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Stability control of nonlinear micromechanical resonators under simultaneous primary and superharmonic resonances

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Fast effects of a slow excitation on the main resonance of a nonlinear micromechanical resonator are analytically and experimentally investigated. We show in particular how the bifurcation topology of an undesirable unstable behavior is modified when the resonator is simultaneously actuated at its primary and superharmonic resonances. A stabilization mechanism is proposed and demonstrated by increasing the superharmonic excitation.

Interests in nonlinear resonance have grown over the last decade leading to many applications, for example, in parametric amplification[3–5] and noise squeezing[2] eventually with bifurcation amplification[4], switching of magnetization in single nanoparticles[5], electromagnetic cancer detection through nonlinear resonance interaction[6], spectroscopy to probe a threaded interface[7], pull-in retarding under secondary resonances for MEMS resonators[8] as well as ultralow superharmonic resonance for functional nanowires[9].

Here, unlike previous schemes under primary or single nonlinear resonance excitation, the mechanical structure is simultaneously driven under primary and superharmonic resonances. The effect of adding an extra harmonic excitation in a system has been investigated by very few authors, even theoretically. In the quasi-static case of a beam, it is shown that the axial buckling load can be increased by adding high-frequency excitation[10]. An analytical study of a Van Der Pol-Mathieu-Duffing resonator[11] showed a way to tune the nature of its nonlinear dynamic behavior by adding a fast harmonic excitation. Nevertheless, the fast effects of a slow harmonic excitation on the main resonance of a nonlinear system have neither been studied nor experimentally shown.

This paper analytically investigates a strategy of stability control based on an energy transfer within a single nonlinear system through simultaneous (primary and superharmonic) resonances. Furthermore, we employ this mechanism for the first time in micromechanical resonators to demonstrate experimentally its stability enhancement.

In order to understand simultaneous resonances, let us consider the following resonator equation, for a single degree-of-freedom x:

$$\ddot{x} + \mu\dot{x} + \omega_n x + \alpha_2 x^2 + \alpha_3 x^3$$
$$+\alpha_5 x^5 = \zeta_1 \cos(\Omega) + \zeta_2 \cos(2\Omega) \tag{1}$$

The dot denotes derivation with respect to time, ω_n the natural frequency, α_2, α_3, α_5, the coefficients of the quadratic, cubic and quintic nonlinearities respectively, μ the constant damping coefficient, ζ_1, ζ_2 the drive amplitudes of the first and the second harmonic of the drive frequency Ω.

The effect of each nonlinear term has been experimentally shown and explained, for both doubly clamped beams[12] and cantilevers[13]. The second-order term is due to the electrostatic force, but of course has no influence on the primary resonance like any even-order term. It is included here in the purpose of showing the effect of a superharmonic resonance. The third-order term is the well-known Duffing term, leading to bistable softening or hardening behavior depending on its sign. The fifth-order term is responsible for the amplitude response to become another multivalued (5 possible amplitudes) function of the frequency, a highly unstable so-called mixed behavior[14]. Micromechanical resonators are very commonly driven via the superposition of a static DC voltage V_{dc} and a time varying AC voltage V_{ac}. The electrostatic force being proportional to the square of the voltage, this scheme involves the presence of two harmonics Ω and 2Ω of Eq. (1), with amplitudes ζ_1 and ζ_2 proportional to $V_{ac}V_{dc}$ and V_{ac}^2 respectively.

When Ω is tuned around ω_n, the primary resonance at ω_n is mainly actuated by the first harmonic Ω, the effect of the second harmonic being negligible. Experimentally, this corresponds to the "1f mode" typically used via a lock-in amplifier in MEMS and NEMS electrical characterizations. It has been previously shown that the onset of bistability of the Duffing behavior can be tuned via electrostatic negative nonlinearity: the third order term coefficient can then eventually be nulled so a stable and linear response is obtained[13,15]. This is unfortunately true up to the onset of the mixed behavior only. Worse, investigation of the bifurcation topology in this "1f mode" showed the invariance of this onset, for a given resonator[14]: the onset of the mixed behavior sets a strict upper limit to the frequency stability of the resonator.

When Ω is tuned around $\frac{\omega_n}{2}$, the resonant response at ω_n is provoked by both the first and the second harmonic 2Ω. More precisely, in this "2f mode" scheme also commonly used, the 2Ω harmonic mainly generates the primary resonance at ω_n, and the Ω harmonic actuates a superharmonic resonance at ω_n. The response at ω_n is thus made of simultaneous primary and superharmonic resonances. The latter one is generated via a "slow" excitation compared to the resonant frequency.

