Strategy sequential difficulty effects: Studies in numerical cognition

Kim Uittenhove

To cite this version:

Kim Uittenhove. Strategy sequential difficulty effects: Studies in numerical cognition. Psychology. Aix-Marseille Université, 2012. English. <tel-00759035>

HAL Id: tel-00759035

https://tel.archives-ouvertes.fr/tel-00759035

Submitted on 29 Nov 2012

DOCTORAT AIX-MARSEILLE UNIVERSITE délivré par AIX-MARSEILLE UNIVERSITE

THESE
pour obtenir le grade de
DOCTEUR D'AIX-MARSEILLE UNIVERSITE

Formation doctorale:
Psychologie

Présentée et soutenue publiquement par
Kim Uittenhove

Effets de difficulté séquentielle stratégique: Etudes en cognition numérique

Directeur de thèse: Patrick Lemaire

JURY

M. Pierre Barrouillet (Universite de Geneve)		Rapporteur
Mme. Laurence Taconnat (Universite Francois-Rabelais)		Rapporteur
Mme. Agnes Blaye (Aix-Marseille Universite)		Examinateur
Mme. Catherine Thevenot (Universite de Geneve)		Examinateur

le 17 décembre 2012

Acknowledgements

Pierre Barrouillet, Laurence Taconnat, Agnes Blaye, and Catherine Thevenot, thank you for having accepted to read and evaluate my dissertation. I see this as an opportunity to benefit from your expertise to critically reflect on my research from the past three years.

Patrick Lemaire, thank you for believing in me, and giving me the freedom to integrate a lot of myself in this thesis. I could always count on your insight and enormous knowledge to direct my efforts. I also appreciate how you managed my professional activities with the interest of my career in mind. I could not have hoped for better conditions to achieve my research and dissertation.

Eléonore, thank you for reading my thesis countless times. I really think that without you I would have never been able to finish it. I was also fortunate enough to discover that you are very funny, friendly, adorable, and caring. It was a lot of fun spending the summer with you in the lab, when everyone else was on holiday. I also enjoyed our little business trip to Geneva, with plenty of adventures on the way. I consider myself lucky to have a friend like you.

Céline, thank you for having been at my side for so many months. I really appreciate your kindness, loyalty, humour, and spontaneity. I remember one day I had written an article in French that was completely hopeless. You were still up in the middle of the night, improving it. That is when I realized how rare it is to find a friend like you. I hope you know you can always count on me.

Mariel, we only really got to know each other at Tours. It was a lot of fun hanging out with you, going to bars and restaurants, and talking day and night. Thank you for your support during the final stages of my writing. I am happy to defend my thesis at the same time as you, and I hope we will celebrate it properly.

Stéphane, Mireille, thank you for your support on more technical levels during my thesis. I could always count on your generosity.

Marina, thank you for teaching me the meaning of the word ‘ambition’. Ambition means learning all the skills to master tower defense games. I also like that you are generous, understanding, easy going, considerate, and creative. We made a lot of good jokes and worthless trinkets together (like the plastic water fountain on cocktail day). I am really happy to count you amongst my friends.

Soazig, thank you for making our little corner in the office feel like home. If I could, I would choose to spend eternity sharing an office corner with you. I like your hidden cakes, and your craziness and imagination. I am proud of the many internal investigations we led, as Dr. Watsoaz & Sherlokim. We always discovered the truth behind every lab mystery. My leaving Marseille is not the end of our friendship I hope, especially since we have shared custody over a hedgehog.

Suzanne, you were the first person I considered a friend in the lab. Thank you for your efforts to integrate me in Marseillan culture. I had a great time with you in the office. **Sophie,** I am happy that our paths crossed in the lab so I could get to know you. I am looking forward to seeing you more when I go to Geneva.

Dash, brownies and backrubs, and a great sense of humour. You were a nice and welcome surprise in the last year of my thesis. I'm happy that it was you that took that desk in our office.

Clément, thanks for a lot of great lunch meetings interspersed with science-fiction conversations, and for a shared affection for almonds and cats.

Yann, it was nice sitting opposite you for so much time. I like how you are relaxed, friendly, and funny. And thanks for eating my almonds from time to time, preventing me from doing an overdose.

Michael, thanks for the long conversations online, and the brief conversations in the corridor. It's always nice to talk to you about books and food. And socks in sandals.

Sylvain, I will always remember when we were at Psychonomics, and you would advertize my poster so people would come to see it. I wasn't supposed to know, but one of them told me. You are also very dedicated to your research, and always trying to improve. Nobody deserves to become a doctor more than you.

Joanna, thanks for your friendliness and your empathy. It was really cool having you around. **Cecile,** we didn't really get to know each other very well, but I like how you are a bit crazy and always distracted (like me). **Olivia,** I think you are really cool and funny. I wish I would have had time to know you better. **Idriss,** you are so spontaneous and friendly, and always ready to help others. I really appreciate that. **Carlos,** thank you for adding some Honduran culture in our office. You really managed to leave your fingerprints everywhere in the lab. I appreciate your sense of humour and friendliness.

My parents, brother, grandparents, and friends, thank you for caring and encouraging me, I look forward to celebrating this achievement with all of you.

François, I don't even know where to begin to thank you. From the very beginning, when I came to France, you have supported me with everything. There is nobody I can rely on more than on you. I could write a thesis about how great you are, but I will restrict myself to saying that I think you are the best partner in the universe.

Résumé étendu

Dans cette thèse, nous commençons par soutenir que les variations de performances liées à l'âge dans des domaines aussi divers que la cognition numérique, la mémoire ou la prise de décision peuvent s'expliquer par les variations stratégiques que les adultes, jeunes ou plus âgés, mettent en oeuvre dans ces domaines. Cette approche stratégique de la résolution de problème s'est révélée fructueuse et a permis de comprendre une grande partie des variations de performances liées à l'âge et aux pathologies (voir par exemple, Lemaire, 2010). La littérature montre que les adultes âgés en bonne santé et les patients souffrant de la maladie d'Alzheimer sont moins efficaces que les jeunes adultes dans la façon dont ils sélectionnent et exécutent des stratégies.

La manière d'exécuter des stratégies est un facteur essentiel pour comprendre les variations stratégiques. Les adultes âgés en bonne santé et les patients souffrant de la maladie d'Alzheimer ont tendance à sélectionner moins souvent les stratégies pour lesquelles leur capacités d'exécution sont altérées (Gandini et al., 2008; Gandini, Lemaire, & Michel, 2009; Mata, Schooler, & Rieskamp, 2007; Bouazzaoui et al., 2010). Les stratégies les plus affectées

chez cette population sont souvent plutôt des stratégies procédurales complexes que des stratégies de récupération plus simples. Il donc est important de mieux comprendre la façon dont les stratégies sont exécutées, étant donné l'impact que cela a sur les performances des participants.

Dans tous les domaines de la cognition, l'exécution stratégique est influencée par les caractéristiques de ces stratégies, par le problème, le contexte et le participant (Lemaire, Arnaud, & Lecacheur, 2004; Siegler & Shrager, 1984; Gandini, Lemaire, & Dufau, 2008; Siegler, 1987; Arnaud et al., 2006; Geary, Bow-Thomas, Liu, & Siegler, 1993).

Dans cette thèse, nous voulions aller au delà de la simple description des conditions liées aux variations de performances dans l'exécution stratégique. Ainsi, en se concentrant sur la cognition numérique, notre domaine d'étude, nous avons analysé en détails comment les caractéristiques des stratégies influencent leur exécution. Ceci nous a permis de mieux situer et comprendre l'impact et la nature de nos résultats par rapport à l'exécution de stratégies numériques.

Nous soutenons l'idée que l'efficacité d'exécution des stratégiques cognitives numériques dépend des processus impliqués dans ces stratégies. Ces processus semblent s'appuyer sur l'accumulation d'expérience et la disponibilité de ressources exécutives: moins d'expérience et/ou des ressources exécutives moins disponibles réduisent l'efficacité d'exécution stratégique. De plus, la littérature semble indiquer que certaines variations d'expertise et de ressources exécutives influent l'efficacité d'exécution.

Une limite importante des recherches précédentes sur l'efficacité de l'exécution stratégique est le peu d’attention porté au contexte séquentiel dans lequel les stratégies sont exécutées. En effet, l’essentiel des recherches que nous avons rencontré se limite à l’exécution stratégique sur des problèmes isolés. Cependant, les stratégies sont souvent exécutées en succession rapide. Ne pas prendre en compte les stratégies exécutées précédemment semble problématique puisqu’elles peuvent influencer les performances des participants. Par exemple, le peu de recherche disponible montre que l’exécution stratégique est sensible aux effets de changements de stratégies. Lemaire et Lecacheur (2010) ont montrés que l’exécution de stratégies numériques est plus efficace quand une même stratégie est répétée que lorsque les stratégies changent au cours de deux problèmes consécutifs. Le changement peut conduire à une diminution des ressources exécutives de telle sorte que les stratégies suivantes sont moins efficaces (Lemaire & Lecacheur, 2010; Luwel et al., 2009; Ardiale & Lemaire, sous presse).

Les effets de séquence d’exécution stratégique ne se limitent pas aux effets de changement de stratégies. Dans une première étude, nous avons montré une efficacité réduite après l’exécution d’une stratégie difficile. Nous avons appelé ce phénomène “effets de difficulté séquentielle stratégique”. Comme pour le changement stratégique, l’exécution d’une stratégie difficile peut réduire temporairement les ressources exécutives, réduisant ainsi l’efficacité d’exécution de la stratégie suivante. Dans nos deux études suivantes, nous avons vérifié le lien entre les effets séquentiels de difficulté et les ressources exécutives. Les effets de difficulté séquentielle peuvent également contribuer aux variations d’exécution de stratégies numériques liées à l’âge; c’est l’objet de notre quatrième étude. Pour étudier ces questions et y répondre, nous avons développé un paradigme d’estimation computationnelle.

Paradigme d'estimation computationnelle

Afin d'atteindre nos objectifs, le choix du paradigme était essentiel. Certaines stratégies s'appuient plus que d'autres sur les ressources exécutives (Imbo & Vandierendonck, 2007) et seront plus à même de produire des effets de séquence sur les performances. Le paradigme que nous avons choisi pour nos expériences est l'estimation computationnelle. Cette technique est utilisée dans les situations où les individus manquent de temps, de connaissances, de ressources et/ou de motivation pour calculer les réponses exactes. Dans ces situations, l'estimation permet souvent d'atteindre une approximation satisfaisante. Par exemple, pour la résolution d'une multiplication telle que 43 x 67, un individu peut se contenter d'arrondir les deux opérandes aux dizaines inférieures les plus proches (ici, 40 x 60) pour ensuite calculer le produit des deux opérandes arrondies (ici, 2400), ce qui est bien plus aisé que de calculer la valeur exacte du produit initial.

L'estimation computationnelle se compose de processus de décomposition et de récupération de faits arithmétiques. Le problème initial (par exemple 43 x 67) est décomposé en un certain nombre de récupérations. Ces récupérations sont constituées 1) de la procédure stratégique (par exemple, pour arrondir à la dizaine supérieure : ignorer le chiffre des unités, incrémenter le chiffre des dizaines, multiplier les chiffres des dizaines), 2) des opérandes arrondies (par exemple, pour un arrondi vers le haut : 50 et 70) et 3) de la solution de leur multiplication (par exemple, pour un arrondi vers le haut : 50 x 70 = 3500).

Différentes stratégies d'arrondi mettront en oeuvre différentes quantités de ressources exécutives lors de ces récupérations. La récupération de la procédure, des opérandes arrondies et du produit devrait être plus aisée pour les arrondis vers le bas et plus difficile pour les arrondis vers le haut et les arrondis mixtes. La raison est que les arrondis vers le haut et les arrondis mixtes nécessitent l'incrémentation d'au moins une des opérandes. Si les effets séquentiels impliquent des ressources exécutives, les arrondis vers le haut et les arrondis mixtes doivent alors produire des effets séquentiels plus prononcés que les arrondis vers le bas. De plus, ils doivent être davantage sujets aux effets séquentiels.

Dans toutes nos expériences, nous avons étudié les performances lors d'arrondis mixtes après avoir exécuté des arrondis vers le bas ou vers le haut. En étudiant l'exécution d'arrondis mixtes après celle d'arrondis vers le haut, nous optimisons nos chances de trouver des interférences provenant de l' exécution de la stratégie précédente. La Figure 1 illustre un essai dans le paradigme d'estimation computationnelle.

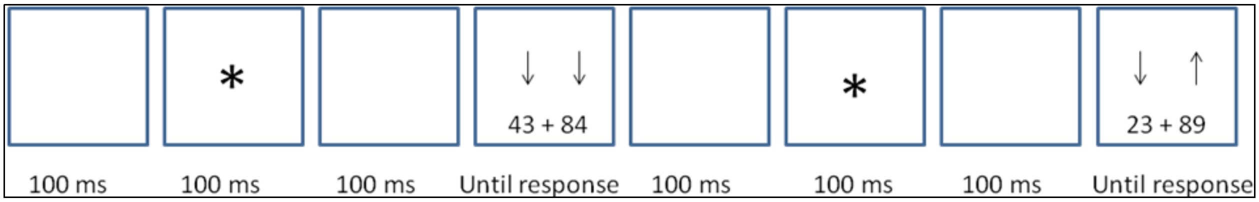


Figure 1. Paradigme d'estimation computationnelle.

Nous allons maintenant brièvement passer en revue les principaux résultats de quatre études sur les effets de difficulté séquentielle stratégique. Dans une première étude, nous avons montré l'existence d'effets de difficulté séquentielle stratégique. Dans une deuxième étude, nous avons montré que des différences de capacités de mémoire de travail entre individus étaient liées à l'intensité des effets de difficulté séquentielle. De plus, il est apparu que donner plus de temps aux participants entre les problèmes réduisait l'intensité des effets observés. Dans une troisième étude, nous avons examiné le déroulement temporel des effets de difficulté séquentielle stratégique en utilisant l'électro-encéphalographie. Dans une quatrième et dernière étude, nous avons observé des effets de difficulté séquentielle stratégique accrus chez les patients souffrant de la maladie d'Alzheimer, une population présentant des ressources exécutives diminuées.

Effets de difficulté séquentielle lors de l'exécution stratégique : une étude en arithmétique (2012, *Experimental Psychology,* 59(5), 295-301)

Nous avons prédit que les performances des participants pour une stratégie donnée seraient plus faibles si la stratégie utilisée pour le problème précédent était difficile. Nous avons constaté que les participants étaient plus lents avec la stratégie d'arrondi mixte après avoir utilisé la stratégie plus difficile d'arrondi vers le haut que la stratégie plus simple d'arrondi vers le bas. Ces effets ont été constaté indépendemment de la difficulté des problèmes, que nous avons contrôlé en variant la taille des opérandes (Figure 2).

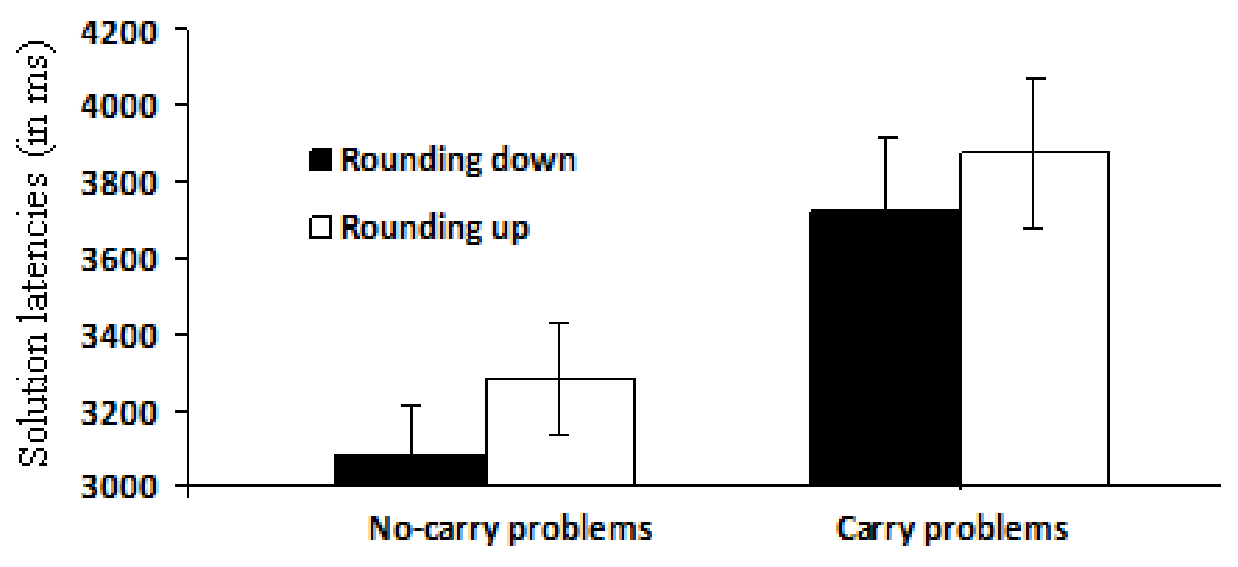


Figure 2. Latence des solutions avec arrondi mixte après arrondi vers le bas et arrondi vers le haut pour des problèmes nécessitant une retenue et pour des problèmes ne nécessitant pas de retenue.

La performance d'une stratégie est ainsi influencée par la difficulté de la stratégie exécutée pour le problème immédiatement précédent. Les mesures actuelles d'efficacité stratégique reflètent ainsi la combinaison de l'efficacité propre de la stratégie et de celle des stratégies immédiatement précédentes. Il est donc nécessaire, pour mesurer les performances stratégiques réelles, de contrôler les effets de difficulté séquentielle. Bien que les résultats de cette étude furent très intéressants, plusieurs questions sont restées sans réponse. La principale question concerne les processus sous-jacents aux effets de difficulté séquentielle : sont-ils liés aux ressources exécutives ? Notre étude suivante s'intéresse à cette question.

Effets de difficulté séquentielle stratégique et capacité de mémoire de travail : une étude de corrélation en arithmétique (en cours de révision, *Psychonomic Bulletin Review*)

Nous avons formé l'hypothèse que les effets de difficulté séquentielle stratégique étaient dûs à une réduction de capacité de la mémoire de travail après l'exécution d'une stratégie difficile. De notre point de vue, une stratégie difficile devrait laisser des traces dans la mémoire de travail, réduisant ainsi sa capacité fonctionnelle et interférant avec l'exécution de la stratégie suivante. Une prédiction qui en découle est que les individus dotés d'une capacité de mémoire de travail réduite devraient être davantage affectés par les effets de difficulté séquentielle stratégique que les individus dotés d'une capacité de mémoire de travail plus importante. Dans cette étude, nous avons testé cette prédiction.

Nous avons mesuré les effets de difficulté séquentielle stratégique chez des individus en calculant les différences entre les temps de latence pour la stratégie d'arrondi mixte après l'exécution d'arrondis vers le haut ou d'arrondis vers le bas. Nous avons évalué les capacités de mémoire de travail chez ces mêmes individus en leur faisant passer les tâches de mise à jour, d'empan d'opérations, et d'empan de lecture. En accord avec nos prédictions, nous avons constaté que les individus ayant une capacité de mémoire de travail réduite présentaient des effets de difficulté séquentielle stratégique significativement supérieurs par rapport aux individus ayant une capacité de mémoire de travail plus grande (Figure 3).

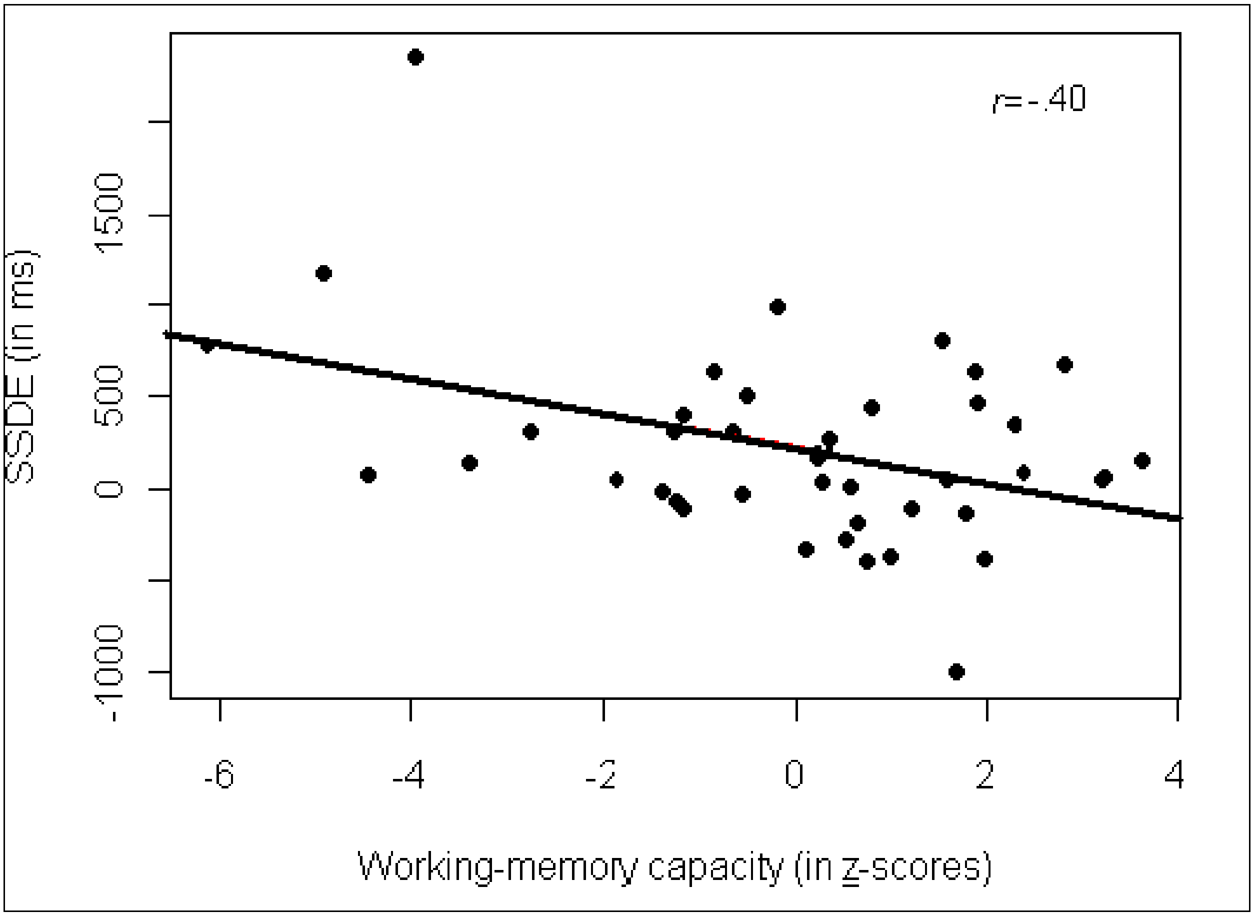


Figure 3. Corrélation entre une mesure composée des capacités de mémoire de travail (z-scores sur les tâches de mise à jour, d'empan d'opérations, et d'empan de lecture) et des effets de difficulté séquentielle stratégique (latences de solution pour arrondi mixte après arrondi vers le haut / arrondi vers le bas).

Nous avons également fait varier l'intervalle entre stimuli de court (300 ms) à long (600 ms), et nous avons constaté que donner aux individus un temps suffisant entre chaque problème leur permettait de récupérer des ressources exécutives, éliminant ainsi les effets de difficulté séquentielle stratégique (Figure 4).

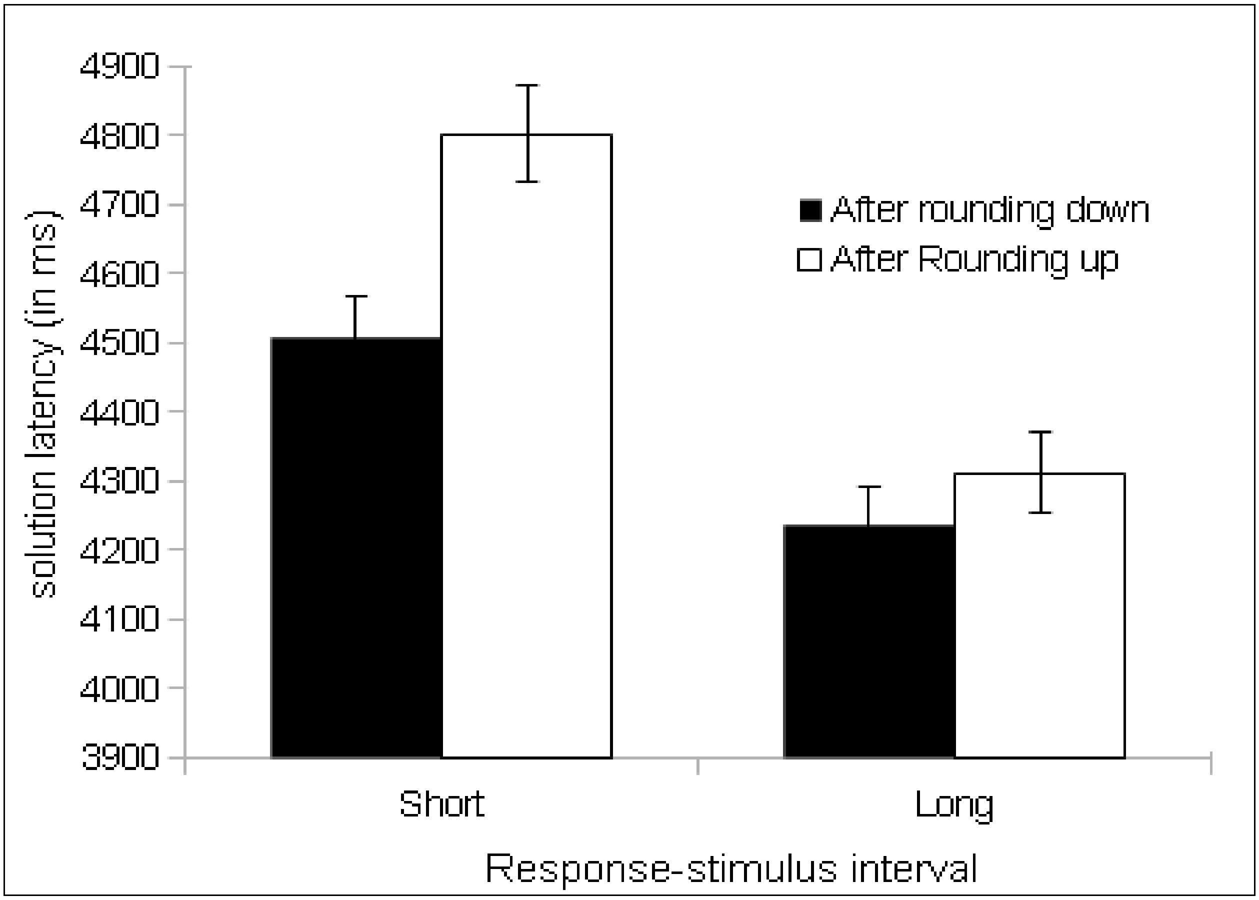


Figure 4. Temps de latence pour des arrondis mixtes après l'exécution d'arrondis vers le bas ou d'arrondis vers le haut dans une situation avec un intervalle entre stimuli court et dans une situation avec un intervalle entre stimuli long.

Le déroulement temporel des effets de difficulté séquentielle stratégique : une étude ERP en arithmétique (soumis, *Experimental Brain Research*)

Dans la deuxième étude, nous avons constaté que les ressources exécutives sont liées à l'intensité des effets de difficulté séquentielle. Cependant, nous ne savons pas quand les effets de difficulté séquentielle interfèrent avec l'exécution stratégique, ni de quelle manière. Nous voulions répondre à ces questions en étudiant le déroulement temporel des effets de difficulté séquentielle stratégique. Si les effets de difficulté stratégique ont comme base les ressources exécutives, nous devrions constater qu'ils interfèrent avec les processus centraux plutôt que périphériques de l'exécution stratégique. De plus, cette interférence doit consister en un effort de traitement plus important après des stratégies difficiles (par opposition par exemple à des critères temporels prolongés) qu'après des stratégies faciles.

Nous avons soumis nos participants à notre paradigme d'estimation computationnelle et nous avons enregistré leur activité cérébrale pendant la tâche. Nous avons constaté une activité cérébrale accrue avec l'arrondi mixte après la stratégie plus difficile d'arrondi vers le haut qu'après la stratégie plus facile d'arrondi vers le bas. Il est intéressant de constater que l'activité cérébrale est la plus apparente immédiatement après l'encodage des caractéristiques du problème, quand les participants récupèrent et maintiennent les étapes de la stratégie devant être exécutée (Figure 5).

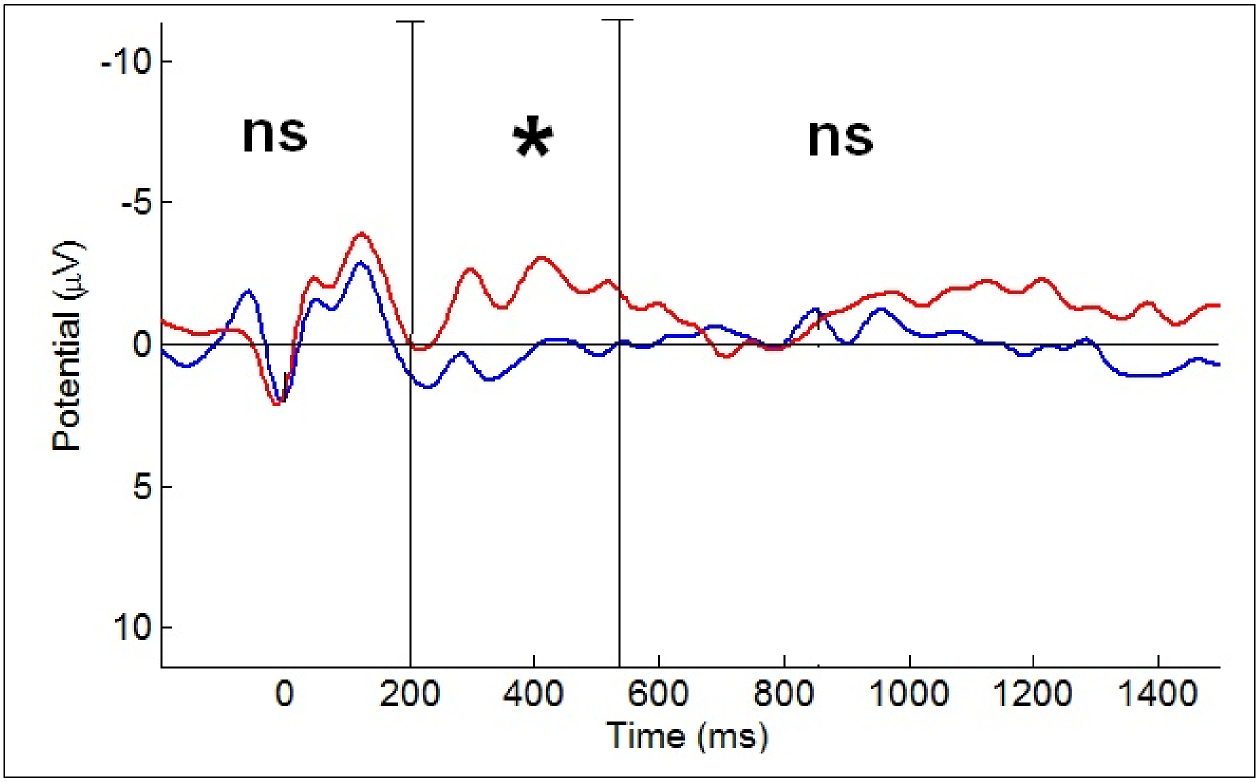


Figure 5. Amplitude des ondes pour l'arrondi mixte après arrondi vers le bas (bleu) et après l'arrondi vers le haut (rouge). Électrode Fp1.

Ainsi, en accord avec nos prédictions, les effets de difficulté séquentielle stratégique semblent correspondre à des efforts de traitement plus importants après l'exécution de stratégies difficiles qu'après l'exécution de stratégies faciles. Ceci est mis en évidence par l'activité cérébrale plus importante après des stratégies difficiles. Par ailleurs, les effets de difficulté séquentielle stratégique semblent interférer principalement avec la récupération et l'exécution de procédures stratégiques, une série de processus centraux.

Effets de difficulté séquentielle stratégique chez les patients souffrant d'Alzheimer: une étude en arithmétique (en cours de révision, *Journal of Clinical and Experimental Neuropsychology*)

De la même façon que nous avons constaté des effets de difficulté séquentielle accrus chez les individus dotés d'une faible capacité de mémoire de travail, nous nous attendions à ce que les populations chez qui les ressources exécutives sont moins efficaces, telles que les adultes âgés et les patients souffrant de la maladie d'Alzheimer, présentent des effets de difficulté séquentielle accrus. Nous avons soumis des adultes âgées en bonne santé et des patients souffrant de la maladie d'Alzheimer à notre paradigme d'estimation computationnelle. Nous avons constaté des effets de difficulté séquentielle comparables entre les adultes âgés en bonne santé et les adultes plus jeunes, mais bien plus importants chez les patients souffrant de la maladie d'Alzheimer (figure 6).

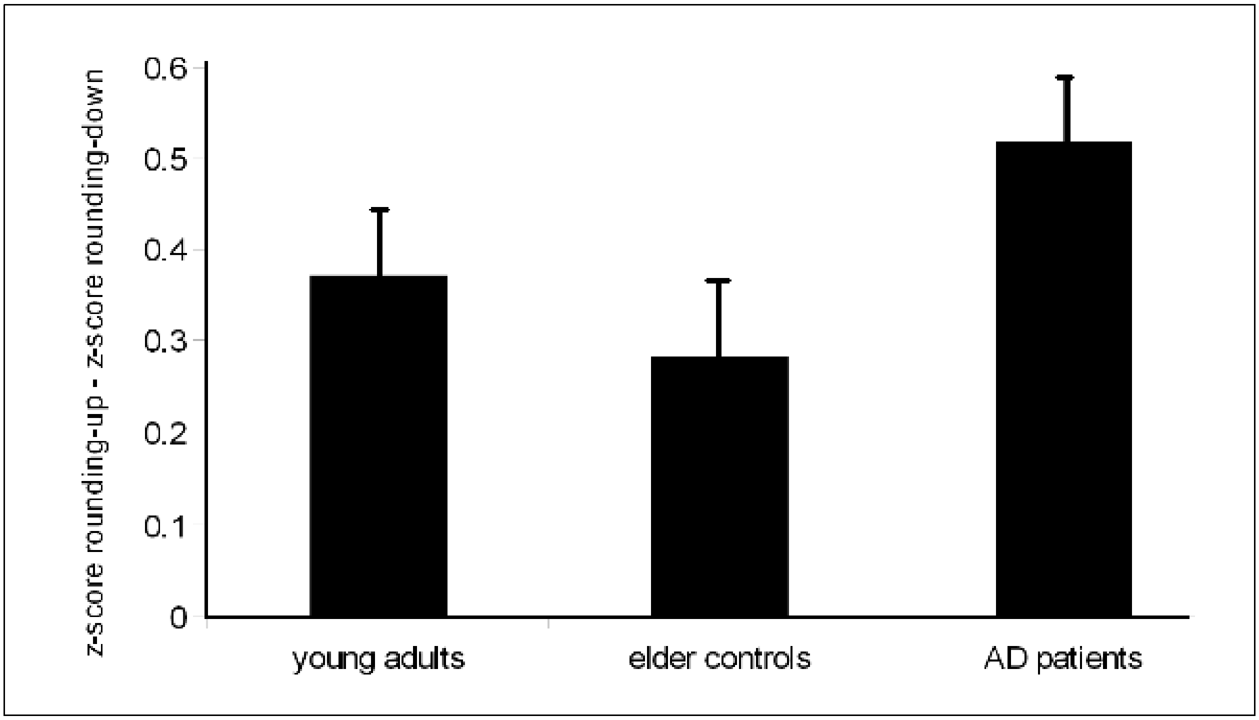


Figure 6. Effets de difficulté séquentielle stratégique ([z-scores après arrondi vers le haut] – [z-scores après arrondi vers le bas]) chez des adultes jeunes, des adultes plus âgés et des patients souffrant de la maladie d'Alzheimer.

En ce qui concerne les effets de difficulté séquentielle comparables chez les jeunes adultes et les adultes plus âgés en bonne santé, il se pourrait que les mécanismes impliqués dans les effets de difficulté séquentielle (comme par exemple la capacité de mémoire de travail) ne soient pas suffisamment affectés chez les adultes plus âgés en bonne santé pour que l'effet soit mesurable. Cependant, nous avons constaté des effets de difficulté séquentielle bien plus importants chez les patients souffrant de la maladie d'Alzheimer. Il est possible que les patients souffrant d'AD éprouvent des difficultés à libérer leur mémoire de travail après l'exécution d'une stratégie (i.e., *deletion inhibition*, Hasher & Zacks, 1999).

Conclusion

Nos recherches ont montré l'existence d'effets de difficulté séquentielle stratégique correspondant à des efforts de traitement plus importants après des stratégies difficiles qu'après des stratégies faciles. De plus, les effets de difficulté séquentielle chez un individu dépendent de sa capacité de mémoire de travail disponible. Les individus ou les populations chez qui la capacité de mémoire de travail disponible est plus faible souffrent davantage des effets de difficulté séquentielle. Les effets de la difficulté de la stratégie précédente interfèrent le plus avec les stades précoces de l'exécution de la stratégie suivante, qui peuvent correspondre à la récupération des procédures stratégiques. Tout au long de l'exécution stratégique, et lorsque le temps entre les problèmes augmente, les effets de difficulté séquentielle se dissipent.

Dans nos études, nous avons montré que l'efficacité de l'exécution stratégique ne dépend pas seulement des processus impliqués dans la stratégie en cours d'exécution, mais également de la stratégie exécutée pour le problème précédent. Si cette stratégie a mis en oeuvre des processus plus gourmands, la stratégie suivante est exécutée plus lentement. Néanmoins, les modèles de sélection stratégique (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) ne prennent pas en compte les effets de difficulté séquentielle stratégique. Nous proposons de combler cette lacune par l'ajout d'un paramètre au modèle. Ce paramètre représenterait la quantité de ressources exécutives disponibles et pourrait fluctuer dynamiquement en fonction des changements de demandes exécutives. Les stratégies difficiles pourraient temporairement consommer les ressources exécutives nécessaires pour la maintenance, la sélection et l'exécution stratégique.

Extended summary

In this thesis, we start by arguing that age-related performance variations in such diverse domains as numerical cognition, memory and decision making can be accounted for by variations in the way young and older adults use strategies in these domains. This strategy approach to problem solving has proven very fruitful and has led to the understanding of a large body of performance variations in aging and pathology (e.g., Lemaire, 2010). We exhibit how healthy older adults and Alzheimer patients (AD) select and execute strategies less efficiently than healthy young adults.

The way strategies are executed is a crucial factor to further understand strategic variations. Healthy older adults and AD patients select less often those strategies on which their execution is most impaired (Gandini et al., 2008; Gandini, Lemaire, & Michel, 2009; Mata, Schooler, & Rieskamp**,** 2007; Bouazzaoui et al., 2010). The type of strategies most impaired in healthy older adults and AD patients are often complex procedural strategies as opposed to easier retrieval strategies. Given the impact of the way in which strategies are executed on participants' performance and on the selection of strategies, it is important to better understand how strategies are executed. In all domains of cognition, strategy execution is influenced by characteristics of strategies, problems, contexts, and participants (Lemaire, Arnaud, & Lecacheur, 2004; Siegler & Shrager, 1984; Gandini, Lemaire, & Dufau, 2008; Siegler, 1987; Arnaud et al., 2006; Geary, Bow-Thomas, Liu, & Siegler, 1993).

In this thesis, we wanted to go beyond a mere description of the conditions under which strategy execution efficiency varies. By focusing on numerical cognition, the domain of our studies, we were able to achieve an in-depth analysis of how aforementioned characteristics determine numerical strategy execution. This detailed account permitted us to better situate and understand the impact and nature of our own results with regard to numerical strategy execution.

We argue that the efficiency of execution of numerical cognition strategies depends on the processes involved in these strategies. We present literature showing how these processes rely on both accumulation of experience and the availability of executive resources. Less experience and/or less available executive resources reduce strategy execution efficiency. Moreover, we present variations in experience and in executive resources that further determine efficiency.

An important limit of previous research on strategy execution efficiency is the little attention for the sequential context in which strategies are executed. Indeed, most of the research we discussed considers strategy execution on isolated problems. However, strategies are often executed in quick succession. Not taking into account strategies executed previously is problematic, since these may influence participants' performance. For example, the limited amount of available research has shown that strategy execution is sensitive to effects of switching. Lemaire and Lecacheur (2010) found that numerical strategy execution is more efficient when we repeat the same strategy than when we switch strategies across two consecutive problems. Switching may reduce available executive resources so that the next strategy execution is less efficient (Lemaire & Lecacheur, 2010; Luwel et al., 2009; Ardiale & Lemaire, in press).

Sequential effects on strategy execution need not be limited to effects of switching. In our first study, we showed a lesser strategy execution efficiency following difficult strategy execution. We named this phenomenon “strategy sequential difficulty effects”. Similar to strategy switching, execution of a difficult strategy could temporarily reduce executive resources, so that the next strategy execution is less efficient. We tested the link between sequential difficulty effects and executive resources in the second and third study. Sequential difficulty effects could also contribute to age-related variations in numerical strategy execution, which we tested in the fourth study. To investigate all of these questions, we developed a computational estimation paradigm.

Computational estimation paradigm

To achieve our objectives, the choice of paradigm was very important. Some strategies rely more than others on executive resources (Imbo & Vandierendonck, 2007) and will be more liable to producing sequential effects on performance. The paradigm we chose for our experiments was computational estimation. This technique is used in situations where people lack time, knowledge, resources, and/or motivation to calculate exact responses. In these cases, estimates often give a sufficient approximation. For example, when solving a problem such as 43 x 67, an individual may satisfy himself rounding both operands down to the nearest decades (i.e., 40 x 60) and retrieve the multiplication of these rounded operands (i.e., 2400), which will be a lot easier than to retrieve the solution to the initial problem.

Computational estimation is composed of processes of decomposition and arithmetic fact retrieval. We decompose the initial problem (e.g., 43 x 67) in a number of subsequent retrievals. These retrievals involve 1) The procedure of the strategy (e.g., for rounding up: discard unit digits, increment ten digits, multiply incremented ten digits), 2) The rounded operands (e.g., for rounding up: 50 and 70), and 3) The solution to their multiplication (e.g., for rounding up: 50 x 70 = 3500).

Different rounding strategies will put different demands on executive resources during these retrievals. Retrieval of the procedure, rounded operands, and multiplication should be easiest for rounding down and most difficult for rounding up and mixed rounding. This is because the latter require the additional step of incrementing at least one operand. If sequential effects involve executive resources, rounding up and mixed rounding should thus engender larger sequential effects than rounding down. Moreover, they should also be more susceptible to sequential effects.

In all our experiments, we contrasted performance with mixed rounding after having executed rounding down or rounding up. By looking at execution of mixed rounding following rounding up, we optimize our chances of finding interference from previous with subsequent strategy execution. See Figure 1 for an illustration of a trial in the computational estimation paradigm.

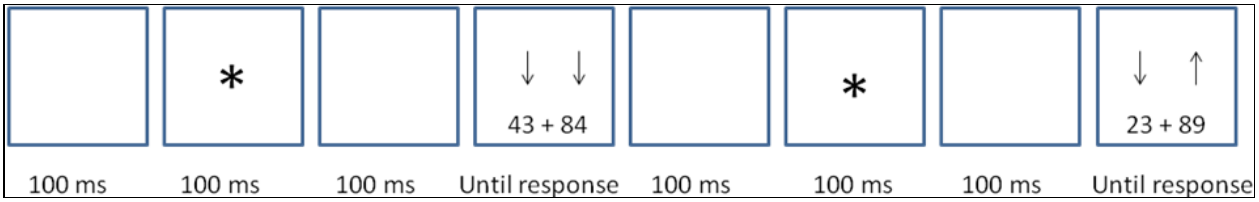


Figure 1: The computational estimation paradigm.

We will now briefly review the main results obtained in four studies on strategy sequential difficulty effects. In the first study, we showed strategy sequential difficulty effects during execution. In a second study, we found that inter-individual differences in working-memory capacities were related to the size of sequential difficulty effects. Moreover, giving participants more time between problems decreased the size of the effects. In a third study, we examined the time course of strategy sequential difficulty effects using electroencephalography. In a fourth and final study, we found increased strategy sequential difficulty effects in AD patients, a population with reduced executive resources.

Sequential difficulty effects during strategy execution: A study in arithmetic (2012, *Experimental Psychology,* 59(5), 295-301)

We predicted that participants' performance with a given strategy would be poorer if the strategy used on the previous problem was difficult. We found that participants were slower using a mixed-rounding strategy after having used the more difficult rounding-up strategy than after having used the easier rounding-down strategy. These effects existed independently of the difficulty of the problem, which we manipulated by varying the size of the operands (See Figure 2).

