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A solution to the anisotropy problem in bouncing cosmologies

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ABSTRACT: Bouncing cosmologies are often proposed as alternatives to standard inflation for the explanation of the homogeneity and flatness of the universe. In such scenarios, the present cosmological expansion is preceded by a contraction phase. However, during the contraction, in general the anisotropy of the universe grows and eventually leads to a chaotic mixmaster behavior. This would either be hard to reconcile with observations or even lead to a singularity instead of the bounce. In order to preserve a smooth and isotropic bounce, the source for the contraction must have a super-stiff equation of state with $P/\rho = w > 1$. In this letter we propose a new mechanism to solve the anisotropy problem for any low-energy value of w by arguing that high energy physics leads to a modification of the equation of state, with the introduction of non-linear terms. In such a scenario, the anisotropy is strongly suppressed during the high energy phase, allowing for a graceful isotropic bounce, even when the low-energy value of w is smaller than unity.

KEYWORDS: alternatives to inflation, string theory and cosmology, quantum cosmology, cosmic singularity.

1. Introduction

The classical problems of flatness and homogeneity that plague the Big Bang cosmology can be successfully addressed by a phase of accelerated expansion. In the standard inflationary scenario, the expansion is driven by a scalar field rolling down its potential [1]. As a bonus, inflation predicts the existence of quantum fluctuations in the initial vacuum state, leading to primordial perturbations seeding the observed cosmic large-scale structures [2]. These primordial fluctuations are endowed with a nearly scale-invariant spectrum, in agreement with observations of the cosmic microwave background [3].

On the other hand, the difficulties of embedding inflation within a quantum gravity theory and the persistence of the initial singularity in the inflationary scenario have motivated several proposals of alternative cosmologies. There is a general consensus on the existence of a high energy cut-off at the order of the Planck scale, at which classical general relativity should be replaced by a quantum gravity theory. For example, in string theory it has been suggested that the finiteness of the string length should act as a natural cut-off for the curvature and the energy density in the early universe [4]. From this point of view, the Big Bang singularity just represents the outcome of the extrapolation of general relativity beyond its domain of applicability, whereas the quantum gravity theory should regularize this singularity, replacing it by a maximum in the curvature and energy density of the universe. As a consequence of the Big Bang regularization the existence of a contraction phase before the Big Bang has been argued in several frameworks [5, 6, 7, 8]. Following this hypothesis, the universe should contract from initial conditions in a low energy regime, evolving into a phase of higher and higher curvature, until the high energy cut-off of the true quantum gravity theory comes into play. This reverses the contraction into a standard decelerated expansion, thus avoiding the general relativistic singularity and replacing it by a cosmic bounce.

An appealing feature of bouncing cosmologies is that a contraction phase would easily solve the flatness and horizon problems of the classical Big Bang model without the need of inflation. However, the contraction phase also inevitably leads to a problematic growth of anisotropy. Einstein equations rapidly become anisotropy dominated, while the contribution of matter becomes negligible, leading to a "velocity dominated" [9, 10] singularity. This typical outcome of general relativity [11] can only be avoided if the energy density of the matter source grows faster than the anisotropy. For a contracting universe, this happens if the source behaves as super-stiff matter, i.e. if the ratio $P/\rho = w$ of the pressure and the energy density is larger than unity. For example, this condition can be realized by a scalar field with a negative exponential potential, as in the Ekpyrotic model [7].

On the other hand, all sources with $w < 1$ grow too slowly and are finally taken over by anisotropies. When this happens, a mixmaster scenario takes place, with the development of chaotic BKL oscillations in the scale factors [9, 10, 11, 12, 13]. It is not clear whether the quantum gravity phase would be able to make its way out of the huge anisotropies generated during the BKL oscillations [14]. The danger is that the universe emerging after the bounce is too anisotropic to be acceptable. Even worse, the universe could not manage to bounce, ending in an anisotropic velocity dominated singularity.

The anisotropy problem in bouncing cosmologies represents a severe shortcoming for all models with $w \leq 1$. This rules out contractions driven by standard forms of matter, such as dust, radiation or scalar fields with positive potentials. The search for viable models of bouncing cosmologies is therefore restricted to sources with $w > 1$, which require scalar fields with peculiar potentials or other more exotic forms of matter.

