



Dynamique des galaxies : gravité newtonienne & gravité modifiée

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**THESE DE DOCTORAT DE
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Olivier Tiret

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**Dynamique des Galaxies :
Gravité newtonienne & Gravité modifiée**

Soutenue le 20 Juin 2008

devant le jury composé de :

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Dynamique des Galaxies : Gravité newtonienne & Gravité modifiée

Mots clés : Gravitation – Matière noire – Galaxies – Simulations

Résumé

La dynamique des systèmes gravitationnels s'appuie traditionnellement sur la physique de Newton. Appliquée à l'échelle des galaxies, la gravitation newtonienne impose l'existence d'une certaine matière, actuellement invisible : la matière noire. Si ce modèle (Λ Cold Dark Matter) rencontre des succès à grande échelle, des difficultés apparaissent à l'échelle des galaxies. Dans ma thèse, j'explore par des simulations numériques une alternative de la gravitation newtonienne : MOND (Modified Newtonian Dynamics), où la loi newtonienne de la gravité est modifiée selon une échelle d'accélération, sans l'intervention de la matière noire. Cette expression de la gravitation est non-linéaire et impose une méthode différente de celle utilisée dans les systèmes avec matière noire. J'ai écrit un code permettant la résolution des deux modèles de gravité, ce qui a permis de les comparer. J'ai testé ainsi l'évolution de galaxies spirales isolées puis en interaction. Ces simulations modélisent aussi la dissipation du gaz froid et la formation d'étoiles. Celles-ci ont montré que les galaxies sont moins stables en gravitation modifiée qu'en gravitation newtonienne, elles forment des barres plus rapidement. Ces simulations ont aussi révélé des différences importantes sur les transferts de moment angulaire lors des formations des barres et sur les effets de friction dynamique qui ralentissent les barres. Ce travail a permis de réaliser, pour la première fois en gravité modifiée, des simulations de galaxies en interaction du type des Antennes. Là encore, les effets de friction dynamique ont un rôle majeur sur la durée du temps de fusion, plus long en gravitation modifiée. Ceci ouvre des horizons vers des simulations cosmologiques qui pourraient valoriser un modèle en analysant la formation hiérarchique des structures à partir des fluctuations de densité primordiales. Par ailleurs, la modélisation de la cinématique des galaxies (naines, spirales et elliptiques) est aussi approfondie. En particulier, l'analyse des courbes de rotation des galaxies spirales montre que celles-ci peuvent contenir un composant de gaz moléculaire froid deux fois plus massif que le composant atomique.

Galactic Dynamics : Newtonian Gravity & Modified Gravity

Keywords : Gravitation – Dark matter – Galaxies – Simulations

Abstract

Dynamics of gravitational systems is traditionally based on Newton's physics. At galactic scale, the Newtonian gravity imposes a special matter to exist : the dark matter. Nowadays, this matter is invisible. If this model (Λ Cold Dark Matter) is successful at large scales, some difficulties appear at galactic scale. In my thesis, I use numerical simulations to explore an alternative to the Newtonian gravity : MOND (Modified Newtonian Dynamics), where the modification of the gravity follows an acceleration scaling law, without invoking any dark matter. This gravity is non linear and needs specific methods than those which are used for the Newtonian gravity with dark matter. I wrote a code able to solve the two gravity models, to compare each other. Then, I tested the evolution of isolated galaxies and interacting galaxies. These simulations take into account the dissipation of cold gas and the star formation. They have shown that galaxies are less stable in modified gravity than in Newtonian gravity, the bars form faster in MOND. These simulations have revealed some important differences concerning the angular momentum exchange during the bar formation and dynamical friction effects which slow down the bars. A simulation of interacting galaxies like the Antennae is feasible for the first time in modified gravity. Again, the dynamical friction effects play a major role about the fusion time-scale, longer in modified gravity. New horizons toward cosmological simulations are opened and could valorized a model by studying hierarchical structure formation starting from primordial fluctuation. Moreover, modelisation of galaxy kinematic (dwarves, spirales, elliptics) is deepened. In particular, the analysis of rotation curves shows that spiral galaxies could contain a molecular gas component two times heavier than the atomic component.

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Chapitre 1

Introduction

1.1 Le modèle cosmologique standard

La notion de matière noire est apparue dans les années 30, tandis que l'astronome F. Zwicky étudiait le mouvement individuel des galaxies dans l'amas de Coma. Il s'est rendu compte que les vitesses des galaxies étaient beaucoup trop importantes pour que l'amas soit à l'équilibre. Une masse invisible devait être présente afin de retenir les galaxies dans la structure de l'amas, et éviter qu'il ne se disloque ; c'est l'hypothèse de la matière noire. A la même période des observations sont réalisées à l'échelle des galaxies. Celles-ci commencent aussi à montrer que la masse totale des galaxies est supérieure à celle de leur contenu en étoiles et en gaz e.g. Rubin et al. (1985). C'est dans les années 70 qu'il devient généralement accepté que toutes les galaxies sont entourées d'un halo de matière noire et que la masse manquante à l'échelle des galaxies et des amas de galaxies doit avoir la même origine.

La nature de la matière noire étant inconnue, de nombreux candidats sont proposés, conduisant à trois grands scénarios sur la formation de l'univers. Les modèles de matière noire chaude (HDM) et de matière noire tiède (WDM) faisant intervenir respectivement des particules de quelques eV et keV. Le candidat le plus adapté à l'époque (années 90) était le neutrino. Le troisième modèle est celui de la matière noire froide (CDM) où les particules de matière noire sont non relativistes lors de leur découplage. Les méthodes de calcul et la puissance des ordinateurs au cours de cette période deviennent suffisamment performantes pour réaliser des simulations numériques de formation des structures dans ces différents modèles cosmologiques. Le modèle de HDM est ainsi éliminé parce que les particules relativistes mises en jeu ne permettent pas l'effondrement gravitationnel à petite échelle : les galaxies ne se forment pas. Le modèle de WDM corrige en partie ce problème. Les petites structures arrivent à se former, mais elles apparaissent beaucoup trop tard par rapport à ce que l'on observe dans notre Univers.

De nos jours il est communément accepté de représenter notre Univers par le modèle Λ CDM (Λ Cold Dark Matter). Celui-ci décrit un univers en expansion dominé par de la matière noire froide. La contribution de la matière ordinaire (baryons) est quant à elle faible, 4% ; de plus que les observations ne détectent que 10% des baryons seulement (Fukugita et al. 1998).

Dans ce modèle, les structures se forment de manière hiérarchique à partir de fluctuations primordiales. Tout un réseau de filaments cosmiques se construit petit à petit par l'intermédiaire de la matière noire, qui entraîne avec elle le gaz chaud primordial. Celui-ci se refroidit alors tout en formant les premières ébauches de galaxies. Ces protogalaxies interagissent entre elles, fusionnent, pour finalement former les galaxies que l'on observe aujourd'hui.

L'augmentation de la puissance de calcul ces dix dernières années a permis une avancée considérable de la compréhension de ce modèle par rapport à ce que notre Univers veut bien nous laisser voir. Ces simulations numériques à haute résolution font du modèle Λ CDM un très grand succès en ce qui concerne l'étude de la formation des grandes structures de l'Univers, mais, à l'échelle des galaxies trois problèmes majeurs restent actuellement non résolus.

En effet, les galaxies ainsi formées ne ressemblent pas aux galaxies observées, notamment les disques obtenus dans les simulations cosmologiques sont trop petits ; ce qui pose le *problème du moment angulaire* (Navarro & Benz 1991; Navarro & Steinmetz 1997, 2000a,b; D'Onghia et al. 2006; D'Onghia & Navarro 2007). De plus, les simulations prédisent que des galaxies comme la Voie lactée devraient être entourées de quelques centaines de galaxies satellites alors que l'on en observe seulement quelques dizaines, ce qui pose le *problème des satellites manquants* (Klypin et al. 1999; Moore et al. 1999a; Power et al. 2003; De Lucia et al. 2004). Egalement, notons le *problème des cuspidés* selon lequel les halos de matière noire sont trop concentrés (Moore et al. 1999b; Hayashi et al. 2007), et reproduisent mal les observations, en particulier pour les galaxies de faible brillance de surface (McGaugh & de Blok 1998a).

Différentes possibilités sont envisagées pour tenter de lever ces problèmes. Tout d'abord la résolution des simulations, les simulations de la fin des années 90 étaient considérées comme suffisamment résolues ainsi les défauts du Λ CDM n'étaient pas dus à des artefacts numériques. Les performances des supercalculateurs ont continué de progresser, de nouvelles simulations encore mieux résolues sont réalisées. Ces dernières montrent qu'il est possible de former des disques plus étendus qu'auparavant (Mayer et al. 2008). Avec des résolutions de quelques centaines de parsec, les galaxies sont suffisamment résolues pour former des bras spiraux, en particulier lors de l'instabilité de barre au cours de laquelle des transferts de moment angulaire s'effec-

tuent entre le disque galactique et son halo de matière noire. Les ondes de densité brassent les étoiles et le gaz des galaxies, les rendant moins compactes. De plus, avec une meilleure résolution, la pente des cusps dans les profils de matière noire a été revue à la baisse. Elle était initialement de 1.5 (Fukushige & Makino 1997; Moore et al. 1999b; Ghigna et al. 2000), et s'abaisse à 1.2 (Diemand et al. 2005); sans toutefois devenir des coeurs comme les observations l'indiquent préférentiellement (Swaters et al. 2003). Différentes formes de rétroactions des supernovae sont alors prises en compte pour éviter aux baryons de tomber dans les puits de matière noire trop concentrés : rétroactions thermique, cinétique, adiabatique, e.g. Mayer et al. (2008). D'autre part, on remarque que si les halos de matière noire prédits par les simulations Λ CDM sont trop concentrés à l'échelle des galaxies, à l'inverse, ils ne le sont pas assez à l'échelle des amas de galaxies (Duffy et al. 2008). Les effets de transfert radiatif commencent aussi à être étudiés dans les simulations et pourraient jouer un rôle dans le problème des satellites dont le nombre ne fait qu'augmenter avec la résolution des simulations (Strigari et al. 2007; Madau et al. 2008). En coupant l'accrétion du gaz sur les structures de faible masse de matière noire, après l'époque de reionisation, les prédictions théoriques peuvent être réconciliées avec les observations (Bullock et al. 2000). Le grand nombre de sous structures existerait toujours mais seulement en matière noire. Des effets de lentilles gravitationnelles devraient donc toujours avoir lieu, et permettre de tester cette hypothèse.

Toutes sortes de nouveaux phénomènes sont pris en compte pour tenter de rétablir les propriétés du Λ CDM en accord avec les observations à petites échelles. Toutefois la communauté des astrophysiciens n'a pas encore abouti à un consensus où ces principaux problèmes seraient résolus. Les solutions apportées sont dans certains cas satisfaisantes alors qu'elles ne font qu'accentuer le problème dans d'autres. La nature de la matière noire reste quand à elle toujours indéterminée, de nombreuses expériences tentent en vain de détecter le moindre indice sur cette matière exotique.

C'est pourquoi d'autres scénarios alternatifs méritent d'être explorés. En particulier des modèles ne faisant pas l'hypothèse sur l'existence de la matière noire, mais remettant en cause les lois de la physique classique. Parmi les modèles proposés, MOND se distingue aujourd'hui comme l'alternative la plus avancée et la plus robuste des théories de la gravitation. Cette théorie partie d'une modification empirique de la gravité s'est construite progressivement, s'étoffant de connaissances théoriques et reproduisant avec simplicité les données observationnelles. C'est cette voie que j'ai poursuivie dans ma thèse.

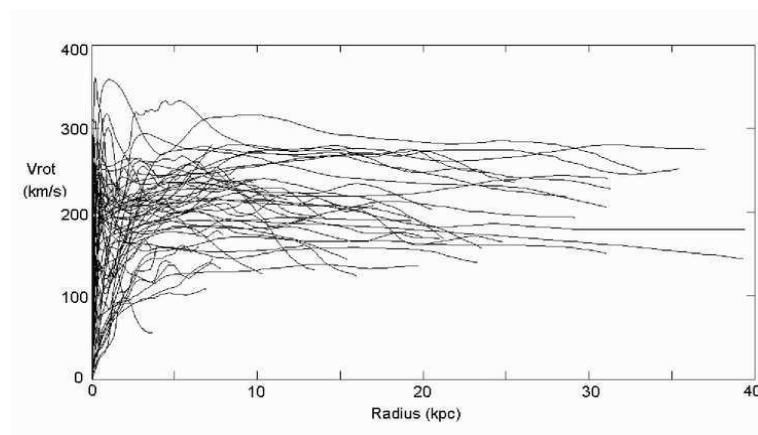


FIGURE 1.1 – Les courbes de rotation des galaxies observées sont anormalement plates si l’on considère le potentiel newtonien créé par la masse visible de la galaxie. Sofue et al. (1999)

1.2 Le Paradigme MOND

MOND (MODified Newtonian Dynamics) a été proposé en 1983 par Milgrom (1983a,b,c) pour décrire les courbes de rotation plates des galaxies sans avoir recours à la matière noire.

MOND distingue deux régimes de gravité, différenciés par leur échelle d’accélération : un régime de faibles accélérations (régime MONDien), et un régime de fortes accélérations (le régime newtonien). La transition est définie pour l’accélération critique :

$$a_0 = 1,2 \cdot 10^{-10} m.s^{-2}.$$

Nous verrons plus loin (sec. 1.3) comment la valeur de cette accélération a été déterminée. Dans la théorie MOND, cette accélération est présentée comme une constante universelle, au même titre que la constante de gravitation universelle G , la constante de Planck h , ou c la vitesse de la lumière.

Le régime MONDien. La principale motivation de Milgrom lorsqu’il a créé MOND était de reproduire la partie plate des courbes de rotation des galaxies (fig. 1.1) en considérant uniquement la matière observée. Pour cela, il propose de modifier l’accélération de la façon suivante :

$$a_M = \sqrt{a_0 a_N},$$

où a_M est l’accélération de MOND et a_N l’accélération newtonienne. En effet, pour une masse ponctuelle M ,

$$a_N = \frac{GM}{r^2},$$

et,

$$a_M = \frac{\sqrt{GMa_0}}{r},$$

ou,

$$a_M = \frac{v_c^2}{r},$$

avec v_c la vitesse circulaire, d'où,

$$v_c = (GMa_0)^{1/4} = \text{constante.} \quad (1.1)$$

Le plateau des courbes de rotation correspond donc à une région des galaxies où l'accélération est très faible devant l'accélération critique, là où la gravité MONDienne domine. D'un point de vue newtonien, le régime MONDien correspond à une région où la masse noire domine.

Dans ce régime, la force entre deux particules devient en $1/r$ au lieu de $1/r^2$. L'accélération critique a_0 est atteinte vers 25 kpc pour notre galaxie, et vers 7000 ua pour le Soleil.

Le régime newtonien. Pour le régime des hautes accélérations, la gravité newtonienne n'est pas modifiée :

$$a_M = a_N.$$

Ce régime correspond à la partie centrale des galaxies géantes, là où la courbe de rotation croît.

La paramétrisation de MOND. Ces deux régimes de gravité peuvent être regroupés dans une seule formule :

$$a_N = a_M \mu(|a_M/a_0|) \quad (1.2)$$

où μ est une fonction d'interpolation entre le régime newtonien et le régime MONDien. Pour retrouver les propriétés précédentes, il faut :

$$\mu(x) \begin{cases} 1 : x \gg 1, & a_M \gg a_0 \\ x : x \ll 1, & a_M \ll a_0 \end{cases}$$

Plusieurs fonctions μ ont été proposées dans la littérature. Celle que l'on rencontre le plus couramment est la fonction dite standard (tab. 1.1)

1.3 Les succès

Je rappelle ici, les principaux succès de MOND, et quelques résultats récents sur le sujet.

μ	Nom usuel	référence
$\frac{x}{(1+x^2)^{1/2}}$	Standard	Milgrom (1983a)
$\frac{x}{1+x}$	Simple	Famaey & Binney (2005)
$1 - e^{-x}$	Exponentiel	Milgrom (1983a)
$\frac{(1+4x)^{1/2}-1}{(1+4x)^{1/2}+1}$	TeVes	Bekenstein (2004)

TABLE 1.1 – Différentes formes de la fonction μ proposées dans la littérature.

Courbes de Rotation. L'élégance de MOND réside dans sa capacité à décrire la très grande diversité des courbes de rotation, une fois fixées la fonction μ et l'accélération critique a_0 (Sanders 1996). Toutes les formes de courbes de rotation sont bien modélisées par MOND, comme celles des galaxies de type précoce (Sanders & Noordermeer 2007). Ces galaxies sont très massives, leur courbe de rotation commence par augmenter pour atteindre un maximum (200-400 km.s⁻¹) et diminuer vers un plateau (200-250 km.s⁻¹). Les galaxies de faible brillance de surface (de Blok & McGaugh 1998) et les galaxies de très faible masse (Milgrom & Sanders 2007), sont aussi en accord avec les prédictions de MOND, ces galaxies sont d'autant plus intéressantes à étudier que leur accélération caractéristique est inférieure à a_0 ou de l'ordre de a_0 . Quelques exemples de courbe de rotation de galaxies, de divers types, sont représentées sur la Fig. 1.2.

Une autre catégorie de galaxies de faible masse a aussi été considérée dans le cadre de MOND, les naines sphéroïdales (dSph) découvertes dans le groupe local. Gerhard & Spergel (1992), Gerhard (1994) ont analysé ces observations et concluent que MOND ne réussit pas ce test, car les rapports masse/luminosité nécessaires ne sont pas réalistes. Toutefois, McGaugh & de Blok (1998b) argumentent sur le fait que le contenu en étoiles de ces petites structures n'est pas contraint avec précision ; de plus la plupart des valeurs trouvées par Gerhard (1994) semblent raisonnables (c'est à dire compris entre 1 et 6, Fig. 1.3).

Une dizaine de galaxies ne peut pas être modélisée par MOND¹. Mais dans ces quelques cas, il s'agit de galaxies pour lesquelles la distance ou l'inclinaison est mal connue, et d'autres présentent des signes d'interaction. Certains auteurs (Gentile et al. 2004) s'intéressent à des modélisations fines (à haute résolution) des courbes rotations en gravité newtonienne et modifiée. Quelques cas semblent difficiles à reproduire par MOND mais ne permettent pas de l'exclure définitivement.

La loi de Tully-Fisher. La loi de Tully-Fisher (Tully & Fisher 1977) est une loi observationnelle qui relie la luminosité L de la galaxie à la vitesse maximale v_c de la

1. <http://www.astro.umd.edu/~ssm/mond/>

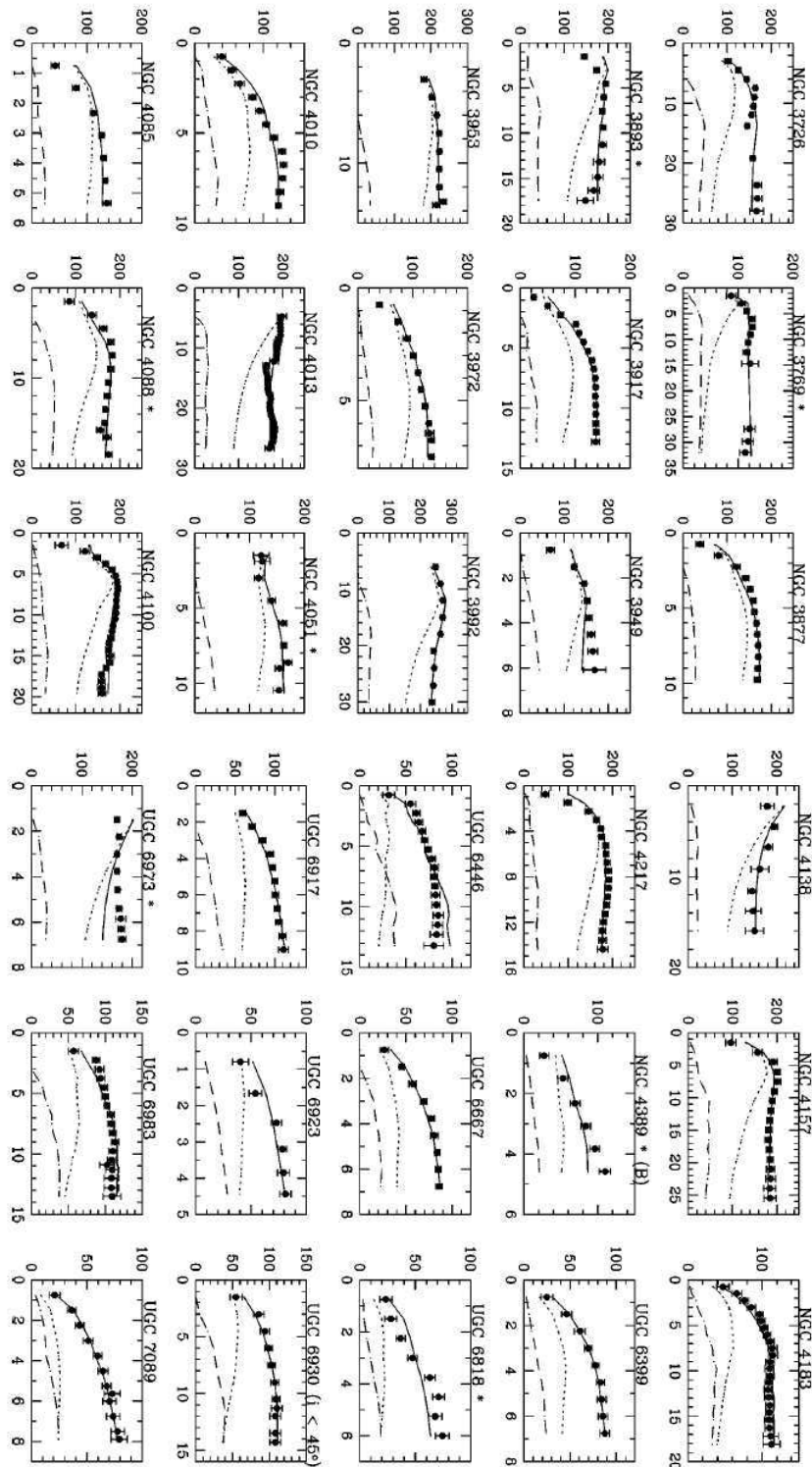


FIGURE 1.2 – Exemples de courbes de rotation ajustées par MOND (trait plein), la composante stellaire est représentée en pointillé et le gaz en tiret. Les distances sont en kpc et les vitesses en km.s^{-1} . Sanders & McGaugh (2002).

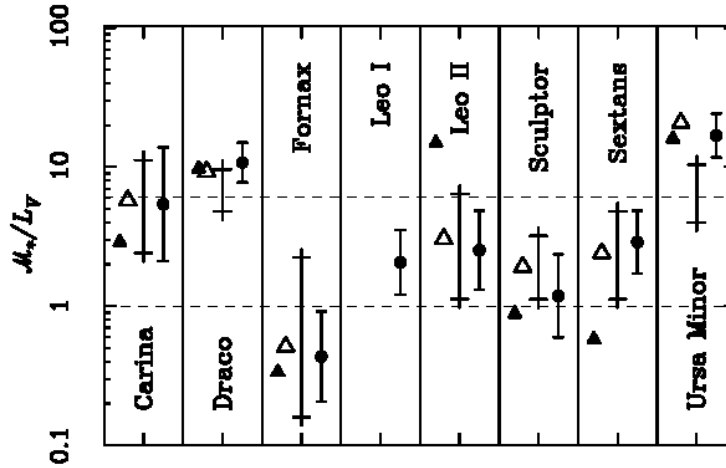


FIGURE 1.3 – Rapports masse/luminosité prévu par MOND pour les naines sphéroïdales du groupe local (Gerhard 1994; Mateo 1997)

courbe de rotation :

$$L \propto v_c^4$$

Cette loi n'avait pas de signification physique convaincante en gravitation newtonienne avec matière noire, tandis que cette relation coïncide parfaitement avec les prédictions de MOND.

En effet, d'après l'équation 1.1 :

$$M \propto v_c^4,$$

ce qui correspond à la loi de Tully-Fisher si l'on suppose un rapport Masse/Luminosité constant.

La formule exacte est en fait :

$$M = \frac{v_c^4}{G a_0}.$$

où l'accélération critique de MOND intervient. Cette loi est une façon d'estimer a_0 à partir de l'observation d'un grand nombre de galaxies (McGaugh & de Blok 1998a).

Vitesse d'échappement dans la Voie Lactée. Considérons une masse ponctuelle M , isolée dans l'Univers, et une particule test suffisamment éloignée de M , pour qu'elle se trouve dans le régime MONDien. Le potentiel gravitationnel ressenti par la particule test s'écrit :

$$\phi(r) = \sqrt{GM a_0} \ln(r)$$

Par conséquent, il n'est pas borné. La vitesse d'échappement v_{esc} que devrait avoir la particule pour sortir du puits de potentiel est infinie :

$$\frac{1}{2}v_{esc}^2 = \phi(r) - \phi(\infty).$$

Dans notre Galaxie, on mesure une troncature dans la distribution des vitesses au niveau du Soleil (projet RAVE, Smith et al. (2007)) correspondant à une vitesse d'échappement :

$$v_{esc} = 544_{-46}^{+64} \text{ km.s}^{-1}$$

Pour résoudre ce problème avec la gravitation modifiée, il faut prendre en compte l'effet d'un champ extérieur (Bekenstein & Milgrom 1984). En effet, MOND ne vérifie pas le principe d'équivalence fort, tout objet isolé tombe de la même manière dans un champ gravitationnel constant (principe d'équivalence faible), mais la dynamique interne du système n'est pas la même en présence ou non d'un champ extérieur (cf. section 4.3). Le champ extérieur permet de retrouver un potentiel newtonien au delà d'un certain rayon.

Famaey et al. (2007) ont montré que notre Galaxie devait être soumise à un champ extérieur de l'ordre de $a_0/100$ pour obtenir le bon ordre de grandeur de la vitesse d'échappement. Or cette valeur correspond justement à l'accélération produite par Andromède, au niveau de la Galaxie.

Amas de Galaxies : “The bullet cluster”. L'observation du double amas de galaxies 1E 0657-56, représentant deux amas en collision (Fig. 1.3, haut), a longtemps été considérés comme une observation allant à l'encontre de MOND. En effet, dans ce système, on distingue dans des régions différentes la répartition du gaz X intra amas, de la matière noire prédite par les cartes de lentilles gravitationnelles (Clowe et al. 2006). Or le gaz chaud est supposé dominer la masse des baryons (masse visible) à cette échelle (David et al. 1990; White & Fabian 1995). Les contours de convergence (κ) issus des cartes de lentilles gravitationnelles devraient être centrés sur le gaz, en gravitation modifiée.

Le problème est même plus général. A l'échelle des amas de galaxies, la masse baryonique n'est pas suffisante pour expliquer la dynamique de ces systèmes avec MOND. Pointecouteau & Silk (2005) ont montré qu'il manquait un facteur 5 entre la masse dynamique prédite par MOND et la masse baryonique observée.

Une solution pour résoudre ce problème de masse manquante à l'échelle des amas de galaxies est de prendre en considération la masse des neutrinos (Sanders 2003), au centre des amas. Les données observationnelles fournies par les amas de galaxies posent alors une forte contrainte sur la masse du neutrino.

Dans le cas de l'amas 1E 0657-56, Angus et al. (2006) trouvent que les cartes de lentilles gravitationnelles s'accordent avec MOND si les neutrinos ont une masse de l'ordre de 2eV. Cette estimation reste actuellement plausible avec les mesures de la

masse du neutrino. Celle-ci est encore mal déterminée, les expériences en physique des particules sur la désintégration β et les oscillations des neutrinos fournissent une limite supérieure s'étendant entre 0.8 eV et 3 eV (Fogli et al. 2004).

Un autre point a aussi été discuté sur l'amas 1E 0657-56, retournant la situation en faveur de MOND. L'analyse de l'onde de choc dans le gaz permet d'évaluer la vitesse relative de collision entre les deux amas : $v_{rel} = 4740_{-550}^{+710}$ km.s⁻¹. Cette vitesse semble très élevée pour être reproduite par les simulations Λ CDM. Hayashi & White (2006) trouvent une probabilité de 1 sur 500 d'obtenir un tel système d'amas dans la simulation Millenium (Springel et al. 2005). Ils estiment que *l'évènement est rare mais pas impossible*.

Toutefois les simulations numériques Λ CDM idéalisées ont montré que la vitesse de l'onde de choc pouvait être supérieure à la vitesse de collision (Springel & Farrar 2007; Milosavljević et al. 2007).

Angus & McGaugh (2008) ont étudié un modèle de chute libre du "bullet cluster", en comparant le Λ CDM et MOND. Ils trouvent que la gravité newtonienne avec matière noire permet d'obtenir une vitesse relative atteignant seulement 3500 km.s⁻¹. En revanche, MOND n'a pas de difficulté à reproduire une vitesse relative de 4700 km.s⁻¹. L'accélération MOND est plus forte et permet d'atteindre cette vitesse élevée plus rapidement sur un temps de Hubble.

Mastropietro & Burkert (2007) ont réalisé une simulation numérique réaliste de l'amas 1E 0657-56, dans le cadre du Λ CDM. Ils arrivent à reproduire les observations si la vitesse relative initiale est de l'ordre de 3000 km.s⁻¹, ce qui est assez élevé. En diminuant la vitesse initiale (2000 km.s⁻¹), la morphologie de l'amas en collision est bien respectée, par contre l'émission en rayon X du gaz chaud intra-amas excité par la collision est moins importante et n'est plus en accord avec les observations.

ABEL 520. Par les mêmes techniques d'observation, un autre amas ABEL 520 (Fig. 1.3, bas) présente cette fois des signatures en faveur de MOND. En effet, les cartes de matière noire prédites par les lentilles gravitationnelles montrent que la distribution de matière noire est corrélée avec la répartition du gaz X. Le gaz chaud domine bien la masse du système. Dans le modèle Λ CDM, on s'attend à ce que la matière noire soit répartie autour des galaxies les plus massives de l'amas, ce qui n'est pas le cas dans ces observations (Mahdavi et al. 2007). Un an après la publication des résultats sur l'amas *Bullet Cluster*, l'observation d'ABEL 520 apporte à son tour un contre-exemple.

Ces données sont récentes, l'amas ABEL 520 n'a pas encore fait l'objet de modélisation particulière à ce jour.

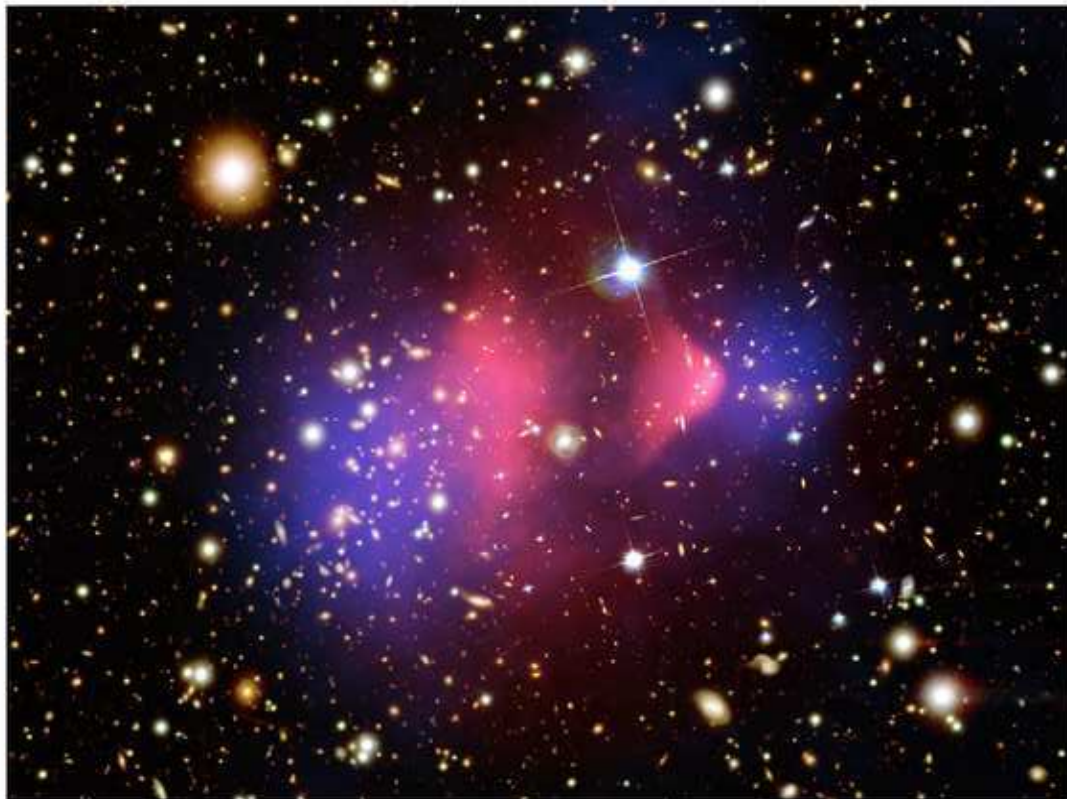


FIGURE 1.4 – L’amas de galaxies 1E 0657-56 *Bullet Cluster*, représente la collision entre de deux amas (Clowe et al. 2006). Cette image composite est la superposition d’une image en visible, de la carte en émission X (rose) et de la carte de matière noire déduite des effets de lentilles gravitationnelles faibles (bleu). Dans cet amas, le gaz X ne suit pas la distribution de matière noire.

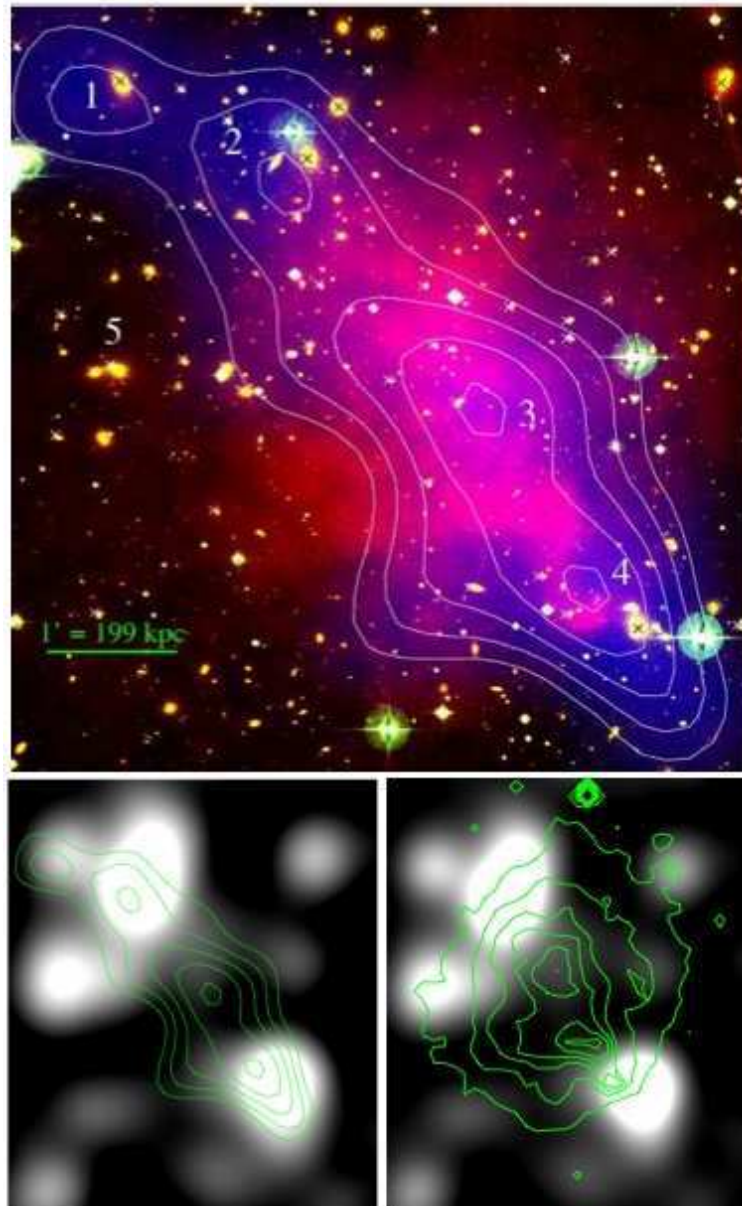


FIGURE 1.5 – ABEL 520, un autre amas de galaxies pour lequel la distribution de matière noire est surprenante (Mahdavi et al. 2007). Dans ce cas, le gaz X suit bien la distribution de matière noire. *Haut* : composition de l’image en visible, de la carte en émission X (rose) et de la carte de matière noire (bleu), les croix indiquent les galaxies les plus massives. Superposition de la carte de lumière des galaxies rouges avec les contours des lentilles gravitationnelles (*en bas, à gauche*) et avec les contours de l’émission X (*en bas, à droite*). Aucune galaxie ne se trouve dans la région du pic maximum des contours de lentilles gravitationnelles.

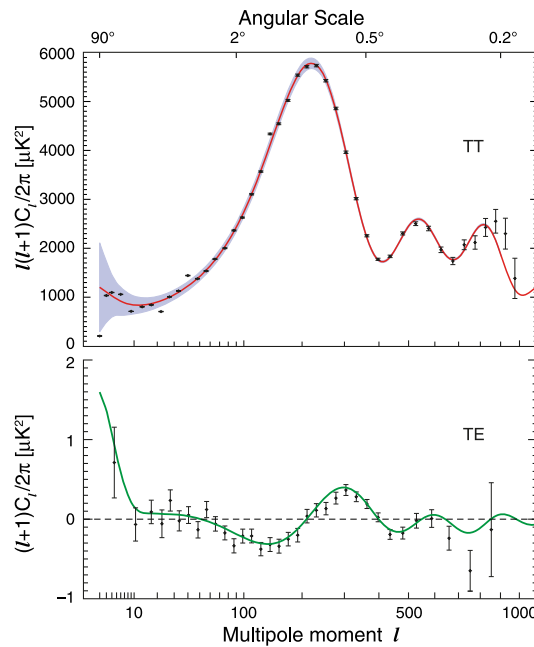


FIGURE 1.6 – Mesure du spectre de puissance (haut) et de la polarisation (bas) par WMAP-5, Hinshaw et al. (2008)

Fond diffus cosmologique (CMB). Les structures de l’Univers que nous observons aujourd’hui (galaxies, amas de galaxies, filaments) sont nées de fluctuations primordiales, qui ont donné lieu aux anisotropies observées dans le fond diffus cosmologique. Elles sont caractérisées par le spectre de puissance des fluctuations de température de ce fond (Fig. 1.6). Plusieurs expériences comme COBE, BOOMERANG, MAXIMA et dernièrement WMAP ont permis de révéler de plus en plus précisément la structure de ces anisotropies. Cette analyse permet de dresser un bilan de la matière présente dans notre Univers. Dans le cadre du modèle Λ CDM, les données récentes WMAP-5 (Hinshaw et al. 2008) permettent d’estimer la quantité de baryons $\Omega_b = 4.4\%$, la quantité de matière noire $\Omega_c = 21.4\%$ et la quantité d’énergie noire $\Omega_\Lambda = 74.2\%$.

Ces données constituent aussi un défi à relever pour MOND, mais fournissent elles vraiment un test sur la loi de la gravitation aux grandes échelles? McGaugh (1999, 2000) s’était intéressé aux fluctuations du CMB dans un Univers sans matière noire (en gravité newtonienne). Le résultat est plutôt remarquable (Fig. 1.7), les deux premiers pics sont très bien ajustés par ce modèle alors que le deuxième pic du Λ CDM est beaucoup trop haut. Les données observationnelles se sont affinées ensuite, rendant incompatible le modèle sans matière noire.

Skordis et al. (2006) se sont intéressés aux fluctuations du fond diffus cosmologique dans le cadre de MOND. La théorie de Bekenstein de MOND (voir section 1.5) permet, elle aussi, de décrire les fluctuations du CMB observées, moyennant

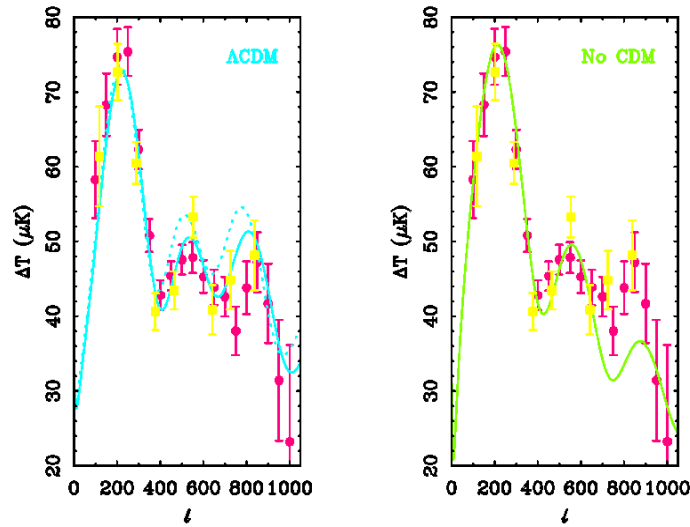


FIGURE 1.7 – Prédiction du spectre de puissance dans un univers newtonien avec matière noire (gauche) et sans matière noire (droite), McGaugh (2000). “And yes, the prediction was in the literature before the data”, S. McGaugh.

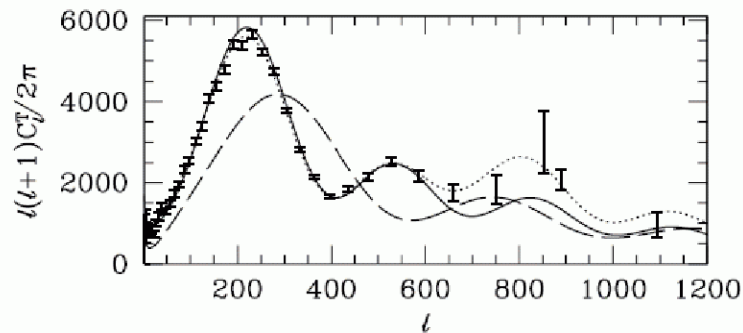


FIGURE 1.8 – Ajustement du spectre de puissance pour un Univers MONDien, $\Omega_b = 5\%$, $\Omega_\nu = 17\%$, $\Omega_\Lambda = 78\%$ (trait plein); $\Omega_b = 5\%$, $\Omega_\Lambda = 95\%$ (tiret); et pour le modèle Λ CDM (pointillé); Skordis et al. (2006).

quelques contraintes. Notons que MOND ne résout pas le problème de l'énergie noire, la constante cosmologique est toujours nécessaire et domine de loin le bilan énergétique. Dans le cadre de MOND, si les baryons représentent $\Omega_b = 5\%$ de l'Univers, l'énergie noire représente alors $\Omega_\Lambda = 95\%$ du total, ce qui aurait pour conséquence de décaler les pics du spectre de puissance angulaire vers de plus grands l (Fig. 1.8). La même solution que pour les amas de galaxies est nécessaire pour résoudre le problème avec MOND. Skordis et al. (2006) montrent qu'en prenant en compte une fraction $\Omega_\nu = 15\%$ de neutrinos dans l'Univers, les anisotropies du fond diffus cosmologique sont bien reproduites par MOND (Fig. 1.8).

Cette analyse aboutit à un test qui permettra peut-être de distinguer MOND du modèle Λ CDM. La hauteur du troisième pic prévu par les deux modèles est sensiblement différente. Pour l'instant ce pic n'est pas bien déterminé par WMAP, mais la mission PLANCK devrait permettre d'améliorer considérablement la précision du spectre dans ce domaine.

1.4 Des problèmes

Au même titre que le Λ CDM rencontre des succès aux grandes échelles et des problèmes aux petites échelles, on peut dire que MOND reproduit bien les petites structures, mais n'est pas entièrement convaincant aux échelles cosmologiques. Le fait de devoir prendre en compte les neutrinos en plus des étoiles et du gaz est toutefois satisfaisant car ces particules existent bien dans l'Univers.

Bien que leur masse reste encore mal déterminée, les neutrinos (de 2eV) nécessaires pour rendre de compte la structure à grande échelle de notre Univers par des théories alternatives de type MOND, frôlent la limite supérieure estimée actuellement entre 0.8 eV et 3 eV. L'expérience KATRIN, dont la mise en route est prévue en 2009, pourra falsifier des neutrinos électroniques de 2 eV à un niveau de confiance de 95%.

D'autre part les théories actuelles de MOND ne permettent pas de fournir une explication sur l'origine de l'accélération de l'Univers. La proportion d'énergie noire nécessaire pour ajuster les données du CMB domine toujours le bilan énergétique de notre Univers (environ 75%).

Pour tenter de se passer de neutrinos massifs et de trouver une origine sur l'accélération de l'univers, plusieurs travaux théoriques dans le cadre des théories alternatives de la gravitation sont en cours. Ces théories décrivent la constante cosmologique Λ par un champ vectoriel, remettant au goût du jour l'éther d'Einstein (Zlosnik et al. 2006, 2007; Zhao 2007; Skordis 2008).

A ce stade nous arrivons à des échelles où nous ne maîtrisons pas toute la physique

mise en jeu. Mais la question la plus cruciale est de comprendre le sens physique de la modification de la gravité. L'origine de la fonction μ reste encore indéterminée. La modification $a_N = \mu(a_M/a_0)ma_M$ peut être interprétée par une modification de la gravité (modification de l'accélération) ou une modification de la loi d'inertie (modification de la masse inertielle, Milgrom (1994)).

Les principales difficultés de MOND relèvent de l'écriture d'une théorie relativiste ayant pour limite MOND dans le régime des faibles vitesses et des faibles accélérations. Dans la section suivante je retrace les principales étapes qui conduisent à l'élaboration de cette théorie.

1.5 MOND en physique théorique

Plusieurs étapes se sont succédées pour recadrer la formulation empirique de MOND dans un contexte théorique rigoureux. Une application de cette formule telle quelle à un ensemble de points ne conserve pas les grandeurs habituelles comme l'impulsion, le moment angulaire, ou l'énergie (Milgrom 1983c; Felten 1984), Fig. 1.9. La première étape a consisté à reformuler MOND à partir d'un lagrangien (Bekenstein & Milgrom 1984) et obtenir une première théorie effective de MOND : AQUAL (AQUAdratic Lagrangian theory).

En gravité newtonienne, le lagrangien s'écrit :

$$L_N = - \int d^3r \left\{ \rho\phi + (8\pi G)^{-1}(\nabla\phi)^2 \right\},$$

et permet de dériver l'équation de Poisson newtonienne,

$$\Delta\phi = 4\pi G\rho \tag{1.3}$$

où ϕ est le potentiel gravitationnel et ρ le terme source. Dans la théorie AQUAL, le lagrangien modifié prend la forme suivante :

$$L_M = - \int d^3r \left\{ \rho\phi + (8\pi G)^{-1}a_0^2\mathcal{F} \left[\frac{(\nabla\phi)^2}{a_0^2} \right] \right\},$$

et l'analogue de l'équation de Poisson en gravité modifiée s'écrit :

$$\nabla \cdot \left\{ \mu \left[\frac{|\nabla\phi|}{a_0} \right] \nabla\phi \right\} = 4\pi G\rho. \tag{1.4}$$

Dans cette théorie, la fonction \mathcal{F} (reliée à μ par $\mu(\sqrt{y}) = d\mathcal{F}(y)/dy$) et a_0 sont ajoutées à la *main*. \mathcal{F} doit avoir les propriétés suivantes :

$$\mathcal{F}(y) \begin{cases} y & y \gg 1 \\ \frac{2}{3}y^{3/2} & y \ll 1 \end{cases}$$

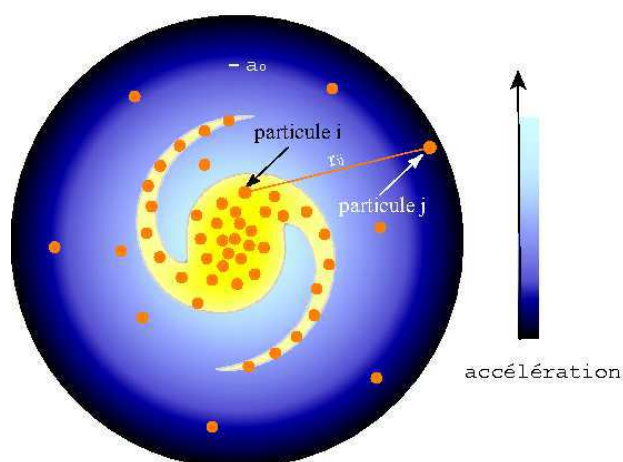


FIGURE 1.9 – Une application naïve de la formule 1.2 de MOND ne conserve pas l’impulsion. La particule i se trouve dans une région de forte accélération (gravité newtonienne), tandis que la particule j se trouve dans région de faible accélération (gravité MONDienne). Si l’on isole la contribution de j sur i et inversement, on a : $a_{j \rightarrow i} = Gm_j/r_{ij}^2$ et $a_{i \rightarrow j} = \sqrt{Gm_i a_0}/r_{ji}$. (On verra Chap. 3 qu’il n’est en fait pas correct d’isoler la contribution de chaque particule, la gravitation MOND n’est pas linéaire avec la masse.)

pour retrouver les bonnes propriétés de la fonction μ .

Les lois de conservation sont alors vérifiées grâce aux propriétés d'invariance du lagrangien par translation dans le temps et par translation/rotation dans l'espace. Une généralisation relativiste d'AQUAL est aussi proposée par Bekenstein & Milgrom (1984). Elle est en accord avec les tests post-newtoniens dans le système solaire. Mais cette théorie est acausale, les ondes peuvent se déplacer plus vite que la lumière.

Ce problème est résolu par la théorie PCG (Phase Coupling Gravity, Bekenstein (1988)). Par contre d'autres incompatibilités apparaissent, la déviation de la lumière prédite par PCG n'est pas cohérente avec les observations, cette déviation est trop faible, ce qui nécessiterait l'ajout de matière noire. De plus, PCG ne permet pas de reproduire correctement le mouvement de précession de Mercure.

Il a fallu attendre Bekenstein (2004) pour obtenir une théorie relativiste covariante de la gravitation, ayant pour limite MOND dans le régime des faibles vitesses et des faibles accélérations. Cette théorie est appelée TeVeS (Tensor Vector Scalar). TeVeS est en accord avec les tests conduits dans le système solaire (développement post-newtonien de la relativité générale, mouvement de Mercure). Les effets de lentilles gravitationnelles sont bien reproduits et ne requièrent pas de matière noire supplémentaire.

Bien que résolvant la plupart des obstacles rencontrés par les théories précédentes, Bruneton & Esposito-Farèse (2007) ont mis en évidence des problèmes de stabilité (l'Hamiltonien n'est pas borné inférieurement) et d'autres problèmes de "fine tuning" sur la fonction libre. Ils concluent que TeVeS est la théorie la plus prometteuse dans le domaine mais qu'elle ne constitue pas une théorie cohérente pour l'instant.

TeVeS est une théorie effective, elle ne donne pas d'explication sur la modification de la gravité. Ces travaux théoriques sont actuellement en cours, en espérant obtenir une compréhension microscopique de la Gravitation.

Sur ce point, on peut relever l'étude de Blanchet (2007), Blanchet & Le Tiec (2008), où MOND est interprété en gravité newtonienne par le phénomène de polarisation bien connu en électromagnétisme.