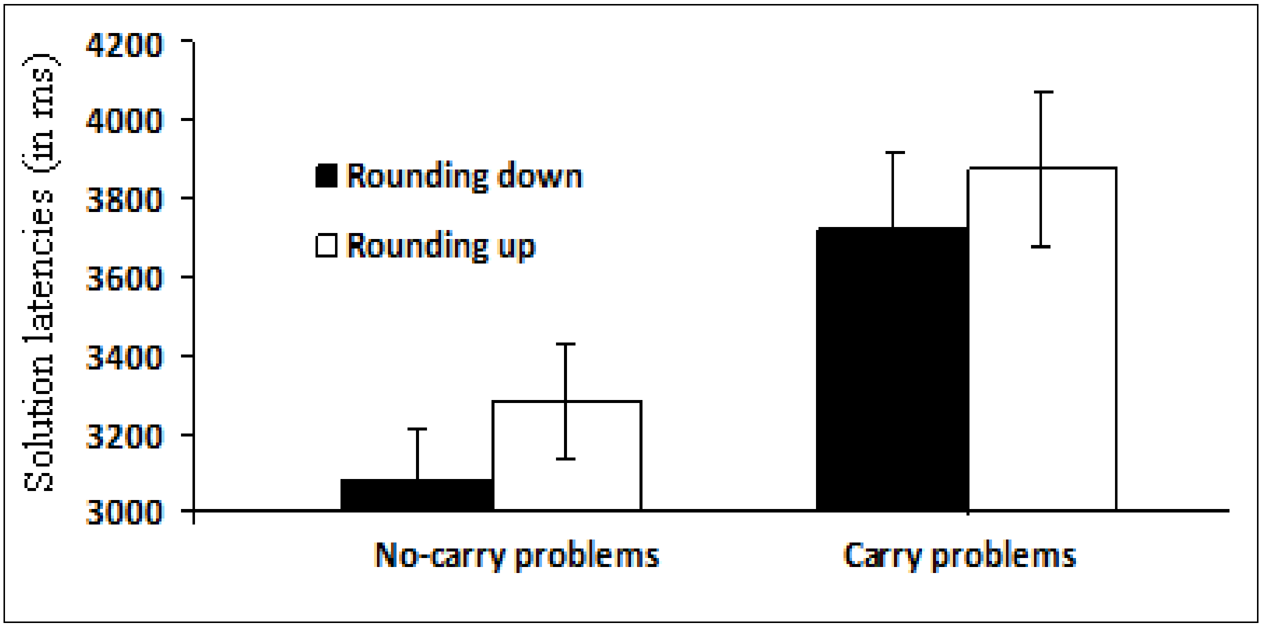


Figure 2. Solution latencies with mixed rounding following rounding down and rounding up on problems involving a carry on the operands and problems involving no carry on the operands.

Strategy performance is thus influenced by the difficulty of the strategy we executed on the immediately preceding problem. Current measures of strategy efficiency thus reflect the combination of actual strategy performance and the effects of strategies we executed immediately before. If we want to measure actual, uncontaminated strategy performance, we need to control sequential difficulty effects. Although the results of this study were very interesting, several questions were left unanswered. The main question being the processes underlying strategy sequential difficulty effects, are they linked to executive resources? Our next studies address this issue.

Strategy Sequential Difficulty Effects and Working-Memory: A correlational study in arithmetic (in revision, *Psychonomic Bulletin Review*).

We hypothesized strategy sequential difficulty effects to be due to decreased working-memory resources following difficult strategy execution. In our view, a difficult strategy would leave traces in working memory that reduce functional working-memory capacities and interfere with execution of the next strategy. A prediction that derives from this is that individuals with lower working-memory capacities should suffer more from strategy sequential difficulty effects than individuals with higher working-memory capacities. In this study, we tested this prediction.

We measured strategy sequential difficulty effects in individuals by taking the difference in solution latencies with the mixed-rounding strategy following execution of rounding up and rounding down. We assessed working-memory capacities in the same individuals by having them perform the operation span, running span, and reading span tests. Consistent with our prediction, we found that individuals with lower working-memory capacities showed significantly larger strategy sequential difficulty effects than individuals with higher working-memory capacities (see Figure 3).

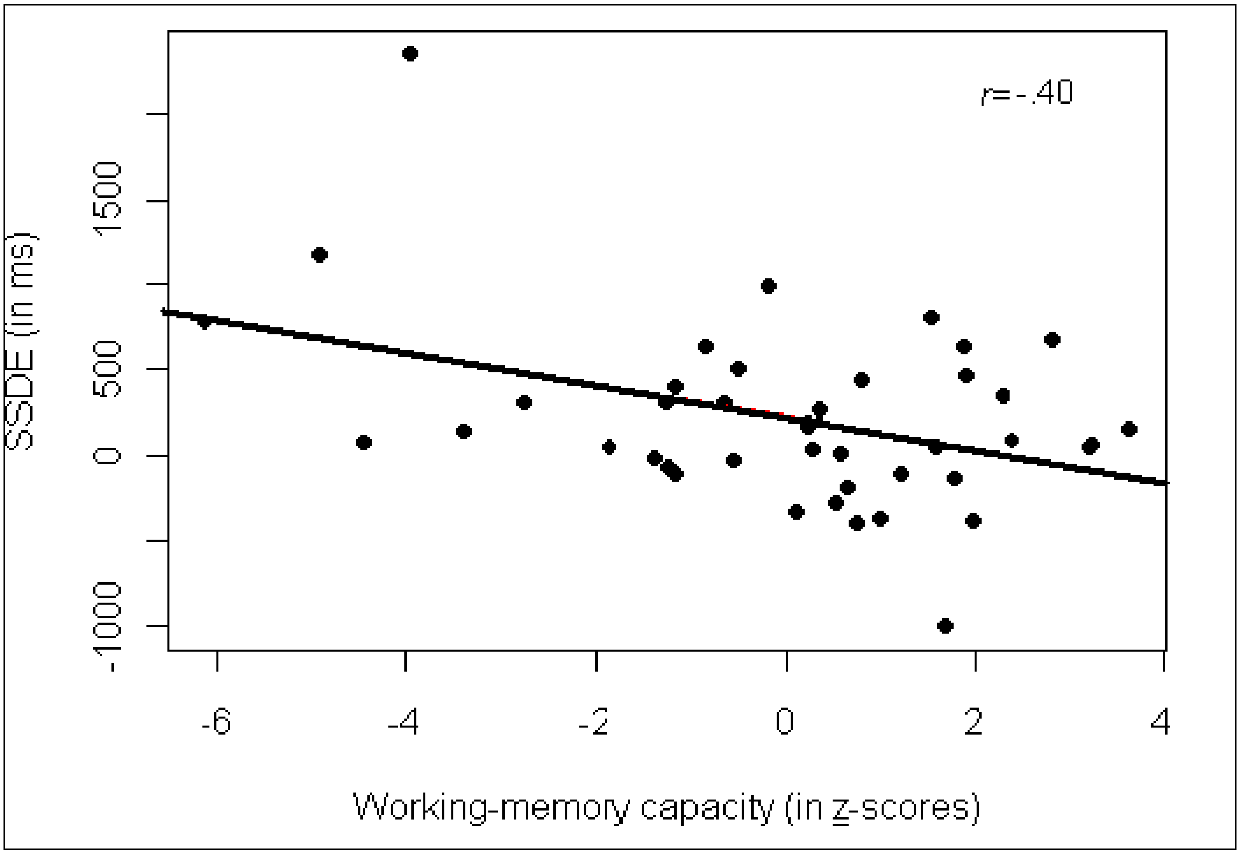


Figure 3. Correlation between a composite measure of working memory capacities (z-scores on the operation span, reading span, and running span) and strategy sequential difficulty effects ([solution latencies with mixed rounding following rounding up] –[following rounding down]).

We also varied the response-stimulus interval from short (300 ms) to long (600 ms), and found that giving individuals sufficient time between problems allowed them to recuperate from reduced executive resources, annihilating sequential difficulty effects (see Figure 4).

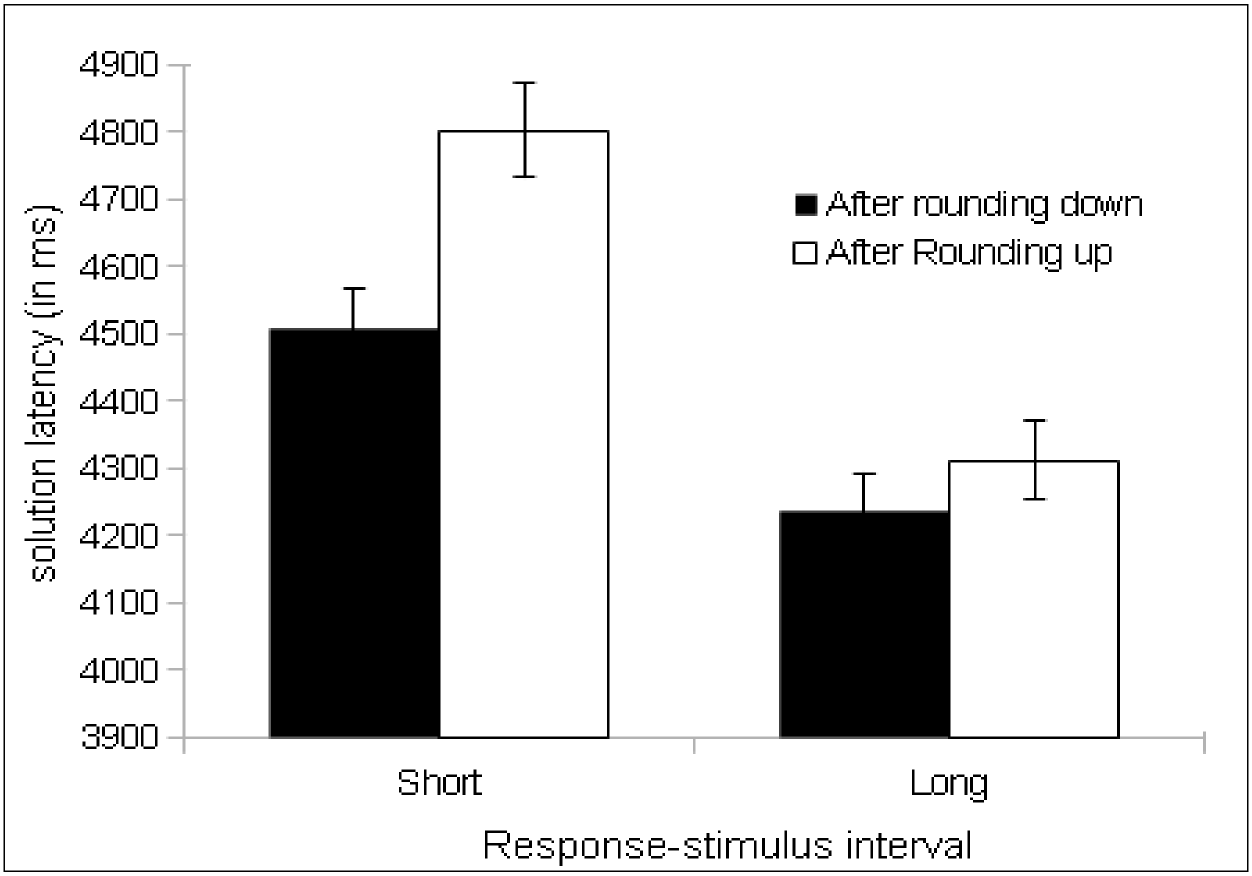


Figure 4. Solution latencies with mixed rounding after having executed rounding down or rounding up in a situation with short response-stimulus interval and a situation with long response-stimulus interval.

The time course of strategy sequential difficulty effects: An ERP study in arithmetic (submitted, *Experimental Brain Research*)

In the second study, we found that executive resources are linked to the size of sequential difficulty effects. However, we do not know when sequential difficulty effects interfere with strategy execution, and in what way. We hoped to answer these questions by investigating the time course of strategy sequential difficulty effects. If sequential difficulty effects have an executive resource basis, we should find that they interfere with central rather than with peripheral processes of strategy execution. Moreover, this interference should consist of more effortful processing following difficult strategies (as opposed to for example elongated time criteria) than following easy strategies.

We presented participants with our computational estimation paradigm and registered their cerebral activities during the task. We found greater cerebral activities with mixed rounding when it followed the more difficult rounding-up strategy than the easier rounding-down strategy. Interestingly, the point in time at which greater cerebral activities were most apparent was immediately after the encoding of the problem characteristics, when participants would be retrieving and maintaining the steps of the strategy to be executed (See Figure 5).

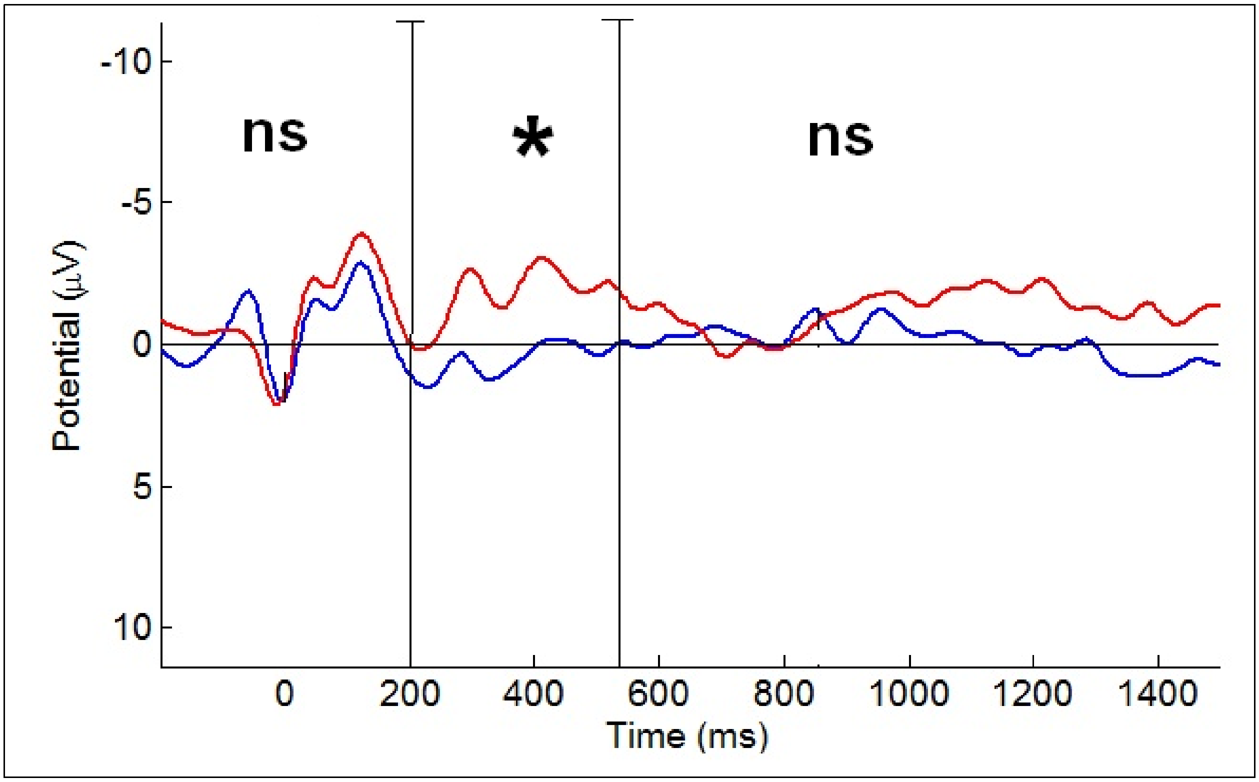


Figure 5. Wave amplitudes when using MR after RD (blue) or after RU (red); Fp1 electrode.

Thus, consistent with our predictions, strategy sequential difficulty effects seem to correspond to more effortful processing following difficult than following easy strategies, as evidenced by greater cerebral activities following difficult strategies. Second, strategy sequential difficulty effects seem to interfere most with the retrieval and execution of strategy procedures, a series of central processes.

Strategy sequential difficulty effects in Alzheimer patients: A study in arithmetic (in revision, *Journal of Clinical and Experimental Neuropsychology*).

Similar to the finding of larger sequential difficulty effects in individuals with low working-memory capacities, we expected populations with less efficient executive resources, such as older adults and AD patients, to show larger sequential difficulty effects. We presented our computational estimation paradigm to AD patients and healthy older adults. We found comparable sequential difficulty effects in young and healthy older adults but dramatically increased sequential difficulty effects in AD patients (See Figure 6).

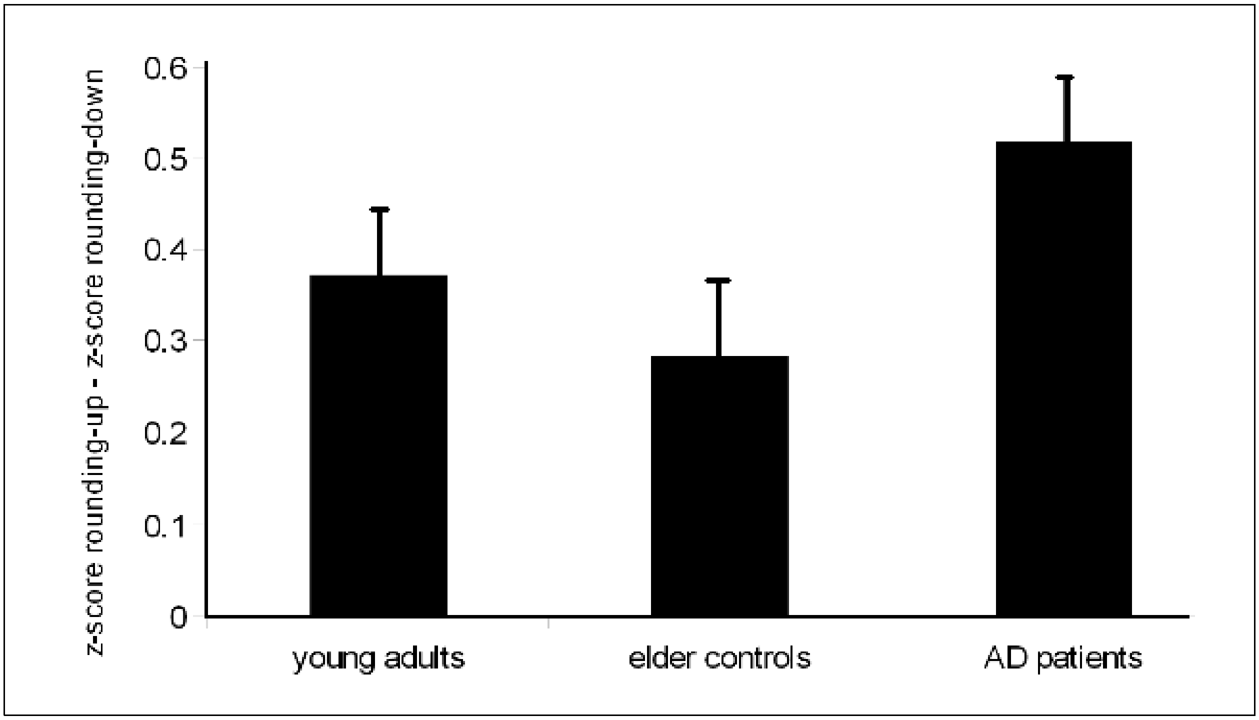


Figure 6. *Strategy sequential difficulty effects ([z-scores after rounding up] – [z-scores after rounding down]) for young and healthy older adults, and AD patients.*

Regarding comparable sequential difficulty effects in healthy older and young adults, it could be that the mechanisms implicated in sequential difficulty effects (e.g., working-memory capacities) were not sufficiently affected in healthy older adults to yield an effect. However, sequential difficulty effects were significantly and dramatically increased in AD patients. AD patients may have deficits in clearing working memory after executing a strategy (i.e., deletion inhibition, Hasher & Zacks, 1999). Since a difficult strategy would take up more working-memory resources, this would explain why AD patients are relatively more impaired following difficult strategies.

Conclusion

Our research showed strategy sequential difficulty effects during strategy execution. This corresponded to more effortful processing following difficult than following easy strategies. Moreover, sequential difficulty effects are dependent on available working-memory capacities of the individual. Individuals or populations with less available working-memory capacities suffer more from sequential difficulty effects. The effects of previous strategy difficulty interfere most with early stages of the next strategy execution, which may correspond to retrievalof strategy procedures. Throughout strategy execution, and with increasing time between problems, sequential difficulty effects dissipate.

In our studies, we showed that strategy execution efficiency is not only dependent on the processes involved in the strategy we are in the course of executing, but also on the strategy we executed on the previous problem. If the strategy we executed on the previous problem contained more demanding processes, we execute the next strategy slower. Nevertheless, models of strategy selection (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) currently do not take into account strategy sequential difficulty effects in any way. We suggested that they could resolve this issue by adding an additional parameter to the model. This parameter could represent the amount of available executive resources and could dynamically fluctuate as a function of changing executive demands during the task. Difficult strategies could temporarily consume executive resources needed for strategy maintenance, selection, and execution.

Table of content

General Introduction

Throughout daily life we are confronted with problems in a variety of domains. For example, how do we memorize important information during a lecture ? Do we write down as much as possible or do we rely on a mental organization strategy ? Between several possible strategies, we need to choose one in particular.

A ‘strategy’ is described by Lemaire and Reder (1999, p. 365) as “a procedure or a set of procedures to achieve a higher level goal”. Strategies are key to understanding problem-solving performance, and performance variations with age. Problem-solving performance varies as a function of the characteristics of the strategy and will be limited by how well the selected strategy is executed.

Two big issues concerning strategies are how they are selected and how they are executed. In this thesis, we will primarily focus on strategy execution. We argue that strategy execution relies on experience but also on executive resources and that the latter depend not only on current task demands but also fluctuate as a function of previous task demands. Furthermore, we suggest that fluctuation of executive resources can vary in healthy and pathological aging.

Imagine a situation in which you have to review a doctoral dissertation. Chances are that you will be more efficient when you just come back from a relaxing walk than when you have just spent a difficult hour on the introduction of your newest article. In this thesis, we will demonstrate so-called ‘sequential difficulty’ effects on a transient trial-to-trial basis, in numerical cognition, with a strategic approach. Moreover, we investigate the possibility that they are enlarged in older adults.

Before introducing experimental works on sequential difficulty effects, two chapters summarize previous studies relative to variations in strategy execution in numerical cognition and in aging. Chapter 1 discusses how a strategy approach can help us understand performance variations in young adults, healthy older adults, and Alzheimer patients (AD). Besides describing and explaining these variations, we also discuss how older adults can compensate for declines.

Chapter 2 focuses on a single strategy dimension in a single domain. We discuss strategy execution in numerical cognition, the domain of our studies. Based on extensive literature research, we attempted to identify what aspects of numerical strategies determine execution efficiency. For this, we looked at main processes involved in numerical strategies. These rely to a variable degree on two main cognitive resources: Numerical knowledge in long-term memory and executive resources.

In the problematic chapter, we propose that sequential difficulty effects can also interfere with strategy execution, diminishing strategy efficiency. We also wondered whether sequential effects during strategy execution could contribute to our understanding of age-related variations in strategy use. Four experimental studies investigated the existence of sequential difficulty effects during numerical strategy execution, and the evolution of such effects with age.

Aging and Performance

The Western population grows increasingly older. In late 1950, at the time of the advent of cognitive science with Broadbent and Chomsky, older adults (> age 60) made up 11% of the population in France (Calot & Chesnais, 1997). In 2005, this proportion had increased to 20%, and is expected to increase to 30% in 2050 (Robert-Bobée, 2007).

Several cognitive afflictions can accompany aging, amongst which the most known may well be Alzheimer's disease, touching 26 million people worldwide in 2005 and expected to quadruple by 2050 (Brookmeyer et al., 2007). Alzheimer's disease (AD) seriously impairs older adults' functioning in society, inducing many social costs.

These predictions about the growing numbers of older adults and of associated pathologies explain the increasing interest in normal and pathological aging among cognitive psychologists since the dawn of this research domain. Indeed, the physiological changes brought about with age induce differences in cognitive performance. Initially, this branch of research focused on the description of age-related changes in cognitive performance. This was helpful in identifying problem areas for older adults and AD patients.

Healthy older adults, and especially AD patients, lose autonomy in certain areas. In healthy older adults, performance has been shown to remain relatively stable or to even improve in some domains, whereas other domains are marked by performance declines (see Lemaire, 2010, for a review). For example, Maylor (1994) found that healthy older adults outperformed younger adults in a quiz testing general knowledge, whereas Salthouse (1992) found that normal aging was associated with worse performance on Raven's Progressive Matrices (r=-.61), with one of the largest age-related declines for any cognitive measure. These age-related declines in tasks such as Raven's Progressive Matrices are exacerbated when aging is accompanied by Alzheimer's disease (Grady et al., 1988). Moreover, in tasks were healthy older adults normally outperform young adults, AD patients show declined performance. For example, Alzheimer patients have more difficulties accessing the meaning of words than healthy older adults (Chobor & Brown, 1990), which may hamper their performance during a quiz testing general knowledge.

Maybe more important than merely describing age-related differences is trying to understand them. By understanding performance declines in older adults and AD patients, we may be able to remedy them better. To understand age-related declines in cognitive domains, it is useful to distinguish between the evolution with age of experience and of executive resources. Experience increases stored knowledge necessary for problem solving, and improves with age. Executive resources allow the brain to be flexible, and process or acquire information but suffer from declines through degradation of the supporting brain regions (e.g., Reuter-Lorenz & Park, 2010). The differential evolution with age of experience and executive resources is the reason why Maylor (1994) found that older adults outperformed their younger counterparts when stored knowledge was concerned, whereas Salthouse (1992) found that they performed a lot worse on Raven's Progressive Matrices.

What if aging is accompanied by AD? In AD, executive resources seem to be more impaired than in healthy older adults (e.g., Grady et al., 1988) and furthermore, stored knowledge is also affected (Chobor & Brown, 1990). That is why AD patients perform even worse than healthy older adults on problem-solving tests such as Raven's Progressive Matrices, and show impairments on tests measuring knowledge, where healthy older adults have good performance.

Differences in experience and executive resources influence problem-solving performance not only quantitatively (e.g., less available executive resources slow down problem solving) but also qualitatively (e.g., less available executive resources lead individuals to use easier strategies). Qualitative differences, or how individuals solve problems, may help explain cognitive performance variations in young adults, healthy older adults, and AD patients (Lemaire, 2010). Thus, in this chapter, we hope to explain performance variations in young adults, healthy older adults, and AD patients by looking at the strategies they use to accomplish cognitive tasks.

1.1 A strategy approach to performance variations

To understand cognitive performance, we need to have a thorough understanding of which and how strategies are used. Do individuals select strategies in an adequate manner and do they execute these strategies efficiently? Strategy variations have been shown a wide variety of cognitive tasks (e.g., Siegler, 2007; Lemaire & Arnaud, 2008; Lemaire, Arnaud, & Lecacheur, 2004; Dunlosky & Hertzog, 2001; Mata, Schooler, & Rieskamp 2007). For example, Siegler (1987) investigated children's performance when solving addition problems and found that it was better understood when taking into account the strategies they used. Lemaire and Siegler (1995) provided a conceptual framework for explaining performance variations from a strategy perspective.

They decomposed strategy variations in four dimensions. The first dimension is strategy repertoire (i.e., which strategies are used). The second dimension is strategy distribution (i.e., relative frequency of use of strategies), the third dimension is strategy execution (i.e., the speed and precision obtained with each strategy). The last dimension is strategy selection (i.e., how we choose a strategy between many available alternatives). Older adults and AD patients have been found to be affected in different ways in all strategy dimensions.

In addition to describing how and explaining why older adults are affected in these dimensions of strategy use, we will show how, in each dimension, they compensate for age-related deficits. We review empirical evidence from memory, decision making, and numerical cognition. However, before tackling age- and pathology-related differences in strategy dimensions, we advance some methodological issues related to the assessment of strategies.

1.2. How to assess strategy variations

To determine which strategies are used, in what proportions they are used, and when they are used, we need to be able to know the strategy a participant uses on each trial. Several methods, all with their weaknesses and strengths, can be combined for assessing strategy use.

A first, very straightforward method, is to simply ask the participant what strategy he or she just used for the problem. However, such reports are not always accurate (Kirk & Ashcraft, 2001). First, there is the problem of validity. Some strategies are difficult to consciously access; participants may thus not always know how they actually solved the problem. Second, there is the problem of reactivity. Asking a participant to report the strategy he or she used may change his or her behavior. For example, participants may use those strategies that are easiest to report.

These problems make verbal reports not the most reliable method for establishing which strategies were used on particular problems. However, often they have found to be valid and to create no interference (e.g., Grabner & De Smedt, 2011). Many studies on strategic variations thus continue using verbal reports as a means for assessing which strategies participants use.

Verbal reports can be complemented by analyzing participants' behavior during problem solving. In numerosity estimation, eye movement patterns have been successfully used to determine what strategy an individual used to determine the number of dots in a display (Gandini, Lemaire, & Dufau, 2008; Green, Lemaire, & Dufau , 2007). For example, the use of counting during this task can be inferred from the sequential fixation of all the dots in a display. In some cases, chronometric data can also be used to infer strategies. Indeed, some strategies lead to longer solution latencies or to better accuracy than other strategies. For example, when solving arithmetic verification problems, an approximate calculation strategy will lead to faster latencies than an exact calculation strategy. This can be seen in the fact that problems that facilitate the use of approximate calculation (problems with a large difference between the size of the hands of an equation, e.g., $7 + 4 < 15$) are solved faster than problems requiring the use of exact calculation (problems with a small difference between the size of the hands of an equation, e.g., $7 + 4 < 12$) (e.g., Duverne & Lemaire, 2005).

However, in many cases it is not so evident to distinguish strategies based on eye movements and solution latencies. For example, in the case of solving addition problems, Thevenot, Fanget, and Fayol (2007) rightfully remark that short solution latencies could indicate use of the fast retrieval strategy, but also fast use of procedural strategies. Recently, these authors took a first step towards distinguishing retrieval and procedural strategies without verbal reports and chronometric measures by using the operand-retrieval paradigm. This paradigm infers the strategies that were used by looking at strategy after effects. These after effects are based on the assumption that traces of the operands of an arithmetic problem will be weaker after the use of a procedural strategy than after the use of retrieval, since during a procedural strategy we have to focus attention on the integration of partial results at the cost of the attention we pay to the initial operands. Consequently, the operands of a problem should be less accurately recognized if an individual used a procedural strategy to solve the problem. Thevenot, Barrouillet, and Fayol (2001) found evidence for this when they observed that participants recognized operands such as 23 and 48 better when they were merely compared than when they had to be added using a procedural strategy. Thevenot, Fanget, and Fayol (2007) thus argued that operand recognition constituted a valuable tool for distinguishing between the use of retrieval and procedural strategies on arithmetic problems (see also Thevenot et al, 2012; Thevenot, Castel, Fanget, & Fayol, 2010; Thevenot, Fanget, & Fayol, 2007).

When the question is not which strategy participants use, but the efficiency participants obtain with a particular strategy, the choice/no choice methodology proves to be a valuable tool (e.g., Siegler & Lemaire, 1997; Imbo & Vandierendonck, 2007; Lemaire & Lecacheur, 2002; Dierckx, Vandierendonck, & Pandelaere, 2003; see Luwel, Onghena, Torbeyns, Schillemans, & Verschaffel, 2009, for a review). This method entails presenting participants with a condition in which they can freely choose among available strategies on each problem and a condition in which the strategy is imposed on all problems. The solution latencies and accuracy obtained with a strategy in the choice condition gives us a measure of strategy efficiency when this strategy is judged appropriate by participants. The solution latencies and accuracy obtained with a strategy in the no choice condition provides a measure of efficiency uncontaminated by strategy preferences, differences in frequency of selection of different strategies, and unbiased by problem type. Both measures give complementary information on strategy execution efficiency.

We explore age- and pathology-related differences in strategy dimensions, using converging evidence from a variety of aforementioned methods.

1.3. Strategy repertoire

Whether young adults, healthy older adults, and AD patients use the same strategies to solve a problem is an important question. Declines in problem-solving performance could be due to the use of less adequate strategies whereas improvements could be due to the use of more adequate strategies.

To observe differences in the strategies young and older adults use in numerical cognition, Lemaire and Arnaud (2008) collected verbal protocols of participants while they were solving two-digit addition problems (e.g., 37 + 58). They observed use of the same nine strategies in young and older adults (see also Hodzik & Lemaire, 2011). Similarly, in AD patients, Arnaud, Lemaire, Allen, and Michel (2008) found the same variability in strategy use in young adults, healthy older adults, and AD patients. All groups used both retrieval and non-retrieval strategies during a simple subtraction task.

However, when Lemaire and Arnaud (2008) took a closer look at their data, they observed that individual young adults used 5.5 strategies on average whereas older adults used only 3.2 strategies. This suggests a reduction with age in the number of strategies employed during a cognitive task (see also Hodzik & Lemaire, 2011; Geary, French, & Wiley, 1993). Furthermore, Gandini, Lemaire, Anton, and Nazarian (2008) found that there were more older than young adults who used a single strategy when assessing the number of dots in a collection (see also Duverne, Lemaire, & Vandierendonck, 2008; El Yagoubi, Besson, & Lemaire, 2005;

Lemaire & Arnaud, 2008; Hodzik & Lemaire, 2011).

When looking at AD patients' data from Arnaud et al. (2008), we see that 15% of older adults and 17% of AD patients compared to 5% of young adults used only retrieval strategies on all problems. The same pattern was found in a study by Gandini, Lemaire, and Michel (2009). In a numerosity estimation task, 7.5% of young adults used the benchmark strategy on more than 94% of problems, compared to 35% of of healthy older adults, and an astonishing 49% of AD patients.

This pattern was also found in other domains, such as decision making. Chen and Sun (2003) found that while younger adults demonstrated flexibility using multiple strategies during a yard sale in which they had to sell objects at the highest price possible, older adults adopted a single strategy.

In the memory domain, like in numerical cognition and decision making, older adults use the same strategies as do younger adults. Dunlosky and Hertzog (1998) had young and older adults memorize word pairs (e.g., king-crown). The results showed that young and older adults used the same three strategies. They either used a mental-image strategy (e.g., constructing a mental image containing the two words of the pair), a sentence-construction strategy (e.g., forming a sentence containing the two words of the pair), or a repetition strategy (e.g., continuously repeating the two words of the pair). However, unlike in numerical cognition and decision making, the average number of mnemonic strategies used per person was the same in young (2.9) and older (2.8) adults (see also Bailey, Dunlosky, & Hertzog, 2009). This discrepancy with data from numerical cognition and decision making could be due to the limited number of strategies used in the task used by Dunlosky and Hertzog (e.g., a floor effect). Indeed, young adults used on average fewer strategies for this task (2.9) than older adults (3.2) for the task used by Lemaire and Arnaud (2008).

Thus, although healthy older adults and AD patients use the same strategies as do younger adults, they generally use fewer strategies within a task (except in the memory domain). A smaller strategy repertoire may reduce their ability to adaptively respond to each and every problem type, reducing the quality of problem solving. Maybe older adults have good reasons for using smaller strategy repertoires. Maybe, their accrued experience leads them to use only the best strategies during problem solving and to discard completely less efficient ones.

However, another explanation is more likely and can be sought in the decline of executive resources with age, and especially in AD. Maintaining multiple strategies simultaneously active during a task relies on working memory and executive functions (Kray & Lindenberger, 2000). Using multiple strategies could thus be very costly for older adults and AD patients.

In numerical cognition, Ardiale, Hodzik, and Lemaire (2012) found increased strategy switch costs in older adults as compared to young adults when the number of strategies to switch between increased. This suggests difficulties in maintaining and switching among multiple strategies in older adults. Although we know of no studies that have investigated strategy switch costs in AD patients, task switching literature has shown that AD patients have larger costs when having to hold multiple tasks online than healthy older adults (Belleville et al., 2008). This suggests that this population may have additional difficulties when using multiple strategies at the same time, compared to healthy older adults. Older adults and AD patients can compensate for these deficits by using smaller sets of active strategies to solve arithmetic problems.

Hodzik and Lemaire (2011) assessed young and older participants’ strategy repertoire and executive functions while they solved two-digit additions. They found that older adults had a smaller strategy repertoire, and that this was entirely predicted by reductions in executive functions (inhibition and flexibility) in older adults (see Figure 1). Whether AD patients compensate in the same way is an issue for which we have no data. The larger percentage of mono-strategic AD patients in Arnaud et al. (2008) and Gandini et al. (2009) suggests that they may compensate just like healthy older adults do. However, this result could also be the consequence of AD patients' failure to disengage from a particular strategy once they started using it.

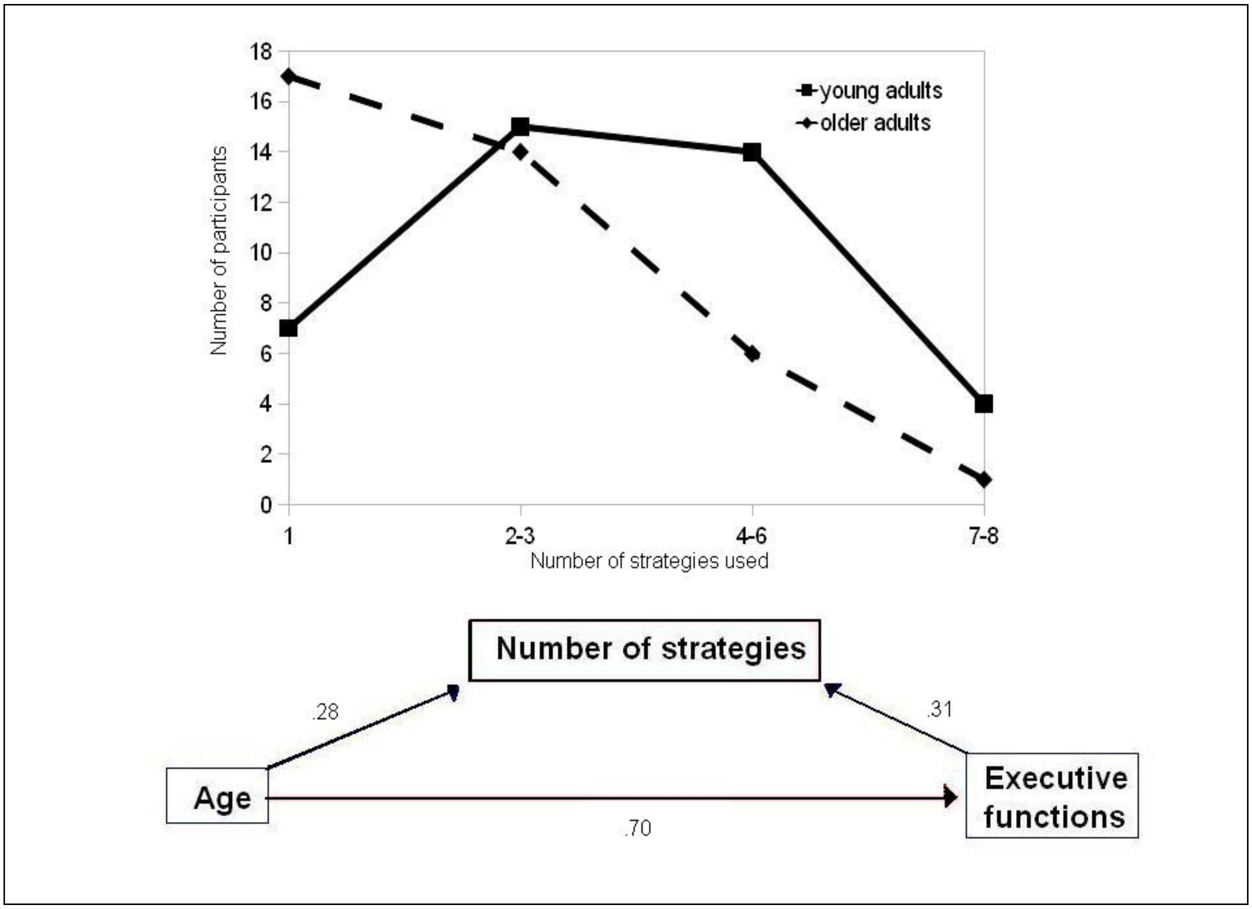


Figure 1. Top: Data from Hodzik &Lemaire (2011) showing that a large proportion of older adults use only one strategy when solving addition problems whereas most young adults use two to six strategies. Bottom: The decrease of the number of strategies with age was fully explained

by decreases in executive functions (inhibition and flexibility) in older adults (correlation coefficients).

1.4. Strategy execution

Even if young, healthy older adults, and AD patients use the same strategies, if they do not execute these the same way, differences in cognitive performance may arise. The fact that healthy older adults and AD patients have difficulties managing many different strategies or procedures at the same time may have repercussions on strategy execution as well. The difficulties maintaining multiple procedures could be found within numerical strategies, when these require a large degree of coordination of different processes (e.g., procedural strategies). Such strategies may be executed less well in healthy older adults and AD patients, whom suffer from reductions in executive resources.

In numerical cognition, Gandini, Lemaire, Anton, and Nazarian (2008) tested execution of numerosity estimation strategies such as the visual estimation strategy and the anchoring strategy. When using a visual estimation strategy to determine the numerosity of a set of items, we retrieve an answer from long-term memory. When using an anchoring strategy, we take a more precise approach, decomposing the itemsets in smaller groups, subitizing these, and adding the number of similar groups. When comparing young and older adults with these strategies, Gandini et al. (2008) found that both populations were as efficient with the visual estimation strategy. However, when using the anchoring strategy, older adults were slower than young adults (see also Gandini, Lemaire, & Michel, 2009). In arithmetic, healthy older adults have also been found to be impaired in the application of retrieval strategies (Arnaud, Lemaire, Allen, & Michel, 2008) and especially of complex procedures (Lemaire & Lecacheur, 2001; Lemaire & Arnaud, 2008; Salthouse & Coon, 1994; Duverne & Lemaire, 2005; Hodzik & Lemaire, 2011).

In AD patients, strategy execution is further impaired (e.g., Grafman et al., 1989; McGlinchey-Berroth et al., 1989; Arnaud, Lemaire, Allen, & Michel, 2008; Kaufmann et al., 2002; Duverne, Lemaire, & Michel, 2003), especially for more procedural strategies (Mantovan et al., 1999). Gandini, Lemaire, and Michel (2009) found that specific impairment of the anchoring strategy was stronger in AD patients than in healthy older adults. AD patients were 5.5 seconds slower when using anchoring than when using visual estimation, compared to 4.2 seconds in healthy older adults. In another example from numerical cognition, Arnaud et al. (2008) showed that AD patients were slower than healthy older adults when using a direct retrieval strategy on simple subtraction problems, but were relatively more impaired when using a simple yet procedural strategy such as counting (Figure 2).

Duverne, Lemaire, and Vandierendonck (2008) found that age-related impairments in the application of strategies on difficult problems were enlarged when working-memory executive components were simultaneously taxed by a secondary task. This suggests that executive resources play an important role in the age-related deficits in strategy execution.

In decision making, Mata, von Helversen, and Rieskamp (2010) (see also Mata & Nunes, 2010; Mata, Wilke, & Czienskowski, 2009) compared a decision environment favorable to the simple TTB strategy (i.e., Take The Best), in which the best out of two alternatives could be chosen relying on the information from a single indicator, to a decision environment favorable to the more complex WADD strategy (i.e., Weighted Additive), in which the best out of two alternatives could only be chosen relying on a combination of the information from multiple cues. When comparing young and older adults in these environments, Mata et al. (2010) found increased strategy execution errors in older adults, especially when using the more complex WADD strategy. The integration of the information from multiple indicators is a process that puts high demands on executive resources, explaining the age deficits when implementing the WADD strategy (see also Cohen & Faulkner, 1983; Finucane et al., 2005).

In the memory domain, Isingrini and Taconnat (2007) suggested that executive resource decreases play an important role in the inadequate usage of mnemonic strategies in older adults (see also Velanova, Lustig, Jacoby, & Buckner, 2007; Bouazzaoui et al., 2012). Taconnat et al. (2007; see also Taconnat et al., 2006; Angel et al., 2010) have shown that when the load on executive resources was increased by providing less cognitive support during retrieval from memory, older adults' performance suffered compared to young adults. Moreover, the extent to which performance was affected was associated to individual declines in executive resources (Figure 2).

Older adults can to some extent compensate for their difficulties with strategy execution. El Yagoubi, Lemaire, and Besson (2005) found that older adults recruited additional contralateral brain regions in comparison to young adults to solve addition problems. Moreover, additional recruitment was associated to successful performance in older adults. Similarly, in a functional MRI study, Gandini, Lemaire, Anton, and Nazarian (2008) found that older adults' strategies in numerosity estimation were supported by different cortical networks than in young adults, which may be a sign of age-related compensation.

