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The charge-exchange induced coupling between plasma-gas counterflows in the heliosheath

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Abstract. Many hydrodynamic models have been presented which give similar views of the interaction of the solar wind plasma bubble with the counterstreaming partially ionized interstellar medium. In the more recent of these models it is taken into account that the solar and interstellar hydrodynamic flows of neutral atoms and protons are coupled by mass-, momentum-, and energy-exchange terms due to charge exchange processes. We shall reinvestigate the theoretical basis of this coupling here by use of a simplified description of the heliospheric interface and describe the main physics of the H-atom penetration through the more or less standing well-known plasma wall ahead of the heliopause. Thereby we can show that the type of charge exchange coupling terms used in up-to-now hydrodynamic treatments unavoidably leads to an O-type critical point at the sonic point of the H-atom flow, thus not allowing for a continuation of the integration of the hydrodynamic set of differential equations. The remedy for this problem is given by a more accurate formulation of the momentum exchange term for quasi- and sub-sonic H-atom flows. With a refined momentum exchange term derived from basic kinetic Boltzmann principles, we instead arrive at a characteristic equation with an X-type critical point, allowing for a continuous solution from supersonic to subsonic flow conditions. This necessitates that the often treated problem of the propagation of interstellar H-atoms through the heliosheath has to be solved using these newly derived, differently effective plasma – gas friction forces. Substantially different results are to be expected from this context for the filtration efficiency of the heliospheric interface.

Key words. Interplanetary physics (heliopause and solar wind termination; interstellar gas) – Ionosphere (plasma temperature and density)

1 Charge-exchange coupling of counterdrifting plasma – gas media

In problems like the mutual interaction of plasma and H-atom gas flows in the heliosheath due to the large local Knudsen numbers Kn, in principle, a kinetic treatment of charge-exchange induced coupling processes is required, as was already emphasized by Osterbart and Fahr (1992) or Baranov and Malama (1993). In kinetic approaches the distribution function of the H-atom gas needs to be described by a Boltzmann-Vlasov integro-differential equation (see, e.g. Ripken and Fahr, 1983; Osterbart and Fahr, 1992; Baranov and Malama, 1993; Pauls and Zank, 1996; Fahr, 1996; McNutt et al., 1998, 1999; Bzowski et al., 1997, 2000). Thus, one would have to start from the following type of a Boltzmann equation:

\[
\frac{df_H(r, v)}{d\tau} = f_p(r, v) \int f_H(r, v') v_{rel}(v, v') \sigma(v_{rel})d^3v' - f_H(r, v) \int f_p(r, v') v_{rel}(v, v') \sigma(v_{rel})d^3v',
\]

where \(f_H(r, v)\) and \(f_p(r, v)\) are the velocity distribution functions of the H-atoms and the protons, respectively, \(r\) and \(v\) are the relevant phase-space variables, \(d\tau\) is the increment of the line element on the associated dynamical particle trajectory, \(v_{rel}\) denotes the relative velocity between collision partners of velocities \(v\) and \(v'\), and \(\sigma(v_{rel})\) is the velocity-dependent charge exchange cross section.

Due to reasons connected with the fairly laborious mathematical tractability of the Boltzmann equation, many authors have preferred to change over from Eq. (1) to a set of hydrodynamic moment equations (for a review, see Zank, 1999). Introducing the lowest moments \(\Phi_0 = m_p\); \(\Phi_1 = m_p v\); \(\Phi_2 = \frac{1}{2}m_p v^2\), then leads to the conventionally used conservation equations for mass, momentum, and energy flows.
First, the following simple equation is obtained from Eq. (1) for the average mass exchange:

\[ \langle \Phi_0 \rangle = Q_0^+ - Q_0^- = 0, \]  

where \( Q_0^+ \) and \( Q_0^- \) are the charge exchange – induced mass gain – and mass loss – rates. Equation (2) simply states that no net mass gains will result from pure charge exchange reactions between particles with identical atomic nuclei.