Is there a way to relax this constraint and recover models with $w < 1$ for the construction of viable contraction phases? As the mixmaster behavior takes place at high energies, when strong curvature effects dominate [10, 11, 13], one possibility is a modification of the effective equation of state (EoS) of matter. Indeed, it may well be that the linear EoS is a valid approximation only for sufficiently low densities and pressures [15]. Then, as the energy density increases, non-linear terms in the EoS will eventually come into play. Thus, before reaching any conclusion on the anisotropy problem in a contracting phase, one should examine the effects of non-linearity of the EoS.

The purpose of this work is to show that the introduction of a simple non-linear term in the EoS can alleviate or even completely solve the anisotropy problem in bouncing cosmologies. A suppression of the anisotropies by many orders of magnitudes can be achieved, depending on the scale at which the non-linearity starts to dominate. By a suitable implementation of this mechanism, it becomes admissible to consider a low energy contraction with $w < 1$. As the simplest working example, we consider here an EoS with a quadratic term.

2. A toy model of fluid with Non-linear EoS

Let us assume that the contraction era is dominated by a perfect fluid with energy density ρ and pressure P, and that gravity in this phase is described by Einstein equations. The general form of the EoS including a quadratic correction is [15]

$$P = \alpha \rho + \epsilon \frac{\rho^2}{\rho_c}, \tag{2.1}$$

where α is a pure number that determines the low energy EoS of the fluid, ρ_c is the scale at which the quadratic term becomes important and ϵ is the sign of the quadratic correction[1]. Here we will only focus on the case $\epsilon = +1$.

One can think of Eq. (2.1) as the second order truncation of a series expansion in powers of ρ. In such case, the linear term would represent the lowest order approximation to the full EoS; ρ_c can thus be interpreted as the non-linearity scale of the microscopic theory of the fluid. It is also interesting to note that that in several higher-order and quantum gravity theories, and in particular in the brane scenario, the corrections terms in the Einstein equations can be re-arranged as quadratic corrections to the energy density (see e.g. [16, 17] and refs. therein). However, in the brane scenario the quadratic correction term appears in the effective 4-D Friedmann constraint equation, whereas in the general relativity framework we adopt here the quadratic EoS affects the energy conservation and

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[1]In general [15], one can of course add to (2.1) a constant p_0 term; however this is only relevant at low energies [22] and we don't need to consider it here.

the Raychaudhuri equations (cf. [15]). Hence, the dynamics of the isotropisation mechanism in the two cases is not the same. In particular, in the brane scenario the effective quadratic term produces isotropy at high energy, but a singularity is unavoidable. Similar corrections, but with the opposite sign, arise in Loop Quantum Gravity, where the quadratic term is responsible for the bounce, see e.g. [18].

In order to test the growth of the anisotropies, the simplest approach is to consider a Bianchi I cosmology, which can be described in terms of two dynamical quantities: the Hubble expansion scalar H and the traceless shear tensor $\sigma_{\alpha\beta}$ (with $\alpha, \beta = 1, \ldots, 3$). Introducing $\sigma^2 = \sigma_{\alpha\beta}\sigma^{\alpha\beta}/2$, the energy conservation equation and Einstein equations take the form (with $8\pi G/c^4 = 1$)

$$\dot{\rho} + 3H\left(\rho + P\right) = 0 \tag{2.2}$$

$$3H^2 - \sigma^2 = \rho \tag{2.3}$$

$$\dot{H} + H^2 + \frac{2}{3}\sigma^2 = -\frac{1}{6}\left(\rho + 3P\right) \tag{2.4}$$

$$\dot{\sigma} + 3H\sigma = 0. \tag{2.5}$$

Along with Eq. (2.1), the above set of equations is closed and can be solved in terms of the scale factor of the universe a as

$$\rho = \frac{(1+\alpha)\rho_c}{\left(\frac{a}{a_*}\right)^{3(1+\alpha)} - 1} \tag{2.6}$$

$$\sigma^2 = \sigma_i^2 \left(\frac{a}{a_i}\right)^{-6}, \tag{2.7}$$

where a here is implicitly defined by $\dot{a} = Ha$.