In memory, Velanova, Lustig, Jacoby, and Buckner (2007) (Figure 2) showed increased recruitment of frontal regions in older adults relative to young adults when a retrieval strategy demanded heavy use of control processes. Moreover, the timing of increased recruitment in older adults occurred at relatively late stages of the retrieval event (see also Angel et al., 2009; 2010a; 2011), suggesting that older adults failed to engage appropriate top-down attentional control at early stages, but were able to compensate for this at later stages (see also Angel et al., 2010b).

This may underlie the often observed retention of high-level cognitive function during advanced aging at the cost of slower performance.

In AD patients however, prefrontal activity during a complex addition and subtraction task has been shown to be reduced compared to healthy controls (Rémy, Mirrashed, Campbell, & Richter, 2003). This reduced activity could suggest failure engaging appropriate top-down control in AD patients, which may explain their greater difficulty executing procedural strategies, which need a lot of this control.

Overall, it appears that at least in some cases, healthy older adults are able to engage a wider network of brain regions to execute the same strategies as younger adults. This could attenuate age-related declines in strategy execution. Without this functional compensation in older adults, perhaps we would see deficits in strategy execution even on simpler strategies. This may be the case for AD patients, who show less signs of functional compensation than healthy older adults. Hence, in this population we see impairments in simpler strategies such as counting (which healthy older adults perform equally well as do younger adults).

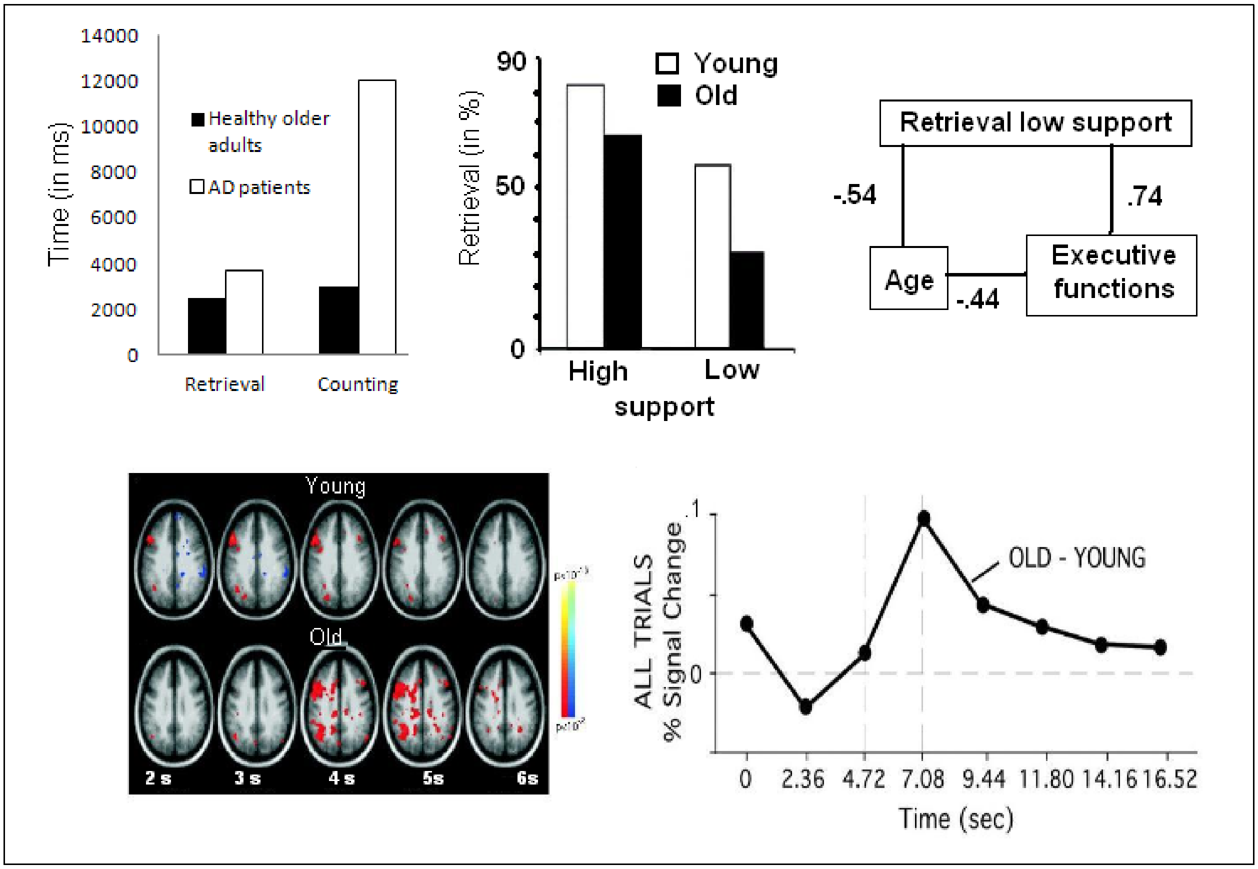


Figure 2. Top left: Data from Arnaud et al. (2006) showing the greater difficulty of AD patients when executing strategies on subtraction problems, especially for the more procedural counting strategy as opposed to the simpler direct retrieval strategy. Top middle: Data from Taconnat et al. (2007) showing that older adults recall a smaller proportion of items at retrieval, and more so when retrieval is made harder by providing low cognitive support. Top right: Data from Taconnat et al. (2007) showing that the impairments in older adults at retrieval with low support are explained by executive functions (correlation coefficients). Bottom left: Data from Velanova et al. (2007) showing that when high cognitive control is needed during retrieval, older adults initially underactivate compared to young adults, but they later compensate by activating more and bilateral frontal and parietal regions. Bottom right: Data from Velanova et al. (2007)

showing young and older adults' activation of frontal regions during retrieval-task performance under high cognitive control. We see an initial underactivation in older adults, following by a compensation that is larger in magnitude than the initial underactivation.

1.5. Strategy distribution

Older adults and AD patients know the same strategies as younger adults (Lemaire & Arnaud, 2008), but seem to execute the more difficult ones less well. They can compensate for this deficit in execution of harder strategies by changes in strategy distributions. With fewer resources, older adults tend to have a bias for easier strategies, even if those strategies do not yield the best performance.

For example, in a numerosity estimation task, older adults more often used the visual estimation strategy than younger adults (68% vs. 46%) (Gandini et al., 2008) Visual estimation is the numerosity estimation strategy older adults execute best (Gandini et al., 2008) because it relies in large part on retrieval from long-term memory, which is rich and well-organized in older adults through accumulated experience.

In AD patients, the shift towards easier strategies is even clearer. Gandini, Lemaire, and Michel (2009) found that the easier visual estimation strategy was used in 75% of cases by AD patients, compared to 62% in healthy older adults and 59% in young adults (see Figure 3). Moreover, the frequency of use of visual estimation is linked to the advantage in speed when using this strategy. Whereas AD patients gain about 5.5 seconds when using the visual estimation strategy compared to the anchoring strategy, young adults gain 'only' about 3.9 seconds (See Figure 3).

The shift to retrieval strategies has also been found when solving addition problems with either procedural or retrieval strategies (e.g., Geary, French, & Wiley, 1993; Thevenot et al., 2012). Thevenot et al. (2012) used the operand-recognition paradigm to assess strategies used by young and older adults on one-digit addition problems with sums larger than ten. They found that older adults used more often direct retrieval strategies on this type of problem than did young adults.

Similar findings of shifts in strategy distributions that can compensate for deficits in strategy execution have been found in other domains. In decision making, Mata, Schooler, and Rieskamp (2007) showed that older adults relied more on simpler strategies than younger adults, regardless of the decision environment. Even when the environment favored the use of a strategy necessitating the integration of information from multiple cues to reach a decision (WADD), older adults more often than young adults relied on a simpler strategy using the information from one cue only (TTB) (Figure 3)(see also Johnson, 1990).

In the memory domain, Bouazzaoui et al. (2010) have shown that the use of external memory strategies in everyday life increased with age whereas the use of internal memory strategies decreased. External memory strategies (e.g., writing down the things to remember) are easier for older adults because they require less executive resources whereas internal memory strategies (e.g., forming visual images) demand a lot of these resources.

Moreover, declines in fluid intelligence explain age-related shifts of strategy distributions towards easier strategies. Mata, Schooler, and Rieskamp (2007) found that the increased use of the easier TTB strategy in older adults was related to measures of fluid intelligence (Figure 3) such as reasoning (figural analogies, letter series and practical problems).

In memory, Bouazzaoui et al. (2010) found that the use of more difficult internal memory strategies in older adults correlated with the level of executive functioning (as measured by perseverative errors in WCST and verbal fluency). The better older adults' executive functioning, the more often they still use internal memory strategies.

It seems that older adults mainly shift to the use of easier strategies because they lack the capacities for implementing the more difficult ones that demand a high level of integration of information or cognitive control. This may be the very reason why AD patients, who are even more impaired in executive resources than healthy older adults, switch to use of simpler strategies even more than healthy older adults.

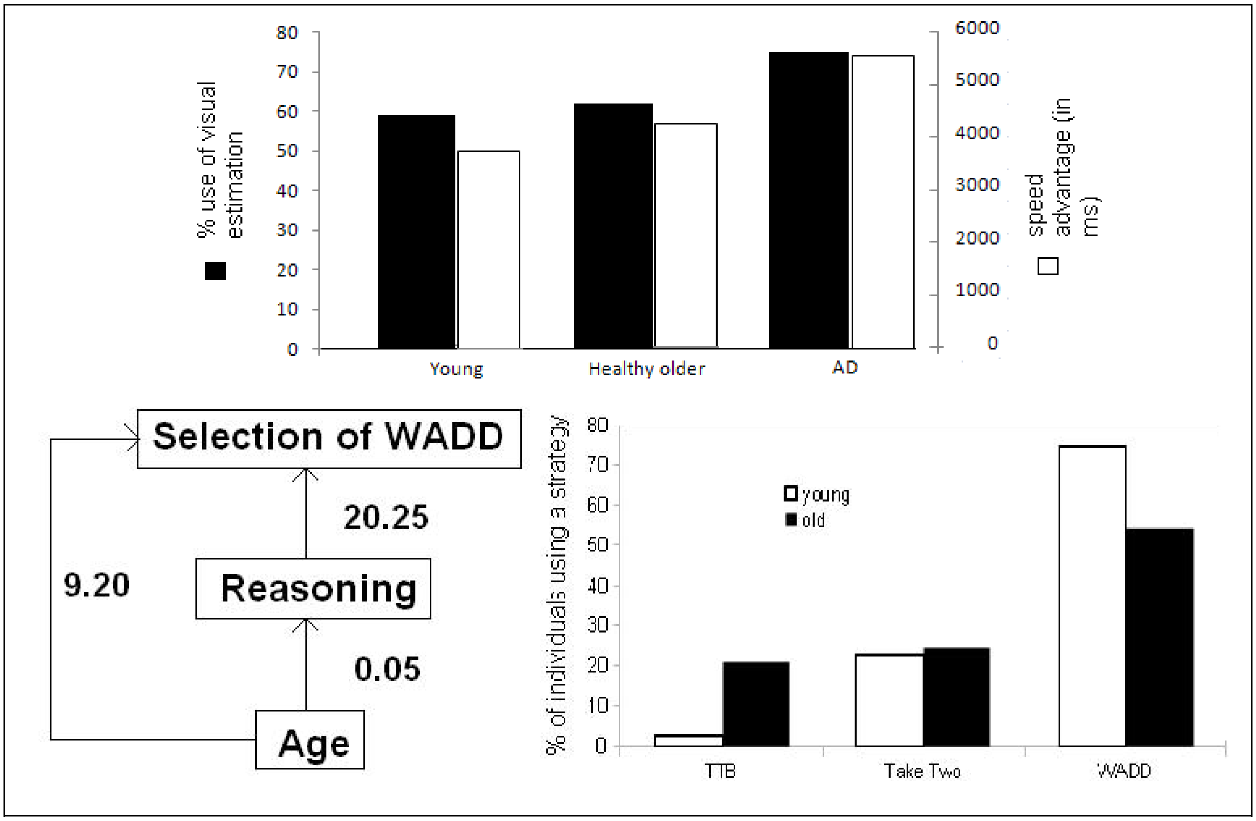


Figure 3. Top: Data from Gandini, Lemaire, & Michel (2009) displaying the percentage of use of the visual estimation strategy during a numerosity estimation task in young, healthy older adults, and AD patients, and the speed advantage obtained by using this strategy in these groups. Bottom left: Data from Mata, Schooler, & Rieskamp (2007) showing that selection of the

WADD is significantly explained by age and reasoning (logistic regressions with G2-values). Age and reasoning significantly predict selection of WADD (p<.01) but in a model including reasoning, age does not add any additional explanatory value (p<.83). Bottom right: Data from Mata, Schooler, & Rieskamp (2007) showing that older adults use less often the complicated WADD strategy than younger adults, and more often the simpler TTB strategy.

1.6. Strategy selection

Do young adults, healthy older adults, and AD patients adjust their strategy use equally well to different problem types? The fact that older adults seem to use easier and fewer strategies during problem solving could interfere with the adaptive selection of strategies as a function of problem characteristics. Independently of differences in strategy execution, if healthy older adults and/or AD patients do not adaptively adjust their strategies to problem and situation characteristics, their performance will suffer.

In healthy older adults, we could expect improvements of adaptive strategy selection, since older adults may have more experience relating strategies to problems and thus may have better problem-strategy associations than younger adults. Indeed, models of strategy selection (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) assume that the primary mechanism by which a strategy gets chosen on a problem is the association strength between the problem and the strategy, which is shaped by experience. However, some research suggests that the ability to select a strategy requires inhibitory control to suppress competing strategies, and flexibility to alternate between strategies (Lemaire & Lecacheur, 2010; Luwel et al., 2009). We could expect these capacities to decline with age, leading to less adaptive strategy selection.

Lemaire, Arnaud, and Lecacheur (2004) tested young and older adults in a computational estimation task. In this task, participants see problems such as 46 x 52 and have to find an approximative product without calculating the answer precisely. Approximate answers are obtained by using rounding strategies. Lemaire et al. (2004) focused on two types of rounding strategies: Rounding down and rounding up. During rounding down, both operands are rounded down to the closest decades (e.g., doing 40 x 50 to solve 43 x5 8). During rounding up, both operands are rounded up to the closest decades (e.g., doing 50 x 60 to solve 43 x 58). The advantage of testing rounding strategies is that it is easy to know which strategy yields the best performance on each problem. The rounding-down strategy will be the best strategy for a problem with small-digit units (e.g., 51 x 62) and the rounding-up strategy will be the best strategy for a problem with large-unit digits (e.g., 37 x 69). The task given to young and older adults by Lemaire et al. (2004) was to choose the best strategy on each problem. The data clearly showed that young adults selected the best strategy on 63% of problems, whereas older adults did so on only 56% of problems (see also Lemaire & Arnaud, 2008; Duverne, Lemaire, and Michel, 2003; Gandini, Lemaire, & Michel, 2009; Hodzik & Lemaire, 2011; Duverne & Lemaire, 2005; 2004; Duverne, Lemaire, & Vandierendonck, 2007; El Yagoubi, Lemaire, & Besson, 2003; Arnaud, Lemaire, Allen, & Michel, 2008).

In AD patients, Duverne, Lemaire, and Michel (2003) revealed that AD patients less systematically chose approximate verification strategies and exact calculation strategies on arithmetic verification problems than did younger adults (see Gandini, Lemaire, & Michel, 2009 for an example in numerosity estimation). However, their performance was as good as that of healthy older adults. Arnaud, Lemaire, Allen, and Michel (2008) showed that AD patients chose retrieval and non-retrieval strategies as systematically during an arithmetic task than did healthy older adults. These findings suggest that pathological aging did not affect strategy selection mechanisms above and beyond normal aging.

In the decision-making domain, Mata, von Helversen, and Rieskamp (2010) have shown that older and younger adults generally use the simpler TTB strategy in environments where information from a single cue suffices to make a valid decision and use the more complex WADD strategy in environments where the information from multiple cues needs to be combined to lead to valid decisions. However, older adults were less able than their younger counterparts to adapt their selection of TTB and WADD from trial to trial using performance feedback.

In the memory domain, Souchay and Isingrini (2004) have shown that older adults were less able than young adults to adjust their study time in accordance to the difficulty of the learning task. Whereas young adults would allocate more study time when the learning task was difficult, older adults would not adjust the amount of study time equally well as did younger adults (see also Dunlosky & Connor, 1997; Schmitt, Murphy, & Sanders, 1981).

Thus, it seems that older adults' strategy selection suffers from declines in executive resources rather than benefit from accrued experience and more fine-tuned problem-strategy associations. Consistent with this, Hodzik and Lemaire (2011) showed that lesser strategy adaptivity in choosing rounding strategies on computational estimation problems in older adults was partly mediated by decreased efficiency of executive functions (inhibition and flexibility) (see Figure 4).

Souchay and Isingrini (2004) also found that the inability of older adults to adjust study time as a function of learning difficulty was mediated by executive functions (see also Taconnat et al., 2009). Similarly, Hayes, Kelly, and Smith (2012) found that working-memory capacity in older adults was related to the ability to selectively encode information as part of a learning strategy. Moreover, Castel, Balota, and McCabe (2009) showed larger deficiencies in selective encoding of information in AD patients, equally related to reduced performance on complex working-memory span tasks.

Sliwinski, Buschke, Kuslansky, and Scarisbrick (1994; see also Lemaire & Lecacheur, 2010, Expt. 2) showed that requiring participants to initiate a new arithmetic operation produced larger age differences than requiring them to repeat an operation (Figure 4), consistent with the view that flexibly shifting between strategies is impaired in older age.

To compensate for their deficiencies in inhibition and flexibility, older adults could repeat strategies across trials more often, leading to less adaptive strategy selection. Ardiale and Lemaire (in press) found that, during within-item strategy switching (i.e., changing strategy on the same item after starting to execute the poorest strategy), older adults tended to continue executing the same strategy more often than young adults, even when it would have been more efficient to switch strategies. Recent work by Lemaire and Leclere (submitted) showed that older adults tended to repeat strategies more often also between items than did younger adults. In extreme cases, older adults will consistently select only one strategy, to avoid switching at all (e.g., Duverne, Lemaire, & Vandierendonck, 2008; El Yagoubi, Besson, & Lemaire, 2005; Lemaire & Arnaud, 2008; Gandini, Lemaire, Anton, & Nazarian, 2008).

Surprisingly, AD patients were not more impaired than healthy older adults in their ability to systematically choose the best strategy on each item in numerical cognition (Duverne, Lemaire, & Michel, 2003; Gandini, Lemaire, & Michel, 2009; Arnaud, Lemaire, Allen, & Michel, 2008), unlike in the memory domain (Castel, Balota, & McCabe, 2009). We would have expected them to be less adaptive than healthy older adults, since they have further reduced executive resources. Leclere and Lemaire recently submitted a paper showing lesser strategy selection adaptivity in AD patients than in healthy older adults in computational estimation. On easy problems (for which the best strategy is obvious), healthy older adults selected the best strategy on 95% of problems compared to 79% in AD patients. Leclere and Lemaire suggest that AD patients' strategy selection adaptivity was impaired in their study because they studied complex arithmetic, which is more impaired in AD patients than simple arithmetic studied in previous studies finding no effect of pathological aging on strategy selection adaptivity (Duverne, Lemaire, & Michel 2003; Arnaud, Lemaire, Allen, & Michel, 2008). Moreover, Leclere and Lemaire found that when repetition was inadequate, healthy older adults repeated in 39% of cases, compared to 71% in AD patients.

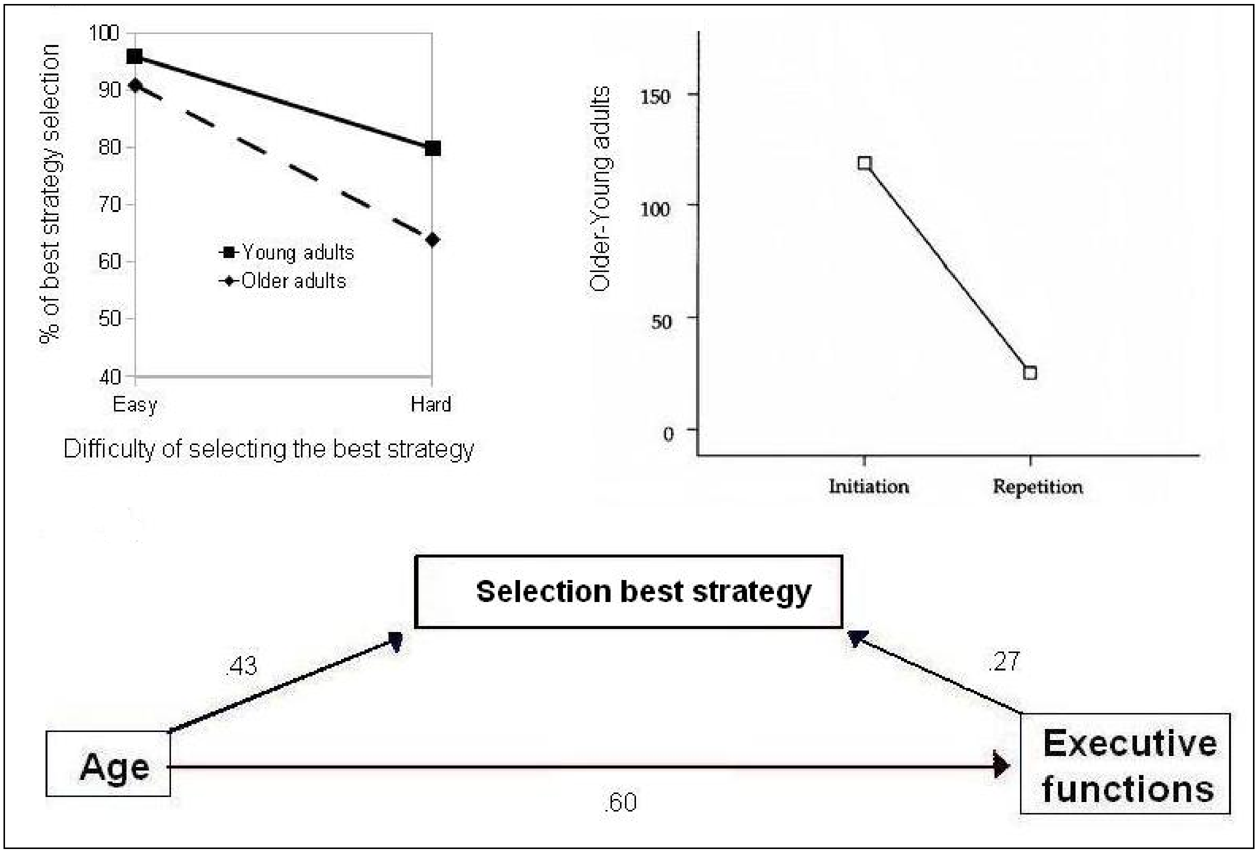


Figure 4. *Top left: Data from Hodzik & Lemaire (2011) showing that older adults choose the best strategy less often than young adults, especially on problems for which choosing the best strategy was difficult. Top right: Reaction time data from Sliwinski et al. (1994) showing that older adults were more impaired when they had to initiate a new arithmetic operation than when they had to repeat the previous arithmetic operation. Bottom: Data from Hodzik & Lemaire (2011) suggesting that the decreased selection of the best strategy in older adults was at least partly explained by decreases in executive functions in older adults (correlation coefficients).*

1.7. Conclusion

The findings presented here clearly demonstrate that, whatever the cognitive task, numerical cognition, decision making or memory, if we want to understand individuals' performance and age-related variations, we have to determine the strategies they use, and how they select and execute them. We investigated different strategy dimensions and found age-related differences in each of them. Moreover, by studying these dimensions, we were able to pinpoint how older adults and AD patients can compensate for deficits related to their age or pathology.

Interesting in our approach is that we did not merely describe differences in strategy dimensions but we also tried to understand how these differences arise. In strategy repertoire, decline of executive resources rather than accrued experience seems to contribute to older adults' and AD patients' reduced strategy repertoire. Decreases in the capacities to flexibly alternate between strategies lead older adults and AD patients to use fewer strategies and sometimes even only a single strategy during problem solving.

Regarding strategy execution, experience leads to some strategies being executed equally efficiently in young, healthy older adults, and AD patients, such as the visual estimation strategy and direct retrieval. Decline in executive resources leads to other strategies being less efficiently executed, such is the case of arithmetic back-up strategies.

However, we need to be cautious when assuming that some strategies are executed as efficiently in older as in younger adults. When comparing brain activities between young and older adults, we often find increased and/or delayed brain activities in older adults when executing even simpler retrieval strategies. Without these increases in brain activities we might find that even simpler strategies are executed less efficiently in older adults. In AD patients, such neuro-functional compensations seem to be non existent. This could be the reason for seeing declines in strategy execution in this population even for simpler strategies such as counting.

Very conveniently, healthy older adults and AD patients will often use those strategies that their experience permits them to execute well, instead of relying on more complex strategies that put a heavy load on their executive resources.

Finally, strategy selection in older adults and AD patients seems to suffer from declined executive resources rather than benefit from increasingly fine-tuned associations. This may be somewhat surprising, since several models of strategy selection (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) propose that the primary mechanism by which a strategy gets chosen on a problem is the association strength between the problem and the strategy. Is this discrepancy between the models and empirical data explicable by the mere involvement of executive functions in strategy selection that prevent older adults from flexibly switching between strategies? Or have older adults' associations between problems and strategies, in spite of accumulation of experience, deteriorated? (see for example Fabre & Lemaire, 2005) Future research could disentangle these alternatives by testing strategy adaptivity in young and older adults, without requiring rapid switching. If given sufficient time between problem presentations, maybe older adults will choose their strategies more adaptively. Recent results in the course of preparation, by Leclere and Lemaire, seem indeed to suggest that with increased response-stimulus intervals, healthy older adults manage to switch strategies as much as young adults (46% vs 47% of strategy switching respectively). However, in their study, Leclere and Lemaire did not test whether this also implied that older adults were as adaptive in their strategy choices as young adults.

Table 1: Overview of studies on strategy repertoire in young and healthy older adults, and AD patients

	Repertoire	
Domain	**Effects**	**References**
Numerical	Young and older adults use the same strategies to solve complex arithmetic problems	Lemaire and Arnaud, 2008; Arnaud, Lemaire, Allen, & Michel, 2008; Hodzik & Lemaire, 2011
	AD patients use the same strategies to solve complex arithmetic problems as young and healthy older adults	Arnaud, Lemaire, Allen, & Michel, 2008
	Young adults use more strategies per person than older adults	Lemaire & Arnaud, 2008; Hodzik & Lemaire, 2011; Geary, French, & Wiley, 1993
	More older adults than young adults are monostrategic	Gandini, Lemaire, Anton, & Nazarian, 2008; Duverne, Lemaire, & Vandierendonck, 2008; El Yagoubi, Besson, & Lemaire, 2005; Lemaire & Arnaud, 2008; Hodzik & Lemaire, 2011
	More AD patients than healthy older adults and young adults are monostrategic	Arnaud, Lemaire, Allen, & Michel, 2008; Gandini, Lemaire, & Michel, 2009
	More difficulties using multiple strategies in older than in young adults	Ardiale, Hodzik, & Lemaire, 2012
	Reduced executive functions explain reductions in the number of strategies older adults use	Hodzik & Lemaire, 2011
Decision making	More older adults than young adults are monostrategic	Chen & Sun, 2003
Memory	Young and older adults use the same strategies to memorize items	Dunlosky & Hertzog, 1998
	Young and older adults use the same number of strategies to memorize items	Dunlosky & Hertzog, 1998; Bailey, Dunlosky, & Hertzog, 2009

Table 2: Overview of studies on strategy execution in young and healthy older adults, and AD patients

Execution

Domain	Effects	References
Numerical	Young and older adults execute retrieval strategies equally well	Gandini, Lemaire, Anton, & Nazarian, 2008
	Older adults are impaired at execution of retrieval strategies	Gandini, Lemaire, & Michel, 2009; Arnaud, Lemaire, Allen, & Michel, 2008
	Older adults are impaired at execution of procedural strategies	Gandini, Lemaire, Anton, & Nazarian, 2008; Gandini, Lemaire, & Michel, 2009; Lemaire & Lecacheur, 2001; Lemaire & Arnaud, 2008; Salthouse & Coon, 1994; Duverne & Lemaire, 2005; Hodzik & Lemaire, 2011
	AD patients are impaired in execution of retrieval strategies	Arnaud, Lemaire, Allen, & Michel, 2008; Grafman et al., 1989; McGlinchey-Berroth, Milberg, & Charness, 1989; Kaufmann et al., 2002; Duverne, Lemaire, & Michel, 2003
	AD patients are more impaired at execution of procedural strategies than retrieval strategies	Mantovan et al., 1999; Gandini, Lemaire, & Michel, 2009; Arnaud, Lemaire, Allen, & Michel, 2008; McGlinchey-Berroth, Milberg, & Charness, 1989
	Strategy execution deficits in older adults are influenced by executive resources	Duverne, Lemaire, & Vandierendonck, 2008
	Older adults use additional brain regions while executing the same strategies as younger adults	El Yagoubi, Lemaire, & Besson, 2005; Gandini, Lemaire, Anton, & Nazarian, 2008
	AD patients do not use additional brain regions while executing the same strategies as healthy older and young adults	Rémy, Mirrashed, Campbell, & Richter, 2003
Decision making	More strategy execution errors and longer execution times in older adults for complexer strategies	Mata, von Helversen, & Rieskamp, 2010; Mata, Wilke, & Czienskowski, 2009; Cohen & Faulkner, 1983; Finucane et al., 2005
Memory	Executive resource decreases lead to impaired strategy execution in older adults	Isingrini & Taconnat, 2007; Velanova, Lustig, Jacoby, & Buckner, 2007; Bouazzaoui et al., 2012; Taconnat et al., 2007; Taconnat et al., 2006; Angel et al., 2010
	Older adults are impaired when retrieval strategies require more cognitive control	Taconnat et al., 2006; 2007; Angel et al., 2010
	Increased and delayed frontal activity in older adults when executing more demanding retrieval strategies	Velanova, Lustig, Jacoby, & Buckner, 2007; Angel; 2009; 2010; 2011

Table 3: Overview of studies on strategy distribution in young and healthy older adults, and AD patients

Distribution

Domain	Effects	References
Numerical	Older adults more often use retrieval strategies	Gandini et al., 2008; Gandini, Lemaire, & Michel, 2009; Thevenot et al., 2012; Geary, French, & Wiley, 1993
	AD patiens more often than healthy older adults use retrieval strategies	Gandini, Lemaire, & Michel, 2009
	Use of retrieval strategies in older adults and AD patients is linked to speed advantages	Gandini, Lemaire, & Michel, 2009
Decision making	Older adults rely more often on simpler strategies	Mata, Schooler, & Rieskamp, 2007; Johnson, 1990
	Use of simpler strategies in older adults is related to declines in fluid intelligence	Mata, Schooler, & Rieskamp, 2007
Memory	Older adults rely more often on strategies requiring less executive resources	Bouazzaoui et al., 2010
	Use of strategies requiring less executive resources in older adults linked to declines in executive functions	Bouazzaoui et al., 2010

Table 4: Overview of studies on strategy selection in young and healthy older adults, and AD patients

Selection

Domain	Effects	References
Numerical	Older adults less often choose the best strategy on each problem than young adults	Lemaire, Arnaud, & Lecacheur, 2004; Lemaire & Arnaud, 2008; Duverne, Lemaire, and Michel, 2003; Gandini, Lemaire, & Michel, 2009; Hodzik & Lemaire, 2011; Duverne & Lemaire, 2005; 2004; Duverne, Lemaire, & Vandierendonck, 2007; El Yagoubi, Lemaire, & Besson, 2003; Arnaud, Lemaire, Allen, & Michel, 2008
	AD patients choose the best strategy on each problem as often as healthy older adults	Duverne, Lemaire, & Michel, 2003; Gandini, Lemaire, & Michel, 2009; Arnaud, Lemaire, Allen, & Michel, 2008
	AD patients less adaptive than healthy older adults	Lemaire & Leclere (submitted)
	Lesser strategy adaptivity in older adults mediated by declines in executive functions	Hodzik & Lemaire, 2011
	Older adults are impaired compared with young adults when having to switch	Sliwinski, Buschke, Kuslansky, & Scarisbrick, 1994; Lemaire & Lecacheur, 2010
	Older adults repeat strategies more often over consecutive trials than young adults	Ardiale & Lemaire (in press); Lemaire & Leclere (submitted)
	AD patients repeat strategies more often over consecutive trials than healthy older adults	Lemaire & Leclere (submitted)
Decision making	Older adults were less able than young adults to adapt their selection of strategies from trial to trial	Mata, von Helversen, & Rieskamp, 2010
Memory	Older adults were less able than young adults to adapt their strategy to problem difficulty	Souchay & Isingrini, 2004; Taconnat et al., 2009; Dunlosky & Connor, 1997; Schmitt, Murphy, & Sanders, 1981
	Lesser strategy adaptivity in older adults mediated by declines in executive functions	Souchay & Isingrini, 2004; Hayes, Kelly, & Smith, 2012; Taconnat et al., 2009
	Strategy adaptivity further reduced in AD patients, mediated by declines in working-memory capacity	Castel, Balota, & McCabe, 2009

Numerical Strategy Efficiency

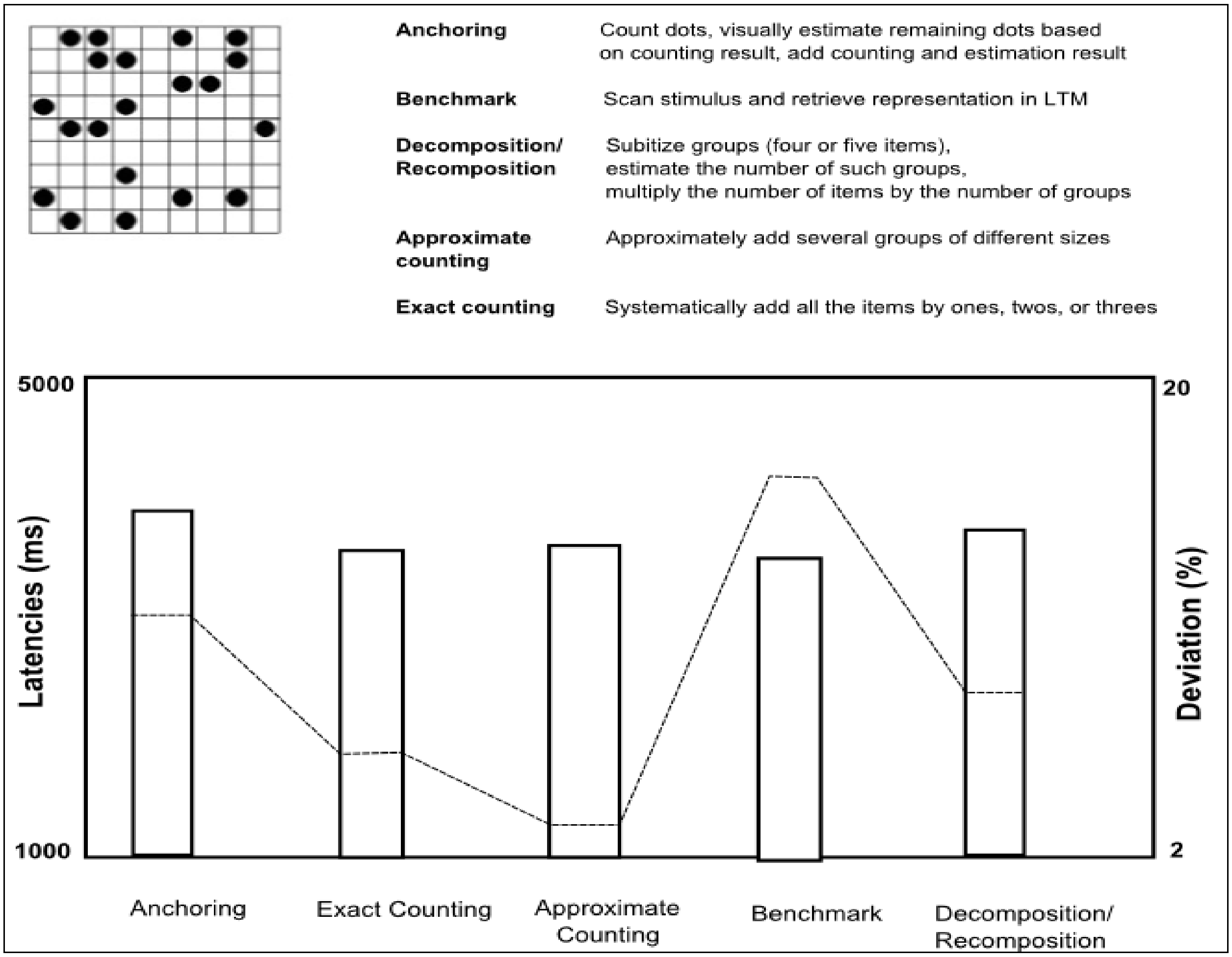
What determines numerical performance? Besides the strategies we choose to solve problems, efficiency with which we execute strategies is an important factor. Strategy efficiency can be characterized by speed and precision. These measures differ between strategies (Lemaire & Siegler, 1995). Before introducing our own experiments studying determinants of numerical strategy execution efficiency, we explore what is already known.

Empirical studies on strategy execution have discovered multiple determinants of strategy execution efficiency. Characteristics of problems (e.g., Lemaire, Arnaud, & Lecacheur, 2004; Siegler & Shrager, 1984), participants (e.g., Arnaud et al., 2006; Geary, Bow-Thomas, Liu, & Siegler, 1993), strategies (e.g., Gandini, Lemaire, & Dufau, 2008), and situations (Siegler, 1987) have been found to influence strategy speed and accuracy. For example, Lemaire, Arnaud, and Lecacheur (2004) found that when participants accomplished a computational estimation task (i.e., finding an approximate product to two-digit multiplication problems like 32 x 47), older adults executed the rounding-down strategy (i.e., doing 30 x 40 to estimate 32 x 41) more slowly under accuracy-pressure conditions than under no-pressure conditions, especially when they solved easy problems. This strategy x problem interaction was even stronger in young adults.

Several models have attempted to formalize the mechanisms involved in strategy selection and execution (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005). Regarding strategy execution, these models state that strategies involving more and more complex procedures are executed slower. For example, in numerosity estimation, people are on average faster when using the 'benchmark strategy' (i.e., visual estimation, or the retrieval of a stored numerical representation) than when using the 'anchoring strategy' (i.e., a combination of visual estimation and counting) (Gandini, Lemaire, & Dufau, 2008; see Figure 5). The slower execution of the anchoring strategy results from the involvement of more complicated processes.

In this chapter, we want to go beyond a mere description of the conditions under which strategy execution efficiency varies. To achieve this, we found it useful to analyze numerical strategies and identify the processes involved in these strategies (i.e., componential analysis, see for example Geary & Lin, 1998). In the decision-making domain, Payne, Bettman, and Johnson (1988) already recognized the value of this approach when they decomposed decision-making strategies (i.e., strategies to determine which of two alternatives to choose) in more elementary components to provide a more objective measure of the effort necessary for using a strategy. Based on such analysis of numerical strategies, we hoped to obtain a framework permitting us to understand why strategy execution varies with characteristics of strategies, participants, problems, and situations.

For example, the anchoring strategy in numerical estimation is composed of counting, visual estimation, decomposition and arithmetic fact retrieval whereas the benchmark strategy mostly consists of visual estimation (see Figure 5, top). We could say that counting, estimation, retrieval, and decomposition are the building blocks that are mixed in specific ways to make up the anchoring strategy, the benchmark strategy, and other numerosity estimation strategies such as exact counting. Numerical strategies used in other tasks, such as mental arithmetic, can be similarly decomposed in more basic processes.



Anchoring	Count dots, visually estimate remaining dots based on counting result, add counting and estimation result
Benchmark	Scan stimulus and retrieve representation in LTM
Decomposition/ Recomposition	Subitize groups (four or five items), estimate the number of such groups, multiply the number of items by the number of groups
Approximate counting	Approximately add several groups of different sizes
Exact counting	Systematically add all the items by ones, twos, or threes

Figure 5. Top: Data from Gandini et al. (2008) that document the strategies used by young adults during a numerosity estimation task. Bottom: The solution latencies and accuracies (% deviation) with these strategies (Gandini et al., 2008).

Here, we discuss two main types of processes involved in numerical strategies. We distinguish between retrieval-based processes (e.g., arithmetic fact retrieval, estimation, etc.) and procedural processes (e.g., problem decomposition, counting, etc.). In Chapter 1, we discussed how healthy older adults performed equally well with most retrieval-based strategies and were impaired when using procedural strategies. Both types of processes require certain cognitive resources, and it is this aspect that affects strategy efficiency, and generates variations with participants, problems, and situations. The most relevant cognitive resources for numerical procedures are numerical long-term memory and executive resources.

Numerical long-term memory refers to stored declarative and procedural numerical knowledge, and executive resources refer to the control processes necessary for maintaining, manipulating, and coordinating information. These resources differently affect strategy execution efficiency. Long-term memory has a large capacity and depends on accumulation of experience, which will render strategy execution more efficient. For executive resources, the most important aspect is that they are limited (Marois & Ivanoff, 2005; Barrouillet, Bernardin, & Camos, 2004; Kane & Engle, 2002), so the more strategies rely on them, the harder it will be to execute them. We look at both types of resources in more detail.

2.1. Numerical long-term memory

On the most basic level, numerical long-term memory involves magnitude representations, supported by neuronal populations in the brain tuned to specific numerosities (Eger et al., 2003). For example, when an individual sees a quantity of four, some neurons in the intra-parietal sulcus will fire at their peak rate, permitting the individual to identify a numerosity of four, different from three or five.

On a higher level, we find symbolic number, which is acquired by mapping symbols to these analog quantities. Symbolic numbers are the basis of an associative network called the arithmetic facts base (Siegler, 1988). For example, when an individual is asked how much is 4 x 5, he will be able to retrieve 20, if this solution is well associated to the problem.

Finally, numerical long-term memory involves conceptual and procedural knowledge (Rittle-Johnson, Siegler, & Alibali, 2001; LeFevre et al., 2006). Conceptual knowledge refers to the understanding of numbers and the relationships between them. For example, most people know that any number times zero equals zero or that any number divided by itself equals one. Procedural knowledge refers to strategies that can be applied on numerical problems. For example, when we have to resolve a long division manually, we will retrieve an appropriate procedure from numerical long-term memory.

All processes involved in numerical strategies will require the acquisition of some level of knowledge. Most evidence for this claim comes from developmental studies investigating the role of instruction on the development of mathematical abilities. For example, Opfer and Siegler (2007) found that children that received instruction became more accurate in number-line estimation (see Siegler & Shrager, 1984, for the role of experience in arithmetic fact retrieval; Camos, Fayol, & Barrouillet, 1999; Pratt & Savoy-Levine, 1998, for acquisition of procedural knowledge necessary for counting and solving of long divisions).

2.2. Executive resources

Baddeley and Hitch (1974) were among the first to pay attention to executive resources, by proposing the existence of a central executive, coordinating slave systems that maintain information representations (i.e., phonological loop and visuospatial sketchpad).

Miyake et al. (2000) proposed that the actions of the central executive can actually be decomposed in a number of related yet distinct executive functions (e.g., inhibition, shifting, and updating). These executive functions are necessary for the active maintenance of information. Indeed, active information processing requires disengaging from previously relevant information and activating newly relevant information (shifting), inhibiting competing information, and updating relevant information.

Miller and Cohen (2001) suggested that executive functions themselves rely on active maintenance of the goals and rules of a task, which would bias attention in favor of task-relevant information and consequently foster inhibition of irrelevant information. Either way, executive functions and active maintenance of information (i.e., working memory) are closely related. Given the interdependence of executive functions and working memory, we will in this chapter refer to both by the term 'executive resources'.

Probably the most limiting factor in human information processing is the fact that these resources are severely limited. The number of chunks (i.e., units of information) that can be held in the focus of attention (i.e., working-memory span) is only seven on average, and less when a concurrent task has to be performed (e.g., arithmetic operations, Ilkowska & Engle, 2010).

Barrouillet, Bernardin, and Camos (2004) proposed the time-based resource sharing model to deal with the limited capacity of executive resources. In their model, processing and maintenance of information both rely on the same resource, which is attention. Maintenance of information thus interferes with active processing of information and vice versa. Barrouillet et al. further assume that attention can only be focused on a single item at a time and that traces of items in working memory decay with time. Attention is needed to refresh these items before their traces have completely disappeared. Thus, attention needs to be rapidly switched between items to maintain or process several items in a more or less concurrent manner. In this view, the limits of working memory would depend on the rapidity with which we can shift our attention between elements and the time for memory traces of these elements to completely decay in the absence of refreshing.