Le champ (E, g) dérive d'un potentiel (ϕ_e, ϕ_g) :

$$E = -\nabla \phi_e \quad | \quad g = -\nabla \phi_g$$

Il existe une équation du champ (équation de Maxwell, équation de Poisson) :

$$\rho_e = -\nabla D \quad | \quad \rho_g = -\nabla g_N$$

En électromagnétisme, on définit la permittivité ϵ , et la susceptibilité χ_e , d'un diélectrique. μ et χ_g serait l'analogue en gravitation :

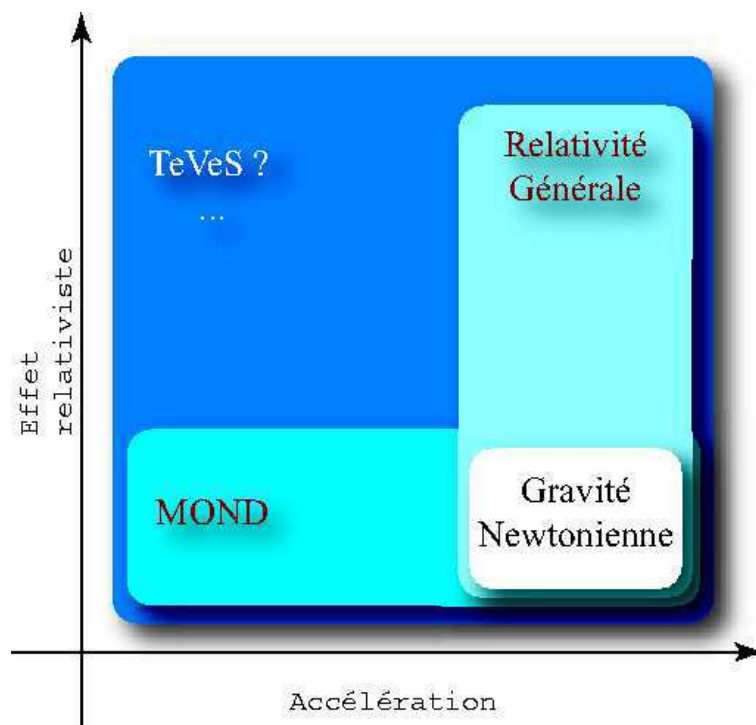


FIGURE 1.10 – Vers une théorie complète de la Gravitation ...

$$\begin{array}{l|l} D = \epsilon E & g_N = \mu g \\ \epsilon = \epsilon_0(1 + \chi_e) & \mu = (1 + \chi_g) \end{array}$$

MOND peut être interprété par un effet de polarisation.

$$D = \epsilon_0 E + P \quad | \quad g_N = g + g_p$$

De la même façon qu'une charge polarise un diélectrique, une masse plongée dans un "certain milieu" polariserait ce dernier. L'intensité de la masse faisant varier la distance entre les deux dipôles, le champ résultant mimerait l'effet MOND.

Toutes ces théories alternatives de la gravitation sont des extensions de la relativité générale d'Einstein (Fig. 1.10), qui reste de nos jours la théorie la plus avancée malgré les problèmes de matière noire et d'énergie noire.

1.6 Simulations numériques

Lorsque j'ai commencé ma thèse, très peu de simulations N-Corps avaient été réalisées en gravité modifiée. Les seules simulations où l'équation de Poisson modifiée était vraiment résolue (cf sect. 3) étaient celles de Brada & Milgrom (1999). Cet article porte sur l'évolution de disques purement stellaires dans un espace à deux dimensions. Ces simulations montrent l'effet stabilisant de MOND par rapport à

la gravitation newtonienne sans matière noire. La formation de gauchissement produit par une galaxie satellite est analysée dans Brada & Milgrom (2000a). Il s'agit de simulations où les étoiles sont lancées sur des orbites dans le potentiel statique MON-Dien d'une galaxie perturbée par un compagnon. Dans Brada & Milgrom (2000b), l'évolution d'une galaxie naine orbitant autour d'une galaxie hôte est présentée. En plus des effets de marée, la galaxie hôte crée un champ extérieur (sect. 4.3); il est montré que la perturbation ressentie par le satellite est plus importante qu'en gravité newtonienne avec matière noire.

Nusser (2002), Knebe & Gibson (2004) ont présenté des simulations cosmologiques avec MOND. Ces simulations sont souvent contestées car le potentiel MON-Dien n'est pas calculé directement à partir de la distribution de matière. Ils utilisent un code newtonien, les accélérations MOND sont calculées à partir des accélérations newtoniennes via la formule de MOND (eq. 1.2).

Ces trois dernières années, une équipe Italienne a aussi développé un code pour traiter la gravitation modifiée (Ciotti et al. 2006; Londrillo & Nipoti 2008). Ils résolvent l'équation de Poisson modifiée sur une grille à géométrie sphérique. Ce code est bien adapté pour l'étude de galaxies sphéroïdales non-collisionnelles (Nipoti et al. 2007a,b, 2008).

La principale motivation dans ma thèse est de réaliser des simulations N-Corps en gravité modifiée et en gravitation newtonienne avec matière noire, afin de les comparer. Il n'y avait pas de code existant pour résoudre l'équation de Poisson modifiée. Ceci m'a amené à en écrire un (Sect.3) et à l'appliquer à diverses situations (galaxies isolées, en présence d'un champ extérieur, ou en interaction, cf Sect. 4). Je me suis aussi intéressé à la modélisation de données, en particulier les courbes de rotation des galaxies spirales et les courbes de dispersion de vitesse dans les galaxies elliptiques, c'est par cet aspect que je commence la présentation de mon travail (Sect. 2).

Chapitre 2

Le problème de la masse manquante, une approche observationnelle

2.1 Modifications de la gravité et baryons noirs

Tiret & Combes 2008, en préparation

Résumé. Dans cet article, la modification de la gravité est présentée à partir des observations des courbes de rotation. En effet, lorsque celles-ci sont modélisées dans le cadre de la gravitation newtonienne, on peut déterminer la masse dynamique de la galaxie et la comparer à sa masse visible. Il s'avère que l'écart entre la masse dynamique et la masse visible est corrélé à l'accélération newtonienne de la composante visible, ce qui est le fondement de MOND (Fig. 2.1). L'analyse newtonienne des courbes de rotation montre que l'accélération est un critère pertinent (et non la distance par exemple) pour modifier la gravité et retrouver une masse visible égale à la masse dynamique. Les galaxies semblent donc obéir à une loi d'échelle sur l'accélération. Cette analyse en gravitation newtonienne permet aussi de contraindre la forme la fonction μ .

Dans une seconde partie, nous discutons de la valeur de l'accélération critique a_0 si une fraction des baryons noirs est prise en compte sous forme de gaz moléculaire H_2 dans les disques des galaxies. En effet, notre Univers contient 5% de baryons (nucléosynthèse primordiale, anisotropies du fond diffus cosmologique), mais la matière que l'on observe dans les galaxies représente seulement 10% de ces baryons (Fukugita et al. 1998). Le bilan total montre qu'au minimum 50% des baryons restent "sombres".

Pour cela, nous utilisons deux approches :

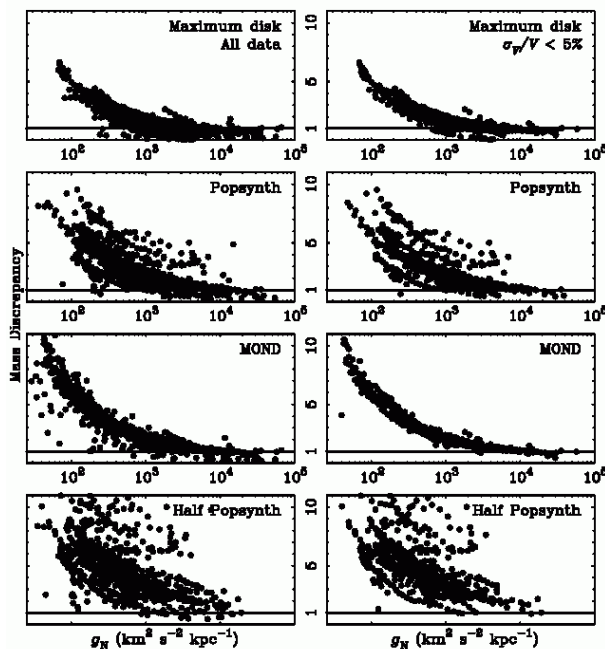


FIGURE 2.1 – Corrélation entre le rapport masse dynamique/masse visible et l'accélération. Cette relation ne dépend pas de la manière dont est estimé le rapport masse luminosité (méthode du disque maximum, population stellaire, MOND, ...). McGaugh (2004).

- les ajustements des courbes de rotation MONDien. On cherche si une fraction de gaz moléculaire suivant la distribution de gaz atomique (HI et He) permet de mieux ajuster les courbes de rotation. Et quel est l'impact sur l'estimation de l'accélération critique a_0 . Cette méthode reprend l'analyse de Hoekstra et al. (2001) où ils analysent 24 courbes de rotations spirales (en gravitation newtonienne) et trouvent que si la densité surfacique de gaz HI est multipliée par 7 en moyenne, il n'y a plus besoin de d'ajouter de matière noire.
- l'optimisation de la dispersion dans la loi de Tully-Fisher baryonique en variant aussi (a_0, c). En effet, on distingue la loi de Tully-Fisher classique qui représente la luminosité (donc la masse en étoiles) en fonction de la vitesse asymptotique de la courbe de rotation, de la loi de Tully-Fisher baryonique (masse en étoiles et en gaz). Dans la loi de Tully-Fisher classique, les galaxies de type tardif ne vérifiaient pas cette relation (Fig. 2.1). En prenant en compte la masse de gaz ces galaxies s'alignent aussi sur la même loi. Dans le cadre des baryons noirs, l'idée consiste à chercher si une fraction de gaz moléculaire améliore aussi la relation (Pfenniger & Revaz 2005). Nous avons interprété cette relation dans le cadre de MOND, avec un échantillon ne contenant que des galaxies de faible masse (type tardifs).

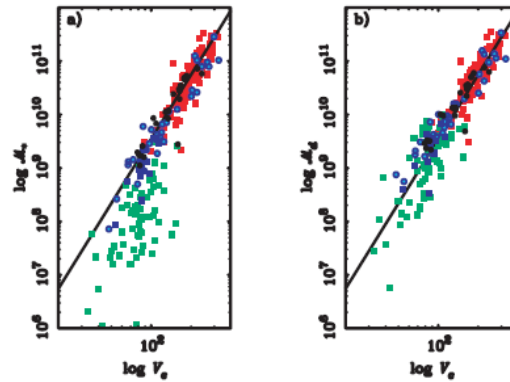


FIGURE 2.2 – McGaugh et al. (2000) représente la loi de Tully-Fisher baryonique dans laquelle la masse prise en compte est la masse en étoiles et la masse en gaz atomique (droite), au lieu de la masse en étoiles seulement (gauche). Les galaxies de type tardif vérifient bien la même loi de Tully-Fisher que les galaxies de types précoces.

Cette analyse montre que les ajustements des courbes de rotation et la minimisation de la dispersion dans la loi de Tully-Fisher sont obtenus pour $a_0 \sim 0.9 - 1 \cdot 10^{-10} \text{m.s}^{-2}$ et $c \sim 3$. L'analyse la plus précise est en fait apportée par les ajustements des courbes de rotation. Il y a bien une accélération critique commune à toutes les galaxies, par contre la proportion de gaz moléculaire peut varier d'une galaxie à l'autre (entre 0 et 6). Ceci est réaliste car le rapport entre gaz atomique et moléculaire n'a pas de raison d'être universel, il dépend du taux de formation d'étoiles, de l'histoire de la galaxie.

MOND and the dark baryons

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ABSTRACT

We consider for the first time the implications on the modified gravity MOND model of galaxies, of the presence of dark baryons, under the form of cold molecular gas in galaxy discs. We show that MOND models of rotation curves are still valid and universal, but the critical acceleration a_0 separating the Newtonian and MONDian regimes has a lower value. We quantify this modification, as a function of the scale factor c between the total gas of the galaxy and the measured atomic gas. The analysis of 43 resolved rotation curves allows us to find the best couple ($a_0 = 0.9 \times 10^{-10}$ m.s $^{-2}$, $c = 3.$), which is also compatible to the couple minimizing the scatter in the baryonic Tully-Fisher relation.

Key words. Galaxies: general — Galaxies: kinematics and dynamics — Galaxies: spiral — Galaxies: structure — Cosmology: dark matter

1. Introduction

The missing mass problem at galactic scales is revealed by rotation curves of spiral galaxies, where stars and gas rotate with speeds larger than expected (e.g. Sofue & Rubin, 2001), in the frame of Newtonian gravity. This missing mass has been called dark matter and has been developed now into a standard cosmological model Λ CDM (e.g. Blumenthal et al. 1984). Dark matter particles collapse to form the first structures of the Universe, and it is in those potential wells that the baryonic gas infalls and cools to form galaxies. As a result, galaxies are embedded in a spheroidal dark matter halo with a mass profile obeying $M(r) \propto r$ in the outer parts, so that the keplerian circular velocity, $v_c = \sqrt{GM(r)/r}$, tends to be constant in the majority of observed rotation curves (e.g. Bosma 1981). In the central parts however, the CDM numerical simulations predict a cuspy dark matter profile, with a relative CDM amount much larger than what is observed (Navarro et al 1997, Navarro & Steinmetz 2000).

An alternative, without invoking any dark matter, is to consider a modification of the Newton gravity law, at low accelerations, so that the luminous mass is sufficient to describe the dynamics of the gravitational system. This was done empirically by Moti Milgrom who proposed the MODified Newtonian Dynamics (MOND) paradigm (Milgrom, 1983). The MOND regime is distinguished from the Newtonian regime when the characteristic acceleration of the gravitational system falls below a critical acceleration $a_0 \sim 1.2 \times 10^{-10}$ m.s $^{-2}$. In the deep MOND regime, the modification a_M of the Newtonian acceleration, a_N , can be written as $a_M = \sqrt{a_0 a_N}$. The asymptotic circular velocity is thus a constant : $v^4 = GMa_0$. This is the Tully-Fisher law, which finds there a justification.

In this paper, we first want to emphasize that the missing mass problem is indeed tightly linked to the critical acceleration a_0 , in compiling a series of observed rotation curves, in particular those of dwarf irregular galaxies, which are dominated by dark matter all along their discs. As al-

ready shown by de Blok & McGaugh (1998, or Milgrom & Sanders (2007)), rotation curves are far from being always flat in the outer parts of galaxies. A detailed modelisation should yield more precision in the determination of the interpolation function $\mu(x)$, describing the passage from the Newtonian to MONDian regimes.

In a second part we consider for the first time how the problem of missing baryons could be made compatible with the MOND phenomenology. In the cosmic baryon budget, it is now well known that only about 6% of them are shining as stars and gas in galaxies (Fukugita et al. 1998). Locally, UV absorption in front of background sources have shown that as much as 30% of baryons could be associated to the Lyman α forest and an uncertain fraction (5-10%) in the Warm-Hot medium (WHIM at 10^5 - 10^6 K, Nicastro et al 2005, Danforth et al 2006). So about half of the baryons are not yet accounted for. Most of them should be in the cosmic filaments, in the intergalactic medium, but it appears unavoidable that a significant fraction of them lies as cold gas in galaxies themselves, as suggested by Pfenniger et al (1994), and Pfenniger & Combes (1994). From the rotation curves, it is possible to put an upper limit to the fraction of dark baryons in galaxies: the dark baryons cannot overpredict the rotation speeds. HI interferometric studies of nearby galaxies have shown that the dark matter distribution necessary to fit the rotation curves of galaxies follows the surface density of the atomic gas (Bosma 1981). Hoekstra et al. (2001) estimate that a factor 7-10 between the dark baryons and atomic gas reproduces the rotation curve of any galaxy, giant or dwarf, high of low surface brightness, without adding any dark matter halo. This scale factor means than there cannot be more than 10% of the total baryons in the form of cold gas in galaxies. The observed HI extended rotation curves roughly require only to double the fraction of baryons in galaxies at maximum. Out of the 50% of the dark baryons still not accounted for, no more than 10% can be in galaxies, but they can be less abundant. From a dynamical point a view, numerical simulations (Revaz et al. 2008) of galaxies in Newton

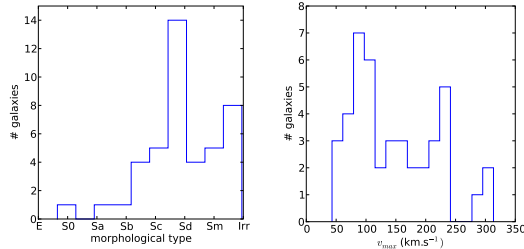


Fig. 1. *Left:* morphological type in the sample made of 43 nearby galaxies. *Right:* distribution of the maximum velocity in the galaxy rotation curves.

gravity with dark matter and cold dark baryons allow up to a scale factor of 5 to reproduce the global behavior of galaxy: the stability against local axisymmetric collapse is ensured through a low dissipation, and still spiral patterns can develop due to the self-gravity of the disc.

It is thus interesting to take into account the possible existence of dark baryons in the framework of MOND at galactic scales. If some fraction of the assumed dark matter is real, the actual critical acceleration a_0 is overestimated. We want to derive the new possible limiting acceleration, and interpolating function μ . The contribution of dark baryons has been discussed for modeling galaxy cluster cores by Milgrom (2007). At galactic scales, Gentile et al. (2007) have studied the contribution of neutrinos in rotation curves.

The paper is organized as follows: in section 2 we describe the galaxy sample. The modeling of galaxies is discussed in section 3. In section 4, we show how the mass discrepancy problem leads to a modification of the Newtonian gravity and in section 5, the implications of the existence of dark baryons with MOND are explored through rotation curve fits and the Tully-Fisher law, then discussed in section 6.

2. Galaxy sample

We have compiled from the literature a galaxy sample made of 43 nearby objects (Tabs. 1). For each of them, the resolved HI kinematical data are available. They are unperturbed (without large asymmetries such as lopsidedness or tidal interactions) so that a reliable rotation curve can be obtained. The galaxies have inclinations on the plane of the sky which vary from 30° to 90° . A stellar light profile is also available in the literature, so that the contribution of the visible mass can be computed with reasonable accuracy.

We have searched for late-type galaxies, where the fraction of gas (and dark matter) is more important. And in particular gathered a large number of dwarf irregulars, which are dominated by dark matter. Therefore, they are a convenient laboratory to determine the radial dark matter distribution, independently of the stellar mass-to-light ratio. The Fig.1 (left) displays an histogram of the morphological type in the sample, 70% of galaxies are late-type. The Fig. 1 (right) shows the distribution of the maximum velocity of galaxies, the median of the sample is $V_{max} = 128 \text{ km.s}^{-1}$.

3. Methods

3.1. Galaxy modeling

The stellar disc is modelled by an exponential surface density:

$$\Sigma_*(r) = \Sigma_{0*} \exp(-r/r_*).$$

and the bulge is represented by an Hernquist profile, of characteristic scale r_b :

$$\rho(r) = \frac{8\rho(r_b)}{(r/r_b)(r/r_b + 1)^3}$$

The parameters Σ_{0*} , r_* , and r_b are deduced from the luminosity profile fits.

The HI gas surface density is taken directly from observations without any particular modeling. Like this, it is particularly constraining to fit the wiggles of rotation curves associated with gas over density.

3.2. Rotation curve modeling

The rotation curves of each component (stellar disc, bulge, gas, dark matter) are computed using the Bessel functions. For a given surface density Σ , the Newtonian circular velocity can be written (Binney & Tremaine, 1994):

$$v_c^2(R) = -R \int_0^\infty S(k) J_1(kR) k dk,$$

with,

$$S(k) = -2\pi G \int_0^\infty J_0(kR) \Sigma(R) R dR.$$

J_1 and J_2 are Bessel functions of order 0 and 1. For an exponential disc, it can be simplified by:

$$v_c^2(R) = 4\pi G \Sigma_{0*} R_* y^2 [I_0(y) K_0(y) - I_1(y) K_1(y)],$$

where $y = R/2R_*$. I and K are the modified Bessel functions.

The MOND rotation curves are deduced from the Newtonian acceleration, using the MOND formula:

$$a_N = a_M \mu(a_M/a_0)$$

and inversely,

$$a_M = a_N \nu(a_N/a_0)$$

When not explicitly notified otherwise, we use the standard μ -function, $\mu(x) = x/\sqrt{1+x^2}$.

3.3. The mass-to-light ratio

To convert the luminosity profile to a mass profile, we define a mass to light ratio Γ_* . In Newtonian gravity with dark matter, the choice of Γ_* may vary along a large range of values, bounded at the top by the maximum disc hypothesis (van Albada & Sancisi, 1986). For giant spiral galaxies, where the spatial resolution does not smooth out the central gradient of the rotation curve, the maximum disc hypothesis yields physically reasonable mass-to-light ratios, and indicate that central parts of galaxies are not dominated by dark matter. In the Sect. 4, we use the maximum disc method to estimate the stellar mass to light ratio.

An other method independent from the rotation curve fits uses the color of the galaxy. With stellar population

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Name	D (Mpc)	L_B (L_\odot)	M_{HI} (M_\odot)	Type	V_{max} (km.s^{-1})	Inclination (degree)	References
DDO154	4.0	3.4E+07	1.8E+08	I	43	32	(1)
DDO168	4.0	2.2E+08	1.8E+08	I	52	80	(2)(3)
DDO170	15.8	3.1E+07	1.7E+09	I	66	80	(4)
IC2574	2.5	5.4E+08	2.1E+08	SABm	67	80	(5)
M33	0.8	2.8E+09	1.8E+09	Sc	134	55	(6)(7)
NGC1003	11.8	5.7E+09	2.5E+09	Sc	112	81	(2)
NGC1560	3.0	6.6E+08	7.0E+08	Scd	78	83	(8)
NGC2366	2.6	3.4E+08	3.4E+08	IB	45	90	(9)
NGC2403	3.6	5.9E+09	3.6E+09	SABc	140	60	(10)(11)(12)
NGC24	6.8	1.1E+09	3.8E+08	Sc	109	70	(13)
NGC247	2.8	2.2E+09	8.0E+08	SABc	107	75	(14) (15)
NGC253	2.6	1.2E+10	6.6E+08	SABc	224	78	(16)
NGC2841	20.0	4.9E+10	2.1E+10	Sb	292	68	(10)(11)
NGC2903	10.4	2.5E+10	5.8E+09	SABb	181	56	(10)(11)
NGC2915	5.6	4.7E+08	7.7E+08	SBa	93	63	(17)
NGC2998	70.7	3.9E+10	2.8E+10	SABc	200	61	(2) (18)
NGC300	2.9	3.5E+09	1.4E+09	Scd	93	39	(19)
NGC3109	1.9	9.8E+08	3.8E+08	SBm	67	80	(11)
NGC3198	11.6	1.1E+10	7.3E+09	Sc	149	70	(11)(20)
NGC3726	14.9	1.3E+10	7.6E+09	Sc	160	49	(9)
NGC4203	15.1	3.9E+09	7.5E+08	E-SO	152	90	(9)
NGC4242	7.3	1.3E+09	4.5E+08	Sd	98	51	(9)
NGC4258	7.3	1.7E+10	3.7E+09	SABb	216	72	(9)
NGC4395	2.7	6.4E+08	4.7E+08	Sm	85	90	(9)
NGC45	5.9	1.2E+09	1.3E+09	SABd	100	60	(13)
NGC4725	11.9	1.5E+10	2.6E+09	SABa	223	54	(9)
NGC5033	18.4	2.5E+10	1.5E+10	Sc	196	66	(10)(18)
NGC5055	10.3	2.3E+10	9.8E+09	Sbc	172	56	(9)
NGC5371	40.0	5.6E+10	9.9E+09	Sbc	313	54	(9) (10)
NGC55	1.6	4.6E+09	6.8E+08	SBm	86	85	(21)
NGC5533	58.3	3.5E+10	3.0E+10	Sab	230	60	(2) (22) (23)
NGC5585	7.6	1.9E+09	1.2E+09	SABc	89	53	(24)
NGC5907	11.0	1.3E+10	2.4E+09	Sc	219	87	(25)
NGC6503	4.8	2.0E+09	1.0E+09	Sc	122	74	(2) (9)
NGC6674	51.8	3.4E+10	2.7E+10	Sb	240	62	(2)
NGC6946	6.7	1.8E+10	7.8E+09	SABc	159	31	(26)
NGC7331	12.8	2.8E+10	7.3E+09	Sbc	241	75	(2)(11)
NGC7793	4.1	2.8E+09	1.0E+09	Scd	90	53	(27)
NGC801	84.0	5.1E+10	1.9E+10	Sc	218	85	(2) (18)
NGC925	6.5	4.1E+09	2.5E+09	Scd	114	61	(9)
UGC128	56.4	1.0E+09	6.2E+09	Sd	128	32	(28)
UGC2259	10.0	2.3E+08	3.7E+08	SBd	90	53	(11)
UGC2885	84.0	5.6E+10	5.9E+10	Sc	298	62	(18)

Table 1. (1) Carignan & Beaulieu (1989); (2) Br oeil (1992a); (3) Sanders (1996); (4) Lake et al. (1990); (5) Martimbeau et al. (1994); (6) Newton (1980); (7) Corbelli (2003); (8) Broeils (1992b); (9) Wevers (1984); (10) Begeman (1987); (11) Kent (1987); (12) Sicking (1997); (13) Chemin et al. 2006; (14) Carignan & Puche (1990b); (15) Carignan (1985); (16) Puche et al. (1991); (17) Meurer et al. (1994); (18) Kent (1986); (19) Puche et al. (1990); (20) Begeman (1989); (21) Puche et al. (1991a); (22) Broeils & Knapen (1991); (23) Kent (1984); (24) Cote et al. (1991); (25) Sancisi & van Albada (1987); (26) Carignan et al. (1990); (27) Carignan & Puche (1990a); (28) de Blok et al. (1985); (29) Carignan et al. (1988); (30) Roelfsema & Allen (1985)

models, the mass to light ratio can be determined from the measured galaxy colors (Bell & de Jong, 2001). This method is used in the Sect. 5.2 to plot the baryonic Tully-Fischer relation.

With MOND, when the μ -function and the critical acceleration a_0 are fixed, Γ_* is the only parameter to fit the rotation curve (see Sect. 5.1).

3.4. The visible molecular gas

In about a dozen galaxies of the sample, there is information on the visible H_2 gas, traced by the CO emission. It is observed in the more massive galaxies or early-types, due to their larger metallicity and consequent larger abundance in

the CO molecule. In these galaxies, where the visible molecular component is a significant fraction of the gaseous mass, the radial distribution of the CO emission is exponential, with about the same scale length as the optical exponential disk (e.g. Young & Scoville 1991). This radial distribution is completely different from the HI distribution, which is more extended, with a surface density varying nearly with the inverse of the radius. In our approach, we therefore include the visible H_2 gas in the stellar component; in these massive and early-type galaxies, it is never larger than 10% of the stellar mass, and enters in the uncertainty on the stellar mass to light ratio. It is automatically included in particular in the maximum disk method.

4

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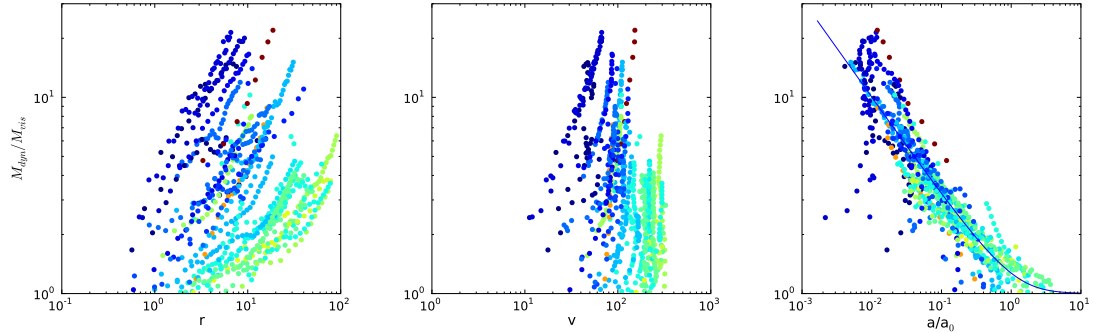


Fig. 2. In Newtonian gravity, the dynamical to visible mass ratio is plotted versus the radius, on the *left*, the velocity, in the *middle*, and the acceleration of the visible matter on the *right*. Note how the latter reduces considerably the scatter of the relation. At large acceleration, there is no need of dark matter. The MOND phenomenology is dictated from the observations of rotation curves, which established a strong correlation between the mass discrepancy and the acceleration. The color codes the type of the galaxies. Late-type galaxies are in dark gray (blue) while early-type are in light gray (yellow). The solid line (blue), on the *right* panel, represents the ν function associated to the standard μ function (see text).

4. The critical acceleration

In this section, we want to explore the dependence of the missing mass problem, as a function of scales (in radius, velocity or acceleration). Let us reason here in the context of Newtonian gravity. The process of rotation curves fitting determines the best mass-to-light ratio, thus the visible mass profile of the different components (stellar disc, bulge, gas):

$$M_{vis} = M_{\star} + M_b + M_g,$$

and the dynamical mass is defined by,

$$M_{dyn} = M_{vis} + M_{DM}$$

We consider that the dark matter has a spherical symmetry:

$$v_c^2(r) - v_{vis}^2(r) = \frac{GM_{DM}(r)}{r}$$

Now, the ratio M_{dyn}/M_{vis} can be plot versus r , v , or a for each galaxies. This is represented on the Fig. 2. The color scale indicates the morphology type of galaxies. Late-type galaxies are in dark gray (blue) while early-type tends to light gray (yellow).

It can be seen that the amount of dark matter relatively increases with radius, as expected for each galaxy. But for a given radius, the amount of dark matter is not the same whatever the galaxy. This does not change, even when the radius is normalized to the characteristic scale of the luminosity, i.e. r_{exp} of R_{25} . The same is true for the velocities. The point at which the dark matter becomes important varies considerably from one galaxy to the next. On the contrary, when the ratio M_{dyn}/M_{vis} is plotted versus the acceleration (the Newtonian acceleration of the visible component), a very strong correlation is observed. Whatever the galaxy considered, the required amount of dark matter to fit the rotation curve is the same for a given acceleration of the visible component.

This does not depend much on the estimation of the mass-to-light ratio Γ_{\star} , as shown by late-type galaxies which are gas dominated. This relation has also been found by

McGaugh (2004), when estimating the mass to light ratio from several ways (maximum disc, stellar populations, ...). Whatever the method, a strong correlation between the mass discrepancy and the Newtonian acceleration is observed.

This relation can be interpreted in two ways:

- either we consider the context of Newtonian gravity and must find why baryons fall in dark matter haloes so that this relation with the acceleration of the visible matter is verified. The Λ CDM model does not give a real clue on this correlation. Van den Bosch & Dalcanton (2000) try to obtain this relation using semi-analytical models of galaxy formation, in the Λ CDM framework. They must then consider star formation feedback effects through supernovae explosions, to reproduce the lack of high surface brightness dwarf galaxies. They however do not succeed to eliminate massive galaxies with low surface brightness. They also tuned their galaxies to fit the observed Tully-Fisher relation in the Λ CDM model, while galaxies automatically reproduce it in the MOND model. It should be interesting to look at this in self-consistent cosmological simulations with baryons.
- or we consider the alternative model of MOND, which is a direct application of the mass discrepancy-acceleration relation. In this frame, the visible mass is the only one, and the gravitation law must be modified to remove the need of dark matter, such that $M_{dyn}/M_{vis} = 1$.

The mass discrepancy-acceleration relation allows to write:

$$\frac{M_{dyn}}{M_{vis}} = f\left(\frac{a_{vis}^{newt}}{a_0}\right)$$

The mass can be expressed in terms of acceleration:

$$\frac{GM_{dyn}}{r^2} = \frac{GM_{vis}}{r^2} f\left(\frac{a_{vis}^{newt}}{a_0}\right)$$

$$a_{vis+DM}^{newt} = a_{vis}^{newt} f\left(\frac{a_{vis}^{newt}}{a_0}\right).$$

A test particle should feel a MONDian acceleration, a^{MOND} equivalent to the Newtonian acceleration with dark matter, a_{vis+DM}^{newt} :

$$a_{vis+DM}^{newt} = a^{MOND},$$

then,

$$a^{MOND} = a_{vis}^{newt} f\left(\frac{a_{vis}^{newt}}{a_0}\right).$$

which is equivalent to the Milgrom's formulation of MOND:

$$a^{MOND} = a_{vis}^{newt} \nu\left(\frac{a_{vis}^{newt}}{a_0}\right)$$

or,

$$a_{vis}^{newt} = a^{MOND} \mu\left(\frac{a^{MOND}}{a_0}\right),$$

with,

$$I(x) = x\mu(x)$$

$$\nu(y) = I^{-1}(y)/y$$

Thus, rotation curves of galaxies tell us how Newtonian gravity should be modified to obtain a ratio M_{dyn}/M_{vis} of one, without any dark matter halo. The μ -function of MOND and the critical acceleration a_0 can be observationally derived from Newtonian interpretation of the rotation curves. The solid line on Fig. 2 (right) represents the function:

$$\nu(x) = \sqrt{0.5 + 0.5\sqrt{1 + (2/x)^2}},$$

associated to the standard μ -function. It is in good agreement with the data.

Let us note that for any model devised to fit rotation curves (as DM and MOND models), it is sufficient to verify the global Tully-Fisher relation, together with the virial theorem, to reveal the mass discrepancy-acceleration relation. The TF relation writes as $M_{vis} \propto V^4$, the virial equilibrium of galaxies as $V^2 \propto M_{dyn}/R$; from the combination of these two relations, can be derived $M_{dyn}/M_{vis} \propto R/V^2 \propto a_0/a$. Although these are only global relations, they yield almost the right slope, obtained for all galaxies, including points internal to each galaxy. The main difference here is that the TF relation is an integral part of the MOND model, while it is fine tuned in the DM+Newton model.

5. Dark baryons

Consider now that part of the missing mass problem in galaxies could be alleviated by the presence of some dark baryons. They could be under the form of cold molecular gas H_2 , in the outer flaring disc. In the frame of Newtonian gravity, this has been developed in a model by Pfenniger & Combes (1994) where this cold gas component served as a fuel to star formation, and helped to interpret secular evolution along the Hubble sequence. Combes & Pfenniger (1997) discussed how this cold gas could be detected directly or indirectly. In this model the amount of non-baryonic DM condensed in galaxies is reduced, but still necessary. Revaz et al. (2008) have performed galaxy simulations using the two gas phases (atomic and molecular) to study the disc stability and the development of spiral structure in low surface brightness galaxies.

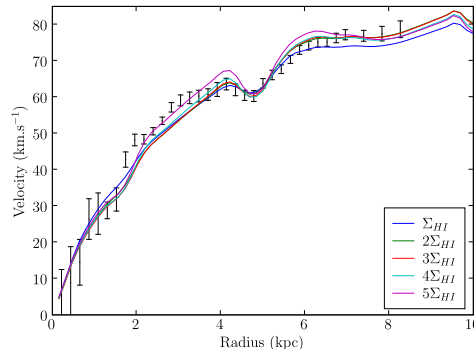


Fig. 3. Rotation curve fit of NGC 1560, in the MOND framework with dark baryons, under the form of cold molecular gas. If this molecular gas represents a mass $M_{mol} = (c - 1)M_{at}$, the value of the actual estimation of the critical acceleration $a_0 = 1.2 \times 10^{-10} \text{ m.s}^{-2}$ must be divided by a factor c , to be in agreement with the observation. The factor c is indicated in the bottom right corner, for each curve fitted.

Is the presence of some dark baryons in galaxies compatible with MOND? Since dark molecular gas increases the baryonic mass, the critical acceleration will be reached at a lower level, and this is not necessarily in agreement with a single acceleration a_0 for all galaxies. But since the required "phantom" dark matter is proportional to the HI gas (Hoekstra et al 2001), it appears quite possible to combine the presence of dark baryons with MOND, by only reducing the value of the acceleration. For example, let us try on the dwarf LSB galaxy NGC 1560, dominated by dark matter in the Newtonian frame. Its rotation curve reaches a plateau at 7 kpc. In accordance with MOND, this maximum velocity can be written:

$$v_c^4 = GMa_0$$

The mass M , in a first approximation corresponds to the mass of the atomic gas (HI+He) M_{at} , since at this outer radius, the stellar content is negligible. If we consider that this galaxy may contain as much molecular gas as atomic gas, which means that the mass M_{at} is multiplied by 2, the actual value a_0 must be divided by 2 to conserve the same asymptotic velocity. With this toy-model, if the total mass of gas is equal to the atomic gas mass time a scale factor c , the critical acceleration a_0 must be divided by c to fit the rotation curve. It works fine for the galaxy NGC1560, as shown in the Fig. 3. The most remarkable is the shape of the rotation curve which is conserved by modifying the values (c, a_0) , not just the maximum velocity. The wiggle at 5 kpc is well reproduced until $c = 5$. Beyond this value, the oscillation is more and more pronounced and does not match anymore the data.

5.1. Rotation curve analysis

We analyzed the 43 galaxies, where a complete set of data is available: the luminosity profile, the HI profile, the HI kinematics. The atomic gas mass is corrected for the primordial helium abundance ($M_{He} = 0.3M_{HI}$). The mass-to-light ratios are estimated with the rotation curve fits in MOND,

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by taking into account the stellar and atomic disc as usual. They are given on Tab. 2, the stellar mass-to-light ratios, in the B-band, vary between 1 and 6 from the late-type to early-type galaxies.

The method consists in varying the scale-factor c :

$$M_{gas}^{tot} = cM_{at} = M_{at} + M_{mol},$$

between the total gas and the atomic gas, to find the critical acceleration a_0 which best fits the data by minimizing the χ^2 . For each galaxy the fit parameters are the couple (a_0, c) . Even if the critical acceleration may vary, according to the amount of dark baryons considered, we expect to obtain a common value for all galaxies. On the other hand, the scale-factor c has no reason to be universal, but the mean value should be consistent with previous works (Hoekstra et al. 2001, Pfnenniger & Revaz 2005). The Fig. 6 shows the best rotation curve fit of all the sample.

5.2. Tully-Fisher relation

An other way to trace the dark baryons in galaxy discs is the Tully-Fisher relation. Originally, this observational law involves the luminosity versus the rotational velocity, which is equivalent to the stellar mass versus the velocity (with a constant mass-to-light ratio). A divergence from this law was found for late-type galaxies. On the Tully-Fisher diagram, the stellar mass of these galaxies is too small for the velocity observed. The problem was solved by taking into account the mass of the atomic gas instead of the sole stellar mass (McGaugh et al., 2000). Pfnenniger & Revaz (2005) have reanalyzed the sample of McGaugh et al. (2000), but considering a fraction of dark baryons (under the form of H_2 gas). They find that a factor $c = 3$ reduces the scatter of the Tully-Fisher law. Begum et al. (2008) carry out the same analysis but focusing on a sample with low-mass galaxy, which are more constraining because these contain generally more gas. They find a higher scale-factor $c = 9$.

We reanalyzed this baryonic Tully-Fisher relation with an other galaxy sample, independent from that of McGaugh et al. (2000) and Begum et al. (2008). We use the GoldMine database, selecting galaxies with $v_c < 100 \text{ km.s}^{-1}$. This sample contains 576 galaxies. As in Begum et al. (2008), these galaxies are low-mass galaxies with a high gas fraction. Effectively they are problematic galaxies in the classical Tully-Fisher relation, and thus they are more constraining for the scale-factor c . The GOLDMine database gives the stellar luminosity in the K-band, which is quite appropriate to plot the Tully-Fisher relation. It gives the $B - V$ color too, from which the mass to light ratio is derived (Bell & de Jong, 2001).

We interpret the Tully-Fisher law in the MOND gravity. In a first approximation we assume that the asymptotic velocity is in the MOND regime. Hence to avoid too many parameters in the fit, we fix the slope of the Tully-Fisher law to 4, in conformity to our hypotheses. The fit parameters are the couple (a_0, c) , as in the rotation curve analysis.

5.3. Results

From the rotation curve analysis, the minimization of χ^2 gives the variation of c with a_0 . We find that when a_0 tends to 0, which is equivalent to the Newtonian gravity, the mean

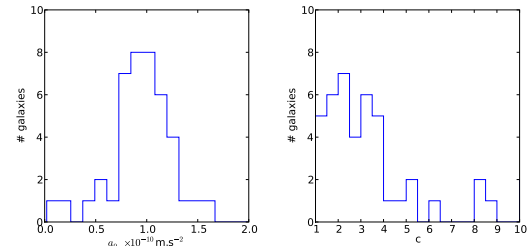


Fig. 4. *Left:* histogram of a_0 obtained when fitting the rotation curve sample including a dark baryon component. The universal critical acceleration is peaked for $a_0 = 0.96 \cdot 10^{-10} \text{ m.s}^{-2}$. *Right:* histogram of c when all the rotation curve are fitted fixing $a_0 = 0.96 \cdot 10^{-10} \text{ m.s}^{-2}$.

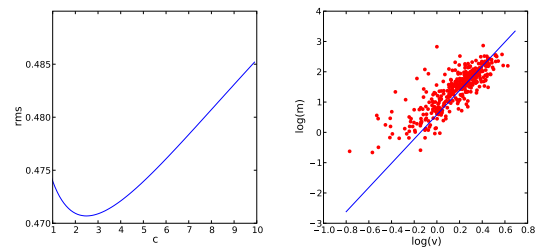


Fig. 5. *Left:* rms of the linear least square fit as a function of c . The minimum is obtained for $c=2.6$. *Right:* the baryonic Tully-Fisher relation including a dark-baryon component.

proportion of dark gas needed to fit the rotation curve is $c = 12 \pm 4$. The dark gas disc plays the role of a dark matter halo. This result is in agreement with what found by Hoekstra et al. (2001). They argue for $c = 7$ which is the more common value in their frequency histogram but their mean value is larger ($c = 11$). With this analysis, we derive the value for the critical acceleration of MOND when a dark baryon component is taken into account. The critical acceleration a_0 increases as the scale-factor decreases. Indeed, in presence of dark baryons, the MOND regime is reached later, at outer radii.

The critical acceleration common to all the sample is estimated to $a_0 = 0.96 \pm 0.39 \cdot 10^{-10} \text{ m.s}^{-2}$ (Fig. 4, left). The mean value of the scale-factor is $c = 3.2 \pm 2.4$. The histogram of c (Fig. 4, right) shows that the majority of the galaxy in the sample (75%) could contain some dark baryons in the proportion $1.5 < c < 6$, while a few percent of galaxies have a scale-factor of 1, equivalent to a model without dark baryons.

The analysis of the Tully-Fisher relation is in agreement with rotation curve fits. The Fig. 5 shows the scatter in the Tully-Fisher as a function of c . The rms is minimized for $c = 2.6 \pm 2.5$ and $a_0 = 0.85 \pm 0.35 \cdot 10^{-10} \text{ m.s}^{-2}$. This result is also in agreement with those of Pfnenniger & Revaz (2005), concerning the parameter c .

6. Conclusion & discussion

We revisit the mass discrepancy-acceleration relation, at the origin of the motivation for MOND. In the frame of this

Name	$(M_*/L)_B$	c	Name	$(M_*/L)_B$	c
ddo154	0.1	1.5	n4258	4.1	4.5
ddo168	0.1	1.5	n4395	3.0	2.5
ddo170	1.2	1.0	n45	3.5	2.0
i2574	0.1	2.5	n4725	7.7	3.0
m33	2.6	3.0	n5033	3.3	2.0
n1003	0.5	1.5	n5055	3.8	3.0
n1560	0.2	2.0	n5371	5.6	5.0
n2366	0.1	1.0	n55	0.1	3.5
n2403	2.1	1.5	n5533	6.1	1.0
n24	2.1	8.0	n5585	0.8	2.0
n247	1.2	4.0	n5907	5.5	10.
n253	3.0	3.5	n6503	3.0	6.0
n2841	5.1	8.5	n6674	7.5	2.5
n2903	2.7	3.0	n6946	2.0	3.0
n2915	5.8	1.0	n7331	3.3	8.0
n2998	3.2	1.0	n7793	1.5	1.5
n300	0.6	2.0	n801	2.4	5.0
n3109	0.1	2.0	n925	1.1	2.5
n3198	2.8	2.5	u128	6.0	1.5
n3726	1.7	2.5	u2259	8.6	3.5
n4203	0.9	10.	u2885	6.4	3.0
n4242	0.9	3.5			

Table 2. c is the scale-factor between the total gas (atomic and molecular) and the atomic gas. $(M_*/L)_B$ is the mass to light ratio in the B-band.

modified gravity theory, this relation is a direct observation, which quantifies the critical acceleration and determines the interpolation function μ . In the frame of Newtonian gravity and dark matter theory, we have to understand the meaning of this strong correlation, which is related to both the Tully-Fisher and virial relation. Until now, only fine tuning, involving baryonic physics and supernovae feedback, has been invoked as an interpretation.

We then consider for the first time how the presence of dark baryons in galaxies could be made compatible with the MOND phenomenology. Only a small fraction of all missing baryons can be present in galaxies, to avoid overpredicting the observed rotation curves. If dark baryons are present in cosmic filaments under the form of cold gas, it is unavoidable to find a small fraction of them in galaxies. We show how the presence of these dark baryons in galaxy discs can reduce the critical acceleration a_0 of MOND. We quantify the best couple (a_0, c) with c being the scale-factor between the total gas and the atomic gas in a galaxy. The analysis is done by rotation curve fits, and minimization of the scatter in the baryonic Tully-Fisher relation. In the study of rotation curve fits, more galaxies with high-quality data, should be included in the sample. Especially the late-type galaxies which are very constraining.

We find that a scale-factor $c = 3_{-2}^{+3}$ optimizes the rotation curve fits and the Tully-Fisher relation. It can be compared to the results of Pfenniger & Revaz (2005), $c = 3_{-2}^{+9}$ and Begum et al. (2008), $c = 9_{-7}^{+19}$. In the fit of the Tully-Fisher relation, we fix the slope to 4 and vary the zero point (a_0). It assumes that the velocity used in the Tully-Fisher relation corresponds to the constant circular velocity in the deep MOND regime. It is possible that the velocity has not yet reached the plateau of the MOND regime. This effect will decrease the value of the slope because these sample of galaxies contains essentially late-types. Their rotation curve is still increasing with radius in the outer measured

radius, and the velocity observed will always be lower or equal to the asymptotic velocity.

A factor c larger than 7 appears less likely from a dynamical point of view (Revaz et al. 2008). With such a large factor, the total mass in gas is about the same order as that in the stellar disc, at a redshift $z=0$. In this case, galaxy discs should be too cold and unstable.

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References

- Begum A., Chengalur J. N., Karachentsev I. D., Sharina M. E.: 2008 MNRAS, tmp291
- Bell E. F., de Jong R. S.: 2001 ApJ 550, 212
- Begeman K.G.: 1987, PhDT, 199
- Begeman K.G.: 1989, A&A, 223, 47
- Binney T., Tremaine S.: 1994 Princeton Series in Astrophysics, Galactic Dynamics
- de Blok W.J.G., van der Hulst J.M., Bothun G.D.: 1995, MNRAS, 274, 235
- Blumenthal G. R., Faber S. M., Primack J. R., Rees M. J.: 1984 Nature, 311, 517
- Bosma A.: 1981, AJ 86, 1825
- Bosma A.: 1981, AJ 86, 1825
- Broeils A.H.: 1992a, PhDT, 258
- Broeils A.H.: 1992b, A&A, 256, 19
- Broeils A.H., Knapen J.H.: 1991, A&AS, 91, 469
- Carignan C.: 1985, ApJS, 58, 107
- Carignan C., Beaulieu S.: 1989 ApJ, 347, 760
- Carignan C., Charbonneau P., Boulanger F., Viallefond F.: 1990, A&A, 234, 43
- Carignan C., Puche D.: 1990a, AJ, 100, 394
- Carignan C., Puche D.: 1990b, AJ, 100, 641
- Carignan C., Sancisi R., van Albada, T.S.: 1988, AJ, 95, 37
- Chemin L., Carignan C., Drouin N., Freeman K.C.: 2006, AJ, 132, 2527
- Combes, F., Pfenniger, D.: 1997 A&A, 327, 453
- Corbelli, E.: 2003, MNRAS, 342, 199
- Cote S., Carignan C., Sancisi R.: 1991, AJ, 102, 904
- Danforth, C. W., Shull, J. M., Rosenberg, J. L., Stocke, J. T.: 2006, ApJ 640, 716
- de Blok W. J. G., McGaugh S. S.: 1998 ApJ, 508, 132
- Fukugita M., Hogan C. J., Peebles P. J. E.: 1998, ApJ 503, 518
- Gavazzi G., Boselli, A., Donati, A., Franzetti, P., Scodreggio, M.: 2003, A&A 400, 451
- Gentile, G., Zhao, H. S., Famaey, B.: 2007 arXiv 0712.1816G
- Hoekstra H., van Albada T. S., Sancisi R.: 2001 MNRAS 323, 453
- Lake G., Schommer R.A., van Gorkom J.H.: 1990, AJ, 99, 547
- Kent S.M.: 1984, ApJS, 56, 105
- Kent S.M.: 1986, AJ, 91, 1301
- Kent S.M.: 1987, AJ, 93, 816
- Martimbeau N., Carignan C., Roy J.R.: 1994, AJ, 107, 543
- McGaugh S.S.: 2004 ApJ, 609, 652
- McGaugh S.S., Schombert J.M., Bothun G.D., de Blok W.J.G.: 2000, ApJ 533, 99
- Meurer G.R., Mackie G., Carignan C.: 1994, AJ, 107, 2021
- Milgrom M.: 1983, ApJ 270, 365
- Milgrom M.: 2007, arXiv 0712.4203M
- Milgrom M., Sanders R.H.: 2007 ApJ, 658, 17
- Navarro, J.F., Frenk C.S., White S.D.M.: 1997, ApJ 490, 493
- Navarro, J.F., Steinmetz, M.: 2000, ApJ 528, 607
- Newton, K.: 1980, MNRAS, 190, 689
- Nicastro, F., Mathur, S., Elvis, M. et al.: 2005, Nature 433, 495
- Pfenniger D., Combes F., Martinet L.: 1994 A&A 285, 79
- Pfenniger D., Combes F.: 1994 A&A 285, 94
- Pfenniger D., Revaz Y.: 2005, A&A 431, 511
- Puche, D., Carignan, C., Bosma A.: 1990, AJ, 100, 1468
- Puche D., Carignan C., van Gorkom J.H.: 1991, AJ, 101, 456
- Puche D., Carignan C., Wainscoat R.J.: 1991b, AJ, 101, 447
- Roelfsema P.R., Allen R.J.: 1985, A&A, 146, 213

- Revaz Y., Pfenniger D., Combes F., Bournaud F.: 2008, arXiv 0801.1180R
Sanders R.H.: 1996, ApJ, 473, 117
Sancisi R., van Albada T.S.: 1987, IAUS, 117, 67
Sicking F.J.: 1997, PhDT, 40
Sofue Y., Rubin V.: 2001, ARAA 39, 137
van Albada, T. S., Sancisi, R.: 1986 RSPTA 320, 447
van den Bosch F. C., Dalcanton, J. J.: 2000 ApJ, 534, 146
Wevers B.M.H.R.: 1984, Groningen
Young J.S., Scoville N.Z.: 1991, ARAA 29, 581

Tiret & Combes: MOND and the dark baryons

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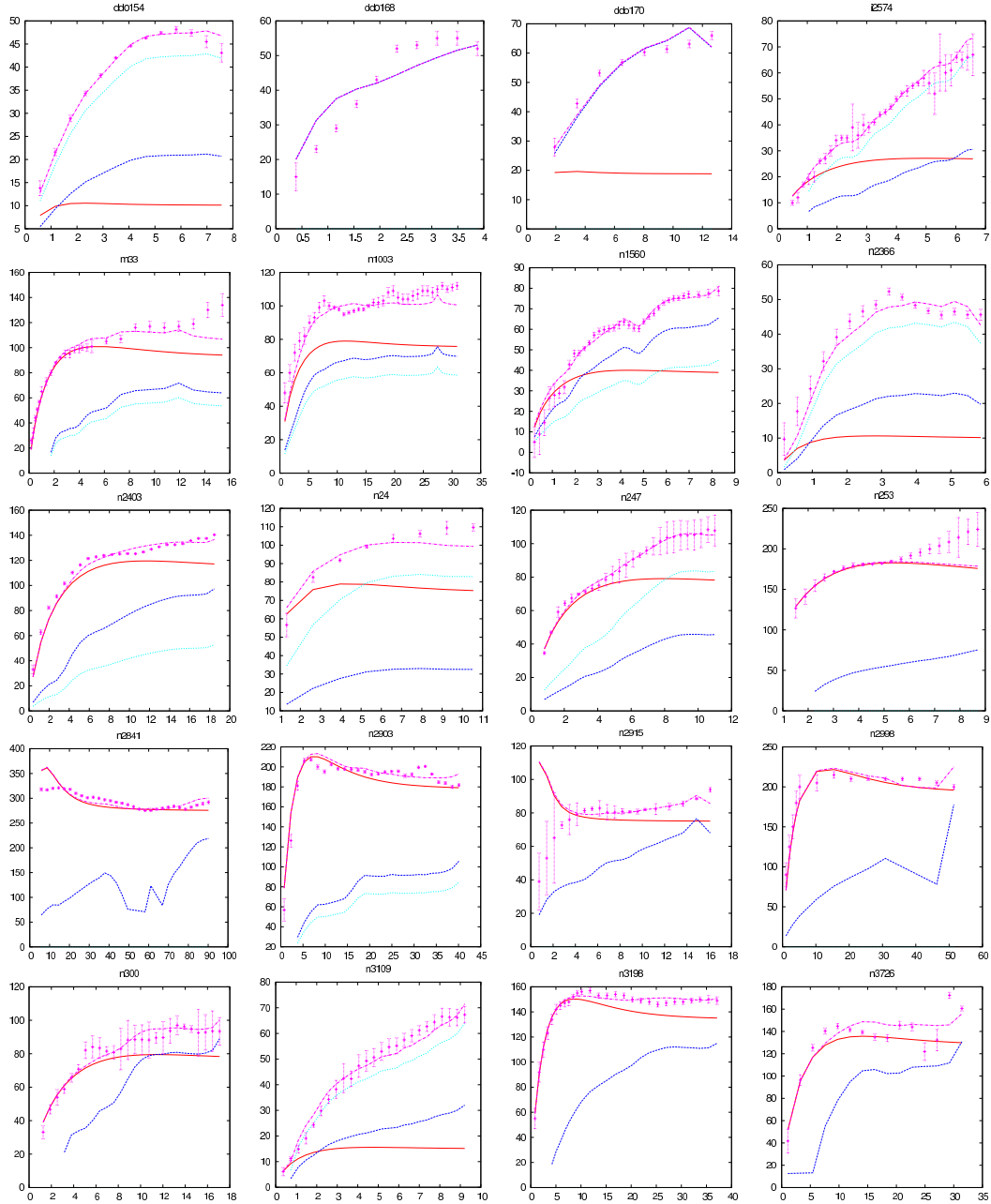


Fig. 6. MOND rotation curve fits: the solid line (red) represents the stellar disc, the atomic gas corresponds to the dash line (blue), the dot line (cyan) is the cold and dark molecular gas. The total modelled rotation curve is in dot-dashed (magenta) while the observed HI velocity corresponds to the symbols and error-bars.

