons and synapses.

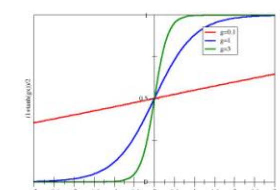


$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

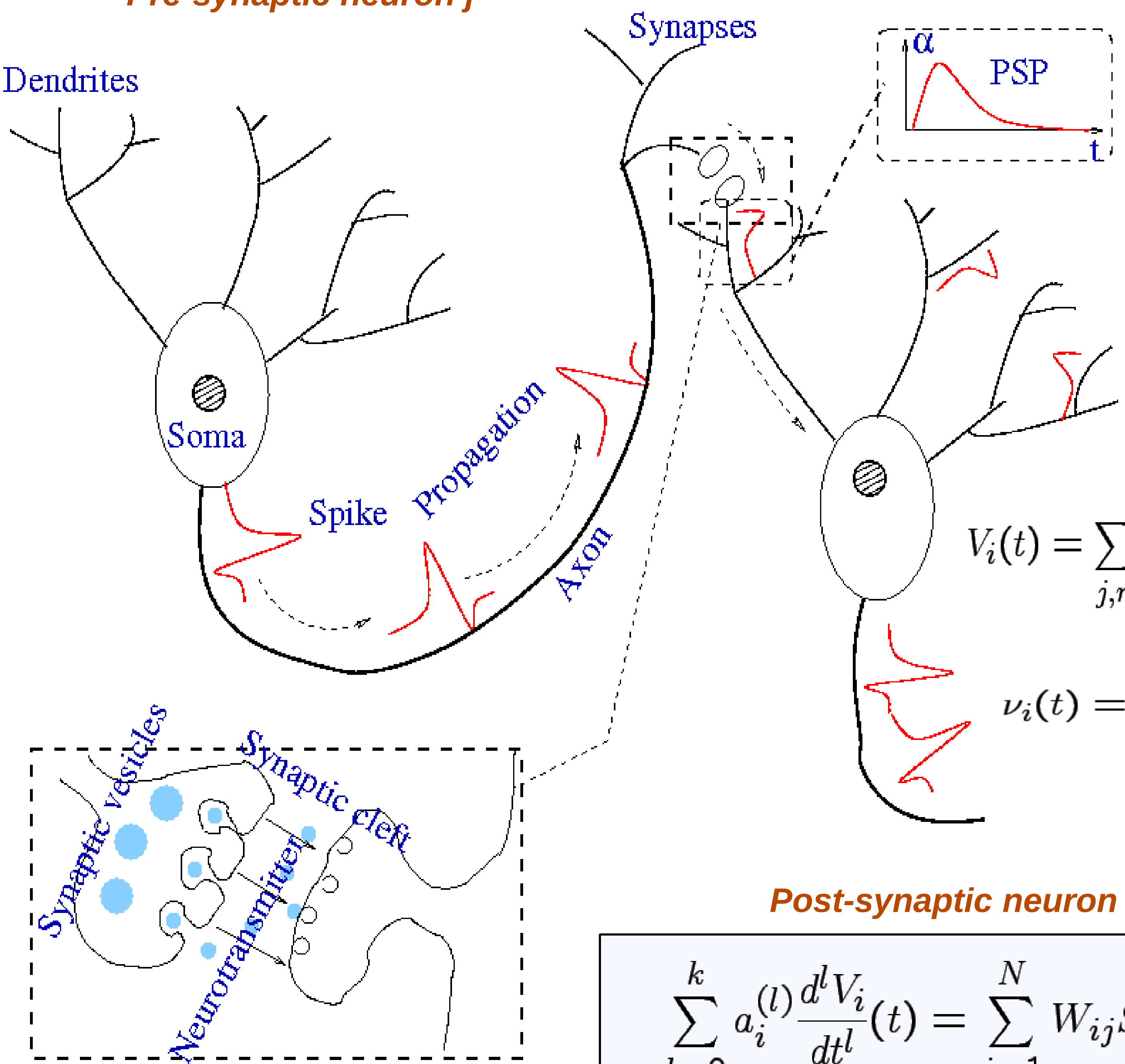
$$\alpha_{ij}(t) = W_{ij}\alpha_i(t)$$

$$V_i(t) = \sum_{j,n} \alpha_{ij}(t - t_j^{(n)})$$

$$\nu_i(t) = S_i(V_i(t))$$



Neurons and synapses.

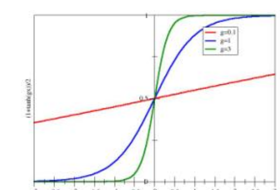


$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l \alpha_i}{dt^l}(t) = \delta(t)$$

$$\alpha_{ij}(t) = W_{ij}\alpha_i(t)$$

$$V_i(t) = \sum_{j,n} \alpha_{ij}(t - t_j^{(n)})$$

$$\nu_i(t) = S_i(V_i(t))$$

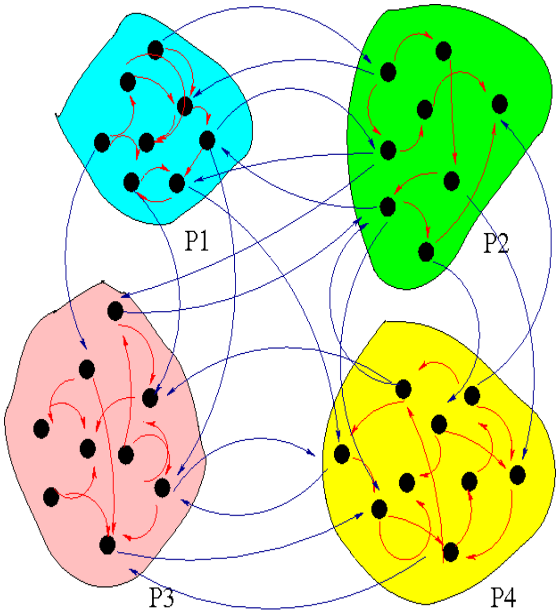


Post-synaptic neuron i

$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{j=1}^{N} W_{ij} S_j(V_j(t)) + I_i(t) + B_i(t).$$

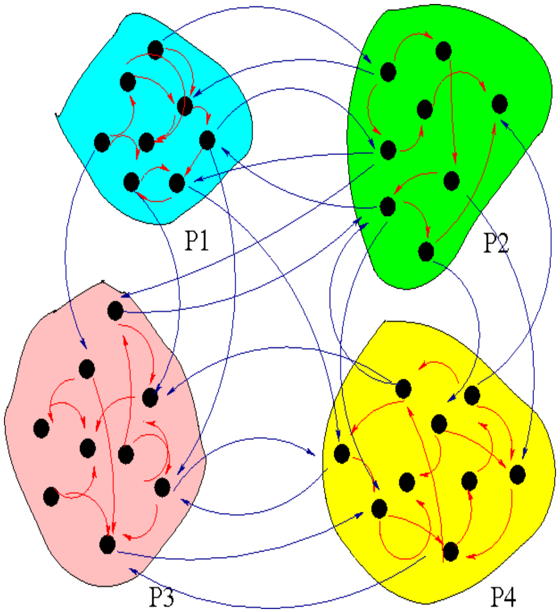
Neural mass model.

Neural mass model.



P populations of neurons, a =1 ... P

Neural mass model.