Numerical strategies, like strategies in other domains, such as memory and decision-making, require executive resources. Most evidence for this claim comes from dual-task and correlational studies. In dual-task studies, additional executive tasks occupy limited executive resources so that numerical strategy execution becomes less efficient. For example, Imbo, Duverne, and Lemaire (2007) had participants execute computational estimation strategies (e.g., doing 40 x 70 to estimate 43 x 72) and found that when they presented a simultaneous choice reaction task demanding executive resources (deciding whether randomly presented tones were high or low), participants executed numerical strategies less well (see also Imbo & Vandierendonck, 2007; Duverne, Lemaire, & Vandierendonck, 2008).

In correlational studies, individual differences in working-memory capacity or executive functions are correlated to efficiency of strategy execution. For example, Andersson (2008) administered tests of executive functions (e.g., verbal fluency, Trails task, Stroop) and working memory (counting span, digit span, Corsi span) to children. He found that performance on these correlated with strategy efficiency on complex arithmetic problems (see also Agostino, Johnson, & Pascual-Leone, 2010; Rasmussen & Bisanz, 2005; Bull & Scerif, 2001).

2.3. Retrieval-based versus procedural processes

Whereas retrieval-based processes will rely relatively more on relevant experience than on executive resources, procedural processes will involve relatively more executive resources

(Tronsky, 2005; Caviola, Mammarella, Cornoldi, Lucangeli, 2012; Imbo & Vandierendonck, 2007; Imbo & Vandierendonck, 2008). Therefore, strategies involving more procedural processes will suffer more from executive resource shortages and will take longer to execute than strategies relying more on retrieval processes.

For example, Tronsky (2005) had participants solve complex multiplication problems (e.g., 3 x 18) under dual-task conditions and found that working memory was more involved in more difficult non-retrieval strategies than in easier retrieval strategies (see also Caviola, Mammarella, Cornoldi, Lucangeli, 2012; Imbo & Vandierendonck, 2007). Imbo and Vandierendonck (2008) found higher correlations between working-memory span and performance when children used procedural strategies than retrieval strategies on addition and multiplication problems (see Figure 6).

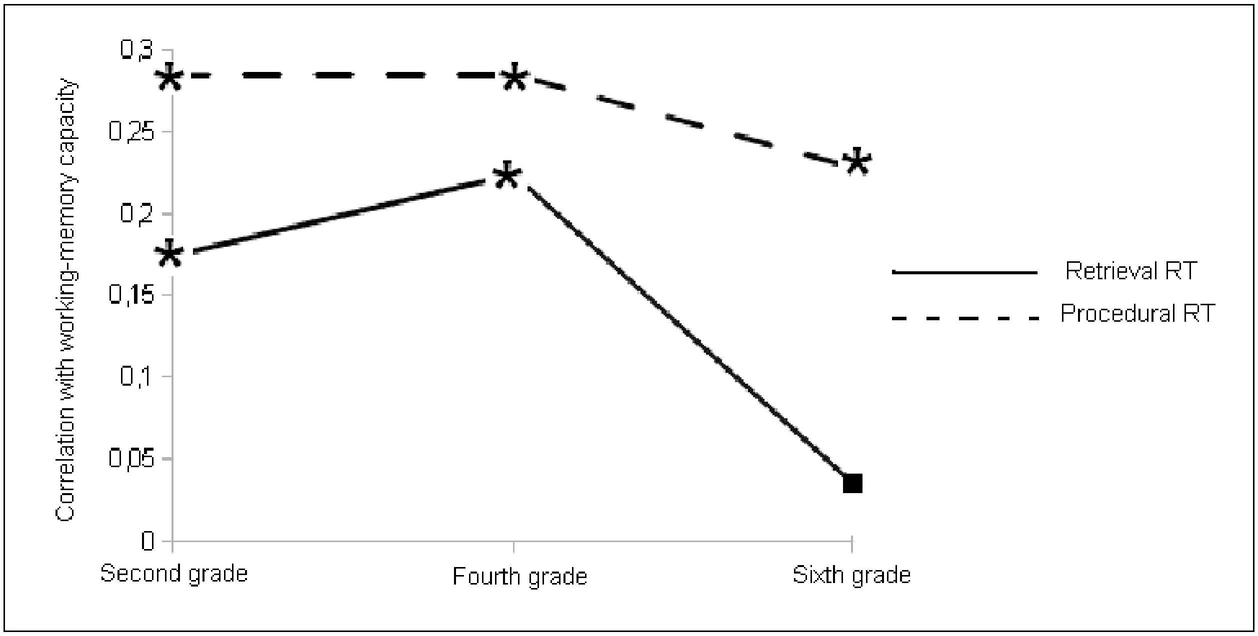


Figure 6. Data from Imbo & Vandierendonck (2008) showing the correlations between working-memory span in children from three grades and execution speed of retrieval and procedural strategies on addition and multiplication problems. Asterixes indicate significant correlations.

The reliance of strategies on more retrieval-based or more procedural processes can thus explain differences in efficiency. We review retrieval-based and procedural processes involved in numerical strategies and how acquired numerical knowledge and executive resources play a role in these processes. Moreover, we show how the availability of numerical knowledge and executive resources can vary and thus generate determinants of strategy execution.

2.3.1. Retrieval-based processes

We review two processes often involved in numerical strategies, which rely heavily on retrieval. Estimation requires the retrieval from long-term memory of a stored numerical representation. Arithmetic fact retrieval requires the retrieval of an arithmetic fact from the arithmetic facts database.

Estimation is part of many numerical strategies. Often, the situation does not permit us to obtain an exact measure of number. For example, we will not calculate the exact total price of all the articles we buy in a supermarket but we might want to keep an approximate count so as to not overspend. The efficiency of strategies that involve estimation will rely on the quality of stored numerical representations (Booth & Siegler, 2008; 2006; Siegler & Opfer, 2003; Gandini, Lemaire, & Michel, 2009).

Ideally, we would assess quality of numerical representations by using single-cell recordings to look at discriminatory firing of neurons in the hIPS to different numerosities. However, such invasive techniques are not available in humans so we have to indirectly assess numerical representations by basing ourselves on behavior.

The numerical distance effect is a behavioral effect resulting from neural boundaries between adjacent quantities being fuzzy, which makes it harder to compare them. In reaction-time experiments, we usually find that reaction times are longer when adjacent quantities are compared in contrast to distant quantities, known as 'the numerical distance effect' (Moyer & Landauer, 1967). For example, we will find longer reaction times when we ask individuals to choose the biggest number between 7 and 8, than when we ask them to choose between 4 and 11. Larger numerical distance effects (i.e., more difficulties discriminating adjacent quantities) supposedly reflect less fine numerical representations, because it means that adjacent quantities are represented less distinctly in the brain. In this logic, numerical distance effects can be used as a tool to indirectly assess the fineness of underlying numerical representations (Holloway & Ansari, 2009).

In children, initial magnitude representation is coarse. Wood, Ischebeck, Koppelstaetter, Gotwald, and Kaufmann (2009) conducted a study investigating the magnitude of numerical distance effects in children, young adults, and older adults. Children showed larger numerical distance effects (see Figure 7) than young and older adults. Moreover, initial numerical representations in children are logarithmic (i.e., smaller numbers are given more weight). Through brain maturation and experience, numerical representations become finer in children (Holloway & Ansari, 2008), and more linear (Siegler & Booth, 2004; see also Ashcraft & Moore, 2012; Booth & Siegler, 2008; 2006; Siegler & Opfer, 2003; Ramani & Siegler, 2008; Siegler & Ramani, 2008).

Providing children with adequate feedback seems to be instrumental in the development of more linear magnitude representations (Opfer & Siegler, 2007, see also Whalen, Gallistel, & Gelman, 1999). For example, Opfer and Siegler (2007) gave children a number-line estimation task which consisted of putting magnitudes such as 20 on a number line with endpoints 0 and 100. They either provided children with feedback concerning the correctness of their estimate or provided no feedback at all. Compared to children who received no feedback, children that had received feedback became more accurate (i.e., linear) in their number-line estimates. The evolution of numerical representations in development is depicted in Figure 7.

With age, children will thus become increasingly efficient at strategies involving numerical representations, such as estimation. For example, when we ask them to execute a plausibility checking strategy to verify an equation such as $3 + 4 < 9$, they will execute this strategy more efficiently if they can distinguish $3 + 4$ and 9 faster. The rapidity with which they can distinguish these depends on the fineness of the underlying numerical representations (Wood, Ischebeck, Koppelstaetter, Gotwald, & Kaufmann, 2009), which improves with experience and feedback.

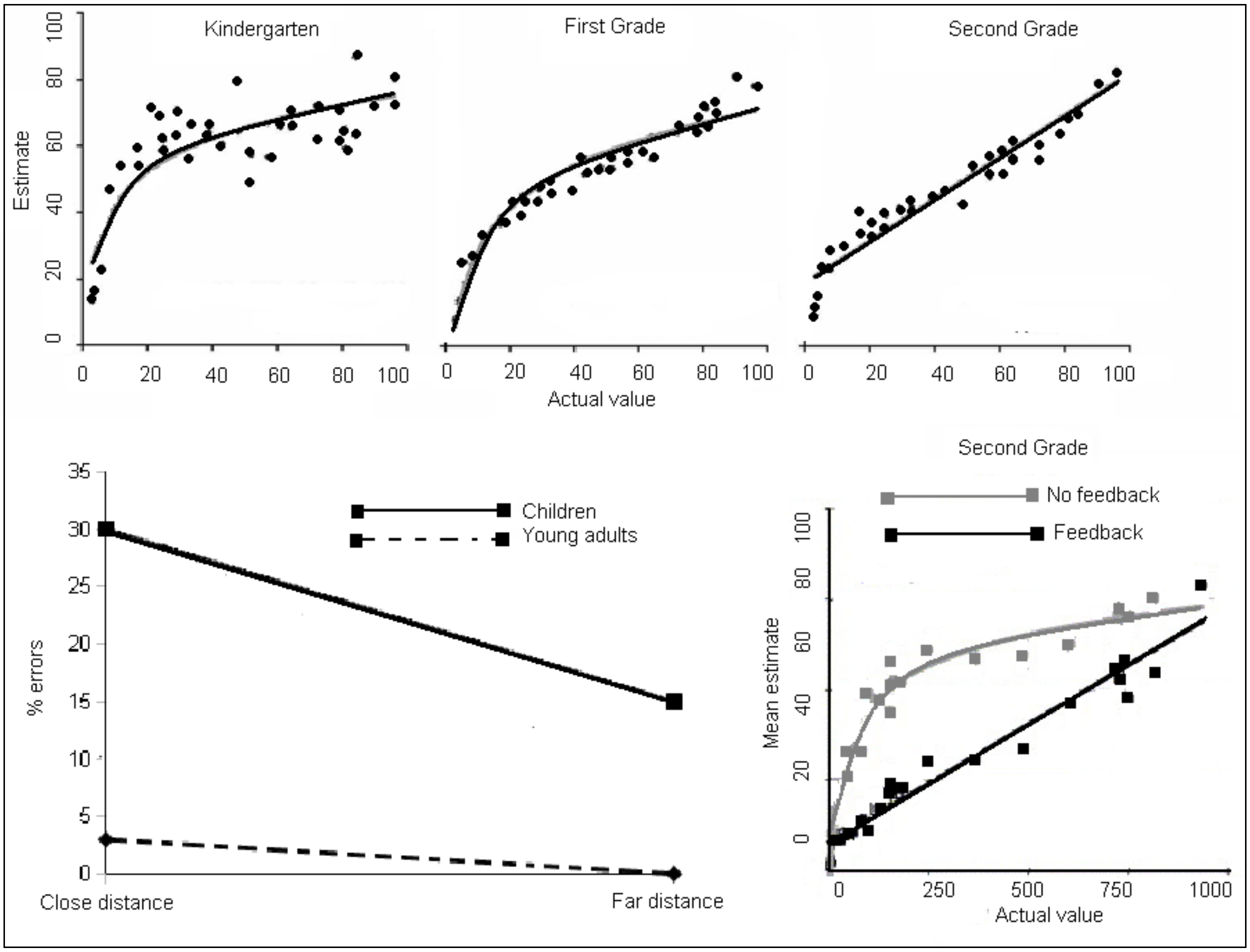


Figure 7. Top: Data from Siegler & Booth (2004) showing increasingly linear magnitude representations in children during a number-line estimation task (actual magnitude on the X-axis and estimated magnitude on the Y-axis). Bottom left: Data from Wood, Ischebeck, Koppelstaetter, Gotwald, & Kaufmann (2009) showing the larger error rates in children than in young adults when distinguishing quantities, especially when adjacent (e.g., 7 and 8 as opposed to 3 and 8). Bottom right: Data from Opfer & Siegler (2007) showing how magnitude representations in second grade children are more linear when performance feedback is

provided compared to when no feedback is provided (actual magnitude on the X-axis and estimated magnitude on the Y-axis).

Arithmetic fact retrieval is crucial to the performing of mental arithmetic. When presented with the operands of a problem (e.g., 3 x 7), activation will spread to several associated solutions, including the correct one (e.g., 21). If the association between the operands and the correct solution is strong enough, this solution will then be retrieved as a response. Efficient execution of arithmetic fact retrieval will thus mainly depend on the association strengths between problems and solutions. When one particular solution is most strongly associated to a problem, it will be easier to retrieve than when many solutions are more evenly associated to a problem. For example, in 4-year olds (see Figure 8; Siegler & Shrager, 1984), the problem 1 + 1 has finer peaked associations than the problem 5 + 4. The solution to 1 + 1 will thus be retrieved faster and more accurately than for 5 + 4 (for which the false answer 5 is equally well associated as the correct answer 9 in 4-year olds).

The association strength between operands and solutions depends on experience and has several important determinants (see Figure 8). First, there is the age of acquisition of the arithmetic facts**.** Arithmetic facts acquired earlier have better associations than later acquired facts (Campbell & Graham, 1985). Second, there is the placement of the problem in the counting string**.** Problems to which the answer can be counted more easily (e.g., 5 + 1 as opposed to 1 + 5) will be answered correctly more often, leading to stronger associations to the correct solution (Siegler & Shrager, 1984). Third, there is the frequency of presentation of the problem**.** The more frequently a problem has been encountered, the stronger it will be associated to the correct solution (Siegler & Shrager, 1984; Ashcraft, 1992). It follows that with age, arithmetic facts should get better associated in children (Lemaire, Barrett, Fayol, & Abdi, 1994). Lastly, there is the sum of operands**.** The larger the sum, the more difficult it is to obtain the answer through procedural back-up strategies, thus decreasing association strength (Siegler & Shrager, 1984; Ashcraft, 1992; Siegler, 1988; Allen, Ashcraft, & Weber, 1992; Lemaire, Barrett, Fayol, & Abdi, 1994). Through experience, some problem-solution associations become increasingly fine-tuned, leading to more efficient execution of the retrieval strategy.

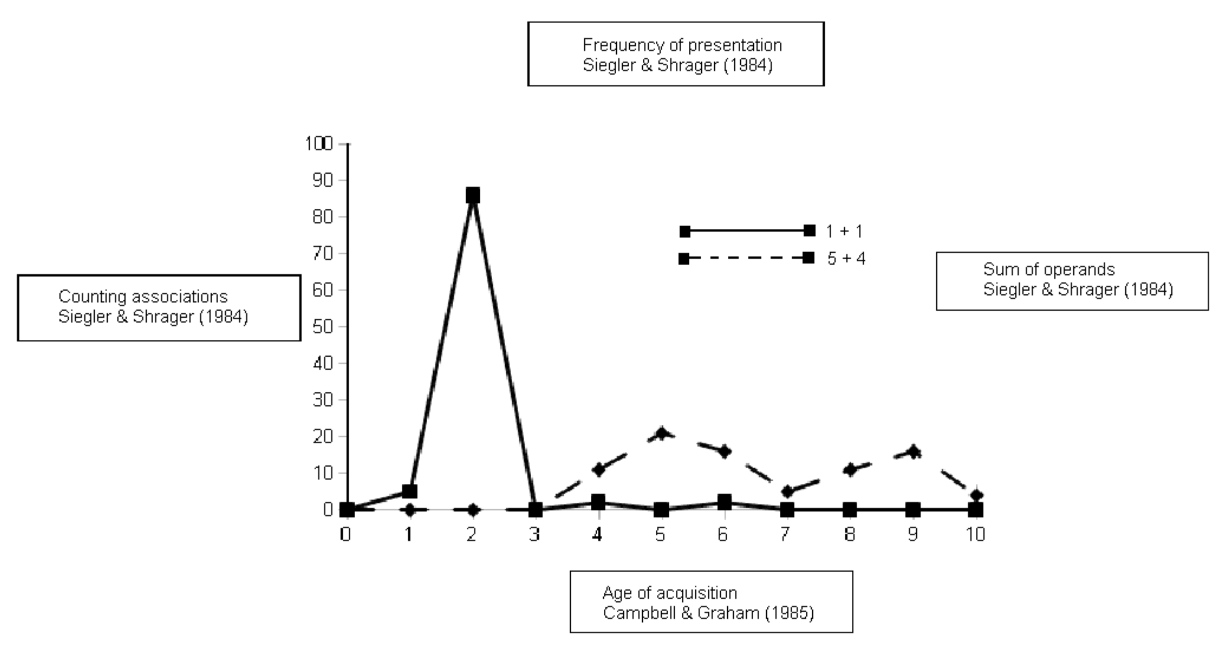


Figure 8. Predictions of association strength for the solutions to 1 + 1 and 5 + 4, from Siegler & Shrager (1984) based on 4-year olds' answers to single-digit addition problems. Around the graph we have added the determinants of association strength between problems and solutions.

Retrieval also necessitates executive resources to bring into working memory the needed facts, inhibit competitors and maintain the needed facts as long as necessary. To find support for the involvement of executive resources in retrieval processes, we focused on studies that studied single-digit addition and multiplication problems in adults, because solutions are usually well associated to operands in these problems, so that individuals are likely to use retrieval. For example, Campbell and Arbuthnott (1996; see also Arbuthnott & Campbell, 2000) have shown that inhibition is important in sequential arithmetic fact retrieval. They found that when individuals retrieve a wrong solution, it rarely corresponds to a solution given just prior. This suggests that a solution is inhibited immediately after responding so as to not interfere with subsequent retrievals. As another example, Seyler, Kirk, and Ashcraft (2003) showed that performing arithmetic fact retrieval decreased the available executive resources for a concurrent executive task (letter recall), suggesting that executive resources were absorbed by performing arithmetic fact retrieval (see also De Rammelaere, Stuyven, & Vandierendonck, 2001; Lemaire, Abdi, & Fayol, 1996; Deschuyteneer & Vandierendonck, 2005; Deschuyteneer, Vandierendonck, & Mullaert, 2006; Imbo & Vandierendonck, 2007; Kaufman, Lochy, Drexler & Semenza, 2004).

However, since the amount of executive resources necessary is fairly limited for this type of process (e.g., Tronsky, 2005; Caviola, Mammarella, Cornoldi, Lucangeli, 2012; Imbo & Vandierendonck, 2007), execution efficiency for strategies involving retrieval-based processes will mainly be shaped by the acquisition of experience (e.g., Booth & Siegler, 2008; 2006; Siegler & Opfer, 2003; Gandini, Lemaire, & Michel, 2009; Siegler & Shrager, 1984; Ashcraft, 1992; Siegler, 1988; Allen, Ashcraft, & Weber, 1992; Lemaire, Barrett, Fayol, & Abdi, 1994).

2.3.2. Procedural processes

We review two processes often involved in numerical strategies, which rely on experience, but more crucially on executive resources. Counting requires the simultaneous coordination of enumeration and keeping track of items. Problem decomposition requires maintaining problem structure and partial results, and integrating partial results. We first discuss the role of experience in these processes and then how the availability of executive resources is important in determining how efficient we will be when executing them.

Counting is used whenever a more precise measure of number is needed. For example, when grading an exam, a teacher will count the number of errors precisely instead of estimating them. Counting relies on the acquisition of symbolic numbers to denominate precise quantities (see Wynn, 1990; Gallistel & Gelman, 1992) and of enumeration principles (Wynn, 1990; Gallistel & Gelman, 1992; Sarnecka & Carey, 2008). Moreover, the counting procedure needs to be acquired. Counting combines enumeration and pointing (to keep track of counted and to-be-counted objects). The efficiency of strategies that involve counting will partly rely on the experience with enumeration and pointing, and how easily they are combined.

In children, enumeration becomes faster and more precise between 6 and 8 years of age through experience and better access to number names (see Figure 9; Camos, Fayol, & Barrouillet, 1999). The pointing procedure also gets increasingly well established with age. Whereas young children find the pointing procedure difficult, older children have no difficulties (see Figure 9; Camos, Fayol, & Barrouillet, 1999). With age, children will thus become increasingly efficient at strategies involving counting. For example, in Siegler (1987), the speed with which children executed the min strategy on simple additions (e.g., to solve 3 + 4, children can start from the larger addend (4) and count up (3) instead of putting up 3 and 4 fingers and counting all fingers) improved between kindergarten, and first and second grade with training (see Figure 9).

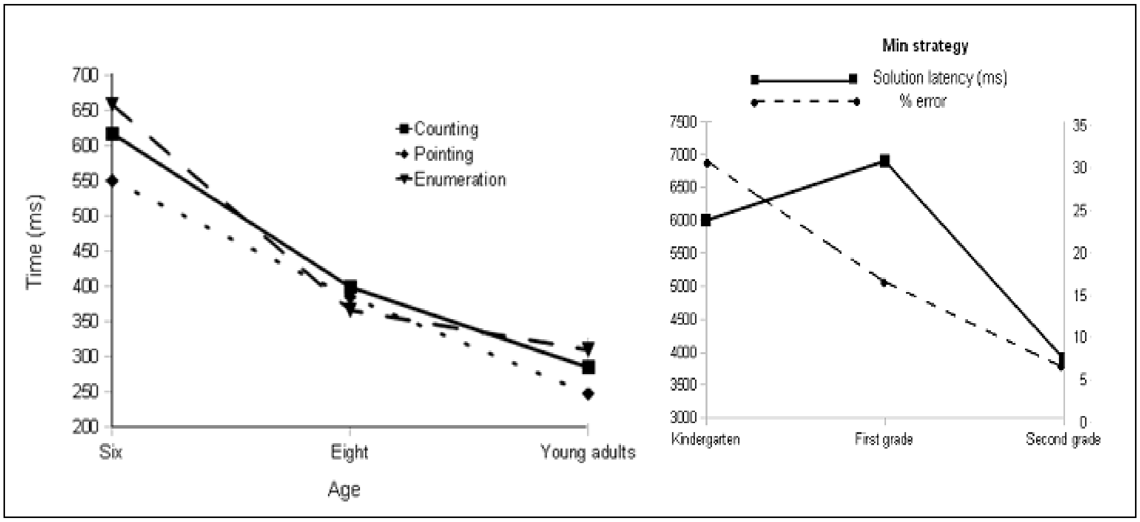


Figure 9. Left: Data from Camos, Fayol, & Barrouillet (1999) demonstrating the increased efficiency of execution of pointing and enumeration, speeding up the counting process from six years to young adulthood. Right: Data from Siegler (1987) Demonstrating how the min strategy gets executed increasingly faster and more accurately in children.

Problem decomposition is used whenever a problem is too large to solve at once. We will then subdivide the problem in parts, solve the parts, and integrate the partial results to obtain the solution to the original problem. For example, when calculating the exact answer to 324 + 254, we first add the units, then the tens, and then the hundreds. We then add these partial results. This first necessitates an understanding of the problem. In our example, we need to understand how we can add parts of the problem and obtain the same result as when adding the two operands at once. The efficiency of strategies that involve decomposition will thus depend on conceptual understanding of the problem and on experience executing appropriate decomposition strategies.

Understanding of problems is acquired through schooling and experience (Rittle-Johnson, Siegler, & Alibali, 2001; Bryant, Christie, & Rendu, 1999; Hecht & Vagi, 2010). Moreover, decomposition procedures are also acquired with experience. For example, Pratt and Savoy-Levine (1998) demonstrated the importance of tutoring children for learning the procedure to solve a long division manually (see Figure 10, see also Hecht & Vagi, 2010; Baroody, 1999). Training makes children more efficient in the execution of procedural strategies (Siegler & Lemaire, 1995; Baroody, 1999). With age, children will thus become better at executing strategies involving the decomposition of the problem and the application of procedures (Lemaire & Callies, 2009).

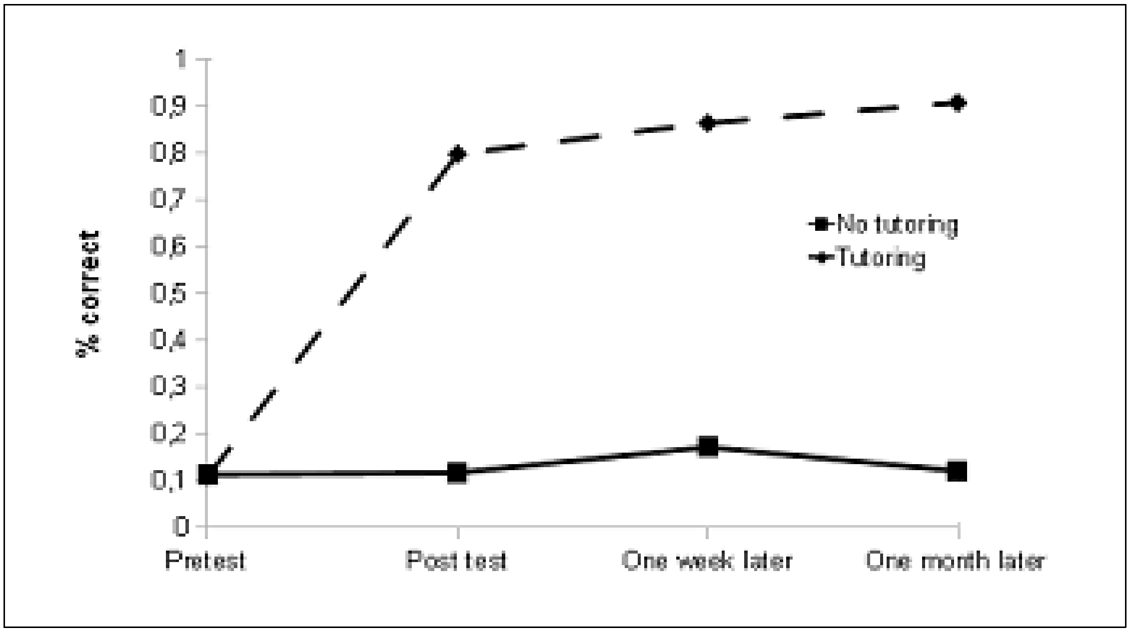


Figure 10. Data from Pratt & Savoy-Levine (1998) showing how tutoring children leads to significant and long-lasting improvements in the application of procedures to solve a long division manually. At the pretest all children scored the same, but at the post test, children that had received tutoring showed significant improvements, which lasted until at least a month after the post test.

Procedural processes rely more on executive resources than retrieval-based processes. For example, to count, we need to iterate number-words, and keep track of the counted and to-be-counted items. Towse and Hitch (1997) demonstrated the involvement of the phonological loop and the visuospatial sketchpad in counting. Compared to retrieval, Hecht (2002) found larger impairments due to working-memory shortage when using counting strategies to solve simple arithmetic problems. Tuholski, Engle, and Baylis (2001) compared young adults with high and low working memory spans on speeded enumeration of 1–12 randomly arranged dots. Counting rate was significantly faster for participants with high working memory span (295 ms/item) than for those with low working memory span (373 ms/item). Counting may be more efficient in individuals with greater working-memory capacity because of better inhibition of previously viewed locations.

The role of executive resources has also widely been shown in other procedures. For example, Logie, Gilhooly, and Wynn (1994) demonstrated that the central executive was involved in the decomposition of two-digit addition problems. Again, compared to retrieval, Imbo and Vandierendonck (2007) found that a concurrent choice reaction task impaired arithmetic performance more when procedural strategies were executed on single-digit addition problems (see also Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Andersson, 2008; Fürst & Hitch, 2000; Agostino; Johnson; Pascual-Leone, 2010; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Hecht, 2002; Tronsky, 2005; Imbo, Vandierendonck, & Vergauwe, 2007; Rasmussen & Bisanz, 2005)

2.4. Variations in experience and executive resources

Experience and executive resources differ between cultures, generations, and according to educational background. Moreover, demands on experience and executive resources vary according to contextual factors such as the problem and the situation. These variations can alter the efficiency of retrieval-based and procedural processes, rendering numerical strategies more or less efficient.

2.4.1. Western and Asian culture.

Reports from Chinese students show that learning mathematics was a more important component of their educational experiences than for Canadian students. This leads to an advantage for Chinese children in many numerical elementary processes. For example, in a number-line estimation task, Siegler and Mu (2008) found that Chinese children were more efficient than Canadian children. The superior estimation skills of Chinese children moreover had repercussions on other numerical abilities such as mental arithmetic skills.

Geary, Bow-Thomas, Liu, and Siegler (1996) measured the impact of differential schooling in Chinese and United States children on the efficiency of strategies to solve simple additions. Chinese and US children from three grades were measured at two time points. Between these two times Chinese children received on average 123 hours of mathematical instruction, whereas US children received only 97 hours. The results showed that Chinese children had better performance on simple addition problems, due to shorter execution times of counting, decomposition and arithmetic fact retrieval (See Figure 11; see also Geary, Hamson, Chen, Fan, Hoard, & Salthouse, 1997). The efficiencies of these three strategies are closely related. The fact that Chinese children are more competent with procedural back-up strategies such as decomposition and counting entails that they will more often find the correct solution to a problem, leading to stronger problem-answer associations in Chinese children (LeFevre & Liu, 1997). Due to these stronger associations, Chinese students can execute the retrieval strategy more rapidly and precisely, and so they use it more often (Campbell & Xue, 2001).

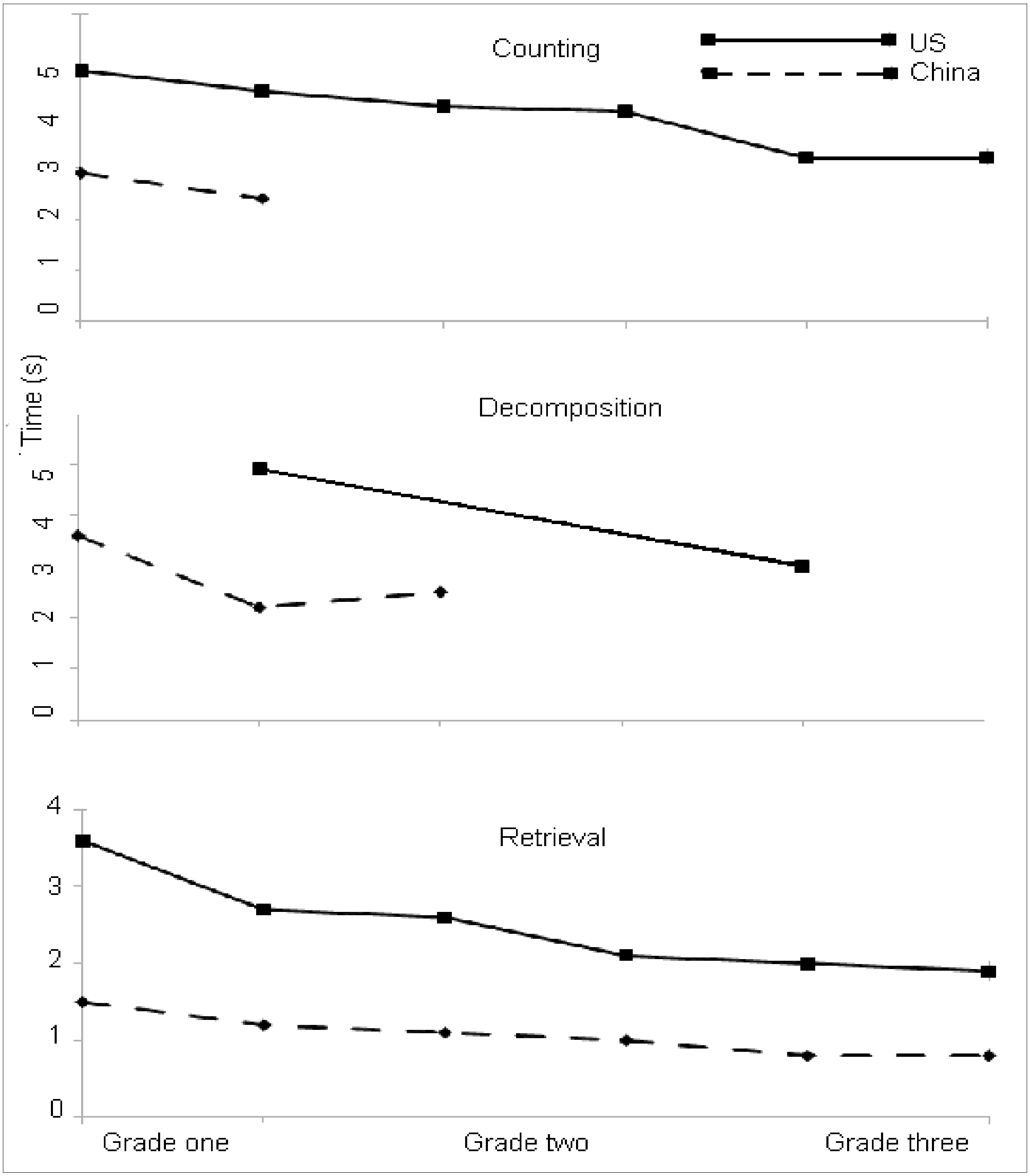


Figure 11. *Data from Geary, Bow-Thomas, Liu, & Siegler (1996) showing the evolution of the superior competence of Chinese children over US children in using counting, decomposition and*

retrieval to obtain the answers to simple additions (time and grade of measurement on x-axis, solution latencies on y-axis).

When comparing strategy execution on two-digit addition problems between western and Asian cultures, Imbo and LeFevre (2009) found that Chinese students were less impaired by concurrent executive load (choice reaction time task) than were Canadian students (See Figure 12; see also Geary, Bow-Thomas, Liu & Siegler, 1996). This latter finding can be explained by the fact that Chinese children seem to have an elevated digit span compared to Canadian children. This may be due to the fact that their language is more economical for number words, permitting them to keep a larger number of digits simultaneously active.

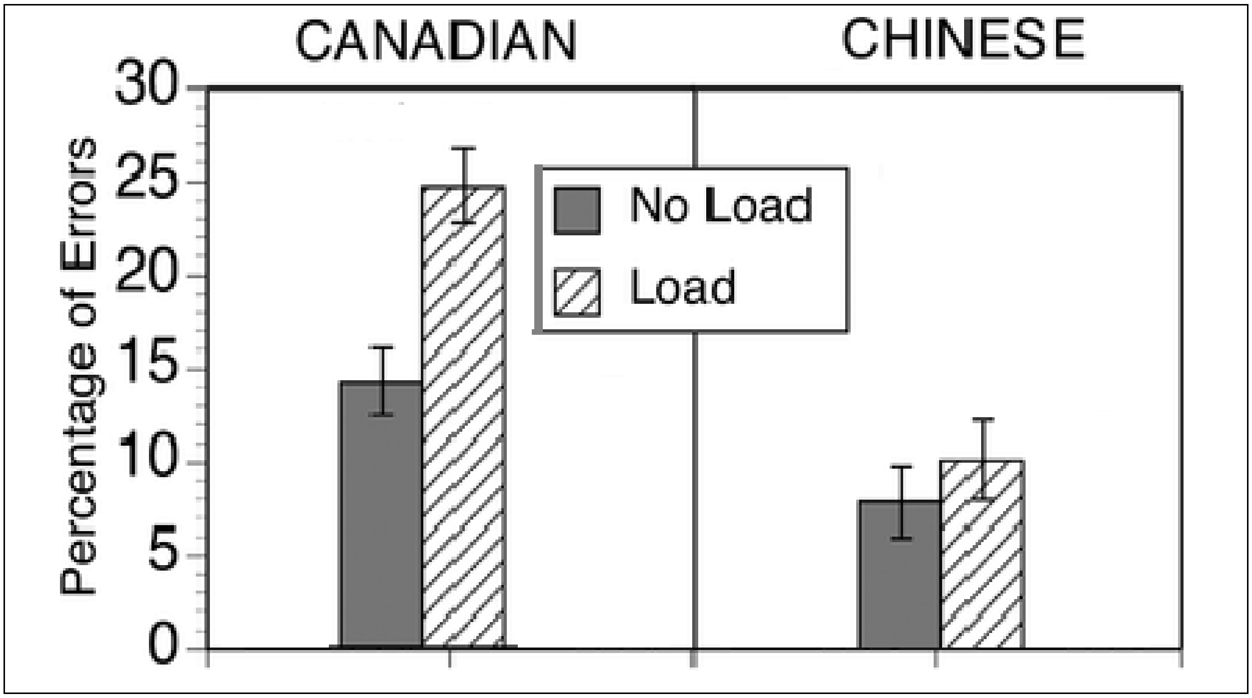


Figure 12. Data from Imbo & Lefevre (2009) showing that when solving two-digit addition problems requiring a carry procedure, Chinese students were less impacted than Canadian students by a secondary executive task.

2.4.2. Socio-economic background.

Reports from children with low and middle socio-economic status (SES) show that low-SES children trail behind in mathematical understanding. They count, add, subtract, and compare magnitudes less well than their peers (Jordan, Kaplan, Olah & Locuniak, 2006). These differences seem to be mostly related to differences in experience. Evidence for this can be found in a study from Siegler and Ramani (2008). These authors first showed that low-SES children had poor numerical representations (i.e., logarithmic instead of linear numerical representations). In a second experiment, they showed how providing low-SES children with additional training improved their performance. The authors had low-SES children play numerical board games for four sessions of 15 minutes, and found that this improved their numerical estimation performance to the level of their peers. Furthermore, a later study by Ramani and Siegler (2008) showed that playing the board games also had repercussions for other numerical processes, such as counting.

2.4.3. Healthy and pathological aging.

Whereas stored knowledge grows through accumulation of experience, executive resources decline through deterioration of the supporting brain regions in older adults (e.g., Reuter-Lorenz & Park, 2010), and even more so in AD patients (e.g., Grady et al., 1988). This differential evolution in age of experience and executive resources makes it hard to predict the effects of aging on numerical strategy efficiency. We review empirical evidence of healthy and pathological aging effects on retrieval-based (arithmetic fact retrieval during simple arithmetic) and procedural (decomposition during complex arithmetic) processes. Aging effects on these numerical processes can explain aging effects in execution of retrieval and procedural strategies discussed in Chapter 1.

What is the effect of healthy and pathological aging on arithmetic fact retrieval? To investigate this question, researchers usually present individuals with simple arithmetic problems to which the answer is likely to be obtained by retrieval (e.g., 3 x 4). Geary and Wiley (1991) found that older adults were slower than young adults (930 vs. 833 ms) when retrieving arithmetic facts in simple addition and subtraction production tasks (e.g., 9 + 8 = ?). Similarly, Allen, Ashcraft, and Weber (1992) found that older adults were slower than young adults when verifying simple multiplications (e.g., 4 x 7 = 28, TRUE).

These longer solution latencies in older adults are surprising, given that experience is relatively more important than executive resources in arithmetic fact retrieval. Moreover, Schaie (1996) has demonstrated that arithmetic facts have been better acquired in older adults than in contemporary young adults (see also Geary & Lin, 1998; Geary et al., 1996; 1997; Green, Lemaire, & Dufau, 2007), constituting another reason for expecting better arithmetic fact retrieval in older adults.

However, even though the acquisition of arithmetic facts relies on experience, executive resources are needed for retrieving these facts and maintaining them. The fact that older adults were slower than young adults could be due to impairments in the executive resources managing the retrieval of arithmetic facts.

To test his explanation, it is useful to distinguish between central and peripheral processes (Cerella, 1985). Central processes are sensitive to problem complexity whereas peripheral processes remain more or less constant across variations of complexity, and usually include encoding and responding processes. Retrieval can be considered a central process, since it varies with the difficulty of the problem. For example, solutions to larger problems are less easily retrieved than solutions to smaller problems.

If central retrieval processes are affected in older adults, we thus expect an interaction with problem difficulty such that the effect of aging is larger on more difficult problems. If peripheral processes are affected in aging, we expect equal effects for simple and more complex problems, since peripheral processes do not vary with problem difficulty. Using this distinction, Allen, Ashcraft, and Weber (1992) found that peripheral but not central processes were affected by normal age in simple arithmetic (see also Geary & Lin, 1998; Geary & Wiley, 1991). They found that the effect of age on the speed of arithmetic fact retrieval was similar for small and large problems (see Figure 13; see also Arnaud, Lemaire, Allen, & Michel, 2008). The difficulties with arithmetic fact retrieval thus seem to stem from a peripheral source in normal aging. It could be that perceptual encoding of the problems or preparation of responses was slowed down in healthy older adults (see also Allen et al. 1997).

In AD patients however, Duverne, Lemaire, and Michel (2003) showed that arithmetic fact retrieval latencies did interact with problem difficulty (Figure 13), suggesting that central retrieval processes are impaired in AD. Moreover, Kaufmann et al. (2002) found that AD patients exhibited decreased arithmetic fact and procedural knowledge (see Pesenti, Seron, & Van der Linden, 1994 for a case study). This demonstrates that, in addition to problems with the retrieval of arithmetic facts, the stored associations between problems and solutions are damaged in AD.

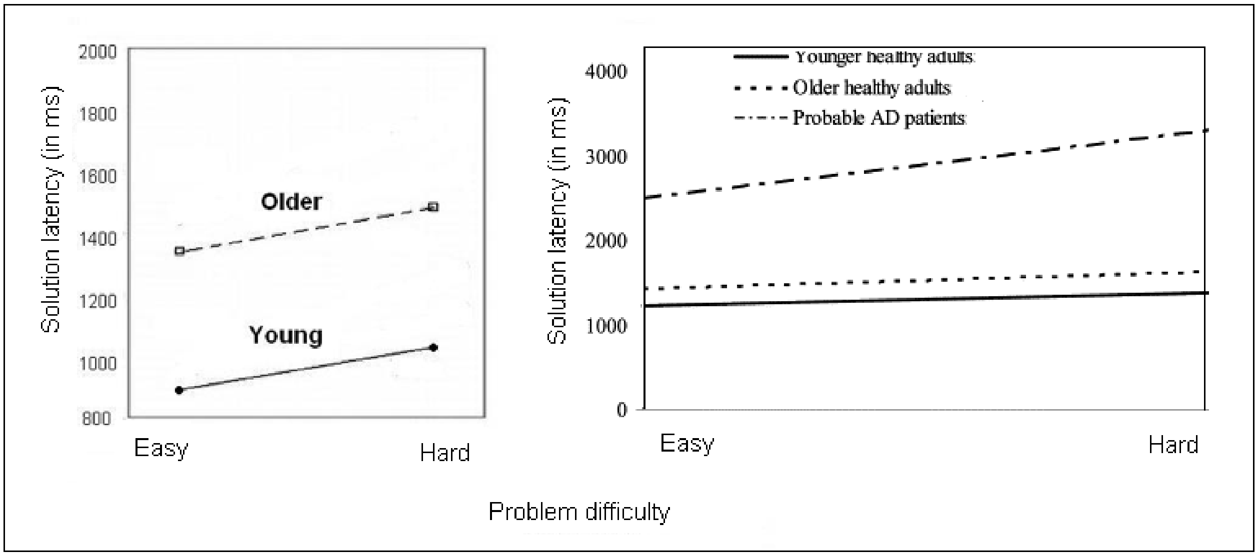


Figure 13. Left: Allen, Ashcraft, & Weber (1992) found longer solution latencies to verify large addition problems than small problems. They also showed significantly longer solution latencies for older compared to young adults. However, age did not interact with problem difficulty, suggesting that age-related slowing stemmed from slowing of encoding or response processes and not from slowing of central retrieval processes. Right: Duverne, Lemaire, and Michel (2003) found that the effect of AD on solution latencies did increase for more difficult problems, suggesting that central retrieval processes were slowed down in this population.

What is the effect of healthy and pathological aging on complex arithmetic requiring decomposition? Complex arithmetic pertains to arithmetic problems that can not be solved by mere arithmetic fact retrieval (e.g., hierarchical operations such as 5 + 3 x 4). These problems typically require multiple retrievals, maintenance of the problem structure and integration of partial results. Consequently, such problems will depend relatively more on executive resources (Trbovich & LeFevre, 2003). Given the declines of executive resources in normal and pathological aging (e.g., Reuter-Lorenz & Park, 2010), complex arithmetic could suffer more from the effects of aging than simple arithmetic.

Salthouse and Coon (1994) demonstrated that older adults were increasingly slowed relative to young adults when the number of arithmetic operations required to verify an equation was increased in a hierarchical problem-format (e.g., 5 + 3 - 1 - (3 + 4) - 1 = 6, FALSE; see Figure 14). For example, when three operations were required, young adults would verify a problem in approximately four seconds, whereas older adults would need six seconds. When seven operations were required, this difference would increase to 10 versus 21 seconds. Duverne and Lemaire (2005) interpreted this as an impairment in the ability to coordinate several procedures in older adults.