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Tiret & Combes: MOND and the dark baryons

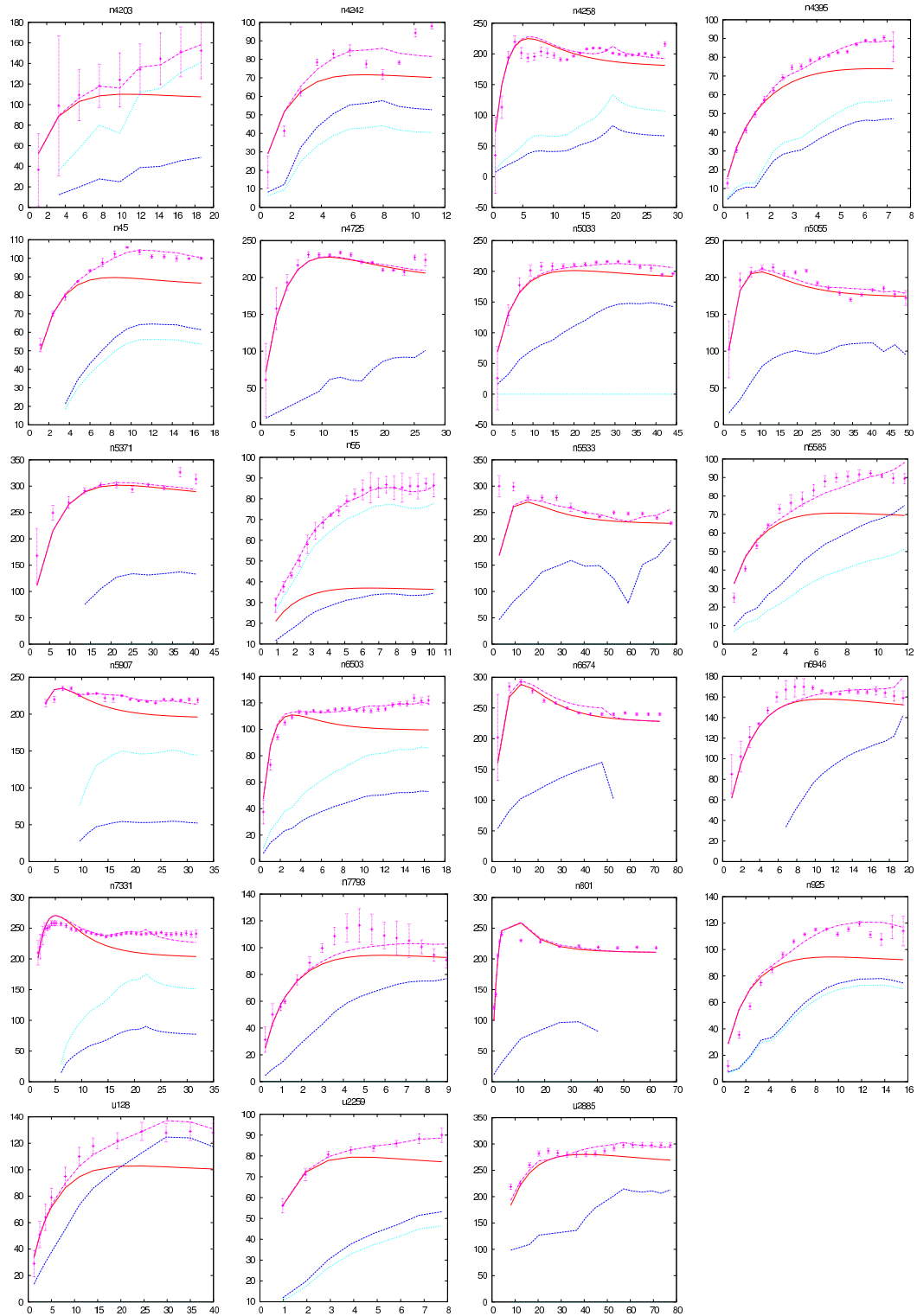


Fig. 7. Continued

2.2 Galaxie elliptique : NGC 3379

Tiret, Combes, Angus, Famaey, Zhao 2007, A&A, 476, 1T

Angus, Famaey, Tiret, Combes, Zhao 2008, MNRAS, 383, 1A

Résumé. Dans Klypin & Prada (2007), les auteurs analysent la dispersion de vitesse des galaxies satellites autour de galaxies massives à partir du Sloan Digital Sky Survey. Dans le cas du modèle Λ CDM, ils comparent les profils de dispersion de vitesse obtenus à partir de simulations cosmologiques (Fig. 2.2, gauche), et pour MOND, le profil de dispersion est déduit de la résolution de l'équation de Jeans :

$$\frac{d\sigma^2}{dr} + \sigma^2 \frac{(2\beta + \alpha)}{r} = -g(r)$$

où σ est la dispersion de vitesse radiale, $\alpha = d \ln \rho / d \ln r$ est la pente du traceur de densité ρ , et l'anisotropie $\beta = 1 - (\sigma_\theta^2 + \sigma_\phi^2) / 2\sigma^2$.

Ils concluent que MOND n'est pas compatible avec ces observations car leur modélisation prédit une dispersion de vitesse constante au-delà de 100 kpc (Fig. 2.2, droite). Ceci est dû principalement à un mauvais choix de la représentation de la distribution des satellites, ils supposent une pente constante ($\alpha = -3$) tandis que les observations autorisent une pente variable de la forme : $\rho(r) \propto (r)^{-1.7}(r + 150)^{-1.5}$ avec r en kpc.

En utilisant une modélisation réaliste de galaxie massive et de ces satellites, nous montrons que MOND est en accord avec les observations. Le cas détaillé de NGC 3379 permet d'analyser la cinématique à plusieurs échelles par le mouvement (i) des étoiles (~ 1 kpc), (ii) des nébuleuses planétaires (~ 20 kpc), et des satellites (~ 200 kpc). Ces trois échelles caractéristiques sont bien modélisées par MOND. Mais ces observations ne peuvent pas fournir un test contraignant. En effet, la grande liberté dans le choix du profil d'anisotropie ne permet pas d'exclure le modèle de MOND ni celui du Λ CDM.

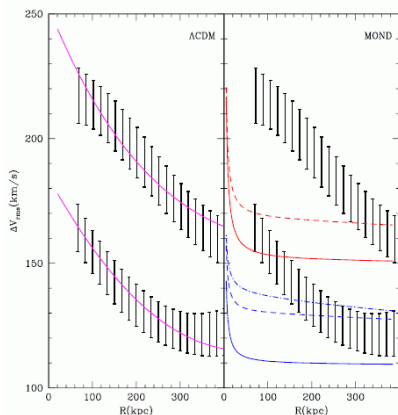


FIGURE 2.3 – Profil de dispersion de vitesse des satellites autour de galaxies massives, Klypin & Prada (2007).

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**Astronomy
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LETTER TO THE EDITOR

Velocity dispersion around ellipticals in MOND

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ABSTRACT

We investigate how different models that have been proposed for solving the dark matter problem can fit the velocity dispersion observed around elliptical galaxies, on either a small scale (~20 kpc) with stellar tracers, such as planetary nebulae, or large scale (~200 kpc) with satellite galaxies as tracers. Predictions of Newtonian gravity, either containing pure baryonic matter, or embedded in massive cold dark matter (CDM) haloes, are compared with predictions of the modified gravity of MOND. The standard CDM model has problems on a small scale, and the Newtonian pure baryonic model has difficulties on a large scale, while a fit with MOND is possible on both scales.

Key words. galaxies: elliptical and lenticular, cD – galaxies: kinematics and dynamics – cosmology: dark matter

1. Introduction

Measuring the velocity field in and around galaxies is the main way to test the dark matter distribution at small and intermediate scales. The observation of what are apparently non-Newtonian rotation curves around spiral galaxies (e.g. Rubin et al. 1980) has been first solved by assuming that galaxies are embedded in dark matter haloes. Numerical simulations, however, predict a radial distribution for the CDM model much more concentrated than what is observed (Gentile et al. 2004; de Blok 2005).

An alternative explanation was proposed by Milgrom (1983), as MODified Newtonian Dynamics (MOND). When the Newtonian acceleration falls below the critical value $a_0 \sim 2 \times 10^{-10} \text{ m s}^{-2}$, the gravity law is empirically modified and then declines in $1/r$ instead of $1/r^2$. Around spherical systems, the modified acceleration g satisfies the relation

$$g\mu(g/a_0) = g_n$$

where g_n is the Newtonian acceleration. For non-spherical geometry, this is only an approximation. However, in this Letter we only consider spherical systems as representing elliptical galaxies and we adopt $\mu(x) = x/\sqrt{1+x^2}$.

This model is very successful on a galactic scale; in particular, it explains a large number of rotation curves of galaxies, with some exceptions (Gentile et al. 2004), and naturally the Tully-Fisher relation, (e.g. Sanders & McGaugh 2002). While the dark matter problem is observationally very clear around spiral galaxies, thanks to their rotation curve measured with the cold hydrogen gas at 21 cm, which is nearly in circular orbits (e.g. Bosma 1981; Verheijen & Sancisi 2001), the situation is much more complex around elliptical galaxies, with little or no rotation.

Recently, planetary nebulae have been used as an efficient tool for measuring the velocity field at large radii in early-type

galaxies, and they complement stellar absorption kinematical studies (Romanowky et al. 2003). In typical elliptical galaxies, the velocity-dispersion profiles were found to decline with radius, up to 5 effective radii, thereby requiring no dark matter at all. Dekel et al. (2005) show that the data are still compatible with the usual dark matter models, if the planetary nebulae tracers have particularly radial orbits in the outer parts, because of a recent merger with small impact parameter. However, the recent results from Douglas et al. (2007) challenge this interpretation, prolonging the decline to more than 7 effective radii.

On larger scales around early-type galaxies, from 50 to 300 kpc, Klypin & Prada (2007), KP07, have proposed to test gravity models with satellite galaxies as a tracer. From the Sloan Digital Sky Survey, they stack several thousand galaxies in 3 luminosity classes and determine the number density of satellites and their velocity dispersion around them. In each mass range, the radial distributions are obtained with around 1500 satellites, although about 1.5 satellite exist around each galaxy.

This large-scale galaxy neighborhood has not been widely tested yet in modified gravity. The well-known difficulty of MOND in clusters has found a possible solution with neutrinos of 2 eV mass (Sanders 2003; Angus et al. 2007a; but see Zhao 2007), and the escape velocity around giant galaxies like the Milky Way was shown to correspond to observations, when including the external field effect (Famaey et al. 2007; Wu et al. 2007).

In this work, we solve the Jeans equation for the distribution of the velocity dispersion around elliptical galaxies, and in particular we fit the NGC 3379 galaxy, where the most extended data is available for the velocities. We also further explore fits at larger radii for the special case of NGC 3379, with satellite galaxies as tracers, as done by KP07 and Angus et al. (2007b). This is only statistically valid around a generic

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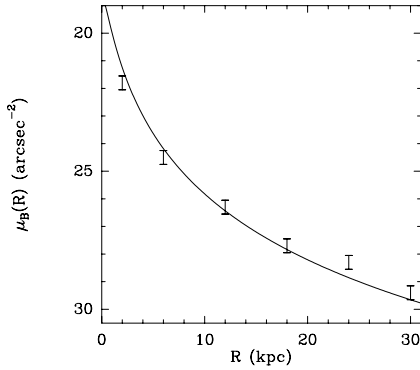


Fig. 1. Fit of the blue-light distribution in NGC 3379. The data points are taken from Douglas et al. (2007). The curve represents the light profile derived with the mass-to-light ratio given in Table 1 and a mass model of cored-Hernquist type (Table 2).

Table 1. Parameters for deriving the mass of NGC 3379.

D (Mpc)	M_B	B_{rc}	M/L_B
9.8	-19.9	10.10	7.0

Table 2. Radial distribution of the tracer density.

	α_0	α_1	r_α	ϵ
stars & PN	-1	-3	1.1	1.2
Sat	-1.7	-1.5	150	0

$\rho(r) \propto (r + \epsilon)^{\alpha_0} (r + r_\alpha)^{\alpha_1}$; ϵ, r_α in kpc.

early-type galaxy, with a mass comparable to NGC 3379. This is the brightest galaxy of a group, but the observed companion velocity is not statistically significant.

2. The Jeans equation

Because of the spherical symmetry, it has been shown (Angus et al. 2007) that the Jeans equation can be written in MOND, as well as in DM:

$$\frac{d\sigma^2}{dr} + \sigma^2 \frac{(2\beta + \alpha)}{r} = -g(r)$$

where σ is the radial velocity dispersion, $\alpha = d \ln \rho / d \ln r$ is the slope of the tracer density ρ , and $\beta = 1 - (\sigma_\theta^2 + \sigma_\phi^2) / 2\sigma^2$ is the velocity anisotropy.

The Jeans equation is usually used for a unique self-gravitating component, where the density ρ appearing in the “pressure” term on the left is the same as the density appearing in the Poisson equation, giving the density field $g(r)$ on the right. On the left side, we use the density and the velocity dispersion of the tracers only, which can be very different from the density producing the potential, in particular for the satellite galaxies, which act as test particles.

In the present approach, we want to compare all models (Newtonian, with or without CDM and MOND) with the same dynamics for the Jeans equation, fitting the density of tracers as close as possible to the observed density distribution. For the CDM models, we consider two different amounts of dark matter: (i) the CDM1 model, reminiscent of what is found in cosmological simulations, where the amount of dark matter inside the

radius of 200 kpc is equal to 60 times the visible matter; (ii) the CDM2 model, more akin to what is required to model rotation curves of spiral galaxies: the amount of dark matter inside the radius of 200 kpc is equal to 20 times the visible matter.

All models are then compared with the same tools, while KP07 compare the CDM model with numerical simulations, and MOND with the Jeans equation, but without exploring all variations of the tracer density and anisotropy profiles.

3. Results

NGC 3379 is modeled by a cored-Hernquist distribution (see Fig. 1). The stellar mass of this galaxy is estimated to be $M = 1 \times 10^{11} M_\odot$, assuming a constant mass-to-light ratio M/L (see Table 1). In MOND (and Newton without DM), this stellar distribution is the only matter contributing to the gravitational potential. In Newtonian dynamics with DM, the DM halo follows the NFW radial profile found in N -body cosmological simulations. We used three kinds of tracers to compare Newton without DM, CDM1, CDM2, and MOND models, for different length scales: stars for the inner part ($r < 1$ kpc, Shapiro et al. 2006), planetary nebulae for the middle part ($1 \text{ kpc} < r < 20$ kpc, Douglas et al. 2007), and satellites ($20 \text{ kpc} < r < 200$ kpc, KP07). The satellites do not correspond to the real ones orbiting NGC 3379, but are a statistical representation from the SDSS (KP07), giving the velocity dispersion of satellites submitted to the gravity of a typical elliptical galaxy in the same mass category as NGC 3379. The parameters of the tracer density are displayed in Table 2.

We numerically solve the Jeans equation for each tracer. The projected velocity dispersion (σ_{los}) is represented in Fig. 2.

3.1. Newton dynamics without DM

- Stars: the projected velocity dispersion (σ_{los}^*) is well-fitted using a small radial anisotropy increasing with radius from $\beta = 0.2$ to $\beta = 0.4$. Since the acceleration here is above the critical a_0 , both MOND and Newton share the same fitting parameters.
- PN: as for the stars, the observed PN velocity dispersion ($\sigma_{\text{los}}^{\text{PN}}$) is also in good agreement with the Newtonian model. It implies a tangential velocity in the central region ($\beta = -0.6$) evolving to radial trajectories ($\beta = 0.8$).
- Satellites: $\sigma_{\text{los}}^{\text{sat}}$ cannot be fitted at all by considering the stellar mass alone. A dark matter component is required.

3.2. CDM1: DM halo mass from cosmological simulations

The mass of the NFW halo is 60 times the stellar mass inside 200 kpc.

- Stars: the cusp of the DM halo does not allow fitting the σ_{los}^* , whatever the anisotropy profile. The predicted velocity dispersion is too large.
- PN: $\sigma_{\text{los}}^{\text{PN}}$ does not fit the entire data set. The dispersion is too large around 10 kpc. If β is increased towards its maximum, up to a purely radial anisotropy profile (which tends to decrease the velocity dispersion), $\sigma_{\text{los}}^{\text{PN}}$ is then too small in the inner part. In the best fit, the value of β is maximum at 1 kpc.
- Satellites: the model reproduces the observations well, if radial anisotropy is maintained all over the region ($\beta > 0.8$).

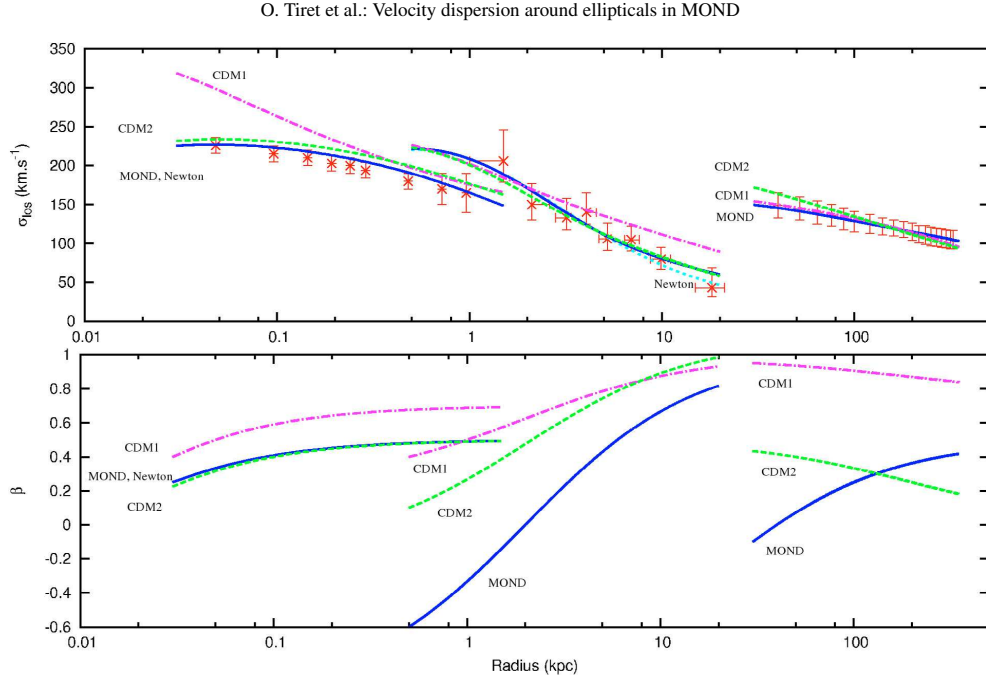


Fig. 2. Fit of the velocity dispersion data around NGC 3379 with pure a Newton model without DM, CDM1, CDM2, and MOND. The data points in the very center ($r < 1$ kpc) are from Shapiro et al. (2006). On intermediate scales they are taken from Douglas et al. (2007) and are based around 10–30 kpc on the planetary nebulae as tracers. On large scales, the data points correspond to the statistical distribution from the satellite galaxies as tracers (KP07), adapting NGC 3379 in their small luminosity class. The tracer density is taken as an Hernquist distribution on a small scale, and fitted to the satellite density distribution on a large-scale. The best fit is obtained with varying $\beta(r)$. Purple/dot-dash: CDM1. Green/dash: CDM2. Blue/full line: MOND, Cyan/short dash: Newton without DM.

3.3. CDM2: DM halo mass compatible with average spirals

The mass of the NFW halo is 20 times the stellar mass inside 200 kpc.

- Stars & PN: they are well-fitted, and the DM halo cusp is no longer a problem in the internal dynamics of the galaxy. For the PN, the velocity dispersion is isotropic near the center ($r < 1$ kpc) and increase to its maximum $\beta = 1$ at 20 kpc.
- Satellites: it is more difficult to fit $\sigma_{\text{los}}^{\text{sat}}$ with less DM. We need to impose an isotropic velocity dispersion ($\beta \sim 0$), to avoid too fast Oa fall in $\sigma_{\text{los}}^{\text{sat}}$. The negative gradient of $\beta(r)$ helps to straighten the slope of $\sigma_{\text{los}}^{\text{sat}}(r)$ which otherwise is too steep.

We also varied the dark-to-visible mass ratio between 10 to 60, and 20 gives the best compromise between the 1–10 kpc and 30–200 kpc regions.

3.4. MOND

All scales are easy to fit with MOND.

- Stars: under 1 kpc, the gravitational potential is purely Newtonian, so MOND and Newton models are identical. The anisotropy needs to be a bit radial ($\beta \sim 0.2$ – 0.4), but always far from the allowed limits.
- PN: until 5 kpc the potential is still Newtonian. A slight difference appears after 8 kpc. When varying $\beta(r)$ from -0.6 (tangential) to 0.8 (radial), the MOND model is in good agreement with the observations.

Table 3. Best fit for the anisotropy distributions.

		MOND	CDM1	CDM2	Newton
Stars	$r_\beta =$	0.01	0.01	0.01	0.01
	$\beta_0 =$	-0.5	-0.5	-0.6	-0.5
	$\beta_1 =$	1	1.2	1.1	1.
PN	$r_\beta =$	2	2	2	2
	$\beta_0 =$	-1	0.25	-0.15	-1
	$\beta_1 =$	2	0.75	1.25	1.8
Sat	$r_\beta =$	20	200	200	–
	$\beta_0 =$	-1	0.98	-0.5	–
	$\beta_1 =$	1.5	-0.22	0.5	–

$\beta(r) = \beta_0 + \beta_1 r / (r + r_\beta)$; r_β in kpc.

- Satellites: $\sigma_{\text{los}}^{\text{sat}}$ is also well-fitted by MOND on this scale, using $\beta = -0.2$ at $r = 20$ kpc to $\beta = 0.2$ at $r = 200$ kpc.

Figure 3 shows how the variations in β can help to fit many versions of the velocity curves. As in all other models, there is still much latitude to the fit. Large spatial variations of β are possible if the ellipticals are the results of mergers (Dekel et al. 2005)

4. Discussion

In contrast to the conclusion of KP07, MOND does not predict constant velocity dispersion with radius in the neighborhood of elliptical galaxies. There is a wide latitude for varying the possible anisotropy parameter according to the scale and the tracer considered. Our best fit starts from a tangential or isotropic configuration near the center and evolves progressively to a radial

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O. Tiret et al.: Velocity dispersion around ellipticals in MOND

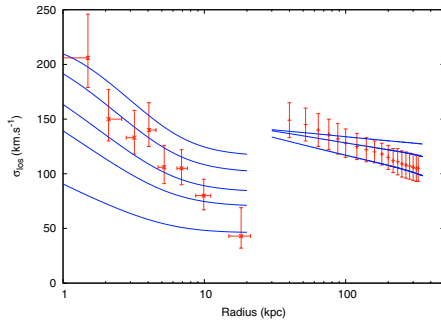


Fig. 3. Fit of the velocity dispersion data around NGC 3379 with MOND, and various values of β . The data points are the same as in Fig. 2. The 4 curves on a small-scale are $\beta = -1, 0, 0.5, 0.9$; and $\beta = -1, 0, 0.5$ on a large-scale.

one for each of the three scales considered, which appears quite realistic.

While the Newtonian model without dark matter has problems in the outer parts, the CDM1 model encounters severe difficulties in the inner parts. The CDM2 model, with a reduced dark matter relative to the visible mass, can also fit the data quite well. Compared to the MOND model, it requires a larger radial anisotropy on each scale, and its β profile is unusual for the satellite tracer with a negative slope. For $\beta = \text{const.}$, the $\sigma_{\text{los}}^{\text{sat}}$ would be too steep compared to the observations. And by introducing anisotropy increasing with radius, the $\sigma_{\text{los}}^{\text{sat}}$ slope is even more increased.

In the MOND regime, the external field effect (EFE, e.g. Wu et al. 2007) would also change the predictions. This will modulate the actual force on the particle tracers, and on the velocity. Since there was still latitude for fitting the observations with MOND, we feel that it is still possible to consider it with other anisotropy parameters. However, we note that the external field effect is not even necessary, for reproducing the observed velocity dispersion slope.

References

- Angus, G. W., Shan, H. Y., Zhao, H. S., & Famaey, B. 2007a, *ApJ*, 654, L13
 Angus, G. W., Famaey, B., Tiret, O., Combes, F., & Zhao, H. S. 2007b, *MNRAS*, in press [arXiv:astro-ph/0709.1966]
 Bosma, A. 1981, *AJ*, 86, 1825
 de Blok, W. J. G. 2005, *ApJ*, 634, 227
 Dekel, A., Stoehr, F., Mamon, G. A., et al. 2005, *Nature*, 437, 707
 Douglas, N. G., Napolitano, N. R., Romanowsky, A. J., et al. 2007 [arXiv:astro-ph/0703047]
 Famaey, B., Bruneton, J. P., & Zhao, H. S. 2007, *MNRAS*, 377, L79
 Gentile, G., Salucci, P., Klein, U., et al. 2004, *MNRAS*, 351, 903
 Klypin, A., & Prada, F. 2007 [arXiv:astro-ph/0706.3554] (KP07)
 Milgrom, M. 1983, *ApJ*, 270, 365
 Navarro, J. F., & Steinmetz, M. 2000, *ApJ*, 528, 607
 Romanowsky, A. J., Douglas, N. G., Amaboldi, M., et al. 2003, *Science*, 301, 1696
 Rubin, V. C., Thonnard, N., & Ford, W. K., Jr. 1980, *ApJ*, 238, 471
 Sanders, R. H. 2003, *MNRAS*, 342, 901
 Sanders, R. H., & McGaugh, S. S. 2002, *ARA&A*, 40, 263
 Shapiro, K. L., Cappellari, M., de Zeeuw, T., et al. 2006, *MNRAS*, 370, 559
 Verheijen, M. A., & Sancisi, R. 2001, *A&A*, 370, 765
 Wu, X., Zhao, H. S., Famaey, B., et al. 2007, *ApJ*, 665, L101
 Zhao, H. S. 2007, *ApJL*, in press [arXiv:astro-ph/0710.3616]

The velocity distribution of Sloan Digital Sky Survey satellites in Modified Newtonian Dynamics

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ABSTRACT

The recent Sloan Digital Sky Survey measured velocity distribution of satellite galaxies has been modelled in the context of Modified Newtonian Dynamics (MOND). We show that even when the extra constraint of adhering to the projected satellite number density profile is added, the two line-of-sight (los) velocity dispersion profiles presented in Klypin & Prada can be matched simply with a radially varying anisotropy. Interestingly, the anisotropies required to fit the los velocity dispersions are remarkably similar to the anisotropies generated by dissipationless collapse simulations in MOND. The mass-to-light ratios of the two host galaxies used are sensible, and positivity of the distribution function is satisfied.

Key words: gravitation – galaxies: clusters – dark matter.

1 INTRODUCTION

Ever since Milgrom proposed the Modified Newtonian Dynamics (MOND) in a series of ground-breaking papers (Milgrom 1983a,b,c; Bekenstein & Milgrom 1984) there have been attempts to falsify the theory using myriad observations and techniques. Many have suffered from poor data, which after being re-analysed was in total agreement with MOND (Milgrom 1995), or they sank because the MOND analysis was performed poorly (see Milgrom & Sanders 2003). However, some issues stand because they are legitimate concerns; most notably the missing mass in clusters (Aguirre, Schaye & Quataert 2001; Sanders 2003, 2007; Clowe et al. 2006; Angus et al. 2007a; Angus, Famaey & Buote 2007b). Thanks to the rapid development of MOND *N*-body codes (Ciotti, Londrillo & Nipoti 2006; Turet & Combes 2007), issues of galaxy stability are being treated and merging time-scales of galaxy pairs have been shown to be borderline high in recent simulations by Nipoti, Londrillo & Ciotti (2007b).

Generally, it is forlorn to attack MOND at the galaxy scale because it so outperforms cold dark matter (CDM) even with zero free parameters, the tidal dwarf galaxies observed by Bournaud et al. (2007) being a great example of this (Gentile et al. 2007; Milgrom 2007), as well as the tight correlation between dark and luminous mass which is inferred under the dark matter paradigm (McGaugh 2005). For a review of recent successes of MOND see Bekenstein (2006). However, the Sloan Digital Sky Survey (SDSS) brings a new dimension to our ability to test MOND. With such a vast archive of galaxies, and thanks to its piercing magnitude range,

Klypin & Prada (2007, hereafter KP07) were able to generate the satellite line-of-sight dispersions for a narrow range of galaxy luminosity. In this small luminosity range, the line-of-sight (los) dispersions of many satellites were stacked together to essentially create a mock galaxy group with a los dispersion known over a range of radii (50–400 kpc). As there is no evidence for dark matter in small MONDian groups (Milgrom 1998, 2002; but see Buote & Canizares 1994, 1996; Buote et al. 2002; Angus et al. 2007) one should expect the velocity dispersions calculated from the MOND gravity of the host galaxy in the Jeans equations to coincide with the observed ones. In their recent preprint KP07 claimed that MOND ‘dramatically fails’ to reproduce the falling velocity dispersions. However, their Jeans modelling is based on fairly crude assumptions. Here we re-examine this issue with detailed models.

In Section 2 we explain our method for correctly solving the Jeans equation in MOND, and then in Section 3 we discuss how we proceeded to fit the data for the two representative host galaxies discussed in KP07.

2 THE JEANS EQUATIONS IN MOND

KP07 chose red galaxies that had mostly spherical geometry as host galaxies for the satellite distribution. Then, irrespective of the gravitational theory, to calculate the radial velocity dispersions $\sigma_r(r)$ of an equilibrium population in a given spherically symmetric gravitational field, we must solve the Jeans equation

$$\frac{d}{dr}\sigma_r^2(r) + \frac{\gamma(r)}{r}\sigma_r^2(r) = -g(r), \quad \gamma(r) = \alpha(r) + 2\beta(r), \quad (1)$$

where $g(r)$ is the modulus of the internal gravity. The function $\beta(r) = 1 - \sigma_t^2(r)/2\sigma_r^2(r)$ is the anisotropy parameter, where σ_t is

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the two-component tangential dispersion and σ_r is the radial dispersion. The function $\alpha(r) = d \ln p(r)/d \ln r$ is the logarithmic gradient of the three-dimensional number density profile $p(r)$ of satellites. Actually, in the case of satellites, $p(r)$ is rather their probability distribution in configuration space given the few numbers of observed satellites per host galaxy. However, in the case of satellites, the mass density $\rho(r)$ that will be used in the Poisson equation is *not* the same as the number density (or probability distribution) $p(r)$ of satellites. Satellites will only be considered as a population of test particles in equilibrium in a dominating external potential generated by the host galaxy. In this respect, the velocity dispersions expected for planetary nebulae at large radii (e.g. Douglas et al. 2007) will be much lower than those of the satellites. The comparison of the large-scale (~ 300 kpc) to small-scale (~ 30 kpc) velocity dispersion and anisotropy will be explored in detail for well-studied galaxies in a future paper (Tiret et al. 2007). Numerical simulations of elliptical galaxy formation through mergers in the MOND regime will also be reported in future work.

We note that the parameter $\alpha(r)$ for the satellite number density is taken to be constant in KP07, but clearly decreases from the observed surface density (fig. 1 of KP07). For this reason, we hereafter consider α and β (and therefore γ) to be functions of r . We take a simple double power-law profile such that the probability density goes like (Zhao 1996)

$$p(r) \propto \left(1 + \frac{r}{r_\alpha}\right)^{-\alpha_0} \quad (2)$$

where α_0 is the asymptotic slope of the probability density and r_α is the break radius of the satellite distribution. It is simple to show that the logarithmic density slope $\alpha(r) = d \ln p/d \ln r$ is given by

$$\alpha(r) = \alpha_0 \frac{r}{r + r_\alpha}. \quad (3)$$

Similarly, we can define the anisotropy $\beta(r)$ as

$$\beta(r) = \frac{r - r_\beta}{r + r_\alpha}, \quad (4)$$

where r_β is the radius where orbits are isotropic. We use the standard technique of solving first-order, linear ordinary differential equations and multiply both sides by an integrating factor. From there it can be solved numerically at any given radius, r_0 , via the equation

$$\sigma_r^2(r_0) = r_0^{2r_\beta/r_\alpha} (r_0 + r_\alpha)^{-\alpha_0 - 2 - 2r_\beta/r_\alpha} \times \int_{r_0}^{\infty} r^{-2r_\beta/r_\alpha} (r + r_\alpha)^{\alpha_0 + 2 + 2r_\beta/r_\alpha} g(r) dr. \quad (5)$$

For the integration limits we set $\sigma_r(100 \text{ Mpc}) = 0$, which physically must be true.

2.1 The Poisson equation and the external field effect

Now, we need to compute the gravity $g(r)$ from the density of the host galaxy and the adopted gravitational theory. In MOND, the Poisson equation reads

$$-\nabla \cdot [\mu(x)g] = 4\pi G\rho, \quad x \equiv \frac{g}{a_0}, \quad (6)$$

where $a_0 = 3.6(\text{km s}^{-1})^2 \text{pc}^{-1}$ is the MOND acceleration constant, and the μ -function is chosen, from Famaey & Binney (2005), to be $\mu(x) = x/(1+x)$. Here, given that satellites are located far away from the bulk of the mass of the host galaxy, we model the density as a point mass.

When analyzing the internal gravity of a system at large radii, as in the case of satellites, a role may be played by the ‘external field effect’ (EFE; see Angus & McGaugh 2007; Wu et al. 2007) linked with the breaking of the Strong Equivalence Principle inherent to any acceleration-based modification of gravity.

However, for these isolated galaxies (unlike the Milky Way), we have no information on the proximity or masses of nearby massive galaxies. In fact, because these galaxies are stacked together, the individual external field would be different from host to host, therefore, it makes no sense to include a single value for it. If there is indeed an EFE, then to remain consistent with the los dispersions it must in general be less than $a_0/100$. Here we do not consider the EFE.

The internal gravity g of an isolated spherical galaxy in MOND is then determined by

$$g\mu(g/a_0) = GMr^{-2} \quad (7)$$

where g is the internal gravitational field of the system that we are interested in, and M is the mass of the host galaxy.

3 RESULTS

Once equations (5) and (7) are solved, the radial dispersions must be cast into los dispersion in order to be compared with the SDSS data. The projected number density $\Sigma(R) = 2 \int_R^\infty r p(r) (dr/\sqrt{r^2 - R^2})$ (see Binney & Mamon 1982) is fitted with $r_\alpha = 40$ kpc and $\alpha_0 = -3.1$ and is shown in our Fig. 1 along with the data points given by KP07.

This left us with only the galaxy mass and $\beta(r)$ as free parameters to vary in an attempt to fit the satellite los velocity dispersion of hosts in both magnitude binnings (as described in their fig. 2) by keeping the mass as close to the two representative galaxies given by KP07. The higher we push the galaxy masses, the easier it becomes to fit the los dispersions, however, the value of M we use has to be physically consistent with what is expected for the host galaxies, i.e. M/L of a few solar units.

The two representative galaxies arise because host galaxies over two ranges of magnitude have their velocity dispersions binned together and a representative mass for that particular range of magnitudes is chosen. The two ranges are g -band luminosities between 2.4 and $4.2 \times 10^{10} L_\odot$ for galaxy 1 and between 4.2 and $6.6 \times 10^{10} L_\odot$ for galaxy 2. KP07 take $M/L_g \sim 2-3$ for galaxy 1 and $M/L_g \sim 3-5$ for galaxy 2. Galaxy 2 actually corresponds to very red galaxies and so there is considerable potential for variability of the chosen mass not just because the binning of galaxies relates to luminosities that vary around their mean by ~ 25 per cent, but also because of the uncertainty in M/L_g .

In Fig. 1 we plot the MOND los velocity dispersion for both galaxies along with the data obtained by KP07. The fit parameters are listed in Table 1.

We found that the projected number density profile was very constraining to possible fits and if we ignored its shape as per KP07, we would require lower host galaxy masses. Remaining consistent with the projected number density, we need a mass exactly that quoted by KP07 for galaxy 1 and 40 per cent higher for galaxy 2, which is compatible with the uncertainties linked with the mass-to-light ratios.

Clearly, the fits require substantially radial biased orbits in the outer parts. We use the same $\beta(r)$ for both galaxies, although galaxy 2 (the more massive one) is the more constraining. Both galaxies require $\beta > 0.6$ for radii greater than 200 kpc. The physics of the solutions is as follows: from the Jeans equation (equation 1), to get

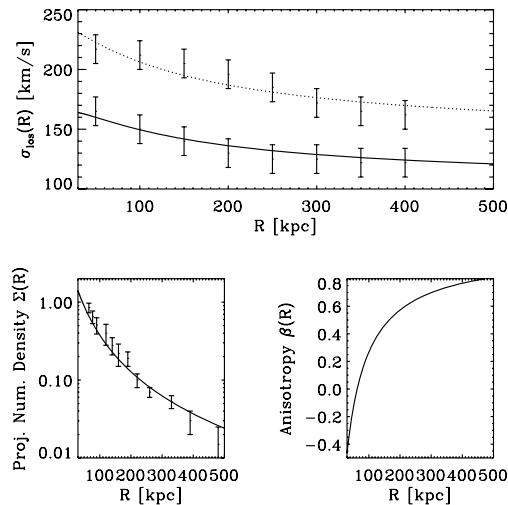


Figure 1. Top panel: the expected los velocity dispersion profiles in MOND for the two host galaxies along with the observed velocity dispersion profiles with 1σ errors. The solid line is for galaxy 1: $M = 8.0 \times 10^{10} M_{\odot}$ for $r_{\alpha} = 40$ kpc, $\alpha_0 = -3.1$ and $r_{\beta} = 63$ kpc. The dashed line is for galaxy 2: $M = 2.8 \times 10^{11} M_{\odot}$, and all other parameters are the same as for galaxy 1. Bottom left: the surface density profile we used for the fits to the velocity dispersions. The fit involves choosing $r_{\alpha} = 40$ kpc and $\alpha_0 = -3.1$ for equation (2) and then integrating along the line of sight. The data points are the observed satellite surface densities from KP07. Bottom right: the variation of the anisotropy parameter, $\beta(r)$ as a function of radius. They transition from more tangentially biased orbits at small radii (<50 kpc) and become radially biased with increasing radius.

Table 1. Shows the parameters used in the MOND simultaneous fitting of the projected number density of satellite galaxies and los dispersions presented by KP07.

Galaxy	Mass [$10^{11} M_{\odot}$]	M/L_g	α_0	r_{α} [kpc]	r_{β} [kpc]
1	0.8	1.9–3.3	-3.1	40	63
2	2.8	4.2–6.6	-3.1	40	63

a high radial dispersion, we need a low absolute value of γ (which from equation 1 must be a negative number in the outskirts where the variation of σ_r is getting nearly constant, cancelling the first term in equation 1). This can be achieved by having a low absolute value of α , but also one tending to get steeper in the outskirts to delay the constancy of σ_r . However, the α needed is too low in absolute value to account for the observed slope. By taking high β , we manage to fit at the same time the slope of $\Sigma(R)$ and the los dispersion. This high radial anisotropy at large radii is exactly what is expected for self-gravitating populations from simulations of elliptical galaxy formation in MOND at radii considerably larger than the half-mass radius of the galaxy (Nipoti, Londrillo & Ciotti 2007a, see their fig. 2). Although it is unclear how valid it is to expand this result to the anisotropy of the satellites test-particle population, we can a priori expect satellites to indeed have such a high radial anisotropy.

An important question is whether the distribution function (DF)

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corresponding to the model is positive everywhere and stable. A necessary condition is that $\gamma(r) < 0$ everywhere, which is satisfied. When the spatial range of radial orbits is wide, there is a risk of radial-orbit instability (e.g. Aguilar & Merritt 1990). However, this is not a concern here because the regions considered are largely outside the self-gravitating part of the elliptical galaxy, and only satellites taken as test particles are orbiting in this radial range.

4 CONCLUSION AND DISCUSSION

Building upon the recent preprint of KP07, we found MOND consistently reproduces the observed declining los dispersions. By solving the Jeans equation in MOND, we have shown that even with the two very constraining data sets consisting of the projected number density profile of satellite galaxies and of their velocity dispersion profile (for a range of host galaxy masses), the data are fully consistent with MOND by simply including an increasing radial anisotropy that requires a single free parameter. These increasing anisotropies are very similar to those found by Nipoti et al. (2007a) from their simulations of dissipationless collapse in MOND. The masses of the host galaxies are reasonable and in accordance with the luminosities of the host galaxies. It is surely possible to find better fits to the projected number density and los dispersions, but the data hardly warrant it.

Finally, we comment that another preprint by Moffat & Toth (2007, hereafter MT07) also suffers from an incomplete analysis of MOND which neglected the radial anisotropy and the correct variation of the slope of the tracer density profile, $\alpha(r)$. MT07 claim that the los velocity dispersions are in full agreement with their favoured gravity theory, dubbed MOG, even though they fit only the los dispersions of the more massive host satellite (galaxy 2, although in fairness this is the more difficult galaxy to use). The goodness of fit looks inferior to the MOND one presented here. Whether their theory can fit the dispersions of the lower mass host with the same set of free parameters remains to be seen. Furthermore, their fit requires a host galaxy mass of $6 \times 10^{11} M_{\odot}$ which is more than twice our value and three times that used by KP07, and which corresponds to an unlikely g -band mass-to-light ratio of 9–14.

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The article is dedicated to the memory of Sarah Jon Main.

REFERENCES

- Aguilar L. A., Merritt D., 1990, ApJ, 354, 33
- Aguirre A., Schaye J., Quataert E., 2001, ApJ, 561, 550
- Angus G. W., McGaugh S. S., 2007, MNRAS, in press (doi:10.1111/j.1365-2966.2007.12403) (arXiv:0704.0381)
- Angus G. W., Shan H. Y., Zhao H. S., Famaey B., 2007a, ApJ, 654, L13
- Angus G. W., Famaey B., Buote D. L. 2007b, MNRAS, submitted, (arXiv:0709.0108)
- Bekenstein J. D., 2006, Contemporary Physics, 47, 387
- Bekenstein J., Milgrom M., 1984, ApJ, 286, 7
- Binney J., Mamon G. A., 1982, MNRAS, 200, 361
- Bournaud F. et al., 2007, Sci, 316, 1166
- Buote D. A., Canizares C. R., 1994, ApJ, 427, 86
- Buote D. A., Canizares C. R., 1996, ApJ, 457, 565

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- Buote D. A., Jeltema T. E., Canizares C. R., Garmire G. P., 2002, *ApJ*, 577, 183
- Ciotti L., Londrillo P., Nipoti C., 2006, *ApJ*, 640, 741
- Clowe D., Bradac M., Gonzalez A. H., Markevitch M., Randall S. W., Jones C., Zaritsky D., 2006, *ApJ*, 648, L109
- Douglas N. G. et al., 2007, *ApJ*, 664, 257
- Famaey B., Binney J., 2005, *MNRAS*, 363, 603
- Gentile G., Famaey B., Combes F., Kroupa P., Zhao H. S., Tiret O., 2007, *A&A*, 472, L25
- Klypin A., Prada F., 2007, preprint (arXiv:0706.3554) (KP07)
- McGaugh S. S., 2005, *ApJ*, 632, 859
- Milgrom M., 1983a, *ApJ*, 270, 365
- Milgrom M., 1983b, *ApJ*, 270, 371
- Milgrom M., 1983c, *ApJ*, 270, 384
- Milgrom M., 1995, *ApJ*, 455, 439
- Milgrom M., 1998, *ApJ*, 496, L89
- Milgrom M., 2002, *ApJ*, 577, L75
- Milgrom M., 2007, *ApJ*, 667, L45
- Milgrom M., Sanders, R. H., 2003, *ApJ*, 599, L25
- Moffat J. W., Toth V. T., 2007, preprint (arXiv:0708.1264)
- Nipoti C., Londrillo P., Ciotti L., 2007a, *ApJ*, 660, 256
- Nipoti C., Londrillo P., Ciotti L., 2007b, *MNRAS*, 381, L104
- Sanders R. H., 2003, *MNRAS*, 342, 901
- Sanders R. H., 2007, *MNRAS*, 380, 331
- Tiret O., Combes F., 2007, *A&A*, 464, 517
- Tiret O., Combes F., Angus G. W., Famaey B., Zhao H. S., 2007, *A&A*, 476, L1
- Wu X., Zhao H. S., Famaey B., Gentile G., Tiret O., Combes F., Angus G. W., Robin A. C., 2007, *ApJ*, 665, L101
- Zhao H. S., 1996, *MNRAS*, 278, 488

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2.3 Galaxies naines de marée

Gentile, Famaey, Combes, Kroupa, Zhao, Tiret 2007, A&A, 472, 25

Résumé. L'observation de débris de collisions de galaxies montre la naissance de galaxies naines de marée (Fig. 2.3). Ces petites structures de $10^9 M_{\odot}$ se forment au cours d'interaction de galaxies, dans les queues de marée. L'autogravité de la matière dans ces extensions est suffisante pour créer un effondrement qui entraîne l'apparition d'un disque en rotation.

Dans le modèle standard du Λ CDM, la matière noire initialement présente autour des galaxies fusionne pour donner un seul halo de matière noire encore plus étendu et plus massif. On ne s'attend donc pas à observer une concentration de matière noire dans les sous-structures comme les galaxies naines de marée. Or, les données observationnelles semblent indiquer le contraire (Bournaud et al. 2007). L'analyse des courbes de rotation de ces objets (en gravitation newtonienne) montre que la masse dynamique est deux fois plus grande que la masse visible. L'interprétation actuellement envisagée, et compatible avec le modèle Λ CDM, serait la présence de matière noire collisionnelle sous forme de gaz moléculaire H_2 dans ces galaxies (baryons noirs). Une autre interprétation peut aussi être apportée par la gravitation modifiée. L'accélération caractéristique de ces objets est de l'ordre de $10^{-11} \text{ m.s}^{-2}$ ($a_0/10$), leur dynamique interne est MONDienne. Dans cet article, nous montrons que les courbes de rotation des galaxies naines de marée sont en accord avec les prédictions de MOND.

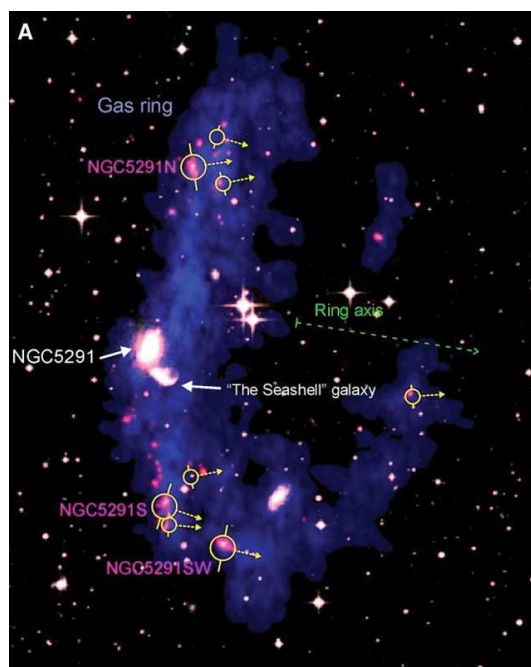


FIGURE 2.4 – Formation de galaxies naines de marée dans un anneau de gaz autour de NGC5291 après une collision de galaxies. (Bournaud et al. 2007)

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LETTER TO THE EDITOR

Tidal dwarf galaxies as a test of fundamental physics

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ABSTRACT

Within the cold dark matter (CDM) framework tidal dwarf galaxies (TDGs) cannot contain dark matter, so the recent results by Bournaud et al. (2007, *Science*, 316, 1166) that 3 rotating TDGs do show significant evidence for being dark matter dominated is inconsistent with the current concordance cosmological theory unless yet another dark matter component is postulated. We confirm that the TDG rotation curves are consistent with Newtonian dynamics only if either an additional dark matter component is postulated, or if all 3 TDGs happen to be viewed nearly edge-on, which is unlikely given the geometry of the tidal debris. We also find that the observed rotation curves are very naturally explained without any free parameters within the modified Newtonian dynamics (MOND) framework if inclinations are adopted as derived by Bournaud et al. We explore different inclination angles and two different assumptions about the external field effect. The results do not change significantly, and we conclude therefore that Newtonian dynamics has severe problems while MOND does exceedingly well in explaining the observed rotation curves of the 3 TDGs studied by Bournaud et al.