The exchange terms resulting for the higher moments like, for example, \( \Phi_{1,2} \) can only be evaluated with some knowledge of the distribution functions \( f_p \) and \( f_H \). Representing these distribution functions as functions of the lowest hydrodynamic moments themselves, e.g. using shifted Maxwellians with isotropic temperatures \( T_p \) and \( T_H \) (as done by Holzer, 1972; Ripken and Fahr, 1983; Fahr and Ripken, 1984; Isenberg, 1986; Pauls and Zank, 1996; Fahr, 1996; Lee, 1997) permits one to present the above expressions in the following forms:

\[ \langle \Phi_1 \rangle = \sigma_{rel} \langle v_{rel} \rangle m_p n_p n_H (V_H - V_p) \]  

and:

\[ \langle \Phi_2 \rangle = \sigma_{rel} \langle v_{rel} \rangle m_p n_p n_H \left[ \frac{1}{\gamma - 1} \left( \frac{P_p}{\rho_p} - \frac{P_H}{\rho_H} \right) - \frac{1}{2} (V_H - V_p)^2 \right], \]

where \( n_H, V_H, P_H \) and \( n_p, V_p, P_p \) are density, bulk velocity, and pressure of the H-atoms and of the protons, respectively, \( \sigma_{rel} = \sigma (v_{rel}) \) is the actual charge exchange cross section, and \( \langle v_{rel} \rangle \) is the double-Maxwellian average of the relative speed between protons and H-atoms as given, for example, by Holzer (1972):

\[ \langle v_{rel} \rangle = \frac{128}{9\pi} \left( \frac{P_p}{\rho_p} + \frac{P_H}{\rho_H} \right) + (V_H - V_p)^2. \]

2 The H-atom gas passage through the heliosheath

The problem at hand with treating the passage of neutral interstellar gas (LISM H-atoms) through the plasma interface ahead of the solar system essentially resembles that of a passage of an H-atom gas flow through a predetermined quasi-static plasma structure simulating the region downstream of the expected outer interstellar shock (see Baranov and Malama, 1993; Zank, 1999; Fahr et al., 2000).

In a one-dimensional approach for the region along the stagnation line (\( z \)-axis!) this LISM plasma ahead of the heliopause, due to its very low sonic Mach number, can be taken as incompressible and nearly stagnating. One could perhaps ask how this type of plasma structure with a constant proton density and temperature extended over a limited space volume in reality might be maintained in space. This question, however, is not of relevance for this very fundamental study aimed at here. We more or less only want to check whether or not we use the correct theoretical instruments to describe the charge-exchange induced adaptation of an H-atom fluid streaming with some initial relative drift velocity into a plasma fluid and we want to describe this process of adaptation in the reference system of the plasma bulk.

Thus, to describe the charge exchange influence of the preheliopause plasma sheath on the H-atom flow, one can make use of the following set of equations:

\[ \frac{d}{dz} (\rho_H V_H) = 0, \]

\[ \rho_H V_H \frac{d}{dz} V_H = -\frac{d}{dz} P_H - \sigma_{rel} V_{rel} n_p \rho_H V_H. \]

\[ \frac{d}{dz} \left[ V_H \left( \frac{\rho_H V_H^2}{2} + \frac{\gamma P_H}{\gamma - 1} \right) \right] = \sigma_{rel} V_{rel} n_p \rho_H \left[ \frac{1}{\gamma - 1} \left( \frac{P_p}{\rho_p} - \frac{P_H}{\rho_H} \right) - \frac{V_H^2}{2} \right], \]

where \( \gamma \) is the polytropic index taken to be identical for both protons and H-atoms, \( \sigma_{rel} = \sigma (v_{rel}) \) is the relevant charge exchange cross section for protons and H-atoms interacting with an average relative speed \( V_{rel} \) which for \( V_p = 0 \) is given by:

\[ V_{rel} = \sqrt{\frac{128}{9\pi} \left( \frac{P_p}{\rho_p} + \frac{P_H}{\rho_H} \right) + V_H^2}. \]

Equation (6) can be integrated to yield the constant mass flow:

\[ C_0 = \rho_0 V_{H0} = \rho_H V_H. \]

In addition, it is suggested to introduce the normalized coordinate \( \xi \) defined by \( z = \xi D \) and the quantity \( \Lambda = D/\lambda = D \sigma_{rel} n_p \) (\( \xi = 0 \) and \( \xi = 1 \) mark the two borders of the plasma wall with the extent \( D \)). With these conventions we then obtain the following so-called characteristic equation:

\[ \frac{d}{d\xi} V_H = \frac{V_{rel} \Delta \rho_p - P_H + \frac{1}{2} C_0 V_H (\gamma + 1)}{\gamma P_H - C_0 V_H}. \]