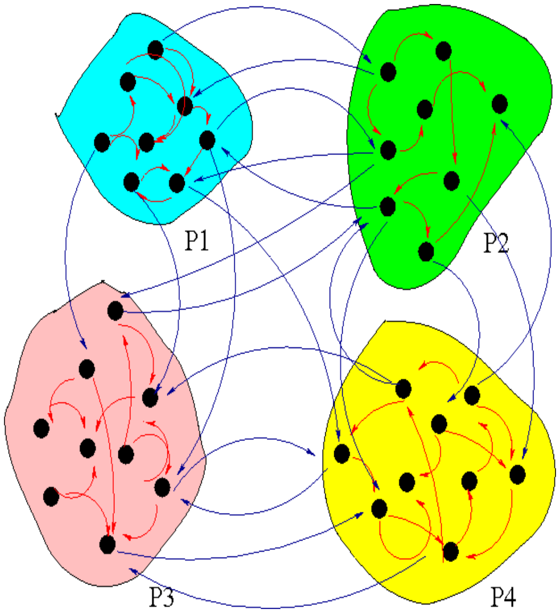


P populations of neurons, a =1 ... P

Voltage-based model

$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{j=1}^{N} W_{ij} S_j(V_j(t)) + I_i(t) + B_i(t).$$

Neural mass model.



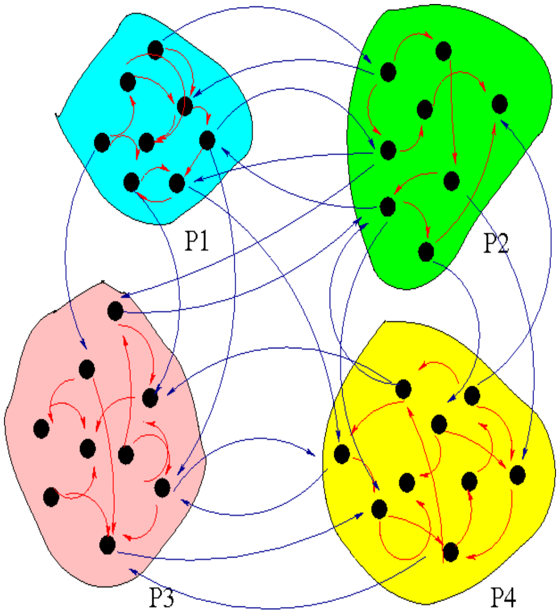
P populations of neurons, a =1 ... P

Voltage-based model

$$\sum_{l=0}^{k} a_i^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{j=1}^{N} W_{ij} S_j(V_j(t)) + I_i(t) + B_i(t).$$

Assumptions:

Neural mass model.



P populations of neurons, a =1 ... P

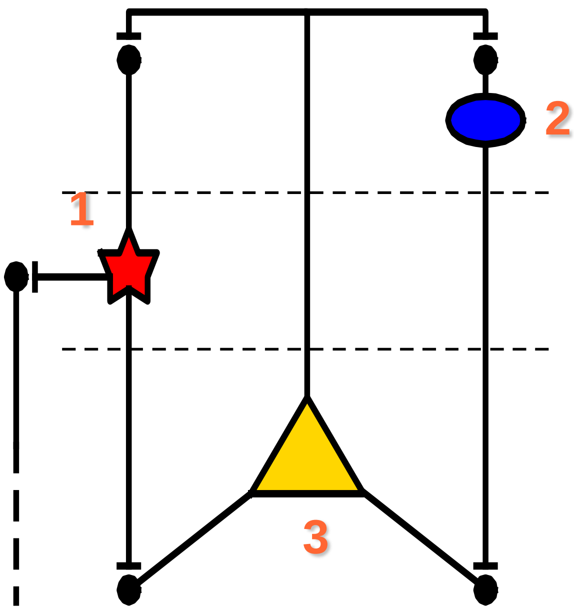
Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Assumptions:

- **Synapse response, current and noise depend only on the neuronal population.**

Neural mass model.



P populations of neurons, a =1 ... P

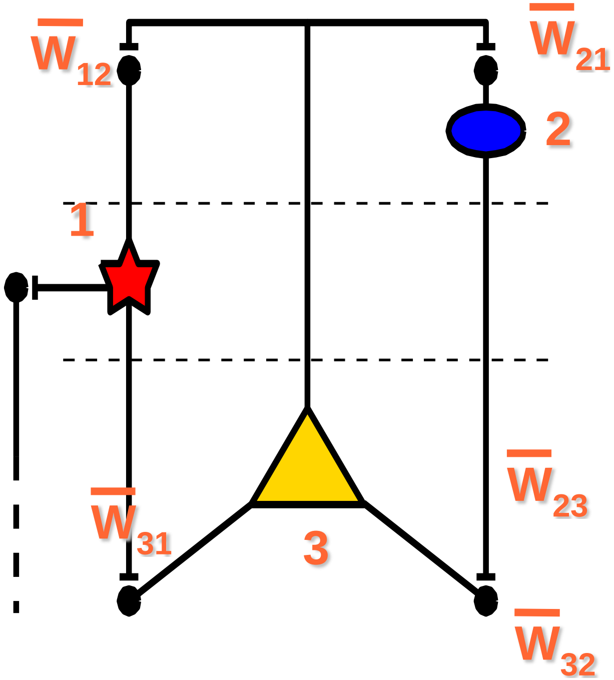
Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Assumptions:

- Synapse response, current and noise depend only on the neuronal population.

Neural mass model.



P populations of neurons, a =1 ... P

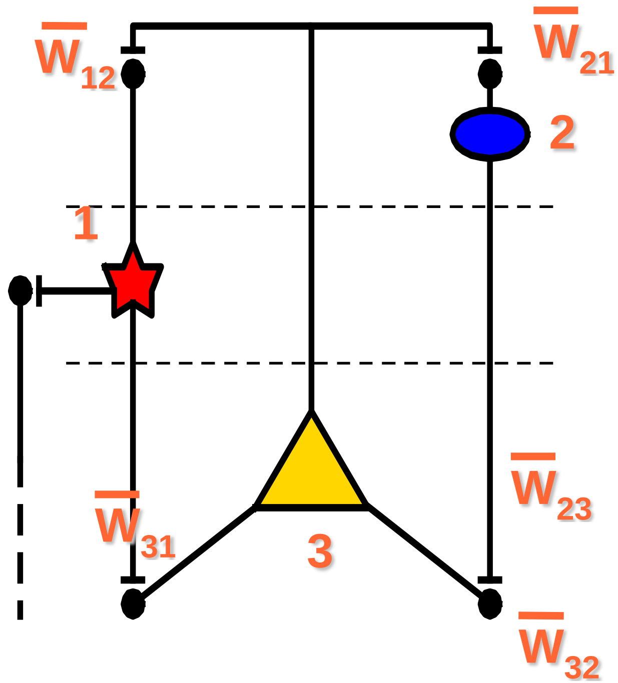
Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Assumptions:

- Synapse response, current and noise depend only on the neuronal population.

Neural mass model.