Key words. gravitation – cosmology: dark matter – galaxies: dwarf – galaxies: kinematics and dynamics

1. Introduction

Dwarf irregular galaxies have the tendency to form during galaxy interactions within the extended tidal tails, and are therefore dubbed Tidal Dwarf Galaxies (TDG, Mirabel et al. 1992; Duc et al. 2000; Braine et al. 2000). Tidal tails transport the angular momentum and energy away from the merging sub-systems which build up the larger galaxies we see today, and gravitational instabilities within the tidal tails lead to local collapse and star formation. Observations of local interacting galaxies have shown the formation of TDGs to be quite common, in some cases dozens of condensations are seen in the tidal tails (Weilbacher et al. 2000), with a few ones having a mass typical of dwarf galaxies (Braine et al. 2000). Based on a sample of 6 local interacting galaxies, Delgado-Donate et al. (2003) estimate that at most a few long-lived TDGs forms per merger. Monreal-Ibero et al. (2007) studied in detail the stability of external star forming regions in ULIRGs, and concluded that they are good TDG candidates. The relation in terms of evolution between TDGs and the other kinds of dwarf galaxies is presently a matter of debate (see, e.g., Duc & Mirabel 1998; Okazaki & Taniguchi 2000; Bournaud & Duc 2006; Metz & Kroupa 2007). In particular, Okazaki & Taniguchi (2000) claim that conservative assumptions about TDG production within a hierarchical CDM structure formation framework imply that all dE galaxies may be TDGs leaving little or no room for traditional dark-matter filled dwarf galaxies (but see Bournaud & Duc 2006). On the other hand, Skillman & Bender (1995) cast some doubt on the idea that all dwarf ellipticals originate from dwarf irregulars after their gas has been blown away.

In any case, in the Cold Dark Matter (CDM) scenario, TDGs should be nearly devoid of dark matter (Barnes & Hernquist 1992), contrary to other kinds of dwarf galaxies.

In a recent paper, Bournaud et al. (2007, hereafter B07) analyse the rotation curves of 3 TDGs belonging to the NGC 5291 system. They find evidence for a mass discrepancy that is unexpected within the CDM framework, and they put forward the hypothesis of baryonic dark matter to explain the observations. Here we investigate the possibility that the observations can be explained within the framework of MOND (Modified Newtonian Dynamics, Milgrom 1983), without the need for baryonic dark matter.

In general, disk galaxies' rotation curves do not decrease in the outer parts as would be expected from the visible matter distribution. In the standard picture, this is explained by dark matter halos around galaxies. However, MOND is an alternative explanation where our understanding of gravity (or inertia) is changed, rather than our understanding of the matter content of the Universe. Milgrom (1983) postulated that for gravitational accelerations below $a_0 \approx 10^{-8} \text{ cm s}^{-2}$ the effective gravitational attraction approaches $(g_N a_0)^{1/2}$ where g_N is the usual Newtonian gravitational field. MOND leads to remarkable fits of galactic kinematics over 5 decades in mass (Sanders & McGaugh 2002) ranging from tiny dwarfs (e.g., Gentile et al. 2007) through our own Milky Way (Famaey & Binney 2005; Famaey et al. 2007) and early-type spirals (e.g., Sanders & Noordermeer 2007), to massive ellipticals (Milgrom & Sanders 2003), without resorting to galactic dark matter. However, possible problems for MOND on galactic scales include the kinematics of Local Group dwarf spheroidals (Łokas et al. 2006) and galaxy merging timescales (Nipoti et al. 2007).

TDGs are thus objects on galactic scales on which to test the MOND paradigm¹. Here we take into account uncertainties on the external field effect (EFE) and the galaxies' inclinations to show that the MOND paradigm is very likely to correctly explain the kinematics of the TDGs.

2. Data

We use the HI rotation curves recently published by B07. They performed HI observations of the NGC 5291 system, using the VLA (Very Large Array) in its BnA and CnB configuration, yielding a spatial resolution of $\sim 2.2 \times 1.6$ kpc. They discuss the ring structure composed of collisional debris that surrounds NGC 5291, and they detect eight clumps where velocity gradients can be observed. Only three of these, however, are resolved enough to allow a more detailed investigation of their kinematics.

The rotation curves of these three TDGs (NGC 5291N, NGC 5291S, and NGC 5291SW) were derived by B07 using an envelope-tracing method (Sancisi & Allen 1979; Sofue 1996; Gentile et al. 2004), which gives reliable velocity estimates for poorly resolved and/or highly inclined disk galaxies. For the purpose of our analysis, we folded the two sides of the rotation curve $V_{\text{rot}}(r)$ and the baryonic contribution $V_{\text{bar}}(r)$, to obtain azimuthally averaged $V_{\text{rot}}(r)$ and $V_{\text{bar}}(r)$ curves. B07 use an inclination i of 45° for the three TDGs, based on the fact that their model reproduces successfully the morphology of the system with $i = 45^\circ$, and that the rotation axis of the clumps differs from that of the ring by less than 15° . The total baryonic masses, as derived by B07, are: $9^{+1.0}_{-0.7} \times 10^8 M_\odot$ for NGC 5291N, $9.3^{+1.1}_{-0.9} \times 10^8 M_\odot$ for NGC 5291S, and $5 \pm 1.5 \times 10^8 M_\odot$ for NGC 5291SW.

3. Fitting procedure

To make a rigorous fit of these galaxies in MOND, two important issues to take into account are their inclination and the EFE of MOND (e.g., Famaey et al. 2007; Wu et al. 2007; Angus & McGaugh 2007). The rotation curves were thus fitted within the MOND framework with five different hypotheses:

- First, we simply applied the MOND prescription to the rotation curve. This approach has no *explicit* free parameter: we used i , the distance and the stellar mass-to-light (M/L) ratio as derived by B07. The M/L ratio is not a crucial assumption since the stellar mass, derived from stellar population synthesis models, is several times lower than the total baryonic mass. We used $a_0 = 1.2 \times 10^{-8} \text{ cm s}^{-2}$ (Begeman et al. 1991).

- Second, we left i as a free parameter. Indeed i is an important parameter influencing the derived dynamical mass, and the inclination determined by B07 is only an estimate. The inclination we used is however constant with radius: i.e., we ignored any possible warps, which are taken into account in the error-bars.

- Third, we added a first estimate for the EFE, while i was left as a free parameter. In modified gravity (Bekenstein & Milgrom 1984), if one considers, as a first approximation, that a galaxy free-falls with a “uniform” acceleration in an external linear potential, the internal potential becomes polarized Keplerian (Milgrom 1986; Zhao & Tian 2006; Zhao & Famaey 2006). Here, to simplify the treatment of the EFE, we follow the less

¹ A preliminary fit just posted on the arXiv by Milgrom (2007) has shown that MOND has the ability to explain the dynamics without resorting to disk dark matter.

rigorous approach of Famaey et al. (2007) in which the polarization in the direction of the external field (EF) as well as a term proportional to the gradient of μ are neglected. We define the internal acceleration as $a_{\text{int}} = V_{\text{obs}}^2/r$, where V_{obs} is the observed rotation velocity and r is the galactocentric radius in the TDGs. Then, following the MOND hypothesis: $a_{\text{int}} = g_N/\mu(x)$, where $g_N = V_{\text{bar}}^2/r$ is the Newtonian acceleration, $\mu(x) = x/(1+x)$ (Famaey & Binney 2005) and $x = (a_{\text{int}} + g_{\text{ext}})/a_0$ (instead of the usual $x = a_{\text{int}}/a_0$), where g_{ext} is the EF. We roughly estimate g_{ext} due to NGC 5291 as follows: from the total HI profile ΔV_{20} (Malphrus et al. 1997), and assuming that the rotation curve of NGC 5291 stays flat out to the distances of the TDGs, we find $g_{\text{ext}} \lesssim (0.5\Delta V_{20}/\sin(i))^2/R_G$, where R_G is the projected distance of the TDGs from the centre of NGC 5291. This gives $g_{\text{ext}} \lesssim 0.3a_0$. We then assume g_{ext} to have a typical value of $0.2a_0$. Hence, the galaxies are mostly dominated by a_{int} until the last data point, so the various approaches to take the EF into account are slightly degenerate, which we illustrate with $x = \max(a_{\text{int}}/a_0, g_{\text{ext}}/a_0)$ as a second parametrization. Our two choices of x for the EFE bracket the range of possible effects of the EF on the rotation curve, the first one giving a larger EFE than the second. A more rigorous treatment of the effect of EF on rotation curves has already been done in Wu et al. (2007). It was found that the internal iso-potential contours are twisted due to the general orientations of the EF, but there are typically less than 10% difference among different orientations (hence the modulus of vector sums) of the EF and internal field. Rotation curves would become normal baryon-only Newtonian RCs if the system accelerates faster and faster in a stronger and stronger EF.

- Fourth, we fitted the rotation curves with Newtonian gravity and no dark matter, as would be expected in CDM, still keeping i as a free parameter.

- Finally, we fixed i at the most favourable inclination for the previous fits, and we derived the baryonic distribution required to fit the rotation curves in Newtonian gravity.

4. Results

In Fig. 1 we show the results of the fits. Overall MOND gives good fits to the rotation curves of these systems. In particular, Fig. 1 (panels a–c) shows that with no explicit free parameter, MOND can explain well the observed kinematics. As could be expected a priori, leaving i as a free parameter improves the quality of the fits (see Fig. 1, panels d–f). The best-fit values are not far from the first assumption of $i = 45^\circ$: we find $i = 50^{+10}_{-7}$ degrees for NGC 5291N, $i = 34 \pm 4$ degrees for NGC 5291S, and $i = 39^{+11}_{-7}$ degrees for NGC 5291SW: their average and scatter is similar to B07. It is interesting to note that all three TDGs fall on the baryonic Tully-Fisher (BTF) relation (McGaugh 2005): the agreement is perfect for i as a free parameter and only slightly worse for $i = 45^\circ$ (see Fig. 2). TDGs add to a long list of classes of galactic objects that satisfy the BTF relation. It is puzzling that an empirical relation works so well for galaxies of a wide range of formation histories unless it is the direct prediction of a realistic law of gravity relating to the instantaneous baryon distribution, as MOND is.

The fits with the two EFE estimates (see Fig. 1, panels g–i, for the first estimate) give good fits, whose best-fit parameters are displayed in Table 1. The first EFE estimate gives slightly worse fits than the second; we thus conclude that the EFE due to NGC 5291 in the 3 TDGs cannot be too large: it has to be smaller than $\sim (0.1-0.2)a_0$.

We realise that the EFE is a function of distance R_G from the host galaxy NGC 5291. The 3 TDGs are at projected distances

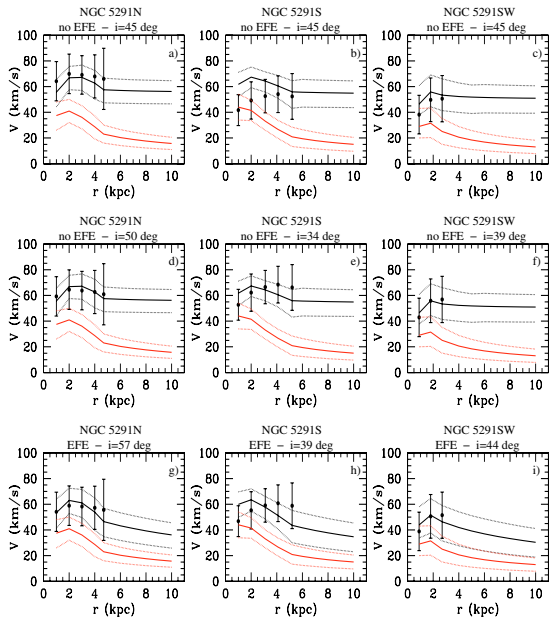


Fig. 1. Rotation curve data (full circles) of the 3 tidal dwarf galaxies (Bournaud et al. 2007). The lower (red) curves are the Newtonian contribution V_{bar} of the baryons (and its uncertainty, indicated as dotted lines). The upper (black) curves are the MOND prediction and its uncertainty (dotted lines). The top panels have as an implicit assumption (following Bournaud et al.) an inclination angle of 45 degrees. In the middle panels the inclination is a free parameter, and the bottom panels show the fits made with the first estimate for the external field effect (EFE) (Sect. 3).

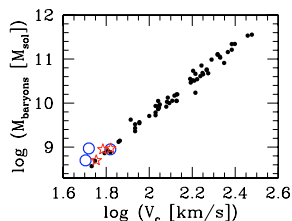


Fig. 2. Baryonic Tully-Fisher relation (baryonic mass vs. circular velocity). The small full circles are the disk galaxy data from McGaugh (2005). The 3 TDGs studied here are shown as blue empty circles ($i = 45^\circ$) and red stars (i as a free parameter).

ranging from 58 to 75 kpc; given the small range of projected distances, and since the 3D distance is unknown, the EFE estimates are made using their average projected distance, 65 kpc, instead of their individual projected distances, which results in errors on the EFE $\leq 15\%$, much smaller than the overall uncertainties on the actual EFE value. We ran models of the EFE with variations of $\sim 15\%$, and found very similar results. Similarly, the difference between the EF effects on the two sides of the TDG rotation curves, is smaller than the uncertainty on its value, so we ignored it and worked on the azimuthally averaged rotation curve.

In Fig. 1 the curves with EFE show a Keplerian fall-off just outside the present data, a signal which is falsifiable if future

Table 1. Best-fit values of the 5 fits for each TDG. i is the inclination, and the EFE estimates refer to Sect. 3. The 1- σ uncertainties in the fitted inclinations are based on the χ^2 statistics.

TDG name	χ^2	i	Note
NGC 5291N	0.60	45 deg	no EFE
	0.16	50^{+10}_{-7} deg	no EFE
	0.30	57^{+9}_{-17} deg	first EFE estimate
	0.17	50^{+10}_{-5} deg	second EFE estimate
	3.60	90^{+8}_{-16} deg	baryons + Newt. gravity
NGC 5291S	5.42	45 deg	no EFE
	1.36	34 ± 4 deg	no EFE
	2.70	39^{+6}_{-5} deg	first EFE estimate
	1.70	35 ± 4 deg	second EFE estimate
	3.25	90^{+0}_{-30} deg	baryons + Newt. gravity
NGC 5291SW	0.45	45 deg	no EFE
	0.09	39^{+11}_{-9} deg	no EFE
	0.19	44^{+16}_{-9} deg	first EFE estimate
	0.09	39^{+11}_{-7} deg	second EFE estimate
	0.42	90^{+0}_{-33} deg	baryons + Newt. gravity

observations could extend the kinematics of these TDGs. This is not due to tidal truncation, but to the EFE. A rotating disk inclined with an angle θ_{EF} with respect to the EF will generally precess around the axis of the EF (Wu et al. 2007; Famaey et al. 2007). The precession means that these TDG disks in MOND are not expected to be exactly aligned with the same inclinations, justifying our fits where i is left as a free parameter. We note that continued accretion from the tidal matter and torquing of the TDGs may, however, affect the shape of the rotation curve at large radii, so the solid lines at large radii only show the case for unperturbed satellites.

We note that large uncertainties still exist in the mass distributions and inclinations of these TDGs, and thus in their dynamics. The scenario proposed by B07 within the CDM paradigm leaves some liberty as to the exact geometry and encounter. In particular, the inclination of the TDG on the sky planes could be more edge-on, and the inferred enclosed mass required to explain the observed velocities smaller, decreasing the need for dark matter or modified gravity. The overall quality of Newtonian, purely baryonic fits with $i = 90^\circ$ and with the baryonic distribution taken from B07 (not shown in Fig. 1) is worse than the MOND fits with a free inclination, but it is only slightly worse than the MOND fits with $i = 45^\circ$: in NGC 5291N the MOND fit is much better, in NGC 5291S it is a bit worse, and in NGC 5291 the two fits are equivalent. However, given the geometry of the NGC 5291 system, it seems unlikely (but possible) that all 3 TDGs studied have such high inclinations. Also, the radial distribution of baryons, essentially the HI gas, is also poorly determined, and can be chosen to fit the observed rotation curves in Newtonian gravity, instead of being selected a priori. We have made such a fit (Fig. 3), by selecting exponential profiles for the surface density of the interstellar gas distributed in a disk. The enclosed gas mass within each radius is also plotted, together with the gas surface density, compatible with the observations. The deduced rotation curves are consistent with the data, within the error bars. Let us note that the fit is still rough, no effort was made to fit more components (such as the stellar component and the molecular gas) in addition to the HI gas.

Given the (relatively) poor quality of the rotation curves, a more rigorous study of the EFE and of the possible warps is difficult, since they are not well constrained observationally. The

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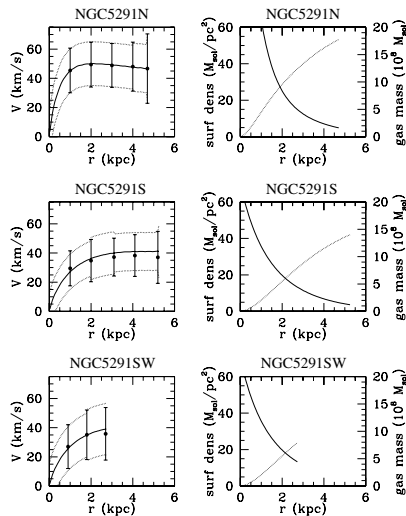


Fig. 3. *Left:* rotation curves with $i = 90^\circ$, with the baryonic distribution that best fits them in Newtonian dynamics. Symbols and curves are like in Fig. 1. *Right:* the corresponding surface density distribution (solid line) and integrated mass (dotted line).

uncertainty on the EFE is unlikely to significantly affect our conclusions given that the EFE and the no-EFE results do not differ much.

5. Conclusions

We analysed the rotation curves of 3 TDGs presented by Bournaud et al. (2007) and we found that MOND explains their kinematics very naturally. Within the MOND framework, no additional unseen matter is required in these 3 TDGs. The dark matter required in Newtonian dynamics implies a large amount of unseen baryonic matter, which is very unconventional in the CDM context.

We explored different possibilities for the inclination angles and the external field effect (EFE) due to NGC 5291. The results do not change significantly: obviously the fits are improved with one extra free parameter (the inclination), and we find the external field to be smaller than $\sim(0.1-0.2)a_0$, within the a priori expectations from the kinematics of NGC 5291. Therefore, MOND can fit the kinematics of the 3 TDGs presented by Bournaud et al. (2007) for inclinations similar to those that they estimated and for realistic values of the external field.

We also found that in the (rather unlikely) case where all 3 TDGs are seen edge-on, Newtonian fits with no dark matter are possible but slightly worse than the MOND fits with $i = 45^\circ$, and not as good as the MOND fits with i as a free parameter. This Letter has thus shown that TDGs pose severe challenges to the current standard cosmological theory and that very recent observations of rotating TDGs are consistent with a non-Newtonian interpretation. TDGs thus hold important clues on fundamental physics and therewith require further research.

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References

- Angus, G. W., & McGaugh, S. S. 2007, MNRAS, submitted [arXiv:0704.0381]
- Barnes, J. E., & Hernquist, L. 1992, Nature, 360, 715
- Begeman, K. G., Broeils, A. H., & Sanders, R. H. 1991, MNRAS, 249, 523
- Bournaud, F., & Duc, P.-A. 2006, A&A, 456, 481
- Bournaud, F., Duc, P.-A., Brinks, E., et al. 2007, Science, 316, 1166
- Braine, J., Lisenfeld, U., Duc, P.-A., & Leon, S. 2000, Nature, 403, 867
- Calura, F., & Matteucci, F. 2004, MNRAS, 350, 351
- Delgado-Donate, E. J., Muñoz-Tuñón, C., Deeg, H. J., & Iglesias-Páramo, J. 2003, A&A, 402, 921
- Duc, P.-A., & Mirabel, I. F. 1998, A&A, 333, 813
- Duc, P.-A., Brinks, E., Springel, V., et al. 2000, AJ, 120, 1238
- Famaey, B., & Binney, J. 2005, MNRAS, 363, 603
- Famaey, B., Bruneton, J.-P., & Zhao, H. 2007, MNRAS, 377, L79
- Gentile, G., Salucci, P., Klein, U., Vergani, D., & Kalberla, P. 2004, MNRAS, 351, 903
- Gentile, G., Salucci, P., Klein, U., & Granato, G. L. 2007, MNRAS, 375, 199
- Łokas, E., Mamon, G., & Prada, F. 2006, EAS Publ. Ser., 20, 113
- McGaugh, S. S. 2005, ApJ, 632, 859
- Metz, M., & Kroupa, P. 2007, MNRAS, 376, 387
- Milgrom, M. 1983, ApJ, 270, 365
- Milgrom, M. 1986, ApJ, 302, 617
- Milgrom, M., & Sanders, R. H. 2003, ApJ, 599, L25
- Milgrom, M. 2007, [arXiv:0706.0875]
- Mirabel, I. F., Dottori, H., & Lutz, D. 1992, A&A, 256, L19
- Monreal-Ibero, A., Colina, L., Arribas, S., & Garcia-Marin, M. 2007, A&A, 472, 421
- Nipoti, C., Londrillo, P., & Ciotti, L. 2007, [arXiv:0705.4633]
- Okazaki, T., & Taniguchi, Y. 2000, ApJ, 543, 149
- Sancisi, R., & Allen, R. J. 1979, A&A, 74, 73
- Sanders, R. H., & McGaugh, S. S. 2002, ARA&A, 40, 263
- Sanders, R. H., & Noordermeer, E. 2007, MNRAS, in press [arXiv:astro-ph/0703352]
- Skillman, E. D., & Bender, R. 1995, Rev. Mex. Astron. Astrof. Conf. Ser., 3, 25
- Sofue, Y. 1996, ApJ, 458, 120
- Weilbacher, P. M., Duc, P.-A., Fritze v. Alvensleben, U., Martin, P., & Fricke, K. J. 2000, A&A, 358, 819
- Wu, X., Zhao, H. S., Famaey, B., et al. 2007 [arXiv:0706.3703]
- Zhao, H. S., & Famaey, B. 2006, ApJ, 638, L9
- Zhao, H. S., & Tian, L. 2006, A&A, 450, 1005

Chapitre 3

Simulations numériques avec MOND

3.1 Code N-Corps newtonien

Les simulations N-Corps de galaxies proposent de modéliser un système gravitationnel (galaxies, galaxies en interaction, amas, ...) par une distribution de particules définies par certaines grandeurs (position, vitesse, masse, ...). Ces particules sont ensuite contraintes d'évoluer selon les lois physiques définies dans le code (loi de la gravitation, équations de l'hydrodynamique, ...). Différentes composantes dans la galaxie comme les étoiles, le gaz ou encore de la matière noire peuvent être représentées, et chaque particule du même type est régie par les mêmes équations. Ainsi, toutes les particules sont soumises à la gravité, mais les particules de type "gaz" peuvent aussi entrer en collision et perdre de l'énergie.

La simulation numérique de dynamique des galaxies permet de résoudre l'équation du mouvement de chaque particule afin d'étudier les propriétés locales (comme les ondes de chocs, explosions de supernovae) ou globales (comme la stabilité d'un disque) des galaxies au cours du temps.

Dans le cas des systèmes gravitationnels, la première grandeur à déterminer est la force gravitationnelle qui s'exerce sur chaque particule. En gravité newtonienne, deux méthodes sont couramment utilisées, donnant lieu à deux types de codes : le développement multipolaire du champ de gravitation du code *en arbre* (Barnes & Hut 1986; Hernquist 1987), où la force ressentie par une particule est la somme directe des forces exercées par les voisins proches et l'approximation multipolaire de la force créée par les régions éloignées ; et la résolution de l'équation de Poisson par transformée de Fourier rapide du code *Particule Maille* (Miller & Prendergast 1968; Hohl & Hockney 1969).

Ces deux méthodes utilisent la propriété de linéarité de la gravitation newto-

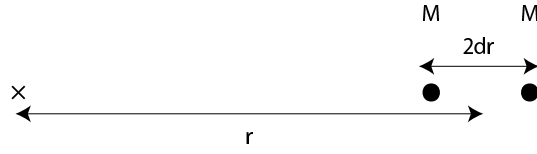


FIGURE 3.1 – Champ de gravité créé par deux particules.

nielle. Le potentiel gravitationnel en un point donné est la somme du potentiel gravitationnel créé par la distribution de points dans la boîte de simulation.

Mais si l'on veut réaliser une simulation en modifiant la gravité à la manière de MOND, il n'est pas exact de calculer l'accélération newtonienne à partir d'un code classique (tree code ou PM) pour finalement appliquer la paramétrisation de MOND, c'est à dire $a_{MOND} = f(a_{Newt})$. En effet, la gravité MONDienne n'est plus linéaire avec la masse. Si l'on considère le champ de gravité de deux particules en régime MONDien ressenti par une particule test à la distance r (Fig. 3.1) :

$$a_N = \frac{GM}{(r+dr)^2} + \frac{GM}{(r-dr)^2} \quad (3.1)$$

$$= \frac{GM}{r^2} [(1+x)^{-2} + (1-x)^{-2}] \quad (3.2)$$

avec $x = dr/r$.

Si la distance entre les deux particules tend vers 0, on a :

$$a_N \simeq \frac{2GM}{r^2}$$

Appliquons la paramétrisation de MOND :

$$a_M \simeq \frac{\sqrt{2GMa_0}}{r}$$

On peut aussi calculer la contribution de l'accélération MONDienne de chaque particule et les rapprocher ensuite :

$$a_M = \frac{\sqrt{GMa_0}}{r+dr} + \frac{\sqrt{GMa_0}}{r-dr} \quad (3.3)$$

$$= \frac{\sqrt{GMa_0}}{r} [(1+x)^{-1} + (1-x)^{-1}] \quad (3.4)$$

si $x \rightarrow 0$,

$$a_M \simeq \frac{2\sqrt{GMa_0}}{r}$$

Contrairement à la gravité newtonienne, la gravité MONDienne n'est plus additive, ce qui rend difficile les prédictions analytiques, les raisonnements et méthodes habituels ne sont plus applicables.

L'issue la plus correcte, respectant les lois de conservation, consiste à résoudre directement l'équation non-linéaire aux dérivées partielles : l'équation de Poisson modifiée (eq. 1.4). C'est probablement l'une des raisons pour laquelle MOND a été très peu testé du point de vue numérique. En effet, la partie des codes (tree-code et PM) qui résout la gravitation newtonienne est l'une des mieux maîtrisée (erreurs numériques) et des plus efficaces (temps de calcul), et ce, depuis des dizaines d'années. Remettre en cause la loi de la gravité pour un dynamiqueur numérique nécessite l'abandon d'acquis, correspondant à la partie la plus robuste des programmes.

3.2 Un nouveau *potential solver*

La difficulté dans la résolution de ces équations complexes est de trouver une méthode applicable sur des grilles dont la taille typique varie entre 256^3 , 512^3 , et qui converge rapidement vers la solution. Pour résoudre ce type d'équation, il existe plusieurs sortes d'algorithmes sur grille, dont la technique multigrille développée par Brandt en 1977 (Press et al. 1993).

3.2.1 Équation de Poisson Modifiée

C'est une méthode sur grille qui va permettre de résoudre l'équation de Poisson modifiée (ou newtonienne). La première étape consiste à discrétiser l'équation. Le schéma que j'ai utilisé est le suivant :

$$\begin{aligned}
4\pi G\rho_{i,j,k} = & \tag{3.5} \\
& ((\phi_{i+1,j,k} - \phi_{i,j,k})\mu_{M_1} - (\phi_{i,j,k} - \phi_{i-1,j,k})\mu_{L_1} \\
& + (\phi_{i,j+1,k} - \phi_{i,j,k})\mu_{M_2} - (\phi_{i,j,k} - \phi_{i,j-1,k})\mu_{L_2} \\
& + (\phi_{i,j,k+1} - \phi_{i,j,k})\mu_{M_3} - (\phi_{i,j,k} - \phi_{i,j,k-1})\mu_{L_3})/h^2
\end{aligned}$$

avec $\rho_{i,j,k}$ et $\phi_{i,j,k}$ la densité spatiale et le potentiel discrétisés sur une grille de pas h , μ_{M_l} , et μ_{L_l} , la valeur $\mu(x)$ aux points M_l et L_l (Fig. 3.2). Les composantes du gradient

$$(\partial/\partial x, \partial/\partial y, \partial/\partial z),$$

intervenant dans $\mu(x)$, sont approximées par

$$\left(\frac{\phi(B) - \phi(A)}{h}, \frac{\phi(I) + \phi(H) - \phi(K) - \phi(J)}{4h}, \frac{\phi(C) + \phi(D) - \phi(E) - \phi(F)}{4h} \right),$$

Ce schéma a été proposé par Brada & Milgrom (1999). La difficulté de l'équation réside dans le terme en $\nabla\phi$ intervenant dans la fonction μ , en plus de la non-linéarité de l'EDP.

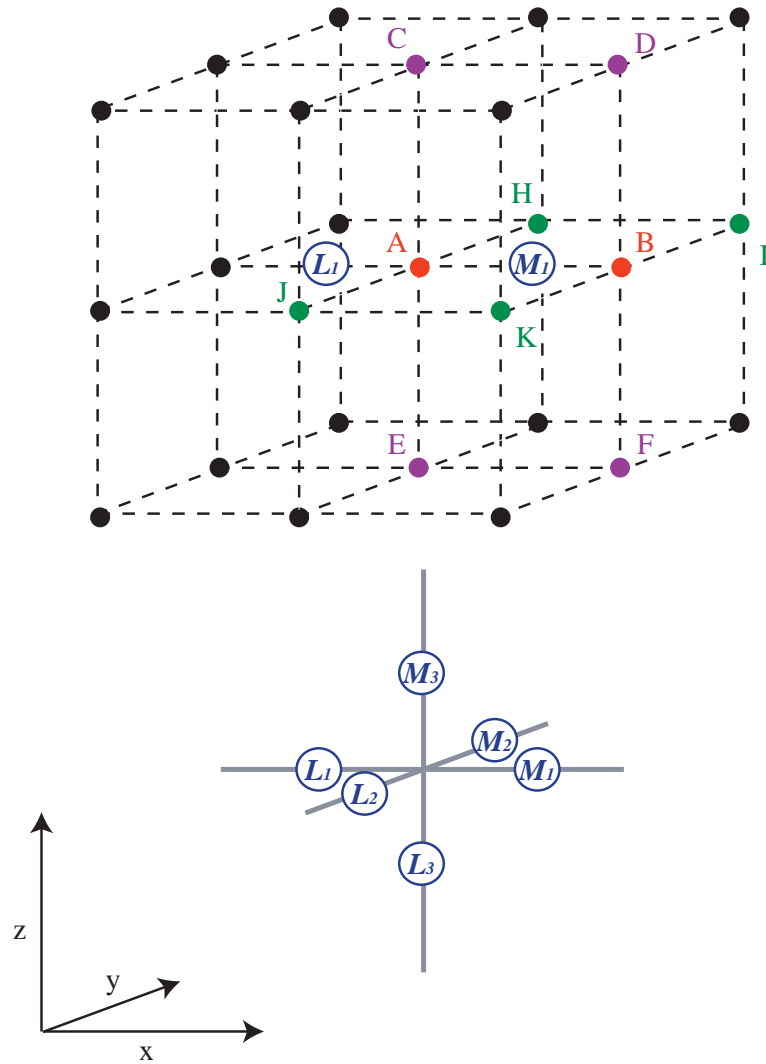


FIGURE 3.2 – Schéma de discrétisation de l'équation de Poisson modifiée proposé par Brada & Milgrom (1999). La densité et le potentiel sont calculés sur les noeuds de grille. Le gradient dans $\mu(x)$ est estimé aux points L_i et M_i .

Pour résoudre numériquement sur grille une équation elliptique de la forme

$$\mathcal{L}u = f,$$

il est usuel de redéfinir le problème sous la forme plus générale d'une équation de diffusion

$$\frac{\partial u}{\partial t} = \mathcal{L}u - f,$$

il s'agit de la méthode de relaxation.

Partant d'un état initial u_i , celui-ci va évoluer jusqu'à un équilibre $u_{eq} : \frac{\partial u_{eq}}{\partial t} = 0$ lorsque $t \rightarrow \infty$; u_{eq} est alors la solution de l'équation elliptique.

Mais les méthodes de relaxation utilisées telles quelles, ont de très faibles performances de convergence; $10^2, 10^4$ itérations peuvent être nécessaires pour des grilles de $256^3, 512^3$ cellules. Il est nécessaire de les coupler à un algorithme qui accélère la convergence. J'ai utilisé l'algorithme FMG-FAS (Full MultiGrid-Full Approximation Scheme) qui s'avère être l'un des plus efficaces pour résoudre des équations elliptique non-linéaires.

3.2.2 Méthode de relaxation : Gauss-Seidel

La technique multigrille est une méthode qui permet d'accélérer la convergence dans la résolution de l'équation aux dérivées partielles, elle est couplée à un algorithme de relaxation. Pour la résolution d'équation elliptique du deuxième ordre, il est conseillé (Numerical recipes) d'utiliser la relaxation de Gauss-Seidel d'ordre deux (*red and black ordering*). Le schéma itératif de cette méthode est le suivant :

$$u_i^{(k+1)} = L_{i,i}^{-1} \left[f_i - \sum_{j<i} L_{i,j} u_j^{(k+1)} - \sum_{j>i} L_{i,j} u_j^{(k)} \right]$$

Contrairement à d'autres schémas de relaxation comme la méthode de Jacobi,

$$u_i^{(k+1)} = L_{i,i}^{-1} \left[f_i - \sum_{j \neq i} L_{i,j} u_j^{(k)} \right]$$

les valeurs $u_i^{(k+1)}$ remplacent dès qu'elles sont calculées les valeurs $u_i^{(k)}$, ce qui évite un tableau de travail temporaire.

Dans le cas de la relaxation de Gauss-Seidel d'ordre 2, l'itération porte d'abord sur les cellules rouges, et ensuite sur les cellules noires (fig. 3.3)

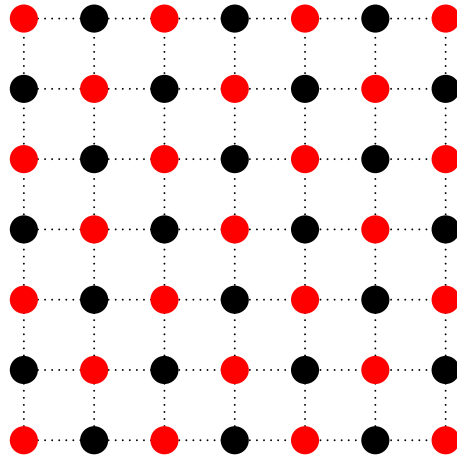


FIGURE 3.3 – Relaxation de Gauss-Seidel du deuxième ordre, sur une grille régulière à 2D. On itère d’abord sur toutes les cases rouges, puis sur toutes les cases noires.

Algorithme n°1:

Relaxation de Gauss-Seidel, *red and black ordering*

```

!   itérations sur les cases rouges
1 : do k = 2, nmax-1
2 :   do j = 2, nmax-1
3 :     do i = 2 + mod (j + k, 2), nmax-1, 2
4 :       // ... //
5 :     enddo
6 :   enddo
7 : enddo
!   itération sur les cases noires
8 : do k = 2, nmax-1
9 :   do j= 2, nmax-1
10 :    do i = 3 - mod (j + k, 2), n-1, 2
11 :      // ... //
12 :    enddo
13 :  enddo
14 : enddo

```

3.2.3 Opérateurs \mathcal{P} et \mathcal{R}

L’opérateur prolongation \mathcal{P} , permet de passer du niveau n à $n + 1$ par interpolation, et réciproquement l’opérateur restriction \mathcal{R} , permet de passer du niveau $n + 1$ à n .

\mathcal{R} doit être l'adjoint de \mathcal{P} :

$$\langle u_H | \mathcal{R}v_h \rangle_H = \langle \mathcal{P}u_H | v_h \rangle_h$$

où,

$$\langle u_h | v_h \rangle_h = h^3 \sum_{x,y,z} u_h(x,y,z)v_h(x,y,z)$$

Dans mon code, j'effectue une interpolation tri-linéaire pour l'opérateur prolongation \mathcal{P} , l'opérateur restriction associé est l'opérateur dit *full weighting*

A titre illustratif, ces opérateurs à 2 dimensions s'écrivent :

$$\mathcal{P} = \frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad \mathcal{R} = \frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad (3.6)$$

3.2.4 L'algorithme FMG-FAS (Full MultiGrid-Full Approximation Scheme)

On veut résoudre une équation du type,

$$\mathcal{L}(u) = f,$$

où \mathcal{L} est un opérateur elliptique (par exemple un Laplacien Δ , dans l'équation de Poisson), et f est le terme source (la densité ρ).

Considérons deux niveaux de grille indexés par h (haute résolution) et H (basse résolution)

Après discrétisation, sur une grille de pas h :

$$\mathcal{L}_h(u_h) = f_h$$

On obtient une première approximation de la solution, u_h^n , de cette équation par une méthode de relaxation (Sect. 3.2.2).

Soit le résidu :

$$r_h^n = f_h - \mathcal{L}_h(u_h^n)$$

Si l'opérateur \mathcal{L} est linéaire :

$$\mathcal{L}_H(u_H^n) - \mathcal{L}_H(\mathcal{R}[u_h^n]_H) = \mathcal{L}_H(u_H^n - \mathcal{R}[u_h^n]_H) = \mathcal{R}[r_h^n]_H,$$

alors, on estime l'erreur e_H en résolvant l'équation :

$$\mathcal{L}_H(e_H^n) = \mathcal{R}[r_h^n]_H, \quad (3.7)$$

la nouvelle solution au niveau h est :

$$u_f^{n+1} = u_f^n + \mathcal{P}[e_H^n]_h$$

Si l'opérateur \mathcal{L} est non linéaire il n'est plus possible de calculer l'erreur directement en résolvant l'équation 3.7. On définit :

$$\tau_H^n = \mathcal{L}_H(u_H^n) - \mathcal{L}_H(\mathcal{R}[u_h^n]_H),$$

pour résoudre :

$$\mathcal{L}_H(u_H^n) = f_H + \tau_H^n.$$

Comme :

$$e_H^n = u_H^n - \mathcal{R}[u_h^n]_H,$$

on corrige finalement la solution au niveau h :

$$u_f^{n+1} = u_f^n + \mathcal{P}[e_H^n]_h$$

Algorithme n°2:

1 cycle FMG-FAS		
1 :	u_H	<i>! solution initiale</i>
2 :	do level=1, lmax	<i>! boucle sur les niveaux de grille</i>
3 :	$\mathcal{P}u_H$	<i>! interpolation</i>
4 :	$\Rightarrow u_h$	<i>! relaxation de GS</i>
5 :	$\mathcal{R}u_h$	<i>! restriction</i>
6 :	$\Rightarrow u_H$	<i>! relaxation de GS</i>
7 :	$e_H = \tilde{u}_H - \mathcal{R}u_h$	<i>! correction</i>
8 :	$u_h^{(new)} = u_h^{(old)} + \mathcal{P}e_H$	<i>! iteration</i>
9 :	// ... //	
10 :	enddo	

3.3 Grille régulière - Grille adaptative

3.3.1 Taille de boîte - Temps de calcul

La simulation d'un système autogravitant à partir d'une résolution sur grille nécessite de définir une boîte de simulation dans laquelle l'objet étudié peut évoluer. Les dimensions de la boîte sont déterminées afin de ne pas perturber le système par des effets de bords (les conditions de bords sont décrites Sect. 3.4), tout en conservant une résolution spatiale permettant de résoudre les phénomènes dynamiques mis en

jeu. Quelques chiffres vont permettre d'éclaircir la situation... Dans le cas d'une simulation de galaxies isolées du type géante (Voie Lactée), le disque d'étoiles est tronqué vers 20 kpc, et le disque de gaz vers 30 kpc. En gravité newtonienne, cette galaxie est plongée dans un halo de matière noire plus ou moins sphérique ; ce halo est tronqué à 45 kpc ; le rayon de troncature du halo est en fait restreint par la taille de la boîte simulation. Celle-ci est déterminée en fonction des moyens de calcul disponibles. En effet, la méthode multigrille que j'ai décrite précédemment permet de réaliser une simulation de galaxie isolée dans un temps *raisonnable* sur des machines ayant 1-2 Go de mémoire, en utilisant une grille régulière de 256^3 noeuds. Ceci est la configuration optimum en ce qui concerne le temps de calcul, la mémoire utilisée, et la résolution de la simulation. En fixant les bords de la boîte à 50 kpc, cela permet d'obtenir une résolution spatiale de 400 pc. Par *raisonnable*, j'entends moins d'une semaine de calcul monoprocesseur. J'ai parallélisé en openMP (mémoire partagée) la version "grille régulière" du code, pour effectuer les simulations de galaxies isolées ; la simulation MONDienne prend deux fois plus de temps que la simulation newtonienne (il y a besoin de plus d'accès mémoire pour résoudre l'équation de Poisson modifiée), mais se parallélise mieux ; soit 160h de temps monoprocesseur, se ramenant à une vingtaine d'heures de temps passé sur 8 processeurs pour la simulation MONDienne et 4 processeurs pour la simulation newtonienne avec matière noire.

Le problème de taille de boîte devient plus critique pour des simulations à des échelles plus grandes comme celles des galaxies en interaction. La boîte de simulation doit contenir les deux galaxies distantes initialement d'au moins 100 kpc. Si l'on place les frontières de la boîte à 150 kpc, la résolution spatiale est de 1,2 kpc sur grille de 256^3 . Pour augmenter d'un facteur 2 la résolution il faut passer à une grille de 512^3 (facteur 8). Les conséquences sont coûteuses pour la mémoire : ~ 12 Go, et pour le temps de calcul : 1 semaine de calcul parallèle... Bien que restant des temps relativement courts pour des simulations N-Corps, il était utile de passer un peu de temps à faire fonctionner le code sur une grille adaptative. Ces simulations à une ou plusieurs galaxies n'ont réellement besoin de résolution que dans le plan galactique ; en-dehors le potentiel est très lisse.

3.3.2 Résolution adaptative

La facilité d'utilisation d'une grille régulière réside dans le passage entre les coordonnées de grille et les coordonnées physiques. Considérons une grille à 3 dimensions découpant un espace cubique dans lequel on se repère par un système de coordonnées cartésiennes $(x, y, z)_{\mathbb{R}^3}$. La grille "virtuelle" définie dans le programme correspond elle à un réseau de parcelles cubiques repérées par les indices $(i, j, k)_{\mathbb{N}^3}$. Supposons que ce maillage représente une densité de matière ρ , et que l'on veuille calculer le potentiel

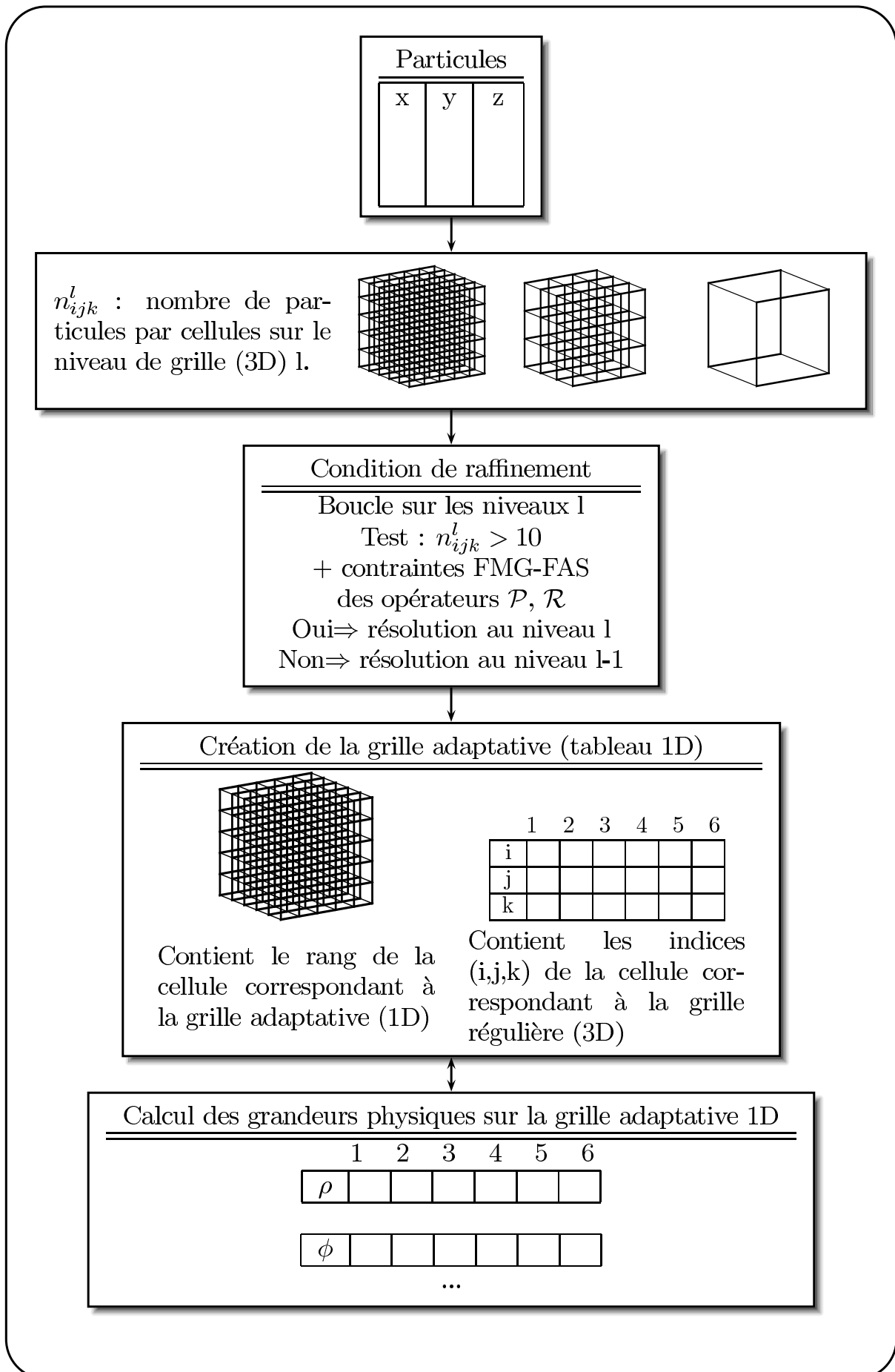


FIGURE 3.4 – Construction de la grille adaptative

gravitationnel newtonien Φ . Nous avons vu dans la Sect. 3.2.2, qu'il fallait pour cela calculer un Laplacien Δ , du type

$$\rho(x + h, y, z) + \rho(x - h, y, z) + \dots$$

il correspond sur une grille de pas h ,

$$\rho_h(i + 1, j, k) + \rho_h(i - 1, j, k) + \dots$$

Dans la grille adaptative, on ne garde en mémoire que certaines cellules (parcelles) qui vérifient un critère de résolution, comme le nombre de particules à l'intérieur de la cellule. L'étape de construction de la grille adaptative doit être entièrement automatique, si une concentration de particules se produit, il faut affiner à cet endroit. Si la condensation se déplace, la résolution par le maillage doit suivre la distribution en allouant de nouvelles cellules et en désallouant d'autres.

La grille adaptative représentant l'espace à 3 dimensions, est un tableau (dynamique) à une dimension (vecteur) ne prenant en compte que les cellules activées. La principale difficulté rencontrée dans l'élaboration du code est le repérage dans ce vecteur. De manière générale, les codes avec résolution adaptative utilisent un algorithme arborescent, et beaucoup de temps est perdu dans le calcul de cette arborescence. Ces méthodes sont surtout utilisées dans les tree-code où la gravité n'est pas calculée sur grille. Je lui ai préféré la façon suivante (Fig. 3.4) :

1- Un tableau à 3D permet de construire la grille adaptative. Il s'agit d'un tableau d'entiers, de dimension $nmax^3$, qui a la résolution la plus fine de la simulation. Ce tableau impose la quantité de mémoire utilisée.

2- Une boucle sur toutes les particules compte le nombre de particules par cellule.

3- On fixe un nombre maximum de particules par cellule (par exemple 10). Toutes les cellules qui contiennent plus de 10 particules auront la résolution maximale. Ces cellules sont trouvées en effectuant une boucle sur les indices du tableau.

Dès que l'on trouve une cellule, on crée son équivalent dans le tableau 1D où sont conservés les indices de positions (i, j, k) . Et la position de la cellule du tableau 1D est conservée dans la cellule (i, j, k) du tableau 3D.

4- A partir de ces deux tableaux, il est très facile de trouver les voisins d'une cellule de la grille adaptative, pour ainsi calculer les différentes grandeurs comme la densité ρ (par interpolation Cloud In Cell), le potentiel ϕ , ...

3.4 Conditions de bords

J'utilise des conditions de bords isolées (problème de Dirichlet). Cela nécessite d'avoir une estimation du potentiel newtonien ou MONDien sur les bords de la

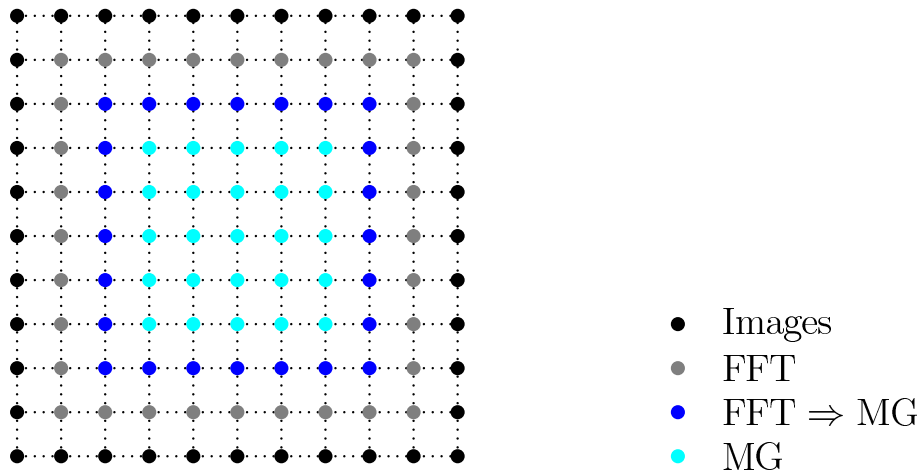


FIGURE 3.5 – Détermination des conditions de bords par imbrication de grilles. L'accélération MONDienne sur les bords de la boîte (MG) est calculée par la formule $a_M = a_N \nu(|a_N/a_0|)$. L'accélération newtonienne est calculée par résolution de l'équation de Poisson (FFT) sur une grille de plus grande dimension.

boîte. J'obtiens ces conditions à partir d'une résolution de l'équation de Poisson (newtonienne) par transformée de Fourier rapide (FFT) avec l'utilisation d'image sur les faces pour éviter les répliques périodiques (James 1977). La taille physique de la boîte *FFT* est supérieure à la taille de la boîte de simulation comme l'indique la figure 3.5.