In the above equation the following denotation was used: \( \Delta \rho = \rho_p/\rho_H \). From Eq. (7) with Eq. (11) one derives the differential equation for the pressure:

\[ \frac{d}{d\xi} P_H = -V_{rel} \Lambda C_0 - C_0 \frac{d}{d\xi} V_H. \]

Starting the integration at \( \xi = 0 \) with supersonic H-atom inflow velocities, i.e. \( V_{H0}^2 \geq \gamma P_{H0}/\rho_{H0} \), one at first with increasing, but small values of \( \xi \), obtains reasonable and physically meaningful results for \( V_H \) and \( P_H \), as shown in Fig. 1. However, proceeding with the integration to some critical point \( \xi_c \geq 0 \), where locally the equality \( \gamma P_{Hc} = C_0 V_{Hc} \) is reached, the integration of the upper system of differential equations can no longer be continued.

The resulting singularity at \( \xi = \xi_c \) within the frame of this set of equations hereby cannot be avoided as usual, by
Fig. 1. Shown as a function of the plasma wall coordinate \( x = z/D, \) \( D = 50 \) AU being the linear extent of the plasma wall, is the H-atom bulk velocity \( V_H \) in units of \([\text{km/s}]\) (left ordinate) and the H-atom pressure \( P_H \) in units of \( \left[10^{-13} \text{dyne/cm}^2\right] \) (right ordinate) calculated for various values of the parameter \( \Lambda = D\sigma_{rel} n_p. \) The H-atom flow in all cases enters the plasma wall at \( x = 0 \) with a velocity of \( V_H(0) = 25 \) km/s. Curves reaching the singular point \( x_c \) where \( \gamma P_{H,c} = C_0 V_{H,c} \) stop at this point.

3 Reinvestigation of the charge-exchange induced plasma-gas friction

Due to the local conversion of protons into H-atoms, and vice-versa, a net exchange of momentum per unit of volume and time results for either of the two fluids. The net momentum change suffered by one fluid species can be summed from two contributions, one is the momentum loss due to losses of particles of this species, and the other is a momentum gain due to gains of particles of this species. As derived in detail by Fahr (2002), the charge-exchange induced momentum loss rate is calculated from the following expression:

\[
\langle Q_1^{-}(v_H) \rangle = \frac{2}{\sqrt{\pi}} n_p n_H m_p \left( \frac{2 K T_p}{m_p} \right) \int_0^\infty \int_0^\infty x \cos \vartheta \sigma(v_{rel}) \sqrt{x_H^2 + x^2 - 2 x H x \cos \vartheta \exp(-x^2) x^2 dx \sin \vartheta d \vartheta. \ (14)
\]

On the other hand, the charge-exchange induced momentum gain rate describing the production of new H-atom mo-

mentum per unit of time and unit of volume, due to newly decharged protons, has to be evaluated from the following expression:

\[
\langle Q_1^{+}(v_H) \rangle = \frac{2}{\sqrt{\pi}} n_p n_H m_p \left( \frac{2 K T_p}{m_p} \right) v_H \left( \left[ x_H^2 + x^2 - 2 x H x \cos \vartheta \exp(-x^2) x^2 dx \sin \vartheta d \vartheta. \right) \right.
\]

The integrands of the double-integrals in Eqs. (14) and (15) contain as a factor function the charge exchange cross section \( \sigma(v_{rel}) \), which itself is a function of the velocity coordinates \( x \) and \( \vartheta_H. \) In view of its weak velocity-dependence we can fairly well Taylor-expand this charge exchange cross section \( \sigma(v_{rel}) \) and obtain the expression:

\[
\sigma(v_{rel}) = \sigma_{rel} + \left[ \frac{d \sigma}{d x} \right]_{rel} (x_{rel} - X_{rel}), \ (16)
\]

where \( \sigma_{rel} = \sigma(V_{rel}), \) and where \( X_{rel} \) and \( x_{rel} \) are the ther-

diagonally normalized velocities \( V_{rel} \) and \( v_{rel} \) given by:

\[
x_{rel} = v_{rel}/\sqrt{2KT_p/m_p} \quad \text{and} \quad X_{rel} = \sqrt{\frac{64}{9\pi} \left( 1 + \frac{T_H}{T_p} \right) + M_p^2},
\]

with \( M_p^2 = \frac{m_p V_p^2}{2KT_p} \)

representing the Mach number of the proton flow. Based on the well-known formula given by Maher and Tinsley (1977) for the charge exchange cross section in the form