P populations of neurons, a =1 ... P

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Synaptic weights

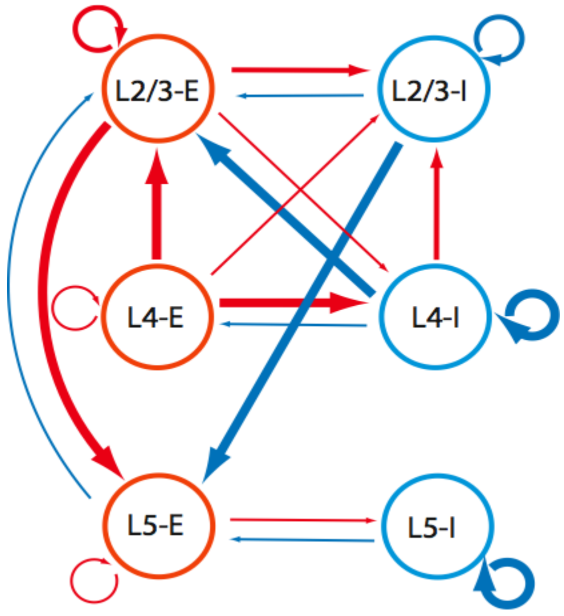
$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

(independent)

Assumptions:

- Synapse response, current and noise depend only on the neuronal population.
- The probability distribution of synaptic efficacies depend only on pre- and post synaptic neuron' population

Neural mass model.



P populations of neurons, a =1 ... P

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Synaptic weights

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

(**independent**)

Assumptions:

- **Synapse response, current and noise depend only on the neuronal population.**
- **The probability distribution of synaptic efficacies depend only on pre- and post synaptic neuron' population**

Dynamic mean-field theory.

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Random (quenched)

Nonlinear

Stochastic (annealed)

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t).$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \left[\sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) \right] + I_a(t) + B_a(t).$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \left[\sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) \right] + I_a(t) + B_a(t).$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \frac{\bar{W}_{ab}}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)).$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \frac{\bar{W}_{ab}}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \rightarrow \phi_b((t))$$

$$N_b \rightarrow \infty$$

Dynamic mean-field equations.

$$\frac{1}{N_a} \sum_{i=1}^{N_a} V_i(t) \rightarrow V_a(t)$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\; i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab}\phi_b(t)$$

$$\frac{1}{N_b}\sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Dynamic mean-field equations.

$$\frac{1}{N_a}\sum_{i=1}^{N_a} V_i(t) \to V_a(t) \longrightarrow \sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} \bar{W}_{ab}\phi_b(t) + I_a(t) + B_a(t),\; a = 1 \ldots P,$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Naive mean-field equations.

$$\frac{1}{N_a} \sum_{i=1}^{N_a} V_i(t) \to V_a(t) \longrightarrow \sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t) + I_a(t) + B_a(t),\ a = 1 \ldots P,$$

$$\phi_b(t) = S_b(V_b(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Naive mean-field equations.

$$\frac{1}{N_a} \sum_{i=1}^{N_a} V_i(t) \to V_a(t)$$

$$\phi_b(t) = S_b(V_b(t))$$

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} \bar{W}_{ab} S_b(V_b(t)) + I_a(t) + B_a(t),\ a = 1 \ldots P,$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$
$$N_b \to \infty$$

Naive mean-field equations.

$$\frac{1}{N_a} \sum_{i=1}^{N_a} V_i(t) \to V_a(t)$$

$$\phi_b(t) = S_b(V_b(t))$$

$$\longrightarrow \sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} \bar{W}_{ab} S_b(V_b(t)) + I_a(t) + B_a(t),\ a = 1 \ldots P,$$

$$\lim_{N_b \to \infty} \frac{1}{N_b} \sum_{i=1}^{N_b} S_b\left(V_i(t)\right) = S_b\left(\lim_{N_b \to \infty} \frac{1}{N_b} \sum_{i=1}^{N_b} V_i(t)\right)$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Local interactions field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \bar{W}_{ab} \phi_b(t)$$

$$\frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) \to \phi_b((t))$$

$$N_b \to \infty$$

Naive mean-field equations.

$$\frac{1}{N_a} \sum_{i=1}^{N_a} V_i(t) \to V_a(t)$$

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} \bar{W}_{ab} S_b(V_b(t)) + I_a(t) + B_a(t),\ a = 1 \ldots P,$$

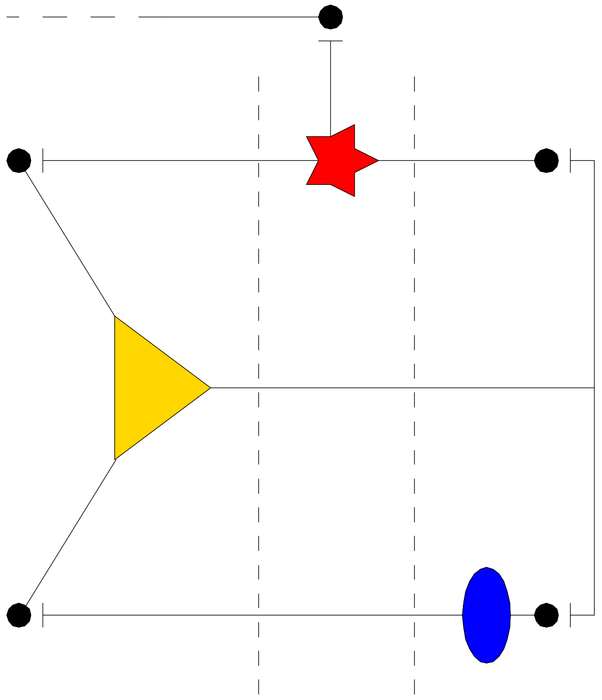
$$\phi_b(t) = S_b(V_b(t))$$

$$\lim_{N_b \to \infty} \frac{1}{N_b} \sum_{i=1}^{N_b} S_b\left(V_i(t)\right) = S_b\left(\lim_{N_b \to \infty} \frac{1}{N_b} \sum_{i=1}^{N_b} V_i(t)\right)$$

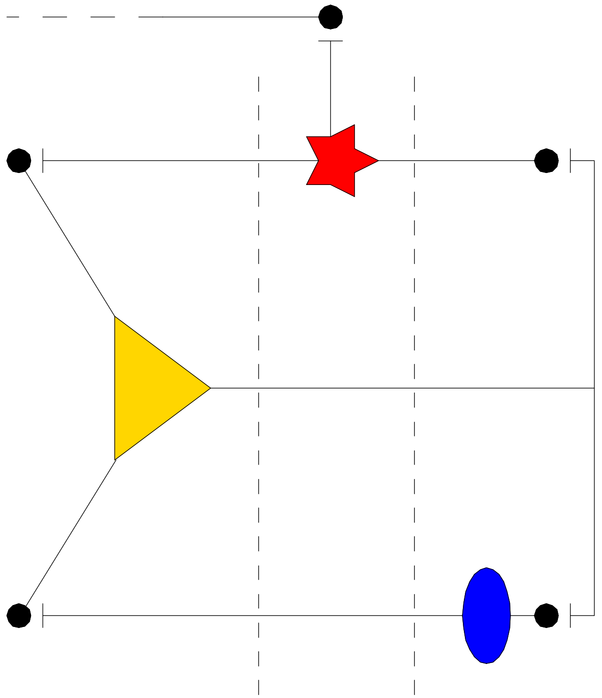
?

Jansen-Ritt equations.

Jansen-Ritt equations.



