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To cite this version:

Blaise Faugeras, Jacques Blum, Cedric Boulbe. EQUILIBRIUM RECONSTRUCTION FROM DISCRETE MAGNETIC MEASUREMENTS IN A TOKAMAK. 6th Inverse Problems, Control and Shape Optimization International Conference (PICOF’12), Apr 2012, Palaiseau, France. <hal-01323138>

HAL Id: hal-01323138
https://hal.archives-ouvertes.fr/hal-01323138
Submitted on 30 May 2016

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EQUILIBRIUM RECONSTRUCTION FROM DISCRETE MAGNETIC MEASUREMENTS IN A TOKAMAK

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ABSTRACT

We describe an algorithm for the reconstruction of the equilibrium in a Tokamak from discrete magnetic measurements. In order to solve this inverse problem we first use toroidal harmonics to compute Cauchy boundary conditions on a fixed closed contour. Then we use these Cauchy boundary conditions to solve a non-linear source identification problem.

1. INTRODUCTION

In this paper we are interested in the numerical reconstruction of the quasi-static equilibrium of a plasma in a Tokamak [1]. The state variable of interest in the modelization of such an equilibrium under the axisymmetric assumption is the poloidal flux $\psi(r, z)$ which is related to the poloidal magnetic field by the relation $B = \frac{1}{r} \nabla \psi \perp$ in the cylindrical coordinate system $(r, z)$.

A poloidal cross section of a Tokamak and the different domains and contours are shown on Fig. 1. The domain $\Omega_0$ contains the poloidal field coils (PFcoils) domains $\Omega_{C_i}$ and the plasma domain $\Omega_p$. It is assumed that $\Omega_0$ does not include any ferromagnetic structure and thus the poloidal flux satisfies the elliptic PDE

$$- \Delta^* \psi = j$$

where the differential operator

$$\Delta^* = \frac{\partial}{\partial r} \left( \frac{1}{\mu_0 r} \frac{\partial}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_0 r} \frac{\partial}{\partial z} \right),$$

is linear and $j$ is the toroidal component of the local current density.

In $\Omega_0 \setminus \{ \Omega_p \cup \Omega_{C_i} \}$ the current is null ($j = 0$), in the PFcoils $\Omega_{C_i}$ it is supposed to be known ($j = I_i/S_i$, $S_i$ is the surface of coil $\Omega_{C_i}$ and $I_i$ the total intensity of the current) and in the plasma $\Omega_p$ it is unknown but takes the form

$$j = rp'(\psi) + 1/\mu_0 (ff')(\psi)$$

In $\Omega_p$ Eq. (1) is called the Grad-Shafranov equation and $p'$ and $ff'$ are unknown functions to be identified.

The plasma domain is unknown, $\Omega_p = \Omega_p(\psi)$. It is a free boundary problem in which the plasma boundary is defined either by its contact with the limiter $\Gamma_L$ (as in Fig. 1) or as a magnetic separatrix (hyperbolic line with an X-point).

In order to achieve the numerical reconstruction of the equilibrium the main inputs we have are magnetic measurements taken at several locations surrounding the vacuum vessel (see Fig. 1): B probes measure the
local value of the poloidal magnetic field and flux loops measure the local value of the flux $\psi$.

The method we propose can be divided into two main steps described in the next section.

2. NUMERICAL METHOD

In a first step we solve Eq. (1) in $\Omega_0 \setminus \Omega_p$ using an analytic solution and a fit to the magnetic measurements. The goal is to transform the set of discrete measurements into Cauchy conditions for $\psi$ on a fixed contour $\Gamma$. This latter defines a domain $\Omega$ of boundary $\Gamma$ and containing the plasma (see Fig. 1). In a second step the value of $\psi$ on $\Gamma$ is used as a Dirichlet boundary condition to solve numerically Eq. (1) in $\Omega$, whereas the remaining Neumann boundary condition on $\Gamma$ is used simultaneously for the identification of the unknown functions $p'$ and $ff'$.

2.1. Step 1 - Compute Cauchy conditions on $\Gamma$

Each of the PF coils is modeled by a sum of filaments of current [3]. Using the analytic expression of the Green function of the operator $\Delta^*$ and the additivity property we can subtract from $\psi$ and from the magnetic measurements the effects of the PF coils.

In the domain $\Omega_0 \setminus \Omega_p$ the resulting corrected flux then satisfies $-\Delta^*\psi = 0$. Using a system of toroidal coordinates with center $C$ inside the plasma domain and a separation of variable technique, any solution of this equation can be shown to be equal to a series of toroidal harmonic functions $T_n$ [5, 6]. Numerically these harmonics can be accurately evaluated [4] and the flux can be efficiently approximated using a truncated series, $\psi = \sum_{n=1}^N a_n T_n$. The coefficients $a_n$ are computed by a least-square fit to the modified magnetic measurements. A regularization term can be added to the least-square cost function for example imposing some regularity on a circular contour surrounding the center of the toroidal coordinates system.

Finally one can evaluate $(\psi, \partial_n \psi)$ on the contour $\Gamma$ and thus provide Cauchy conditions $(g, h)$ to the resolution of the problem in the domain $\Omega$.

2.2. Step 2 - Reconstruction in $\Omega$

The Dirichlet boundary condition $g$ is used to solve the boundary value problem:

$$
\begin{cases}
-\Delta^* \psi &= \lambda \frac{r}{R_0} A(\bar{\psi}) + \frac{R_0}{r} B(\bar{\psi}) \chi_{\Omega_p}(\psi) \quad \text{in } \Omega \\
\psi &= g \quad \text{on } \Gamma 
\end{cases}
$$

where the unknown functions $A$ and $B$ are related to $p'$ and $ff'$, $\bar{\psi}$ is a normalized flux, and $\lambda$ and $R_0$ are normalizing coefficients. The Neumann boundary condition is used to identify $A$ and $B$ by minimizing the cost function $J(A, B) = \int_{\Gamma} (\partial_n \psi - h)^2 ds + R$ where $R$ is a Tikhonov regularization term.

An iterative strategy involving a finite element method for the resolution of the direct problem [3] and an optimisation procedure for the identification of the non-linearity is proposed [2]. It is important to achieve this identification within a few ms so as to be able in the future to control the current profile in real time. The main ideas are: pre-computation of the inverse of the finite element stiffness matrix and of all the elements that are not modified by the non-linearities, Picard iterations for these non-linearities, reduction of the functions to be identified in small dimension basis and least-square resolution by normal equations.

3. CONCLUSION

The method presented here has led to the development of a software, EQUINOX, which enables to follow in real-time the quasi-static evolution of the plasma equilibrium in any Tokamak. It has already been validated on TORE SUPRA (the CEA-EURATOM Tokamak at Cadarache), JET (Joint European Torus) or ITER configurations.

4. REFERENCES