Algorithme n°3:

Calcul des conditions de bords

```

!      intégration sur le temps
1 :   do t=1,tmax
2 :     // ... //
3 :     calcul de  $\rho$ 
4 :     calcul de  $\phi_N$  par FFT
5 :     calcul de  $a_N$ 
6 :      $a_M = a_N \nu(a_N/a_0)$ 
7 :     Intégration de  $a_M = -\nabla \phi_M \Rightarrow \phi_M$ 
8 :      $\Rightarrow$  Conditions de bords pour FMG-FAS
9 :     call FMG-FAS
10 :    // ... //
11 :   enddo

```

A partir du potentiel newtonien, l'accélération newtonienne est la dérivée de celui-ci. Dans le cas de la simulation MOND, l'accélération MOND est calculée par :

$$a_M = a_N \nu(|a_N/a_0|).$$

Dans le cas le la fonction standard :

$$\mu(x) = \frac{x}{\sqrt{1+x^2}},$$

$$\nu(x) = \sqrt{0.5 + 0.5\sqrt{1 + (2/x)^2}},$$

et pour la fonction simple :

$$\mu(x) = \frac{1}{1+x},$$

$$\nu(x) = 0.5 \left(1 + \sqrt{1 + 4/x} \right)$$

Et le potentiel MONDien est ensuite déduit par intégration de l'accélération MONDienne sur les bords de la boîte MG .

Remarques : l'approximation $a_m = f(a_N)$ est vérifiée si la galaxie est loin des bords de la boîte. Dans ces simulations, il y a un facteur 10 entre l'échelle de longueur du disque et les frontières de la boîte (Tiret & Combes 2007).

3.5 Potentiel adouci

Il est nécessaire de trouver un moyen d'adoucir le potentiel dans une résolution sur grille, pour éviter la relaxation à deux corps dûe au nombre limité de particules. Dans les tree-codes, la formulation de la force est entrée analytiquement, donc l'adoucisement est simplement un paramètre ϵ dans l'expression de la force :

$$F = \frac{GMm}{(r + \epsilon)^2}$$

De même dans les codes PM, où la résolution par FFT utilise la convolution du potentiel en $1/(r + \epsilon)$ au lieu de $1/r$.

Pour obtenir un effet similaire dans une résolution par différence finie (MG-FAS), le potentiel gravitationnel est lissé par une gaussienne. L'écart-type de la gaussienne est choisi en fonction du nombre de particules et de la taille des cellules.

3.6 Modélisation du gaz

La méthode du "Sticky-Particles" (SP) ou "particules collantes" est une recette permettant de modéliser du gaz froid ($T < 5000\text{K}$) qui n'est pas dominé par des forces de pression ; c'est le cas du milieu interstellaire (Combes & Gerin 1985).

Le SP modélise des collisions inélastiques entre des nuages de gaz. Considérons deux particules $(\vec{r}_1, \vec{v}_1, m_1)$, $(\vec{r}_2, \vec{v}_2, m_2)$, dans le référentiel du centre de masse. Avant la collision, les vitesses relatives radiales et tangentielles sont (\vec{v}_r, \vec{v}_t) , et après la collision (\vec{v}'_r, \vec{v}'_t) :

$$\begin{aligned}\vec{v}'_r &= \beta_r \vec{v}_r \\ \vec{v}'_t &= \beta_t \vec{v}_t.\end{aligned}$$

La vitesse de chaque particule après la collision s'écrit :

$$\begin{aligned}\vec{v}'_1 &= \frac{m_2}{M} [\beta_t(\vec{v}_1 - \vec{v}_2) + (\beta_r - \beta_t)((\vec{v}_1 - \vec{v}_2) \cdot \vec{e}_r)\vec{e}_r] \\ \vec{v}'_2 &= \frac{m_1}{M} [\beta_t(\vec{v}_2 - \vec{v}_1) + (\beta_r - \beta_t)((\vec{v}_2 - \vec{v}_1) \cdot \vec{e}_r)\vec{e}_r]\end{aligned}$$

où $M = m_1 + m_2$, et $\vec{e}_r = (\vec{r}_2 - \vec{r}_1) / \|(\vec{r}_2 - \vec{r}_1)\|$.

β_r et β_t sont les coefficients qui permettent de régler le taux de dissipation. Pour conserver le moment angulaire on peut montrer que β_t doit être égal à 1. Il ne reste plus que β_r comme paramètre à fixer. Plusieurs façons de procéder sont possibles, soit les particules collisionnent souvent et perdent peu d'énergie à chaque collision, soit elles collisionnent peu mais perdent beaucoup d'énergie à chaque fois. Les deux cas donnent des résultats similaires pour des simulations de galaxies (isolées ou en interaction). La grandeur à comparer par rapport aux observations est la dispersion de vitesse dans le disque de gaz. Celle-ci doit être aux alentours de 10 km.s^{-1} .

Ce schéma de dissipation est simple et efficace. Il se prête bien à une implémentation dans un code de type grille (et grille adaptative), contrairement aux méthodes SPH (Smooth Hydrodynamics Particles) qui modélisent un fluide continu et nécessitent l'utilisation d'un arbre (code TreeSPH).

3.7 Conditions initiales

Pour toutes les simulations présentées dans ce travail, le disque stellaire est modélisé par un disque de Miyamoto-Nagai :

$$\rho_\star(r, z) = \left(\frac{b_\star^2 M_\star}{4\pi} \right) \frac{a_\star r^2 + (a_\star + 3\sqrt{z^2 + b_\star^2})(a_\star + \sqrt{z^2 + b_\star^2})^2}{\left[r^2 + (a_\star + \sqrt{z^2 + b_\star^2})^2 \right]^{5/2} (z^2 + b_\star^2)^{3/2}}.$$

Le disque de gaz est représenté par un disque de Toomre :

$$\Sigma_g(r) = \frac{a_g M_g}{2\pi(r^2 + a_g^2)^{3/2}}.$$

Le bulbe correspond à une sphère de Plummer :

$$\rho_b(r) = \left(\frac{3M_b}{4\pi b_b^3} \right) \left(1 + \frac{r^2}{b_b^2} \right)^{-5/2}.$$

Les vitesses initiales sont calculées par la méthode d'Hernquist (1993). En gravitation newtonienne, le halo de matière noire est modélisé par un profil de Plummer de telle façon que la courbe de rotation de la galaxie soit identique dans les deux modèles de gravité.

3.8 Test

Le code newtonien a été testé en comparant l'évolution de galaxies isolées avec un code PM (FFT). Les structures (bras spiraux, barres, anneaux de résonances) formées à partir des deux codes étaient similaires et apparaissaient dans des temps comparables.

Dans la version gravitation modifiée, seule la procédure calculant le potentiel change. Quelques cas de distribution simple ont pu être comparés à des formules analytiques MONDiennes (masse ponctuelle, disque de Kuzmin). Pour une distribution quelconque, j'ai aussi comparé le potentiel obtenu par l'algorithme FMG, à une méthode itérative proposée dans la librairie NAG, mais très peu efficace pour les tailles de grille utilisée (Tiret & Combes 2007).

Chapitre 4

Application du code

4.1 Évolution des galaxies spirales I

Tiret & Combes 2007, A&A, 464, 517

Résumé. Dans cet article, nous présentons notre nouveau code qui permet de réaliser des simulations N-Corps en gravitation newtonienne et modifiée. Puis, nous analysons la stabilité de disque stellaire de galaxies géantes. Nous étudions la formation des barres, des anneaux de résonances et leur évolution en comparant pour chaque galaxie, son évolution en gravité newtonienne avec matière noire par rapport à la gravitation modifiée. Les principales différences montrées par ces simulations sont l'interaction entre le halo de matière noire et le disque de galaxie qui n'existe évidemment plus avec MOND. Cela se traduit par une vitesse constante de la rotation de la barre avec MOND, en effet, celle-ci n'est plus ralentie par les effets de friction dynamique due à la présence d'un halo massif. D'autre part les transferts de moment angulaire lors de la formation de la barre se font entre la région interne et externe du disque, en gravitation modifiée, tandis que le moment angulaire est transmis au halo de matière noire en gravitation newtonienne. Il est évacué par les bras spiraux et a pour effet d'étendre radialement le disque galactique en MOND et le halo de matière noire en gravitation newtonienne. Nous terminons par une comparaison sur la fréquence d'apparition des barres dans une séquence de Hubble simulée par les deux modèles. Dans ces simulations où les disques de galaxies sont essentiellement composés d'étoiles, les barres se développent plus rapidement (moins d'un milliard d'années) en gravité modifiée qu'en gravité newtonienne ; en particulier dans le cas des galaxies de type tardif où le halo de matière noire domine en masse et stabilise la galaxie.

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**Astronomy
&
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Evolution of spiral galaxies in modified gravity

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ABSTRACT

We compare N -body simulations of isolated galaxies performed in both frameworks of modified Newtonian dynamics (MOND) and Newtonian gravity with dark matter (DM). We have developed a multigrid code able to efficiently solve the modified Poisson equation derived from the Lagrangian formalism AQUAL. We take particular care of the boundary conditions that are a crucial point in MOND. The 3-dimensional dynamics of initially identical stellar discs is studied in both models. In Newtonian gravity the live DM halo is chosen to fit the rotation curve of the MOND galaxy. For the same value of the Toomre parameter (Q_T), galactic discs in MOND develop a bar instability sooner than in the DM model. In a second phase the MOND bars weaken while the DM bars continue to grow by exchanging angular momentum with the halo. The bar pattern speed evolves quite differently in the two models: there is no dynamical friction on the MOND bars so they keep a constant pattern speed while the DM bars slow down significantly. This affects the position of resonance like the corotation and the peanut. The peanut lobes in the DM model move radially outward while they keep the same position in MOND. Simulations of (only stellar) galaxies of different types on the Hubble sequence lead to a statistical bar frequency that is closer to observations for the MOND than the DM model.

Key words. galaxies: general – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure – cosmology: dark matter

1. Introduction

As has been emphasized in the last years, the concordance Λ CDM cosmological model is very successful in accounting for large-scale structure formation (e.g., Silk 2004), but encounters severe problems at galactic scale: in particular the highly peaked dark matter (DM) distribution predicted by numerical simulations (Navarro et al. 2004) is not compatible with most observed rotation curves of galaxies (de Blok 2005); the predicted angular momentum of baryons condensed in galaxies is much too low (Steinmetz 2003), and the number of predicted satellites around a given giant galaxy is more than an order of magnitude larger than what is observed (Moore et al. 1999). One solution to these problems has been searched for in the energetic feedback provided either by violent star formation (e.g., Kravtsov et al. 2004) or by an AGN (Croton et al. 2006). However, even large variations of these parameters have not been successful in solving the problems significantly for all galaxy types. Another kind of solution is resorting to the modified Newtonian dynamics (MOND), proposed by Milgrom (1983) as an empirical modification of gravity, when the generated acceleration falls below a universal value $a_0 \sim 2 \times 10^{-10} \text{ m s}^{-2}$. In this model, there is no DM anymore, but the visible mass in the inner parts of galaxies produces a much boosted gravity force in the outer parts, with a longer range effect. Bekenstein & Milgrom (1984) developed a self-consistent Lagrangian theory, where the Poisson equation is transformed into:

$$\nabla[\mu(|\nabla\Phi|/a_0)\nabla\Phi] = 4\pi G\rho, \quad (1)$$

where $\mu(x)$ is a function that is equal to unity at large x (Newtonian regime), and tends to x when $x \ll 1$ in the MOND regime. Far in this regime, and assuming some symmetry (spherical, cylindrical, or plane) it can be shown that the

MOND acceleration g_M satisfies the relation (Brada & Milgrom 1995):

$$g_M^2 = a_0 g_N, \quad (2)$$

where g_N is the Newtonian acceleration. This model has large success at galactic scale, in particular explaining all rotation curves of galaxies, and naturally the Tully-Fisher relation, as developed in the excellent review by Sanders & McGaugh (2002).

Interest has grown in the MOND theory since the proposition by Bekenstein (2004) of a Lorentz-covariant theory (TeVeS), able to replace general relativity, accounting for gravitational lensing and passing elementary tests of gravity in the solar system. Simulations have been attempted to explore the large-scale structure formation, with encouraging results (Knebe & Gibson 2004; Nusser & Pointecouteau 2006). More recently, weak lensing observations of the bullet merging cluster 1E0657-56 (Clowe et al. 2006) claim that the spatial separation between the main baryonic component (X-ray gas) and the total mass shows direct evidence for the existence of collisionless DM. They find that any modified gravity model, considering only the baryonic mass, fails to reproduce the observations. However, Angus et al. (2007) have re-analyzed these observations in the context of modified gravity and show that the data are also compatible with the Bekenstein model of MOND, in which some collisionless dark matter exists under the form of ordinary hot neutrinos of 2 eV.

The most stringent constraints on the choice of the interpolation function $\mu(x)$ are expected to be obtained on a small scale however. To better fit the rotation curve of the Milky Way, the function $\mu(x) = x/(1+x)$ has been proposed by Famaey & Binney (2005), in place of the empirical initial function $\mu(x) = x/(1+x^2)^{1/2}$. In addition, physical constraints and the external field effect further reduce the choice of the interpolating function (Zhao & Famaey 2006).

Since the motivation of MOND and its best success concern the galactic scales, and in particular the rotation curves fit without dark matter, more physical constraints should be explored at these scales. In particular, the stability of spiral galaxies in this model, the secular evolution taking into account spiral waves and bars have to be investigated, to compare the dynamical behavior of a typical galaxy in the Newtonian CDM model and the MOND frame. Brada & Milgrom (1999, hereafter BM99) have begun to tackle this problem, and have shown that the Toomre Q -parameter could be chosen lower than in the Newtonian case, to obtain the same stability level. The modified acceleration provided a comparable stability level with respect to bars as does a dark matter halo in the Newtonian case. There are, however, limitations in their model, since they considered infinitely thin discs and ignored the z -structure, acceleration, and dynamics, which are very different in Newtonian and MOND regimes.

In this work, we present numerical simulations of several spiral galaxy models, representing the whole Hubble sequence and a large mass range, in both CDM Newtonian and MOND models. The goal is to find specific tests and constraints to the gravity theory, to be applied on a global statistical basis and confront them to the observations. The diagnostics are to be found in the bar frequency, the spiral morphology, the thickness of discs and their box/peanut shapes, the surface density profiles, and the angular momentum distribution. In this first approach, pure stellar discs are considered, while gas and star formation will be investigated in a future work. In the next section, we describe the numerical code developed to solve the difficult problem of MOND dynamics, and in Sect. 3 the analysis and diagnostics we applied to the simulations results. Initial conditions for spiral galaxies described in Sect. 4, are selected to be as close as possible in the plane for the two compared models: in particular they have the same radial baryonic distribution and the same rotation curve and velocity dispersion. Results are presented in Sect. 5 and then discussed in Sect. 6 to emphasize the fundamental differences in galaxy evolution for the two competing dynamics.

2. Numerical model

The non-linearity of the MOND gravity leads us to use different techniques than the usual ones for the potential solver (or force solver).

2.1. Multigrid (MG) potential solver

The modified Poisson equation is a non-linear elliptic partial differential equation (PDE). This kind of equation can be solved efficiently using multigrid (MG) techniques. We have written an N -body code in which we implemented a full multigrid algorithm (FMG) with full approximation scheme (FAS) for the potential solver (see Numerical Recipes, Press et al. 1992). Brada & Milgrom (BM99) used such a code to solve (1).

Up to some point, the code works like a particle-mesh (PM) code. Particles evolve in a 3D Cartesian grid. Density is computed using the cloud in cell interpolation, the potential is deduced by MG techniques, the equation of motion is solved by the leapfrog scheme. The only difference from a classical PM-code occurs in the potential solver.

The MG computes the solution on finer and finer grids (Fig. 1) by calculating correction terms on each level and converges even more quickly than by solving the same equation directly on the finest grid. We use the Gauss-Seidel relaxation with red and black ordering (Press et al. 1992) to solve the system of equations obtained by discretisation. This step is called

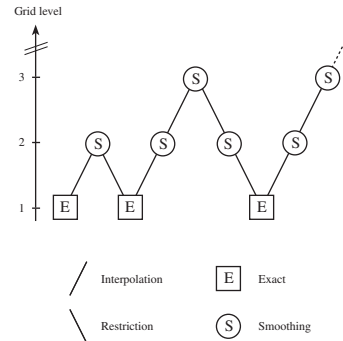


Fig. 1. Full multigrid (FMG) algorithm is used to accelerate the convergence in the resolution of the modified Poisson equation (see text).

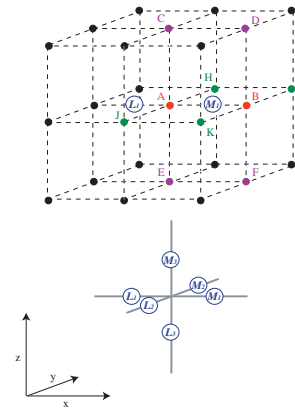


Fig. 2. Discretisation scheme of the modified Poisson equation proposed in BM99. Density and potential is calculated on the grid nodes. The gradient components in $\mu(x)$ are estimated at the L_i and M_i points.

smoothing. To go from the grid level n to $n + 1$ we make a trilinear interpolation (prolongation operator, P), and inversely, the full-weighting operator (R) is used to go to the level $n + 1$ to n . The number of pre-/post-relaxations were chosen to $\nu_{\text{pre}} = 2$ and $\nu_{\text{post}} = 1$.

Here is the discrete form of (1):

$$4\pi G\rho_{i,j,k} = (\phi_{i+1,j,k} - \phi_{i,j,k})\mu_{M_1} - (\phi_{i,j,k} - \phi_{i-1,j,k})\mu_{L_1} \\ + (\phi_{i,j+1,k} - \phi_{i,j,k})\mu_{M_2} - (\phi_{i,j,k} - \phi_{i,j-1,k})\mu_{L_2} \\ + (\phi_{i,j,k+1} - \phi_{i,j,k})\mu_{M_3} - (\phi_{i,j,k} - \phi_{i,j,k-1})\mu_{L_3} / h^2 \quad (3)$$

with $\rho_{i,j,k}$ and $\phi_{i,j,k}$ the spatial density and potential discretized on a grid of step h , μ_{M_i} , and μ_{L_i} , the value of $\mu(x)$ at points M_i and L_i (Fig. 2). The gradient component ($\partial/\partial x, \partial/\partial y, \partial/\partial z$), in $\mu(x)$, are approximated by $(\frac{\phi(B)-\phi(A)}{h}, \frac{\phi(I)+\phi(H)-\phi(K)-\phi(J)}{4h}, \frac{\phi(C)+\phi(D)-\phi(E)-\phi(F)}{4h})$, it is the stable numerical scheme proposed in BM99.

In the Newtonian case, the interpolation function is constant: $\mu(x) = 1$, so that Eq. (3) becomes:

$$4\pi G\rho_{i,j,k} = (\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,j+1,k} + \phi_{i,j-1,k} \\ + \phi_{i,j,k+1} + \phi_{i,j,k-1} - 6\phi_{i,j,k}) / h^2. \quad (4)$$

We recognize the discrete Poisson equation with a 7-point Laplacian stencil.

To avoid the usual 2-body relaxation in simulated galaxies with insufficient number of particles, the gravitational potential is softened through a convolution with a Gaussian function ($\sigma = 1.2$ cells). This value of the softening suppresses efficiently high spatial frequency noise, without introducing any bias (Dehnen 2001; Zhan 2006). For the following simulations, the calculation was made on a 257^3 grid. The radius of the simulation box is 50 kpc, generally the galactic disc is truncated at 20 kpc. The same code can solve Poisson and Modified Poisson equations (it is just the coefficients of the PDE that are constant in Newton and variable in MOND).

2.2. Boundary conditions

In classical PM-code, fast Fourier transform (FFT) implies periodic boundary conditions. For isolated galaxy simulations, the interactions with periodic images can be suppressed using screening masses (James 1977). What kind of boundary conditions are possible to use for simulations of isolated galaxies with MG codes? This point is not trivial and is not developed in BM99 code. It is particularly important in MOND where the gravitational potential scales as $\log(r)$ far from the galaxy. It might appear similar in Newton gravity with the dark matter halo, but the latter is nearly spherical in general, and the influence of mass exterior to the box is considered negligible. In MOND in the contrary, the potential at large distance is due to the baryonic disc (with spiral arms or bar).

Periodic conditions are not realistic at this scale and using a box eight time larger to suppress the periodic images is too costly in CPU time. The most natural way is to use isolated boundary conditions. But this supposes we know the potential at the border of the box. To solve this problem we have to make an approximation. We use the MOND formula (Eq. (2) in the deep MOND regime), which links the MOND acceleration to the Newtonian acceleration. If $\mu(x) = x/(1+x)$, one has more generally:

$$a_{\text{MOND}} = a_{\text{N}} \nu(a_{\text{N}}/a_0) \quad (5)$$

with

$$\nu(x) = 0.5 \left(1 + \sqrt{1 + 4/x} \right). \quad (6)$$

It is critical to use this expression directly in the Newtonian code principally because it is not true if the system has no symmetry (planar, cylindrical, or spherical). It is a concern even for an isolated galaxy. During its evolution, spiral arms and bars are formed and destroyed. The particle configuration is then not symmetric. However, we are interested only in the outer parts of the galaxy. We just need to determine the MOND potential on the boundary of the simulation box, that is far from the galactic center and its gravitational instabilities.

Brada & Milgrom (1995) proposed a test to check if the approximation $a_{\text{MOND}} = f(a_{\text{N}})$ is justified. They showed that $|\nabla\phi_{\text{N}}|$ must be a function of ϕ_{N} out of the disc. So by plotting $|\nabla\phi_{\text{N}}|$ versus ϕ_{N} at different positions in the box simulation we obtain Fig. 3. The approximation is good for low ϕ that is far from the galactic center. This is expected since the potential is more spherical. Hence, our solution to solve the boundary conditions problem is to compute the Newtonian potential by the FFT technique on a larger grid, then the Newtonian acceleration on each edge of the simulation box. Finally we use the MOND formula

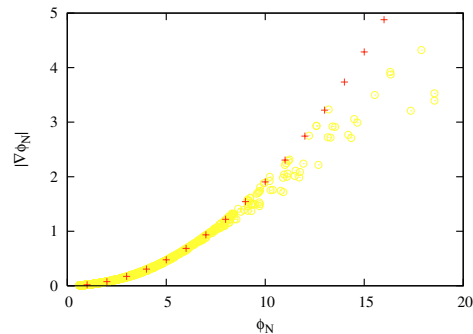


Fig. 3. Plot of $|\nabla\phi_{\text{N}}|$ versus ϕ_{N} for a barred galaxy with grand design spiral. The crosses represent $|\nabla\phi_{\text{N}}| = \phi_{\text{N}}^2/GM$ (this is the spherical approximation where the galaxy is approximated by a point mass).

to deduce the MOND acceleration and compute the MOND potential on the border.

In this way, we obtain boundary conditions that are not fixed in time and that are not required to be homogeneous. We can take a small perturbation to the spherical symmetry like the disc or bar shape of the galaxy into account. Even if the correction is not very important to dynamical evolution, this makes the code more realistic.

2.3. Tests

We have made several tests to check the validity of the solution obtained by the MG technique: the analytical solution of a mass point, the Kuzmin disc. We present here a more demanding test for a totally non-symmetric system. It is the potential of a galaxy where a bar is formed during the simulation. We compute the MOND potential on the one hand with the MG technique and on the other hand with a classical relaxation scheme (NAG library). The second method is very inefficient (several hours when it takes less one minute with MG). We plot the potential along the bar and perpendicular to the bar. The two different methods are in complete agreement. This plot also demonstrates the high symmetry at the outer boundary of the galaxy. Even if there is a bar (5 kpc), the potential at 25 kpc is quasi-spherical (within a few %).

We have tested the dynamical evolution of a stellar disc in Newtonian gravity with the MG code compared to a classical FFT-PM-code. We obtained the same result for the time evolution of the bar. The code has been parallelized in open-MP since the G-S relaxation with red and black ordering allowed this. All red cells can be updated independently and this the same for the black cells.

3. Analysis techniques

3.1. Fourier analysis in the galactic plane

The potential in the galactic plane is developed with the basis of the cosine function ($\Phi_m(r)$) and a phase term ($\phi_m(r)$):

$$\Phi(r, \theta) = \Phi_0(r) + \sum_{m=1} \Phi_m(r) \cos[m\theta - \phi_m(r)]$$

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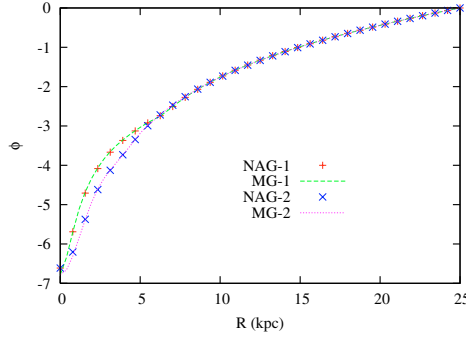


Fig. 4. Test of the MG solver compared to the NAG routine *DOECF* to solve modified Poisson equation for a barred system with a grand design spiral structure: (1) potential perpendicular to the bar, (2) potential parallel to the bar.

to calculate the maximum strength (Q_m) of the m mode. We use the maximum force ratio:

$$Q_m = \max \left| \frac{F_{\theta,m}}{F_r} \right|$$

with the radial force:

$$F_r = -\frac{d\Phi_0}{dr}$$

and the tangential force:

$$\begin{aligned} \max(F_{\theta,m}) &= \max \left(\frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) \\ &= \frac{m}{r} \Phi_m(r). \end{aligned}$$

The bar strength is the maximum strength of the mode $m = 2$. In general, the phase term, ϕ_2 , gives the rotation speed of the bar Ω_b and the derivative:

$$\Omega_b = \frac{\partial \phi_2}{\partial t}.$$

But the mode $m = 2$ could correspond to a two-arm spiral structure. Then, it is more informative to calculate the Fourier transform $\hat{\phi}_2(r, \Omega)$ from $\phi_2(r, t)$. One can distinguish the angular velocity (Ω) of a structure like a bar or a spiral arm versus the radius (r). A bar is identified by a solid rotation in the central part of the galaxy:

$$\Omega(r) = \text{const.} = \Omega_b.$$

3.2. Resonance

We estimate the position of resonant orbits using the epicyclic approximation (Fig. 10). To do that, we need to determine the angular velocity of the stellar disc (Ω),

$$\Omega^2 = \frac{1}{r} \frac{d\Phi_0}{dr}$$

the epicyclic frequency (κ),

$$\kappa^2 = r \frac{d\Omega^2}{dr} + 4\Omega^2$$

and the vertical frequency (ν_z),

$$\nu_z^2 = \left. \frac{\partial^2 \Phi}{\partial z^2} \right|_{z=0}.$$

3.3. Heating

The heating of disc is computed by averaging the radial velocity dispersion, $\sigma_r(t)$, normalized by the initial σ_{crit} (see Sect. 4.2) inside the 5 kpc of the galaxy, giving the averaging Toomre parameter:

$$Q_T = \left\langle \frac{\sigma_r}{\sigma_{\text{crit}}} \right\rangle_{r < 5 \text{ kpc}}$$

with σ_{crit} , the critical velocity dispersion derived from the Toomre stability criterion,

$$\sigma_{\text{crit}} = \frac{3.36G\Sigma}{\kappa}$$

Σ is the stellar surface density.

3.4. Units

In our code we use a unit system where the universal constant of gravity is: $G = 1$, and the mass unity is $U_m = 2.26 \times 10^9 M_\odot$. The length unity is $U_r = 1.02 \text{ kpc}$ and the velocity unit is $U_v = 100 \text{ km s}^{-1}$. The time unit is $U_t = 10 \text{ Myr}$. In this paper, when the units are not indicated, they are in this unit system.

4. Initial conditions

To study the stability differences for galaxies in the MOND and DM models, we construct a sequence of galaxies from early type to late type (Sa, Sb, Sc, Sd). Each type of galaxy corresponds to a set of two model galaxies, one for MOND and the other for DM. A galaxy, for a given type, has the same spatial density for the baryonic disc and bulge in the two models. The stellar disc is modeled by a Miyamoto-Nagai disc:

$$\rho_d = \left(\frac{b_d^2 M_d}{4\pi} \right) \frac{a_d R^2 + \left(a_d + 3\sqrt{z^2 + b_d^2} \right) \left(a_d + \sqrt{z^2 + b_d^2} \right)^2}{\left[R^2 + \left(a_d + \sqrt{z^2 + b_d^2} \right)^2 \right]^{5/2} (z^2 + b_d^2)^{3/2}}$$

with M_d the mass of the disc (at infinity), a_d and b_d the characteristic length and height, and the bulge by a Plummer sphere:

$$\rho_b = \left(\frac{3M_b}{4\pi b_b^3} \right) \left(1 + \frac{r^2}{b_b^2} \right)^{-5/2}$$

with M_b the mass of the bulge (at infinity) and b_b the characteristic length.

4.1. Rotation curves

From an observer's point of view, a galaxy must have the same rotation curve in MOND and in DM. The shape of the rotation curve is imposed by the MOND model. To obtain the same rotation curve (in the galactic plane) in DM we adjust a Plummer dark matter (live) spherical halo to fit the MOND rotation curve. Parameters of the dark matter halos are given in Table 1. The error on the fit parameters is about 2%.

4.2. Velocity dispersion

We used the same value for the Toomre parameter (Q_T) for MOND and DM. The radial velocity dispersion is initialized by:

$$\sigma_r = Q_T \sigma_{\text{crit}}.$$

Table 1. Parameters. The characteristic length of the bulge for Sa, Sb, Sc galaxies is 1 kpc. For the characteristic height of the Miyamoto-Nagai disc, we choose $b_d/a_d = 1/10$. The given mass is the truncated mass. The Toomre parameter value is the same in the DM and MOND model: $Q_T = 2$. The disc is made of 2×10^5 particles, the mass of the bulge particles is equal to the mass of the disc particle. The mass of dark matter particles is three times the mass of the disc particle.

Run	M_d	M_b	a_d	M_h	b_h
Sa	40	12.65	4	206.4	14.8
Sb	30	5	5	173.7	14.6
Sc	20	2	6	148.8	14.5
Sd	10	–	6	129.7	13.1

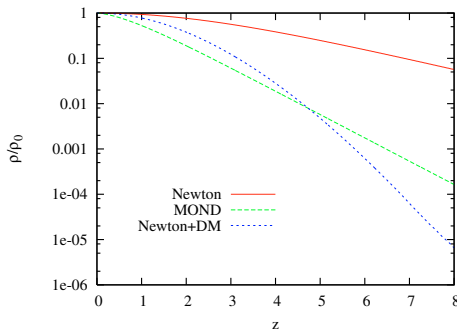


Fig. 5. Vertical structure at the gravitational equilibrium of a disc in Newtonian gravity, MOND, and Newtonian with a dark matter halo.

The tangential velocity dispersion (σ_θ) is deduced from the epicyclic approximation,

$$\sigma_\theta = \sigma_r \frac{\kappa}{2\Omega}.$$

For the vertical velocity dispersion (σ_z), the hydrostatic equilibrium of an isothermal infinite stellar disc in Newtonian gravity gives:

$$\sigma_z^2 = H\pi G\Sigma(r)$$

where H is the characteristic height and $\Sigma(r)$ is the surface density.

We have calculated the vertical density profile ($\rho(z)$) in MOND for an isothermal infinite stellar disc (we consider the problem in one dimension). The equivalent pressure of the gas of star is $P = \rho\sigma_z^2$. The gravitational potential Φ is given by the modified Poisson equation. The gravitational equilibrium ($\nabla P = -\rho\nabla\Phi$) is obtained when:

$$\frac{d}{dz} \left[\mu \left(\frac{|d\Phi/dz|}{a_0} \right) \frac{d\Phi}{dz} \right] = 4\pi G\rho \quad (7)$$

with

$$\frac{d\Phi}{dz} = -\sigma_z^2 \frac{1}{\rho} \frac{d\rho}{dz}. \quad (8)$$

σ_z is a constant of z but varies with r . We solved numerically Eq. (7), the result is plotted in Fig. 5. It shows also the vertical profile in Newtonian gravity: $\rho(z) = \rho_0 \operatorname{sech}^2(z/H)$, and in Newtonian gravity with a dark matter halo. For this plot, we selected $\rho_0 = 2.3 \times 10^{-6} M_\odot \text{ kpc}^{-3}$, which is a typical value of the outer disc. The dark matter halo is a Plummer sphere with

$M_h = 6 \times 10^{11} M_\odot$ and $b_h = 15 \text{ kpc}$ ($r = 8 \text{ kpc}$). In our model we choose $H = \text{const.}$ in the DM model as well as in the MOND model. We want to keep the same initial height for a galaxy in DM and MOND. Figure 5 shows that the vertical profile in MOND or in Newton with a dark matter halo are quite similar. The initial vertical velocity dispersion in MOND is the same as in the DM model. The stellar rotational velocity is not exactly the circular velocity (v_c), but $v_c - v_a$ where v_a is the asymmetric drift deduced from Jeans equations applied to an infinitely thin disc. The system is relaxed initially in its axisymmetric potential to have a well stable virialized initial state.

5. Results

5.1. Bar growth

5.1.1. Dark matter model

Figure 6 (left panel) shows the evolution of an Sa galaxy in the DM model. At first sight, the initial Miyamoto-Nagai disc develops a bar instability in several Gyr. The bar length grows to 6 kpc until 2 Gyr (Fig. 8); its shape is rather squarish. For this run, we do not clearly see a grand design spiral structure during the bar growth. They exist, but they are more visible for a colder disc ($Q_T = 1.5$). After $t = 2 \text{ Gyr}$, transient spirals are developed between 10 kpc and 20 kpc, while a ring appears at the end of the bar. The bar length continues to grow until 4 Gyr, as the ring extends, too (6–7 kpc). During the period between 5 Gyr and 8 Gyr, the bar changes its morphology and takes a butterfly shape. Its length does not increase contrary to the ring that has extended to 10 kpc. Spirals that developed at $t = 2 \text{ Gyr}$ have driven particles to a pseudo ring at 15 kpc, for $t = 4 \text{ Gyr}$, to 25 kpc, at $t = 8 \text{ Gyr}$. The nature of these rings will be discussed in Sect. 5.2.2.

Figure 7 (left) displays the maximum strength of the bar as a function of time. One can distinguish three parts on this plot. First, Q_2 begins to increase until 2 Gyr. Then, this growth stops suddenly, the bar strength drops in 500 Myr. After 2.5 Gyr the bar strength grows again until 5 Gyr and appears to be constant until 8 Gyr.

5.1.2. MOND model

In MOND, the same galaxy (with the same value for Q_T) shows quite different structures (Fig. 6, right). First, a multi-spiral pattern appears rapidly after 0.4 Gyr to give place to a grand design two-arm spiral at 0.6 Gyr. This spiral structure persists until 3 Gyr. The spiral arms have spread out particles up to 30 kpc. After 4 Gyr, the bar begins to be rounder and weakened. No rings are clearly visible in the MOND simulations.

$Q_2(t)$ is plotted in Fig. 7 (right), the bar develops very soon (1 Gyr), compared to the DM model. The bar strength is constant until 4.5 Gyr where a drop occurs suddenly (like in DM at 2.5 Gyr). Afterwards, the bar strength remains low until the end of the simulation. However, the bar length is still constant (6 kpc) even if the bar strength is weakened (Fig. 8).

5.2. Pattern speed and resonance

5.2.1. Pattern speed

The bar pattern speed is represented in Fig. 9, still for the Sa-type galaxy ($Q_T = 2$), for the MOND model, and for the DM model with a live and analytic halo. The bar in DM with a live halo is considerably slowed down during the simulation

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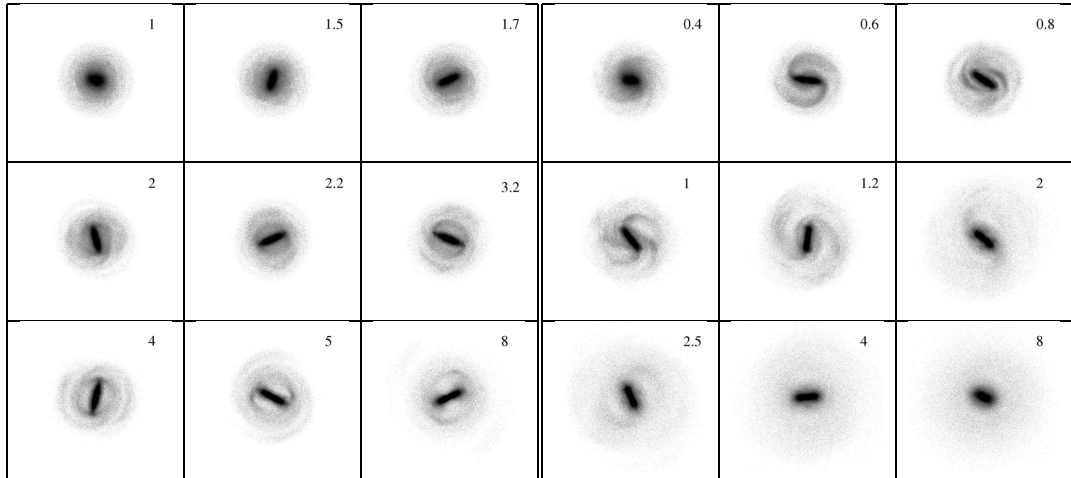


Fig. 6. Bar growth of Sa type ($Q_T = 2$) in the DM model (*left panel*) and the MOND model (*right panel*). The size of the box is $80 \text{ kpc} \times 80 \text{ kpc}$. In the DM model, the bar develops in several Gyr. It can be noticed that the bar is surrounded by a ring at the end of the simulation. Particles are confined in the disc. In the MOND model, the bar appears quite rapidly (in less than 1 Gyr), and a lot of particles are spread out around it up to 30 kpc.

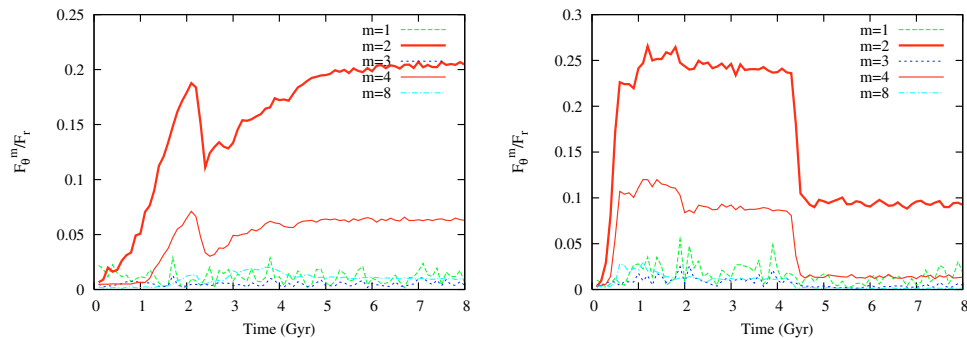


Fig. 7. Time evolution of Q_m for $m = 1, 2, 3, 4, 8$ of galaxy Sa in DM model (*left*) and in MOND model (*right*). In the DM model, the bar strength increases progressively compared to the MOND model where the bar reaches its maximum after 1 Gyr. The same drop appears at $t = 2.5$ Gyr in DM and $t = 4.5$ Gyr in MOND. After that, the bar strength increases again in the DM model, but not in MOND.

($25 \text{ km s}^{-1} \text{ kpc}^{-1}$ to $10 \text{ km s}^{-1} \text{ kpc}^{-1}$), while in MOND, the pattern speed is constant ($25 \text{ km s}^{-1} \text{ kpc}^{-1}$). This plot emphasizes the dynamical friction effects experienced by the stellar bar against the DM particles. To confirm this result we perform a second simulation with the DM model using an analytical dark matter halo instead of a live halo. In this case, the pattern speed of the bar is still constant and corresponds to the MOND result.

5.2.2. Corotation and ILR

The bar pattern speed determines the position of resonant orbits in the reference frame of the bar rotating at Ω_b . Because of velocity dispersion, stars do not just have a circular motion around the galactic center, they oscillate with the epicyclic frequency κ (parallel to the galactic plane); likewise, they oscillate in z with the frequency ν_z . In most numerical simulations (with gas and dark matter) and in galaxies where it was possible to determine

the bar pattern speed, the bar extends to its corotation (e.g., Buta & Combes 1996).

In the DM model, during the two first Gyr, the bar pattern speed is about $25 \text{ km s}^{-1} \text{ kpc}^{-1}$ so that the corotation is nearly 13 kpc (Fig. 10, top), while the bar ends at 7 kpc, but as the bar slows down, the corotation is shifted out to 20–25 kpc at $t = 8$ Gyr (Fig. 10, middle). Hence the ring surrounding the bar from $t = 2$ Gyr until the end of the simulation does not correspond to the corotation resonance as it might be expected. The epicyclic approximation (Fig. 10, middle) indicates that we should have an outer and an inner Lindblad resonance (OILR, IILR), that is where Ω_b (the bar pattern speed) intercepts $\Omega \pm \kappa/2$. The IILR is located very near the center of the galaxy. The OILR is about 12 kpc, where the ring is observed at the end of the simulation. The bar appears to end nearly at the OILR.

Between the two ILR x_2 , orbits must exist and destroy the bar. Figure 11 displays the velocity field in the reference frame rotating with the bar, and the potential outline indicating the bar orientation. Corotation is well identified when vectors change

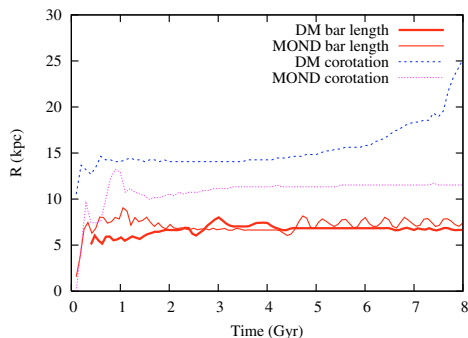


Fig. 8. Bar length and corotation radius in the DM and MOND models. The bar length is defined by the radius where the bar strength ($Q_2(r)$) is equal to half its maximum. The difference between the end of the bar and the corotation radius in MOND is constant and smaller than in the DM model. The corotation radius in the DM model is shifted outward because the bar slows down (see Sect. 5.2).

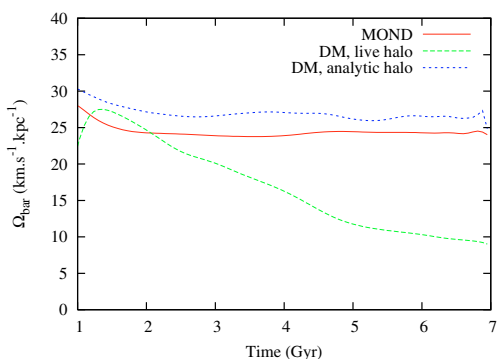


Fig. 9. Rotation speed of the bar for MOND and DM models. In MOND, the bar turns with the same velocity from the beginning to the end. In DM, particles of the halo slow down the rotation of the bar by dynamical friction.

orientation ($r \sim 25$ kpc). The trajectory of particles are parallel to the bar potential like x_1 orbits, not perpendicular (like x_2 orbits). We have performed an orbit analysis to determine the existence of x_1 and x_2 orbits in this bar potential. The result is that the bar is not dominated yet by x_2 orbits. We have launched particles in the bar potential rotating at $\Omega_b = 10 \text{ km s}^{-1} \text{ kpc}^{-1}$. The value of the Jacobi's integral,

$$E_j = \frac{1}{2} \dot{r}^2 + \Phi - \frac{1}{2} |\Omega_b \times r|^2$$

of particles varies between $h_{\min} = -38$ (in our system units, $G = 1$ see Sect. 5.2.2), the bottom of the potential well, to $h_{\max} = -13$, the potential nearly the corotation. x_2 orbits exist between $r = 3-3.5$ kpc in an energy range about $-20 < E_j < -18$. For lower Ω_b like $5 \text{ km s}^{-1} \text{ kpc}^{-1}$, x_2 orbits appears clearly and more frequently between $r = 2.5-9$ kpc in an energy range of $-26 < E_j < -13$. Figure 10 gives just an indication on the resonance with the epicyclic approximation. It can be noted that the drop in bar strength between 2 Gyr and 2.5 Gyr is correlated with ILR formation and its analogue in the z -direction (peanut formation, see the next section). In the MOND model the bar always

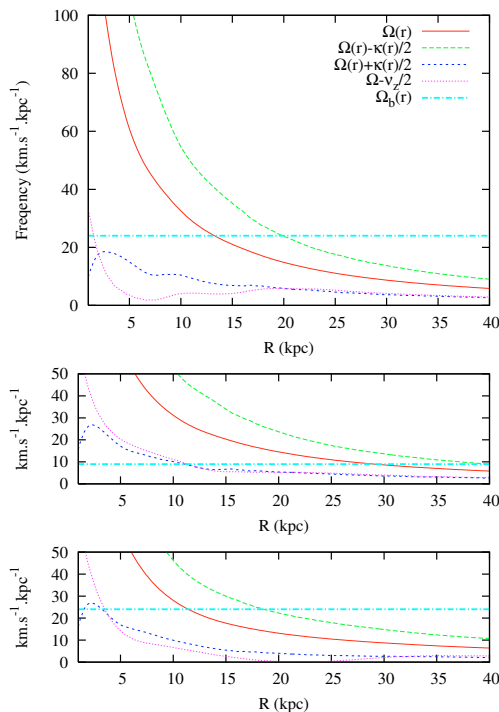


Fig. 10. $\Omega(r)$ curve, and curves combined with the epicyclic frequency $\Omega_b = \Omega \pm \kappa/2$ and vertical frequency $\Omega_b = \Omega - v_z/2$: $t = 2.5$ Gyr (top) and $t = 8$ Gyr (middle) in the DM model, $t = 8$ Gyr (bottom) in the MOND model.

ends near its corotation (Fig. 8), while the DM bar is relatively shorter.

5.2.3. Vertical resonance and warp

It can be shown that v_z and κ have a similar evolution. An equivalent resonance of the ILR exists in the z -direction if $\Omega_b = \Omega - v_z/2$. When particles resonate both in the plane and perpendicular to it, their vertical oscillations can be amplified and a peanut shape results.

The drop in the evolution of the bar strength coincides with the peanut resonance, in DM as well as in MOND. At this moment particles are elevated out of the galactic plane. Stars are less bound and orbits become more oval, the bar strength is thus weakened. Figure 12 illustrates the moment when the peanut is formed. At $t = 2.5$ Gyr in the DM model, and $t = 4.5$ in the MOND model, particles between 2 kpc and 8 kpc resonate and are detached from the galactic plane.

To confirm that the drop in the bar strength is really due to the peanuts, we have performed a 2D-simulation for the DM and MOND models. In this case the bar strength is not weakened during its evolution Fig. 13. There is no drop at $t = 2.5$ Gyr in the DM model or at $t = 4.5$ Gyr in the MOND model.

Like ILR and corotation in the DM model, the position of the z -resonance is also shifted out when the bar pattern speed decreases. In our simulation, dynamical friction acts quite progressively on the bar. The radial shifting of the peanut lobes is

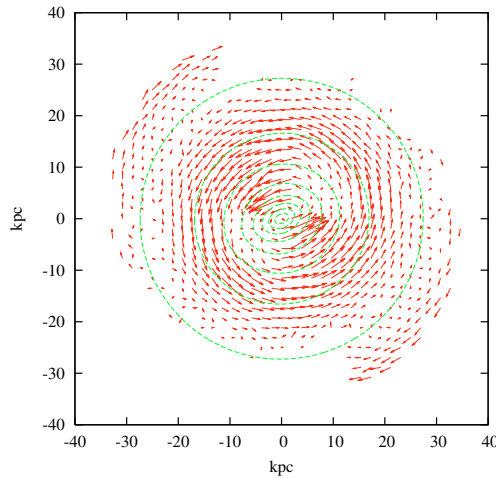


Fig. 11. Velocity field in the reference frame rotating with the bar. Vectors are parallel to the isopotential lines, indicating that the majority of orbits are x_1 type.

continuous. Martinez-Valpuesta et al. (2006) have observed similar phenomena in the formation of a peanut galaxy, but they distinguish two episodes. They obtained a short bar due to chaotic orbits that appear between the ILR and vertical-ILR.

Figure 14 shows several edge-on views of the galaxy in the DM and MOND models. In MOND simulation, the galactic disc is more easily warped than in DM. The disc begins to take a U-shape, to finally flare. The ratio h/h_r , where h is the equivalent characteristic height and h_r the characteristic length for an exponential disc, is about 0.26 in the MOND model and 0.22 in the DM model at $r = 25$ kpc. It is not as different as can be expected because h_r in MOND is larger (7 kpc) than in the DM model (5.5 kpc). Particles are ejected radially further than in the DM model because of the angular momentum transfer (see Sect. 5.4); the disc is thus less compact. The origin of the flare comes from the vertical velocity dispersion that is more important in MOND for outer regions than in the DM model (Fig. 15). In MOND, vertical instabilities are developed because of the self-gravity, and the disc heats more.

We have seen that the weakening of the bar coincided with the peanut's occurrence. Let us note that the peanut is not the only way to weaken a bar in a pure stellar disc. If the disc is too cold, it develops a bar instability very quickly so that the stars have no time to follow a typical orbit with a constant bar strength. The corresponding disordered motions of the stars weaken the bar. This can be shown in a 2D (planar geometry) simulation (Fig. 16).

5.3. Heating

The problem now is to understand why the bar strength continues to increase after the z -resonance in DM and not in MOND. Part of the explanation can be found by following the evolution of Q_T for the two models. The value of the Toomre coefficient indicates the heating rate of the disc. In these simulations, Q_T starts at a value of 2 in the whole disc. The evolution in the DM model and MOND model (Fig. 17) is differentiated from the beginning like the evolution of the bar strength.

Heating in the DM model. In the DM model, the disc heats progressively. When the peanut is forming, it weakens the bar and $Q_T \sim 2.7$. This value is not enough to avoid bar formation, which is why bar strength increases again. A disc in DM model needs a value of about 4 for Q_T to be stable and not form a bar.

Heating in the MOND model. In MOND, Q_T increases to 3.5 in a few Gyr. The apparition of a z resonance weakens the bar strength. At this time, the MOND disc is thus more stable because particles have more velocity dispersion. The bar strength does not increase anymore. We have performed another MOND simulation with $Q_T = 3.5$ from the beginning. A weak bar is formed with a strength of about 0.12. That corresponds with the bar obtained at the end of the simulation with $Q_T = 2$ initially.

In MOND, all the matter participates to the dynamics; the galactic disc is completely self-gravitating, hence it heats up more than a disc rotating in a dark matter halo. The fact that no ring is clearly visible in the MOND simulation can be understood since the disc is hotter and might not sustain these features. The pitch angle of a density wave depends on Q_T . For a hot system, the theory predicts that the spiral will be more open than for a cold one; it is thus more difficult to form a ring.

5.4. Angular momentum exchange

Another crucial point for the bar formation is the exchange of angular momentum. For the bar to grow, particles of the disc have to lose angular momentum to fall in the inner region and have an elliptical orbit instead of a circular one. Angular momentum can be exchanged between the inner and outer parts.

Angular momentum and dark matter. In DM, the halo can receive angular momentum from the disc. It is well illustrated in Fig. 18: the disc loses about 30% of angular momentum in the halo. In other terms if the halo increases its angular momentum, it will be less compact and will inflate. Figure 19 represents the time evolution of several radii comprising a fraction of the mass between 10% to 90% by 10%. One can notice the expansion of the radius below 60%. At $t = 1$ Gyr, 90% of the mass is included in a sphere of 29 kpc; at $t = 8$ Gyr this mass is in a sphere of 32 kpc radius.