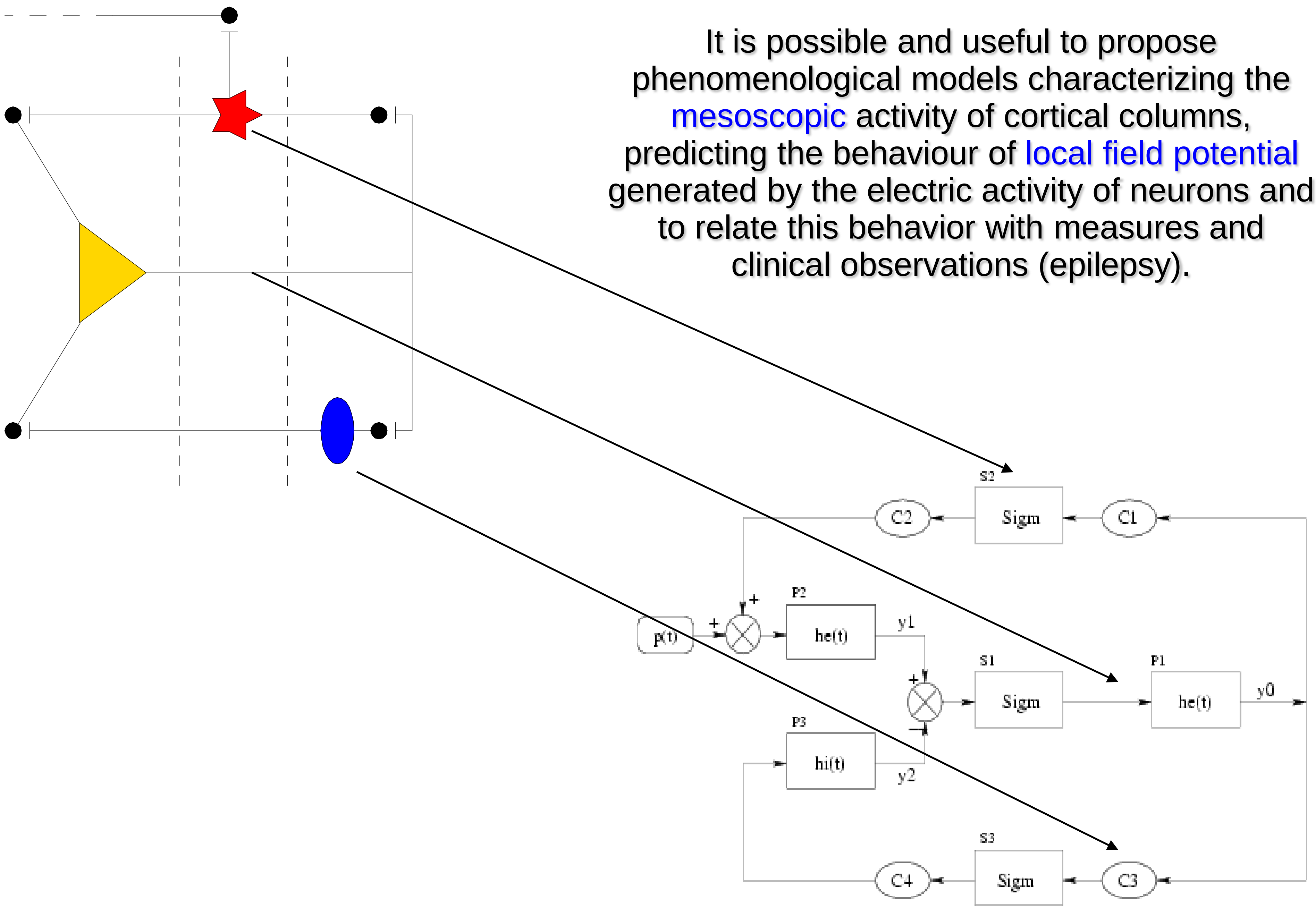
Jansen-Ritt equations.



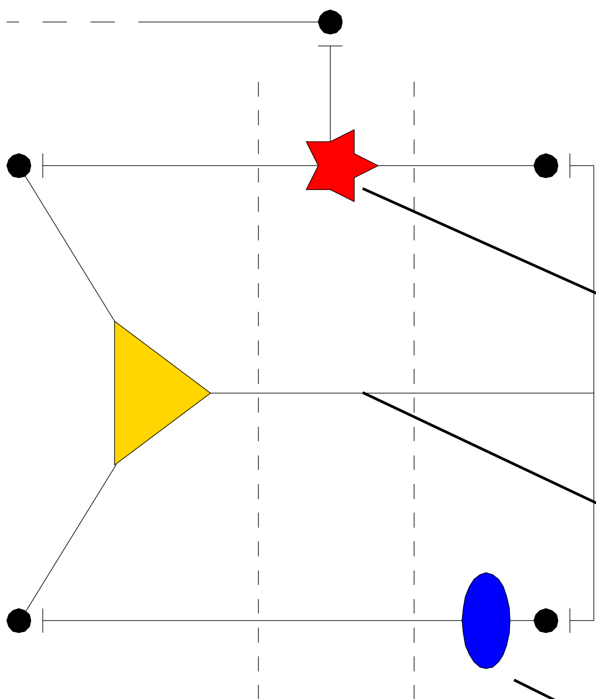
It is possible and useful to propose phenomenological models characterizing the mesoscopique activity of cortical columns, predicting the behaviour of local field potential generated by the electric activity of neurons and to relate this behavior with measures and clinical observations (epilepsy).

Jansen-Ritt equations.

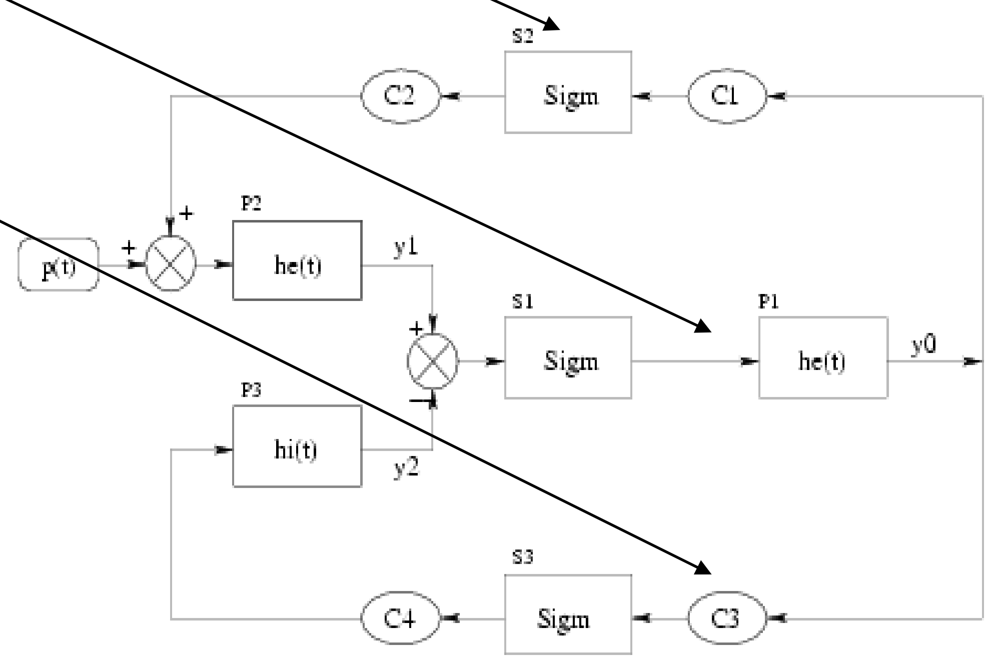
It is possible and useful to propose phenomenological models characterizing the mesoscopic activity of cortical columns, predicting the behaviour of local field potential generated by the electric activity of neurons and to relate this behavior with measures and clinical observations (epilepsy).



