New Branches of Massive Gravity

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The basic building block for Lorentz invariant and ghost free massive gravity is the square root of the combination $g^{-1}\eta$, where g^{-1} is the inverse of the physical metric and η is a reference metric. Since the square root of a matrix is not uniquely defined, it is possible to have physically inequivalent potentials corresponding to different branches. We show that around Minkowski background the only perturbatively well defined branch is the potential proposed by de Rham, Gabadadze and Tolley. On the other hand, if Lorentz symmetry is broken spontaneously, other potentials exist with a standard perturbative expansion. We show this explicitly building new Lorentz invariant, ghost-free massive gravity potentials for theories that in the background preserve rotational invariance, but break Lorentz boosts.

Introduction. Much progress has been recently made in understanding massive gravity. Attempts to give a mass to the graviton date back to the work of Fierz and Pauli (FP) in 1939 [1]. They considered a mass term for a spin two field which is uniquely determined requiring the absence of ghost degrees of freedom (dof). The mass term breaks the diffeomorphism invariance of General Relativity (GR), leading to a graviton with five degrees of freedom instead of the two of GR. In 1972, Boulware and Deser (BD) found that the ghostly sixth mode, removed by the FP tuning, reappears at non-linear level or around non-trivial backgrounds [2]. It is now well known that it is possible to avoid the presence of the BD ghost by choosing a suitable potential [3]. Such a ghost free theory can be constructed by adding to the Einstein-Hilbert action a potential V function of the symmetric polynomials of the square root of the matrix $X^\mu_\nu \equiv g^{\mu\alpha} f_{\alpha\nu}$, where $f_{\mu\nu}$ is a fixed reference metric [4]. In the following we will consider the case $f_{\mu\nu} = \eta_{\mu\nu}$, i.e. the reference metric coincides with the Minkowski metric. This theory, dubbed de Rham-Gabadadze-Tolley (dRGT) massive gravity, is Lorentz invariant (LI) in the unitary gauge[1]. More in general, the whole class of rotationally invariant massive gravity theories with five degrees of freedom can be constructed [6, 7] by using the canonical analysis, which can be extended to any non-derivative modified gravity theory [8].

In this work we present alternative consistent branches for LI massive gravity. We show how their perturbative expansion and construction is related to the spontaneous breakdown of the Lorentz symmetry. We also briefly comment on some of their phenomenological consequences.

Ghost free potentials. From the canonical analysis of a generic, non-linear massive deformation of GR in four dimensions [6]

$$ S = \int d^4x \sqrt{g} M_{pl}^2 \left(R - m^2 V \right) \equiv S_{EH} + S_V, \quad (1) $$

it follows that, in the unitary gauge, in order to have 5 propagating dof, the potential V has to satisfy two conditions. In terms of the ADM variable (N, N^i, γ_{ij}) they read

$$ \text{Rank}|\mathcal{V}_{AB}| = 3, \qquad \chi^{0^2} \mathcal{V}_i + 2\chi^A \chi^j \frac{\partial \mathcal{V}_A}{\partial \gamma^{ij}} = 0; \quad (2) $$

where we define $\mathcal{V} \equiv N V$. $\mathcal{V}_A$ is the derivative of $\mathcal{V}$ with respect to $N^A = N_A = (N, N^i)$ whose components are the lapse and shifts; χ^A is the eigenvector associated to the null eigenvalue of $\mathcal{V}_{AB}$. Requiring that V has a residual Lorentz invariance besides rotational invariance forces the potential to be a function of the eigenvalues λ_i of $X = g^{-1}\eta$ [9]. We could consider a more general reference metric in X, or even promote it to be dynamical in the context of bigravity.

In the simple case of a 2d space-time, we can solve the partial differential equations in (2) under the assumption of Lorentz invariance and find that only two potentials are allowed [6]

$$ V_\pm = a_0 + a_1 \left(\sqrt{\lambda_1} \pm \sqrt{\lambda_2} \right) = a_0 + a_1 \text{Tr} \left(\sqrt{X} \mid_\pm \right). \quad (3) $$

With $\sqrt{X} \mid_\pm$ we denote the two possible different branches of the square root of X in two dimensions (modulo an overall sign). The two potentials are therefore associated with the two different branches of the matrix square root[2]. Extending this observation to four dimen-

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[1] Invariance under coordinate transformation can be restored by introducing a suitable set of Stückelberg fields [5].