In order to investigate the fast effects of a slow excitation, we use the multiple scales method[15] to solve Eq. (1). A first-

order uniform solution is sought in the form

$$x(t, \varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + \cdots \quad (2)$$

where ε is the small nondimensional bookkeeping parameter, $T_0 = t$ and $T_1 = \varepsilon t$. Since both harmonics are linked by the drive frequency Ω, in order to analyze the nonlinear response under simultaneous primary and superharmonic resonances, we express the nearness of Ω to $\frac{\omega_n}{2}$ by introducing the detuning parameter σ according to

$$2\Omega = \omega_n + \varepsilon\sigma \quad (3)$$

Substituting Eq. (2) into Eq. (1) and equating coefficients of like powers of ε yields

for order ε^0

$$\zeta_1 \cos\left(\sigma T_1 + \frac{T_0 \omega_n}{2}\right) + \omega_n^2 x_0 + x_0^{(2,0)} = 0 \quad (4)$$

for order ε^1

$$\zeta_2 \cos(\sigma T_1 + T_0\omega_n) + x_0^2\alpha_2 + x_0^3\alpha_3 + x_0^5\alpha_5$$
$$+x_1\omega_n^2 + cx_0^{(1,0)} + 2x_0^{(1,1)} + x_1^{(2,0)} = 0 \quad (5)$$

where $x_i^{(j,k)} = \dfrac{\partial^k}{\partial T_1^k}\left(\dfrac{\partial^j x_i}{\partial T_0^j}\right)$.

The general solution of Eq. (4) can be written as

$$x_0 = X\cos(\omega_n T_0 + \Phi) - \frac{4\zeta_1}{3\omega_n^2}\cos\left(\frac{\omega_n T_0}{2} + \sigma T_1\right) \quad (6)$$

Equation (6) is then substituted into Eq. (5) and the trigonometric functions are expanded. The elimination of the secular terms yields two first order non-linear ordinary-differential equations which describe the amplitude X and phase β modulation of the response and allow for a stability analysis

$$\dot{X} = -\frac{4\epsilon\zeta_1^2 \sin\beta\left(20X\alpha_5\zeta_1^2\cos\beta + 9\alpha_2\omega_n^4\right)}{81\omega_n^9}$$
$$-\frac{\epsilon\mu X}{2} + O(\varepsilon^2) \quad (7)$$

$$\dot{\beta} = -\frac{2\epsilon\zeta_1^2\left(2\alpha_2\cos\beta + 6\alpha_3 X + 15\alpha_5 X^3\right)}{9\omega_n^5 X}$$
$$+2\sigma - \frac{40\epsilon\alpha_5\zeta_1^4\{6 + \cos(2\beta)\}}{81\omega_n^9}$$
$$-\frac{6\epsilon\alpha_3 X^2 + 5\epsilon\alpha_5 X^4}{16\omega_n} + O(\varepsilon^2) \quad (8)$$

where $\beta = 2\sigma T_1 - \Phi$. The steady-state motions occur when $\dot{X} = \dot{\beta} = 0$, which corresponds to the singular points of Eqs. (7) and (8). Thus, the frequency-response equation under simultaneous primary and superharmonic excitations can be written and plotted parametrically with respect to the phase β.

The resonator design considered for analytical simulations is described in Fig. 1(a). It uses 2 electrodes: electrode 1 for

actuation and electrode 2 for sensing which allows for 2 port electrical measurements.

All numerical simulations were carried out with the following set of parameters: $l_r = 200\,\mu m$, $b = 4\,\mu m$, $h = 2\,\mu m$, $l_a = 20\,\mu m$, $g_a = 1\,\mu m$, $l_d = 112\,\mu m$, $g_d = 750\,nm$, where h and b are the width and thickness of the microbeam, g_a and g_d are the actuation and sensing gaps. V_{ac} and V_{dc} were used for parametric investigations.
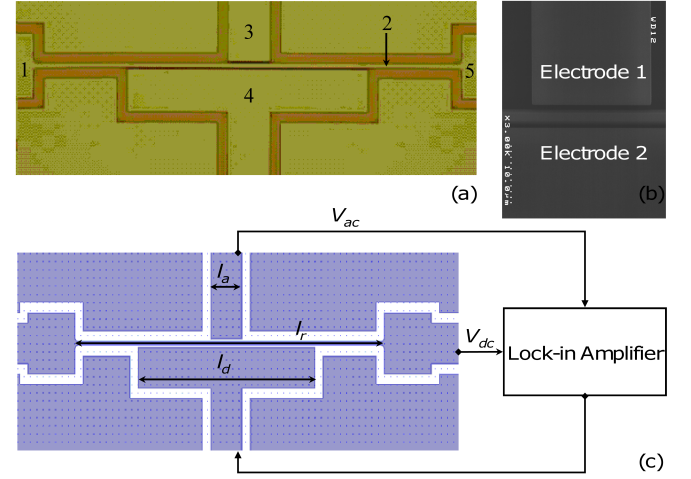

FIG. 1: (a): Optical microscope image of the device. 1 and 5: anchors, 2: microbeam, 3: actuation electrode, 4: sensing electrode. (b): SEM image of the device. (c): Connection layout for the electrical characterization.