This exchange, which is efficient after 2 Gyr, contributes to bar formation (especially after the drop). Then the disc is not too hot and density waves can propagate. One can notice that the core of the halo is unaffected.

Angular momentum in MOND. In MOND, the disc does not lose lot of angular momentum. Angular momentum is exchanged inside the disc itself. Figure 20 shows the evolution of the same radius (10–90%) of the mass versus time for the disc. The inner part of the disc loses angular momentum as expected because of bar formation (contraction of the disc below 8 kpc), and the outer part of the disc receives angular momentum from the inner region (90% of the mass is inside a 15 kpc at the beginning and extends at 20 kpc at the end). This occurs during the first 3 Gyr, and the transfer is mediated by the spiral arms seen in Fig. 6. They evacuate angular momentum from the inner part to the outer part of the disc, and spread out particles around the disc. This is possible when the disc is not too hot. After this phase there is a saturation when the disc becomes stable and no density wave can propagate and increase the bar strength.

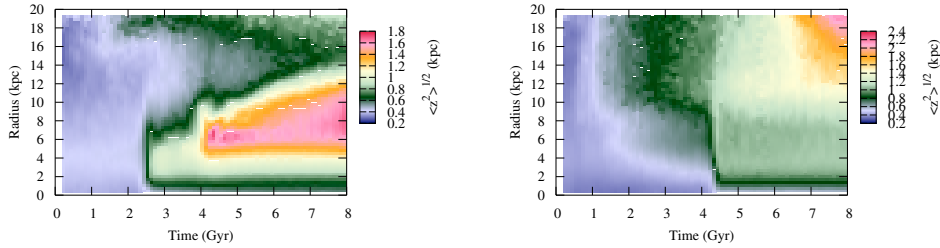


Fig. 12. Peanut formation in the DM (*left*) and MOND (*right*) models. These plots display $\langle z^2 \rangle^{1/2}$ of particles at several radii in a function of time. The peanut appears at $t = 2.5$ in the DM model and $t = 4.5$ in the MOND model. At these times, particles get out of the galactic plane in the range between 2–8 kpc. As the bar slows down in the dark matter halo, more and more particles resonate, and the peanut lobes extend to 12 kpc at $t = 8$ Gyr. In the MOND model the peanut stays at the same place all along the simulation.

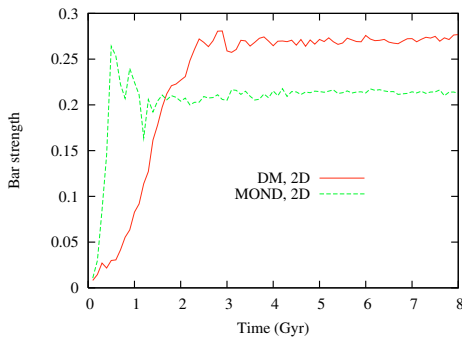


Fig. 13. 2D simulation of the Sa galaxy in DM and MOND models. The bar strength increases until a maximum and remains roughly constant.

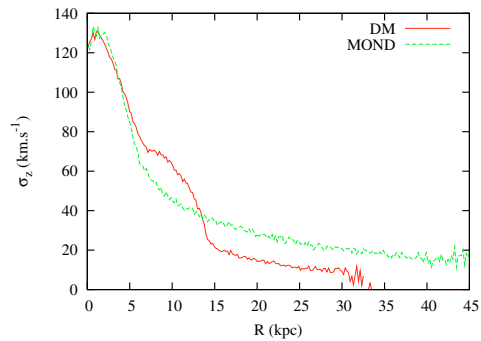


Fig. 15. Vertical velocity dispersion (σ_z) of the disc for the DM and MOND models at $t = 8$ Gyr. In DM, the peanuts makes σ_z increase around 12 kpc. After 15 kpc, σ_z in MOND is larger than in DM.

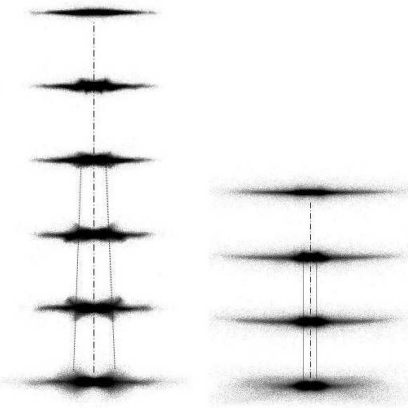


Fig. 14. Edge-on view of an Sa galaxy, shows the characteristic peanut shape. In the DM model (*left*), the position of the peanut lobes are radially shifted out as the bar slows down. In the MOND model (*right*), the peanut keeps the same size, one can notice the warp and flaring of the disc.

5.5. Dark matter compared to MOND along the Hubble sequence

A series of simulations have been run to explore the parameters of galaxies along the Hubble sequence according to Table 1. In

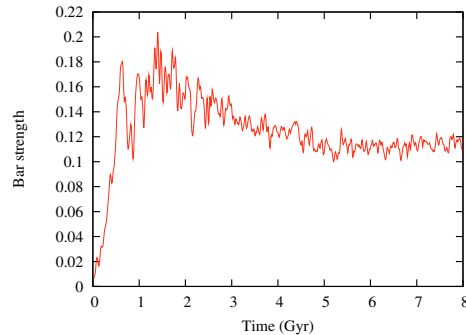


Fig. 16. 2D simulation of a cold stellar disc ($Q_T = 1$) evolving in a small dark matter halo. The bar is formed quickly in a few galactic rotation. Bar weakens because of non-adiabatic bar growth (see text).

the DM model, from the early-type to late-type galaxies the ratio between the visible mass and the dark matter inside the optical radius increases. Thus, late-type galaxies are less self-gravitating than early types, so they are more stable (Fig. 21).

In MOND, galactic discs are cold and form a bar in a few Gyr whatever their type. The evolution scheme seen for the Sa type is reproduced for the Sb, Sc, and Sd type too. Even if the disc in MOND is cold and unstable at the beginning, it heats quickly and stabilizes itself along its evolution (Fig. 21).

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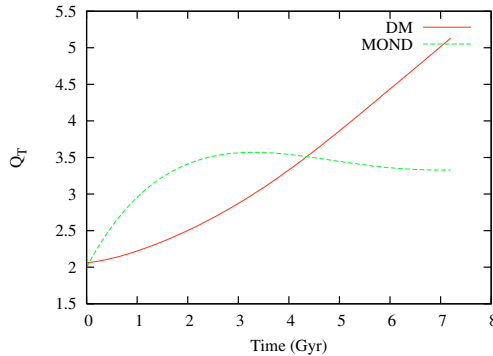


Fig. 17. Evolution of the Toomre coefficient value in function of time. The disc heats up in a few Gyr in the MOND model because of a complete self-gravity.

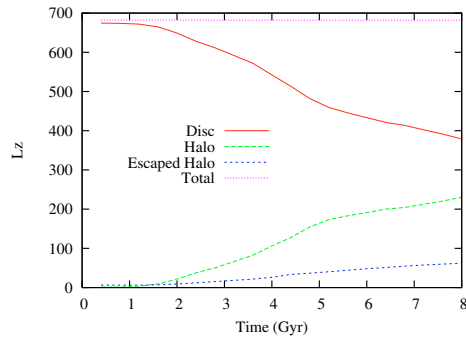


Fig. 18. Vertical component of the angular momentum in DM model. L_z is exchanged from the disc to the halo during the bar formation. A non negligible proportion of particle (dark matter particles) escaped from the simulation box. Their motions are treated in a Keplerian potential as if all the matter inside the box is concentrated into a point mass.

Peanuts are formed for galaxies with a sufficiently massive bulge like Sa and Sb galaxies. In this case, the peanut occurrence weakens the bar. But in the DM model, the bar strength increases again because of the angular momentum transfer between the disc and the dark matter halo. While in MOND, the bar strength keeps low, and the disc heats up because of instability and stabilizes itself.

For Sc and Sd galaxies in the MOND model, the bar is formed too quickly (a few galactic rotations) because the system is too unstable. The stars have no time to settle in orbits supporting the bar at a given bar strength, since the orbital structure of the bar varies on a time scale shorter than the orbital period. These galaxies present a strong bar during a short time at the beginning of simulation to finish with a weak bar.

The pattern speed of the bar is plotted in Fig. 22. In MOND the pattern speed is always constant for a given galaxy. Early-type galaxies have a higher bar pattern speed than late types (the disc is more massive). In DM, the bar is always slowed down by dynamical friction due to the halo. Late-type galaxies need more time to form a bar (Fig. 21), so their pattern speeds are less slowed down than for early types.

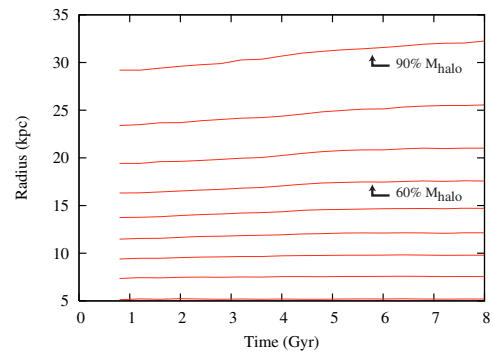


Fig. 19. In the DM model, the dark matter halo inflates because of angular momentum exchange from the disc to the halo.

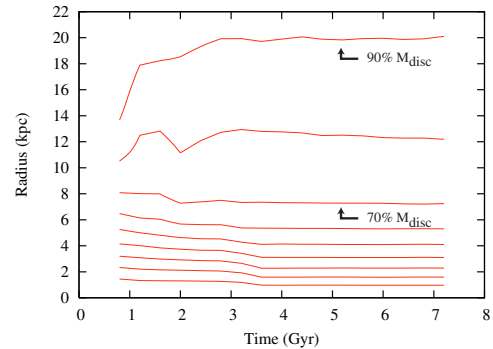


Fig. 20. In MOND, the outer part of the disc inflates while the center contracts. Angular momentum is transferred between these two regions during the bar growth.

We have made a statistical study of the bar strength for typical galaxies of the Hubble sequence. Figure 23 shows the bar frequency obtained using the time spent by a galaxy with a given bar strength. Two tendencies are clear from this plot. First, galaxies in the MOND model have stronger bars ($Q_2 > 0.25$) than in the DM model. The MOND discs are more unstable at the beginning so they form a strong bar very quickly. Secondly, there is a hole at low bar strength in the MOND model that is not present in the DM model. This is due to the dark matter halo that stabilizes the disc at the beginning, so it takes more time to a galaxy to form a bar. The bar strength distribution obtained from the observations presents some characteristics that are reproduced with the MOND model. In particular, there is a small proportion of galaxies with a very weak bar, and a few galaxies have very strong bar (e.g., Block et al. 2002; and Whyte et al. 2002).

6. Discussion and conclusion

In this paper, the dynamical evolution of pure stellar discs in MOND is compared to Newtonian gravity with DM, using numerical simulations. We have developed an N -body code that solves the modified Poisson equation in three dimensions using MG technique for the potential solver. The simulations in the DM models have been performed with the same code by solving Poisson equation with the same MG technique.

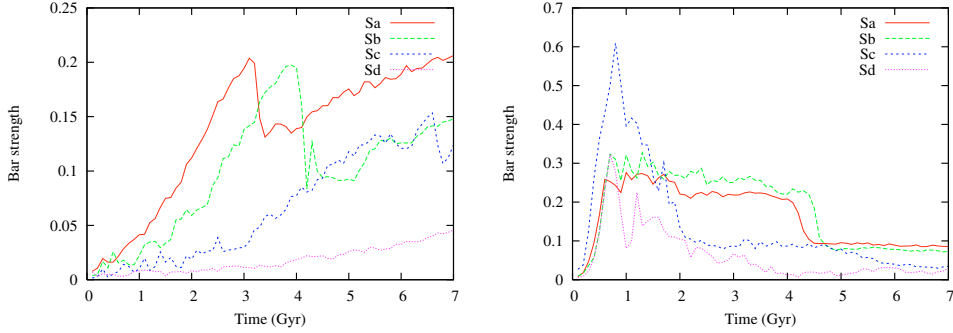


Fig. 21. Evolution of the bar strength in the DM (*left*) and MOND (*right*) models. Late-type galaxies in the DM model are more stable than early types; they need a larger proportion of dark matter to obtain the same rotation curve as in MOND. Late-type galaxies in MOND are too unstable; the bar destroys itself.

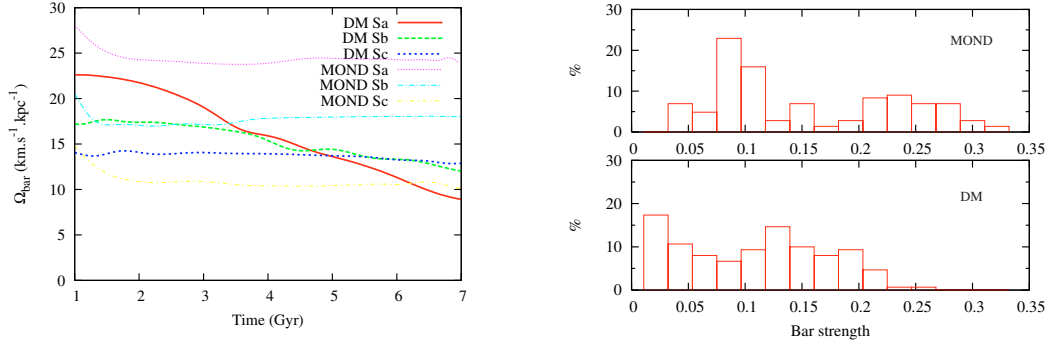


Fig. 22. Bar pattern speed in DM and MOND models. Dynamical friction effects in the DM model slow down the bar. In MOND, the bar pattern speed is still constant.

Fig. 23. Bar frequency in the simulated Hubble sequence in MOND (*top*) and in DM (*bottom*). Bars are stronger with MOND and there is a dearth of galaxies without bars in MOND, but not in the DM model.

For isolated galaxy evolution, the main difference between the MOND gravity and the Newtonian gravity with dark matter is the self-gravity of the disc. Even if the acceleration in MOND scales as $M^{1/2}$ instead of M in Newtonian gravity (BM99), the dark matter halo in the DM model stabilizes more efficiently the disc. From a given initial state, the MOND disc is more unstable than the DM disc in the sense that it develops a bar instability sooner, for the same Toomre parameter value.

One of the main effects of the dark matter halo is the dynamical friction experienced by the stellar bar against the DM particles. The bar pattern speed is slowed down in the DM model. This does not exist in MOND. The bar pattern speed in MOND keeps constant all along the evolution, thus higher than in the DM model. This has consequences on the position of the resonances like corotation. Bar lengths are often compared to the corotation radius. In this case, bars obtained with MOND end closer to the corotation radius.

The 3D simulations reveal several differences between MOND and Newtonian gravity with dark matter. Peanut bars are formed in the DM model as well as in the MOND model, but peanut lobe positions, which correspond to the z -inner Lindblad resonance, depend on the bar pattern speed. In MOND, the peanut always remains the same size ($\Omega_b = \text{const.}$), contrary to the DM model where the lobes are radially shifted far from the center (about 12 kpc). In MOND, successive instabilities due to

self-gravity make the vertical velocity dispersion higher, in the outer region of galaxies, than in the DM model. There is a higher tendency for MOND discs to warp and flare.

Two mechanisms to weaken a bar have been described. First, for galaxies with a massive bulge (early type) a peanut resonance can be formed. This vertical motion of stars dilutes the bar concentration in the plane and makes the bar strength decrease. Secondly, if the disc is cold and unstable, it forms a bar so quickly that the orbital structure of the bar varies on a time scale shorter than the orbital period, and the stars cannot settle on orbit supporting the bar.

The present simulations reveal that the dark matter halo has two contradictory influences on the disc stability. On the one hand, the DM halo stabilizes the disc and delays the bar formation; on the other hand, it can reinforce the bar growth when the bar is forming by accepting the angular momentum from the disc stars, in particular after the peanut's formation. In contrast, peanut galaxies in MOND should have low bar strength.

Statistically, the MOND bar frequency corresponds better to the observations than to the DM model. Indeed, there is a hole in the barred galaxy distribution for low bar strength and more galaxies distributed at high bar strength. But in this work, only stellar discs are considered without any gas component. Bar formation and destruction is affected by the gas component in the spiral galaxies. In particular gas accretion allows galaxies to

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have several bar cycles (Bournaud & Combes 2002). Gas components will be added in future works.

Through this work, we help to develop numerical tools for testing MOND. Using this code, many physical situations could be simulated. More complex systems will be studied, such as interacting galaxies where MOND might reveal larger differences compared with the DM model.

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References

- Angus, G. W., Shan, H., Zhao, H., & Famaey, B. 2007, *ApJ*, 654, L13
 Bekenstein, J. D. 2004, *Phys. Rev. D*, 70, 3509
 Bekenstein, J. D., & Milgrom, M. 1984, *ApJ*, 286, 7
 Block, D., Bournaud, F., Combes, F., Puerari, I., & Buta, R. 2002, *A&A*, 394, 35
 Bournaud, F., & Combes, F. 2002, *A&A*, 392, 83
 Brada, R., & Milgrom, M. 1995, *MNRAS*, 276, 453
 Brada, R., & Milgrom, M. 1999a, *ApJ*, 519, 590 (BM99)
 Brada, R., & Milgrom, M. 1999b, *ApJ*, 512, L17
 Brada, R., & Milgrom, M. 2000a, *ApJ*, 531, L21
 Brada, R., & Milgrom, M. 2000b, *ApJ*, 541, 556
 Brandt, A., & Lanza, A. 1988, *Class. Quantum Grav.*, 5, 713
 Buta, R., & Combes, F. 1996, *Fund. Cosm. Phys.*, 17, 95
 Clowe, D., Bradac, M., Gonzalez, A. H., et al. 2006, *ApJ*, 648, 109
 Croton, D. J., Springel, V., White, S. D. M., et al. 2006, *MNRAS*, 365, 11
 Debattista, V. P., & Sellwood, J. A. 2000, *ApJ*, 543, 704
 Dehnen, W. 2001, *MNRAS*, 324, 273
 de Blok, W. J. G. 2005, *ApJ*, 634, 227
 Famaey, B., & Binney, J. 2005, *MNRAS*, 363, 603
 James, R. A. 1977, *J. Comput. Phys.*, 25, 71
 Knebe, A., & Gibson, B. K. 2004, 347, 1055
 Kravtsov, A. V., Gnedin, O. Y., & Klypin, A. A. 2004, *ApJ*, 609, 482
 Martinez-Valpuesta, I., Shlosman, I., & Heller, C. 2006, *ApJ*, 637, 214
 Milgrom, M. 1983, *ApJ*, 270, 365
 Moore, B., Ghigna, S., Governato, F., et al. 1999, *ApJ*, 524, L19
 Navarro, J. F., Hayashi, E., Power, C., et al. 2004, *MNRAS*, 349, 1039
 Nusser, A. 2002, *MNRAS*, 331, 909
 Nusser, A., & Pointecouteau, E. 2006, *MNRAS*, 366, 969
 Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, *Numerical Recipes in Fortran 77*, 2nd edn. (Cambridge University Press)
 Sanders, R. H., & McGaugh, S. S. 2002, *ARA&A*, 40, 263
 Silk, J. 2004, in *The Mitchell Symposium on Observational Cosmology AIPC*, 743, 33
 Steinmetz, M. 2003, *Ap&SS*, 284, 32
 Whyte, L. F., Abraham, R. G., Merrifield, M. R., et al. 2002, *MNRAS*, 336, 1281
 Zhan, H. 2006, *ApJ*, 639, 617
 Zhao, H. S., & Famaey, B. 2006, *ApJ*, 638, L9

4.2 Évolution des galaxies isolées : II-Dynamique du gaz

Tiret & Combes 2008, accepté dans A&A

Résumé. Dans cet article, nous reprenons l'étude de l'évolution des galaxies en ajoutant une composante gazeuse dans le disque et de la formation d'étoiles (loi de Schmidt). La dissipation du gaz est modélisée par le schéma des *sticky-particles*. Nous testons différentes fonctions μ (simple et standard). La dynamique des galaxies géantes n'est pas sensible au choix de cette fonction. Par contre, en utilisant la fonction standard avec MOND, cela permet d'ajouter moins de matière noire en gravitation newtonienne pour obtenir la même courbe de rotation. En gravité modifiée, les temps de formation de barre étaient rapides (< 1 Gyr), la présence de gaz n'a pas modifié de façon significative ces temps caractéristiques. Par contre, le fait d'avoir moins de matière noire dans les nouvelles simulations en gravitation newtonienne accélère la formation des barres. L'histogramme de fréquence de barre redevient alors similaire entre le modèle de MOND et la gravité newtonienne avec matière noire. Nous discutons aussi la formation d'anneaux de gaz sur les orbites résonnantes de Lindblad ainsi que la résonance verticale de type cacahuète.

Evolution of spiral galaxies in modified gravity: II- Gas dynamics

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ABSTRACT

The stability of spiral galaxies is compared in MOND and Newtonian dynamics with dark matter. We extend our previous simulations that involved pure stellar disks without gas, in dealing with gas dissipation and star formation. We also vary the interpolating μ -function between the MOND and Newtonian regime. Bar formation is studied and compared in both dynamics, from initial conditions identical in visible component morphology and kinematics (same density profile, same rotation curve and velocity dispersion). One first result is that the MOND galaxy evolution is not affected by the choice of the μ -function, it develops bars with the same frequency and strength. The choice of the μ -function changes significantly the equivalent Newton models, in changing the dark matter to visible mass ratio, and therefore the stability. The introduction of gas shortens the time-scale for bar formation in the Newton model, but not significantly in the MOND model, since it was already small. As a consequence, the bar frequency histograms are now more similar between MOND and Newton, with gas than without gas. The thickening of the plane occurs through vertical resonance with the bar and peanut formation, even more quicker with gas. Since the mass gets more concentrated with gas, the radius of the peanut is smaller, and the appearance of the pseudo-bulge more boxy. The bar strength difference is moderated by saturation, and feedback effects, like the bar weakening or destruction by gas inflow due to gravity torques. Averaged over a series of models representing the Hubble sequence, the MOND models have still more bars, and stronger bars, than the equivalent Newton models, fitting better the observations. Gas inflows driven by bars produce accumulations at rings or pseudo-rings at Lindblad resonances, and MOND models can reproduce quite well observed morphologies, as was found before in the Newtonian dynamics.

Key words. Galaxies: general — Galaxies: kinematics and dynamics — Galaxies: spiral — Galaxies: structure — Cosmology: dark matter

1. Introduction

Galaxy dynamics and evolution are depending strongly on the amount of dark matter assumed, on its nature, shape and radial distribution (e.g. Dubinski et al 1996, Debattista & Sellwood 2000, Athanassoula 2002, Valenzuela & Klypin 2003, Avila-Reese et al. 2005, Colín et al. 2006). They are thus expected to be quite different under the hypothesis of modified gravity, built to be free of any dark matter. Simulations of galaxy dynamics compared to the observations could then provide constraints to the models.

Galaxy stability was investigated in MOND by Brada & Milgrom (1999), through simulations of dissipationless stellar discs. While it is known from Ostriker & Peebles (1973) that dark matter haloes have a stabilising influence on the disc dynamics, they show that MOND discs without any dark matter are also more stable than their analogs in Newtonian gravity. In other words, they compare the stabilizing effects of the MOND dynamics with those of dark matter haloes, and conclude that they are both efficient, for the same disc “temperature”, or the fraction of the total kinetic energy that is in random motions. This temperature is quantified by the ratio $t = T_{rot}/|W|$ of the rotational kinetic energy T_{rot} and the absolute value of the total gravitational energy W . If the system is stationary, the latter is equal to twice the total kinetic energy, through the virial theorem. The maximum value of t is 0.5, and Ostriker & Peebles (1973) proposed a criterion for stability of $t < 0.14$. This criterion t based on the fraction of ordered to total ki-

netic energy is only one way to define the “temperature” of a disc, to be compared in the two dynamics.

The Toomre Q parameter is often used as the criterion for axis-symmetric stability of a galaxy disc, of surface density Σ , radial velocity dispersion σ_r , and epicyclic frequency κ . Its value in Newtonian dynamics, which is:

$$Q = \frac{\sigma_r \kappa}{3.36 G \Sigma} \quad (1)$$

has to be modified in MOND, by replacing G by $G/\mu^+(1+L^+)^{1/2}$, where μ^+ is the value of the MOND interpolating function just above the disc, and $L = d \ln \mu(x)/d \ln x$ (Milgrom 1989). The function $\mu(x)$ satisfies $\mu(x \ll 1) \sim x$ and $\mu(x \gg 1) \sim 1$.

The two criteria t and Q are therefore not equivalent in MOND, since they depend on the total mass (Brada & Milgrom 1999). If the criterion of $t < 0.14$ corresponds to about $Q > 2.5$ for Newtonian galaxy models, or MOND models of high mass, they correspond to much higher Q for dwarf galaxies, deep in the MOND regime. This explains the conclusion by Brada & Milgrom (1999) that discs are locally more stable in the deep MOND regime. This conclusion, however, pertains to the *local* stability, based on the modified Toomre criterion. For the *global* instability, Brada & Milgrom (1999) find that the MOND galaxies have a larger growth rate than the Newtonian equivalent. Physically, the extra stability in the MOND regime comes from the fact that the gravity force is proportional to the square of the density, instead of the density. However, the

fact that the disc is fully self-gravitating and not embedded in a stabilizing halo component acts in the other direction.

The two proposed “temperature” parameters are not easy to compare with observations. The modified Toomre parameter is model dependent, and for the first case, the ordered to total kinetic energy ratio t , the total energy involves the random motions of the dark matter halo in the Newtonian case, which is not observed. To compare MOND and Newtonian dynamics with DM, we prefer to adopt the same properties for the visible component, both in morphology and kinematics. In addition to identical morphology, the initial discs have the same rotation curve and the same radial velocity dispersion. This “temperature” parameter is closer to observations, although the two models have not always the same “temperature” with all possible definitions. In a previous work, considering only stellar dynamics, we have concluded that galaxies were more unstable in MOND dynamics (Tiret & Combes 2007, hereafter TC07). The overall conclusion, averaged over a series of models representing the Hubble sequence, was that the bar frequency is higher in the MOND dynamics, and more compatible with observations.

In the present work, we consider in addition the gas dynamics, which has a major effect on disc stability. We also adopt a different interpolating function μ , which is the standard one proposed by Milgrom (1983) for galaxy rotation curves. This one reduces the amount of “phantom” dark matter, i.e. the dark matter required in the Newtonian dynamics to obtain the same rotation curve as in the MOND disc. In our previous work (TC07) we had adopted the $\mu(x) = x/(1+x)$ function, proposed by Zhao & Famaey (2006), for which the “phantom” dark matter is almost doubled. We therefore recompute all models, with and without gas, to provide better comparisons.

In the next section, we describe the numerical techniques used to solve the modified Poisson equation, and also the model of sticky particles adopted for the gas dynamics. Galaxy models and initial conditions are displayed in Section 3, results are presented in Section 4 and discussed in Section 5, to establish the new bar frequency in the MOND and Newtonian models.

2. Numerical model

Gravitation. We use our Potential Solver described in TC07 to solve the Poisson equation in the case of Newton gravity (eq. 2) and MOND (eq. 3).

$$\Delta\phi = 4\pi G\rho \quad (2)$$

$$\vec{\nabla} \cdot \left[\mu \left(\frac{|\vec{\nabla}\phi|}{a_0} \right) \vec{\nabla}\phi \right] = 4\pi G\rho \quad (3)$$

It is a grid solver using multigrid techniques (Numerical Recipes, Press et al., 1992), the grid characteristics are the same than in TC07. The simulation box covers a 100 kpc cube, the spatial resolution is about 400 pc (256^3 nodes).

Gas dissipation. These new simulations include a gas component. The dissipation between *gas clouds* is modelled by a sticky-particle scheme. This allows to control more directly the gas dissipation, through a rebound, or elasticity parameter β (e.g. Combes & Gerin 1985).

Run	M_d	M_b	Gas	a_d	a_g	M_h	b_h
Sa	40	12.7	2%	4	6	146.3	29.4
Sb	30	5	5%	5	7.5	118.9	24.8
Sc	20	2	7%	6	9	92.8	20.9

Table 1. Parameters of the Sa, Sb, Sc galaxy models. The unit system is $G = 1$: the mass unity is $U_m = 2.26 \cdot 10^9 M_\odot$, the length unity is $U_r = 1.02 \text{ kpc}$.

Consider two particles $(\mathbf{r}_1, \mathbf{v}_1, m_1)$, $(\mathbf{r}_2, \mathbf{v}_2, m_2)$ in their mass center frame. The radial and tangential relative velocities are $(\mathbf{v}_r, \mathbf{v}_t)$ before the collision, and $(\mathbf{v}'_r, \mathbf{v}'_t)$ after the collision:

$$\mathbf{v}'_r = \beta_r \mathbf{v}_r$$

$$\mathbf{v}'_t = \beta_t \mathbf{v}_t$$

or,

$$\mathbf{v}'_1 = \frac{m_2}{M} [\beta_t(\mathbf{v}_1 - \mathbf{v}_2) + (\beta_r - \beta_t)((\mathbf{v}_1 - \mathbf{v}_2) \cdot \mathbf{e}_r) \mathbf{e}_r]$$

$$\mathbf{v}'_2 = \frac{m_1}{M} [\beta_t(\mathbf{v}_2 - \mathbf{v}_1) + (\beta_r - \beta_t)((\mathbf{v}_2 - \mathbf{v}_1) \cdot \mathbf{e}_r) \mathbf{e}_r]$$

with $M = m_1 + m_2$ and $\mathbf{e}_r = (\mathbf{r}_2 - \mathbf{r}_1) / \|\mathbf{r}_2 - \mathbf{r}_1\|$

We choose $\beta_t = 1$ to conserve the angular momentum, and $\beta_r = -0.85$. $\beta_r < 0$ ensures that two particles after the collision go in opposite ways than before the collision. This also introduces a randomisation in velocities.

Selecting $\beta_r > 0$ results in a model with colder gas, with lower velocity dispersion.

The collision grid, used to find the neighbours for collisions is in 2D due to the small thickness of the gas layer. The collision cell size is the same than the gravitational one so that dissipation cools at the same resolution scale than gravity heats. An individual gas cloud is typically subject to 10 – 20 collisions along a galactic rotation.

Star formation. The star formation follows the Schmidt law:

$$\frac{dN_\star}{dt} = C \cdot \rho_{gas}^{1.2}$$

C is calibrated so that the gas consumption time through star formation is $\tau \sim 2$ Gyr for a spiral galaxy. We use the hybrid particle technique to form stars from the gas. Since the amount of stellar material formed at a given location is always smaller than the particle mass-resolution, only a fraction of the gas particle is transformed into stars, the rest remaining gaseous, until several cycles used up the total gas content of a particle. For the dissipation rate, in the sticky-particle scheme, it is the effective mass fraction of gas that is taken into account. When the gas fraction of a given particle is less than 0.5, β_r is chosen positive (the particles cross each other and do not rebound during the collision). As the gas fraction decreases, the dissipation is progressively reduced, and the hybrid-particle takes a stellar behaviour.

3. Initial conditions

3.1. The μ -function

In TC07, we used the *simple* form of the μ -function: $\mu(x) = x/(1+x)$, proposed by Zhao & Famaey (2006). This function has been found a better compromise with theory

and observations of the Milky Way than other functions like the *standard* one: $\mu(x) = x/\sqrt{1+x^2}$. However, most of galaxy rotation curves are reproduced with the standard function (e.g. Sanders & McGaugh 2002). Coupled with $a_0 = 1.2 \times 10^{-10} \text{ m.s}^{-2}$, the well known value for the critical acceleration of MOND, the rotation curves of galaxy is thus fixed if one gives a density profile. In our study, we will always compare any MOND galaxy model with the Newtonian gravity & dark matter model (DM) having the same rotation curve. In this paper, we use the *standard* μ -function.

3.2. The galaxy models

We study in the present paper high surface brightness galaxies, while low surface brightness (LSB) ones will be modeled in future works. LSB systems require a full study, since bar formation is qualitatively different there (e.g. Mayer & Wadsley 2004).

As in TC07, the stellar disc is modeled by a Miyamoto-Nagai disc. The initial distribution of the gas follows a Toomre disc:

$$\Sigma_g(r) = \frac{a_g M_g}{2\pi(r^2 + a_g^2)^{3/2}}.$$

The 3D structure is given by:

$$\rho_g(r, z) = \Sigma_g(r) \text{sech}^2(z/H_g),$$

a_g and H_g being the radial and vertical gas scale-lengths. The bulge and the DM halo are initially distributed in Plummer spheres live haloes. Compared to TC07 the stellar matter (disc+bulge) keeps the same characteristics. Now we add a gas disc of 2% to 7% of the total visible mass. The mass and the characteristic radius of the “phantom” dark halo (in the corresponding Newtonian dynamics) have changed because we now use the standard μ -function instead of the simple μ -function. Simulations without gas have been repeated with this new parameters for the DM and MOND models. The characteristic values are given in the Table 1. The characteristic length of the bulge for Sa, Sb, Sc galaxies is 1 kpc. For the characteristic height of the Miyamoto-Nagai disc, we choose $b_d/a_d = 1/10$. The given mass is the truncated mass. The Toomre parameter value is the same in the DM and MOND model: $Q = 2$. We use the same definition of Q for the Newtonian and MOND simulations: $Q = \frac{\sigma_r \kappa}{3.36 G \Sigma}$, where G is not varied. Instead of trying to adapt the stability status (cf Introduction section), we build the initial galaxy to have the same morphological and kinematical characteristics in both models, in all what concerns the visible component. The number of particle is 2.10^5 for the stellar disc, 5.10^4 for the Sa/Sb gas discs, and 10^5 for the Sc gas disc. The mass of a bulge particle is equal to the mass of a disc particle. The mass of dark matter particles is three times the mass of the disc particle. The initial distribution of star, gas, and dark matter particles are respectively truncated at 25 kpc, 30 kpc, and 45 kpc.

4. Results

4.1. The couple of parameters (μ, a_0)

In TC07, we found that the bar instability occurs in a much longer time-scale in the DM model than in MOND (several Gyr compared to less than one). By adding gas, the

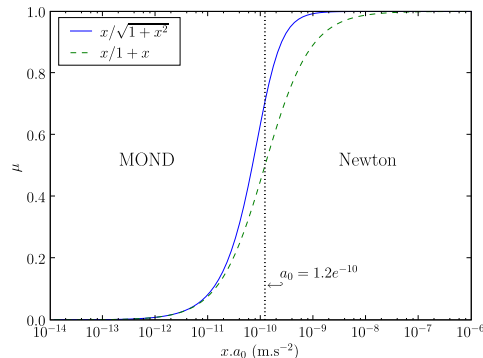


Fig. 1. The simple μ -function ($\mu(x) = x/(1+x)$, used by TC07) transfers in the MOND regime one order of magnitude earlier in acceleration, than the standard μ -function ($\mu(x) = x/\sqrt{1+x^2}$), used in the present work.

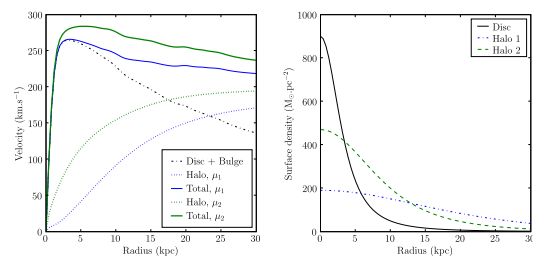


Fig. 2. Left: rotation curves of an Sa-type galaxy using the *standard* (μ_1 , this work) and *simple* (μ_2 , TC07) μ -functions. In each case, the halo needed in the DM model to obtain the MOND rotation curve is plotted too (Halo, μ_1 , Halo, μ_2 curves). Right: Surface density of the stellar disc compared to the dark matter halo (in the Newtonian model) needed to fit the MOND rotation curve if the free function is the standard one (1, this work) or the simple one (2, TC07).

bar instability could be expected to develop faster (about 1 Gyr) especially in the DM model. We first run several simulations, keeping the “simple” μ -function; the bar appeared sooner with gas but not as soon as could have been expected. The dark matter halo needed to fit the MOND rotation curve (using the simple μ -function) is very massive and stabilizes too much the galactic disc.

With the simple μ -function, the acceleration keeps Newtonian until $a = 10^{-8} \text{ m.s}^{-2}$ (Fig. 1). This means that below $a = 10^{-9} \text{ m.s}^{-2}$ in the DM-model, the gravitation begins to be dominated by the dark matter halo.

To increase the Newtonian regime, and avoid introducing too much dark matter for the DM model, there are two possibilities: either change the value of the critical acceleration a_0 , or change the standard function and keep $a_0 = 1.2 \times 10^{-10} \text{ m.s}^{-2}$. The latter possibility is adopted in the present work. Figure 2 (left) shows an example of rotation curve computed with the simple and standard function. At first sight the difference between the two curves does not seem very important (about 25 km.s^{-1}), but the dark matter mass needed to fit the MOND rotation curve

with the standard μ -function is twice the dark matter mass necessary for the simple μ -function (Fig. 2, right), inside the visible radius. This plot corresponds to an Sa-type galaxy, where the more massive and concentrated dark matter (halo 2) multiply by a factor 2 the time to form the bar (see TC07). It is even more sensitive for the late-type galaxies which have an even larger fraction of dark matter. We do not think that this stabilising effect is due to a different swing amplification: with the first developing pattern, we computed the parameter $X = \lambda / \sin i / \lambda_{crit}$ where i is the pitch angle of the spiral arms, and $\lambda_{crit} = 4\pi^2 G\mu / \kappa^2$. The value for the halo 1 and halo 2 models are comparable: $X \sim 1$.

4.2. Bar strength

By losing energy during collision (dissipation), the gas remains a cold component which tends to destabilize the galaxy disc. With gas, bar formation is thus expected to be faster. In gas-poor early type galaxies (Sa, Sb), the gas contribution to the galaxy evolution is weak (fig. 4). The difference appears more clearly for the Sc galaxy (with 7% of gas). With gas, the bar forms sooner and is stronger. The bar strength is estimated by the quantity $Q_2 = \text{Max}|F_\theta(r)/F_r(r)|$ where F_r is the radial force and F_θ the tangential force corresponding to the Fourier component $m = 2$. In the DM model, it is maximum at $t = 6$ Gyr ($Q_2 = 0.3$) without gas instead of $t = 7.5$ Gyr ($Q_2 = 0.2$) with gas. In the Sa, Sb simulations, the bar is weakened by the apparition of a peanut resonance (see TC07). This occurs sooner when there is gas. Quite rapidly, the gas falls in the center of the galaxy, increasing the potential well through its mass concentration. We observe that galaxies in MOND spend more time with a stronger bar than in the DM model. In the MOND-Sa simulation the peanut which weakened the bar occurs later ($t = 4$ Gyr) than in the DM-Sa simulation ($t = 2$ Gyr). In the DM-Sb simulation, the bar is strongly weakened after the vertical buckling: $Q_2 = 0.1$, while it keeps at $Q_2 = 0.2$ during several gigayears after the resonance in the MOND simulation.

The MOND-Sc simulation shows that the bar is destroyed immediately after its formation because of gravity torques and subsequent gas inflow, and angular momentum exchange, as already shown by Bournaud et al. (2005). To maintain or reform bars, gas accretion should then be considered (Bournaud & Combes, 2002). Fig. 5 shows the angular momentum lost by the gas along one galactic rotation.

4.3. Bar pattern speed and Resonance

Many authors have studied the dynamical friction between the galactic bar and the dark matter halo, and the subsequent bar slow down and angular momentum transfer (Weinberg 1985, Little & Carlberg 1991, Hernquist & Weinberg 1992, Debattista & Sellwood 1998, 2000, O'Neill & Dubinski 2003, Valenzuela & Klypin 2003, Sellwood & Debattista 2006).

As in TC07, we make a comparison between the DM and MOND models about the bar pattern speed and its implication on the resonances.

The bar pattern speed (Ω_{bar}) is calculated from the Fourier transform $\widehat{\phi}_2(r, \Omega)$ of $\phi_2(r, t)$, the phase term of

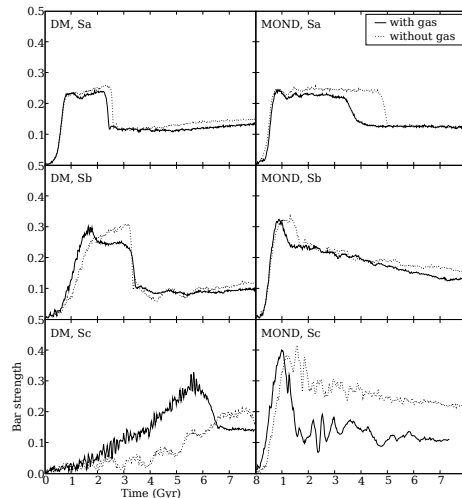


Fig. 4. Several cases of bar strength evolution in DM and MOND models, with and without gas. Bars are formed in 1 – 2 Gyr in all models, except for the Sc galaxy in DM, where the halo is relatively more dominant and stabilizes the disc.

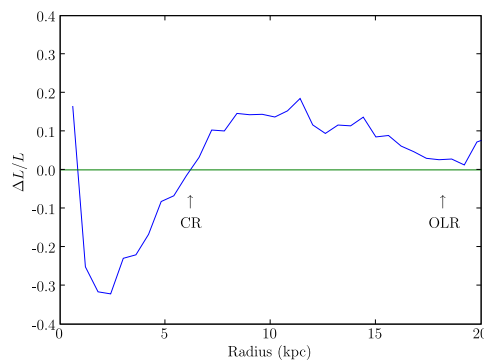


Fig. 5. Effects of the gravity torques created by the stellar bar during the bar dissolution (MOND, Sc; $t \sim 1.2$ Gyr). It corresponds to the variation of angular momentum over one galactic rotation normalized to $L(r)$. The gas loses 15-30% in region of the bar during a short period of 100 Myr.

the potential Fourier decomposition (see TC07). In the DM model, the bar pattern speed loses $5-10 \text{ km.s}^{-1}.\text{kpc}^{-1}$ after its formation, while it keeps constant in the MOND model (Fig. 6, left). In TC07, we attribute this slow down to dynamical friction effects. The slow down is less pronounced than in the previous work because the dark matter halo is less massive and the dynamical friction experienced by the stellar bar against the dark matter halo is less important.

Let us come back to the dominant difference between MOND and DM models, as far as dynamical friction is con-

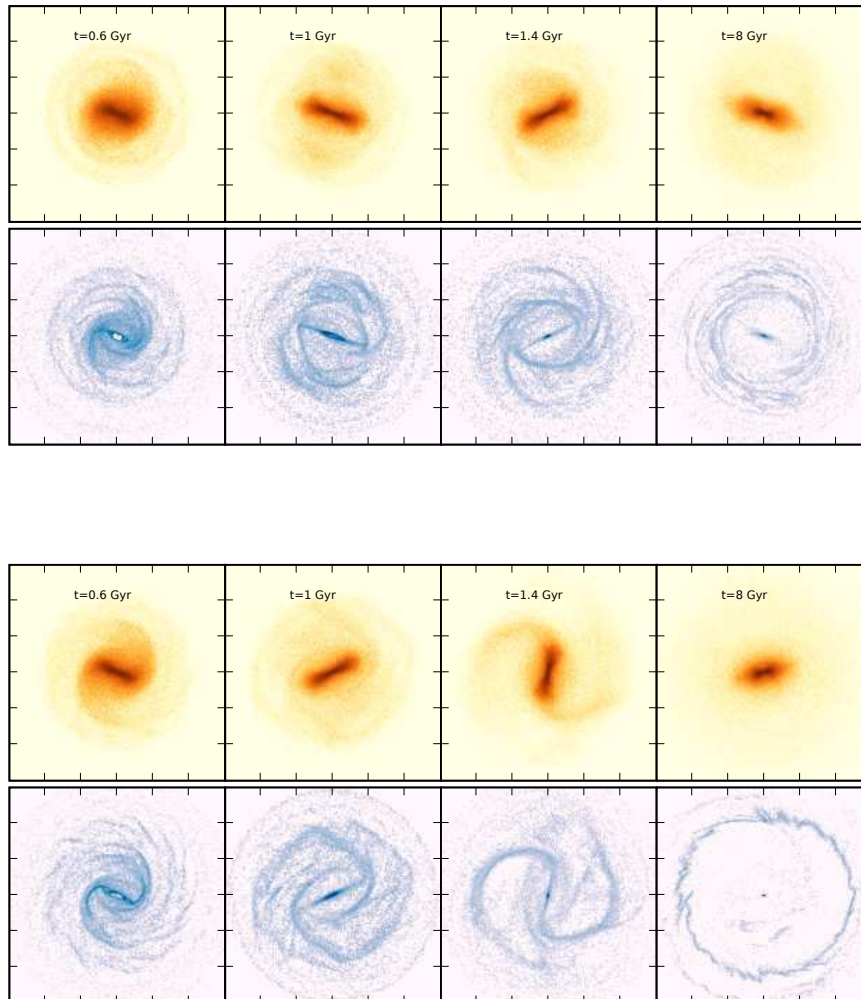


Fig. 3. Snapshots of an Sa type galaxy in the DM model (top) and in the MOND model (bottom). In each row, the stellar component is shown above, and the gas below, for times going from 0.6 to 8 Gyr. These evolutions show in DM and in MOND the bar instability, with the formation of grand design spiral arms, and the successive apparitions of ILR, UHR, and OLR rings (see text). The size of the box is 80 kpc \times 80 kpc.

cerned. A previous analysis by Ciotti and Binney (2004) has shown that, due to the much longer range of the MOND gravity, dynamical friction is more efficient. However, there is no dark matter particles to transfer angular momentum, and this reduces considerably the friction for MOND. Recently Nipoti et al. (2008) studied the slow down of a rigid bar, built as a small perturbation (a few percent in mass) in an otherwise stellar disc and find that the dynamical friction is more efficient in MOND than in DM. But again this does not reproduce the realistic case of a mas-

sive bar in an unstable disk. In MOND, the massive bar cannot transfer its angular momentum to halo particles, and the friction is then much lower, even negligible with respect to the DM bar model.

Fig. 3 shows different snapshots of an Sa-type galaxy, evolving in the DM and MOND model. The stellar and gaseous components have been separated. Resonant rings are easily identified in the gas disc. During the first gigayear, we notice the formation of an ILR (the small ring inside the bar). In this high density region, the gas

6 Tiret & Combes: Evolution of spiral galaxies in modified gravity: II- Gas dynamics

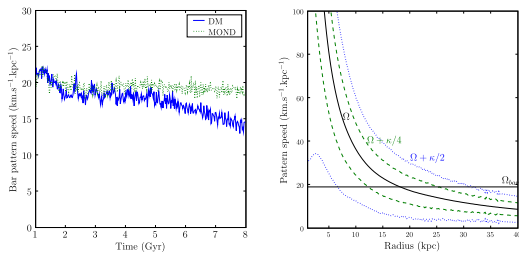


Fig. 6. Left: in the DM model, the stellar bar is submitted to the dynamical friction against the dark matter particles in the halo; the bar pattern speed slows down in the DM model and keeps constant in the MOND model. Right: Pattern speed and location of the Lindblad resonances in the MOND model at $t = 8$ Gyr.

particles collide frequently and lose angular momentum. Progressively, gas particles lose energy through collisions, and the ring migrates inward with time, while particles infall towards the center of the galaxy. This depends on many factors, such as the density of particles in the various radial zones, dominated by the $x1$ or $x2$ orbits, the mass concentration, and the relative strength of gravity torques and viscous torques (e.g. Schwarz 1984, Combes & Gerin 1985, Piner et al 1995, Regan & Teuben 2003). Around $t = 1 - 1.2$ Gyr, an other ring is forming at the UHR (the elongated ring surrounding the bar). After $t = 2$ Gyr this ring is diluted in the DM model; while in the MOND model transition spirals continue to appear and drag particles outward, until the OLR (at $t = 8$ Gyr). The figure 6 (right) represents the rotation curve $\Omega(r)$ and combination with the epicyclic frequency $\kappa(r)$. The OLR is located at 30 kpc, that is the intersection between Ω_{bar} and $\Omega + \kappa/2$.

This evolution in MOND is a good illustration of the bar formation and angular momentum transfer. The initial disc is not stable enough to avoid the bar instability. In the central region, particles which were on quasi circular orbits lose angular momentum, and run on elongated orbits. The total angular momentum has to be conserved so it is transferred to the external region of the disc (by the spiral arms). Then, particles accumulate either in the center or in the periphery (OLR) of the galaxy. In the simulation, the first resonance to appear in the ILR, then the UHR, and then the OLR. In a real galaxy, it is possible to have the three simultaneously, if accretion of external gas is taken into account, to replenish the gas disc. The non-existence of a massive halo in MOND allows particles to settle down on stable resonant orbits. The bar pattern speed keeps constant so the resonances remain at the same positions. Particles are trapped on these orbits more easily. Fig. 7 displays the formation of rings and pseudo rings in MOND simulations compared to the observations.

4.4. Vertical structure

The edge-on views of galaxies are plotted on fig. 8, in the MOND and DM models, comparing the simulations with gas and without gas. Those views correspond to the final snapshot of each simulation ($t = 8$ Gyr). For all the simula-

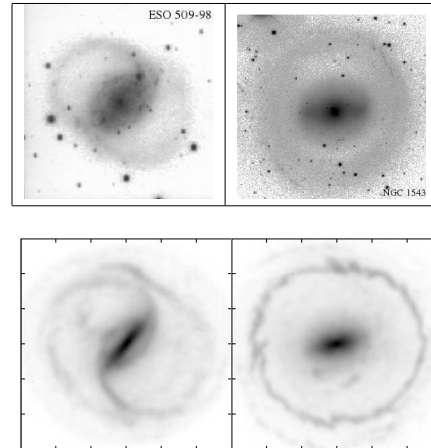


Fig. 7. Two examples showing the morphological structures of ESO 509-98 and NGC 1543 (top panel) compared to simulated galaxies in MOND (bottom panel). Rings and pseudo-rings structures are well reproduced with modified gravity. The size of the box in the simulation (bottom panel) is 80 kpc \times 80 kpc.

tions with gas (in the MOND and DM models) the peanut resonance is already well settled.

For the simulations without gas, all the DM-simulations and the MOND-Sa simulation clearly show the buckling, while the peanut looks like a boxy bar for the MOND-Sb and MOND-Sc. In the DM simulation the peanut is not symmetric because it is still forming, as the bar formation is slower (fig. 4). The Fig. 8 plots are the stellar component only. The gas responds to the peanuts resonance and forms a vertical structure too, but quite rapidly (1 Gyr) it comes back to the galactic plane by collisions. Moreover, by dissipation and angular momentum transfer during the bar formation, the gas peaks in the nucleus and creates a large concentration in the stellar component too. Then the bar pattern speed is higher in simulation with gas and the resonances (bar in the plane, peanut vertically) occur closer to the galactic center (fig. 10). It is why the models with gas appear less extended (like the MOND-Sc).

On fig. 9, the thickness is traced by the root mean square of the vertical position of particles inside 15 kpc. For the MOND-Sa, the buckling occurs around 4-5 Gyr, the mean thickness increases from 0.8 kpc to 1.4 kpc, as in the DM simulation, where it appears at 2 Gyr and thickens the disc from 0.6 kpc to 1.2 kpc. In the MOND simulation, the disc continues to heat, while yield a thick disc (fig. 8). In the DM simulation, the buckling is shifted outward in the disc while the bar slows down (fig.6), more and more particles get out from the galactic plane (after 6 Gyr for the DM-Sa galaxy).