[2] In general, an $n \times n$ matrix with n distinct non-zero eigenvalues has 2^n square roots, according to each of the possible choice in the sign of the square root of its eigenvalues. Assuming that a matrix M with eigenvalues λ_i can be diagonalized by a matrix U, $M = U M_D U^{-1}$, we have that $\sqrt{M} = U \sqrt{M_D} U^{-1}$ where $\sqrt{M_D} = \text{diag}(\pm\sqrt{\lambda_1}, \dots, \pm\sqrt{\lambda_n})$. We will assume that $\sqrt{\lambda_i}$ are real and positive.

sions, one can check that a large class of potentials satisfy the conditions (2):

$$V_{\text{br}} = \sum_{n=0}^{3} \frac{a_n}{n!} \, S_n \mid_{\text{br}}, \qquad (4)$$

where $S_n \mid_{\text{br}}$ are the symmetric polynomials of $\sqrt{X} \mid_{\text{br}}$ once a choice of branch for the square root is made

$$\begin{aligned} S_0 &= 1, \quad S_1 = \tau_1, \quad S_2 = \tau_1^2 - \tau_2, \\ S_3 &= \tau_1^3 - 3\,\tau_1\,\tau_2 + 2\,\tau_3, \end{aligned} \qquad (5)$$

with $\tau_n = \text{Tr}\left(\sqrt{X}\mid_{\text{br}}{}^n\right)$. The subscript $\mid_{\text{br}}$ implies that different options for the square root of X can be chosen, according to the $\pm$ sign in front of each of the square root of the eigenvalues. On the other hand, once a choice is made, the symmetric polynomials should be constructed consistently and no mixing between the different branches is possible without violating the conditions (2), i.e. without reintroducing the 6th ghost mode[3].

The dRGT potential [3] corresponds only to a single branch among all those in (4), i.e. the branch where $\sqrt{X_D} = \text{diag}(+\sqrt{\lambda_1}, +\sqrt{\lambda_2}, +\sqrt{\lambda_3}, +\sqrt{\lambda_4}) \equiv \sqrt{X_D}\mid_{\text{dRGT}}$. The other choices represent *different* potentials through we can realize non-linear ghost free theories of massive gravity.

In the following we present two different approaches to study different branches for massive gravity. The first one is based on the construction of $\sqrt{X}$ by using ADM variables. Alternatively, we can construct $\sqrt{X}$ perturbatively around a chosen background.

ADM approach. The ADM approach joins together the definition of $\sqrt{X}$ given in [10] and the auxiliary variable used in [7] to solve the first of (2). Following [10], the square root can be expressed separating the dependence on the lapse:

$$N\sqrt{X} = \mathbb{A} + N\,\mathbb{B}, \qquad (6)$$

where $\mathbb{A}$ and $\mathbb{B}$ are independent from N. Squaring the above equation one gets a quadratic set of equations for $\mathbb{A}$ and $\mathbb{B}$, leading to different possible branches. The choice made in [10] corresponds to the dRGT potential; on the other hand different choices are possible. Let us show how different branches arise and their properties. It is useful to introduce a new shift variable ξ^i such that, in the new variables, the action becomes linear in the lapse. The transformation is written implicitly as

$$N^i = N\,\xi^i + \mathcal{Q}^i, \qquad (7)$$

where $\mathcal{Q}^i = \mathcal{Q}^i(\gamma_{jk}, \xi^j)$ will be specified later. Solving equation (6) for $\mathbb{A}$ and $\mathbb{B}$ translates in the following

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[3] Indeed one can check that, while (2) are satisfied for any choice of sign among $\pm\sqrt{\lambda_i}$, the very same equations are not satisfied when the combination of two branches is considered. For instance, taking $V_{\text{br}_1} + b\,V_{\text{br}_2}$, to have 5 dof requires $b = 0$.

quadratic equations

$$\mathbb{A}^2 = \begin{pmatrix} 1 & \mathcal{Q}^i \\ -\mathcal{Q}^j & -\mathcal{Q}^i\,\mathcal{Q}^j \end{pmatrix}, \qquad \mathbb{B}^2 = \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{K}_{(3\times3)} \end{pmatrix},$$

$$\mathbb{A}\,\mathbb{B} + \mathbb{B}\,\mathbb{A} = \begin{pmatrix} 0 & \xi^i \\ -\xi^j & -\mathcal{Q}^i\,\xi^j - \mathcal{Q}^j\,\xi^i \end{pmatrix}, \qquad (8)$$

where $\mathcal{K} \equiv \mathcal{K}^{ij} = \gamma^{ij} - \xi^i\,\xi^j$. We can focus for instance on two different branches for $\mathbb{A}$ parametrised as follow