Jansen-Ritt equations.



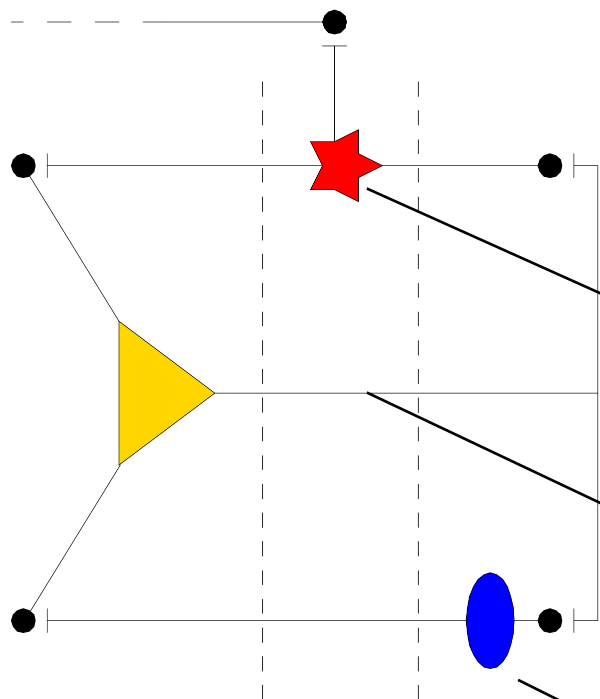
It is possible and useful to propose phenomenological models characterizing the mesoscopique activity of cortical columns, predicting the behaviour of local field potential generated by the electric activity of neurons and to relate this behavior with measures and clinical observations (epilepsy).



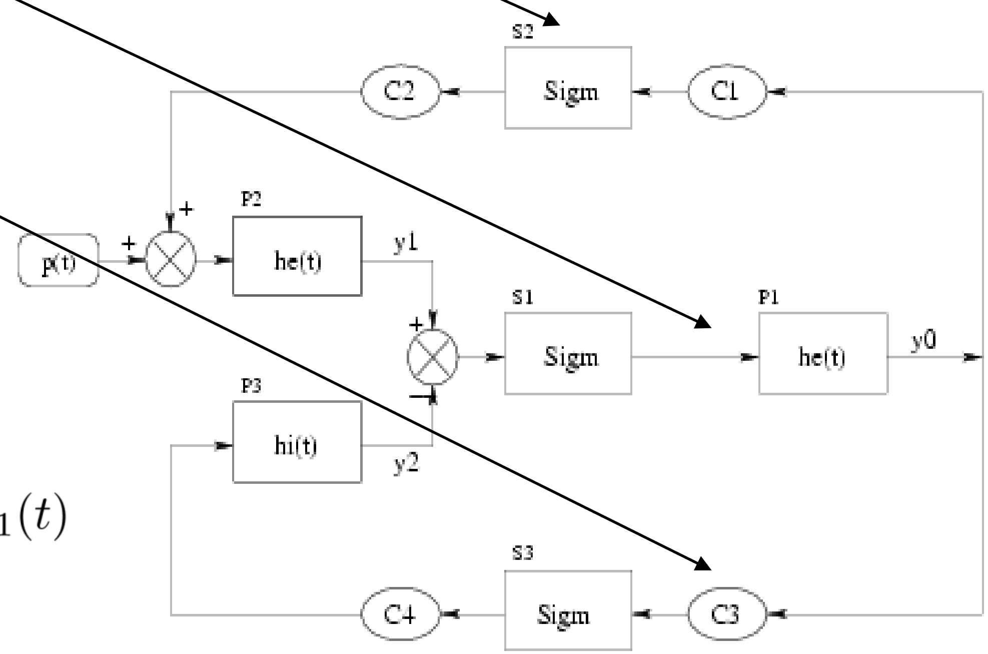
Jansen – Rit model (1995).

$$\begin{cases} \dot{y}_0 = -a y_0(t) + A f(y_1(t) - y_2(t)), \\ \dot{y}_1 = -a y_1(t) + A\left[p(t) + C_2 f(C_1 y_0(t))\right], \\ \dot{y}_2 = -b y_2(t) + B C_4 f(C_3 y_0(t)). \end{cases}$$

Jansen-Ritt equations.



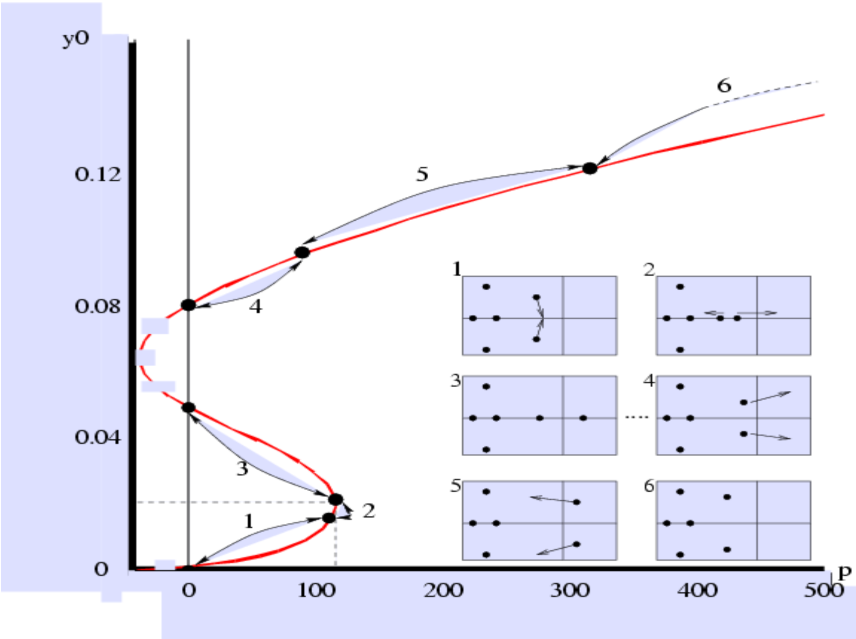
It is possible and useful to propose phenomenological models characterizing the mesoscopique activity of cortical columns, predicting the behaviour of local field potential generated by the electric activity of neurons and to relate this behavior with measures and clinical observations (epilepsy).