AD patients are more impaired in complex arithmetic than healthy older adults (Mantovan et al., 1999). For example, an AD patient tested by McGlinchey-Berroth, Milberg, and Charness (1989) showed impairments when asked to apply a procedure to square two-digit numbers. His impairments were in particular due to his difficulties in combining and integrating the multiple steps of the procedure.

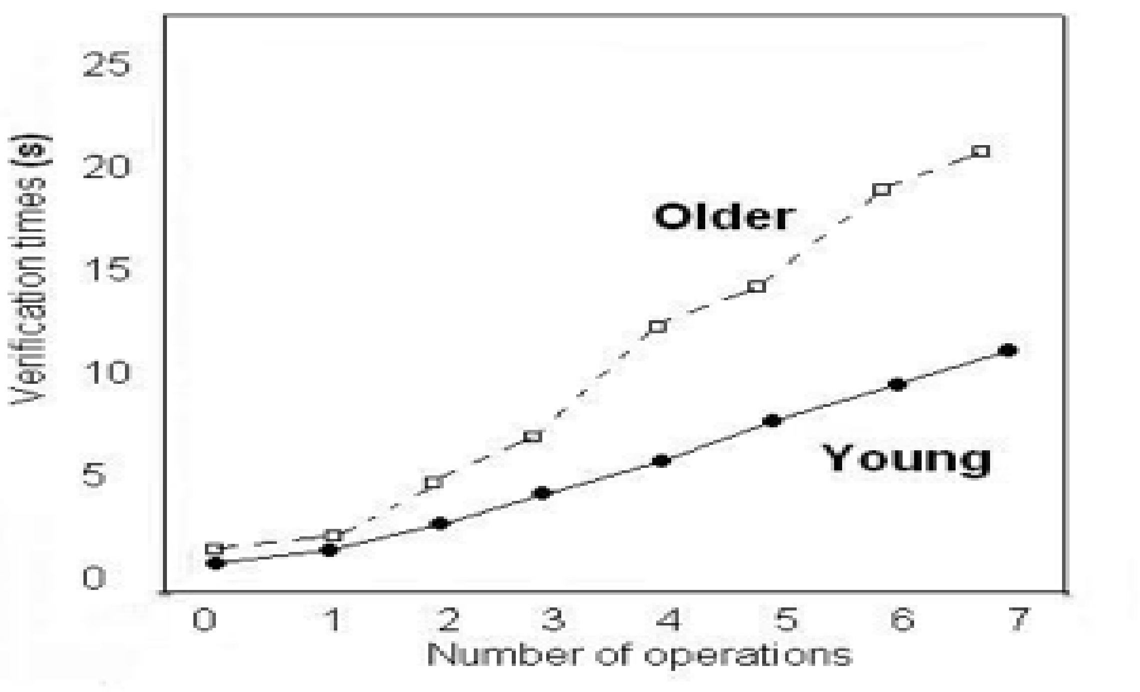


Figure 14. Verification times in younger and older adults from Salthouse & Coon (1994). We see that older adults are relatively more impaired than young adults as the number of operations (in hierarchical format, e.g., (4 + 3) / 2 - (1 + 8)) increases.

2.4.4. Task & stimuli demands

People can vary the amount of cognitive resources they allocate to strategy execution as a function of task or situational demands. They can choose to monitor strategy execution more closely, if avoiding errors is important. This will require more executive resources, and strategy execution will be slower. Alternatively, they can invest less executive resources and obtain quicker strategy execution. For example, Siegler (1987) found that when second graders were presented subtraction problems under conditions in which they were rewarded solely for speed, they executed the same strategy faster but less accurately than when accuracy was rewarded. In the case of rapid strategy execution, less executive capacities are used to monitor accuracy of the execution so that strategy execution is easier (see also Trbovich & LeFevre, 2003; Campbell & Austin, 2002).

Stimuli can alter the demands numerical processes put on executive resources. One of the main aspects of stimuli that can increase or decrease the need for executive resources is problem difficulty (Furst & Hitch, 2000; Imbo, Vandierendonck, & Vergauwe, 2007; Imbo, Vandierendonck, & De Rammelaere, 2007; Duverne, Lemaire, & Vandierendonck, 2008; Lemaire & Fayol, 1995; Campbell, 1999; Campbell & Austin, 2002; Green, Lemaire, & Dufau, 2007; Arnaud, Lemaire, Allen, & Michel, 2003; see Zbrodoff & Logan, 2005, for a review). In children, addition problems with larger sums will lead to longer solution times with different types of strategies such as counting, decomposition, or arithmetic fact retrieval (Siegler & Shrager, 1984). For example, it is more difficult to count 5 and 4 to solve 5 + 4 than 2 and 3 to solve 2 + 3. The fact that counting is more difficult on this type of problem will lead to more wrong answers, so that solutions will end up less well associated to problems for larger sums (Siegler & Shrager, 1984; Ashcraft, 1992; Siegler, 1988; Allen, Ashcraft, & Weber, 1992; Lemaire, Barrett, Fayol, & Abdi, 1994; Zbrodoff, 1995). When performing arithmetic fact retrieval, there will then be more competitors, so that executive resources are more needed to inhibit these competitors (Niedeggen & Rosler, 1999; Campbell & Arbuthnott, 1996; Arbuthnott & Campbell, 2000; Seyler, Kirk, & Ashcraft, 2003).

Stimulus format is also important in how efficiently a strategy will be applied on the problems. For example, LeFevre et al. (2001) found that strategy execution was less efficient on spoken problems than on visually presented problems. Campbell and Alberts (2009) found that strategy execution is also less efficient on problems in word form than on problems in symbolic form (see also Campbell, 1999). This could be linked to larger difficulties processing spoken information and word problems in working memory.

2.4.5. Interaction between experience and executive resources.

Although experience and executive resources can be considered as two separate influences on strategy execution efficiency, they also influence one another. More available executive resources enhance the learning of relevant declarative and procedural knowledge (e.g., Andersson, 2008; Bull & Scerif, 2001). In the other direction, experience reduces the need for executive resources during strategy execution. When we are inexperienced with a procedure, the association between the different steps is weak. In that case we rely on top-down support to retrieve and maintain all the steps. However, practice can strengthen the associations between the steps of a procedure. Over time, the steps can be sequentially retrieved just by their association strength and without much top-down support. In that case the procedure has become automatic (Miller & Cohen, 2000). For example, Towse and Hitch (1997) found no implication of the central executive in the execution of the combination of pointing and enumeration in children, a highly automatized procedure. Camos, Fayol, and Barrouillet (1999) showed that executing pointing and enumeration at the same time even facilitated the individual procedures, so that the necessary time for executing the two processes together was shorter than the time needed for executing the individual procedures.

2.4.6. Problem-strategy associations

Different problems elicit the use of different types of strategies. The adequacy of the pairing between a problem and a strategy determines the efficiency of the strategy on that problem. Usually, strategies get used on problems on which they work well (Lemaire & Siegler, 1995; Siegler & Lemaire, 1997; Siegler & Shipley, 1995; Luwel et al., 2003). Models of strategy selection (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) explain this by assuming that strategies get chosen based on associative force, shaped by previous successes with a strategy on a problem type.

Experience is very important in the fine-tuning of these problem-strategy associations. Through experience people learn which strategies work well on a problem, and so they can optimize their performance by choosing appropriate strategies. Characteristics of problems (Campbell & Xue, 2001; LeFevre, Sadesky & Bisanz, 1996; Luwel, et al., 2003; Gandini, Lemaire, & Dufau, 2008) are very important in determining the strategies that are appropriate. For example, in numerosity estimation, a dot display where the items are organized will facilitate use of the anchoring strategy, whereas a random dot display will not facilitate the usage of this strategy and will have to be resolved by another strategy, such as counting (Gandini, Lemaire, & Dufau, 2008). This can be explained by the fact that an organized dot display lowers the demands of the anchoring strategy on executive resources (i.e., items are more readily grouped and more easily counted).

Stronger problem-strategy associations in and off themselves also seem to render strategy execution more efficient, regardless of whether the problem makes the application of a certain strategy easier. For example, in computational estimation (e.g., estimating the solution to problems such as 34 + 62 by rounding the operands), two-digit addition problems with small unit-digits (e.g., digits that are smaller than 5, for example '34 + 62') will be solved faster with the rounding-down strategy (e.g., rounding both operands down) than with the rounding-up strategy (e.g., rounding both operands up) even if these strategies are imposed (Lemaire, Arnaud, & Lecacheur, 2004). It could be that the steps of a strategy are retrieved more efficiently when this strategy is strongly associated to the problem, thereby increasing strategy efficiency.

2.4.7. Sequential effects

We have shown the importance of executive resources in the processes involved in numerical strategies. Moreover, we have shown that the availability and role of these resources differ between populations, with experience, and with contextual factors. But what if available executive resources also dynamically fluctuate in function of prior cognitive engagements? Such fluctuations can stem from previous strategy executions**.** Switching strategies or prior high-intensity executive engagement may temporarily reduce available executive resources and thus interfere with strategy execution. We review empirical evidence supporting this claim from strategy switching and problem sequential difficulty effects.

Strategy switch costs are the finding that a strategy is executed slower when it is different from the previous strategy. Luwel et al. (2009), in a numerosity judgment task, had participants switch between an addition strategy (i.e., counting the number of dots in a grid) and a subtraction strategy (i.e., counting the number of empty blocks in a grid and subtracting this from the total number of blocks). They found that people were slower when executing different strategies on successive trials than when they executed the same strategy. In Lemaire and Lecacheur (2010), these effects differed according to the strategy. They had participants switch between use of two rounding strategies on computational estimation problems (e.g., 43 x 68) and found a switch cost only when switching from the harder rounding-up strategy to the easier rounding-down strategy (see also Ardiale & Lemaire, in press).

Switching effects on strategy execution can be interpreted as the result of the extra demands switching puts on executive resources (e.g., Monsell, 2003), thus interfering with strategy execution, which also requires these resources (Imbo, Duverne, & Lemaire, 2007; Imbo & Vandierendonck, 2007; Duverne, Lemaire, & Vandierendonck, 2008; Andersson, 2008; Agostino, Johnson, & Pascual-Leone, 2010; Rasmussen & Bisanz, 2005; Bull & Scerif, 2001).

Sequential effects on performance are not limited to effects of switching. Problem sequential difficulty effects are the finding of longer execution times following difficult problems. Schneider and Anderson (2010) showed the existence of problem sequential difficulty effects in arithmetic. In three experiments they had participants switch between tasks (addition and subtraction) in an arithmetic verification paradigm, while manipulating the difficulty of the problems (problems with or without carry-over in Expt. 1, vertical or horizontal format in Expt. 2 and true or false solution in Expt. 3). They found that, regardless of task switch or repetition, solution latencies were longer following difficult than following easy problems.

Sequential difficulty effects can be interpreted as a temporary depletion of executive resources by difficult cognitive operations (Schneider & Anderson, 2010), which interferes with subsequent cognitive operations.

2.5. Conclusion

In the preceding chapter, we attempted to identify what determines the efficiency individuals obtain when they execute a strategy. We looked at retrieval-based and procedural processes involved in numerical strategies. We found that experience is necessary for each of these processes and thus partly determines strategy efficiency. We also showed how executive resources are important in these processes, adding additional variation to strategy execution efficiency. Available experience and executive resources moreover vary between individuals, populations, and contexts. Through the role these factors have on available experience and executive resources, they constitute determinants of strategy execution efficiency.

Table 5: Overview of studies showing a role for experience in retrieval-based and procedural processes

Role of experience					
Retrieval-based processes			**Procedural processes**		
Estimation	Importance of experience for developing linear numerical representations in children	Opfer & Siegler, 2007; Whalen, Gallistel, & Gelman, 1999; Siegler & Booth, 2004; Ramani & Siegler, 2008; Siegler & Ramani, 2008; Ashcraft & Moore, 2012; Booth & Siegler, 2008; 2006; Siegler & Opfer, 2003	Importance of experience for learning pointing and enumeration in counting	Wynn, 1990; Gallistel & Gelman, 1992; Wynn, 1990; Gallistel & Gelman, 1992; Sarnecka & Carey, 2008; Camos, Fayol, & Barrouillet, 1999	Counting
Arithmetic fact retrieval	More frequently encountered arithmetic facts are acquired better	Siegler & Shrager, 1984; Ashcraft, 1992; Lemaire, Barrett, Fayol, & Abdi, 1994	Importance of experience for conceptual understanding of problems	Rittle-Johnson, Siegler, & Alibali, 2001; Bryant, Christie, & Rendu, 1999; Hecht & Vagi, 2010	Problem Decomposition
	Earlier acquired arithmetic facts are better retrieved	Campbell & Graham, 1985	Importance of experience for learning procedures	Pratt & Savoy-Levine, 1998; Hecht & Vagi, 2010; Baroody, 1999; Siegler & Lemaire, 1995; Lemaire & Callies, 2009	
	Arithmetic facts associated to more easily solved problems are retrieved better	Siegler & Shrager, 1984; Ashcraft, 1992; Siegler, 1988; Allen, Ashcraft, & Weber, 1992; Lemaire, Barrett, Fayol, & Abdi, 1994			

Table 6: Overview of studies showing a role for executive resources in retrieval-based and procedural processes

Role of executive resources					
	Retrieval-based processes		**Procedural processes**		
Arithmetic fact retrieval	Central executive	De Rammelaere, Stuyven, & Vandierendonck, 2001; Lemaire, Abdi, & Fayol, 1996; Imbo & Vandierendonck, 2007; Kaufman, Lochy, Drexler & Semenza, 2004	Central executive	Logie, Gilhooly, & Wynn, 1994; Imbo & Vandierendonck, 2007; Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Andersson, 2008; Fürst & Hitch, 2000	Problem Decomposition
	Response selection	Deschuyteneer & Vandierendonck, 2005	Memory updating	Agostino; Johnson; Pascual-Leone, 2010	
	Memory updating	Deschuyteneer, Vandierendonck, & Mullaert, 2006	Working memory	Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Hecht, 2002; Tronsky, 2005	
	Inhibition	Campbell & Arbuthnott, 1996; Arbuthnott & Campbell, 2000	Phonological loop & Visuospatial sketchpad	Imbo, Vandierendonck, & Vergauwe, 2007; Rasmussen & Bisanz, 2005; Fürst & Hitch, 2000	
	Working memory	Seyler, Kirk, & Ashcraft, 2003			
			Working memory	Hecht, 2002; Tuholski, Engle, & Baylis, 2001	Counting
			Phonological loop & Visuospatial sketchpad	Towse & Hitch, 1997	

Research Problematic

An important limit of previous research on strategy execution efficiency is the little attention for the sequential context in which strategies are executed. Indeed, most of this research considered strategy execution on isolated problems. However, strategies are often executed in quick succession. Not taking into account previously executed strategies is problematic, since these may influence participants' performance on subsequent strategies. For example, the limited amount of available research has shown that strategy execution is sensitive to effects of switching. Lemaire and Lecacheur (2010) found that numerical strategy execution is more efficient when we repeat the same strategy than when we switch strategies. However, sequential effects on strategy execution need not be limited to effects of switching. Indeed, Schneider and Anderson (2010) found that arithmetic problem solving was slower after a difficult than after an easy previous arithmetic problem.

The main goal of this thesis was to study effects of prior high-intensity cognitive engagement on subsequent processing. Schmeichel (2007) found that efforts at executive control temporarily reduced subsequent available executive resources. In this thesis, we studied such effects from trial to trial, with arithmetic strategies. The strategies under study here vary in executive control requirements. We wanted to determine how execution of a difficult arithmetic strategy would influence execution of the next arithmetic strategy.

One important assumption of models of strategy selection and execution (Lovett & Andersons ACT–R, 1996; Lovett & Schunns RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) is that participants execute strategies on each problem, independently of the strategy that was selected and executed on the previous problem. Thus, only factors characterizing the current problem and strategy are crucial parameters of strategy execution on a given trial. Finding sequential difficulty effects would contest such a view on strategy execution.

Moreover, our work also aimed to study the evolution of such effects in normal and pathological aging. Previous research has abundantly shown that strategy execution is altered in healthy and pathological aging (Gandini, Lemaire, Anton, & Nazarian, 2008; Gandini, Lemaire, & Michel, 2009). Furthermore, such age-related changes are often related to reductions in necessary processing resources in healthy and pathological aging (Taconnat et al., 2007). These reductions may also contribute to larger effects of previous strategy execution on subsequent strategies in these populations.

Computational estimation paradigm

To achieve our objectives, the choice of paradigm was very important. Some strategies rely more than others on executive resources (Imbo & Vandierendonck, 2007) and will be more liable to producing sequential effects on performance. The paradigm we chose for our experiments was computational estimation. This technique is used in situations where people lack time, knowledge, resources, and/or motivation to calculate exact responses. In these cases, estimates often give a sufficient approximation. For example, when solving a problem such as 43 x 67, an individual may satisfy himself rounding both operands down to the nearest decades (i.e., 40 x 60) and retrieve the multiplication of these rounded operands (i.e., 2400), which will be a lot easier than to retrieve the solution to the initial problem.

Computational estimation is composed of processes of decomposition and arithmetic fact retrieval. We decompose the initial problem (e.g., 43 x 67) in a number of subsequent retrievals. These retrievals involve 1) The procedure of the strategy (e.g., for rounding up: discard unit digits, increment ten digits, multiply incremented ten digits), 2) The rounded operands (e.g., for rounding up: 50 and 70), and 3) The solution to their multiplication (e.g., for rounding up: 50 x 70 = 3500).

Different rounding strategies will put different demands on executive resources during these retrievals. Retrieval of the procedure, rounded operands, and multiplication should be easiest for rounding down and most difficult for rounding up and mixed rounding. This is because the latter require the additional step of incrementing at least one operand. If sequential effects involve executive resources, rounding up and mixed rounding should thus engender larger sequential effects than rounding down. Moreover, they should also be more susceptible to sequential effects.

In all our experiments, we contrasted performance with mixed rounding after having executed rounding down or rounding up. By looking at execution of mixed rounding following rounding up, we optimize our chances of finding interference from previous with subsequent strategy execution. See Figure 15 for an illustration of a trial in the computational estimation paradigm.

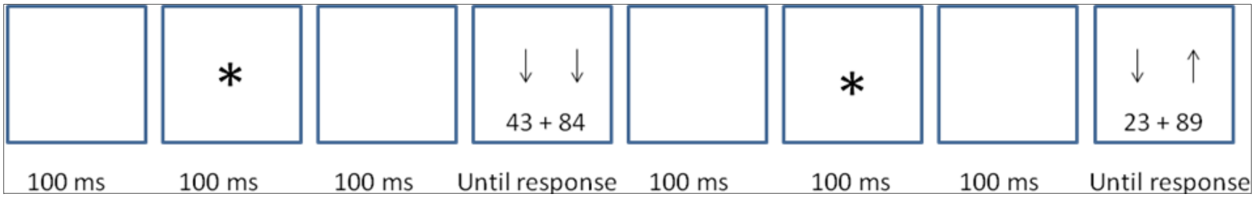


Figure 15. The computational estimation paradigm

Research hypotheses and predictions

Consistent with the ideas of Schneider and Anderson (2010), we hypothesized that difficult strategy execution would temporarily consume cognitive resources, interfering with the next strategy execution. More precisely, a difficult strategy could leave traces in working memory, complete decay (or deletion) of which could overlap with the next strategy execution. Functional working-memory capacity would thus be reduced for the next strategy, translating in lesser execution efficiency following difficult strategies than following easy strategies. The first three studies aimed to put in evidence sequential difficulty effects during strategy execution, and to understand these effects. To do this, we tested a number of predictions derived from our theoretical account of sequential difficulty effects.

The first prediction tested was the possibility that strategy execution is slower following a more difficult than following an easier strategy. Moreover, we varied the size of the operands of the arithmetic problems to test whether strategy sequential difficulty effects existed independently of problem sequential difficulty effects.

The second prediction tested was that individuals with lower working-memory capacity should suffer more from strategy sequential difficulty effects than individuals with higher working-memory capacity. We measured participants' working-memory capacities by administering them the operation span, reading span, and running span tests. We correlated a composite measure of these tasks with the size of strategy sequential difficulty effects (i.e., RT after rounding up – RT after rounding down).

The third prediction tested was that giving participants more time between problems should allow them to free working-memory resources so that occupation of working memory by the previous strategy does no longer overlap with next strategy execution. We tested this by varying the response-stimulus interval from short (i.e., 300 ms) to long (i.e., 600 ms).

The final prediction concerned the moment in strategy execution during which sequential difficulty effects would interfere. If sequential difficulty effects involve executive resources, we should find that they interfere with central rather than with peripheral processes of strategy execution. Moreover, this interference should consist of more effortful processing following difficult strategies (as opposed to for example elongated time criteria) than following easy strategies. To test this prediction, we registered participants' cerebral activities during the computational estimation paradigm.

The fourth study aimed to investigate the evolution of sequential difficulty effects in normal and pathological aging. We expected populations with less efficient executive resources, such as older adults and AD patients, to suffer more from sequential difficulty effects. We presented them the computational estimation paradigm and compared sizes of sequential difficulty effects between young adults, healthy older adults, and AD patients.

In conclusion, in our research we attempted to show strategy sequential difficulty effects during strategy execution, corresponding to slower processing following difficult than following easy strategies. Moreover, we wanted to determine whether this is dependent on available working-memory capacity of the individual, the time between strategy executions and the moment during strategy execution. Lastly, we wanted to determine the evolution of such effects with healthy and pathological aging.

Sequential difficulty effects during strategy execution

Sequential difficulty effects during strategy execution: A study in arithmetic

Uittenhove, K. & Lemaire, P.

Aix-Marseille Université, IUF, & CNRS, Marseille, France

2012

Experimental Psychology, 59(5), 295-301

Corresponding author:
Patrick LEMAIRE
Université de Provence & CNRS
Case D
3 Place Victor Hugo
13331 Marseille France
Email: patrick.lemaire@univ-provence.fr
Tel. +33.4.13.55.09.85

Authors' Note

This research was supported in part by the CNRS (French NSF) and by a grant from the Agence Nationale de la Recherche (Grant # BLAN07-1_196867). Correspondence about this paper should be directed to Patrick Lemaire, CNRS & Université de Provence, 3 Place Victor Hugo, Case D, 13331 Marseille, France (email: patrick.lemaire@univ-provence.fr).

Abstract

In two experiments, we tested the hypothesis that strategy performance on a given trial is influenced by the difficulty of the strategy executed on the immediately preceding trial, an effect that we call *strategy sequential difficulty effect*. Participants' task was to provide approximate sums to two-digit addition problems by using cued rounding strategies. Results showed that performance was poorer after a difficult strategy than after an easy strategy. Our results have important theoretical and empirical implications for computational models of strategy choices and for furthering our understanding of strategic variations in arithmetic as well as in human cognition in general.

An important aspect of human cognition is that in a wide range of cognitive domains performance depends on the strategies that people use. A strategy is “a procedure or a set of procedures to achieve a higher level goal” (Lemaire & Reder, 1999, p. 365). In this study we focus on strategy execution. Theoretical and empirical works have shown the importance of the number and difficulty of procedures included within each strategy for strategy execution: More procedures lead to longer execution times and higher error rates (e.g., Lemaire, Arnaud, & Lecacheur, 2004; Bajic & Rickard, 2009).

Strategy execution depends on other factors, such as the situation, person, and problems (Siegler, 2007). These factors act individually and in interaction with each other to influence strategy execution performance. For example, Lemaire et al. (2004) found that older adults executed the rounding-down strategy more slowly under accuracy-pressure conditions than under no-pressure conditions, especially when they solved easy problems. This interaction was even stronger in younger adults.

This omits the fact that strategies are often part of a sequence, as participants solve a series of problems by changing or repeating strategies on successive problems. The present work rests on the general sequential hypothesis that the execution of a strategy in such a sequence can be influenced by the execution of previous strategies in that sequence. Consistent with this hypothesis, Lemaire and Lecacheur (2010) have recently found that executing two different strategies yielded poorer performance than executing the same strategy on successive problems. Such sequential effects may not be restricted to effects of switching or repetition. In the word-naming literature, the mixing of easy and difficult stimuli slows down performance on the easy stimuli (Lupker, Kinoshita, Coltheart, & Taylor, 2003; see Kiger & Glass, 1981, for similar results in sentence verification). Taylor and Lupker (2001) established that the slowdown on easy stimuli is contingent upon the presentation of difficult stimuli on the immediately preceding trials. Furthermore, Schneider and Anderson (2010) recently found that solving arithmetic problems was slowed down after difficult problems. These effects can be referred to as problem sequential difficulty effects.

Sequential difficulty effects could result from temporary depletion of relevant cognitive resources by difficult cognitive operations (Schneider & Anderson, 2010). This explanation rests on the assumption that certain processing resources need time to be remobilized and thus can be temporarily depleted, and that difficult problems use these resources more than easy problems. Consequently, when solving a problem after a previous difficult problem, the larger amount of resources required and the time needed for remobilizing these resources reduce the amount of resources available for the next problem. Schneider and Anderson, based on problem sequential difficulty effects, postulated the existence of task sequential difficulty effects: With tasks of different difficulty (e.g. addition and subtraction), participants would be slower following performance of the difficult task . Strategies differ from tasks in the sense that the goal remains constant, and what differs is the procedure to attain this goal. Problem-solving is limited by how well a strategy is selected and executed on the problem. That is why we aimed at testing the hypothesis that strategy execution is influenced by strategy sequential difficulty effects. This hypothesis predicts that participants' performance with a given strategy would be poorer if the strategy used on the previous problem was difficult.

Finding strategy sequential difficulty effects would be important for two types of reasons. First, from an empirical perspective, strategy sequential difficulty effects would be an additional predictor of strategy performance, which has never been taken into account in previous research on strategies. Second, strategy sequential difficulty effects would be theoretically interesting for models of strategy choices. Indeed, none of the assumptions of current computational models of strategy selection accounts for the fact that participants obtain poorer performance on a problem with a given strategy after solving a preceding problem with a difficult strategy than with an easier strategy. Computational models of strategy choices (e.g., Lovett and Anderson's 1996 ACT-R model; Lovett and Schunn's 1999 RCCL model, Payne, Bettman, and Jonhson's 1993 adaptive decision maker model, Rieskamp and Otto's 2006 SSL model, or Siegler and Arraya's 2005 SCADS* model) share several core assumptions regarding how we choose among strategies and execute the selected strategy on a given problem. All models propose that choosing among multiple strategies crucially involves associative mechanisms like activating relative costs/benefits of each strategy and selecting the strategy that works best for a given problem on the basis of problem and strategy characteristics. All models also assume that strategies including fewer and/or simpler procedures (e.g., retrieving correct solution of arithmetic problems like 3x4 directly from memory) are easier to execute than strategies including more and/or more complex procedures (e.g., adding 3 four times). These assumptions proved sufficient to account for most findings on strategy choices and strategy execution but would not be sufficient to account for strategy sequential difficulty effects.

Here, we tested strategy sequential difficult effects by asking participants to accomplish computational estimation tasks in which they had to provide estimates to two-digit arithmetic problems (e.g. 43 + 68) with one of the following strategies: mixed-rounding (i.e., rounding the first operand down and the second operand up to the nearest decade; 40+70=110), rounding-down (i.e., rounding both operands down to the nearest decade; 40+60=100), or rounding-up (rounding both operands up to the nearest decade; 50+70=120). Previous research has shown that these strategies differ in difficulty (e.g., Dowker et al., 1996; Imbo et al., 2007; LeFevre et al., 1993). The rounding-down strategy is easiest because it does not require the extra step of incrementing operands and keeping them in working memory. Both the rounding-up and mixed-rounding strategy are more difficult, because the rounding-up strategy requires incrementing and maintaining two operands in working memory and the mixed-rounding strategy requires a switch of operations (rounding the first operand down and the second one up).

The hypothesis that strategy execution is influenced by relative difficulty of the previous strategy predicts that participants will take less time to execute a strategy after using the easier rounding-down strategy than after using the harder rounding-up strategy. Executing the same hard strategy on consecutive trials should also induce sequential difficulty effects. However, these sequential difficulty effects could be partly overshadowed by repetition benefits. In a first experiment we tested these predictions and in a second experiment we replicated it with a different set-up and tested the interaction between strategy sequential difficulty and problem difficulty. Solving difficult problems could reduce the amount of cognitive resources available, so impairment on execution of the next strategy after a difficult strategy could be magnified.

Experiment 1

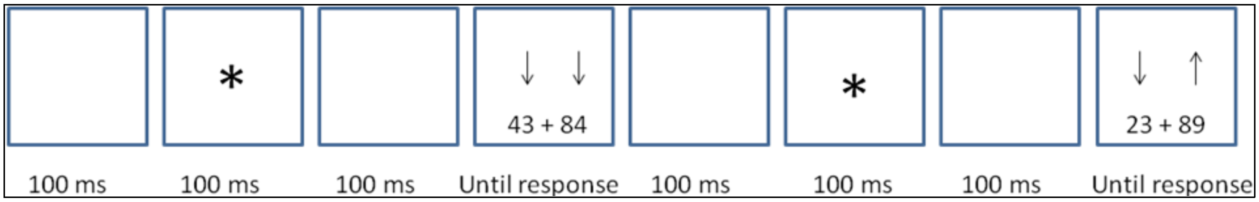
Method

Participants. Twenty-five undergraduates from Aix-Marseille Universite (12 females; 18-28 years, mean: 24 years, 7 months) participated in this experiment

Stimuli. Sets of two-digit addition problems (e.g., 32+68) were created. These sets included rounding-down problems, rounding-up problems, and mixed-rounding problems. Unit digits of both operands were smaller than 5 for rounding-down problems (e.g., 43+64) and larger than 5 for rounding-up problems (e.g., 47+68). Unit digit was smaller than 5 in the first operand and larger than 5 in the s second operand for mixed-rounding problems (e.g., 43+69). Following previous findings in arithmetic (see Campbell, 2005, for overview), we controlled the following factors: (a) no operands contained a 0, 5, or repeated digit (e.g. 44), (b) no reverse orders of operands were used (e.g., 43+82 and 82+43), (c) the first operand was larger than the second in half the problems, (d) no operand would round to 0, 10, or 100, (e) the operands of a problem would never round to the same decade, (f) the problems had a comparable mean exact sum per item condition, (g) the conditions were matched for the difference between correct sums and estimates, (h) the conditions had a comparable number of problems with carry-over on the tens (50%), and (i) during the experiment, the estimated sum of two successive problems was never the same.

Procedure. The stimuli were presented in a 72-point font on a 1280 x 800 screen. Participants were told that they were going to see addition problems to which they had to estimate the answer using one of three strategies. The rounding-down strategy was explained as rounding both operands down to the smaller decades (e.g. 43+24=40+20=60); the rounding-up strategy was described as rounding both operands up to the larger decades (e.g. 48+29=50+30=80); and the mixed-rounding strategy was presented as rounding the first operand down to the smaller decade and the second operand up to the larger decade (e.g. 43+28=40+30=70). Participants were told that they should use the indicated strategy on each trial. Strategies were indicated by two arrows pointing in the direction in which the operand needed to be rounded. They were instructed to say the estimate of each problem out loud. Participants saw three blocks of 26 trials each. Each trial was made up of two problems, yielding a total of 156 problems per participant. On the first problem of each trial, participants had to randomly execute either the rounding-down, rounding-up, or mixed-rounding strategy. On the second problem, they had to execute the mixed-rounding strategy. Each problem matched the cued strategy: For the rounding-down strategy, rounding-down problems were presented; for the rounding-up strategy, rounding-up problems were presented; and for the mixed-rounding strategy, mixed-rounding problems were presented. All problems were separated by a 100 ms blank screen followed by a 100 ms fixation cross followed by another 100 ms blank screen. The trial-procedure is displayed in Figure 1. The time until each response was measured by experimenter key-press, occurring as soon as possible after the response. To avoid experimenters' expectations influencing the response time measurement, we used a double blind procedure. Errors were recorded by having the experimenter write down the answers of the participants so errors could later be identified.

Figure 1. Trial procedure in Experiments. 1 and 2.



Results

The first analysis was aimed at checking the relative difficulty of our strategies such that the rounding-down strategy yielded best and rounding-up worst performance. To achieve this end, we conducted repeated measures ANOVAs on participants' mean solution times and percent errors on the first problem with strategy as a within-participants variable. The second analysis aimed at testing strategy sequential difficulty effects. We conducted repeated measures ANOVAs on participants' solution times and percent errors on the second problem with the strategy on the first problem as a within-participants variable. Prior to analyses on solution latencies, values exceeding the mean + 2 x standard deviation (4.5%) and all trials containing an error (9.5%) were removed. All reported effects are significant with $\underline{p}<.05$.

Relative strategy difficulty. Solution latencies varied with strategies, $F(2,48)=19.1$, $MSe=82158$, $\eta p^2=0.44$. Planned comparisons showed that participants were slower when executing the mixed-rounding (3475 ms) than when executing the rounding-down strategy (3117 ms), *F*(1,24)=27.7, *MSe*=57964. They were not significantly slower when executing the rounding-up strategy (3600 ms) than when executing the mixed-rounding strategy (3475 ms), *F*(1,24)=3.2, *p*=0.09, *MSe*=60783.1.

Analyses of mean percent errors showed a main effect of strategy, *F*(2,48)=8.9, *MSe*=38.6, ηp^2=0.27. Planned comparisons showed that participants erred more when executing the rounding-up strategy (10.5%) than when executing the mixed-rounding strategy (6.4%), *F*(1,24)=5.5, *MSe*=38.2, and erred more when executing the mixed-rounding strategy (6.4%) than when executing the rounding-down strategy (3.1%), *F*(1,24)=6, *MSe*=22.3.

Strategy sequential difficulty effects. Solution latencies on the second problem differed as a function of the strategy used on the first problem, *F*(2,48)=15.8, *MSe*=30590, ηp^2=0.40. Planned comparisons showed that participants were slower after executing the rounding-up strategy (3647 ms) than after executing the mixed-rounding strategy (3482 ms), *F*(1,24)=10.7, *MSe*=31503. Furthermore, they were slower after executing the mixed-rounding strategy (3482 ms) than after the rounding-down strategy (3370 ms), *F*(1,24)=7.3, *MSe*=21729.6.

Mean percent errors on the second problem differed with the strategy used on the first problem, *F*(1,24)=4.6, *MSe*=16.1, ηp^2=0.16. Planned comparisons showed that participants erred more after the mixed-rounding strategy (8.5%) than after the rounding-up strategy (5.2%), *F*(1,24)=7.6, *MSe*=17.7). The difference after the rounding-up strategy (5.2%) and the rounding-down strategy (6%) was not significant, *F*<1.

Discussion

Experiment 1 confirmed relative strategy difficulty and revealed strategy sequential difficulty effects. The rounding-up strategy was the most difficult strategy and rounding-down was the easiest strategy. Moreover, participants were slower after using rounding-up than after using mixed-rounding, and were fastest after executing rounding-down. This is consistent with our hypothesis that strategy execution on a given trial is influenced by the difficulty of the strategy that was executed on the previous trial, even in the case of repetition of strategies. Interestingly, when repeating the mixed-rounding strategy, there were also significantly more errors. We believe that this could be due to fast mixed-rounding initiation on the second problem in the case of strategy repetition. This fast initiation in combination with only partly remobilized resources could then have led to more errors during strategy execution. Stated otherwise: Accuracy could be traded against speed in the case of a repetition of the mixed-rounding strategy, which could make sequential difficulty effects visible on the error rates.

We did not look at how strategy sequential difficulty effects interacted with problem-difficulty, which has been shown to yield sequential difficulty effects (Schneider & Anderson, 2010). Half of the problems in each condition required carry-over on the tens and could be considered difficult. Analysis of the interaction between problem-difficulty and strategy sequential difficulty was not possible in this design since the transition between carry and no-carry problems was not controlled. Experiment 2 aimed at testing how and if problem-difficulty interact with strategy sequential difficulty effects.

Furthermore, it could be argued that execution of the mixed-rounding strategy was affected by its frequent execution and predictability compared to other strategies. However, since practice effects and predictability of the mixed-rounding strategy were equal in all conditions we would not expect this to have caused strategy sequential difficulty effects. Nevertheless, Experiment 2 aimed at confirming the effects in a set-up with unpredictable and equal proportions of strategies.

Finally, one could argue that experimenter key press for measuring solution times was not an optimal measuring procedure because it involved additional experimenter-related reaction time. We chose this measuring procedure rather than voice key registering because during calculation, especially when it is difficult, participants have the tendency to verbalize or to make other unintentional sounds. This sets off the voice key prematurely which leads to a lot of incorrect reaction time registrations and adds extra-stress in participants (because the problems disappear from the screen). Using key-press registering avoided these problems. However, our double-blind procedure made it unlikely that these extra reaction times would be unequally distributed across conditions. Furthermore, previous works have shown that data from this measurement procedure converge with voice key data (Lemaire & Lecacheur, 2010). Nevertheless, Experiment 2 employed a different measuring procedure to test whether the effects are independent from the measuring procedure used in Experiment 1.

Experiment 2

In this experiment, we addressed limitations of Experiment 1. Regarding response time measurement, the participants themselves pressed a key during their verbal response. Verbal protocols collected from every participant indicated that this double response was easy to execute. Furthermore, we presented participants with equal proportions of each strategy, thereby decreasing predictability of the mixed-rounding strategy. A convergence of results from Experiments 1 and 2 would suggest reliability of the individual procedures.

Additionally, we tested whether strategy sequential difficulty effects found in Experiment 1 interacted with difficulty of problems. We tested carry versus no-carry problems (e.g., Deschuyteneer, De Rammelaere & Fias , 2005) in separate blocks. We expected that separating these problems in blocks would maximize the differences between carry and no-carry problems since in the carry block processing resources would be constantly taxed, reducing performance on the ensemble of problems whereas in the no-carry block resources would be constantly freed, improving performance on the level of the block. We expected that this would maximize our chances for observing an interaction between problem difficulty and strategy sequential difficulty effects. We predicted larger strategy sequential difficulty effects in carry blocks than in no-carry blocks, arising from larger amounts of working-memory resources required to solve carry problems (Fürst & Hitch, 2000; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994), maximizing effects of difficult strategies on current strategy execution.

Method

Participants. Thirty-six undergraduates from the University of Provence (22 females; 19-30 years, mean: 23 years, 6 months) participated in this experiment. We excluded two participants because of high error rates (> 50%).

Stimuli. We used the same type of problems as in Experiment 1.

Procedure. The set-up was the same as in Experiment 1, with the following exceptions: Participants were told to execute a concurrent key press when giving their verbal response to the problems; there was an equal number of rounding-down (RD), rounding-up (RU), and mixed-rounding (MR) problems; and carry and no-carry problems were presented in separate blocks (counterbalanced).

We presented participants with 16 RD-MR and 16 RU-MR experimental trials, and 8 RD-RU and 8 RU-RD filler-trials per block, yielding 192 problems in total. The order of trial types was semi-random, with constraints to control for sequential effects over longer sequences of items: The RD and RU strategies preceding the MR strategy in the experimental trials were preceded by RD in 25% of trials, by RU in 25% of trials, and 50% by MR

Results

Our data-analysis design was identical to Experiment 1, with inclusion of the carry/no-carry variable. Prior to analyses on solution latencies, values exceeding mean + 2 x standard deviation (4.4%) and all trials containing an error (9.1%) were removed.

Relative strategy performance. Participants were slower when executing the rounding-up strategy (3886 ms) than when executing the rounding-down strategy (3033 ms). *F*(1,33)=86.8, *MSe*=285471, ηp^2=0.72. Also, participants took more time to solve carry problems (3923 ms) than no-carry problems (2996 ms), *F*(1,33)=58.3, *MSe*= 501494, ηp^2=0.64.

Analyses of mean percent errors revealed that participants erred more on carry (9.1%) than on no-carry problems (6.9%), *F*(1,33)=6, *MSe*=27.4, ηp^2=0.15.

Strategy sequential difficulty effects. Participants executed the mixed-rounding strategy more slowly after executing the rounding-up strategy (3580 ms) than after executing the rounding-down strategy (3401 ms), *F*(1,33)=8.3, *MSe*=130810, ηp^2=0.20. Participants were slower to solve carry problems (3800 ms) than no-carry problems (3180 ms), *F*(1,33)=58.4, *MSe*=223871, ηp^2=0.64. The Carry x Strategy interaction was not significant, *F*<1. Finally, participants erred more on carry (5.4%) than on no-carry problems (3.4%), *F*(1,33)=4.8, *MSe*=26.9, ηp^2=0.13. No other effects were significant.

Discussion

Experiment 2 replicated relative strategy difficulty, as participants were slower with the rounding-up than with the rounding-down strategy. Moreover, Experiment 2 replicated strategy sequential difficulty effects: Participants executed the mixed-rounding strategy more slowly after using the rounding-up strategy than after using the rounding-down strategy.

Moreover, participants took 774 ms longer to solve the carry problems as compared to the no-carry problems, which is much larger than the carry effect in Experiment 1 (302 ms), indicating that our carry manipulation by block did result in larger carry effects. Finally, strategy sequential difficulty effects were not modulated by problem-difficulty. This suggests that the extra cognitive resources required to solve carry problems did not modulate strategy sequential difficulty effects, which seem to exist independently from the problem sequential difficulty effects demonstrated by Schneider and Anderson (2010). The fact that strategy sequential difficulty effects did not increase when cognitive resources were taxed more is somewhat surprising. It could be that the specific resource that we taxed is not implicated in producing sequential difficulty effects or that we did not tax it sufficiently to produce effects.

General discussion

The primary result of this study showed the existence of strategy sequential difficulty effects as longer solution latencies after the more difficult rounding-up strategy. How could a difficult strategy slow down execution of a subsequent strategy? Schneider and Anderson (2010) proposed that working memory might need time to be cleared. Since the difficult rounding-up strategy relies more on working memory than the easy rounding-down strategy, less working memory resources would be available following this strategy, which would slow down execution of the next strategy. Additionally, uncleared working memory content could interfere with current strategy execution.

Our results also bring new lights to previously found sequential effects during strategy execution. In Experiment 1, we found that the mixed-rounding strategy was executed more slowly after the mixed-rounding strategy than after the rounding-down strategy. Repetition of the same strategy should have yielded benefits in line with previous strategy switching research (e.g., Lemaire & Lecacheur, 2010; Luwel et al., 2009). Repetition benefits of the mixed-rounding strategy could have been partly overshadowed by sequential difficulty effects associated with this strategy. Furthermore, in Lemaire and Lecacheur (2010), strategy switch costs were absent when switching from the easy rounding-down to the difficult rounding-up strategy. Analogical to Schneider and Anderson (2010) explanation of asymmetric task switch costs and asymmetries found in task repetitions (Bryck & Mayr, 2008), a sequential difficulty account of the effects is plausible: Execution of a task or strategy after an easy task or strategy will undergo less sequential difficulty effects than execution after a difficult strategy, leading to fully or partially neutralized switch costs following easy strategies and enhanced switch costs following difficult strategies.

Our findings help further our understanding of how strategies are executed. Previous theoretical and empirical works revealed that strategy, problem, situation, and participants' characteristics influence how participants execute strategies. The present findings suggest that strategy execution is also influenced by the order in which strategies are executed. This is important, as in many cognitive tasks, participants execute strategies in sequences. Given that executing a difficult strategy not only yields longer execution times in and of itself but also for the following strategy, sequential difficulty effects could accumulate and lead to severe slowdown of strategies later in the sequence. Future studies will investigate how strategy sequential difficulty effects evolve in longer sequences, with varying inter-trial intervals and with strategies that show less interference. Moreover, given the relationships between strategy choices and strategic execution performance, it is possible that during strategy selection, participants select strategies so as to minimize the detrimental effects of sequential difficulty. People may use a difficult strategy less often where it is expected to impair subsequent performance. Establishing that the effects of sequential difficulty are taken into account during strategy selection would lead models of strategy selection (Strategy Choice And Discovery Simulation, Siegler & Arraya, 2005; Represent Construct Choose Learn, Lovett & Schunn, 1999 and Strategy Selection Learning; Rieskamp & Otto, 2006) to be revised. Currently, these models explain strategy performance as a result of the number and type of procedures included in each strategy (i.e., more procedures or harder procedures in one strategy result in longer latencies). The present data suggest that the difficulty of the strategy executed immediately before the current strategy modulates the role of the number and type of procedures within each strategy. This modulation likely occurs via working-memory and/or executive control resources. Including a parameter for these resources within currently available computational models of strategies would enable these models to account and to simulate the present strategy sequential difficulty effects.

Table 1: Mean solution latencies in ms (and percent errors) for the first problem as a function of the strategy that was executed, in Experiments 1 and 2.