$$\mathbb{A} = \frac{\epsilon_{\mathbb{A}}}{\sqrt{1 - \mathcal{Q}^k\mathcal{Q}^k}} \begin{pmatrix} 1 & \mathcal{Q}^i \\ -\mathcal{Q}^j & -\mathcal{Q}^i\mathcal{Q}^j \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\mathcal{K}}\mid_{\text{br}} \end{pmatrix} \qquad (9)$$

where $\epsilon_{\mathbb{A}}^2 = 1$ and

$$\mathcal{Q}^i = \epsilon_{\mathbb{A}}\,\sqrt{1 - \xi^2}\,\left(\mathcal{K}^{-1/2}\mid_{\text{br}}\right)^{ij}\,\xi^j, \qquad (10)$$

with $\xi^2 = \gamma_{ij}\xi^i\xi^j$. The various branches of the square root of X are controlled by $\epsilon_{\mathbb{A}}$ and by the branches of $\sqrt{\mathcal{K}}$ which is kept formal and what follows applies to any of its branches. Other choices of square roots are available besides this simple one, however they typically lead to a spontaneous breaking of rotational invariance.

Notice that (7) is the most general transformation that trivializes the requirement $\det(\mathcal{V}_{AB}) = 0$ (Monge-Ampère equation [11]), which is the key property of potentials with 5 dof. Such a transformation has to be invertible due to the fact that the equations of motion already impose a constraint on the shifts and such requirement reads

$$\det\left(N\,\delta_i^j + \partial_{\xi^j}\mathcal{Q}^i\right) \neq 0. \qquad (11)$$

Once one finds a consistent background and perturbations around it are considered, (11) has to be satisfied order by order to get a standard expansion.

Take now a flat background for g. Though both the reference metric and the background physical metric are Minkowski space, in the unitary gauge we allow the possibility that they are not aligned, namely

$$\bar{g}_{\mu\nu} = \text{diag}(-c^2, 1, 1, 1). \qquad (12)$$

The misalignment is measured by c. Clearly, by a coordinate transformation one can transform (12) in $\eta_{\mu\nu}$, however the reference metric will not be Minkowski anymore when $c \neq 1$. Thus, even though V is Lorentz invariant, the background value of metric breaks "spontaneously" the Lorentz invariance $SO(3,1)$ while the rotational invariance $SO(3)$ is preserved. At the background level, for the ADM variables, we have $\bar{N} = c$, the spatial metric $\bar{\gamma}_{ij} = \delta_{ij}$ and the shifts are zero, $\bar{N}^i = 0$.

- If ξ^i is zero at the background level ($\bar{\xi}^i = 0$), to preserve rotational symmetry we need

$$\overline{\sqrt{\mathcal{K}}}\mid_{\text{br}} = \sqrt{\mathbf{1}}\mid_{\text{br}} = \epsilon_{\mathbf{1}}\,\mathbf{1}_3, \qquad (13)$$

with $\epsilon_{\mathbf{1}}^2 = 1$. Hence

$$\overline{\sqrt{X}} = \mathrm{diag}(\epsilon_{\mathbb{A}}\, c^{-1},\ \epsilon_{\mathbf{1}}\, 1, \epsilon_{\mathbf{1}}\, 1, \epsilon_{\mathbf{1}}\, 1)\,. \qquad (14)$$

Eq. (11) gives

$$c + \epsilon_{\mathbb{A}}\, \epsilon_{\mathbf{1}} \neq 0\,. \qquad (15)$$

When $c = 1$, Lorentz invariance is not broken and $\epsilon_{\mathbb{A}} = \epsilon_{\mathbf{1}}$. Thus $\overline{\sqrt{X}} = \epsilon_{\mathbb{A}}\, \overline{\sqrt{X}}\,|_{\mathrm{dRGT}}$ and this is just the dRGT branch up to an irrelevant overall sign. For $c \neq 1$, Lorentz invariance is broken, then eq. (15) is satisfied when $\epsilon_{\mathbb{A}} = \pm\epsilon_{\mathbf{1}}$. In this case the choice of the negative sign is not the dRGT one and is non-linearly connected with the new branch:

$$\sqrt{X_D}\,|_{\mathrm{New}} = \epsilon_{\mathbb{A}}\, \mathrm{diag}(-\sqrt{\lambda_1},\, \sqrt{\lambda_2},\, \sqrt{\lambda_3},\, \sqrt{\lambda_4})\,, \quad (16)$$

this choice produces a genuine new potential.