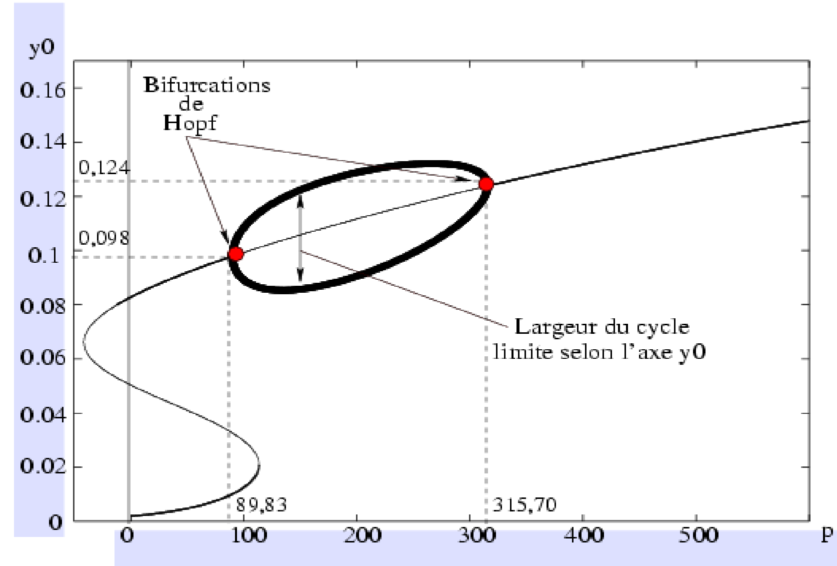
Jansen – Rit model (1995).

$$
\begin{cases}
\dot{y}_0(t) = y_3(t) \\
\dot{y}_3(t) = a(ASig(y_1(t) - y_2(t)) - 2y_3(t) - ay_0(t)) \\
\dot{y}_1(t) = y_4(t) \\
\dot{y}_4(t) = a(A\,(p + C_2 Sig(C_1 y_0(t)))\, - 2y_4(t) - ay_1(t) \\
\dot{y}_2(t) = y_5(t) \\
\dot{y}_5(t) = b(BC_4 Sig(C_3 y_0(t)) - 2y_5(t) - by_2(t)
\end{cases}
$$

Bifurcation analysis

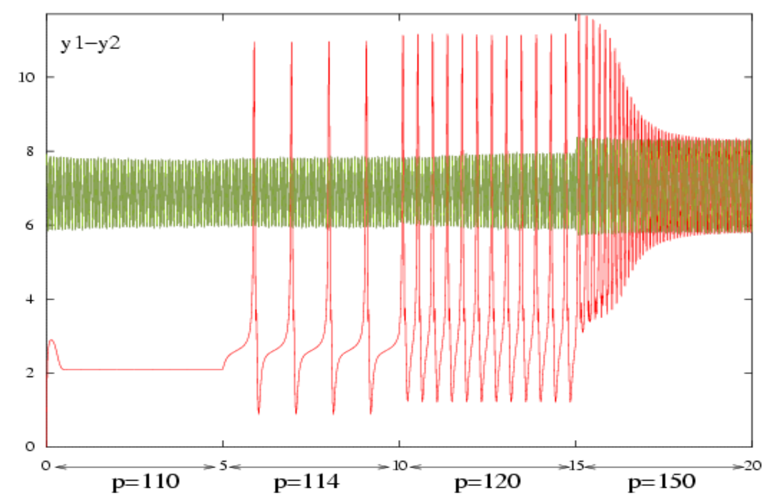


Eigenvalues of the Jacobian of the dynamic system when the thalamic entry varies



Some bifurcations of the system

The bifurcations can predict neural behaviours



Grimbert, Faugeras. Neural Computation 2006

Grimbert, Faugeras:06b, "Bifurcation Analysis of Jansen's Neural Mass Model",Neural Computation, 18, 2006.

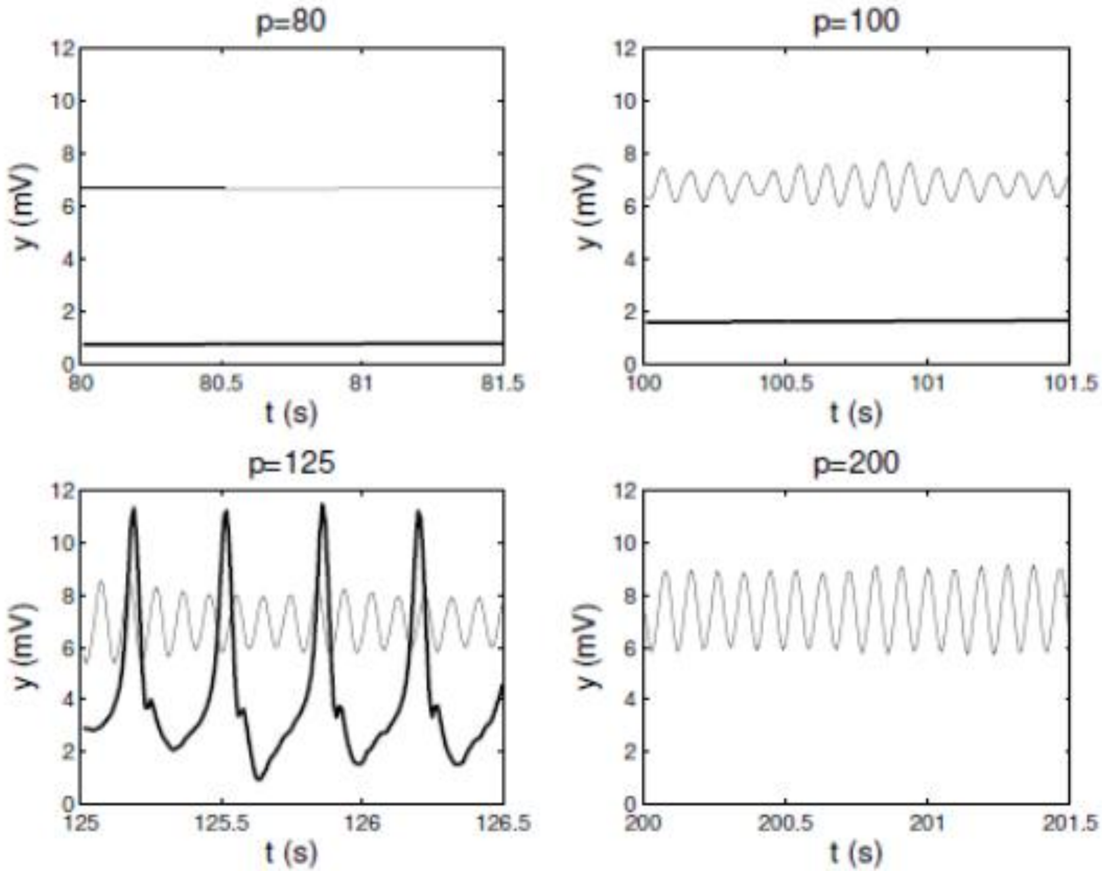


Figure 6: *Activities produced by Jansen's neural mass model for typical values of the input parameter* p *(see text). The thin (respectively thick) curves are the time courses of the output* y *of the unit in its upper (respectively lower) state. For* $p > 137.38$*, there is only one possible behaviour of the system. Note: in the case of oscillatory activities we added a very small amount of noise to* p *(a zero mean Gaussian noise with standard deviation* 0.05*).*

Grimbert, Faugeras:06b, "Bifurcation Analysis of Jansen's Neural Mass Model",Neural Computation, 18, 2006.