Strategies	**Experiment 1**	**Experiment 2**	
		No-Carry Problems	**Carry Problems**
Rounding Down	3117 (3.1)	2573 (7.7)	3492 (8.3)
Rounding Up	3600 (10.5)	3419 (6.6)	4354 (9.4)
Mixed Rounding	3475 (6.4)	--	--
Rounding Up – Rounding Down	483 (7.4)	846 (-1.1)	862 (1.1)

Table 2: Mean solution latencies in ms (and percent errors) for the second problem as a function of the strategy that was executed on the first problem, Experiments 1 and 2.

Strategies used on the first problem	**Experiment 1**	**Experiment 2**	
		No-Carry Problems	**Carry Problems**
Rounding Down	3370 (6.0)	3076 (3.2)	3725 (4.9)
Rounding Up	3647 (5.2)	3284 (3.9)	3875 (6.0)
Mixed Rounding	3482 (8.5)	--	--
Rounding Up – Rounding Down	277 (-0.8)	208 (0.7)	150 (1.1)

Strategy Sequential Difficulty Effects and Working Memory

Strategy Sequential Difficulty Effects and Working Memory: A correlational study in arithmetic.

Kim Uittenhove[1] & Patrick Lemaire[1]

[1]Laboratoire de Psychologie Cognitive, Centre National de la Recherche Scientifique and Aix-Marseille Universite, Marseille, France

Address correspondence to Patrick Lemaire,
LPC-CNRS and Aix-Marseille Universite,
3 place Victor Hugo, Bâtiment 9, case D,
Marseille 13331, France.
Phone: +33 (0)4 88 57 69 01;
Email: Patrick. Lemaire@univ-provence.fr

Authors' Note

This research was supported in part by the CNRS (French NSF) and as grant from the Agence Nationale de la Recherche (Grant # BLAN07-1_196867). Correspondence about this paper should be directed to Patrick Lemaire, CNRS & Aix-Marseille Universite, 3 Place Victor Hugo, Case D, 13331 Marseille, France (email: Patrick.Lemaire@univ-provence.fr).

Abstract

Strategy Sequential Difficulty effects are the finding that when participants execute strategies, performance is worse when following a difficult strategy than when following an easy strategy (Uittenhove & Lemaire, in press). Strategy sequential difficulty effects effects are hypothesized to be due to decreased working-memory resources following difficult strategy execution. In this study we found a correlation between individuals' working-memory and strategy sequential difficulty effects in arithmetic, supporting a working-memory account of these effects. Furthermore, we varied response-stimulus intervals to investigate the time-course of strategy sequential difficulty effects. Implications for strategic variations are discussed.

Sequential difficulty effects in arithmetic have first been discovered by Schneider and Anderson (2010). In three experiments, participants had to switch between tasks (addition and subtraction) in an arithmetic problem verification task. The authors manipulated the difficulty of problems (problems with or without carry-over in Expt. 1; vertical or horizontal format in Expt. 2; and true or false problems in Expt. 3). They found that participants took more time to solve a problem if this problem followed a hard problem than if it followed an easy problem. Similar effects have been found in the language domain: In word-naming tasks, the mixing of easy and difficult stimuli slowed down participants' performance on the easy stimuli (Lupker, Kinoshita, Coltheart, & Taylor, 2003; see Kiger & Glass, 1981, for similar results in sentence verification). Taylor and Lupker (2001) established that the slowdown on easy stimuli is contingent upon the presentation of difficult stimuli on the immediately preceding trials.

Uittenhove and Lemaire (in press) took a strategy approach to problem-solving and proposed that problem sequential difficulty effects could be explained by sequential difficulty effects associated to the different strategies used on different problems. For example, in Schneider and Anderson's Experiment 2, vertical presentation of arithmetic problems could have elicited a columnar-retrieval strategy whereas horizontal presentation could have elicited a more demanding procedural strategy (Geary, Frensch, & Wiley, 1993; Green, Lemaire, & Dufau, 2007).

Sequential difficulty effects in Schneider and Anderson (2010) could have been due to the difficulty of the strategies elicited by the difficulty manipulations. Uittenhove and Lemaire (in press) presented participants a computational estimation task (e.g., estimating the solution to 43+57) with imposed rounding strategies. They found that participants were faster following the easier rounding-down strategy (e.g., doing 40+50 to estimate 43+57) than following the harder rounding-up strategy (e.g., doing 50+60 to estimate 43+57). Uittenhove and Lemaire suggested that working-memory (WM) needs time to be cleared or overwritten after occupation (see also Schneider & Anderson, 2010). During execution of a strategy it is important to keep online information that is relevant to strategy execution and ignore irrelevant information, thus putting a burden on WM. For example, in experiments where WM is blocked by a secondary task, strategy implementation is disrupted (Robins et al., 1996; Duverne, Lemaire & Vandierendonck, 2008). Since difficult strategies rely more on WM than easy strategies (Imbo, Duverne, & Lemaire, 2007; Fürst & Hitch, 2000; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994), Uittenhove and Lemaire (in press) assumed that less WM would be available following difficult strategies because of the time needed to clear or overwrite the content of WM, which would slow down execution of the next strategy. Additionally, uncleared WM-content from a previous strategy could interfere with the information-retrieval for current strategy execution.

In this study, we aimed at testing the WM-explanation of strategy sequential difficulty (SSD) effects. To do this, we investigated the time-course of SSD effects and we tested whether SSD effects in arithmetic are correlated to individuals' WM. This was expected to inform us on whether WM is freed immediately after completion of strategy execution or is overwritten only when a next strategy requires WM-resources, and on whether individuals with low WM-span would show larger SSD effects.

WM was measured by the quantity of information that could be actively managed in WM. This is tested by how many items (words, digits, letters) a person can keep online for recall in a situation that requires resisting interference, shifting attention, and manipulating information. Note that this type of test involves not only an information-maintenance component, but also information-manipulation and WM-managing components. We tested SSD effects by asking participants to provide estimates to two-digit arithmetic problems (e.g., 43+68) with one of the following strategies: Mixed-rounding (i.e., rounding the first operand up and the second operand down to the nearest decades; 50+60=110), rounding-down (i.e., rounding both operands down to the nearest decades; 40+60=100), or rounding-up (rounding both operands up to the nearest decades; 50+70=120). Previous research has shown that these strategies differ in difficulty (e.g., LeFevre, Greenham, & Waheed, 1993; Lemaire, Arnaud, & Lecacheur 2004). The rounding-down strategy is easiest because it does not require the extra step of incrementing operands and keeping them in WM. The rounding-up strategy is more difficult, because it requires incrementing and maintaining two operands in WM. With this paradigm, Uittenhove and Lemaire (in press) have shown that executing mixed-rounding is slower following the rounding-up strategy than after the rounding-down strategy.

The hypothesis that SSD effects are related to temporarily reduced WM-resources predicts that individuals with low WM would have larger SSD effects than individuals with high WM, because they would have more difficulties in clearing or overwriting WM. Furthermore, we varied response-stimulus interval (RSI), which is the duration between participants' response on a given problem and the display of the next problem, from short (300 ms) to long (600 ms). This RSI-factor enabled us to test whether WM is cleared immediately after strategy execution or whether content lingers until WM-resources need to be overwritten for the next strategy. We manipulated this variable within participants, and blockwise because if in our blocked design we find diminished SSD effects in the block with long RSI, we cannot attribute it to the adoption of a meta-strategy in response to the presence of short RSI trials. We predicted that SSD effects would diminish following longer RSI because of temporal decay of the contents of WM

(Barouillet, De Paepe, & Langerock, 2012) or because longer RSI gives participants more time to clear WM for the next strategy.

Method

Participants Sixty (15 men; mean age: 20 years; age range: 17-31 y.o.) undergraduates from Aix-Marseille Université (France) received course credit for their participation. The participants were unaware of the goal of the study. Two participants were excluded for extensive talking or coughing during the experiment, distorting the solution latencies.

Material

WM-tasks: WM-capacities were tested with three tasks: The operation span, the running span, and the reading span task. The operation span (Unsworth, Heitz, Schrock, & Engle, 2005) required participants to recall a series of 2 to 7 letters when these were alternated by arithmetic verification problems (e.g., “(4+2)-1” followed by “1” and “true?” or “false?”). The score for operation span was calculated as the total number of letters recalled correctly in all of the trials. The participants were told to focus on the arithmetic part and had to maintain above 80% accuracy in order for the results to be valid. The running span (Broadway & Engle, 2010) required participants to recall the last three to six letters of a series that contained either the same number or more letters (e.g., participants were asked to recall the last three letters of the following series: a g h t). The score for running span was calculated as the total number of letters recalled correctly. The reading span (Daneman & Carpenter, 1980) required participants to recall the last words of a series of two to five sentences in the correct order while having to perform semantic judgment on the sentences (e.g., “Dans le lac nagent des saladiers” followed by “Correct?” or “Incorrect?”). We used a French version of the task (Delaloye, Ludwig, Borella, Chicherio, & de Ribaupierre, 2007). The score for reading span was calculated as the total number of words recalled correctly in all of the trials.

Computational estimation task: We used the same stimuli as Uittenhove and Lemaire's Experiment 2 (in press). Sets of two-digit addition problems (e.g., 32+68) were created. These sets included rounding-down problems, rounding-up problems, and mixed-rounding problems. Unit digits of both operands were smaller than 5 for rounding-down problems (e.g., 43+64) and larger than 5 for rounding-up problems (e.g., 47+68). Unit digit was larger than 5 in the first operand and smaller than 5 in the second operand for mixed-rounding problems (e.g., 49+63). We presented participants with 32 RD-MR and 32 RU-MR experimental trials. We also had 16 RD-RU and 16 RU-RD filler-trials, to ensure that each strategy was executed equally often. Moreover, (1) the problems had comparable mean exact sums per condition as well as comparable differences between mean exact sums and mean correct estimates, (2) all conditions had carry-over on the tens in half the problems, (3) each transition between carry-over and no carry-over problems appeared equally often, and (4) during the experiment, estimates to two successive problems were never the same. Following previous findings in arithmetic (see Campbell, 2005, for overview), we also controlled the following factors: (a) no operands contained a 0, 5, or repeated digits (e.g., 44), (b) no reverse orders of operands were used (e.g., 43+82 and 82+43), (c) the first operand was larger than the second in half the problems, (d) no operand would round to 0, 10, or 100, (e) the operands of a problem would never round to the same decade,

Procedure All participants first completed the computational estimation task, and then the operation span, reading span, and running span tests. In the 41 participants that were included in the correlation study, we always presented the short RSI-block first, to avoid noise in the SSD effects due to the order of presentation of RSI-blocks. We tested 17 additional participants with the long RSI-block presented first, to neutralize the effect of order of presentation of RSI-blocks for the analysis of the effect of RSI on SSD effects.

Computational estimation: The stimuli were presented in a 72-point font on a 1280x800 computerscreen. Participants were told that they were going to see addition problems to which they had to estimate the answer with one of three strategies. The rounding-down strategy was explained as rounding both operands down to the smaller decades (e.g., 43+24=40+20=60); the rounding-up strategy was described as rounding both operands up to the larger decades (e.g., 48+29=50+30=80); and the mixed-rounding strategy was presented as rounding the first operand up to the larger decade and the second operand down to the smaller decade (e.g., 48+23=50+20=70). Participants were told that they should use the indicated strategy on each trial. Each problem matched the cued strategy: Rounding-down problems were cued to be solved by the rounding-down strategy, rounding-up problems by the rounding-up strategy, and mixed-rounding problems by the mixed-rounding strategy. Strategies were indicated by two arrows pointing in the direction in which the operand needed to be rounded. Participants were instructed to say the estimate of each problem out loud. They saw two blocks of 48 trials each. Each trial was made up of two problems, yielding a total of 192 problems per participant. The order of trials was semi-random, with constraints to control for sequential effects over longer sequences of items: The RD and RU strategies in the experimental trials (RD-MR and RU-MR) were each preceded by RD in 25% of trials, by RU in 25% of trials, and in 50% of trials by MR. One of the blocks consisted of trials with short RSI and one of the blocks consisted of trials with long RSI. In 41 participants the short RSI-block was presented first and we tested 17 additional participants with the long RSI-block presented first. In the block with short RSI, the problems were separated by a 100 ms blank screen followed by a 100 ms fixation cross followed by another 100 ms blank screen. In the block with long RSI, the problems were separated by a 200 ms blank screen followed by a 200 ms fixation cross followed by another 200 ms blank screen. The time until each response was measured by instructing participants to execute a concurrent key press when giving their verbal response to the problems.[1] Errors were recorded by having the experimenter write down the answers of the participants so errors could later be identified.

Results

The first analysis was aimed at checking the relative difficulty of our strategies such that the rounding-down strategy yielded best and rounding-up worst performance and whether this varied with RSI. The second analysis aimed at testing SSD effects and the effects of short and long RSI's. The third and final analyses aimed at testing the correlation between participants' SSD effects and their WM. Prior to analyses on solution latencies, values exceeding the mean + 2 x standard deviation (4%) and all trials containing an error (11.7%) were removed for each participant. All reported effects are significant with $p<.05$.

---

[1] We used self-executed key-press registering instead of voice key registering because during calculation, participants have the tendency to verbalize or to make other unintentional sounds, setting off the voice key prematurely. We do not expect our measurement procedure to have induced systematic differences between conditions since it was applied in the same way on the same strategy in both conditions. Moreover, Lemaire and Lecacheur (2010) have compared voice key data to experimenter key press data and have found identical patterns of results with both measurement procedures in a strategy switching experiment.

Relative Strategy difficulty

We conducted a repeated-measures ANOVA on all participants' mean solution times and percent errors on the first problem of each trial with strategy and RSI as a within-participants variable (See Table 1).

Table 1: Mean solution latencies in ms (and percent errors) for the first problem as a function of the strategy that was executed, for short (300 ms) and long (600 ms) RSI.

	Rounding Down	**Rounding Up**	**Rounding Up – Rounding Down**
Short RSI (300 ms)	4305 (9.5)	5044 (11)	739 (1.5)
Long RSI (600 ms)	4041 (7.4)	4690 (7.3)	649 (-0.1)
Mean	4173 (8.5)	4867 (9.2)	694 (0.7)

Solution latencies varied with strategies, *F*(1,57)=104.8, *MSe*=266655, ηp^2=0.65. Participants were slower when executing the rounding-up (4867 ms) than when executing the rounding-down strategy (4174 ms). Solution latencies also varied with RSI, *F*(1,57)=11.2, *MSe*=495587, ηp^2=0.16. Participants were slower when solving problems in the short RSI block (4675 ms) than in the long RSI block (4366 ms). The effect of strategy on solution latencies did not vary with RSI, *F*<1.

Percent errors did not vary with strategies, *F*<1, but varied with RSI, *F*(1,57)=14.7, *MSe*=0.0034, ηp^2=0.20. Participants erred more when solving problems in the short RSI block (10.2%) than in the long RSI block (7.3%). The effect of strategy on percent errors did not vary with RSI, *F*<1.4.

Strategy sequential difficulty effects

Preliminary analyses revealed only an interaction between order of presentation of RSI-blocks and RSI on solution latencies, *F*(1,56)=22.8, *MSe*=6033277, ηp^2=0.29, with participants solving problems more slowly with short RSI (4860 ms) than with long RSI (4276 ms) only when the short RSI-block was presented first, *F*(1,56)=52.9, *MSe*=14000000. We conducted a repeated measures ANOVA on all 58 participants' solution times and percent errors on the second problem with the strategy on the first problem and RSI as within-participants variables (See Table 2).

Solution latencies on the second problem varied with strategies on the first problem, *F*(1,57)=4.5, *MSe*=264435, ηp^2=0.07. Participants were slower after executing the rounding-up (4704 ms) than after executing the rounding-down strategy (4560 ms). These SSD effects almost significantly varied with RSI, *F*(1,57)=3.3, *p*=0.07, *MSe*=214940, ηp^2=0.05. Participants were significantly slower after solving problems with the rounding-up strategy (4947 ms) than after solving problems with the rounding-down strategy (4693 ms) in the short RSI-block, *F*(1,57)=6.2, *MSe*=1872162, but not in the long RSI-block, *F*<1. Analyses of mean percent errors revealed no significant main or interaction effects (*F*-values<2.1) .

Table 2: Mean solution latencies in ms (and percent errors) for the second problem as a function of the strategy that was executed on the first problem, for short (300 ms) and long (600 ms) RSI.

Strategy used on the previous problem	**Rounding Down**	**Rounding Up**	**Rounding Up – Rounding Down**
Short RSI (300 ms)	4693 (6.8)	4947 (7.2)	281 (0.4)
Long RSI (600 ms)	4427 (6.5)	4460 (4.8)	33 (-1.7)
<u>Mean</u>	4560 (6.7)	4704 (6)	157 (-0.7)

WM-capacity and SSD effects

We used the 41 participants who saw the short RSI-block first for this analysis. We calculated SSD effects per participant by taking the difference between the average solution latency for the mixed-rounding strategy following rounding-up and following rounding-down divided by the mean solution latency following rounding-down. For WM, for each individual, we calculated a *z*-score for each test. We performed a correlation test on participants' SSD effects and the three measures of WM (See Table 3).

Table 3: Correlations between Strategy Sequential Difficulty Effects (SSDE) and Operation, Reading, Running span, and a component-score based on the latter three testscores.

	SSDE	**Operation span**	**Reading span**	**Running Span**	**WM-component**
SSDE	- -				
Operation span	-0.29	- -			
Reading span	-0.20	0.32*	- -		
Running span	-0.22	0.48**	0.32*	- -	
WM-component	-0.31*	0.80**	0.68**	0.80**	- -

$p<.05$; **$p<.01$

SSD effects were negatively correlated with operation span ($r=-.29$), reading span ($r=-.20$) and running span ($r=-.22$). The difference between these correlations was not significant, $p<.62$. The WM-capacities tests positively correlated amongst each other (rs>.31). Given that these three tests are assumed to measure the same underlying factor, we conducted a principal component analysis to determine the loadings of the three tests on a common component that we assumed were WM-capacities. The proportion of variance explained by the one-component model was .58. We used these loadings to reconstitute a component score for each participant and found that it correlated significantly negatively with SSD effects, $r=-.31$, $p<0.05$.

Finally, we performed an ANOVA with the strategy on the first problem as a within-participants variable and WM component scores (low or high) as a between-participants variable on second problem solution latencies. This test revealed that individuals with low WM (415 ms) had significantly larger SSD effects than individuals with high WM (29 ms), *F*(1,39)=5.9, *MSe*=128550, ηp^2=0.13.

Discussion

We found that low WM was associated with larger SSD effects. This supports the hypothesis that SSD effects are related to WM. Based on our results, we assume that SSD effects arise because of incomplete restoring of WM following strategy execution. 'Restoring' could refer to clearing or overwriting of the contents of WM. More complicated strategies use WM-resources more and hence these resources take longer to be restored than after easier strategies. The more efficient is the managing-component of WM, the faster WM-resources can be restored. Individuals with high WM seem to be efficient enough to restore WM fast enough so as not to suffer from SSDE (29 ms), whereas low WM individuals have notable problems, resulting in much larger SSDE (415 ms). This has implications for inter-individual differences in strategy execution: Individuals or populations suffering from declines in WM (for example older adults and AD patients) can be expected to suffer more from SSD effects, which may hamper their problem-solving performance.

Our results are relevant to a recent account of Hofmann, Schmeichel, and Baddeley (2012), whom argued that a temporary reduction in WM can result from either concurrent task load or prior high intensity engagement. They based this hypothesis on findings from Schmeichel (2007) who found that efforts at executive control temporarily undermined subsequent efforts at executive control. For example, taking a working-memory test undermined performance on subsequent tests of working-memory. Our findings show that such effects do not only manifest from one test to another (e.g. general fatigue) but can also be present transiently on a trial-to-trial basis.

The last interesting finding here was decreased SSD effects with longer RSI, indicating that content of the previous strategy does not linger in WM until it is overwritten by the content for a next strategy execution. Rather, the previous strategy content in WM undergoes temporal decay (Barouillet, De Paepe, & Langerock, 2012) in the period immediately following strategy execution or participants start clearing WM immediately after strategy execution. This process could be time-consuming, hence introducing SSD effects at short RSI that diminish with increasing RSI. This further suggests that SSD effects on problem-solving performance can be avoided by allowing sufficient time between problems to allow restoring of the necessary processing resources. This is supported by the fact that participants erred more and were slower in general in the short RSI-block than in the long RSI-block.

We propose that strategies relying more on WM either postpone or elongate the execution of the next strategy, once the previous strategy is finished. The postponement hypothesis suggests that participants delay processing of the next strategy until WM-resources are cleared. The elongation hypothesis suggests that they will immediately start overwriting WM for the next strategy execution but that this process will be more difficult following execution of a difficult strategy.

Our findings have implications for how strategies are executed. Models of strategy selection (Strategy Choice And Discovery Simulation, Siegler & Arraya, 2005; Represent Construct Choose Learn, Lovett & Schunn, 1999 and Strategy Selection Learning; Rieskamp & Otto, 2006) explain strategy performance as a result of the number and type of procedures included in each strategy (i.e., more procedures or harder procedures in one strategy result in longer latencies). This is because execution of these procedures relies on limited WM-resources. The present data suggest that WM-resources also dynamically vary in function of the difficulty of the strategy executed just before. Including a parameter for these resources within currently available computational models would enable these models to account and to simulate strategy sequential difficulty effects and their link to WM.

The time course of strategy sequential difficulty effects

The time course of strategy sequential difficulty effects: An ERP study in arithmetic

Kim Uittenhove[1], Celine Poletti[1], Stephane Dufau[1], & Patrick Lemaire[1]

[1]Laboratoire de Psychologie Cognitive, Centre National de la Recherche Scientifique,

and Aix-Marseille Universite, Marseille, France

Address correspondence to Patrick Lemaire,

LPC-CNRS and Aix-Marseille Universite,

3 place Victor Hugo, Bâtiment 9, case D,

Marseille 13331, France.

Phone: +33 (0)4 88 57 69 01;

Email: Patrick.Lemaire@univ-provence.fr

Authors' Note

This research was supported in part by the CNRS (French NSF) and as grant from the Agence Nationale de la Recherche (Grant # BLAN07-1_196867). Correspondence about this paper should be directed to Patrick Lemaire, CNRS & Aix-Marseille Universite, 3 Place Victor Hugo, Case D, 13331 Marseille, France (email: patrick.lemaire@univ-amu.fr).

Abstract

Uittenhove and Lemaire (2012) found that we are slower when executing a strategy following a difficult strategy than when executing the same strategy following an easier strategy (i.e., strategy sequential difficulty effects). Uittenhove and Lemaire suggested that difficult strategies temporarily reduce available executive capacities, interfering with the next strategy execution. In this study, we used ERP to determine the time course of these effects. In a computational estimation task, we found greater cerebral activities during strategy execution following a more difficult compared to an easier strategy. Interestingly, greater cerebral activities were most apparent immediately after the encoding of the problem, and not during encoding or in later stages of processing. This suggests that strategy sequential difficulty effects interfere most with the retrieval of procedures in contrast to execution of these procedures. We discuss implications of these findings for further understanding of execution of cognitive strategies.

One of the characteristics of human cognition is that individuals use multiple strategies to accomplish a wide variety of cognitive tasks (see Siegler, 2007, for a review). Strategies, defined by Lemaire and Reder (1999, p. 365) as "a procedure or a set of procedures to achieve a higher level goal" are key to understanding cognitive performance. In all domains, performance depends on the type of strategy used by participants and is limited by how well the selected strategy is executed on each item. In this paper, we focus on strategy execution.

Previous data on strategy execution showed that it is influenced by characteristics of problems, participants, and situations (Siegler, 2007). These factors act individually and in interaction with each other to influence strategy execution performance. For example, Lemaire, Arnaud, and Lecacheur (2004) found that when participants accomplished a computational estimation task (i.e., finding an approximate product to to two-digit multiplication problems like 32 x 47), older adults executed the rounding-down strategy (i.e., doing 30 x 40 to estimate 32 x 41) more slowly under accuracy-pressure conditions than under no-pressure conditions, especially when they solved easy problems. This strategy x problem interaction was even stronger in young adults.

Several models have been proposed to formalize strategy execution and selection (Siegler & Shipley's ASCM, 1995; Lovett & Schunn's RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005). Regarding strategy execution, these models assume that strategies involving more and more complex procedures are executed less efficiently.

One important additional assumption of these models is that participants execute strategies on each problem, independently of the strategy that was selected and executed on the previous problem. Thus, only factors characterizing the current problem and strategy in addition to factors characterizing participants and situations are crucial parameters of strategy execution on a given trial.

Recent data challenge the view that strategy execution is uninfluenced by previous strategies and/or problems. Switching strategies or prior use of difficult strategies have been found to influence strategy execution performance on the next trial. For example, Luwel et al. (2009), in a numerosity judgment task, had participants switch between an addition strategy (i.e., counting the number of dots in a grid) and a subtraction strategy (i.e., counting the number of empty blocks in a grid and subtracting this from the total number of blocks). They found that people were slower when executing different strategies on successive trials than when they executed the same strategy (see also Lemaire & Lecacheur, 2010).

As another example, Uittenhove and Lemaire (2012) found that people were slower with a strategy when it was preceded by a difficult strategy than when it was preceded by an easy strategy. They asked participants to accomplish a computational estimation task (e.g., estimate the sum of an addition such as 43 + 57) by imposing three types of rounding strategies: Rounding down (RD), rounding up (RU), and mixed rounding (MR). When analyzing performance for problems on which the MR strategy was imposed (e.g., doing 40 + 60 to estimate 43 + 57), they were able to show that participants were faster after executing an RD strategy (e.g., doing 40 + 50 to estimate 43 + 57) than after executing an RU strategy (e.g., doing 50 + 60 to estimate 43 + 57). These data suggest that using strategies in short succession may affect how we execute a given strategy.

In the present study, we focused on strategy sequential difficulty effects established by Uittenhove and Lemaire (2012). We aimed at determining at what point in time sequential difficulty effects occur. This would permit us to know with what processes of strategy execution these effects interfere. Such knowledge may be instrumental in advising how models of strategy selection and execution could be modified to account for sequential difficulty effects.

The usefulness of using EEG to study execution of arithmetic strategies has been demonstrated in previous studies (Grabner & De Smedt, 2011; El Yagoubi, Lemaire, & Besson, 2003). For example, El Yagoubi, et al. used EEG to determine the time-course of strategies during arithmetic problem verification (e.g., 2+3 < 9?). They found that a more demanding verification strategy (exact calculation compared to approximate calculation) elicited greater negativity between 300 and 600 ms post-stimulus presentation (see also Dehaene et al., 1999; Stanescu-Cosson et al., 2000). These findings suggest that EEG may help to identify differing cognitive demands of strategies in early stages of strategy execution. If strategy execution is more difficult after the difficult rounding-up strategy than after the easy rounding-down strategy, we expect increased negativity in early stages of strategy execution.

In the present study, we used the same computational estimation task as Uittenhove and Lemaire (2012). Participants had to estimate sums of two-digit addition problems (e.g., 32 + 68) using one of three strategies: RD (e.g., 43 + 24 = 40 + 20 = 60), RU (e.g., 48 + 29 = 50 + 30 = 80) or MR (e.g., 43 + 28 = 40 + 30 = 70). We measured electrophysiological activities during execution of MR. Consistent with more difficult processing following difficult strategies, we expected to see larger cerebral activities following the difficult RU than following the easier RD strategy. Moreover, we expected to be able to determine whether strategy sequential difficulty effects act during stimulus-encoding phases, during central execution processes (i.e., retrieval of strategy procedures, calculation processes), or during post-execution processes.

Method

Participants

Seventeen young adults (7 men; 20 to 29 years of age; mean age: 25) participated in this experiment. All had normal to corrected vision and were paid 20 euros for their participation.

Stimuli

Stimuli consisted of 240 two-digit addition problems (e.g., 32 + 68). These problems were constructed so that a third of the problems were best estimated with the RD strategy, with both operands including unit digits smaller than 5 (e.g., 43 + 64), another third were best estimated with the RU strategy, with both operands including unit-digits larger than 5 (e.g., 47 + 68), and the final third were best solved with the MR strategy with the unit digit of the first operand being smaller than 5 and the unit-digit of the second operand being larger than 5 (e.g., 43 + 69). During the experiment, we presented participants with 40 “RD-MR” trials where the MR strategy was preceded by the RD strategy (e.g., 43 + 64 ; 43 + 69) and 40 “RU-MR” trials where the MR strategy was preceded by the RU strategy (e.g., 47 + 68 ; 43 + 69). To ensure that each strategy was executed the same number of times, we complemented these trials with 20 “RD-RU” trials where the RU strategy was preceded by the RD strategy and 20 “RU-RD” trials where the RD strategy was preceded by the RU strategy. Only the “RU-MR” and “RD-MR” trials were analyzed.

Following previous findings in arithmetic (see Campbell, 2005, for an overview), the following additional constraints were imposed on the selection of problems: (a) No operands included a repeated digit (e.g., 44) or a 0 or 5 digit (e.g., 20 or 25), (b) No problems included reversed operands (e.g., 43 + 82 and 82 + 43), (c) The first operand was larger than the second operand in half the problems, and smaller in the other half, (d) No operands could be rounded to

0, 10, or 100, (e) The operands of the same problem could never be rounded to the same decade (e.g. 43 + 41), (f) The mean exact sums of problems were equal in all cells of the design, (g) The mean difference between exact sums and estimated sums of problems was equal in every cell of the design, (h) Half the problems involved a carry operation (e.g., 64 + 73) in each cell of the design, (i) All transitions between problems involving carry operations and no-carry problems were presented equally often in all cells of the design, and (j) The estimated sums of successive problems were never the same (e.g., 43 + 72 followed by 32 + 83).

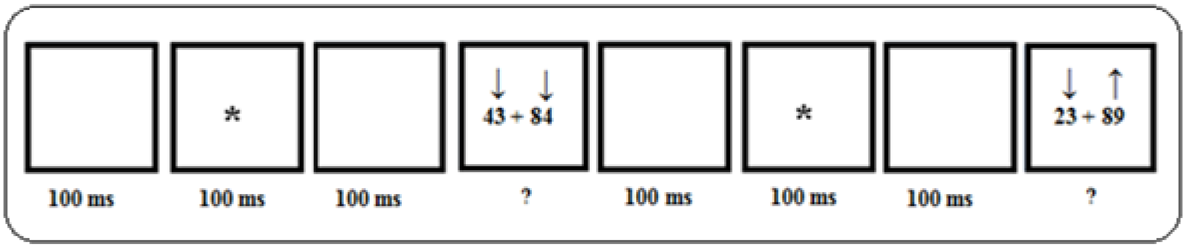
Procedure

After having filled an informed consent, participants were comfortably seated in a quiet and dimly-lit room. Stimuli were presented on a 800x600-resolution screen in a 17-point arial font. When presented with the addition problems, participants had to estimate the response as fast and accurately as possible. They had to do this using the strategy that was indicated. Prior to the experiment, strategies were described to participants. They were told to round both operands down to the closest smaller decades (e.g., 43 + 24 = 40 + 20 = 60) for the RD strategy. They were told to round both operands up to the closest larger decades (e.g., 48 + 29 = 50 + 30 = 80) for the RU strategy. Finally, they were told to round the first operand down to the closest smaller decade and the second operand up to the closest larger decade (e.g., 43 + 28 = 40 + 30 = 70) for the MR strategy.

For each problem, the strategy to use was indicated by two arrows presented above the operands of the problem, with the direction of the arrows indicating how the operands had to be rounded (i.e., an arrow pointing down indicated that the operand had to be rounded down and an arrow pointing up indicated that the operand had to be rounded up). Each problem matched the indicated strategy. That is, the RD strategy was indicated on problems with unit digits smaller than 5; the RU strategy was indicated on problems with unit digits larger than 5; and the MR strategy was imposed on problems including a first small unit-digit operand and a second large unit-digit operand. Participants had to give answers out loud, registered with a dictaphone for later identification of errors. In total, participants solved 240 problems divided in two blocks containing 60 trials with two addition problems each. Presentation order of trials was semi-random to control for sequential effects over longer series of trials. The experimental “RD-MR” and “RU-MR” trials were each preceded by a RD strategy in 25% of cases, by a RU strategy in 25% of cases, and by a MR strategy in 50% of cases. This avoided that RU-MR trials would suffer from different effects over long sequences than RD-MR trials.

All problems were separated by a 100-ms blank screen, followed by a 100-ms fixation cross, and then another 100-ms blank screen (see Figure 1). Estimation time was measured from the moment the stimulus appeared on screen until the participant clicked on the mouse button immediately after having answered. Before the experiment, 20 training trials were presented that contained the same characteristics as the experimental task, to familiarize participants with the experimental task.

Figure 1. Experimental procedure



EEG-procedure

Electrophysiological activity was continuously recorded by a « Biosemi Active Two » system with 64 active electrodes positioned on an elastic cap (Electro-Cap-Inc) following the 10-10 international system (Fp1/2, AF7/8, AF3/4, F7/8, F5/6, F3/4, F1/2, FT7/8, FC5/6, FC3/4, FC1/2, T7/8, C5/6, C3/4, C1/2, TP7/8, CP5/6, CP3/4, CP1/2, P9/10, P7/8, P5/6, P3/4, P1/2, PO7/8, PO3/4, CMS/DRL, O1/2, FPz, AFz, Fz, FCz, Cz, CPz, Pz, POz, Oz, Iz). Six additional external electrodes were used: Two on the left and right mastoids for the offline reference, and four electrodes placed on the left and right temples and below the eyes, to identify eye blinks and horizontal eye movements. The signal was continuously registered at a frequency of 256 Hz and processed using EEGLAB software (Delorme & Makeig, 2004). The signal was filtered (20 Hz low-band, 1 Hz high-band, 24dB/octave), epoched (from stimulus display to 1500 ms post display) and baselined (from 200 ms prior to stimulus display). All electrodes were re-referenced after registration using the mean of the left and right mastoid electrodes. Event-related potentials containing horizontal eye movements or activity exceeding 50 V were rejected. Eye-blink related activities were removed using independent component analysis in EEGLAB (runica algorithm).

Results

Behavioral data

We conducted ANOVAs on solution latencies and errors on the addition problems solved with the MR strategy preceded either by execution of the RD or the RU strategy. The strategy executed just before (RD or RU) was a within-participants factor. Latencies larger than the mean + 2 standard deviations (4.6%) were removed as well as all trials containing an error on either the first or the second problem (11%). Participants were significantly slower after the RU strategy

(4440 ms) than after the RD strategy (4100 ms), $F(1,16)=9.76$, $MSe=100421$, $\eta p^2=0.38$. Percent errors did not yield any significant effects as a function of previous strategy execution ($F<1$). (See Table 1 for means).

Table 1: Solution latencies in ms (and percent errors) on the second problem as a function of the strategy executed on the previous problem.

Strategy used on the previous problem	**Mean solution latency (% errors) with MR on the current problem**
Rounding down	4100 (5.9)
Rounding up	4440 (5.9)
SSDE	340 (0)

Note: SSDE = Strategy Sequential Difficulty Effects (Performance after RU - Performance after RD)

Electrophysiological data

Only event-related potentials corresponding to correct answers were analyzed. We calculated mean amplitudes of latency windows that were determined by a combination of visual inspection and statistical analyses (t-tests) of every 50-ms window after display of the stimulus for a total duration of 3000 ms, with a baseline corresponding to the 200 ms prior to stimulus presentation. Using this method, we defined four windows of interest (See Figure 2). We conducted ANOVA's in each window, on the 44 lateral electrodes, and on the 12 central electrodes (See Figure 3).

Figure 2. Latency windows selected for ANOVAs, with the wave amplitudes when using MR after RD (blue) or after RU (red); Fp1 electrode.

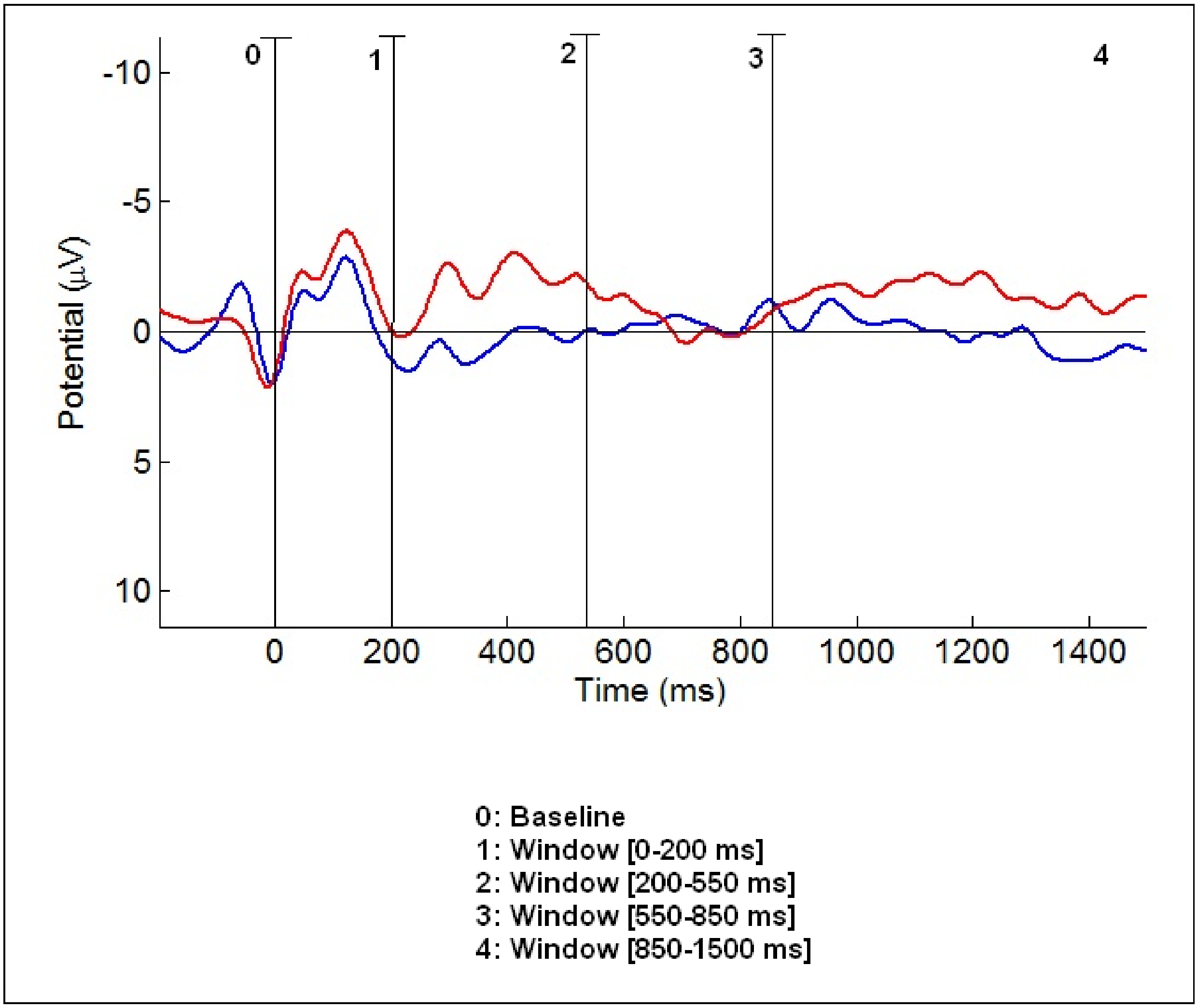
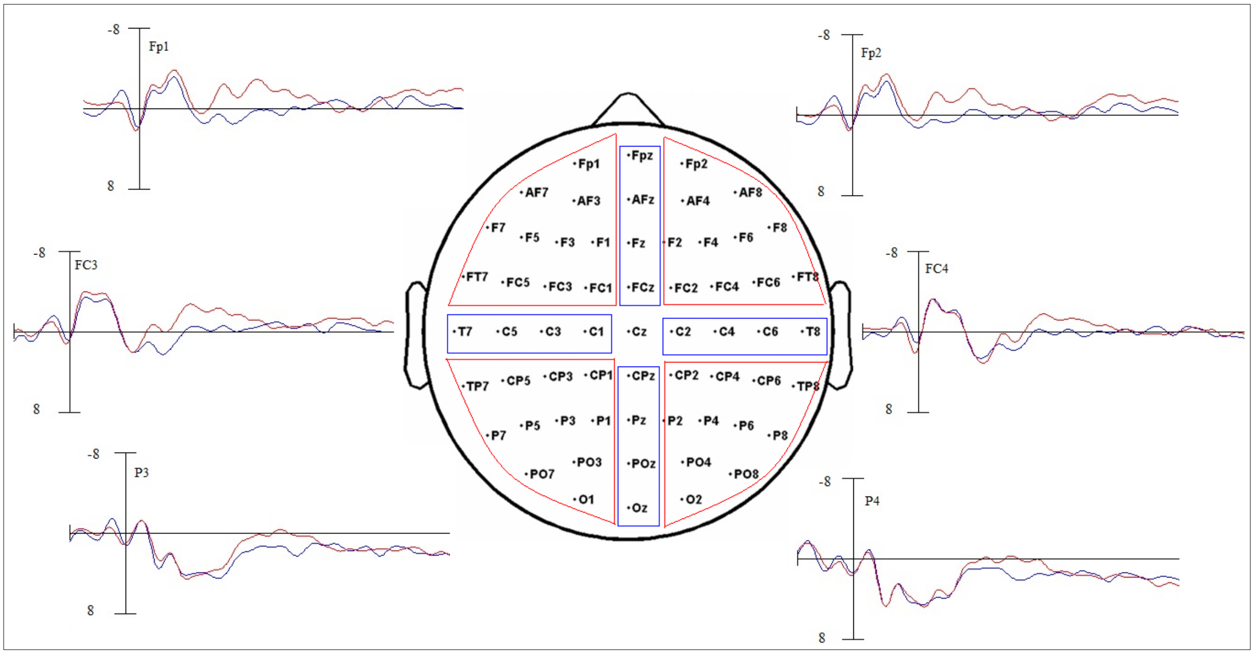


Figure 3. *Wave amplitudes when using MR after RD (blue) or RU (red) for prefrontal left (Fp1) and right (Fp2) over centro-frontal left (FC3) and right (FC4) to posterior left (P3) and right (P4) electrodes. We analyzed 44 lateral electrodes (red) and 12 central electrodes (bleu).*

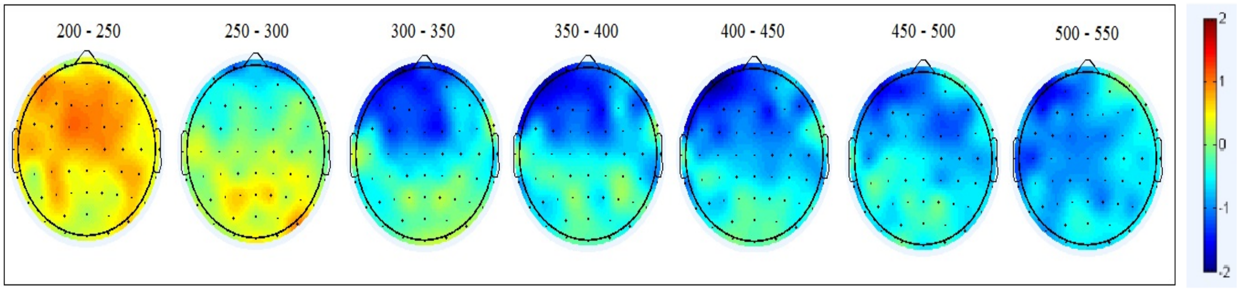


We conducted repeated measures ANOVAs on mean amplitudes of electrophysiological activities during the problems solved with the MR strategy preceded by either the RD or the RU strategy. The strategy executed just before (RD or RU) was treated as a within-participants factor. Additional factors for analysis on lateral electrodes were laterality (left or right) and frontality (frontal or posterior) of electrodes, for 11 electrodes. Additional variables for analysis on central electrodes was position (front, back, left or right), for 4 electrodes.

ANOVA revealed no significant effects (*F*s<1.7, *p*s>.18) that included the strategy executed just before in the first (0-200 ms), third (550-850 ms), or fourth (850-1500 ms) window on either lateral or central electrodes. In the second window (200-550 ms), the effect of strategy executed on the previous problem was marginally significant, for lateral electrodes, $F(1,16)=3.3$, $p=.08$, $MSe=90.4$, $\eta p^2=.17$, and the interaction with laterality was significant, $F(1,16)=4.5$, $p=.05$, $MSe=4.5$, $\eta p^2=.22$. Contrasts revealed that the effect of strategy executed on the previous problem was significant in the left hemisphere, $F(1,16)=6.9$, $p=.02$, $MSe=4.3$, $\eta p^2=.30$, but not in the right hemisphere, $F(1,16)=1.47$, $p=.24$, $MSe=5.1$, $\eta p^2=.08$. Furthermore, the effect of strategy executed on the previous problem was significant in the anterior region, $F(1,16)=4.4$, $p=.05$, $MSe=5.1$, $\eta p^2=.21$, but not in the posterior region, $F(1,16)=2.6$, $p=.12$, $MSe=4.1$, $\eta p^2=.14$. The locus of the effect of the strategy executed on the previous problem thus seems to be in the anterior left region of the brain, $F(1,16)=7.8$, $p=.01$, $MSe=2.5$, $\eta p^2=.33$, with marginally significant effects in the posterior left region, $F(1,16)=3.8$, $p=.07$, $MSe=2.8$, $\eta p^2=.19$, and no significant effects in other regions (Fs<1.9, ps>.18). In the anterior left region, from 200 to 550 ms post-stimulus presentation, mean wave amplitudes of the electrodes Fp1, AF3, AF7, F1, F3, F5, F7, FC1, FC3, FC5, and FT7 were significantly more negative after execution of the RU (-.9 µv) than after execution of the RD strategy (.6 µv) (See Figures 3 and 4).