- If $\bar{\xi}^i$ is not zero, computing the inverse of $\mathcal{K}^{ij}$ on the background, we have $\bar{\mathcal{Q}}_{\pm}^i = \pm\epsilon_{\mathbb{A}}\, \bar{\xi}^i$, thus $\partial_{\bar{\xi}^j} \bar{\mathcal{Q}}_{\pm}^i = \pm\epsilon_{\mathbb{A}}\, \delta_{ij}$, where the additional $\pm$ comes from the branches of $\mathcal{K}^{-1/2}$ in (10). From the transformation (7), the fact that $\bar{N}^i = 0$ gives $(c \pm \epsilon_{\mathbb{A}}) = 0$; however, condition (11) requires that $(c \pm \epsilon_{\mathbb{A}}) \neq 0$. Thus, the case with $\bar{\xi}^i \neq 0$ is inconsistent. Indeed, although the transformation (7) is non-linearly well defined, being perturbatively non-invertible when $\bar{\xi}^i \neq 0$ implies that we cannot determine $\bar{\xi}^i$ in terms of the background value of the old variables. This leaves $\overline{\sqrt{X}}$ non-uniquely determined[4].

Summarising, around a Minkowski background the only consistent branch of the matrix $\sqrt{X}$ that preserve Lorentz symmetry is the usual dRGT one. On the other hand, we have shown that there is also a ghost-free branch of massive gravity associated with a flat rotationally invariant background that breaks Lorentz boosts. Following the same logic, other consistent branches can be constructed choosing less symmetric backgrounds.

Perturbative expansion. The very same results are obtained starting from a perturbative definition of the square root around a background solution of (4). To have a well defined perturbative expansion for this class of potentials, it is necessary that the perturbations of the matrix square root can be expressed in terms of the ones of the original matrix, i.e. that the Sylvester equation has a unique solution[5]

$$\delta\sqrt{X} \cdot \sqrt{X} + \sqrt{X} \cdot \delta\sqrt{X} = \delta X\,. \qquad (17)$$

It is known [12], and recently brought back to the attention [13, 14], that in order to have a unique solution for $\delta\sqrt{X}$ in (17), the spectrum of the eigenvalues of $\sqrt{X}$, $\sigma(\sqrt{X})$, and of $-\sqrt{X}$, $\sigma(-\sqrt{X})$, should not intersect, i.e.

$$\sigma(\sqrt{X}) \cap \sigma(-\sqrt{X}) = \varnothing\,. \qquad (18)$$

This result selects the backgrounds around which a given potential can be expanded according to the Sylvester theorem. Notice that the case $\bar{\xi}^i \neq 0$ of the previous section exactly violates the condition (18) in the background.

For instance, if we wish to have a Minkowski background for g, this implies that at the background level $\overline{X}$ is the identity matrix. Therefore there are only two possible allowed background values for $\overline{\sqrt{X}}$ consistent with condition (18), namely $\overline{\sqrt{X}}\,|_{\pm} = \pm\mathbf{1}_4$. The resulting perturbative construction reproduces to all orders the dRGT potential[6]. Thus, dRGT is the only potential among the class in (4) that allows a standard perturbative expansion around Minkowski.

On the other hand, if we require for $\overline{\sqrt{X}}$ only rotational invariance instead of the full $SO(3,1)$ invariance, new non-trivial different branches are possibile. Taking as background (12), in this case $\bar{X} = \mathrm{diag}(c^{-2}, 1, 1, 1)$, hence up to an overall sign, we can have two different branches for $\overline{\sqrt{X}}$ in accordance with (18), namely $\overline{\sqrt{X}}\,|_{\pm} = \mathrm{diag}(\pm c^{-1}, 1, 1, 1)$. While the branch $\overline{\sqrt{X}}\,|_{+}$ is the 0-order dRGT one, the branch $\overline{\sqrt{X}}\,|_{-}$ will lead to the new one given in (16).

Of course, following the same lines, we can go further and consider a background where also the rotational invariance is broken producing new ghost-free potentials corresponding to other branches of (4). Again, the idea consists of selecting a background value for the metric that removes the common eigenvalues between the spectra of $\overline{\sqrt{X}}$ and $-\overline{\sqrt{X}}$, satisfying therefore the Sylvester theorem (18). We will not consider these other branches here, however they can be potentially interesting when studying Bianchi type cosmological solutions in the context of massive gravity.