\[
\sigma(v) = (A + B \log(v/v_0))^2, \ (17)
\]

with \( A \) and \( B \) being constants, one then explicitly obtains the expression (16) in the following explicit form:

\[
\sigma(v_{rel}) = \sigma_{rel} \left[ 1 + \frac{2 B}{\sigma_{rel}} \left( 1 - \frac{x_{rel}}{X_{rel}} \right) \right], \ (18)
\]

where velocity normalizations, for example, as in \( x_{rel} = v_{rel}/c_p, \) are carried out with the thermal proton velocity \( c_p = \sqrt{2KT_p/m_p}. \)

The net momentum exchange rate resulting from summing up the two above expressions given in Eqs. (14) and (15) with this above representation then evaluates to (for details, see Fahr, 2002):

\[
\langle Q_1^{+}(x_H) \rangle_{H} + \langle Q_1^{-}(x_H) \rangle_H = \Pi \left[ C(M_H) \right.
\sqrt{\pi M_H \left( -9g_1 - 2g_2 + 5ag_1 + 2ag_1 M_H^2 \right)} \left], \ (19)
\]

with \( \Pi \) being defined as:

\[
\Pi = 2/(\sqrt{\pi}) n_p n_H \sigma(X_{rel}) \sqrt{2KT_p/m} \sqrt{2KT_H/m}. \ (20)
\]
Here, the function $C(M_H)$ is defined by:

$$C(M_H) = \frac{\pi}{8 M_H^2} \int_0^\infty \Sigma(x) \exp \left(-\left(x^2 + M_H^2\right)\right) \cdot \left[2x M_H \cosh (2x M_H) - \sinh (2x M_H)\right] dx.$$  \hspace{1cm} (21)

In the above expressions the following notations have been used:

$$\alpha = T_H / T_p ; \quad M_H^2 = \frac{\rho_H V_H}{\gamma P_H} ; \quad g_1 = \frac{1}{15} \left(1 + \frac{B}{\sqrt{\sigma_{rel}}}\right) ; \quad g_2 = g_1 - \frac{1}{\sqrt{\sigma_{rel}}} \frac{2 B}{\sqrt{\sigma_{rel}}} \frac{1}{\chi_{rel}}$$

In addition, it can be proven that expression (19), which is valid for moderate and small Mach numbers $M_H$ for its usage in successive numerical integration procedures, can be simplified into an algebraic representation yielding the following mathematically better manageable form:

$$Q_{sub} = \Pi M_H \left[\frac{7}{3} g_1 - \sqrt{\pi} \left(-9 g_1 - 2 g_2 + 5 \alpha g_1 + 2 \alpha g_1 M_H^2\right)\right].$$ \hspace{1cm} (22)

Here, the quantity $\Pi$ was introduced in Eq. (20)

It should be noted that, compared to the above expression which is valid for small and moderate Mach numbers $M_H$, the analogous simplified expression, which, in principle and rigorously taken, is only justified for the case that high Mach numbers $M_H$ prevail, but which is always applied in the up-to-now literature, attains, when written in the above introduced quantities, the following form:

$$Q_{super} = -\Pi M_H \sqrt{\frac{4\pi}{9}(1 + \alpha) + \alpha M_H^2}. \hspace{1cm} (23)$$

In Figs. 2 and 3 we show the two different representations of the momentum transfer rate between H-atoms and protons, $Q_{sub}$ and $Q_{super}$, and the ratio $R = (Q_{sub} / Q_{super})$, respectively, as a function of the Mach number $M_H$. As is becoming evident from the comparisons given in these two figures, it clearly turns out that, at very low Mach numbers $M_H$, the revised, improved rates $Q_{sub}$ tend to be smaller than those $Q_{super}$ taken in the up-to-now hydrodynamical conventional theories, whereas in contrast at moderate Mach numbers $M_H \geq 1$ the improved rate $Q_{sub}$ tends to become larger.

This evidently means that at moderate and small Mach numbers $M_H$, the plasma-gas friction force was up to now depending on the Mach number $M_H$ either substantially over- or underestimated in published hydrodynamical theories treating the problem of the interstellar H-atom penetration through the heliosheath plasma wall ahead of the heliopause. One should, however, also mention here that theoretical treatments that use a kinetic description for the H-atom fluid, like those presented by Baranov and Malama (1993, 1995) or Izmodenov, Lallement, and Geiss (1999), may perhaps not suffer so much from this fact, since with their Monte Carlo code, used to kinetically treat the H-atoms which are involved in this charge exchange business, the momentum exchange should be treated as correctly as possible within the framework of Monte-Carlo-induced numerical errors.