Figure 2 shows the analytical nonlinear forced response function (FRF) of the beam resonator on its fundamental mode at ω_n for $\Omega = \frac{\omega_n}{2}$ (2f mode) for a fixed $V_{ac} = 0.3\,V$ (hence a fixed 2Ω-excitation amplitude) and for different values of V_{dc}. $V_{dc} = 0$ means no superharmonic resonance and an increased value increases the superharmonic resonance effect. The latter exists when the second-order term becomes of significant value, and hence also when the higher-order terms do. Like Duffing primary resonance, bistability may be reached for simultaneous resonances at intermediate drive levels, and we will call its onset the critical amplitude. At these levels, the behavior is unchanged by simultaneous resonances: typical Duffing-like hardening (or softening) behaviors are observed, and the measured critical amplitude is equal to the primary resonance-only case: the slow excitation has little or no effect on the fast dynamics. When the drive level is still increased (and the fifth-order terms become non negligible), the question is wether or not simultaneous resonances response also exhibits the mixed behavior like primary-only resonance, and wether or not the onset of this mixed behavior can be tuned.

Figure 3 shows the response for different values of V_{dc} (6, 8 and 10 V), and for a fixed $V_{ac} = 1\,V$, high enough to display a mixed behavior. The amplitude of the P bifurcation point, called the onset of the mixed behavior is shown. As opposed to the actuation under primary resonance only (see Fig. 4), the P point amplitude is shifted up by the increase of the AC voltage. This result opens up new ranges of resonator operation, in terms of control, stability and drive amplitude.
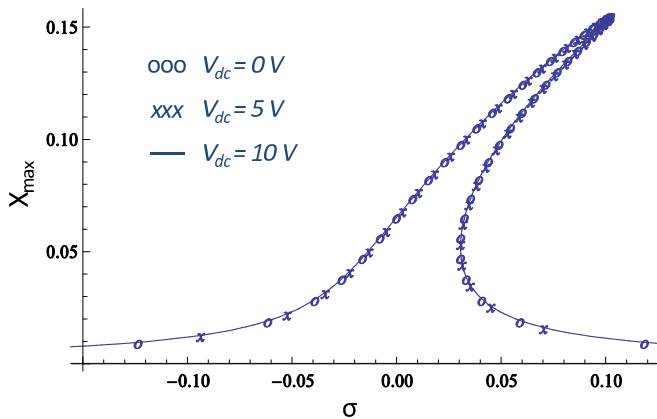
FIG. 2: Analytical frequency responses under simultaneous primary and superharmonic excitations at intermediate drive levels $V_{ac} = 0.3V$, showing hardening behavior, with varying V_{dc}. X_{max} is the displacement of the beam normalized by the gap g_d at its middle point and is traced vs the detuning parameter σ to get rid of the frequency tuning due to change in V_{dc}.
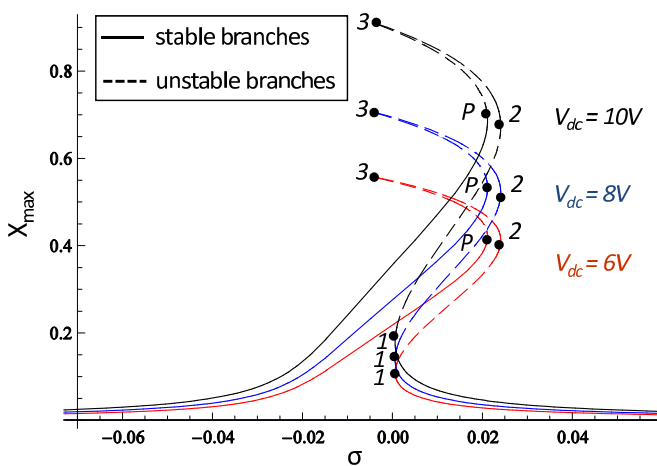

FIG. 3: Analytical frequency responses showing mixed behaviors under simultaneous primary and superharmonic excitations for $V_{ac} = 1V$; the location of the different bifurcation points and the effect of the DC voltage on the onset of the mixed behavior.