The new potential. Let us now focus on the potential V_{New} of (4) realised through the branch (16) (let us set $\epsilon_{\mathbb{A}} = 1$). Expanding around (12) and setting $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ we get that all linear terms are absent (e.g. $\bar{g}$ is a solution of the equations of motion) when

$$a_0 + 3\,a_1 + 3\,a_2 + a_3 = 0\,, \quad a_1 + 2\,a_2 + a_3 = 0\,. \quad (19)$$

Notice that c represents a flat direction and is not determined by background equations. This stems from the fact that the lapse is a Lagrange multiplier [15]. As expected, we need two tunings instead of the one of the LI case. Expanding the potential part $S_{V_{\mathrm{New}}}$ of the action

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[4] Actually a formal, non-standard expansion for $\sqrt{X}$ exist, where the degeneracy of each order in perturbations is removed by the next order. The potentials however feature the peculiar presence of square-roots of combinations of metric fluctuations in the perturbative expansion. We leave the development of this method for a future work.

[5] From now on, we will leave understood the subscript $|_{\mathrm{br}}$ for the $\sqrt{X}$ when we refer to a general branch.

[6] The minus case is equivalent to the plus one, modulo a trivial redefinition of the coefficients a_n.

we get

$$S_{V_{\text{New}}}^{(2)} = \int d^4x \frac{(a_2 + a_3)(c + 1)}{8} M_{pl}^2 m^2 \left[h_{ij} h^{ij} - h_{ii} h_{jj} \right].$$
(20)

It should be stressed that, although from (20) the limit $c \to 1$ looks regular, this limit is in fact ill defined: indeed in the perturbative construction of $\sqrt{X}$ singularities are encountered in this limit. This is in agreement with the result from the existence of a unique solution for the Sylvester equation and from the ADM analysis. Therefore, the limit $c \to 1$ in the $\sqrt{X}$ and in its symmetric polynomials do not commute.

Given the form of the quadratic action (20), one can check that – due to the lack of mass term m_1^2 for vectors perturbations h_{0i} – only 2 dof are present in the linearized theory [16–18]. On the other hand we know that non-perturbatively 5 dof are present, therefore 2 vector modes and a scalar one (classified according $SO(3)$ group) are strongly coupled around the flat background (12). The vanishing of m_1^2 is rather general [18] and stems from the Lorentz invariance of the potential V. Indeed, following the same method used in a classic paper by Goldstone, Salam and Weinberg [19] (see for instance chapter 19.2 of QFT Weinberg's book [20]) to prove Goldstone theorem, using a multi-index notation $g_{\mu\nu} \to g_A$, residual LI of V gives

$$\frac{\partial V}{\partial g_B} (T \cdot g)_B = 0,$$
(21)

where T is a generic generator of $SO(3,1)$. Differentiating the previous relation with respect to g_A and evaluating the result on the solution (12) of the equations of motion, the first derivative of V vanishes and we get

$$\left. \frac{\partial^2 V}{\partial g_A \partial g_B} \right|_{\bar{g}} T_{BC} \, \bar{g}_C = 0.$$
(22)

Thus the mass matrix has a zero eigenvalue for each of the generator of $SO(3,1)$ that does not annihilate $\bar{g}$. The broken generators are precisely the 3 boost while rotations are unbroken. As a result the h_{0i} direction represents a non trivial eigenvector with zero eigenvalue of the mass matrix and then $m_1^2 = 0$.

Concerning cosmological solutions, we find that at the background level this potential shares the same unfortunate feature of dRGT, namely it only admits open FRW solutions. Also in this case however, the problem can be overcome in the context of bigravity [21]. It would be interesting to check whether the instabilities found in [22] and [23] are present when the new potential is used[7]. Also, since the vDVZ discontinuity [24] is absent for the background (12) that spontaneously breaks Lorentz boosts [16], it would be useful to check whether new exact spherically symmetric solutions exist as the ones investigated in [25].

Conclusions. Building on the general canonical analysis of massive gravity, we have shown that new ghost free, Lorentz invariant massive gravity theories exist. They are associated with the different branches of the square root of $X = g^{-1}\eta$ that enters in the construction of the massive gravity potential. The dRGT potential is the unique that produces a Lorentz invariant expansion around Minkowski space. However, there are other branches, different from dRGT, characterised by the fact that the expansion around flat space breaks "spontaneously" the Lorentz invariance. Given the number of parameters, a flat background exists if at least an $SO(2)$ of $SO(3,1)$ is left unbroken. We have discussed the explicit case where the residual group is $SO(3)$. For this case, symmetry arguments show that the vector perturbations (h_{0i}) have vanishing mass. It will be of great interest to study the stability of perturbations for the cosmological solutions of this new potential. Moreover, since this theory is expected not to exhibit the vDVZ discontinuity, it will be interesting to study the features of its spherically symmetric solutions. Finally, it will be important to investigate the ghost free character of the new branches using the first order formalism.

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[7] Notice that in the case of the background used in [22], i.e. the axisymmetric Bianchi type–I metric, also other different potentials can be considered.

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