For the MOND-Sc galaxy, the peanut seems to appears only in the gas model at 5 Gyr. The disc thickens from 0.8 kpc to 1. kpc. Without gas no sudden thickening is measured with this indicator but the galaxy shows a boxy shape along the long and short axis of the bar.

In the DM-Sc simulation, when the bar growth rate is slow, two episodes of buckling can be distinguish (Martinez-

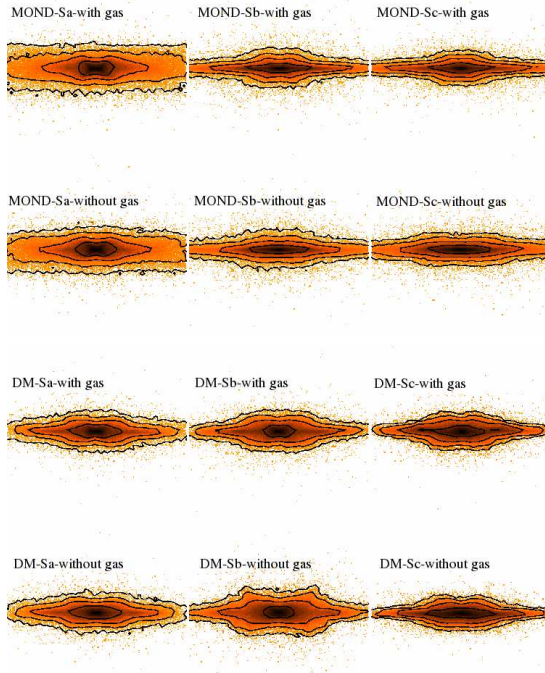


Fig. 8. Edge-on views of the component of galaxies after 8 Gyr, where the bar is perpendicular to the line of sight. The DM simulations show a peanut structure for the Sa and Sb models, with and without gas; the Sc model begins to bend at this time. For the MOND simulations, the Sa model forms a well-defined peanut as the Sb model with gas. The Sb and Sc models without gas are rather boxy. For each galaxy type, the contour levels are the same.

Valpuesta et al. 2006); one around 3 Gyr and the other at 7 Gyr, the disc thickens from 0.6 kpc, 0.8 kpc and 1.3 kpc.

In these simulations, it can be observed that the peanut resonance occurs sooner in the DM-model than in the MOND-model. On the average, it appears 2-3 Gyr after the beginning of the simulation in the DM model against 4-5 Gyr in the MOND model.

Bars are weakened during the buckling (Berentzen et al. 2007), but this is not the only way. The MOND-Sc simulation with gas does not have a strong peanut but the bar is weakened because the instability is too violent, during a short time: the bar strength increases to 0.4 in less than 1 Gyr. The orbits of particles have then no time to follow regular patterns, and chaos reduces the bar strength. Equivalent simulations have been performed in 2D and confirm that it is not an effect of the vertical motion (TC07)

Strong peanuts appear preferentially in early-type galaxies (in MOND as well in DM) while late-types give a boxy bar. This feature should be compared with observation (Lütticke et al. 2000), but within the statistical uncertainties, it is not possible yet to say whether some morphological type has more tendency to form a peanut or not.

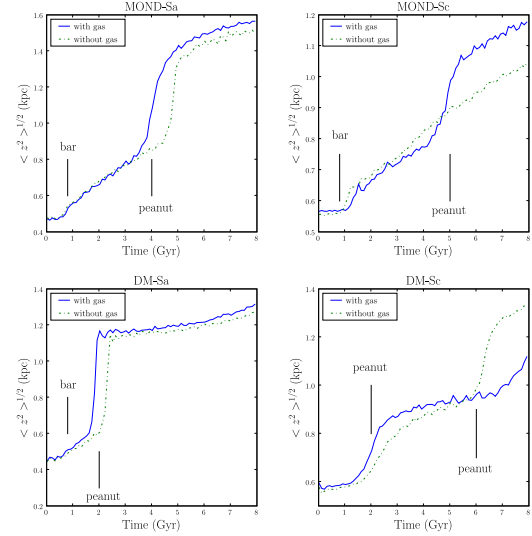


Fig. 9. Evolution of the stellar disc thickness, $\langle z^2 \rangle^{1/2}$, for the Sa and Sc galaxies, in the DM and MOND models. The formation of spiral arms and the bar instability heat the disc which thickens progressively. The steps correspond to the peanut resonance when the stellar particles leave suddenly the galactic plane.

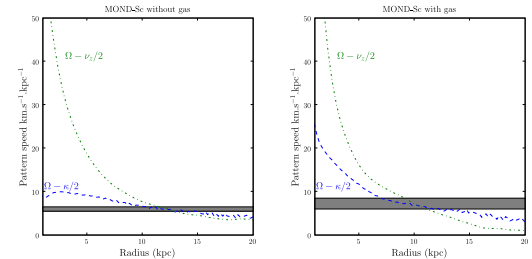


Fig. 10. Localisation of the peanut resonance for the MOND-Sc galaxy without gas (left) and with gas (right). With gas, the bar pattern speed is about $8 \text{ km.s}^{-1}.\text{kpc}^{-1}$ (the gray region marks the variation of the bar pattern speed during time, see text) so the peanut resonance occurs near 9 kpc, while it appears at 12 kpc without gas.

4.5. Statistics of bar frequency

Several groups have estimated bar frequency from observations (Block et al 2002, Laurikainen et al 2004, Buta et al 2005), and found a dearth of weak bars, with an extended wing of strong bars, in the bar strength histogram. In TC07, we estimated the bar frequency in a sample of pure stellar galaxy discs. The obtained bar frequency with MOND was more similar to the observed histogram, since strong bars are more frequent than in the DM model. Similar diagrams are now obtained using numerical simulations of galaxies including the gas behaviour and the star formation. We also run new simulations of galaxies without gas for the sake of

comparison with the new choice of the standard μ -function in MOND, and new dark matter halo distribution in DM.

Without gas, the new DM distributions have not changed the global result of the statistical study in the bar frequency for pure stellar galaxies. There are still less galaxies in MOND with a low bar strength and more galaxies which are strongly barred compared to DM. This can be understood in terms of the bar formation which is always faster in MOND than in Newton+DM in these types of galaxies.

Simulations show that the maximum of bar strength, obtained just after the first instability, corresponds to the largest growth rate of the bar, for the same galaxy and varying the Toomre parameter Q . The more unstable the galaxy is (for lower Q values) the faster the bar grows, and the higher is the maximum reached for the bar strength. When the galaxy is violently unstable, the bar is then weakened by feedback and chaotic effects (like the MOND-Sc one).

By adding gas, the bar frequency in the DM model increases, and tends to approach the MOND bar frequency. The bars were occurring so fast in MOND that the time scale of bar formation cannot be shortened, contrary to the Newtonian case. In DM + gas the proportion of low bar strength galaxies decreases because of the faster bar formation. In MOND with gas, the bar formation is not changed much due to saturation. In the bar strength evolution, we see (fig. 4) a slight deviation only for the Sc galaxy (with 7% of gas). The maximum of the bar strength, in DM and MOND, is the same with and without gas. So the part of the histogram with high bar strength does not change anymore. The difference appears in the middle of the distribution. Instead of having a plateau between 0.1 and 0.3 in bar strength, the distribution is peaked at 0.15, 0.20. In addition, bars are weakened or destroyed faster if there is gas because of gravity torques, and gas inflow.

To summarise, the low bar strength are constrained by the time-scale to form a bar, the strongly barred galaxies correspond to the capacity for a galaxy to have a high bar strength after the bar formation. And the middle of the distribution shows the evolution of the bars after their formations. Several factors concur to minimize the difference from previous results without gas: saturation effects in bar formation, bar weakening due to gas inflows.

5. Discussion and conclusion

In this paper, we present new simulations of isolated galaxies in both frameworks, of Newtonian gravity with dark matter and MOND, to compare their stability and bar frequency, and try to constrain either one by confrontation to observations. The present simulations now include the gas dissipation in the disc, and star formation. For the MOND model, the standard μ -function is used instead of the simple one (as in TC07). We will then discuss also the particular effects of varying μ -functions, in addition to the effect of the gas on galaxy evolution.

First, one remarkable result is that the galaxy evolution in MOND is not affected by the choice of the μ -function. Whatever the adopted interpolating function (standard or simple), a galaxy forms a bar in the same time-scale, and with the same bar strength. This hypothesis that the exact behaviour of μ -function is not important, but the asymptotic behavior is, was made by Milgrom and had not yet

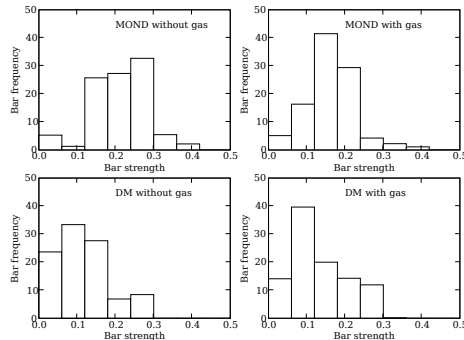


Fig. 11. Histograms of bar strength from the simulated galaxies in the MOND (top) and DM (bottom) models, without (left) and with gas (right).

been tested with numerical simulations. It could be interesting to extend this test with all the family of μ -functions existing. We have not done this comparison to reduce the number of parameters, and make the comparison with the Newtonian gravity more clear.

In our simulations, we constrain the initial galaxies to have the same rotation curve in both MOND and Newtonian dynamics. We also choose the same velocity dispersion in the discs for the two dynamics. If the choice of the standard or the simple μ -function does not affect the MOND model, it however changes significantly (by a factor 2) the mass of the “phantom” dark matter halo inside the visible radius, required in the Newtonian dynamics, which stabilizes the galaxy disc against bar formation. Since there is now less dark matter in the Newton models, the bar pattern speeds are no longer slowed down in large proportions, as was the case on our previous work (TC07). This is now more compatible with observations.

As expected, the gas makes the disc more unstable, decreasing the time-scale to form a bar especially in Newton+DM, where it was previously longer than in MOND. With gas, the histogram of bar frequency for the DM model tends to be more similar to the MOND one. Still galaxies in MOND are more often barred, and also more strongly barred. The bar frequency histogram is compatible with the data in both models, although the MOND results give a better fit.

The introduction of gas accelerates the bar formation, and at the same time the peanut formation. Contrary to what was found by Berentzen et al (2007), the introduction of gas does not damp the buckling instability, in the models considered here (both in MOND and DM). In MOND, without gas, the mass concentration in the Sc-model is lower in the center, and the position of the vertical instability occurs at a larger radius, leading to a less conspicuous box-peanut shape in the edge-on view.

The bar gravity torques drive the gas inwards inside corotation (e.g. Buta & Combes 1996), and outwards outside, producing gas accumulation in resonant rings. Young stars are formed in these rings, which are then conspicuous in optical images. We show that MOND can reproduce as well as Newtonian gravity these resonant structures, rings

and pseudo-rings, and inner and outer Linblad resonances, or ultra-harmonic resonance, near corotation.

This work constitutes a more realistic step from the pure stellar disc simulations, for isolated galaxies. Future simulations (Tiret & Combes 2007b, Tiret & Combes in prep.) will deal with galaxy interactions, and dynamical friction, where gas dissipation and star formation plays a significant role, and which are a challenge for MOND.

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References

- Athanassoula, E.: 2002, ApJ 569, L83
 Avila-Reese, V., Carrillo, A., Valenzuela, O., Klypin, A.: 2005, MNRAS, 361, 997
 Berentzen, I., Shlosman, I., Martinez-Valpuesta, I., Heller, C. H.: 2007 ApJ, 666, 189
 Brada, R., Milgrom, M.: 1999, ApJ 519, 590
 Block, D. L., Bournaud, F., Combes, F., Puerari, I., Buta, R.: 2002 A&A 394, L35
 Bournaud, F., Combes, F.: 2002, A&A 392, 83
 Bournaud, F., Combes, F., Semelin, B.: 2005, MNRAS 364, L18
 Buta, R. & Combes, F.: 1996, Fundamentals of Cosmic Physics, Volume 17, pp. 95-281
 Buta, R., Vasylyev, S., Salo, H., Laurikainen, E.: 2005 AJ 130, 506
 Ciotti, L., Binney, J.: 2004, MNRAS 351, 285
 Colin, P., Valenzuela, O., Klypin, A.: 2006, ApJ, 644, 687
 Combes, F., Gerin, M.: 1985, A&A 150, 327
 Debattista, V. P., Sellwood, J. A.: 1998, ApJ, 493, 5D
 Debattista, V. P., Sellwood, J. A.: 2000, ApJ, 543, 704
 Dubinski, J., Mihos, J.C., Hernquist, L.: 1996, ApJ 462, 576
 Hernquist, L., Weinberg, M. D.: 1992, ApJ 400, 80
 Laurikainen, E., Salo, H., Buta, R., Vasylyev, S.: 2004 MNRAS 355, 1251
 Little, B., Carlberg, R. G.: 1991, MNRAS 251, 227
 Lütticke R., Dettmar R.-J., Pohlen M.: 2000, A&AS, 145, 405
 Martinez-Valpuesta, I.; Shlosman, I.; Heller, C.: 2006 ApJ 637, s214M
 Mayer, L., Wadsley, J.: 2004, MNRAS, 347, 277
 Milgrom, M.: 1983, ApJ 270, 371
 Milgrom, M.: 1989, ApJ 338, 121
 Nipoti, C., Ciotti, L., Binney, J., Londrillo, P.: arXiv, 0802.1122
 O'Neill, J. K., Dubinski, J.: 2003, MNRAS, 346, 251
 Ostriker J., Peebles, P.J.E.: 1973, ApJ 186, 46
 Stone J.M., Teuben P.J.: 1995, ApJ 449, 508
 Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., Numerical Recipes in Fortran 77, second edition, Cambridge University Press, 1992
 Regan, M. W., Teuben, P.: 2003 ApJ 582, 723
 Sanders R.H., McGaugh S.S.: 2002, ARAA 40, 263
 Schwarz M.P.: 1984, MNRAS 209, 93
 Sellwood, J. A.; Debattista, V. P.: 2006 ApJ, 639, 868
 Tiret, O., Combes, F.: 2007a, A&A 464, 517, TC07
 Tiret, O., Combes, F.: 2007b, arXiv, 0712.1459
 Valenzuela, O., Klypin, A.: 2003, MNRAS, 345, 406
 Weinberg, M. D.: 1985, MNRAS, 213, 451
 Zhao, H.S., Famaey, B.: 2006, ApJ, 638, L9

4.3 Effet d'un champ extérieur (EFE)

Wu, Zhao, Famaey, Gentile, Tiret, Combes, Angus, Robin, 2007, ApJ, 665, 101

Résumé. Dans cet article, nous analysons la cinématique de galaxies en gravitation modifiée, lorsque celles-ci sont soumises à un champ de gravité extérieur constant. Nous repartons de l'analyse de Famaey et al. (2007) où ils estiment un champ extérieur de l'ordre de $a_0/100$ pour reproduire la vitesse d'échappement mesurée dans la galaxie. Ils utilisent une méthode analytique (approximation sphérique) pour calculer le champ de gravité MONDien. Ici, nous résolvons numériquement l'équation de Poisson modifiée, en présence d'un champ de gravité extérieur pour calculer le potentiel MONDien et déduire la vitesse d'échappement de la Voie Lactée. Ce calcul a pu être réalisé par les deux codes existants actuellement (Tiret & Combes 2007; Londrillo & Nipoti 2008) pour résoudre l'équation de Poisson modifiée. Les résultats sont identiques avec les deux codes et sont en accord avec les résultats Famaey et al. (2007). Un champ extérieur de l'ordre de $a_0/100$ créé par Andromède permet d'obtenir une vitesse d'échappement $v_{esc} \sim 544 \text{ km.s}^{-1}$. De plus, les simulations N-corps d'une galaxie en présence d'un champ extérieur constant ont aussi montré que le disque galactique est soumis à un couple gravitationnel qui fait précesser la galaxie, ce phénomène pourrait être intéressant à étudier dans les amas de galaxies où l'effet du champ extérieur vient se cumuler aux forces de marée usuelles. Les galaxies de faible masse sont plus vulnérables à cet effet MONDien et auraient tendance à être très perturbées (Brada & Milgrom 2000a).

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LOSS OF MASS AND STABILITY OF GALAXIES IN MODIFIED NEWTONIAN DYNAMICS

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ABSTRACT

The self-binding energy and stability of a galaxy in MOND-based gravity are curiously decreasing functions of its center-of-mass acceleration (of the order of 10^{-12} to 10^{-10} m s $^{-2}$) toward neighboring mass concentrations. A tentative indication of this breaking of the strong equivalence principle in field galaxies is the RAVE-observed escape speed in the Milky Way. Another consequence is that satellites of field galaxies will move on nearly Keplerian orbits at large radii (100–500 kpc), with a declining speed below the asymptotically constant naive MOND prediction. But the consequences of an environment-sensitive gravity are even more severe in clusters, where member galaxies accelerate fast; no dark halo-like potential is present to support galaxies, meaning that extended axisymmetric disks of gas and stars are likely unstable. These predicted reappearances of asymptotic Keplerian velocity curves and disappearances of “stereotypic galaxies” in clusters are falsifiable with targeted surveys.

Subject headings: dark matter — galaxies: kinematics and dynamics — galaxies: structure — gravitation

1. INTRODUCTION

The exact nature of dark matter is an outstanding puzzle despite our ability to carry out increasingly realistic simulations. So far, no astroparticle or gravitational theory can satisfactorily account for its various effects on both galactic and large scales (e.g., Zhao 2006).

In relatively isolated (field) galaxies, observations of a tight correlation between the mass profiles of baryonic matter and dark matter at all radii (McGaugh et al. 2007; Famaey et al. 2007c) are most often interpreted as supporting the modified Newtonian dynamics (MOND; Milgrom 1983; Bekenstein & Milgrom 1984). Indeed, without resorting to galactic dark matter, this simple prescription reproduces (to amazing consistency) the kinematics of galaxies over five decades in mass (e.g., Sanders & McGaugh 2002; Bekenstein 2006; Milgrom & Sanders 2003; Sanders & Noordermeer 2007; Nipoti et al. 2007; Famaey et al. 2007b; Gentile et al. 2007a, 2007b). The recently devised relativistic counterparts of MOND (Bekenstein 2004; Zlosnik et al. 2007) also enabled us to study the cosmic microwave background (Skordis et al. 2006) as well as the gravitational lensing of galaxies and galaxy clusters (e.g., Zhao et al. 2006; Chen & Zhao 2006; Angus et al. 2007; Famaey et al. 2007a). However, it is also important to observationally and theoretically examine how the internal properties of galaxies, like their rotation curve and morphology, should change with the environment.

Indeed, MOND-based theories generate different degrees of dark matter-like effects depending on the absolute acceleration. Like the Milky Way, most galaxies are in the field where they accelerate slowly with respect to the cosmic microwave back-

ground, typically at a rate of $0.01a_0$ to $0.03a_0$ (Famaey et al. 2007b; Milgrom 2002; Angus & McGaugh 2007). But in X-ray clusters, galaxies accelerate much faster, from $0.3a_0$ to $3a_0$ (Angus et al. 2007; Pointecouteau et al. 2005).

This external gravitational field has wider and more subtle implications for the internal system in MOND than in Newton-Einstein gravity, for the very reason that MOND breaks the strong equivalence principle. In particular, it is well known that MOND potentials are logarithmic for isolated distributions of finite mass and, consequently, infinitely deep but that the internal potential becomes “polarized Keplerian” at large distances (Bekenstein & Milgrom 1984; Zhao & Tian 2006) when an external field is applied.

In this contribution, we *numerically* solve the MOND Poisson equation for systems embedded in several different environments, ranging from the field to galaxy clusters, and show (1) that for the Milky Way (embedded in a weak gravitational field), the local escape speed is numerically compatible with the observations as analytically predicted in Famaey et al. (2007b), (2) that rotation curves of Milky Way-like galaxies would have a rapid Keplerian falloff when residing close to the center of clusters, while this falloff in field galaxies would happen at 100–500 kpc, and (3) that usual low surface brightness disks should not exist in MONDian clusters.

2. BINDING ENERGY OF AN ACCELERATING MILKY WAY

Galaxies free fall, but with slowly changing systematic (center-of-mass) velocity $v_{\text{com}}(t)$. Their present nonzero systematic velocity is mainly the accumulation of the acceleration by the gravity from neighboring galaxies over a Hubble time.

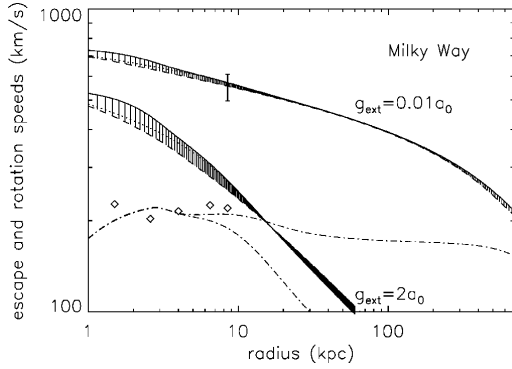


Fig. 1.—Model of a Milky Way–like galaxy in weak ($0.01a_0$) and strong ($2a_0$) external fields. Escape speeds in the disk plane for various field directions (solid and dotted curves) are compared with the error bar for the local escape speed measured from the RAVE survey (Smith et al. 2007). The predicted circular speed curves (dot-dashed curves) are also compared with data (diamonds; Caldwell & Ostriker 1981). Note that for an external field of $0.01a_0$, the transition radius between MOND and the Keplerian regime is at ~ 500 kpc, while for an external field of $0.03a_0$, it is at ~ 150 kpc.

Consider, as a first approximation, that a galaxy is stationary in a noninertial frame (in the Galilean sense), which free falls with a “uniform” systematic acceleration $\dot{\mathbf{v}}_{\text{com}} = g_{\text{ext}}\hat{X} = \text{const}$ due to an external linear potential, say, $-g_{\text{ext}}X$ along the X -direction, where the overdot means time derivatives. Let $\dot{\mathbf{v}}_{\text{int}} = (\dot{X}, \dot{Y}, \dot{Z})$ be the peculiar acceleration of a starlike test particle in the coordinates relative to the center of a nonevolving galaxy internal mass density $\rho(X, Y, Z)$; then

$$\dot{\mathbf{v}}_{\text{int}} = \mathbf{g} - \dot{\mathbf{v}}_{\text{com}} = -\nabla\Phi_{\text{int}}(X, Y, Z), \quad (1)$$

where \mathbf{g} is the absolute acceleration satisfying the MOND Poisson equation

$$-\nabla \cdot [\mu(x)\mathbf{g}] = 4\pi G\rho(X, Y, Z), \quad x \equiv \frac{|\mathbf{g}|}{a_0}. \quad (2)$$

One can define an “effective potential” as $\Phi_{\text{int}}(X, Y, Z)$ (called “internal” potential) and an “effective energy” as $E_{\text{eff}} = v_{\text{int}}^2/2 + \Phi_{\text{int}}(X, Y, Z)$, where E_{eff} is conserved along the orbit of the test particle that is effectively moving in a force field $-\nabla\Phi_{\text{int}}(X, Y, Z)$, which is curl-free and time-independent because the absolute gravity \mathbf{g} is curl-free, the center-of-mass acceleration $\dot{\mathbf{v}}_{\text{com}}$ is assumed a constant, and the galaxy density $\rho(X, Y, Z)$ is assumed time-independent.

Far away from the center of the free-falling system, we have $|\dot{\mathbf{v}}_{\text{int}}| \ll |\dot{\mathbf{v}}_{\text{com}}|$; hence, $\mu \rightarrow \mu_m \equiv \mu(\dot{\mathbf{v}}_{\text{com}}/a_0) = \text{const}$, and the equation reads (Bekenstein & Milgrom 1984; Milgrom 1986; Zhao & Tian 2006; Zhao & Famaey 2006)

$$\nabla^2\Phi_{\text{int}} + \Delta \frac{\partial^2}{\partial X^2}\Phi_{\text{int}} \rightarrow 4\pi G\rho/\mu_m, \quad (3)$$

where Y, Z denote the directions perpendicular to the external field X -direction and $\Delta = [d \ln \mu / d \ln x]_{x=|\dot{\mathbf{v}}_{\text{com}}|/a_0}$ is a dilation factor (note that $1 \leq 1 + \Delta \leq 2$). So, at large radii, where the external field dominates and the equation is linearizable, the

potential satisfies a mildly anisotropic Poisson equation, and the solution at large radii¹ goes to

$$\Phi_{\text{int}}^\infty(X, Y, Z) = -\frac{GM_{\text{int}}}{\mu_m \sqrt{(1 + \Delta)(Y^2 + Z^2) + X^2 + s^2}}, \quad (4)$$

where we included a softening radius s , comparable to the half-light radius of a galaxy. Hence, the internal potential Φ_{int} is finite and approaches zero at large radii.

The escape speed at any location \mathbf{r} in the system can then be meaningfully defined by

$$0 = E_{\text{eff}} = \frac{v_{\text{esc}}^2(X, Y, Z)}{2} + \Phi_{\text{int}}(X, Y, Z). \quad (5)$$

Such an escape speed is a scalar that is independent of “the path to escape” because the “effective energy” E_{eff} is conserved, and a particle with E_{eff} equaling zero (the maximum value of Φ_{int}) will reach an infinite distance from the system and never return; hence, it will be lost into the MOND potential of the background (from which it cannot escape). However, equal escape speed contours across a disk galaxy are generally not axisymmetric, meaning that the escape speeds differ on opposite symmetric locations of the Galaxy.

Hereafter, we numerically solve equation (2) using the MOND Poisson solver developed by the Bologna group (Ciotti et al. 2006); the results based on spherical grids are also confirmed with the Cartesian grid-based code of the Paris group (Tiret & Combes 2007), with very different spatial resolutions. We program in the mass density of the internal system, solving the MOND Poisson equation as if it were isolated, except for requiring a boundary condition on the total gravity as $-\mathbf{g} \rightarrow g_{\text{ext}}\hat{X} - \nabla\Phi_{\text{int}}^\infty(X, Y, Z)$ on the last grid point (X, Y, Z) . Note, finally, that in our models hereafter, we use the parametric μ -function $\mu(x) = x/(1+x)$, which fits well the rotation curve of the Milky Way (Famaey & Binney 2005) as well as external galaxies (Famaey et al. 2007c; Sanders & Noordermeer 2007).

We use the Besançon Milky Way model (Robin et al. 2003) to simulate high surface brightness (HSB) galaxies. This model is a realistic representation of the Galaxy, explaining currently available observations of different types (photometry, astrometry, spectroscopy) at different wavelengths. The stellar populations included in the model are as follows: a thin disk made of seven isothermal layers, each having a different age between 0.1 and 10 Gyr; a 11 Gyr old thick disk with a modified exponential density law; a spheroid with a power-law density that is slightly flattened; and a prolate old bulge modeled by a triaxial density law. We logically remove the dark matter halo in our simulations. We then apply the MOND Poisson solver using $512 \times 64 \times 128$ grid points, where the grid points in the radial direction are chosen as $r_i = 50.0 \tan \{(i + 0.5)[0.5\pi/(512 + 1)]\}$ kpc.

As a first application, the Radial Velocity Experiment (RAVE) solar neighborhood escape speed 544_{-46}^{+64} km s⁻¹ is well reproduced by our fully numerical model galaxy (see Fig. 1 for a typical external field of $0.01a_0$), as analytically anticipated in Famaey et al. (2007b). When the direction of the external gravity changes, the escape speed changes in a narrow range [545, 558] km s⁻¹ in the solar neighborhood. Moreover, we point out that satellites of Milky Way–like galaxies will move on nearly Kep-

¹ Throughout the Letter, by “large radii” we mean a distance large enough to neglect the internal field $g_{\text{int}} \ll \dot{v}_{\text{com}} = g_{\text{ext}}$ but small enough that the external field g_{ext} can be treated as a constant.

lerian orbits at very large radii (>100 – 500 kpc, depending on the external field strength), with a speed much below the asymptotically flat naive MOND prediction.

We also model NGC 1560, a benchmark low surface brightness (LSB) disk galaxy (Broeils 1992). We use an exponential stellar disk of $1.97 \times 10^8 M_{\odot}$ and a multi-Gaussian gaseous component of $1.07 \times 10^9 M_{\odot}$ to match the observed baryon distribution (Broeils 1992). A MOND Poisson solver (Ciotti et al. 2006) with $256 \times 64 \times 64$ grid points is applied, the radial grid being $r_i = 10.0 \tan \{(i + 0.5)[0.5\pi/(256 + 1)]\}$ kpc. Again, it is found that an acceleration of $0.01a_0$ is compatible with the rotation curve of NGC 1560, which is an isolated galaxy.

3. FAST-ACCELERATING GALAXIES IN CLUSTERS

Now, let us consider boosting the Milky Way's systematic acceleration *suddenly* to match the environment in a galaxy cluster. Figure 1 shows that, for an external field of $2a_0$, the escape speed of stars is much reduced, falling Keplerian-like $300 \times (5 \text{ kpc}/r)^{1/2} \text{ km s}^{-1}$ outside 5 kpc, where half of the stars and gas of the Milky Way are located. All of the dwarf satellites of the Milky Way and the outer disk rotating with 200 km s^{-1} would then barely be kept from flying away.

In fact, the instantaneous hypothetical circular speed must also be lowered by the sudden boost of acceleration, and the outer galaxy (>5 kpc) should exhibit a Keplerian falling rotation curve (Fig. 1). Outer disk stars and gas should enter elliptical or parabolic orbits of the same angular momentum, if allowed to respond to a suddenly reduced gravity, and precess with a preferred direction of instantaneous systematic acceleration that thickens the disk. In any case, observing asymptotically flat rotation curves for purely baryonic Milky Way-like galaxies residing in such environments would falsify MOND. A galaxy with the same luminosity would have lower velocities, consistent with the trend observed with the cluster Tully-Fisher relation (Sanders & McGaugh 2002).

Now let us consider suddenly boosting the acceleration of our benchmark LSB galaxy to $\dot{v}_{\text{com}} = 0.3a_0$ or $\dot{v}_{\text{com}} = 2a_0$ (typical of outer and inner parts of galaxy clusters; the real orbit of a member galaxy would pass both regions at apocenter and pericenter, respectively). The circular speed (Fig. 2, *lowest dot-dashed curve*) of a fast-accelerating LSB is very much reduced. All previous disrupting effects are even more severe on an LSB galaxy, with the escape speed falling as low as 50 km s^{-1} . Outer stars with original circular speeds of 50 – 80 km s^{-1} would enter parabolic orbits, and inner stars would move outward on severely elongated nonplanar orbits. Actually, the dynamics resembles a purely Newtonian disk without a round stabilizing dark halo, meaning that the galaxy would become extremely bar-unstable (Mihos et al. 1997). Such an LSB would lose its MOND support and would be subject to strong distortions, even before the traditional tidal effect becomes important.

4. CONCLUSION AND DISCUSSION

The external field effect is a generic prediction of the modified gravity theories, in which the modification is acceleration-based and violates the strong equivalence principle, e.g., the relativistic versions of the “aquadratic Lagrangian” (AQUAL) theory of Bekenstein & Milgrom (1984). The effect is helpful in that it allows for high-velocity stars (1) to escape from Milky Way-like field galaxies and (2) to decrease the orbital velocities of their satellite galaxies at very large radii (100 – 500 kpc), contrary to the naive expectation of MOND without the external field effect, i.e., that rotation curves should be asymptotically flat.

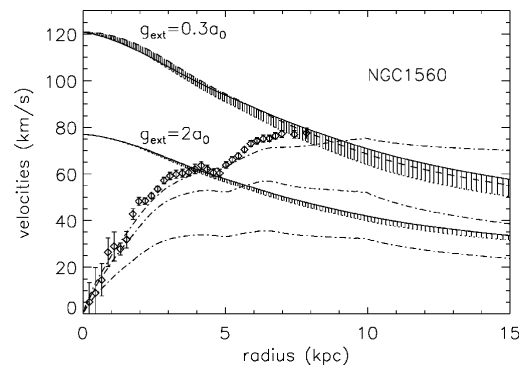


FIG. 2.—Similar to Fig. 1, but for models of a NGC 1560-like LSB galaxy. Circular speed curves for $g_{\text{ext}} = 0$ (no escape), $g_{\text{ext}} = 0.3a_0$ (middle), and $g_{\text{ext}} = 2a_0$ (bottom) are compared with the observed rotation curve of NGC 1560.

totically flat. In this respect, the data of Klypin & Prada (2007) may be useful in the future for identifying the effect.

On the other hand, the internal dynamical structure of a field galaxy would be transformed when entering a cluster. Classical relations of field galaxies (such as the Tully-Fisher relation), the galaxy luminosity functions, the Hubble-type distribution, etc., are expected to be strongly modified in clusters. The effects are most destructive for classical LSB galaxies; curiously, their field counterparts have been a legendary success for MOND in terms of well-fitted rotation curves.

Thus, we argue that it would be extremely valuable to analyze the kinematics of a *sample* of HSB galaxies and to search for LSB galaxies in nearby clusters using deep H I surveys. The study of a sample of galaxies would be needed because of the uncertainty of the determination of the real distance (as opposed to the projected distance) of a galaxy from the cluster center. An obvious difficulty would be that cluster galaxies are H I-deficient (Solanes et al. 2001). An example of such an H I database is the VIVA (VLA Imaging of Virgo in Atomic Gas; Chung et al. 2007) survey. We also predict that a future detection of any undistorted HSB late-type disk galaxy near the center of a galaxy cluster would be extremely surprising in the context of MOND. A null detection of thin LSB disks is predicted in clusters because they most probably have been turned into gas-poor dwarf ellipticals if not fully disrupted. Low surface density gas in galaxies also suffers ram pressure stripping while moving in gaseous clusters; gas is easily stripped in the reduced MONDian internal gravity, further reducing the available mass for self-gravity.

Surely similar effects occur in the context of cored dark halos. Some simulations show that LSB disks and dwarf irregulars get harassed (Moore 1999) and transformed into dwarf ellipticals or ultracompact dwarf ellipticals (Evstigneeva et al. 2007; Cortese et al. 2007) in the densest part of the cluster, coinciding with the region where the external field is the highest. To our knowledge, the properties of cluster disk galaxies (such as their Tully-Fisher relation) have not been extensively simulated. An important assumption for existing simulations is that a large core for the cold dark matter (CDM) halo of the LSB exists; the harassment becomes much less effective if the cluster member LSB has started with a dense CDM cusp density (L. Mayer 2007, private communications).

As for the gas-poor nonrotating dwarf spheroidals (e.g., Sex-

tans), they are expected to have a central CDM density of $\sim 0.1 M_{\odot} \text{pc}^{-3}$, a factor of 100 denser than the galaxy cluster; hence, they might survive the tidal harassment in the CDM. If in MOND a spheroidal object of $M = 2 \times 10^5 M_{\odot}$ and a half-mass radius of $s \sim 500 \text{pc}$ is suddenly introduced into a galaxy cluster, it would have a central binding energy of only $\sim (GM/s)(1 + \Delta)^{-2/3} \mu_m^{-1} \sim (5 \text{ km s}^{-1})^2$, much less than its initial internal random motion energy of $\frac{1}{2} \times 3 \times (10 \text{ km s}^{-1})^2$; hence, it is quickly dispersed (perhaps anisotropically).² In short, any discovery of a sample of classical LSB galaxies in clusters would favor a cuspy CDM and would falsify MOND or cored dark halos.

The tidal harassment effect exists in MOND as well; cluster galaxies suffer from tides in addition to the unique destructive effect of the external field. An even more curious distortion to the MONDian LSB or HSB disk happens when the disk is

² For similar reasons, open clusters in the solar neighborhood are predicted to be internally unbound in MOND.

misaligned by an angle θ_m with the instantaneous direction of the external field, which generally changes amplitude and direction along the orbit of an LSB on timescales of 0.2–1 Gyr. The elliptical potential of equation (4) creates a differential force with a component normal to the disk, hence a specific torque $-\mathbf{r} \times \nabla\Phi$. This causes differential precession of the disk angular momentum vector with an angular speed proportional to $\mu_m^{-1} (GM/r^3)^{1/2} \Delta \sin(2\theta_m)$; an LSB disk is likely shredded by one precession, and an HSB disk is thickened. The *precession, asymmetric dilation, and reduction of inner circular velocity curves* (e.g., Fig. 2) are confirmed by N -body simulations using the code of the Paris group (Tiret & Combes 2007) in MOND, but they are generally forbidden by Newtonian laws in the dark matter framework.

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REFERENCES

- Angus, G. W., & McGaugh, S. S. 2007, MNRAS, submitted (arXiv:0704.0381)
 Angus, G. W., Shan, H. Y., Zhao, H. S., & Famaey, B. 2007, ApJ, 654, L13
 Bekenstein, J. D. 2004, Phys. Rev. D, 70, 083509
 ———. 2006, Contemp. Phys., 47, 387
 Bekenstein, J. D., & Milgrom, M. 1984, ApJ, 286, 7
 Broeils, A. H. 1992, A&A, 256, 19
 Caldwell, J. A. R., & Ostriker, J. P. 1981, ApJ, 251, 61
 Chen, D. M., & Zhao, H. S. 2006, ApJ, 650, L9
 Chung, A., van Gorkom, J. H., Kenney, J. D. P., & Vollmer, B. 2007, ApJ, 659, L115
 Ciotti, L., Londrillo, P., & Nipoti, C. 2006, ApJ, 640, 741
 Cortese, L., et al. 2007, MNRAS, 376, 157
 Evstigneeva, E. A., Gregg, M. D., Drinkwater, M. J., & Hilker, M. 2007, AJ, 133, 1722
 Famaey, B., Angus, G. W., Gentile, G., & Zhao, H. S. 2007a, A&A, submitted (arXiv:0706.1279)
 Famaey, B., & Binney, J. 2005, MNRAS, 363, 603
 Famaey, B., Bruneton, J. P., & Zhao, H. S. 2007b, MNRAS, 377, L79
 Famaey, B., Gentile, G., Bruneton, J.-P., & Zhao, H. 2007c, Phys. Rev. D, 75, 063002
 Gentile, G., Famaey, B., Combes, F., Kroupa, P., Zhao, H. S., & Tiret, O. 2007a, A&A, submitted (arXiv:0706.1976)
 Gentile, G., Salucci, P., Klein, U., & Granato, G. L. 2007b, MNRAS, 375, 199
 Klypin, A., & Prada, F. 2007, preprint (arXiv:0706.3554)
 McGaugh, S. S., de Blok, W. J. G., Schombert, J. M., Kuzio de Naray, R., & Kim, J. H. 2007, ApJ, 659, 149
 Mihos, J. C., McGaugh, S. S., & de Blok, W. J. G. 1997, ApJ, 477, L79
 Milgrom, M. 1983, ApJ, 270, 365
 ———. 1986, ApJ, 302, 617
 ———. 2002, ApJ, 577, L75
 Milgrom, M., & Sanders, R. H. 2003, ApJ, 599, L25
 Moore, B. 1999, Philos. Trans. R. Soc. London, A357, 1763
 Nipoti, C., Londrillo, P., Zhao, H. S., & Ciotti, L. 2007, MNRAS, 379, 597
 Pointecouteau, E., Arnaud, M., & Pratt, G. W. 2005, A&A, 435, 1
 Robin, A. C., Reyle, C., Derriere, S., & Picaud, S. 2003, A&A, 409, 523
 Sanders, R. H., & McGaugh, S. S. 2002, ARA&A, 40, 263
 Sanders, R. H., & Noordermeer, E. 2007, MNRAS, 379, 702
 Skordis, C., Mota, D. F., Ferreira, P. G., & Boehm, C. 2006, Phys. Rev. Lett., 96, 1301
 Smith, M. C., et al. 2007, MNRAS, 379, 755
 Solanes, J. M., Manrique, A., García-Gómez, C., González-Casado, G., Giovanelli, R., & Haynes, M. P. 2001, ApJ, 548, 97
 Tiret, O., & Combes, F. 2007, A&A, 464, 517
 Zhao, H. 2006, preprint (astro-ph/0610056)
 Zhao, H., Bacon, D. J., Taylor, A. N., & Horne, K. 2006, MNRAS, 368, 171
 Zhao, H. S., & Famaey, B. 2006, ApJ, 638, L9
 Zhao, H., & Tian, L. 2006, A&A, 450, 1005
 Zlosnik, T. G., Ferreira, P. G., & Starkman, G. D. 2007, Phys. Rev. D, 75, 044017

4.4 Interaction entre galaxies : Les Antennes

(Tiret & Combes, en préparation)

4.4.1 Présentation

Après m'être intéressé à l'évolution de galaxies isolées en comparant la dynamique newtonienne avec matière noire et MOND, il était naturel d'étendre cette étude sur de plus grandes échelles : les galaxies en interactions. Les simulations présentées dans cette section portent sur des interactions entre galaxies de masse comparable, les *major mergers*.

Le système des *Antennes* (Fig. : 4.1) est l'illustration typique de galaxies en interaction. Observé dans plusieurs domaines de longueurs d'ondes (Whitmore & Schweizer 1995; Hibbard et al. 2001; Gordon et al. 2001; Wang et al. 2004), ce système a la particularité de présenter deux queues de marée symétriques, qui s'étendent sur plusieurs dizaines de kiloparsecs. Cette configuration caractéristique en fait un modèle d'étude de référence.

Il a été simulé à plusieurs reprises en gravitation newtonienne par Toomre & Toomre (1972), Barnes (1988), Mihos et al. (1993), apportant des indications sur les mécanismes de fusion (échelle de temps, orbites, ...), ou encore sur la morphologie des queues de marée (Dubinski et al. 1996) pour laquelle la concentration des halos de matière noire est contraignante.

En gravitation newtonienne, le halo de matière noire joue un rôle majeur lors de l'interaction des galaxies, c'est un réservoir d'énergie. Avant la fusion chaque galaxie est en rotation dans son halo. Celui-ci est maintenu à l'équilibre du fait du mouvement désordonné des particules de matière noire, elles n'ont pas de mouvement d'ensemble. Au cours de l'interaction, le disque d'étoiles et de gaz des galaxies subit les effets de friction dynamique des particules de matière noire. Le moment orbital des galaxies est alors transféré en grande partie en moment angulaire dans le halo de matière noire. Ce processus permet de réduire la distance séparant les galaxies, qui finissent par fusionner lorsque le moment orbital est nul. Au travers de ce mécanisme de friction dynamique, la matière noire permet des temps de fusion courts (de l'ordre d'un milliard d'années), mais les galaxies semblent perdre trop de moment angulaire (les disques formés dans les simulations cosmologiques sont trop petits) au profit du halo de matière noire qui prend un léger mouvement de rotation, ou qui augmente de taille.

Qu'en est-il avec MOND ? Les galaxies ne sont plus immergées dans un halo de matière. La friction dynamique est-elle quand même suffisante pour obtenir la fusion des deux galaxies ? Ciotti & Binney (2004) trouvent que les forces de friction dyna-

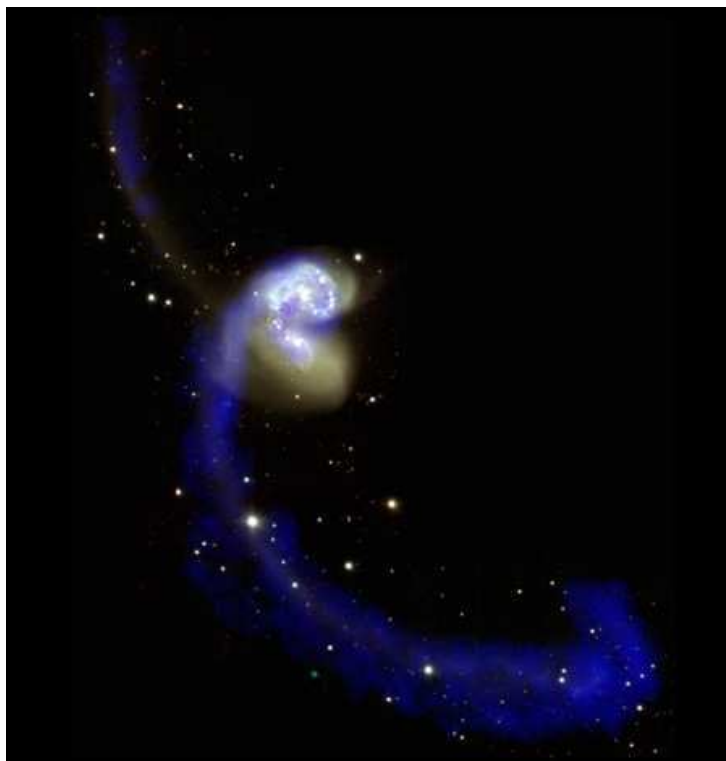


FIGURE 4.1 – Les Antennes observées dans le visible (étoiles) et en radio (gaz HI), Hibbard et al. (2001).

miques ressenties par une particule au sein d'un milieu homogène sont plus fortes sous MOND qu'en gravité newtonienne. Le cas complexe de galaxies en interaction nécessite la réalisation de simulations numériques N-Corps. Aucun modèle n'avait encore traité ce problème d'interaction qui apparaissait comme un défi pour MOND.

4.4.2 Méthodes

La boîte de simulation recouvre un cube de 300 kpc de côté. La résolution de l'équation de Poisson (newtonienne ou modifiée) :

$$\begin{aligned} \Delta\phi &= 4\pi G\rho \\ \vec{\nabla} \cdot \left[\mu \left(\frac{|\vec{\nabla}\phi|}{a_0} \right) \vec{\nabla}\phi \right] &= 4\pi G\rho \end{aligned}$$

est réalisée sur une grille adaptative (Sect.3.3.2). La valeur de l'accélération critique de MOND est : $a_0 = 1.2 \cdot 10^{-10} \text{ m.s}^{-2}$; la fonction d'interpolation correspond à la fonction standard, $\mu(x) = x/\sqrt{1+x^2}$.

Le plus haut niveau de raffinement équivaut à une grille régulière de 1024^3 noeuds. Cela permet d'atteindre une résolution spatiale de 290 pc au niveau des galaxies.

Run	M_d	M_b	Gas	a_d	a_g	M_h	b_h
Galaxie 1	30	5	5%	5	15	118.9	24.8
Galaxie 2	30	5	0%	5	-	118.9	24.8

TABLE 4.1 – Paramètres des galaxies. Le système d’unité correspond à $G = 1$: l’unité de masse vaut $U_m = 2.26 \cdot 10^9 M_\odot$, et l’unité de longueur correspond à $U_r = 1.02 \text{ kpc}$.

L’évolution de l’interaction est simulée pendant 6 milliards d’années, le pas de temps d’intégration de l’équation du mouvement est de 1 million d’années.

Les galaxies comportent un disque d’étoiles et de gaz. La dissipation du gaz est modélisée par l’algorithme des *sticky-particles*. Les grandeurs caractéristiques des galaxies sont indiquées dans le Tab. 4.1. Au début de la simulation, les galaxies sont distantes de 100 kpc et ont une vitesse d’ensemble tangentielle de $+30 \text{ km.s}^{-1}$ et -30 km.s^{-1} chacune. Les galaxies initiales ont évolué pendant 1 milliard d’années de façon isolée avant d’interagir ; à $t=0$ elle présente déjà une structure spirale et une barre.

4.4.3 Résultats

Morphologie & cinématique. Grâce à ces simulations, nous montrons que MOND permet aussi de reproduire le système des Antennes. Comme on peut le voir sur la Fig. 4.2 (droite), la morphologie des queues de marée obtenue en gravitation modifiée est en accord avec les observations ; il s’agit d’une carte de densité des étoiles (en jaune) superposée au gaz (en bleu) à $t=1.8 \text{ Gyr}$. L’extension des queues de marée devient maximale juste après le deuxième passage au périégée. Les deux galaxies finissent par fusionner à $t=2.5 \text{ Gyr}$ (Fig. 4.4).

Les données radio de Hibbard et al. (2001) permettent de contraindre encore plus la modélisation des Antennes en comparant les cartes de vitesse et de dispersion de vitesse. La figure (fig. 4.3) montre cette comparaison entre les observations HI de la vitesse du gaz, en se plaçant du même point de vue que pour la carte de densité.

Friction dynamique. Bien que MOND reproduise la morphologie et la cinématique des Antennes comme cela était déjà le cas en gravité newtonienne avec matière noire, l’évolution du même système peut être très différente d’un modèle à l’autre. La Fig. 4.4 montre l’évolution de la distance intergalactique avec le temps pour différentes distances de premier passage au périégée. Pour les quatre trajectoires simulées la vitesse radiale est nulle à $t=0$, mais la vitesse tangentielle relative varie entre 20 km.s^{-1} et 80 km.s^{-1} , par pas de 20 km.s^{-1} . En gravitation newtonienne, quelle que soit la vitesse initiale, l’évolution est très vite dominée par les effets de friction dy-

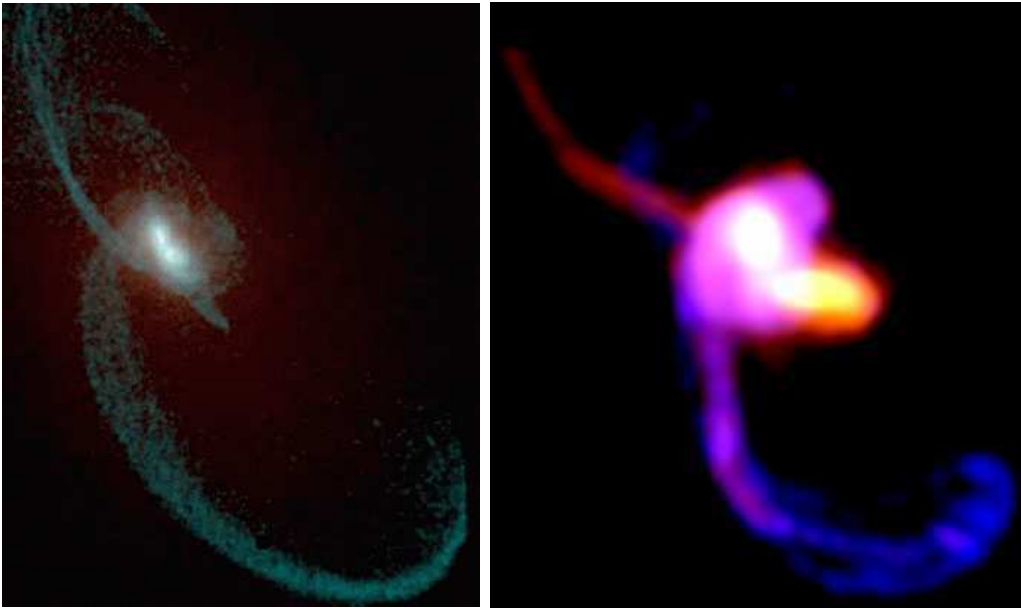


FIGURE 4.2 – Simulation de Antennes en gravité newtonienne avec matière noire (gauche, Barnes (1988)), et avec MOND (droite).

namique du halo de matière noire (effet de sillage). Dans ces simulations, les temps de fusion sont équivalents quelle que soit la vitesse initiale.

Avec MOND la situation est complètement différente. Tant que les galaxies ne se sont pas croisées il n’y a que la friction dynamique à distance qui déforme les galaxies et leur fait perdre du moment orbital. Ce processus est moins efficace que la friction dynamique due à la matière noire en gravitation newtonienne. Pour les mêmes conditions initiales que dans la simulation newtonienne le temps de fusion peut être doublé en MOND (pour $\Delta v = 80 \text{ km.s}^{-1}$). Si bien que deux galaxies initialement distantes de 100 kpc, qui partent sur des orbites circulaires, ne fusionneront pas en un temps de Hubble. Il faut que les disques de galaxies se rencontrent pour fusionner rapidement. Par contre, une collision de plein fouet aura la même échelle de temps quel que soit le modèle.