4 Revised form of the characteristic equation

With the newly derived expression given by Eq. (22) we now analogously also obtain, instead of Eq. (20), the new characteristic equation derived from the hydrodynamical set of differential equations (i.e. Eqs. 6, 7, and 8) in the following
form:
\[
\frac{dV_H}{d\xi} = \Lambda V_{rel} \left( \Delta p - P_H - \frac{1}{2} C_0 V_H (\gamma - 1) \right) + \gamma D V_H Q_{sub} \gamma P_H - C_0 V_H.
\]

(24)

Now, however, the critical point condition for a simultaneous vanishing of the numerator and the denominator of Eq. (24) leads to the new requirement:
\[
1 + \frac{1}{2} \gamma (\gamma - 1) \geq \frac{\gamma^2}{2} \pi \left( \frac{\Delta g_1 - \sqrt{\pi} (-9 g_2 - 2g_2 + 5a g_1 + a g_1 \gamma^2)}{\pi} \right)
\]

As an explicit example we may adopt numerical values for the above parameters as those extracted from Fahr (2000), namely adopting for \( T_P = 20,000 \) K, for \( \alpha = 0.4 \) and for \( V_H = 26 \) km s\(^{-1} \). In addition, with \( \gamma = 5/3 \), one then finds \( V_{rel} = 41.5 \) km s\(^{-1} \) yielding \( \sigma_{rel} = 6.4 \times 10^{-15} \) cm\(^2\) and \( M_p = 1.43 \). When inserting values for \( g_1 \) and \( g_2 \) and for the cross-sectional constants \( A \) and \( B \), as given for the charge exchange reaction between H-atoms and protons by Maher and Tinsley (1977), one finally obtains from Eq. (24) the algebraic request:
\[
\frac{14}{9} = 1.56 \geq 1.39.
\]

As is evident, this clearly means that the above requirement can be fulfilled and that a continuous integration of the set of differential equations is now enabled.

This fact is clearly demonstrated in Fig. 4, where we have shown solutions which should be compared with the analogous solutions shown for identical sets of data in Fig. 1, but now calculated with the newly derived momentum coupling term given in Eq. (22), instead of that one given by Eq. (23).

Integrating the newly derived characteristic Eq. (24) towards the left and the right direction from the critical point, one now obtains solutions for the LISM H-atom gas penetration through a plasma wall, which are different from those shown in Fig. 1, but in our view are the more correct representations of the gas properties at the passage through the prescribed plasma structure.

5 Conclusions

It is clearly manifest from the study of works by Baranov and Malm (1993) Baranov et al. (1998), Fahr (2000) or Fahr et al. (2000) that the Mach numbers \( M_H \) of the relative flows between the LISM proton plasma and the LISM H-atom fluid in the heliosheath region (i.e. post-bow-shock Mach numbers \( M_H \)), in all cases considered so far, are smaller than \( M_H = 2 \). Consequently and strictly speaking, in this Mach number range the specific momentum exchange term for hydrodynamic approaches needs to be given in the newly derived form given by Eq. (22), instead of the conventionally used form given by Eq. (23). Since the effective momentum exchange rate described by this new expression at identical thermodynamical conditions is different from that described by the conventionally used term, one can presume that the adaptation of the LISM H-atom flow to the nearly stagnating LISM proton plasma in this interface region ahead of the heliopause is now operating differently, i.e. partly less and partly more effectively decelerating the LISM H-atoms down to the nearly vanishing LISM proton flow velocities ahead of the heliopause.

The use of an inadequate momentum exchange term in hydrodynamic approaches may thus lead to the erroneous claim for higher LISM proton or H-atom densities to reach the same amount of reduced LISM H-atom bulk flow velocity. Hence, more reliable quantitative interpretations of the H-atom flow through the heliosphere in terms of needed LISM parameters (see papers by Scherer and Fahr, 1996; Scherer et al., 1997, 1999; Izmodenov et al., 1997; Izmodenov, 2000) should be obtained with the application of the new term given in Eq. (22). LISM parameters claimed on the basis of these earlier interpretations may thus need substantial revisions.

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