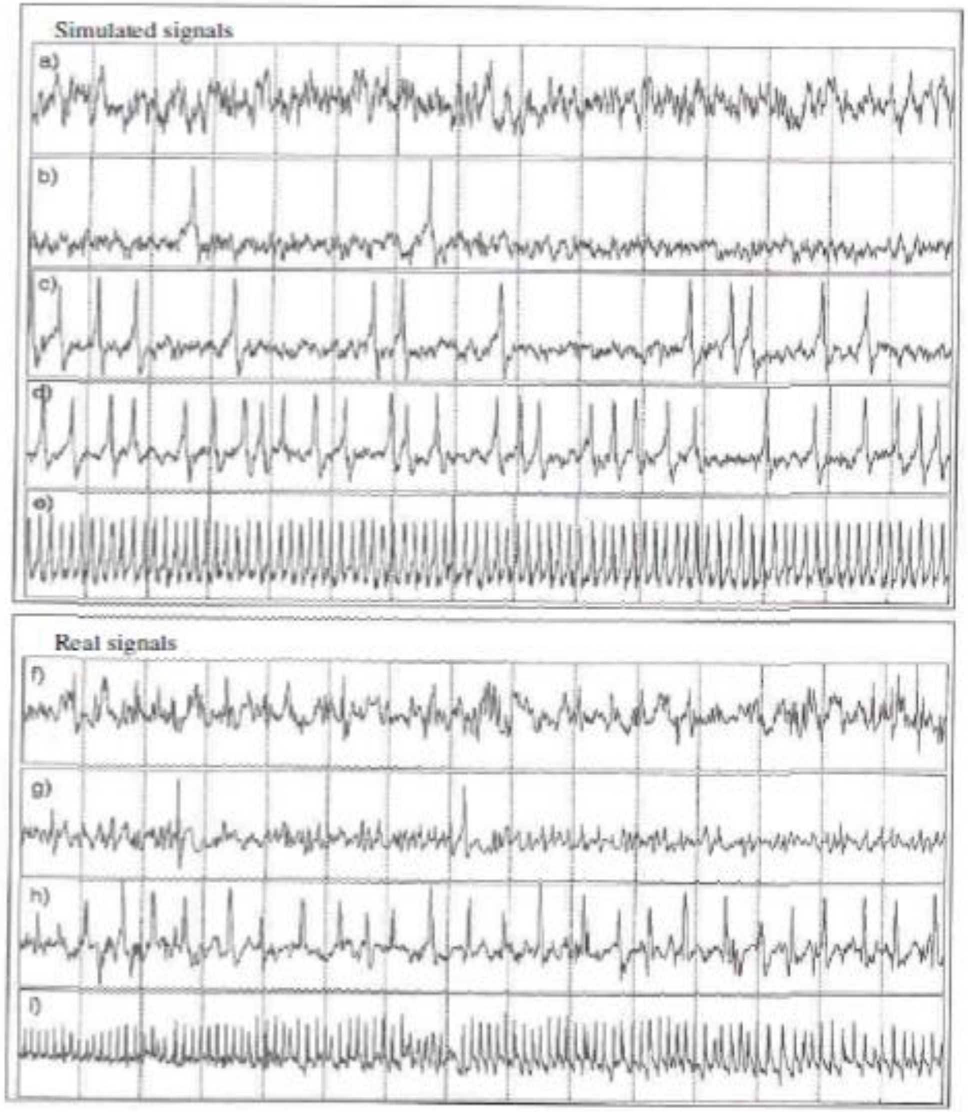


Figure 3: *(a)-(e) Activities of the unit shown in figure 1 when simulated with a white Gaussian noise as input (corresponding to an average firing rate between 30 and 150 Hz). The authors varied the excitation/inhibition ratio A/B. As this ratio is increased we observe sporadic spikes followed by increasingly periodic activities. (f)-(i) Real activities recorded from epileptic patients before (f,g) and during a seizure (h,i) (From [Wendling et al., 2000]).*

Touboul, Chauvel, Wendling, Faugeras, "Neural Mass Activity, Bifurcations and Epilepsy", Neural Computation, 23:12, 2011.

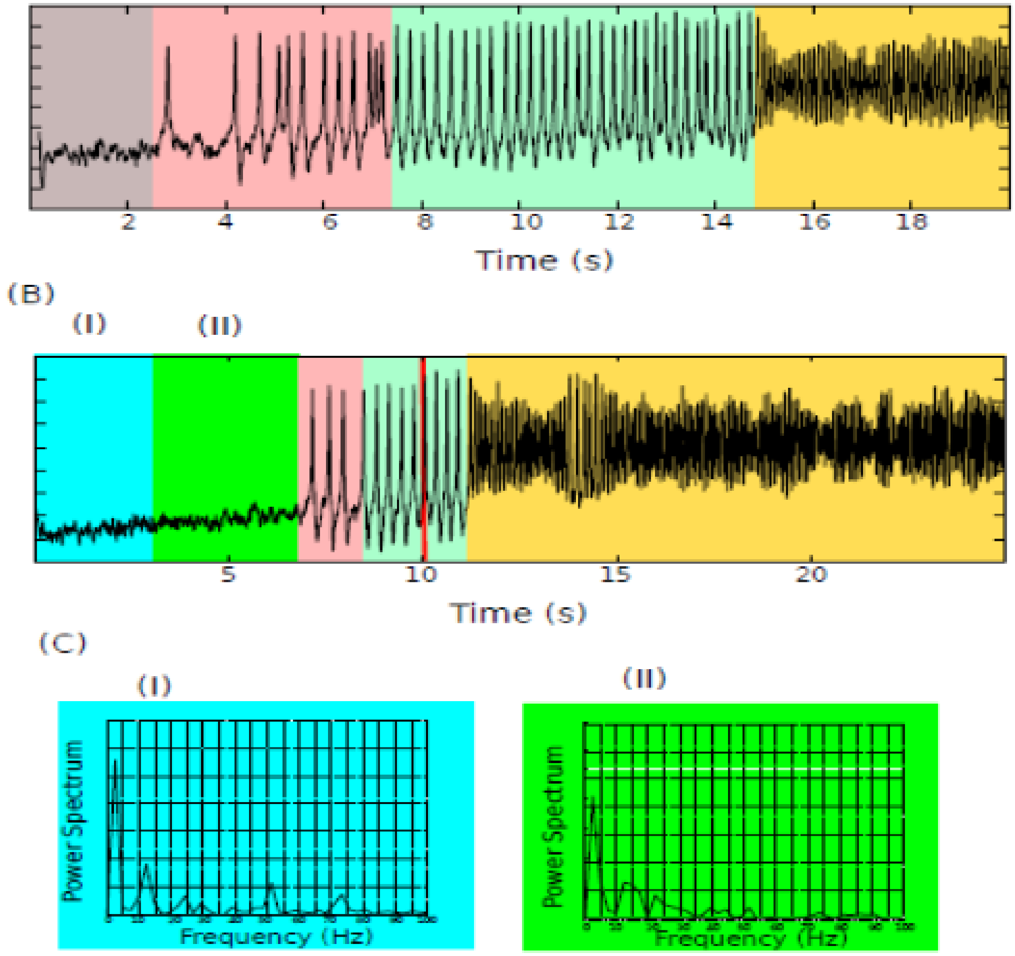


Figure 11: A seizure in the slightly hyperinnerved case ($j = 12.7$). Input $P = \mu_0 + 10^{-3}t + \sigma B_t$ where B_t is a Brownian motion, (A): $\mu_0 = 1.5$ and (B): $\mu_0 = 1.2$ in order to see the different pre-onset phases. $\sigma = 0.4$. (A): (B) Same parameters, with simulation of anti-GABA convulsant injection (decrease α_2 from 0.25 to 0.23) at a time shown by the red bar. This has the effect of stopping rhythmic spike and triggers the seizure alpha activity phase. The differences in pre-onset activity are evidenced during the pre-onset phase by the power spectra (C).