In the second window (200-550 ms), the effect of strategy executed on the previous problem was marginally significant, for central electrodes, $F(1,16)=3.2$, $p=.09$, $MSe=34$, $\eta p^2=.17$, and the interaction with position was significant, $F(3,48)=2.9$, $p=.04$, $MSe=1.7$, $\eta p^2=.15$. Contrasts revealed that the effect of strategy executed on the previous problem was not significant in left, posterior or right regions (Fs<2.4, ps>.14). Contrasts also revealed that the effect of strategy executed on the previous problem was significant in the frontal region, $F(1,16)=5.4$, $p=.03$, $MSe=13.6$, $\eta p^2=.25$. In the frontal region, from 200 to 550 ms post-stimulus presentation, mean wave amplitudes of the electrodes Fpz, Afz, Fz, and Fcz were significantly more negative after execution of the RU (-.8 µv) than after execution of the RD strategy (.6 µv) (See Figures 3 and 4).

Figure 4. Localization of differences in wave-amplitudes during the MR strategy following the RU and RD strategy from 200 to 550 ms after stimulus presentation (every 50 ms).



Discussion

In the present study, behavioral data replicated sequential difficulty effects during strategy execution previously found by Uittenhove and Lemaire (2012). Participants were faster after executing the easier RD strategy on the previous problem than after executing the harder RU strategy. In this study, for the first time, we found electrophysiological signatures of strategy sequential difficulty effects. Cerebral activities during execution of MR were larger when it followed the more difficult RU strategy than the easier RD strategy, indicating more difficult processing following the RU strategy. Our results rule out the alternative that solution latencies are simply elongated following execution of a difficult strategy, due to changes in time criteria (i.e., a difficult strategy could change criteria for execution speed so that the person allows more time for strategy execution on the next problem).

Interestingly, the time at which greater cerebral activities were most apparent was immediately after encoding the problem, when participants would be retrieving and maintaining the steps of the strategy to be executed. Thus, strategy sequential difficulty effects interfered with central (i.e., retrieval of procedures and arithmetic facts) rather than with peripheral processes (i.e., stimulus encoding and responding) of strategy execution. This is supportive of Uittenhove and Lemaire's (2012) account of sequential difficulty effects in terms of executive resources being temporarily consumed by difficult strategies so that processing of the next strategy is more difficult (see also Schneider and Anderson, 2010).

However, sequential difficulty interference was originally found here in only initial stages of strategy execution and not in later stages. This may correspond to lower demands of executive resources in later stages of execution in combination with strategy sequential difficulty effects dissipating with time. Indeed, data from Uittenhove and Lemaire (in revision) showed that magnitude of sequential difficulty effects decreases with increasing response-stimulus intervals, so strategy sequential difficulty effects are more likely to be seen early during strategy execution.

The greater negativity following difficult strategies as compared to easier strategies between 200 and 550 ms post-stimulus display is reminiscent of the greater negativity found between 300 and 600 ms by El Yagoubi, Lemaire, and Besson (2003) when participants executed a more difficult strategy. A number of studies in literature suggests that the N400 is related to the difficulty of integrating information. For example, N400 amplitude is especially large for words that are difficult to integrate in a sentence context because they are semantically unexpected or incongruous (e.g., Benau, Morris, & Couperus, 2011, Kutas & Hillyard, 1980; Kutas & Federmeier, 2011). Larger amplitudes between 200 and 550 ms when participants are forced to execute MR after RU may indicate that lingering information from this strategy interferes with the integration of the content for the current strategy, resulting in greater negativity in a time frame corresponding to the N400.

In yet another study (Niedeggen & Rosler, 1999), N400 amplitude during arithmetic fact retrieval was related to the difficulty of retrieving an arithmetic fact. In a multiplication verification task, when proposed with an incorrect solution, N400 in participants was larger than for correct solutions. Moreover, the effect was even larger when the proposed incorrect solution was related to the correct solution. The N400 could then reflect increased interference during retrieval. In our case, increased N400 amplitude after the difficult strategy could reflect the fact that the still-present information from the previous strategy interfered with the retrieval of the procedures for the current strategy.

Furthermore, topographical analyses suggested that the effects had frontal rather than posterior origins. This is interesting, given the link between frontal regions and executive functions (Kane & Engle, 2002). However, these results are merely suggestive, given the limits of EEG-data for providing spatial information. Future research may couple EEG-data on strategy sequential difficulty effects with MRI data. This would enable us to more precisely determine the locus of strategy sequential difficulty effects. If executive resources are indeed involved in strategy sequential difficulty effects, fMRI data would show that strategy sequential difficulty effects are associated with cerebral activation in prefrontal regions.

On a theoretical level, our results have implications for how models of strategy selection Siegler & Shipley's ASCM, 1995; Lovett & Schunn's RCCL, 1999; Rieskamp & Otto's SSL, 2006; Siegler & Araya's SCADS*, 2005) need to take into account strategy sequential difficulty effects. These models assume that strategies involving more and more complex procedures are executed less efficiently and that strategies are executed on a problem-by-problem basis, independently of the strategy that was executed on the previous trial. Our data show that central processes of strategy execution are not only sensitive to effects of the number and type of processes involved in the current strategy, but also to those involved in the previous strategy. We suggest that the models could resolve this issue by adding an additional parameter, representing the amount of available cognitive resources during strategy execution. More or less cognitive resources could be available as a function of the difficulty of both the current and the preceding strategy, influencing strategy execution efficiency.

Strategy sequential difficulty effects in Alzheimer patients

Strategy sequential difficulty effects in Alzheimer patients: A study in arithmetic.

Kim Uittenhove[1] & Patrick Lemaire[1,2]

[1]Laboratoire de Psychologie Cognitive, Centre National de la Recherche Scientifique; and Aix-Marseille Université, Marseille (France)

[2] Institut Universitaire de France

Address correspondence to Patrick Lemaire,
LPC-CNRS and Aix Marseille Université,
3 place Victor Hugo, Bâtiment 9, case D,
Marseille 13331, France.
Phone: +33 (0)4 88 57 69 01;
Email: Patrick.Lemaire@univ-provence.fr

Authors' Note

This research was supported in part by the CNRS (French NSF) and by a grant from the Agence Nationale de la Recherche (Grant # BLAN07-1_196867). Correspondence about this paper should be directed to Patrick Lemaire, CNRS & Aix-Marseille Université, 3 Place Victor Hugo, Case D, 13331 Marseille, France (email: Patrick.Lemaire@univ-provence.fr). We thank Eve Benchetrit for her help in collecting data from AD patients.

Abstract

Objective: Consistent with Uittenhove and Lemaire (in press), we expected that strategy execution would be slower following execution of a difficult strategy than after an easy strategy (Strategy Sequential Difficulty effects). Moreover, consistent with a working-memory account of these effects (Uittenhove & Lemaire, in press; see also Schneider & Anderson, 2010) we expected larger SSD effects in older adults than in young adults, and especially in AD patients, a population with marked working-memory impairments (Baddeley et al., 1986; 1991).

Method: 25 Young and older (41 AD and 25 healthy) adults were asked to execute rounding strategies to solve arithmetic problems (e.g. solving 43 + 68 by rounding operands down or up, for example 40 + 70 = 110). We measured solution latencies and percent errors with a strategy as a function of the difficulty of the just executed strategy.

Results: Solution latencies were significantly shorter following the easier rounding-down strategy than following the harder rounding-up strategy ($\underline{F}(2,156)=35.8$). Moreover, this effect was significantly larger in AD patients ($\underline{F}(1,78)=117.4$).

Conclusions: We found comparable SSD effects in young and healthy older adults but dramatically increased SSD effects in AD patients. This has implications to further our understanding of strategic aspects underlying decreased cognitive functioning in AD patients.

Keywords: Alzheimer's disease – Strategy Sequential Difficulty Effects – Strategy execution – Perseveration – Arithmetic

Strategy sequential difficulty effects (SSD effects) are the finding that when young adults execute strategies, performance is worse after a difficult strategy as compared to following an easy strategy (Uittenhove & Lemaire, in press). This suggests that strategies relying more on central resources (e.g., difficult or complicated strategies) increase duration of execution of the next strategy relative to less demanding strategies. Uittenhove and Lemaire proposed that working-memory needs time to be cleared after strategy execution (see also Schneider & Anderson, 2010). Since difficult strategies rely more on WM-resources than easy strategies (Logie, Gilhooly, & Wynn, 1994), fewer resources would be available immediately afterward, which would slow down execution of the next strategy. Hence, SSD effects will be dependent on the efficiency of executive WM processes, more specifically: The facility with which an individual can clear the contents of WM. Executive and working-memory functions decline during normal aging (Goldman-Rakic, 1996; Kane & Engle, 2002) and even more so in Alzheimer's disease (Baddeley et al., 1986; 1991). We expected that the resource-diminishing effect of previous strategy difficulty on current strategy execution might be enlarged in healthy older adults and even more so in Alzheimer patients. The present project tested this prediction.

Finding larger SSD effects in AD patients than in older healthy adults can contribute to variations in strategy execution in these populations. Much of the performance decrements in older adults and Alzheimer patients result from executing strategies less efficiently. For example, Arnaud, Lemaire, Allen, and Michel (2008) found that execution of the retrieval strategy on subtraction problems was impaired in older adults in comparison to younger adults, and even more so in AD patients (see also Gandini, Lemaire, Anton, & Nazarian, 2008; Lemaire &

Arnaud, 2008; Gandini, Lemaire, & Dufau, 2008). Mantovan et al. (1998) looked at errors in written calculations of multi-digit multiplications and found that AD patients erred more than healthy older adults when executing complex written procedures. For example, when solving '56x32' on paper (in vertical format), AD patients would often only multiply the unit-digits (e.g., 6x2) and ten-digits (e.g., 50x30) and not cross-multiply unit- and ten-digits (e.g., 3X30). (see also Grafman et al., 1989). These difficulties in strategy execution experienced by healthy older adults and AD patients are associated to decreases in necessary processing resources. Strategies contain steps and intermediary results that need to be kept online while resisting interference and hence rely on several resources like WM and/or executive control (Robins et al., 1996; Duverne, Lemaire, & Vandierendonck, 2008; Imbo, Duverne, & Lemaire, 2007; Imbo & Vandierendonck, 2007). SSD effects may further decrease available WM-capacities in older adults and especially in AD patients, leading to additional difficulties in strategy execution in these populations.

Finding larger SSD in AD patients would have additional important implications. It may be related to decreased flexibility (e.g., as tested with task switching) and increased perseveration (e.g., as seen in perseverative errors in the Wisconsin Card Sorting task) in AD patients. Belleville, Bherer, Lepage, Chertkow, and Gauthier (2008) compared AD patients with healthy older adults and found that AD patients were impaired in the reconfiguration of actions (e.g., local switch costs). Furthermore, Cullen et al. (2005) described repetitive behaviors and perseverative errors in AD patients (e.g., AD patients may keep repeating the same question) and found that the degree of repetitiveness was related to executive dysfunction. In this paper, we argue that AD patients may not be very efficient in clearing the content of a previous strategy, thought, or action from WM, which may lead to larger SSD effects and also to repetitive behavior and perseveration.

In this study, we tested AD patients and healthy older adults with the same computational estimation task as Uittenhove and Lemaire (in press) so as to test the prediction that SSD effects increase with age, especially in AD patients. We tested SSD effects by asking participants to provide estimates to two-digit arithmetic problems (e.g., 43 + 68) with one of the following strategies: mixed-rounding (i.e., rounding the first operand down and the second operand up to the nearest decades; 40+70=110), rounding-down (i.e., rounding both operands down to the nearest decades; 40+60=100), or rounding-up (rounding both operands up to the nearest decades; 50+70=120). Previous research has shown that these strategies differ in difficulty (e.g., LeFevre, Greenham, & Waheed, 1993; Lemaire, Arnaud, & Lecacheur 2004). The rounding-down strategy is easiest because it does not require the extra step of incrementing operands and keeping them in WM. Both the rounding-up and mixed-rounding strategies are more difficult, because the rounding-up strategy requires incrementing and maintaining two operands in WM, and the mixed-rounding strategy requires a switch of operations (rounding the first operand down and the second one up). We predicted that SSD effects would be larger in healthy older adults and in AD patients. Compared to young adults, older healthy adults would be more impaired on the execution of the mixed-rounding strategy following a rounding-up strategy than young adults, and this effect would be even larger in AD patients.

Method

Participants. Three groups of participants were selected according to their age and their health-status: 25 healthy young adults (12 women), 25 healthy older adults (12 women), and 41 individuals diagnosed with probable AD (26 women). Probable AD patients were recruited from the Department of Geriatric Neurology, Sainte Marguerite Hospital (Marseille, France) and Saint Germain Daycentre (Paris, France). Diagnoses were based on extensive medical and psychiatric examinations and have been confirmed with a wide range of psychometric and neuropsychological tests. The inclusion criteria of probable AD in the present study were based on the NINCDS-ADRDA (McKhann et al., 1984) and the DSM IV (DSM, 1996) criteria. We ensured that none of the healthy older adults was affected by any diseases that might affect cognition; they had a minimum score of 28 on the MMSE (Folstein et al., 1975). We excluded ten AD patients for failing to execute rounding strategies on the addition problems. The addition and the subtraction–multiplication subtests of the French Kit (French et al., 1963) were used in order to assess participants' arithmetic skills with an independent paper-and-pencil test. Number of correctly solved items was taken as an index of their arithmetic skills. Participants also completed a French version of the Mill-Hill Vocabulary Scale (MHVS; Deltour, 1993 and Raven et al., 1986) so as to control for their verbal ability. Characteristics of the three groups are summarized in Table 1. The experimental procedures were approved by the local ethic committee (approval reference number: 2010-A00150-39), and all participants gave their written informed consent.

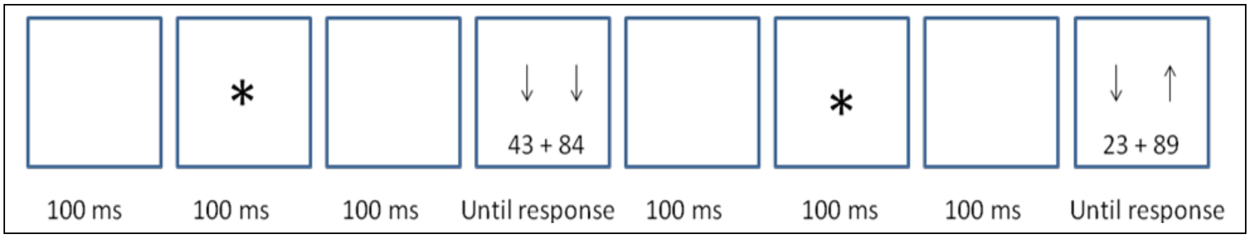
Table 1: Participant's characteristics

	Young Adults	**Healthy older Adults**	**Probable AD**
N	25	25	41
Age Mean(*sd*)	24 (2.43)	68 (5.15)	75 (3.51)
Mill Hill Mean (*sd*)	26 (2.9)	28 (3.5)	24 (3)
French Kit Mean (*sd*)	53 (17.7)	82 (26.1)	54 (9.6)
MMSE	--	> 28	19.6 (2.1)

Stimuli. Sets of two-digit addition problems (e.g., 32+68) were created. These sets included rounding-down problems, rounding-up problems, and mixed-rounding problems. Unit digits of both operands were smaller than 5 for rounding-down problems (e.g., 43+64) and larger than 5 for rounding-up problems (e.g., 47+68). Unit digit was smaller than 5 in the first operand and larger than 5 in the s second operand for mixed-rounding problems (e.g., 43+69). Following previous findings in arithmetic (see Campbell, 2005, for overview), we controlled the following factors: (a) no operands contained a 0, 5, or repeated digit (e.g., 44), (b) no reverse orders of operands were used (e.g., 43+82 and 82+43), (c) the first operand was larger than the second in half the problems, (d) no operand would round to 0, 10, or 100, (e) the operands of a problem would never round to the same decade, (f) the problems had a comparable mean exact sum per item condition, (g) the conditions were matched for differences between correct sums and estimates, (h) the conditions had a comparable number of problems with carry-over on the tens (50%), and (i) during the experiment, the estimated sums of two successive problems were never the same.

Procedure. The stimuli were presented in a 72-point font on a 1280 x 800 screen. Participants were told that they were going to see addition problems to which they had to estimate the answer using one of three strategies. The rounding-down strategy was explained as rounding both operands down to the smaller decades (e.g., 43+24=40+20=60). The rounding-up strategy was described as rounding both operands up to the larger decades (e.g., 48+29=50+30=80). The mixed-rounding strategy was presented as rounding the first operand down to the smaller decade and the second operand up to the larger decade (e.g., 43+28=40+30=70). Participants were told that they should use the indicated strategy on each trial. Strategies were indicated by two arrows pointing in the direction in which the operands needed to be rounded. They were instructed to say the estimate of each problem out loud. Participants saw three blocks of 26 trials each. Each trial was made up of two problems, yielding a total of 156 problems per participant. On the first problem of each trial, participants were randomly cued to execute either the rounding-down, rounding-up, or mixed-rounding strategy. On the second problem, they had to execute the mixed-rounding strategy. Each problem matched the cued strategy: Rounding-down problems were cued to be solved by the rounding-down strategy, rounding-up problems by the rounding-up strategy, and mixed-rounding problems by the mixed-rounding strategy. All problems were separated by a 100-ms blank screen, followed by a 100-ms fixation cross, followed by another 100-ms blank screen. The trial-procedure is displayed in Figure 1. The time until each response was measured by experimenter key-press, occurring as soon as possible after the response. To avoid experimenters' expectations influencing the response time measurement, we used a double-blind procedure. Errors were recorded by having the experimenter write down the answers of the participants so errors could later be identified.

Figure 1. Trial procedure.



Results

The first analysis was aimed at checking the relative difficulty of our strategies such that the rounding-down strategy yielded best and rounding-up worst performance, and whether relative strategy difficulty varied with groups. The second analysis aimed at testing SSD effects and how they differed between groups. Prior to analyses on solution latencies, values exceeding the mean + 2 x standard deviations (4.5%) and all trials containing an error (9.4%) were removed. All reported effects are significant to at least p<.05.

Relative strategy difficulty. We conducted repeated measures ANOVAs on participants' mean solution times and percent errors on the first problem with strategy as a within-participants variable and group as a between-participants variable (see Table 2).

Table 2: Mean solution latencies in ms (and percent errors) for the first problem as a function of the strategy that was executed, for healthy young and elder adults and Alzheimer patients.

Strategies	**Young Adults**	**Healthy Older Adults**	**AD Patients**
Rounding Down	3117 (3.1)	3722 (6.3)	7710 (5.2)
Rounding Up	3600 (10.5)	3979 (7.7)	9173 (7.9)
Mixed Rounding	3475 (6.4)	3897 (5.7)	8351(5.6)
Rounding Up – Rounding Down	483 (7.4)	255 (1.4)	1463 (2.7)

Solution latencies varied with group, *F*(2,78)=59.9, *MSe*=11020400. Planned comparisons showed that AD patients (8411 ms) were significantly slower than healthy older adults (3866 ms), *F*(1,78)=83.6, *MSe*=1092000, and the latter were not significantly slower than young adults (3397 ms), *F*<1. Solution latencies also varied with strategies, *F*(2,156)=47.3, *MSe*=228640. Planned comparisons showed that participants were slower when executing the mixed-rounding (5241 ms) than when executing the rounding-down strategy (4850 ms), *F*(1,78)=24.2, *MSe*=253383. They were also significantly slower when executing the rounding-up strategy (5584 ms) than when executing the mixed-rounding strategy, *F*(1,78)=18.3, *MSe*=257698. Finally, variations in solution latencies with strategies were modulated by group, *F*(4,156)=13.3, *MSe*=228640. Both young adults and healthy older adults executed the rounding-down strategy (3420 ms) more quickly than the rounding-up strategy (3789 ms), *F*(1,78)=15, *MSe*=226968. AD patients executed the rounding-down strategy (7710 ms) a lot faster than the rounding-up strategy (9173 ms), *F*(1,78)=189.8, *MSe*=174838. To test whether this difference in relative difficulty between healthy adults and AD patients existed independent from slower processing speed in AD patients, we contrasted *z*-scores for the rounding-down and the rounding-up strategy. We found that the difference in relative difficulty between Alzheimer patients and healthy adults did not reach significance when analyzing *z*-scores, $F<1$.

Analyses of mean percent errors showed no difference between groups, $F<1$. We found a main effect of strategy, $F(2,156)=9.6$, $MSe=32.4$. Planned comparisons showed that participants erred more when executing the rounding-up strategy (8.7%) than when executing the mixed-rounding strategy (5.9%), $F(1,78)=9$, $MSe=34.6$ or when executing the rounding-down strategy (4.9%), $F(1,78)=15.6$, $MSe=36.9$. Participants did not significantly err more with the mixed-rounding strategy than with the rounding-down strategy, $F(1,78)=1.6$, $MSe=25.8$. Variations in accuracy with strategy was the same in all three groups, $F(4,156)=2.0$, $MSe=32.4$.

Strategy sequential difficulty effects. We conducted repeated measures ANOVAs on participants' solution times and percent errors on the second problem with the strategy on the first problem as a within-participants variable and group as a between-participants variable (see Table 3).

Table 3: Mean solution latencies in ms (and percent errors) for the second problem as a function of the strategy that was executed on the first problem, for healthy young and older adults and Alzheimer patients.

Strategy executed on the first problem	**Young**	**Healthy older adults**	**AD patients**
Rounding Down	3370 (6.0)	3751 (6.0)	7708 (5.5)
Rounding Up	3647 (5.2)	3931 (4.4)	9299 (7.1)
Mixed Rounding	3482 (8.5)	3800 (5.1)	8266 (5.9)
Rounding Up – Rounding Down	277 (-0.8)	180 (-1.6)	1591 (1.6)

Solution latencies on the second problem differed between groups, *F*(2,78)=60.2, *MSe*=10844464. Planned comparisons showed that AD patients (8424 ms) were significantly slower than healthy older adults (3827 ms), *F*(1,78)=82.5, *MSe*=10432200, but the latter were not significantly slower than young adults (3500 ms), *F*<1. Solution latencies on the second problem also differed as a function of the strategy used on the first problem, *F*(2,156)=35.8, *MSe*=269029. Planned comparisons showed that participants were slower after executing the rounding-up strategy (5626 ms) than after executing the mixed-rounding strategy (5183 ms), *F*(1,78)=32.7, *MSe*=239280. Furthermore, they were slower after executing the mixed-rounding strategy than after the rounding-down strategy (4943 ms), *F*(1,78)=9.7, *MSe*=259431. Variations in solution latencies on the second problem with strategy differed between groups, *F*(4,156)=17.1, *MSe*=269029. Healthy older adults and young adults executed mixed-rounding more quickly after the rounding-down strategy (3560 ms) than after the rounding-up strategy (3789 ms), *F*(1,78)=3.9, *MSe*=334283, *p*=0.05. AD patients executed mixed-rounding 1591 ms faster following rounding-down than following rounding-up, *F*(1,78)=117.4, *MSe*=334283. To test whether this difference in SSD effects between healthy adults and AD patients was independent from slower processing speed in AD patients, we contrasted *z*-scores for mixed-rounding after the rounding-down and rounding-up strategy. We found that the differences in SSD effects between Alzheimer patients and healthy adults were still significant when analyzing *z*-scores, *F*(1,78)=4.3, *MSe*=0.08. Moreover, SSD effects expressed in fractions of solution latencies for each individual (i.e., ([RT after rounding-up]-[RT after rounding-down])/[RT after rounding-down]) tended to correlate with individuals' scores on the MMSE, *r*=-0.35, *p*=0.05, in AD patients.

Mean percent errors on the second problem did not differ with group, *F*<1, strategy used on the first problem, *F*<1, and the interaction between these two factors was not significant, *F*(4,156)=1.5, *MSe*=24.

Discussion

SSD effects were dramatically increased in AD patients in comparison to healthy older controls but were of comparable magnitude in healthy older and young adults. The lack of difference between healthy older and young adults could be due to compensation mechanisms in older adults. Healthy older adults obtained larger arithmetic scores at the independent arithmetic test. Their better skills could have helped them compensate for SSD effects. However, arithmetic scores did not correlate with SSD effects in young or healthy older adults (*r*s<.18). Another possibility is that the mechanisms implicated in SSD effects (e.g., WM-capacities) were not sufficiently affected in healthy older adults, explaining comparable SSD effects in healthy older and young adults. Most interestingly, SSD effects were significantly and dramatically increased in AD patients. The increase in SSD effects was still observed when we controlled for general slowing.

We suggest that the reason for these increased SSD effects in AD patients is specific impairments of their WM-capacities. AD patients may have deficits in clearing WM after executing a strategy, in the line of deficits in executive WM components (Baddeley et al., 1986; 1991). Since a difficult strategy would take up more WM resources, this would explain why AD patients are relatively more impaired following difficult strategies. Although deficient clearing of WM is a plausible underlying mechanism of increased SSD effects in AD patients, evidence supporting this claim is still circumstantial. Future studies may want to test both SSD effects and WM capacity in AD patients, to assess whether WM capacity in AD patients is linked to SSD effects. Moreover, deficits in clearing WM could be linked to diminished flexibility and perseverative errors in AD patients. Indeed, if WM is occupied with the content of a previous strategy and is not efficiently cleared, it will interfere with what AD patients do next, and they may persevere in executing this strategy. This may also be true for deficits in flexibility and perseveration in other domains (e.g., repeating the same question). Future research could test this hypothesis with a correlation between SSD effects and perseverative errors in AD.

Increased SSD effects in Alzheimer patients question the practice of using sequential problem-solving tests for assessing cognitive functioning in this population. SSD effects during sequential problem-solving in this group may lead to underestimation of true problem-solving capacities because they interfere with strategy execution. Hence, neuropsychological tests should allow Alzheimer patients sufficient time to recover between problems to more accurately estimate their functioning. Neuropsychologists should also consider that Alzheimer patients may use inadequate strategies because of SSD effects associated to more resource-demanding strategies. Lastly, future research may investigate whether SSD effects are a reliable hallmark for the diagnosis of AD, as has been the case for more general arithmetic impairment (Deloche et al., 1995; Grafman et al., 1989; Mantovan et al., 1998). Deloche et al. (1995) found that calculation performance of patients with beginning AD correlated with Mini Mental State Examination (MMSE) scores but not with memory. SSD effects could be an early marker of difficulties in restoring of WM-capacities which may precede difficulties with calculation or procedural strategies. Similar to Deloche et al. (1995) we found that SSD effects in Alzheimer patients tended to correlate with their scores in the Mini Mental State Examination, indicating that SSD effects in Alzheimer could be a sensitive marker of the degree of more general cognitive dysfunctioning.

General Discussion

In this thesis, we looked at strategy variations underlying performance differences with aging. Crucial to explaining strategy variations is understanding how they are executed. We studied numerical cognition and aimed to better understand strategy execution efficiency in this domain and its relations to cognitive resources.

We decomposed numerical strategies in retrieval-based and procedural processes and summarized empirical evidence for the involvement of numerical long-term memory and executive resources in these processes. We argued that these processes differ in the amount of executive resources they require, and that this determines strategy efficiency. If numerical processes rely more on limited executive resources, they will be harder to execute (e.g., Tronsky, 2005; Caviola, Mammarella, Cornoldi, Lucangeli, 2012; Imbo & Vandierendonck, 2007; Imbo & Vandierendonck, 2008).

Moreover, we provided evidence from literature that strategy execution is influenced by characteristics of strategies, problems, participants, and situations (Lemaire, Arnaud, & Lecacheur, 2004; Siegler & Shrager, 1984; Arnaud et al., 2006; Geary, Bow-Thomas, Liu, & Siegler, 1993; Siegler, 1987). We argued that the influence of these characteristics on strategy execution can be explained by variations they induce in available experience and executive resources.

However, one important assumption in most of the literature is that participants use strategies on each problem, independently of the strategy that was selected and executed on previous problems. Our data (and other recent data on strategy switch costs, Lemaire & Lecacheur, 2010, Luwel et al., 2009; Ardiale, Hodzik, & Lemaire, 2012) challenge the view that strategy execution is uninfluenced by previous strategies and/or problems. In the studies presented in this thesis, we have shown that strategy execution suffers following the execution of difficult strategies in numerical cognition.

We propose that strategy sequential difficulty effects may act through induced variations in executive resources. We suggest that difficulty strategies temporarily consume executive resources, reducing functional executive capacities for the next strategy execution. This claim is supported by our findings of increased strategy sequential difficulty effects in individuals and populations (AD patients) with lower working-memory capacities. Moreover, an electrophysiological study showed that sequential difficulty effects interfere with central stages of strategy execution.

Our findings have implications on the empirical level. It means that we have to modify our interpretation of the performance individuals obtain when executing strategies. Performance with a strategy can no longer be considered isolated from prior strategy executions. The complexity of the strategy executed previously, in combination with individual differences in available executive resources, influences strategy execution efficiency on the current problem.

Furthermore, other dimensions of strategy use could also be affected by sequential difficulty effects. Selection and repertoire are two aspects of strategies that are linked to executive resources (see Chapter 1); these dimensions are thus likely to present sequential difficulty effects. Moreover, sequential difficulty effects could be present in other domains than numerical cognition in which executing strategies requires executive resources. We discuss our recent data on strategy sequential difficulty effects in the memory domain.

Our results also have theoretical implications. We discuss how models of strategy selection could be revised to take into account strategy sequential difficulty effects on the different dimensions of strategy use. Furthermore, we wonder how strategy sequential difficulty effects could be reconciled with the notion of forgetting in models of working memory (Barrouillet, Bernardin & Camos, 2004; Oberauer & Lewandowsky, in press; Oberauer et al., 2012).

7.1. Implications for assessment of strategy execution performance

Sequential difficulty effects are important to take into account when assessing individuals' strategy execution efficiency. If strategies are executed in short succession, performance estimates may be affected by which strategies we just executed (Lemaire & Lecacheur, 2010; Uittenhove & Lemaire, 2012; Luwel et al., 2009). Thus, if we want to have a true estimate of strategy execution performance, we need to provide individuals with sufficient time between problems. We have shown that increasing time between problems decreases strategy sequential difficulty effects (Uittenhove & Lemaire, in revision). Moreover, some people might need more time between problems than others. For example, we found larger sequential difficulty effects in individuals with lower working-memory capacities (Uittenhove & Lemaire, in revision) and in AD patients (Uittenhove & Lemaire, in revision). To estimate true strategy performance in low working-memory individuals and AD patients, more time would be needed between problems than in high working-memory individuals and healthy adults.

To determine the perfect spacing of problems in time, we need to obtain some additional information on sequential difficulty effects. For example, we need to know more precisely how they disappear over time. We know that sequential difficulty effects are reduced at longer response-stimulus intervals. However, we do not know if and how much time it takes for them to completely dissipate. To obtain this information, we could manipulate the response-stimulus interval more finely to obtain a more exhaustive picture of time course of sequential difficulty effects. Most likely, the time-course is dependent on the amount of executive resources devoted to the previous operation. In addition to manipulating response-stimulus intervals, we could vary the amounts of information to process in working memory. This paradigm would allow us to determine precisely how available executive resources fluctuate with immediately prior task demands and passing time.

7.2. The influence on several dimensions of strategy use

The literature on strategy repertoire, distribution, and selection suffers the same shortcomings as the literature on strategy execution. Most of this literature assumes that a strategy gets selected on each problem, independently from previous problems. Our data show that this assumption is invalid (see also Lemaire & Lecacheur, 2010). It is thus equally important to look at sequential difficulty effects in strategy dimensions other than strategy execution.

For example, strategies that impair subsequent performance may be selected less often than strategies that do not impair subsequent performance. Such biases in strategy selection would cause a shift in strategy distribution.

Strategy sequential difficulty effects could also influence strategy repertoire. The literature on strategic variations suggests that the size of strategy repertoire is related to the ease with which we maintain multiple procedures active during the task (Ardiale, Hodzik, & Lemaire, 2012; Hodzik & Lemaire, 2011). What if the costs of maintaining the individual procedures in active state increases? Complex procedures are more difficult to maintain than simple procedures. When we attempt to keep multiple procedures active at the same time, we may be less able to do so when these procedures are more costly to maintain.

Finally, strategy sequential difficulty effects could influence the adaptivity of strategy selection. Strategy selection requires executive resources (e.g., inhibition and flexibility) (Hodzik & Lemaire, 2011; Souchay & Isingrini, 2004; Hayes, Kelly, & Smith, 2012). A previous difficult strategy may consume these capacities so that individuals have more difficulties switching to another strategy and/or analyzing problem characteristics to select an appropriate strategy.

We discuss potential effects of strategy sequential difficulty effects on strategy distribution, repertoire, and selection in more detail. Regarding the question of sequential difficulty effects on distribution, we have collected empirical data that we discuss. For strategy repertoire and selection, we discuss experimental paradigms that would allow us to investigate the influence of strategy sequential difficulty effects.

7.2.1. Strategy distribution

Individuals may choose difficult strategies less often in a sequence, so that subsequent performance does not suffer. This would implicate that strategy distributions are different in contexts where problems are solved sequentially than in contexts where problems are solved in an isolated fashion. We conducted yet another experiment with our computational estimation paradigm (in preparation).

In this experiment, we tested the hypothesis that people anticipate sequential difficulty effects by choosing less often a difficult strategy in a sequence of problems. To find this, we presented participants addition problems in an isolation condition and in a sequence condition (see Figure 16). In the isolation condition, participants had to choose a strategy on every problem, separated by long breaks. In the sequence condition, participants had to choose a strategy on every first problem, which was immediately followed by a second problem on which a strategy was imposed. After the second problem, a long break followed. Participants could choose to either use the rounding-up strategy or the rounding-down strategy.

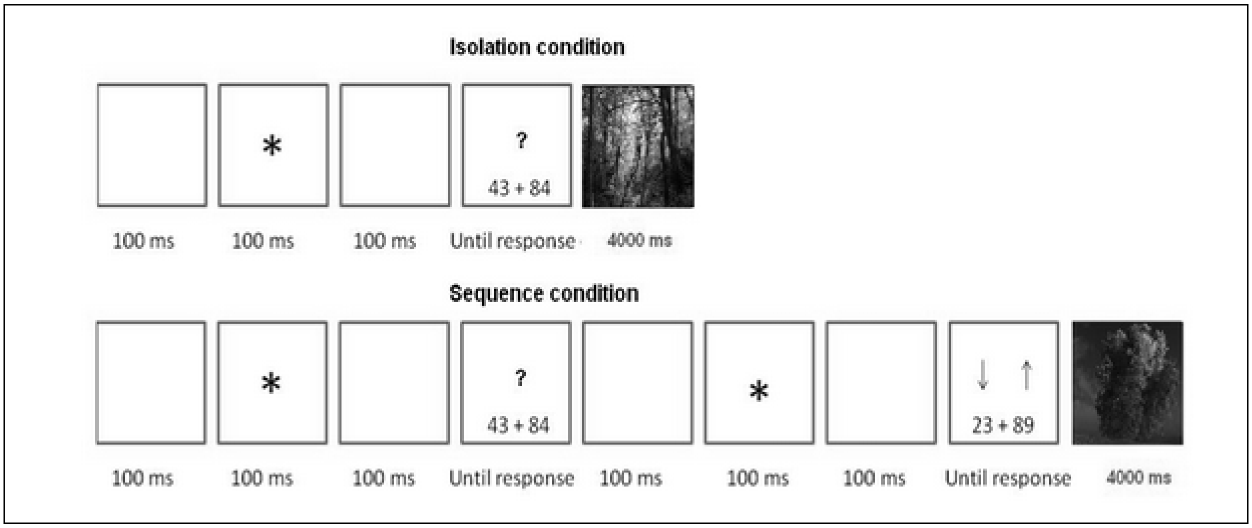


Figure 16. A trial in the isolation condition and a trial in the sequence condition.

We predicted that, in both conditions, participants would choose the rounding-up strategy less often than the rounding-down strategy because rounding up is more difficult. Moreover, we expected that the difference would be larger on the first problem in the sequence condition than in the isolation condition, because choosing to execute the rounding-up strategy could impair performance on the next problem in the sequence condition. Consistent with this prediction, we found that people selected the difficult rounding-up strategy less often than the rounding-down strategy, and especially when other problems followed immediately. In the sequence condition participants selected the rounding-up strategy on 17% of trials compared to 29% in the isolation condition.

The finding of decreased use of rounding up in the sequence condition supports our hypothesis that people can optimize over sequences of problems by choosing easier strategies to make subsequent processing easier. We thus find a shift in strategy distribution when people are solving problems in a sequential context. This type of shift could be enlarged in populations that show larger strategy sequential difficulty effects, such as individuals with low working-memory capacity and AD patients. In Chapter 1 we discussed how AD patients have larger deficits than healthy older adults when executing difficult strategies and thus switch to the use of easier strategies more often than healthy older adults. Similarly, AD patients may shift their strategy distributions more in sequential contexts than healthy adults, to compensate for enlarged strategy sequential difficulty effects.

7.2.2. Strategy repertoire

Individuals may keep fewer strategies simultaneously active when these strategies are difficult than when they are easy. In other words, the strategy repertoire may shrink when strategies are difficult. This may stem from complex procedures being more difficult to maintain than simple procedures, and so less different procedures can be kept active simultaneously during the task.

Inspired by Ardiale, Hodzik, and Lemaire (2012), we propose to test this by measuring switch costs when switching between multiple strategies. The crucial difference would be that we not only manipulate the number of strategies to switch between, but also the difficulty of the strategies between which we need to switch. Switch costs should be larger when switching between more strategies (Ardiale, Hodzik, & Lemaire, 2012) but also when switching between more difficult strategies. Additionally, we predict an interaction, the increase in switch costs when switching between more as compared to fewer strategies may be larger when these strategies are more difficult. Finding this would prove that maintaining several difficult strategies is more difficult than maintaining several easy strategies.

7.2.3. Strategy selection

Strategy selection requires executive resources to activate the to-be-executed strategy and to inhibit competing strategies (Hodzik & Lemaire, 2011; Souchay & Isingrini, 2004; Hayes, Kelly, & Smith, 2012). Strategy sequential difficulty effects may interfere with this process by consuming executive resources so that the wanted strategy can not be readily activated and competitors not adequately inhibited. Individuals may thus repeat the same strategy they just executed and/or they may analyze problem characteristics less well. The net result could be a reduction in the adaptivity of strategy choices. People may less often select the best strategy on each problem.

Unfortunately, testing the hypothesis of larger percentages of strategy repetition following a difficult strategy is not a simple task. Indeed, participants could be expected to switch more often towards an easier strategy following a difficult strategy because they generally prefer easier strategies, but also to avoid additional strategy sequential difficulty effects, and because they temporarily lack the capacities to execute more difficult strategies. This could neutralize the larger percentages of strategy repetition that could be caused by the lack of executive resources needed for switching. The only solution to this problem is to create a situation with strategies of equal difficulty. Will individuals switch strategies more often in a situation containing only equally easy strategies than in a situation containing only equally difficult strategies?

What about the analysis of problem characteristics? Executing a difficult strategy on the previous trial could interfere with the analysis of problem characteristics on the next trial, so that individuals are less able to select the best strategy. However, to test this we need to avoid the confounding effects of the tendency to repeat strategies. One way to do this would be to present participants with two different alternating tasks, in which different types of strategies are used. We could then test whether participants who have just executed a difficult strategy on one task to be less able to choose an adequate strategy as a function of problem characteristics on the next task.

7.3. Strategy sequential difficulty effects in other domains

In the memory domain, like in numerical cognition, people use strategies of varying difficulty to memorize lists of words (e.g., Dunlosky & Hertzog, 1998; 2001). Moreover, mnemonic strategies have also been shown to require executive resources (Bouazzaoui et al., 2010; Isingrini & Taconnat, 2008; Velanova, Lustig, Jacoby, & Buckner, 2007; Bouazzaoui et al., 2012; Taconnat et al., 2007). Strategy sequential difficulty effects found in numerical cognition could thus potentially be generalized to the memory domain.

Uittenhove, Burger, Lemaire, and Taconnat (in preparation) tested the possibility of strategy sequential difficulty effects with mnemonic strategies. They used a variation of the computational estimation paradigm (See Figure 17), in which a sentence-construction strategy (e.g., to memorize 'cat', we construct a sentence such as 'The cat is on my lap') was preceded by either an easy repetition strategy (e.g., to memorize 'cat', we mentally repeat the word 'cat cat cat cat') or a difficult mental image strategy (e.g., to memorize 'cat', we mentally picture a cat chasing a mouse).

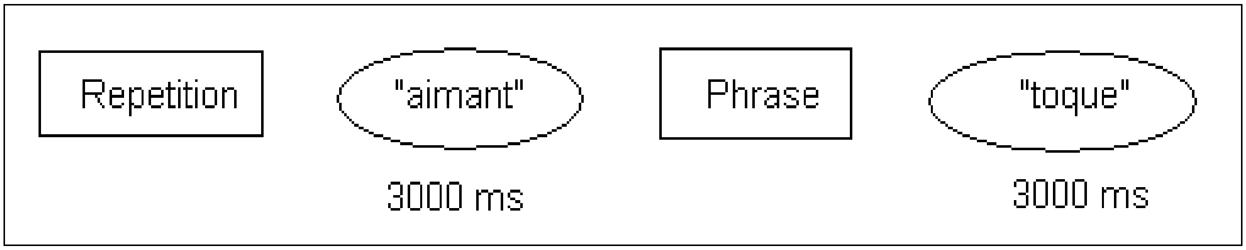


Figure 17. The word-learning paradigm. The strategy participants had to use was cued before the appearance of the problem. Then, the problem appeared for a duration of 3000 ms. Immediately after, the strategy for the next problem was cued, after which the next problem appeared.

They found that recall of words that were memorized with a sentence-construction strategy was better when participants had used an easier repetition strategy to memorize the previous word than when they had used the more difficult mental imagery strategy. After having used a repetition strategy, participants recalled on average 5.8 words compared to 4.4 words after a mental image strategy.

We interpret these results as stemming from less executive resources being available during the memorization of a word with the sentence strategy after the more demanding mental-image strategy than after the easier repetition strategy. The reduced availability of executive resources interfered with proper execution of the sentence strategy so that words were memorized less adequately.

7.4. Models of strategy selection

Several models have been proposed to formalize strategy selection and execution in many cognitive domains. Lovett and Andersons (1996) proposed the ACT–R, Siegler and Shipley advanced the ASCM (1995), Lovett and Schunn (1999) contributed the RCCL, Rieskamp and Otto (2006) proposed the SSL model, and Siegler and Araya (2005) focused on SCADS*. These models assume that strategies involving more and more complex procedures are executed less efficiently and that strategies are executed on a problem-by-problem basis, independently of the strategy that was executed on the previous trial. Our data show that strategy execution is not only sensitive to effects of the number and type of processes involved in the current strategy, but also to those involved in the previous strategy.

Moreover, regarding strategy selection, if we would solely rely on the models, we would have to predict that as experience accumulates, strategy selection would approach perfection. This follows from the models' common assumption that strategies get chosen based on association strengths, which are shaped by the experienced success of strategies on problems (see Figure 18). Accumulation of experience would tune associations so they asymptotically reach perfection .