Both slow (superharmonic) and fast (primary) dynamics are present in Eqs. (7) and (8), and modulate the amplitude X and the phase β of the system, due to terms proportional to $\cos\beta$ or $\sin\beta$ and to $\cos 2\beta$ respectively. Moreover, these terms are proportional either to α_5 or α_2, coefficients of the quintic and quadratic nonlinearities producing the mixed behavior or internal resonance into the system respectively. The AC voltage sets the 2Ω excitation, while the DC voltage only amplifies the Ω excitation. The alteration of the bifurcation topology in the present simultaneous resonances configuration is thus only due to the increase of the superharmonic resonance.

The experimental investigation was carried out by placing the device in a vacuum chamber at ambient temperature. The discrepancy between the theoretical and measured resonance frequencies are explained by a residual stress of $15\,MPa$. A

lock-in amplifier was used to measure the frequency response as shown in Fig. 1(c).
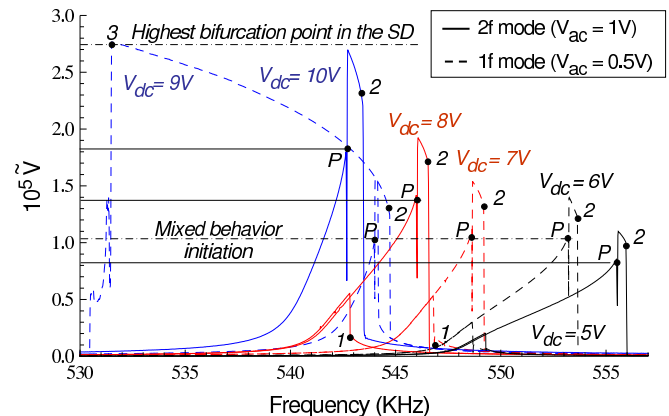

FIG. 4: Frequency responses showing mixed behaviors measured under primary resonance as well as under simultaneous primary and superharmonic excitations, the location of the different bifurcation points and the effect of the DC voltage on the P point.

For each 1f mode measurement, the traced voltage V_{out} is the result of the vectorial substraction of the measurement with zero DC voltage from the AC+DC overall measurement, so the feedthrough voltage due to parasitics is gotten rid of and V_{out} is only due to the mechanical motion of the resonator. For 2f mode measurements, the background due to feedthrough is already at a negligible level. Figure 4 shows six nonlinear mixed behavior peaks, dashed and solid curves corresponding to "1f mode" and "2f mode" respectively. The dimensionless voltage $\tilde{V} = \frac{V_{out}}{V_{dc}}$ is plotted in Fig. 4. On the three 1f-mode curves where $V_{ac} = 0.5\,V$, the P point vertical location is unchanged with respect to the drive amplitude, proportional to $V_{ac}V_{dc}$. Incidentally, one can observe that the shape of the peak changes as the quality factor slightly varies with V_{dc} due to ohmic losses[16]. The onset of the mixed behavior is set only by the gap-to-width ratio of the device[14].

The three 2f-mode curves were obtained for $V_{ac} = 1V$, and a Vdc voltage increased from 6 to 10V. Since the Ω- and 2Ω-excitation amplitudes are proportional to VacVdc and V_{ac}^2 respectively, only the Ω-excitation amplitude was amplified. This increases significantly the quadratic nonlinearity, proportional to $V_{dc}^2 + \frac{1}{2}V_{ac}^2$[12], which in turn amplifies the superharmonic resonance. As a result, and due to the nonlinear interaction between the two harmonics, the slow dynamics transfers more energy into the resonance at ω_n. The bifurcation topology is thus modified, and the onset of the mixed behavior in particular can be retarded as shown in Fig. 4: the slow Ω-excitation allows one to tune it; the simultaneous resonance shows the fast effect of a slow nonlinear resonance on the resonator bifurcation topology around the mixed behavior.

To conclude, generally the geometry of a resonator is designed to satisfy some goals in terms of performances (mass sensitivity, force sensitivity, frequency stability. . .). These performances are limited by the signal to noise ratio, and hence the maximum vibration amplitude, usually set at the onset of bistability for stable operation and noise mixing rea-

sons. One can use a capacitive gate to cancel out the third order nonlinear terms and operate linearly beyond the Duffing critical amplitude, which is possible in a certain range of DC and AC voltage, and width over gap ratio. This new stability domain is small and is jeopardized by the onset of the mixed behavior which cannot be tuned with only primary resonance.

The use of simultaneous resonances (primary+superharmonic) has been shown here to overcome this limitation, by stabilizing the dynamic behavior of the resonator when operated at high drives. In fact, the effect of the high order nonlinearities can be retarded by an energy transfer between fast and slow dynamics which enlarges the stability domain beyond the onset of the mixed behavior. As a result, a great gain in performance is expected via a large increase of the carrier power of the device. This should be particularly true with nanoscale devices which display reduced dynamic ranges or signal to noise ratio.

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