Nipoti et al. (2007c) ont réalisé des simulations de systèmes sphériques (purement stellaires) en interaction et trouvent aussi que les échelles de temps de fusion sont plus longues avec MOND qu’en gravité newtonienne avec matière noire.

Moment angulaire. En gravité modifiée, tout le moment orbital des deux galaxies initiales est converti en moment angulaire dans la galaxie finale issue de la fusion. On s’attend donc à obtenir, avec MOND, des galaxies comportant un plus grand moment angulaire total qu’en gravitation newtonienne, où le moment orbital est aussi transféré en moment angulaire dans le halo de matière noire. Cela peut se traduire par une galaxie plus étendue dans l’espace, où les étoiles et le gaz ne se

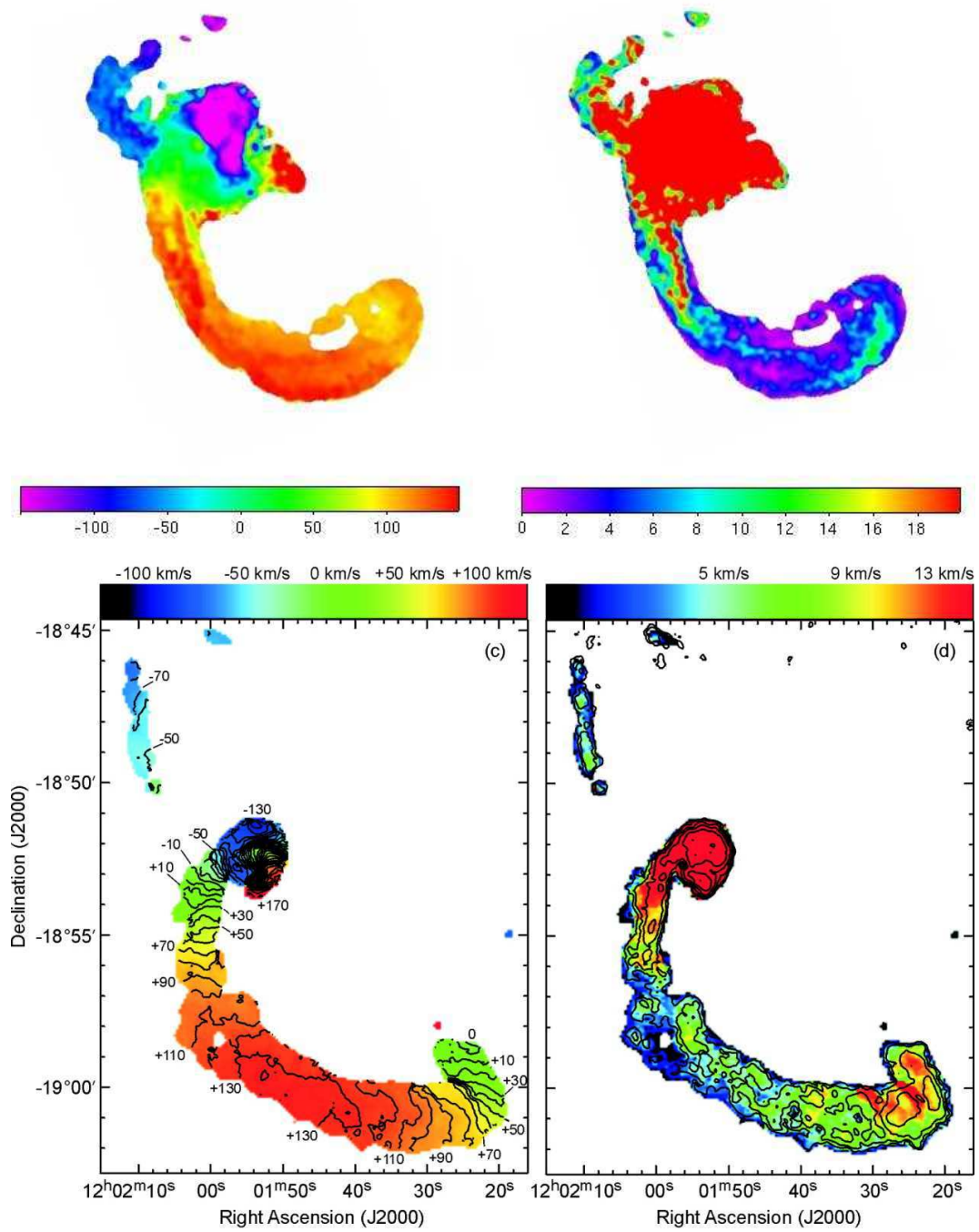


FIGURE 4.3 – Comparaison des cartes de vitesses radiales (gauche) et de dispersion de vitesse (droite) modélisées par MOND (haut) et observées (bas).

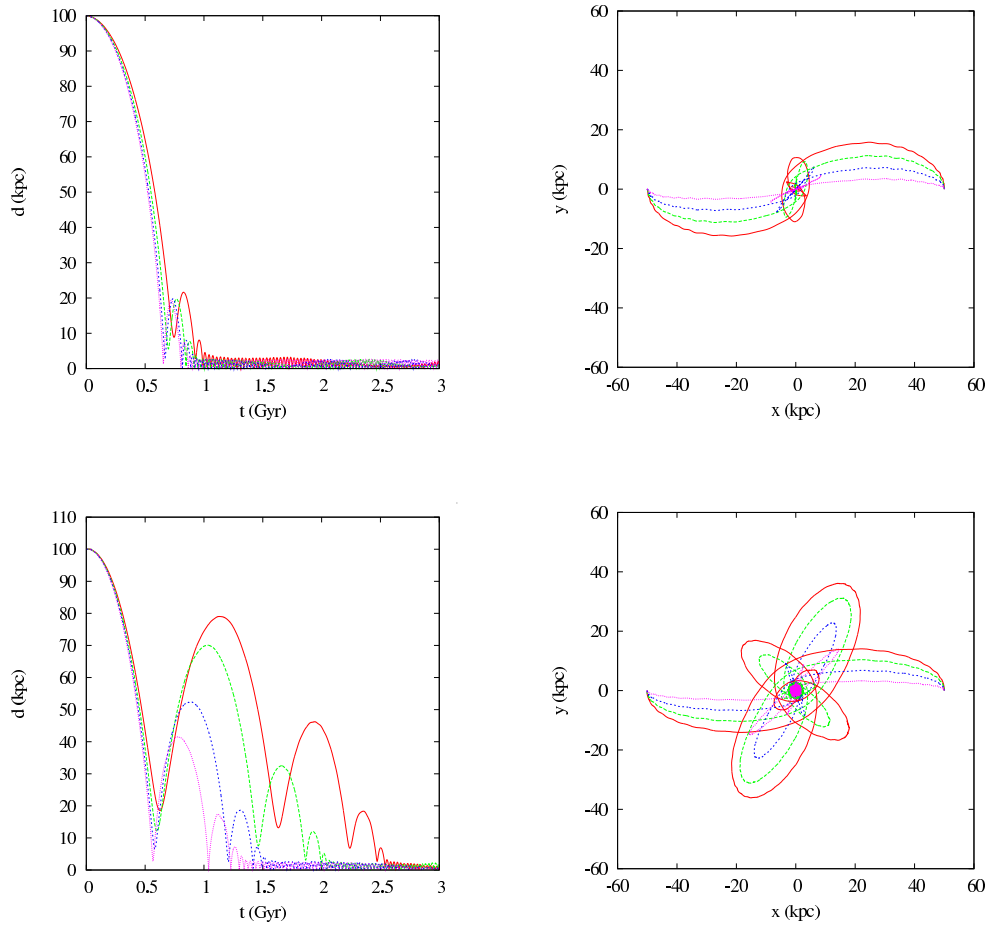


FIGURE 4.4 – Comparaison des effets de friction dynamique entre la gravité newtonienne avec matière noire (haut) et MOND (bas) pour différentes trajectoires.

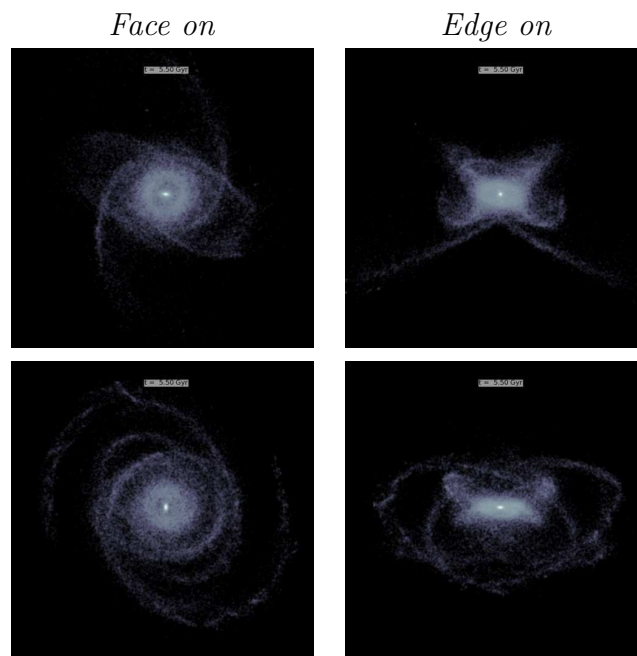


FIGURE 4.5 – Reformation d’un disque de gaz après la coalescence en gravitation newtonienne avec matière noire (haut) et en gravité modifiée (bas).

sont pas effondrés dans le bulbe. La Fig. 4.5 montre une vue de face et de profil du résultat de la fusion dans les deux modèles de gravité ; les conditions initiales étaient identiques ($\Delta v = 80 \text{ km.s}^{-1}$). La redistribution du moment angulaire, dans le modèle MONDien, permet de reformer un disque avec une structure spirale plus marquée et qui s’étend sur une plus grande distance que dans le modèle avec matière noire. En gravité newtonienne, le gaz relaxe dans le potentiel sphéroïdal du halo et reforme plus difficilement un disque. MOND pourrait aussi résoudre le problème du moment angulaire lors de la formation des galaxies. Des simulations cosmologiques avec MOND seraient nécessaires pour prendre en compte toute l’histoire de la formation hiérarchique des structures. L’accrétion de gaz froid extérieur peut aussi aider à reformer un disque.

Naines de marées. Les galaxies naines de marée sont des petites galaxies de l’ordre de $10^9 M_{\odot}$ qui se forment pendant les interactions de galaxies, à l’extrémité des queues de marées (Fig., 4.6, gauche). En gravitation newtonienne, Duc et al. (2004) ont montré que ces structures pouvaient se former si les halos de matière noire étaient suffisamment étendus pour réduire l’autogravité de la matière présente dans la queue de marée et ainsi éviter la formation d’un chapelet de surdensités.

Avec MOND, le potentiel est naturellement amplifié à grande distance et suffit pour former ces débris de collisions massifs. La Fig. 4.6 (droite) représente un instant d’une simulation d’interaction de galaxies en gravitation modifiée. Après le premier



FIGURE 4.6 – Gauche : Observation de ARP 105, une naine de marée s’est formée à l’extrémité de la queue de marée. Droite : Simulation de formation de galaxies naines de marée en gravitation modifiée, le potentiel MOND créé par les galaxies est suffisamment important pour retenir et accumuler le gaz dans les naines.

passage au périégée, les deux queues de marée se sont formées et sont étirées vers l’extérieur. Le gaz présent dans ces extensions s’accumule à l’extrémité de la queue de marée et finit par s’effondrer gravitationnellement en formant un petit disque en rotation. L’unique condition qui a changé par rapport à la simulation des Antennes est la quantité de gaz qui représente 10% de la masse du disque stellaire (pour chacune des galaxies). On peut aussi voir la formation de deux autres naines de marée qui se sont formées dans le pont de matière entre les deux galaxies.

4.5 Matière noire Fantôme

4.5.1 Principe

L'approche classique pour tester MOND et la matière noire consiste à faire une étude séparée de chacune de ces théories, pour ensuite essayer de déterminer laquelle correspond le mieux à la réalité. Une autre approche consiste à chercher une signature propre à MOND à partir des observations, c'est le test de la "Matière noire Fantôme". Il part du principe que les observations (qui tracent un potentiel gravitationnel ϕ_{obs} , et mesurent ρ_{vis}) sont en premier lieu interprétées dans le cadre de la gravitation newtonienne, par des "newtoniens". Elles permettent donc d'estimer la répartition de la matière noire, ρ_{DM} (par exemple, dans le cisaillement produit par les lentilles gravitationnelles). Connaissant la répartition de la matière visible ρ_{vis} , MOND prédit un potentiel, ϕ_M . Quelle serait la répartition de la matière à ajouter si le potentiel MONDien, ϕ_M est interprété en gravitation newtonienne? C'est cette matière que l'on appelle la matière noire fantôme ρ_{PDM} . Si MOND est effectivement la bonne théorie de la gravitation, on a

$$\phi_{obs} \iff \phi_M,$$

et,

$$\rho_{DM} \iff \rho_{PDM}.$$

Nous allons voir quelques cas précis où MOND prédit une répartition de matière noire fantôme peu courante dans le modèle Λ CDM.

4.5.2 Matière noire fantôme autour d'une masse ponctuelle.

Considérons la densité de matière visible ρ_{vis} , celle-ci engendre un potentiel gravitationnel MONDien ϕ_M :

$$\nabla \cdot \left\{ \mu \left[\frac{|\nabla \phi_M|}{a_0} \right] \nabla \phi_M \right\} = 4\pi G \rho_{vis}.$$

En gravité newtonienne, l'effet de MOND est compensé par de la matière noire fantôme ρ_{PDM} tel que :

$$\Delta \phi_M = 4\pi G (\rho_{vis} + \rho_{PDM})$$

$$\rho_{PDM} = \frac{\Delta \phi_M}{4\pi G} - \rho_{vis}$$

On peut ainsi montrer que toute masse ponctuelle isolée en gravitation modifiée sera entourée d'un halo de matière noire fantôme, pour un newtonien, dont la densité

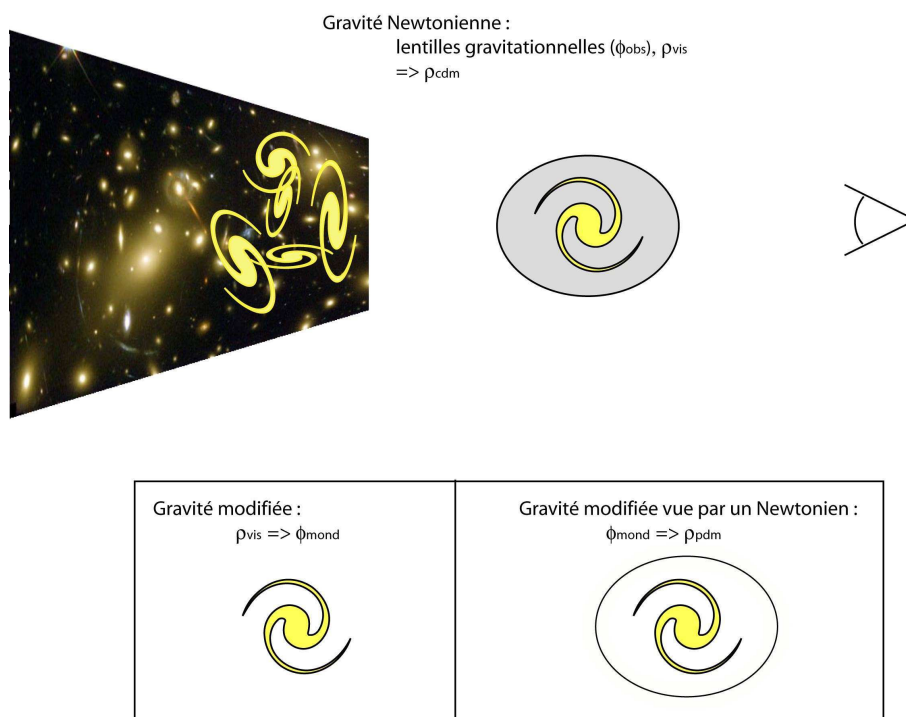


FIGURE 4.7 – L'analyse des déformations de galaxies d'arrière plan par les lentilles gravitationnelles permet de tracer des cartes de matière noire. Quelle serait la distribution de matière noire fantôme si la gravitation modifiée est interprétée par de la gravitation newtonienne ?

décroit en r^{-2} . En effet, les accélérations newtonienne et MONDienne s'écrivent :

$$a_N = \frac{GM}{r^2}$$

et

$$a_M = \frac{\sqrt{GMa_0}}{r}$$

En tout point, à l'exception de l'endroit où se trouve la masse ponctuelle, on a :

$$\frac{1}{r^2} \frac{\partial}{\partial r} [r^2 a_M(r)] = 4\pi G \rho_{PDM}(r)$$

d'où,

$$\rho_{ph(r)} = \left(\frac{Ma_0}{G} \right)^{1/2} \frac{1}{4\pi r^2}$$

Ce profil correspond au profil de matière noire fantôme dans le régime purement MONDien. Si l'on traite le cas de manière exacte, en intégrant l'accélération telle que $a_N = a_M \mu(a_M/a_0)$, MOND prédit en réalité une structure en coquille (Milgrom 2007). La densité de matière noire fantôme augmente jusqu'à un horizon : $r_H = \sqrt{GM/a_0}$, qui est le rayon de transition entre le régime newtonien et MONDien, pour retomber ensuite en r^{-2} . Dans l'exemple représenté sur la fig. 4.8, j'ai calculé le profil de matière noire fantôme (courbe en bleu) autour d'une masse ponctuelle comme le trou noir central de la Galaxie de $3 \cdot 10^6 M_\odot$. Par comparaison, la courbe rouge représente la densité du bulbe de notre Galaxie, modélisée par un profil de Hernquist de 1 kpc de rayon caractéristique et de masse $M = 10^{10} M_\odot$. La détection de cette surdensité de matière noire fantôme autour du trou noir central de la galaxie paraît très improbable étant donné la densité du bulbe qui est 300-400 fois plus dense.

Par contre, Milgrom (2007) montre que ces structures en coquille pourraient peut-être se détecter dans les courbes de rotation des galaxies, où en fonction de la densité de la galaxie et de la fonction d'interpolation choisie, l'effet de matière noire fantôme pourrait être significatif. Un autre aspect original dans cette analyse consiste à interpréter l'anneau de matière noire autour de l'amas Cl 0024+17 (fig. 4.9), par de la matière noire fantôme. Jee et al. (2007) interprétaient cette structure comme une onde en forme d'anneau due à la collision de face entre deux amas (comme la galaxie de la "Roue de charrette").

4.5.3 Matière noire fantôme négative ?

Dans quelques cas il est possible d'avoir $\Delta\phi_M < 0$, ce qui se traduit par de la matière noire fantôme négative. Ces quelques cas avaient été discutés par Milgrom (1986) à partir de calculs analytiques. Grâce à la simulation numérique, il est possible de calculer ces cartes de matière noire fantôme pour une distribution de sources visibles

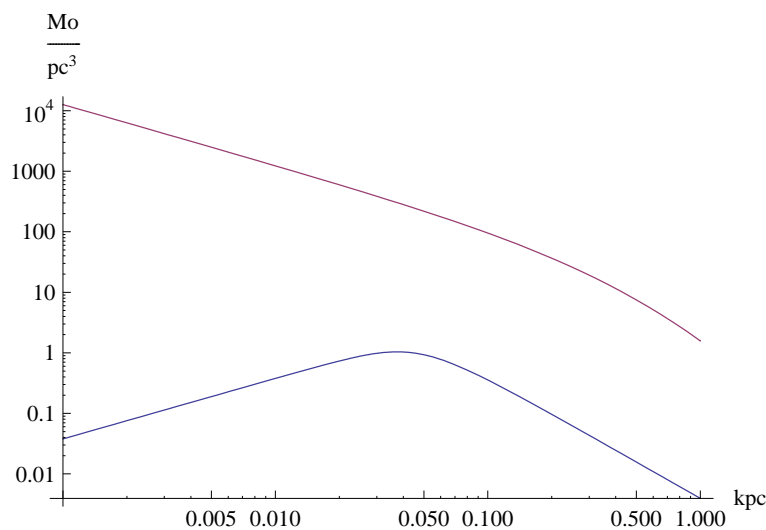


FIGURE 4.8 – Répartition de la matière noire fantôme autour du trou noir central de la Galaxie $M = 3 \times 10^6 M_{\odot}$, courbe bleue; par comparaison la courbe rouge représente la densité du bulbe (profil de Hernquist, $M = 10^{10} M_{\odot}$, $r_b=1\text{kpc}$)

quelconques. La Fig. 4.10 représente une coupe de la densité de matière fantôme. Les régions blanches correspondent à des densités négatives comme le prédisait Milgrom. Ce phénomène de masse fantôme négative pourrait être observé à partir de la déviation de la lumière. Une masse négative serait l'équivalent d'une lentille gravitationnelle divergente. Mais la proportion de cette matière fantôme négative est très faible devant la masse réelle, rendant cette détection très peu probable.

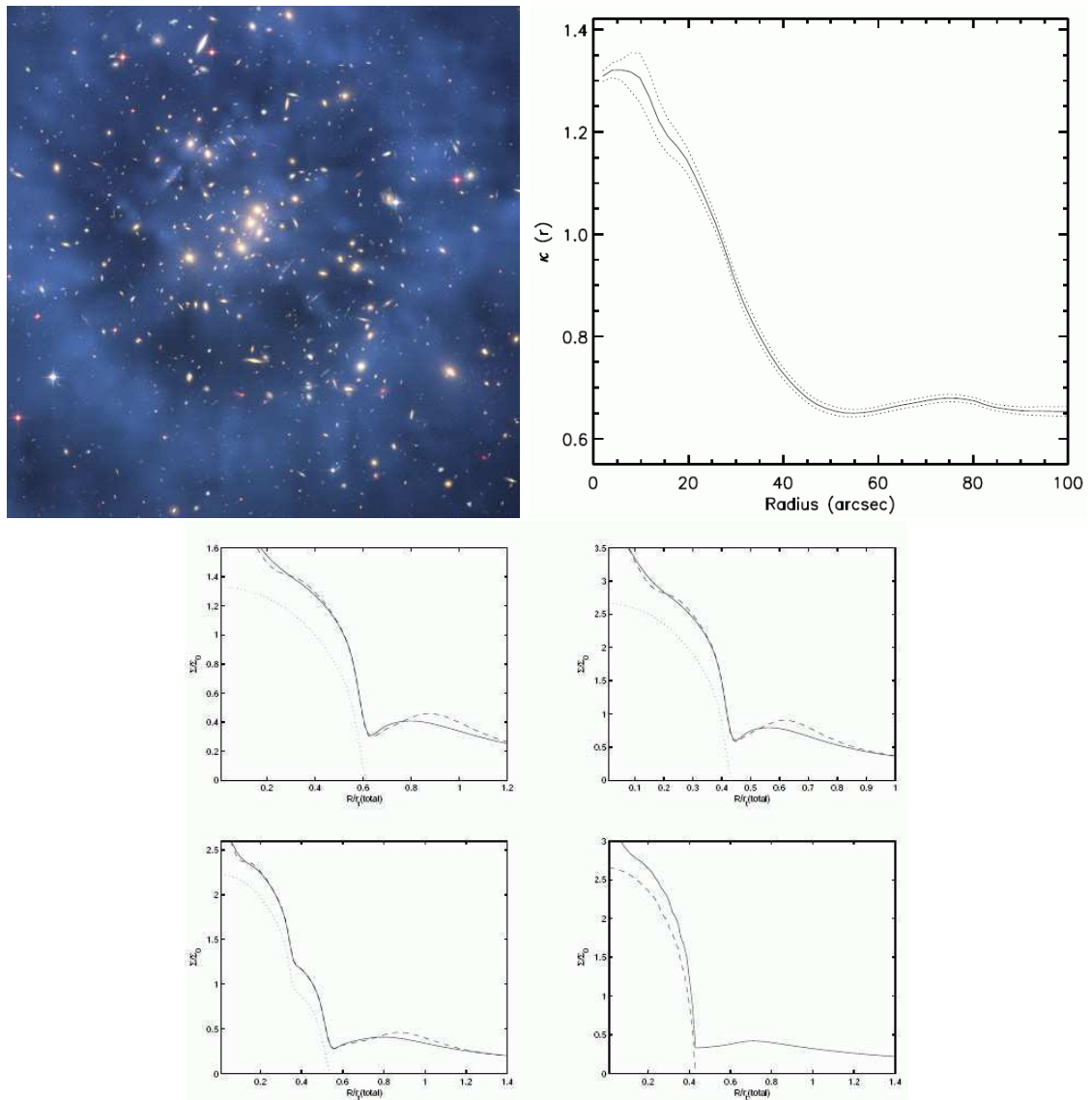


FIGURE 4.9 – Découverte d’un anneau de matière noire dans l’amas de galaxies Cl 0024+17 (Jee et al. 2007). Il peut s’agir d’un anneau produit par la collision de deux amas, ou bien un anneau de matière noire fantôme ? Les quatre graphiques du bas représentent le profil de matière noire fantôme produit par un ou deux amas sur la ligne visée, pour différentes formes de fonctions μ (Milgrom 2007).

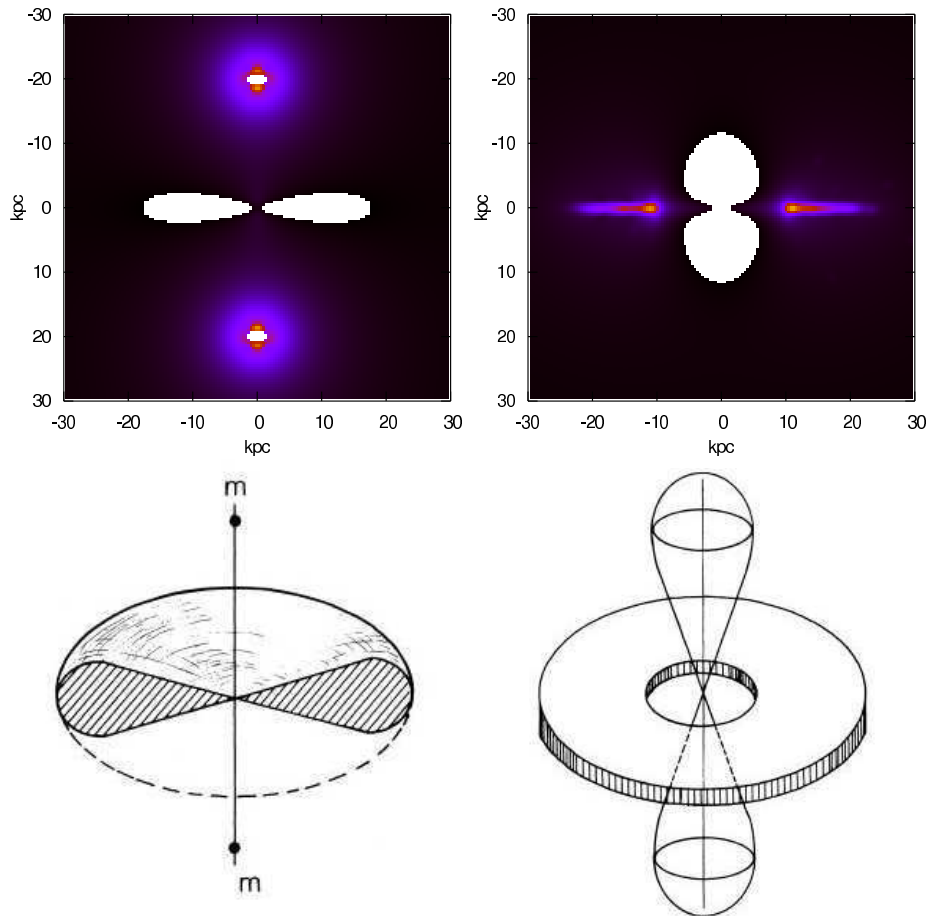


FIGURE 4.10 – Carte de matière noire fantôme générée par deux masses ponctuelles (gauche), et par un disque avec un trou de matière au centre (droite). Les régions où la matière noire fantôme est négative sont en blanc. Les schémas sont extraits de Milgrom (1986).

Chapitre 5

Conclusion

L'un des principaux enjeux de ma thèse était de réaliser des simulations numériques de galaxies en gravitation modifiée. Par le caractère non-linéaire de cette gravité, les programmes déjà existants en gravité newtonienne ne pouvaient être utilisés. Mon travail a commencé par la réalisation d'un nouveau code qui permette à la fois de résoudre l'équation de Poisson classique et modifiée afin de pouvoir comparer l'évolution d'une même galaxie, toutes choses égales par ailleurs. J'ai réalisé des simulations qui ont permis de montrer pour la première fois l'évolution 3D de galaxies isolées avec MOND, en prenant en compte la physique du gaz. Je ne m'attendais pas à trouver des différences remarquables entre MOND et la gravité newtonienne avec matière noire puisque MOND a été construit initialement pour ces échelles. Ces simulations étaient nécessaires pour tester mon nouveau code et m'ont aussi permis de me familiariser avec la dynamique des galaxies en général comme la stabilité, les ondes de densité ou encore les résonances de type barre, anneaux et cacahuète. Bien que ces simulations ne soient pas discriminantes envers l'un des deux modèles de gravité testé, il est intéressant de relever l'interaction entre le disque de la galaxie et son halo de matière noire en gravitation newtonienne qui n'existe plus en gravité modifiée. Par exemple, les transferts de moment angulaire lors de la formation de la barre avec MOND se font entre les régions internes et externes du disque ; ou encore la vitesse de rotation de la barre reste constante avec MOND, elle n'est plus freinée par effet de friction dynamique dû à un halo massif.

Faire interagir des galaxies était une étape cruciale pour MOND. Là encore, la morphologie et la cinématique des disques de galaxies sont cohérentes avec les observations et mais ne permettent toujours pas de préférer un modèle à l'autre. Toutefois, l'évolution de système de galaxies en interaction peut être radicalement différente. Les effets de friction dynamique ressentis par les disques de galaxies sont beaucoup plus faibles avec MOND qu'en gravitation newtonienne où le halo de matière noire joue un rôle majeur. En gravitation modifiée, le paramètre d'impact et le moment or-

bital initial qui aboutit à une fusion dans des temps raisonnables (quelques milliards d'années) sont beaucoup plus contraints qu'en gravitation newtonienne. L'histoire de la formation des galaxies dans un contexte cosmologique où les petites structures fusionnent entre elles pour en former de plus massives devrait montrer ces différences entre les deux modèles. En effet, un simple comptage des galaxies en interactions à $z = 0$ ne permet pas de lever la dégénérescence entre MOND où les galaxies interagissent moins, mais le temps passé dans l'interaction est plus long, et la gravité newtonienne où les interactions sont plus fréquentes, mais l'échelle de temps est plus courte. Dans les deux cas les galaxies sont autant perturbées, elles présentent des bras et des ponts de marée.

Nous avons maintenant un outil numérique pour tester MOND. Les effets non-linéaires de MOND peuvent ainsi être étudiés sans avoir à simplifier le problème par une géométrie élémentaire. Effets du champ extérieur, matière noire fantôme, lentilles gravitationnelles fourniront peut être des contraintes discriminantes.

Ces derniers mois de thèse ont été consacrés à une approche observationnelle de MOND. L'intérêt portait sur la contribution de baryons noirs dans le paradigme MOND. L'analyse des courbes de rotation des galaxies en gravitation modifiée montre qu'une composante de gaz moléculaire (H_2) deux fois plus importante que le gaz atomique serait compatible avec les observations. Cela implique aussi de réévaluer l'accélération critique de MOND à $a_0 \sim 0,9 \cdot 10^{-10} \text{ m.s}^{-2}$. Il n'y a actuellement pas de projet dédié à la détection (directe ou indirecte) de gaz moléculaire H_2 . Si cette estimation de quantité de gaz moléculaire rentre dans la quantité totale des baryons visibles, il resterait encore 50% de baryons noirs à détecter ...

Nous avons vu que l'analyse de la cinématique des galaxies elliptiques ou des naines de marée était compatible avec MOND, mais qu'elle ne donne pas de contrainte forte pour distinguer MOND de la matière noire.

Envisageons quelques implications multi-échelles de MOND :

Système solaire. Les deux sondes spatiales, Pioneer 1 et 2, constituent l'expérience de laboratoire s'étendant sur la plus grande distance jamais réalisée. L'analyse de leur trajectoire a révélé qu'elles étaient soumises à une accélération d'intensité supérieure à celle produite par le soleil. L'origine de cette anomalie reste encore inconnue de nos jours. Pour certains, il s'agirait d'un écart des lois de la gravitation classique (relativité générale) à l'échelle du système solaire. Il est alors tentant pour les théories alternatives de la gravitation de savoir si elles peuvent reproduire cet effet. L'anomalie de Pioneer est compatible avec les théories de type MOND. Mais cet écart se trouve dans le régime transitoire d'accélération entre la gravitation newtonienne et le régime MOND. C'est plutôt la forme de la fonction μ qui détermine si

l'anomalie est reproduite ou non. Mais cette fonction n'est pas contrainte rigoureusement par la théorie... De plus, on n'est pas encore certain que cette anomalie soit liée à une modification de la gravité. D'autres expériences doivent être menées pour confirmer cette mesure. C'est l'un des buts du projet SAGAS (Wolf et al. 2007) qui pourrait permettre une mesure de l'accélération dans le système solaire atteignant une précision de $5 \cdot 10^{-12} \text{ m.s}^{-2}$.

Amas globulaires. Les amas globulaires sont des objets particulièrement intéressants à étudier car ils sont supposés ne pas contenir de matière noire. Swaters et al. (2003); Scarpa et al. (2007) ont observé que les courbes de dispersion de vitesse tendaient vers une constante. En gravitation newtonienne, ces courbes de dispersion de vitesse devraient décroître continûment, alors que MOND prédit un plateau au-delà de l'accélération critique (comme pour les galaxies elliptiques). Aussi, les simulations numériques newtoniennes ont montré que les effets de marée entre l'amas globulaire et la galaxie dans laquelle il orbite pouvait aussi produire un plateau sur la courbe de dispersion. Des simulations numériques réalistes de l'évolution de l'amas dans la galaxie pourraient apporter un autre test sur les lois de la gravitation. Ces systèmes ont des accélérations caractéristiques faibles, de plus le champ extérieur produit par la galaxie peut avoir un impact non négligeable sur la dynamique interne, du fait de la non-linéarité de MOND.

Galaxies Naines. L'étude des galaxies de faible luminosité de surface (LSB) devrait être analysée spécialement. Avec MOND, on ne peut pas appliquer un facteur d'échelle sur le temps comme en gravité newtonienne. L'accélération typique de ces objets n'est pas la même que celle des galaxies géantes, et la physique de MOND change en fonction de l'accélération.

Ces galaxies en gravité newtonienne sont dominées par un halo de matière noire avec un coeur et non un cusp (Gilmore et al. 2008). Celui-ci a tendance à stabiliser le disque contre la formation de barres (Mayer & Wadsley 2004). Or, ces galaxies montrent qu'elles ont été perturbées dans le passé (Mihos et al. 1997). MOND ne devrait pas avoir de difficulté à former des barres dans ces galaxies et reproduire ces observations. Comme nous avons pu le voir pour les galaxies géantes, MOND ne stabilise pas autant qu'un halo de matière noire. L'évolution d'une galaxie LSB devrait ressembler à celle d'une galaxie de type tardif (Sd), qui contient beaucoup de gaz. Le disque est très instable au début, et forme une barre qui entretient l'excitation des ondes spirales dans le disque de gaz.

Groupes compacts de galaxies. Les simulations de galaxies en interaction ont montré de grandes différences entre la gravitation newtonienne avec matière noire

et MOND. Les effets de friction dynamique sont nettement moins importants en MOND. Ceci pourrait être une solution au problème des groupes compacts de galaxies (Hickson 1997), pour lesquels le modèle Λ CDM prédit une faible durée de vie. La plupart de ces groupes devraient avoir fusionné. Avec MOND, ces groupes peuvent se former tôt, lorsque les galaxies apparaissent, sans s'effondrer en quelques milliards d'années.

L'Univers à grande échelle. Après les simulations de galaxies isolées puis de groupes de galaxies en interaction, l'étape suivante serait de réaliser des simulations cosmologiques en gravité modifiée. Cela nécessiterait quelques changements dans le code actuel, en particulier il faut ajouter dans les équations le paramètre d'expansion de l'univers, et utiliser des conditions de bords périodiques. Un point plus délicat à traiter concerne la physique qu'il faut modéliser à ces échelles. En effet, en gravitation newtonienne, les baryons ne représentent que quelques pourcents de la masse totale, ce qui justifiait de réaliser, en première approximation, des simulations purement en matière noire (comme la simulation Millenium, Springel et al. (2005)). La matière noire forme les premières structures de l'Univers, dans lesquelles le gaz s'accrète pour former les galaxies. Avec MOND, il y aura initialement du gaz ($\Omega_b \sim 5\%$) et des neutrinos ($\Omega_\nu \sim 15\%$). La modélisation du gaz dans les simulations numériques ne semble pas encore bien maîtrisée, les différentes techniques (SPH ou AMR) ne donnent pas des résultats toujours similaires (Agertz et al. 2007). Quant à la physique des neutrinos, ces particules de quelques électrons-volts sont relativistes lorsque l'Univers est jeune. Klypin et al. (1993) se sont intéressés à mimer leurs comportements dans le cadre de modèle C+HDM (mélange de matière noire chaude et froide). Avec MOND, la formation des grandes structures serait dictée par les neutrinos tandis que les petites structures seraient dominées par les baryons.

Les grandes expériences qui permettront des avancées dans les années à venir sont :

- le LHC qui détectera peut-être des particules de matière noire non-baryoniques.
- KATRIN pourra mesurer la masse des neutrinos jusqu'à 0.3 eV.
- PLANCK améliorera la mesure du spectre de puissance du CMB, en particulier la hauteur du troisième pic.

La physique théorique cherche quant à elle, à construire une théorie relativiste robuste de MOND qui soit une extension de la relativité générale.

Les simulations numériques vont elles aussi évoluer. Il me semble que les simulations cosmologiques pourront lever la dégénérescence entre la gravitation newtonienne et MOND. Ces simulations pourront tester les différentes cosmologies proposées par les théories alternatives de la gravitation qui se développent de plus en plus. Cela nécessite d'utiliser des codes appropriés à ces nouvelles équations qui sont en

général non-linéaires avec la masse.

Est-ce que MOND peut résoudre les trois principaux problèmes du Λ CDM ? On peut penser que oui : (i) l'absence de matière noire élimine les cuspidés, (ii) tout le moment angulaire reste dans les disques des galaxies au cours de leur formation, (iii) si la persistance des petits satellites est liée à la présence de cuspidés dans les micro-halos de matière noire, les forces de marée MONDienne pourraient elles, être suffisantes pour déchirer ces sous-structures... Cette vision très optimiste n'attend que d'être testée par des simulations cosmologiques ! Il ne va pas sans dire que d'autres problèmes vont venir se greffer, en particulier la physique des neutrinos à l'échelle des amas de galaxies. Si MOND ne fonctionne pas, c'est l'histoire de la formation des grandes structures qui peut le montrer. Malgré tout, il restera à comprendre pourquoi à petite échelle, les galaxies obéissent à une loi d'échelle fondée sur l'accélération. Les simulations comparatives entre MOND et la matière noire de la dynamique des galaxies sont donc à poursuivre pour comprendre cette énigme.

La Gravitation est l'une des plus belles théories qui illustre la *Démarche Scientifique*. La distinction entre le Monde Sublunaire et le Monde Céleste d'Aristote, le Principe d'Inertie de Galilée, la Gravitation Universelle et le principe d'équivalence de Newton ont constitué une suite d'étapes qui a mené à une théorie de la Gravitation la plus avancée à ce jour : la Relativité Générale d'Einstein. Mais les instruments modernes montrent qu'une importante composante sombre domine toujours notre Univers. Serait-ce là un signe de la limite de notre description actuelle de la Gravitation ? La modification de la gravité de Milgrom est une élégante façon d'aborder ces incompréhensions pour s'aventurer dans de nouveaux domaines de la physique.

Bibliographie

- Agertz, O., Moore, B., Stadel, J., et al. 2007, MNRAS, 380, 963
- Angus, G. W., Famaey, B., & Zhao, H. S. 2006, MNRAS, 371, 138
- Angus, G. W. & McGaugh, S. S. 2008, MNRAS, 383, 417
- Barnes, J. & Hut, P. 1986, Nature, 324, 446
- Barnes, J. E. 1988, Apj, 331, 699
- Bekenstein, J. & Milgrom, M. 1984, Apj, 286, 7
- Bekenstein, J. D. 1988, Physics Letters B, 202, 497
- Bekenstein, J. D. 2004, Phys. Rev. D, 70, 083509
- Blanchet, L. 2007, Classical and Quantum Gravity, 24, 3529
- Blanchet, L. & Le Tiec, A. 2008, ArXiv e-prints, 804
- Bournaud, F., Duc, P.-A., Brinks, E., et al. 2007, Science, 316, 1166
- Brada, R. & Milgrom, M. 1999, Apj, 519, 590
- Brada, R. & Milgrom, M. 2000a, Apj, 541, 556
- Brada, R. & Milgrom, M. 2000b, ApjL, 531, L21
- Bruneton, J.-P. & Esposito-Farèse, G. 2007, Phys. Rev. D, 76, 124012
- Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, Apj, 539, 517
- Ciotti, L. & Binney, J. 2004, MNRAS, 351, 285
- Ciotti, L., Londrillo, P., & Nipoti, C. 2006, Apj, 640, 741
- Clowe, D., Bradač, M., Gonzalez, A. H., et al. 2006, ApjL, 648, L109
- Combes, F. & Gerin, M. 1985, A&A, 150, 327

- David, L. P., Arnaud, K. A., Forman, W., & Jones, C. 1990, *Apj*, 356, 32
- de Blok, W. J. G. & McGaugh, S. S. 1998, *Apj*, 508, 132
- De Lucia, G., Kauffmann, G., Springel, V., et al. 2004, *MNRAS*, 348, 333
- Diemand, J., Zemp, M., Moore, B., Stadel, J., & Carollo, C. M. 2005, *MNRAS*, 364, 665
- D’Onghia, E., Burkert, A., Murante, G., & Khochfar, S. 2006, *MNRAS*, 372, 1525
- D’Onghia, E. & Navarro, J. F. 2007, *MNRAS*, 380, L58
- Dubinski, J., Mihos, J. C., & Hernquist, L. 1996, *Apj*, 462, 576
- Duc, P.-A., Bournaud, F., & Masset, F. 2004, *A&A*, 427, 803
- Duffy, A. R., Schaye, J., Kay, S. T., & Dalla Vecchia, C. 2008, *ArXiv e-prints*, 804
- Famaey, B. & Binney, J. 2005, *MNRAS*, 363, 603
- Famaey, B., Bruneton, J.-P., & Zhao, H. 2007, *MNRAS*, 377, L79
- Felten, J. E. 1984, *Apj*, 286, 3
- Fogli, G. L., Lisi, E., Marrone, A., et al. 2004, *Phys. Rev. D*, 70, 113003
- Fukugita, M., Hogan, C. J., & Peebles, P. J. E. 1998, *Apj*, 503, 518
- Fukushige, T. & Makino, J. 1997, *ApjL*, 477, L9+
- Gentile, G., Salucci, P., Klein, U., Vergani, D., & Kalberla, P. 2004, *MNRAS*, 351, 903
- Gerhard, O. E. 1994, in *Dwarf Galaxies*, ed. G. Meylan & P. Prugniel, 335–+
- Gerhard, O. E. & Spergel, D. N. 1992, *Apj*, 397, 38
- Ghigna, S., Moore, B., Governato, F., et al. 2000, *Apj*, 544, 616
- Gilmore, G., Zucker, D., Wilkinson, M., et al. 2008, *ArXiv e-prints*, 804
- Gordon, S., Koribalski, B., & Jones, K. 2001, *MNRAS*, 326, 578
- Hayashi, E., Navarro, J. F., & Springel, V. 2007, *MNRAS*, 377, 50
- Hayashi, E. & White, S. D. M. 2006, *MNRAS*, 370, L38
- Hernquist, L. 1987, *Apjs*, 64, 715

- Hernquist, L. 1993, *Apjs*, 86, 389
- Hibbard, J. E., van der Hulst, J. M., Barnes, J. E., & Rich, R. M. 2001, *Aj*, 122, 2969
- Hickson, P. 1997, *ArA&A*, 35, 357
- Hinshaw, G., Weiland, J. L., Hill, R. S., et al. 2008, *ArXiv e-prints*, 803
- Hoekstra, H., van Albada, T. S., & Sancisi, R. 2001, *MNRAS*, 323, 453
- Hohl, F. & Hockney, R. W. 1969, *Journal of Computational Physics*, 4, 306
- James, R. A. 1977, *Journal of Computational Physics*, 25, 71
- Jee, M. J., Ford, H. C., Illingworth, G. D., et al. 2007, *Apj*, 661, 728
- Klypin, A., Holtzman, J., Primack, J., & Regos, E. 1993, *Apj*, 416, 1
- Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, *Apj*, 522, 82
- Klypin, A. & Prada, F. 2007, *ArXiv e-prints*, 706
- Knebe, A. & Gibson, B. K. 2004, *MNRAS*, 347, 1055
- Londrillo, P. & Nipoti, C. 2008, *ArXiv e-prints*, 803
- Madau, P., Diemand, J., & Kuhlen, M. 2008, *ArXiv e-prints*, 802
- Mahdavi, A., Hoekstra, H., Babul, A., Balam, D. D., & Capak, P. L. 2007, *Apj*, 668, 806
- Mastropietro, C. & Burkert, A. 2007, *ArXiv e-prints*, 711
- Mateo, M. 1997, in *Astronomical Society of the Pacific Conference Series*, Vol. 116, *The Nature of Elliptical Galaxies; 2nd Stromlo Symposium*, ed. M. Arnaboldi, G. S. Da Costa, & P. Saha, 259–+
- Mayer, L., Governato, F., & Kaufmann, T. 2008, *ArXiv e-prints*, 801
- Mayer, L. & Wadsley, J. 2004, *MNRAS*, 347, 277
- McGaugh, S. S. 1999, *ApjL*, 523, L99
- McGaugh, S. S. 2000, *ApjL*, 541, L33
- McGaugh, S. S. 2004, *Apj*, 609, 652

- McGaugh, S. S. & de Blok, W. J. G. 1998a, *Apj*, 499, 41
- McGaugh, S. S. & de Blok, W. J. G. 1998b, *Apj*, 499, 66
- McGaugh, S. S., Schombert, J. M., Bothun, G. D., & de Blok, W. J. G. 2000, *ApjL*, 533, L99
- Mihos, J. C., Bothun, G. D., & Richstone, D. O. 1993, *Apj*, 418, 82
- Mihos, J. C., McGaugh, S. S., & de Blok, W. J. G. 1997, *ApjL*, 477, L79+
- Milgrom, M. 1983a, *Apj*, 270, 371
- Milgrom, M. 1983b, *Apj*, 270, 384
- Milgrom, M. 1983c, *Apj*, 270, 365
- Milgrom, M. 1986, *Apj*, 306, 9
- Milgrom, M. 1994, *Annals of Physics*, 229, 384
- Milgrom, M. 2007, *ArXiv e-prints*, 712
- Milgrom, M. & Sanders, R. H. 2007, *ApjL*, 658, L17
- Miller, R. H. & Prendergast, K. H. 1968, *Apj*, 151, 699
- Milosavljević, M., Koda, J., Nagai, D., Nakar, E., & Shapiro, P. R. 2007, *ApjL*, 661, L131
- Moore, B., Ghigna, S., Governato, F., et al. 1999a, *ApjL*, 524, L19
- Moore, B., Quinn, T., Governato, F., Stadel, J., & Lake, G. 1999b, *MNRAS*, 310, 1147
- Navarro, J. F. & Benz, W. 1991, *Apj*, 380, 320
- Navarro, J. F. & Steinmetz, M. 1997, *Apj*, 478, 13
- Navarro, J. F. & Steinmetz, M. 2000a, *Apj*, 538, 477
- Navarro, J. F. & Steinmetz, M. 2000b, *Apj*, 528, 607
- Nipoti, C., Ciotti, L., Binney, J., & Londrillo, P. 2008, *MNRAS*, 472
- Nipoti, C., Londrillo, P., & Ciotti, L. 2007a, *Apj*, 660, 256
- Nipoti, C., Londrillo, P., & Ciotti, L. 2007b, *MNRAS*, 381, L104

- Nipoti, C., Londrillo, P., & Ciotti, L. 2007c, MNRAS, 381, L104
- Nusser, A. 2002, MNRAS, 331, 909
- Pfenniger, D. & Revaz, Y. 2005, A&A, 431, 511
- Pointecouteau, E. & Silk, J. 2005, MNRAS, 364, 654
- Power, C., Navarro, J. F., Jenkins, A., et al. 2003, MNRAS, 338, 14
- Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1993, Numerical Recipes in Fortran : The Art of Scientific Computing, 2nd edn. (Cambridge University Press)
- Rubin, V. C., Burstein, D., Ford, Jr., W. K., & Thonnard, N. 1985, Apj, 289, 81
- Sanders, R. H. 1996, Apj, 473, 117
- Sanders, R. H. 2003, MNRAS, 342, 901
- Sanders, R. H. & McGaugh, S. S. 2002, ArA&A, 40, 263
- Sanders, R. H. & Noordermeer, E. 2007, MNRAS, 379, 702
- Scarpa, R., Marconi, G., Gilmozzi, R., & Carraro, G. 2007, The Messenger, 128, 41
- Skordis, C. 2008, ArXiv e-prints, 801
- Skordis, C., Mota, D. F., Ferreira, P. G., & Boehm, C. 2006, Physical Review Letters, 96, 011301
- Smith, M. C., Ruchti, G. R., Helmi, A., et al. 2007, MNRAS, 379, 755
- Sofue, Y., Tutui, Y., Honma, M., et al. 1999, Apj, 523, 136
- Springel, V. & Farrar, G. R. 2007, MNRAS, 380, 911
- Springel, V., White, S. D. M., Jenkins, A., et al. 2005, Nature, 435, 629
- Strigari, L. E., Bullock, J. S., Kaplinghat, M., et al. 2007, Apj, 669, 676
- Swaters, R. A., Madore, B. F., van den Bosch, F. C., & Balcells, M. 2003, Apj, 583, 732
- Tiret, O. & Combes, F. 2007, A&A, 464, 517
- Toomre, A. & Toomre, J. 1972, Apj, 178, 623

Tully, R. B. & Fisher, J. R. 1977, *A&A*, 54, 661

Wang, Z., Fazio, G. G., Ashby, M. L. N., et al. 2004, *Apjs*, 154, 193

White, D. A. & Fabian, A. C. 1995, *MNRAS*, 273, 72

Whitmore, B. C. & Schweizer, F. 1995, *Aj*, 109, 960

Wolf, P., Bordé, C. J., Clairon, A., et al. 2007, *ArXiv e-prints*, 711

Zhao, H. 2007, *ApjL*, 671, L1

Zlosnik, T. G., Ferreira, P. G., & Starkman, G. D. 2006, *Phys. Rev. D*, 74, 044037

Zlosnik, T. G., Ferreira, P. G., & Starkman, G. D. 2007, *Phys. Rev. D*, 75, 044017