Why is it working so well ?

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Prop 1.

*η_i is V-conditionally **Gaussian** with mean and covariance.*

$$E\left[\eta_i(t) | V\right] = \sum_{b=1}^{P} \bar{W}_{ab} \frac{1}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t))$$

$$Cov\left[\eta_i(t)\eta_j(s) | V\right] = \delta_{ij} \sum_{b=1}^{P} \frac{\sigma_{ab}^2}{N_b} \sum_{j=1}^{N_b} S_b(V_j(t)) S_b(V_j(s))$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{\bar{W}}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

DMFT

$$\xrightarrow[N_b \to \infty]{}$$

$$\eta_a(t) = \sum_{b=1}^{P} U_{ab}$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

DMFT

$$\xrightarrow[N_b \to \infty]{}$$

$$\eta_a(t) = \sum_{b=1}^{P} U_{ab}$$

where U_{ab} is a Gaussian process with mean and covariance

$$E\left[U_{ab}(t)\right] = \bar{W}_{ab} E\left[S_b(V_b(t))\right];$$

$$Cov(U_{ab}(t), U_{ab}(s)) = \sigma_{ab}^2 E[S_b(V_b(t)) S_b(V_b(s))] \delta_{ac;bd}$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) \xrightarrow[N_b \to \infty]{\text{DMFT}} \eta_a(t) = \sum_{b=1}^{P} U_{ab}$$

where U_{ab} is a Gaussian process with mean and covariance

$$E\left[U_{ab}(t)\right] = \bar{W}_{ab} E\left[S_b(V_b(t))\right];$$

$$Cov(U_{ab}(t), U_{ab}(s)) = \sigma_{ab}^2 E[S_b(V_b(t)) S_b(V_b(s))] \delta_{ac;bd}$$

Dynamic mean-field equations.

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} U_{ab}(t) + I_a(t) + B_a(t)$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x \frac{\sqrt{v_b(u) v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y \frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a} [v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} W_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} W_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t)V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t)V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} {}^{2} \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x \frac{\sqrt{v_b(u) v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y \frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b},\ i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t)V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \cdots^{2} \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) +$$ [redacted] $$), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) +\right.$$ [redacted] $$\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P}$$ [redacted] $$\int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv \left.\right]$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + \sigma_a \eta_i(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t)V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a \sigma_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\sigma_{ab} = 0 \Rightarrow C_{ab}(t,s) = 0 \Rightarrow v_b(t) = 0$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + \quad , \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a \sigma_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\sigma_{ab} = 0 \Rightarrow C_{ab}(t,s) = 0 \Rightarrow v_b(t) = 0$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + \dots(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, \; i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} S_b\left(\mu_b(t)\right) + I_a(t)$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\sigma_{ab} = 0 \Rightarrow C_{ab}(t,s) = 0 \Rightarrow v_b(t) = 0$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Non random synaptic weights.

$$W_{ij} = \frac{\bar{W}_{ab}}{N_b}, i \in a, j \in b.$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} S_b\left(\mu_b(t)\right) + I_a(t)$$

Naive mean-field equations.

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{\bar{W}}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x \frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y \frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Non Markovian process, depending on the whole past.

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$v_a(t) = Var\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Simple model

$$\frac{dV_i}{dt} = -\frac{V_i}{\tau_a} + \sum_{b=1}^{P}\sum_{j=1}^{N_b} W_{ij} S_b(V_j(t)) + I_a(t) + B_a(t), \quad i \in a$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Non Markovian process, depending on the whole past.

Dynamic mean-field equations.

$$\mu_a(t) = E\left[V_a(t)\right]$$

$$\frac{d\mu_a}{dt} = -\frac{\mu_a}{\tau_a} + \sum_{b=1}^{P} \bar{W}_{ab} \int_{-\infty}^{+\infty} S_b\left(h\sqrt{v_b(t)} + \mu_b(t)\right) Dh + I_a(t)$$

$$Dh = \frac{e^{-\frac{h^2}{2}}}{\sqrt{2\pi}} dh$$

$$v_a(t) = Var\left[V_a(t)\right]$$

Evolution is not ruled anymore by EDOs but by a mapping on a space of trajectories.

$$C_{ab}(t,s) = Cov\left[V_a(t) V_b(s)\right]$$

$$C_{ab}(t,s) = \delta_{ab} e^{-(t+s)/\tau_a}\left[v_a(0) + \frac{\tau_a s_a^2}{2}\left(e^{\frac{2s}{\tau_a}} - 1\right) + \sum_{b=1}^{P} \sigma_{ab}^2 \int_0^t \int_0^s e^{(u+v)/\tau_a} \Delta_b(u,v)\, du\, dv\right]$$

$$\Delta_b(u,v) = \int_{R^2} S_b\left(x\frac{\sqrt{v_b(u)v_b(v) - C_{bb}(u,v)^2}}{\sqrt{v_b(v)}} + y\frac{C_{bb}(u,v)}{\sqrt{v_b(v)}} + \mu_b(u)\right) S_b\left(y\sqrt{v_b(v)} + \mu_b(v)\right) Dx\, Dy,$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Th. (Faugeras, Touboul, Cessac, 2008)

Dynamic mean-field equations.

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} U_{ab}(t) + I_a(t) + B_a(t)$$

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field equations.

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} U_{ab}(t) + I_a(t) + B_a(t)$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Th. (Faugeras, Touboul, Cessac, 2008)

- Existence and uniqueness of solutions in finite time.

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field equations.

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} U_{ab}(t) + I_a(t) + B_a(t)$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Th. (Faugeras, Touboul, Cessac, 2008)

- Existence and uniqueness of solutions in finite time.
- Existence and uniqueness of stationary solutions in a specific region of the macroscopic parameters space.

Dynamic mean-field theory.

Voltage-based model

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_i}{dt^l}(t) = \eta_i(V,t) + I_a(t) + B_a(t).$$

Local interaction field.

$$\eta_i(V,t) = \sum_{b=1}^{P} \sum_{j=1}^{N_b} W_{ij} S_b(V_j(t))$$

Dynamic mean-field equations.

$$\sum_{l=0}^{k} a_a^{(l)} \frac{d^l V_a}{dt^l}(t) = \sum_{b=1}^{P} U_{ab}(t) + I_a(t) + B_a(t)$$

Random synaptic weights.

$$W_{ij} \sim \mathcal{N}\left(\frac{\bar{W}_{ab}}{N_b}, \frac{\sigma_{ab}^2}{N_b}\right)$$

Th. (Faugeras, Touboul, Cessac, 2008)

- Existence and uniqueness of solutions in finite time.
- Existence and uniqueness of stationary solutions in a specific region of the macroscopic parameters space.
- Constructive proof => Simulation algorithm.

Are there new and measurable effects here ?

Are there new and measurable effects here ?

How to study DMFT equations and their bifurcations ?

Are there new and measurable effects here ?

How to study DMFT equations and their bifurcations ?

What is synaptic weights are correlated ?