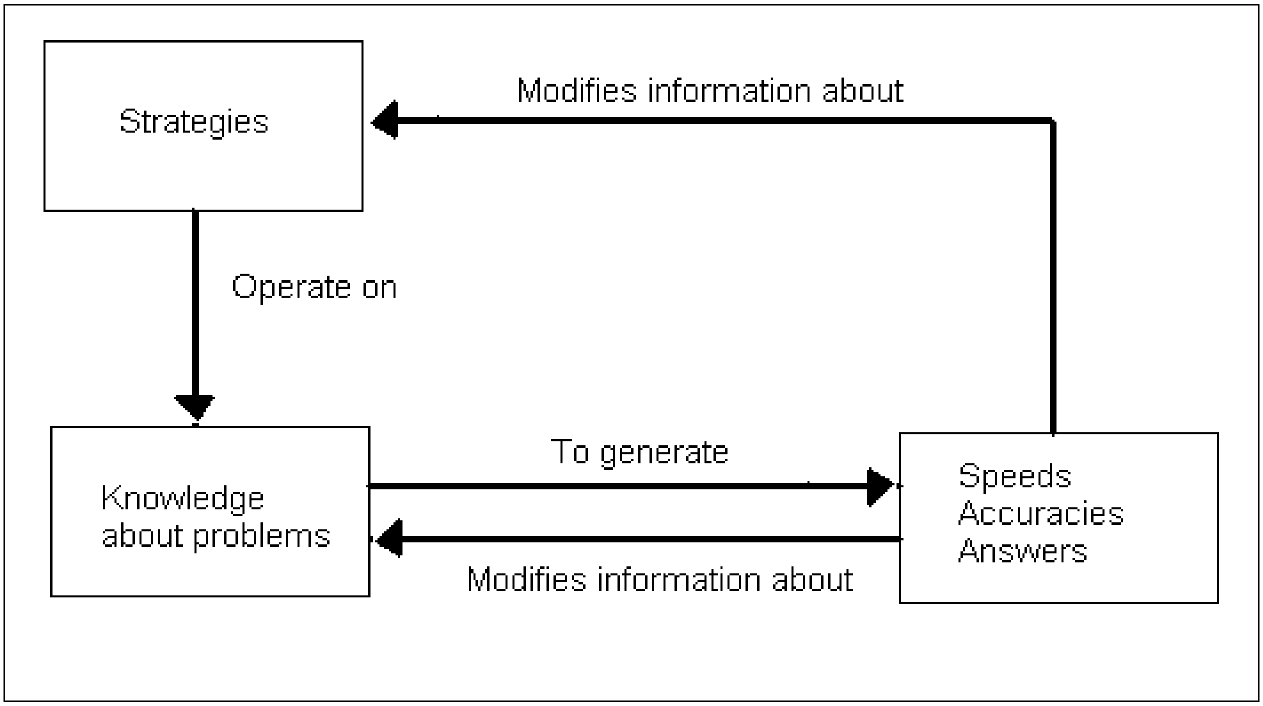


Figure 18. Overview of the Adaptive Strategy Choice Model (ASCM), the basic architecture of which is common to many models of strategy selection.

Yet, empirical data show that people do not always select the strategy that we would expect them to select in the context. For example, they sometimes repeat strategies over two consecutive trials, whereas using a different strategy on the second problem would have been better (e.g., Luwel, Verschaffel, Onghena, & De Corte, 2009; Lemaire & Lecacheur, 2010, Expt. 3). As another example, in a computational estimation task, young adults choose the best strategy for the problem (e.g., rounding-down for problems such as 51 x 62 and rounding-up for problems such as 57 x 69) in 65% of cases only (Lemaire, Arnaud, & Lecacheur, 2004). Associative mechanisms can not account for these findings. To resolve this gap in the assumptions made by models of strategy selection, we need to assume that other mechanisms are also at play during strategy selection.

Other mechanisms could be executive functions. Indeed, selecting a strategy involves activating this strategy and inhibiting competitors (Leclere & Lemaire, submitted; Lemaire & Lecacheur, 2010; Ardiale, Hodzik, & Lemaire, 2012; Ardiale & Lemaire, in press; Duverne, Lemaire, & Vandierendonck, 2008; El Yagoubi, Besson, & Lemaire, 2005; Lemaire & Arnaud, 2008; Gandini, Lemaire, Anton, & Nazarian, 2008; Sliwinski, Buschke, Kuslansky, & Scarisbrick, 1994). Executive resources thus seem to be an important factor to take into account in models of strategy selection. Some of the more recent models such as SCADS* (Siegler & Araya, 2005) included some mechanisms that have an executive nature such as attentional control and an interruption mechanism. However, we feel that these mechanisms could be refined to take into account strategy sequential difficulty effects.

Strategy sequential difficulty effects could temporarily consume executive resources needed for strategy maintenance, selection, and execution. We suggest that the models could incorporate this by adding an additional parameter, representing the amount of available cognitive resources during strategy execution. More or less cognitive resources could be available as a function of the difficulty of both the current and the preceding strategy, influencing strategy execution efficiency and strategy selection.

Less available resources due to previous difficult strategies could introduce repetition biases, impair analysis of problem characteristics, and impair strategy execution. Moreover, this could change the relative success of strategies on problems with repercussions on problem-strategy associations in sequential contexts. When participants find themselves in a similar sequential context in the future, they could less often select a strategy that yielded poor performance on the level of the sequence of problems (see our results on strategy sequential difficulty effects and strategy distribution).

7.5. Strategy sequential difficulty effects and forgetting from working memory

Crucial to an integration of strategy sequential difficulty effects in models of strategy selection is an understanding of the mechanisms involved in these effects. Strategy sequential difficulty effects seem to be related to residual activation of previous strategies, interfering with later strategy executions. We looked at literature on working memory to try and elaborate our understanding of strategy sequential difficulty effects.

What happens to no-longer relevant information in working memory? Do we actively delete (i.e., deletion inhibition, Hasher & Zacks, 1988) or passively forget this information? Whether forgetting from working memory is intentional or not, we distinguish two main views on how it can happen. The first view advocates that the only way to forget information from working memory is interference from new information (Oberauer & Lewandowsky, in press; 2008; Lewandowsky & Oberauer, 2009; Lewandowsky, Oberauer, & Brown, 2009). In this view, new information overwrites the traces of old information in working memory (or uses the neural circuitry that kept attention focused on the old information). The second view advocates that traces of elements in working memory can get forgotten through temporal decay, as soon as attention is no longer allocated to them (Barrouillet, Bernardin, & Camos, 2004, Portrat, Barrouillet, & Camos, 2008; Barrouillet et al., 2011).

Our account of strategy sequential difficulty effects assumes that following a difficult strategy, traces could be left in working memory, complete decay of which may overlap with the execution of the next strategy. The still active traces from the previous difficult strategy could compete for attention with the current strategy, explaining the more difficult processing following difficult strategies. This suggests that elements in working memory can draw attention to them in a bottom-up fashion (i.e., without the context requiring to pay attention to these elements).

This seems to be in violation with time-based forgetting from working memory (Barrouillet, Bernardin, & Camos, 2004). Indeed, without interference from new elements, and existing traces demanding attention in a bottom-up fashion, traces in working memory could continuously be refreshed, and never decay unless new information enters working memory. Our results therefore favour theories that assume that interference is the central mechanism through which forgetting from working memory happens (Oberauer & Lewandowsky, in press; 2008; Lewandowsky & Oberauer, 2009).

7.6. Conclusion

In this thesis, we have shown the existence of an interesting phenomenon. It extends on the finding that efforts at cognitive control reduce subsequent executive resources (Schmeichel, 2007). Indeed, we show that such effects can transiently occur on a trial-to-trial basis. This has repercussions on empirical and theoretical levels. On the most basic level, it has implications for our understanding of the workings of working memory. Attached to this are implications for strategy execution efficiency.

References

Agostino, A., Johnson, J., Pascual-Leone, J., (2010). Executive functions underlying multiplicative reasoning: problem type matters. *Journal of Experimental Child Psychology,* 105(4), 286-305.

Allen, P.A., Ashcraft, M.H., & Weber, T.A. (1992). On mental multiplication and age. *Psychology and Aging,* 7(4), 536-545.

Allen, P. A., Smith, A. F., Jerge, K. A., & Vires-Collins, H. (1997). Age differences in mental multiplication: Evidence for peripheral but not central decrements. *Journal of Gerontology*, 52, 81–90.

Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in children: the importance of central executive functions. *The British Journal of Experimental Psychology,* 78(2), 181-203.

Angel, L., Fay, S., Bouazzaoui, B., Granjon, L., & Isingrini, M. (2009). Neural correlates of cued recall in young and older adults: an event-related potential study. *Neuroreport,* 20(1), 75-79.

Angel, L., Fay, S., Bouazzaoui, B., & Isingrini, M. (2010). More data-driven processing at retrieval reduces age-related memory deficits. *Canadian Journal of Experimental Psychology*, 64(2), 117-123.

Angel, L., Fay, S., Bouazzaoui, B., & Isingrini, M. (2010). Individual differences in executive functioning modulate age effects on the ERP correlates of retrieval success. *Neuropsychologia,* 48(12), 2540-3553.

Angel, L., Fay, S., Bouazzaoui, B., & Isingrini, M. (2011). Two hemispheres for better memory in old age: role of executive functioning. *Journal of Cognitive Neuroscience,* 23(12), 3767-3777.

Angel, L., Isingrini, M., Bouazzaoui, B., Taconnat, L., Allan, K., Granjon, L., & Fay, S. (2010). The amount of retrieval support modulates age effects on episodic memory: evidence from event-related potentials. *Brain Research,* 1335, 41-52.

Arbuthnott, K.D., & Campbell, J.I.D. (2000). Cognitive inhibition in selection and sequential retrieval. *Memory and Cognition,* 28(3), 331-340.

Ardiale, E., Hodzik, S., & Lemaire, P. (2012). Aging and strategy switch costs: A study in arithmetic problem solving. *L'Année Psychologique,* 112(3), 345 - 360.

Ardiale, E., & Lemaire, P. (in press). Within-item strategy switching: An age-comparative study in adults. *Psychology and aging.*

Arnaud, L., Lemaire, P., Allen, P., & Michel, B.F. (2008). Strategic aspects of young, healthy older adults' and Alzheimer patients' arithmetic performance. *Cortex ,* 44*,* 119-130.

Ashcraft, M.H. (1992). Cognitive arithmetic: a review of data and theory. *Cognition,* 44(1-2), 75-106.

Ashcraft, M.H., & Moore, A.M. (2012). Cognitive processes of numerical estimation in children. *Journal of Experimental Child Psychology,* 111(2), 246-267.

Baddeley, A. D., Logie, R., Bressi, S., Della Sala, S., & Spinnler, H. (1986). Senile dementia and working memory. *Quarterly Journal of Experimental Psychology,* 38A, 603-618

Baddeley, A. D., Bressi, S., Della Sala, S., Logie, R. H., & Spinnler, H. (1991). The decline of working memory in Alzheimer's disease: A longitudinal study. *Brain,* 114, 2521-2542.

Baddeley, A. D., & Hitch, G. J. (1974). *Working memory*. In G. H. Bower (Ed.), The psychology of learning and motivation (pp. 47-89). New York, NY: Academic Press.

Bajic, D., & Rickard, T. C. (2009). The temporal dynamics of strategy execution in cognitive skill learning. *Journal of Experimental Psychology*, 35(1), 113-121.

Bailey, H., Dunlosky, J., & Hertzog, C. (2009). Does differential strategy use account for age-related deficits in working-memory performance? *Psychology and aging,* 24(1), 82-92.

Baroody, A.J., (1999). The roles of estimation and the commutativity principle in the development of third graders' mental multiplication. *Journal of Experimental Child Psychology,* 74(3), 157-193.

Barrouillet, P., Bernardin, S., & Camos, V. (2004). Time constraints and resource sharing in adults' working memory spans. *Journal of Experimental Psychology,* 133(1), 83-100.

Barouillet, P., De Paepe, A., & Langerock, N. (2012). Time causes forgetting from working memory. *Psychonomic Bulletin Review,* 19(1)*,* 87-92.

Barrouillet, P., Portrat, S., Vergauwe, E., Diependaele, K., & Camos, V. (2011). Further evidence for temporal decay in working memory: reply to Lewandowsky and Oberauer (2009). *Journal of Experimental Psychology,* 37(5), 1302-1317.

Belleville, S., Bherer, L., Lepage, E., Chertkow, H., & Gauthier, S. (2008). Task switching capacities in persons with Alzheimer's disease and mild cognitive impairment. *Neuropsychologia,* 46(8), 2225-2233.

Benau, E.M., Morris, J., & Couperus, J.W. (2011). Semantic processing in children and adults: incongruity and the N400. *Journal of Psycholinguistic Research*, 40(3), 225-239.

Booth, J.L., & Siegler, R.S. (2008). Developmental and individual differences in pure numerical estimation. *Developmental Pscyhology,* 42(1), 189-201.

Booth, J.L., & Siegler, R.S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development,* 79(4), 1016-1031.

Bouazzaoui, B., Fay, S., Taconnat, L., Vanneste, S., & Isingrini, M. (2012). Differential Involvement of Knowledge Representation and Executive Control in Episodic Memory Performance in Young and Older Adults. *Canadian Journal of Experimental Psychology,* in press.

Bouazzaoui, B., Isingrini, M., Fay, S., Angel, L., Vanneste, S., Clarys, D., Taconnat, L. (2010). Aging and self-reported internal and external memory strategy uses: the role of executive functioning. *Acta Psychologica,* 135(1), 59 - 66.

Bryant, P., Christie, C., & Rendu, A. (1999). Children's understanding of the relation between addition and subtraction: inversion, identity, and decomposition. *Journal of Experimental Child Psychology,* 74(3), 194-212.

Broadway, J.M., & Engle, R.W. (2010). Validating running memory span: Measurement of working memory capacity and links with fluid intelligence. *Behavioral Research Methods,* 42 (2), 563-570.

Brookmeyer, R., Johnson, E., Ziegler-Graham, K., Arrighi, M.H. (2007) Forecasting the global burden of Alzheimer's disease. *Alzheimer's and Dementia,* 3(3),186-91.

Bryck, R. & Mayr, U. (2008). Task selection cost asymmetry without task switching. *Psychonomic Bulletin and Review, 15*, 128-134.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: inhibition, switching, and working memory. *Developmental Neuropsychology*, 19(3), 273-293.

Calot, G., Chesnais, J.-C. (1997). Le vieillissement démographique dans l'union européenne à l'horizon 2050. *TRP, Futuribles, 6.*

Camos, V. Fayol, M., & Barrouillet, P., (1999). L'activité de dénombrement chez l' enfant: Double tâche ou procédure? *L'Année Psychologique*, 99, 623-645.

Campbell, J.I.D. (1999). The surface form x problem size interaction in cognitive arithmetic: evidence against an encoding locus. *Cognition,* 70(2), 25-33.

Campbell, J. I. D. (2005). Handbook of Mathematical Cognition. New York: Psychology Press.

Campbell, J.I.D., & Alberts, N.M. (2009). Operation-specific effects of numerical surface form on arithmetic strategy. *Journal of Experimental Psychology,* 35(4), 999-1011.

Campbell, J.I.D., & Arbuthnott, K.D. (1996). Inhibitory processes in sequential retrieval: evidence from variable-lag repetition priming. *Brain and Cognition,* 30(1), 59-80.

Campbell, J. I. D., & Austin, A. (2002). Effects of response time deadlines on adults' strategy choices for simple addition. *Memory & Cognition*, 30, 988-994.

Campbell, J.I.D., & Graham, D.J. (1985). Mental multiplication skill: Structure, process and acquisition. *Canadian Journal of Psychology*, *39*, 338-366.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology*, 130, 299-315.

Castel, A.D., Balota, D.A., McCabe, D.P. (2009). Memory efficiency and the strategic control of attention at encoding: impairments of value-directed remembering in Alzheimer's disease. *Neuropsychology,* 23(3), 297-306.

Caviola, S., Mammarella, I.C., Cornoldi, C., & Lucangeli, D. (2012). The involvement of working memory in children's exact and approximate mental addition. *Journal of Experimental Child Psychology,* 141-160.

Cerella, J. (1985). Information processing rates in the elderly. *Psychological Bulletin,* 89(1), 67-83.

Chen, Y., & Sun, Y. (2003). Age differences in financial decision-making: Using simple heuristics. *Educational Gerontology,* 29(7), 126 - 47.

Chobor, K.L., & Brown, J.W. (1990). Semantic deterioration in Alzheimer's: the patterns to expect. *Geriatrics,* 45(10), 68-70.

Cohen G., & Faulkner, D. (1983). Age differences in performance on two information-processing tasks: strategy selection and processing efficiency. *Journal of Gerontology,* 38(4), 447-454.

Cullen, B., Coen, R.F., Lynch, C.A., Cunningham, C.J., Coakley, D., Robertson, I.H., Lawlor, B.A. (2005). Repetitive behaviour in Alzheimer's disease: Description, correlates and functions. *International journal of Geriatric Psychiatry,* 20(7), 686-693.

Daneman, M., Carpenter, P.A., 1980. Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior*, 19, 450–466.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence. *Science*, 284, 970-974.

Deloche, G., Hannequin, D., Carlomagno, S., Agniel, A., Pasquier, F., Pellat, J., Denis, P., Beauchamp, D. (1995). Calculation and number processing in mild Alzheimer's disease. *Journal of clinical and experimental neuropsychology,* 17(4), 634-639.

Delaloye, C., Ludwig, C., Borella, E., Chicherio, C., & de Ribaupierre, A. (2007). L'empan de Lecture comme épreuve mesurant la capacité de mémoire de travail: Normes basées sur une population francophone de 775 adultes jeunes et âgés. *Revue Européenne de Psychologie Appliquée,* 58(2), 89-103.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: are the phonological loop and the central executive involved? *Memory and Cognition,* 29(2), 267-273.

Deschuyteneer, M., De Rammelaere, S., & Fias, W. (2005). The addition of two-digit numbers: Exploring carry versus no-carry problems. *Psychology Science*, 47,74−83.

Deschuyteneer, M., & Vandierendonck, A. (2005). The role of response selection and input monitoring in solving simple arithmetical products. *Memory and Cognition,* 33(8), 1472-1483.

Deschuyteneer, M., Vandierendonck, A., & Muyllaert, I. (2006). Does solution of mental arithmetic problems such as 2 + 6 and 3 x 8 rely on the process of "Memory updating"? *Experimental Psychology,* 53(3), 198-208.

Dierckx, V., Vandierendonck, A., & Pandelaere, M. (2003). Is model construction open to strategic decisions? An exploration in the field of relational reasoning. *Thinking and Reasoning*, 9, 97-131.

Dowker, A., Flood, A., Griffiths, H., Harriss, L., & Hook, L. (1996). Estimation strategies of four groups. *Mathematical Cognition*, 2, 113-135.

DSM (1996). *Manuel diagnostic et statistique des troubles mentaux.* Masson, Paris.

Dunlosky, J,. & Connor, L. T. (1997). Age differences in the allocation of study time account for age differences in memory performance. *Memory & Cognition,* 25, 691–700.

Dunlosky, J., & Hertzog, C. (1998). Aging and deficits in associative memory: What is the role of strategy use? *Psychology and Aging*, 13, 597-607.

Dunlosky, J., & Hertzog, C. (2001). Measuring strategy production during associative learning: The relative utility of concurrent versus retrospective reports. *Memory & Cognition*, 29, 247-253.

Duverne, S., & Lemaire, P. (2004). Age-related differences in arithmetic problem verification strategies. Journal of Gerontology: Psychological Sciences, 59B, 135-142.

Duverne, S., & Lemaire, P. (2005). Arithmetic split effects reflect strategy selection: An adult age comparative study in addition comparison and verification tasks. *Canadian Journal of Experimental psychology*, 59, 262-278.

Duverne, S., Lemaire, P., & Michel, B. F. (2003). Alzheimer's disease disrupts arithmetic facts retrieval processes but not arithmetic strategy selection. *Brain and Cognition*, 52, 302-318.

Duverne, S., Lemaire, P., & Vandierendonck, A. (2008). Do working-memory executive components mediate the effects of age on strategy selection or on strategy execution? Insights from arithmetic problem solving. *Psychological Research* 72, 27-38.

Eger E, Sterzer P, Russ MO, Giraud AL, Kleinschmidt A (2003) A supramodal number representation in human intraparietal cortex. *Neuron,* 37, 719-725.

El Yagoubi, R., Lemaire, P., & Besson, M. (2003). Different brain mechanisms mediate two strategies in arithmetic: evidence from Event-Related brain Potentials. *Neuropsychologia*, 41, 855-862.

El Yagoubi, R., Lemaire, P., & Besson, M. (2005). Effects of aging on arithmetic problem solving: An event-related brain potential study. *Journal of Cognitive Neuroscience*, 17, 37-50.

Fabre, L., & Lemaire, P. (2005). Age-related differences in automatic stimulusresponse associations: Insights from young and older adults' parity judgment. *Psychonomic Bulletin & Review*, 12(6), 1100-1105.

Finucane, M.L., Mertz, C.K., Slovic, P., & Schmidt, E.S. (2005). Task complexity and older adults' decision-making competence. *Psychology and Aging,* 20(1), 71-84.

Folstein, M.F., Folstein S.E., & McHugh, P.R. (1975). "Mini-mental state". A practical method for grading the cognitive state of patients for the clinician. *Journal of Psychiatric Research,* 12(3), 189-198.

French, J.W., Ekstrom, R.B., & Price, I.A. (1963). *Kit of reference Tests for Cognitive Factors.* Educational Testing Service, Princeton: NJ.

Fürst, A.J., & Hitch, G.J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory and Cognition*, 28, 774-782.

Gallistel, C.R., Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition,* 44(1-2), 43-74.

Gandini, D., Lemaire, P., Anton J.L., & Nazarian B. (2008). Neural correlates of approximate quantification strategies in young and older adults: an fMRI study. *Brain Research,* 1246, 144-157.

Gandini, D., Lemaire, P., & Dufau, S. (2008). Older and young adults' strategies in approximate quantification. *Acta Psychologica,* 129, 175-189.

Gandini, D., Lemaire, P., & Michel, B-F. (2009). Approximate quantification in young, healthy older adults, and Alzheimer patients. *Brain and Cognition*, 70, 53-61.

Geary, D. C., Bow-Thomas, C. C., Liu, F., & Siegler, R. S. (1993). Even before formal instruction, Chinese children outperform American children in mental addition. *Cognitive Development*, 8, 517-529.

Geary, D. C., Frensch, P. A., & Wiley, J. G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology and Aging*, 8, 242-256.

Geary, D. C., Hamson, C. O., Chen, G-P., Liu, F., Hoard, M. K., & Salthouse, T. A. (1997). Computational and reasoning abilities in arithmetic: Cross-generational change in China and the United States. *Psychonomic Bulletin & Review*, 4, 425-430.

Geary, D.C., Hoard, M.K., Byrd-Craven, J., & DeSoto, M.C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology,* 88(2), 121-151.

Geary, D. C., & Lin, J. (1998). Numerical cognition: Age-related differences in the speed of executing biologically-primary and biologically-secondary processes. *Experimental Aging Research,* 24, 101- 137.

Geary, D. C., Salthouse, T. A., Chen, G-P. & Liu, F. (1996). Are East Asian versus American differences in arithmetical ability a recent phenomenon? *Developmental Psychology*, 32, 254-262.

Geary, D.C., & Wiley, J.G. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and aging,* 6(3), 474-483.

Goldman-Rakic, P.S. (1996). The prefrontal landscape: Implications of functional architecture for understanding human mentation and the central executive. *Philosophical transactions of the royal society of London,* 351(1346), 1445-1453.

Grabner, R.H., & De Smedt, B. (2011). Neurophysiological evidence for the validity of verbal strategy reports in mental arithmetic. *Biological Psychology,* 87(1), 128-136.

Grady, C.L., Haxby, J.V., Horwitz, B., Sundaram, M., Berg, G., Schapiro, M., Friedland, R.P., Rapoport, S.I. (1988). Longitudinal study of the early neuropsychological and cerebral metabolic changes in dementia of the Alzheimer type. *Journal of Clinical and Experimental Neuropsychology*, 10(5), 576-596.

Grafman, J., Kampen, D., Rosenberg, J., Salazar, A.M., & Boller, F. (1989). The progressive breakdown of number processing and calculation ability: A case study. *Cortex, 25,* 121-133.

Green, H., Lemaire, P., & Dufau (2007). Eye movement correlates of younger and older adults' strategies for complex addition. *Acta Psychologica,* 125, 257-278.

Hasher, L., & Zacks, R.T. (1988). *Working memory, comprehension, and aging: A review and a new view.* In G.H. Bower (Ed.), The psychology of learning and motivation (Vol. 22, pp. 193-225). San Diego, CA: Academic Press.

Hayes, M.G., Kelly, A.J., Smith, A.D. (2012). Working Memory and the Strategic Control of Attention in Older and Younger Adults. *The Journals of Gerontology,* in press.

Hecht, S.A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory and Cognition,* 30(3), 447-455.

Hecht, S.A., & Vagi, K.J. (2010). Sources of Group and Individual Differences in Emerging Fraction Skills. *Journal of Educational Psychology,* 102(4), 843-859.

Hitch, G.J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, 10, 302-323.

Hodzik, S., & Lemaire, P. (2011). Inhibition and shifting capacities mediate adults' age-related differences in strategy selection and repertoire. *Acta Psychologica,* 137(3), 335-344.

Hofmann, W., Schmeichel, B.J., & Baddeley, A.D. (2012). Executive functions and self-regulation. *Trends in Cognitive Sciences,* 16(3), 174-180.

Holloway, I.D., & Ansari, D. (2008). Domain-specific and domain-general changes in children's development of number comparison. *Developmental Science,* 11(5), 644-649.

Holloway, I.D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental child psychology* 103(1), 17-29.

Ilkowska, M., & Engle, R. W. (2010). *Trait and state differences in working memory capacity*. In: A.Gruszka, G.Matthews, & B.Szymura (Eds.), Handbook of Individual Differences in Cognition. Attention, Memory, and Executive Control (pp.295-320). Springer, NY.

Imbo, I., Duverne, S., & Lemaire, P. (2007). Working memory, strategy execution, and strategy selection in mental arithmetic. *Quarterly Journal of Experimental Psychology,* 60, 1246-1264.

Imbo, I., & LeFevre, J.A. (2009). Cultural differences in complex addition: efficient Chinese versus adaptive Belgians and Canadians. *Journal of Experimental Psychology,* 35(6), 1465-1476.

Imbo, I., & Vandierendonck, A. (2007). The development of strategy use in elementary school children: Working memory and individual differences. *Journal of Experimental Child Psychology*, 96, 284-309.

Imbo, I., & Vandierendonck, A. (2008). Effects of problem size, operation, and working-memory span on simple-arithmetic strategies: differences between children and adults? *Psychological Research,* 72(3), 331-346.

Imbo, I., Vandierendonck, A., & De Rammelaere, S. (2007). The role of working memory in the carry operation of mental arithmetic: number and value of the carry. *Quarterly Journal of Experimental Psychology,* 60(5), 708-731.

Imbo, I., Vandierendonck, A., & Vergauwe, E. (2007). The role of working memory in carrying and borrowing. *Psychological Research,* 71(4), 467-483.

Isingrini, M., & Taconnat, L. (2007). Episodic memory, frontal functioning, and aging. *Revue Neurologique,* 164(3), 91 – 95.

Johnson, M. M. S. (1990). Age differences in decision making: A process methodology for examining strategic information processing. *Journal of Gerontology*, 45, 75-78.

Jordan, N.C., Kaplan, D., Nabors, Olah, L., & Locuniak, M.N. (2006). Number sense growth in kindergarten: a longitudinal investigation of children at risk for mathematics difficulties. *Child Development,* 77(1), 153-175.

Kane, M.J., & Engle, R.W. (2002). The role of prefrontal cortex in working-memory capacity, executive attention, and general fluid intelligence: an individual-differences perspective. *Psychonomic Bulletin Review,* 9(4), 637-671.

Kaufmann, L., Lochy, A., Drexler, A., & Semenza, C. (2004). Deficient arithmetic fact retrieval--storage or access problem? A case study. *Neuropsychologia,* 42(4), 482-496.

Kaufmann, L., Montanes, P., Jacquier, M., Matallana, D., Eibl, G., & Delazer, M. (2002). About the relationship between basic numerical processing and arithmetics in early Alzheimer's disease: A follow-up study. *Brain and cognition,* 48(2-3), 398-405.

Kiger, J. I., & Glass, A. L. (1981). Context effects in sentence verification. *Journal of Experimental Psychology: Human Perception and Performance*, 7, 688-700.

Kirk, E. P., & Ashcraft, M. H. (2001). Telling stories: the perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology*, 27, 157-175.

Kray, J., & Lindenberger, U. (2000). Adult age differences in task switching. *Psychology and Aging,* 15(1), 126 - 147.

Kutas, M., & Federmeier, K.D. (2011). Thirty years and counting: finding meaning in the N400 component of the event-related brain potential (ERP). *Annual Review of Psychology,* 62, 621-647.

Kutas, M., & Hillyard, S.A. (1980). Reading senseless sentences: brain potentials reflect semantic incongruity. *Science,* 207(4427), 203-205

LeFevre, J.-A., Greenham, S.L., & Waheed, N. (1993). The development of procedural and conceptual knowledge in computational estimation, *Cognition and Instruction,* 11, 95-132.

LeFevre, J.A., Lei, Q., Smith-Chant, B.L., Mullins, D.B. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology,* 55(4), 277-284.

LeFevre, J.A., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, 3, 31-62.

LeFevre, J. A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem-size effect in adults. *Journal of Experimental Psychology,* 22, 216-230.

Lefevre, J.A., Smith-Chant, N.L., Fast, L., Skwarchuk, S.L., Sargla, E., Arnup, J.S., Penner-Wilger, M., Bisanz, J., Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology,* 93(4), 285-303.

Lemaire, P. (2010). Cognitive strategy variations during aging. *Current Directions in Psychological Science*, 19(6), 363-369.

Lemaire, P., Abdi, H; & Fayol, M. (1996). The role of working-memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, 8, 73-103.

Lemaire, P., & Arnaud, L. (2008). Young and older adults' strategies in complex arithmetic. *American Journal of Psychology, 121,* 1-16.

Lemaire, P., Arnaud, L., & Lecacheur, M. (2004). Adults' age-related differences in adaptivity of strategy choices: Evidence from computational estimation. *Psychology and Aging,* 10(3), 467-481.

Lemaire, P., Barrett, S.E., Fayol, M., & Abdi, H. (1994). Automatic activation of addition and multiplication facts in elementary school children. *Journal of Experimental Child Psychology*, 57, 224-258.

Lemaire, P., & Callies, S. (2009). Children's strategies in complex arithmetic. *Journal of Experimental Child Psychology,* 103(1), 49-65.

Lemaire, P., & Fayol, M. (1995). When plausibility judgments supersede fact retrieval: the example of the odd-even effect on product verification. *Memory and Cognition,* 23(1), 34-48.

Lemaire, P., & Lecacheur, M. (2001). Older and younger adults' strategy use and execution in currency conversion tasks: insights from French franc to euro and euro to French franc conversions. *Journal of Experimental Psychology*, 7(3), 195-206.

Lemaire, P., & Lecacheur, M. (2002). Applying the Choice/No-Choice Methodology: The Case of Children's Strategy Use in Spelling. *Developmental Science,* 5, 43-48.

Lemaire, P., & Lecacheur, M. (2010). Strategy switch costs in arithmetic problem solving. *Memory & Cognition,* 38(3), 322-332.

Lemaire, P., & Reder, L. (1999) What affects strategy selection in arithmetic ? An example of parity and five effects on product verification. *Memory & Cognition*, 22, 364-382.

Lemaire, P., & Siegler, R.S. (1995). Four aspects of strategic change: contributions to children's learning of multiplication. *Journal of Experimental Psychology*, 124(1), 83-97.

Lewandowsky, S., & Oberauer, K. (2009). No evidence for temporal decay in working memory. *Journal of Experimental Psychology,* 35(6), 1545-1551.

Lewandowsky, S., Oberauer, K., & Brown, G.D. (2009). No temporal decay in verbal short-term memory. *Trends in Cognitive Science,* 13(3), 120-126.

Logie, R.H., Gilhooly, K.J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, 22, 395-410.

Lovett, M. C., & Anderson, J. R. (1996). History of success and current context in problem solving: Combined influences on operator selection. *Cognitive Psychology,* 31, 168-217.

Lovett, M. C., & Schunn, C. D. (1999). Task representations, strategy variability, and base-rate neglect. *Journal of Experimental Psychology,* 128(2), 107-130.

Lupker, S. J., Kinoshita, S., Coltheart, M., & Taylor, T. E. (2003). Mixing costs and mixing benefits in naming words, pictures, and sums. *Journal of Memory and Language*, *49*, 556–575.

Luwel, K., Onghena, P., Torbeyns, J., Schillemans, V., Verschaffel, L. (2009). Strengths and weaknesses of the choice/no-choice method in research on strategy use. *European Psychologist,* 14, 351-362.

Luwel, K., Schillemans, V., Onghena, P., & Verschaffel, L. (2009). Does switching between strategies within the same task involve a cost? *British Journal of Psychology,* 100(4), 753-771.

Luwel, K., Verschaffel, L., Onghena, P., & De Corte, E. (2003). Flexibility in strategy use: Adaptation of numerosity judgement strategies to task characteristics. *European Journal of Cognitive Psychology*, 15, 247-266.

Mantovan, M.C., Delazer, M., Ermani, M., & Denes, G. (1999). The breakdown of calculation procedures in Alzheimer's disease. *Cortex,* 35(1), 21-38.

Marois, R. & Ivanoff, J. (2005). Capacity limits of information processing in the brain. *Trends in Cognitive Sciences,* 9(6), 296-305.

Mata, R., & Nunes, L. (2010). When less is enough: Cognitive aging, information search, and decision quality in consumer choice. *Psychology and aging,* 25(2), 289-298.

Mata, R., Schooler L. J., & Rieskamp, J. (2007). The aging decision maker: cognitive aging and the adaptive selection of decision strategies. *Psychology and Aging*, 22, 796-810

Mata, R., von Helversen, B., & Rieskamp, J. (2010). Learning to choose: Cognitive aging and strategy selection learning in decision making. *Psychology and aging,* 25(2), 299-309.

Mata, R., Wilke, A., & Czienskowski, U. (2009). Cognitive aging and adaptive foraging behavior. *Journal of Gerontology*, 64B, 474-481.

Maylor, E.A. (1994). Ageing and the retrieval of specialized and general knowledge: performance of Masterminds. *British Journal of Psychology,* 85(1), 105-114.

McGlinchey-Berroth, R., Milberg, W.P., & Charness, N. (1989). Learning of a complex arithmetic skill in dementia: Further evidence for a dissociation between compilation and production. *Cortex,* 25(4), 697-705.

McKhann, G., Drachmann, D.A., Folstein, M.F., Katzman, R., Price, D.L., & Stadlan, E. (1984). Clinical diagnosis of Alzheimer's disease: report of the NINCDS-ADRDA work group under the auspices of the Department of Health and Human Services task force on Alzheimer's disease. *Neurology,* 34, 939-944.

Miller, E.K., & Cohen, J.D. (2001). An integrative theory of prefrontal cortex function. *Annual review of Neuroscience*, 24(2001), 167-202

Miyake, A., Friedman, N. P., Emerson, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology,* 41, 49-100.

Monsell, S. (2003). Task switching. *Trends in Cognitive Sciences*, 7(3), 134-140.

Moyer, R.S., & Landauer, T.K. (1967). Time required for judgements of numerical inequality. *Nature,* 215(5109), 1519-1520.

Niedeggen, M., & Rosler, F. (1999). N400 effects reflect activation spread during retrieval of arithmetic facts. *Psychological Science,* 10, 271-276.

Oberauer, K., & Lewandowsky, S. (2008). Forgetting in immediate serial recall: decay, temporal distinctiveness, or interference? *Psychological Review,* 115(3), 544-76.

Oberauer, K., & Lewandowsky, S. (in press). Evidence Against Decay in Verbal Working Memory. *Journal of Experimental Psychology*.

Oberauer, K., Lewandowsky, S., Farrell, S., Jarrold, C., Greaves, M. (2012). Modeling working memory: An interference model of complex span. *Psychonomic Bulletin Review,* 19(5), 779-819.

Opfer, J.E., & Siegler, R.S., (2007). Representational change and children's numerical estimation. *Cognitive Psychology,* 55(3), 169-195.

Park, D.C., Polk, T.A., Mikels, J.A., Taylor, S.F., & Marshuetz, C. (2001). Cerebral aging: integration of brain and behavioral models of cognitive function. *Dialogues in clinical neuroscience,* 3(3), 151-165.

Payne, J.W., Bettman, J.R., and Johnson, E.J. (1988). Adaptive Strategy Selection in Decision Making. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 14*, 534-552.

Pesenti, M., Seron, X., & Van der Linden, M. (1994). Selective impairment as evidence for mental organisation of arithmetical facts: BB, a case of preserved subtraction? *Cortex* 30(4), 661-671.

Portrat, S., Barrouillet, P., & Camos, V. (2008). Time-related decay or interference-based forgetting in working memory? *Journal of Experimental Psychology,* 34(6), 1561-1564.

Pratt, M. W., & Savoy-Levine, K. M. (1998). Contingent tutoring of long-division skills in fourth and fifth graders: Experimental tests of some hypotheses about scaffolding. *Journal of Applied Developmental Psychology,* 19(2), 287-304.

Ramani, G.B., & Siegler, R.S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development,* 79(2), 375-394.

Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology,* 91(2), 137-157.

Raven, J.C., Court, J., & Raven, J. (1986). *Manual for Raven's Progressive Matrices and Vocabulary Scales.* Oxford Psychologists Press.

Remy, F., Mirrashed, F., Campbell, B., & Richer, W. (2005). Verbal episodic memory impairment in Alzheimer's disease: a combined structural and functional MRI study. *Neuroimage,* 25(1), 253-266.

Reuter-Lorenz, P.A., & Park, D.C. (2010). Human neuroscience and the aging mind: a new look at old problems. *The journals of Gerontology,* 65(4), 405 - 415.

Rieskamp, J., & Otto, P. E. (2006). SSL: A theory of how people learn to select strategies. *Journal of Experimental Psychology,* 135(2), 207-236.

Rittle-Johnson, B., Siegler, R.S. & Alibali, M.W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93, 346-362.

Robbins, T.W., Anderson, E.J., Barker, T.R., Bradley, A.C., Fernyhough, C., Henson, R., & Hudson, S.R. (1995). Working-memory in chess. *Memory and Cognition,* 24(1), 83-93.

Robert-Bobée, I. (2007). Projections de population 2005-2050 Vieillissement de la population en France métropolitaine. *Economie et Statistique*, 408-409, 95-112.

Sarnecka, B.W., & Carey, S. (2008). How counting represents number: what children must learn and when they learn it. *Cognition,* 108(3), 662-674.

Salthouse, T.A. (1992). Influence of working memory on adult age differences in matrix reasoning. *British Journal of Psychology,* 84(2), 171-199.

Salthouse, T.A., & Coon, V.E. (1994). Interpretation of differential deficits: The case of aging and mental arithmetic. *Journal of Experimental Psychology* 20(5), 1172-1182.

Schaie, K. W. (1996). Intellectual development in adulthood: The Seattle longitudinal study. New York: Cambridge University Press.

Schmeichel, B.J. (2007). Attention control, memory updating, and emotion regulation temporarily reduce the capacity for executive control. *Journal of experimental Psychology,* 136(2), 241-255.

Schmitt, F. A., Murphy, M. D., & Sanders, R. E. (1981). Training older adult free recall rehearsal strategies. *Journal of Gerontology*, 36, 329-337.

Schneider, D.W., & Anderson, J.R. (2010). Asymmetric Switch Costs as Sequential Difficulty Effects. *Quarterly Journal of Experimental Psychology,* 63, 1873-1894 .

Seyler, D.J., Kirk, E.P., & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology,* 29(6), 1339-1352.

Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology*, 116, 250-264.

Siegler, R. S. (1988). Strategy Choice Procedures and the Development of Multiplication Skill. *Journal of Experimental Psychology,* 117, 258-27

Siegler, R. S. (2007). Cognitive variability. *Developmental Science*, 10, 104– 109.

Siegler, R.S., & Arraya, R. (2005). *A computational model of conscious and unconscious strategy discovery*. In R. V. Kail (Ed.), Advances in child development and behaviour (pp. 1-42). Oxford, UK: Elsevier.

Siegler, R.S., & Booth, J.L. (2004). Development of numerical estimation in young children. *Child Development,* 75(2), 428-444.

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology*, 126, 71-92.

Siegler, R.S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science,* 19(8), 759-763.

Siegler, R.S., & Opfer, J.E. (2003). The development of numerical estimation: evidence for multiple representations of numerical quantity. *Psychological Science,* 14(3), 237-243.

Siegler, R.S., & Ramani, G.B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Scienc*e, 11(5), 655-661.

Siegler, R. S., & Shipley, C. (1995). *Variation, selection, and cognitive change.* In T. J. Simon & G. S. Halford (Eds.), Developing cognitive competence: New approaches to process modeling (pp. 31-76). Hillsdale, NJ, England: Lawrence Erlbaum Associates.

Siegler, R. S., & Shrager, J. (1984). *Strategy choices in addition and subtraction: How do children know what to do ?* In C. Sophian (Ed.), Origins of cognitive skills (pp. 229293). Hillsdale, NJ: Erlbaum.

Sliwinski, M., Buschke, H., Kuslansky, G., Senior, G., Scarisbrick, D. (1994). Proportional slowing and addition speed in old and young adults. *Psychology and aging,* 9(1), 72-80.

Stanescu-Cosson, R., Pinel, P., Van de Moortele, P.F., Le Bihan, D., Cohen, L., & Dehaene, S. (2000). Understanding dissociations in dyscalculia: A brain imaging study of the impact of number size on the cerebral networks for exact and approximate calculation. *Brain*, 123, 2240-2255.

Souchay, C., & Isingrini, M. (2004). Age related differences in metacognitive control: role of executive functioning. *Brain and cognition,* 56(1), 89-99.

Taconnat, L., Baudouin, A., Fay, S., Calrys, D., Vanneste, S., Tournelle, L., Isingrini, M. (2006). Aging and implementation of encoding strategies in the generation of rhymes: the role of executive functions. *Neuropsychology,* 20(6), 658-665.

Taconnat, L., Clarys, D., Vanneste, S., Bouazzaoui, B., Isingrini, M. (2007). Aging and strategic retrieval in a cued-recall test: the role of executive functions and fluid intelligence. *Brain and cognition,* 64(1), 1-6.

Taconnat, L., Raz, N., Toczé, C., Bouazzaoui, B., Sauzéon, H., Fay, S., and al. (2009). Aging and organization strategies in free recall: The role of cognitive flexibility. *European Journal of Cognitive Psychology*, 21, 347−365.

Taylor, T. E., & Lupker, S. J. (2001). Sequential effects in naming: A time-criterion account. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *27*, 117–138.

Thevenot, C., Barrouillet, P., & Fayol, M. (2001). Algorithmic solution of arithmetic problems and operands-answer associations in long-term memory. *Quarterly Journal of Experimental Psychology,* 54(2), 599-611.

Thevenot, C., Castel, C., Danjon, J., Fanget, M., & Fayol, M. (2012). The Use of the Operand-Recognition Paradigm for the Study of Mental Addition in Older Adults. *The Journals of Gerontology,* in press.

Thevenot, C., Castel, C., Fanget, M., & Fayol, M. (2010). Mental subtraction in high and lower-skilled arithmetic problem solvers: Verbal report vs. operand-recognition paradigms. *Journal of Experimental Psychology*, 36, 1242-1255.

Thevenot, C., Fanget, M., & Fayol, M. (2007). Retrieval or nonretrieval strategies in mental arithmetic? An operand recognition paradigm. *Memory and cognition,* 35(6), 1344-1352.

Towse, J.N. & Hitch, G.J. (1997). Integrating information in object counting: A role for a central coordinating process? *Cognitive Development,* 12, 393-422.

Trbovich, P. L., & LeFevre, J. A. (2003). Phonological and visual working memory in mental addition. *Memory & Cognition,* 31, 738-745.

Tronsky, L.N. (2005). Strategy use, the development of automaticity, and working memory involvement in complex multiplication. *Memory and cognition,* 33(5), 927-940.

Tuholski, S.W. (2001). Individual differences in working memory capacity and enumeration. *Memory and Cognition,* 29(3), 484-492.

Uittenhove, K., & Lemaire, P. (2012) Strategy sequential difficulty effects on strategy execution: A study in arithmetic. *Experimental Psychology,* 59(5), 295-301.

Unsworth N, Heitz RP, Schrock JC, Engle RW. (2005). An automated version of the operation span task. *Behavior Research Methods,* 37, 498–505.

Velanova, K., Lustig, C., Jacoby, L.L., Buckner, R.L. (2007). Evidence for frontally mediated controlled processing differences in older adults. *Cerebral cortex,* 17(5), 1033-1046.

Whalen, J., Gallistel, C.R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science,* 10(2), 130-137.

Wood, G., Ischebeck, A., Koppelstaetter, F., Gotwald, T., & Kaufmann, L. (2009). Developmental trajectories of magnitude processing and interference control: an fMRI study. *Cerebral Cortex,* 19(11), 2755-2765.

Wynn, K. (1990). Children's understanding of counting. *Cognition,* 36(2), 155-193.

Zbrodoff, N.J. (1999). Why is 9 + 7 harder than 2 + 3? Strength and interference as explanations of the problem-size effect. *Memory and Cognition,* 23(6), 689-700.

Zbrodoff, N. J., & Logan, G. D. (2005). *What everyone finds: The problem size effect*. In J. I. D. Campbell (Ed.), Handbook of mathematical cognition (pp. 331-345). New York: Psychology Press.