We are interested in a contraction phase driven by a source satisfying the energy conditions, therefore we restrict to the case $\alpha > -1$. The solution has two branches: here we focus on the branch with $a > a_*$ with positive energy density.

For $a \to a_*$, the energy density (2.6) diverges, signaling the presence of a type III singularity (cf. [15] and refs. therein). Of course we expect that quantum gravity will dramatically change Eqs. (2.3)-(2.5) when ρ reaches a cut-off scale ρ_M, which can be expected to be of the order of the Planck density $\rho_P = 4.6 \times 10^{113}$ J m^{-3}. At this energy level, quantum gravity should intervene and drive the universe across the bounce towards a standard decelerated expansion phase. The details of this quantum phase are all to be established, although several indications seem to point in the right direction [6, 8]. We will not deal with this problem and just assume that the transition from contraction to expansion occurs in this very high energy regime, when ρ is of the order of ρ_M.

The precise value of a_* can be determined from the initial conditions of the universe. In particular, if the universe starts with a scale factor a_i and energy density ρ_i, a_* is given by

$$a_* = a_i \left[(1+\alpha)\frac{\rho_c}{\rho_i} + 1\right]^{-1/3(1+\alpha)}. \tag{2.8}$$

Similarly, once the cut-off scale ρ_M is fixed, the scale factor at the onset of the bounce is simply

$$a_M = a_* \left[(1+\alpha) \frac{\rho_c}{\rho_M} + 1 \right]^{1/3(1+\alpha)}. \qquad (2.9)$$

3. Anisotropy suppression

The level of anisotropies in the universe can be estimated by comparing the contribution of the shear term to that of the matter source term in Eq. (2.3). It is clear that the contraction of the universe will be driven by the larger term. So, if we want to avoid an anisotropic approach to the bounce, we need that the shear term be much smaller than the matter term at the onset of the bounce. More explicitly, we wish to have $\sigma_M^2/\rho_M \ll 1$.

In our simple model, it is very easy to extract the growth of the anisotropies during the contraction phase. By simple algebra on the exact solutions (2.6)-(2.7) and imposing the hierarchy $\rho_i \ll \rho_c \ll \rho_M$, we can immediately write down the following result

$$\frac{\sigma_M^2}{\rho_M} \simeq \frac{\sigma_i^2}{\rho_i} \left(\frac{\rho_c}{\rho_i} \right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{\rho_c}{\rho_M} \right). \qquad (3.1)$$

The final (dimensionless) anisotropy fraction σ_M^2/ρ_M is given by the initial anisotropy σ_i^2/ρ_i, multiplied by an α-dependent growth factor arising in the low energy phase (when the linear term in the EoS dominates) and a suppression factor ρ_c/ρ_M arising in the high energy phase (dominated by the quadratic term). Actually, the exponent of the growth factor depends on the barotropic coefficient α of the linear term in the EoS and is transformed into an additional suppression factor when $\alpha > 1$, as for super-stiff matter, as it is well known. In this sense, Eq. (3.1) above recovers the well known suppression mechanism of super-stiff matter, and shows that this is an unnecessarily strong requirement provided that the EoS contains a quadratic term, giving rise to the ρ_c/ρ_M suppression term.

The efficiency of growth in the linear phase and suppression in the quadratic phase depend on the respective length of the two phases. This is basically determined by the position of the transition scale ρ_c. If we push it very close to the bounce scale ρ_M, then the anisotropy suppression disappears and only the growth due to the linear phase remains. If instead we push ρ_c very close to ρ_i assuming e.g. that the entire pre-bounce is dominated by a purely quadratic EoS, then the growth factor shrinks to one and the suppression factor becomes huge.

Of course, the success of a quadratic EoS in retrieving an isotropic universe crucially depends on the initial amount of anisotropies in the initial conditions. If the universe is already fairly isotropic, we do not need a very long quadratic phase in order to wash out the shear generated during the low energy linear phase. Then we can have ρ_c fairly close to ρ_M. On the other hand, if the universe starts in a very anisotropic state, we need a longer quadratic phase, in order to ensure that the universe does not enter a mixmaster regime.

It might now be useful to make a numerical example with some familiar quantities, just to illustrate the effectiveness of the quadratic EoS in the isotropisation of the universe in a given case. First, we have to choose an initial value ρ_i for the energy density at

the beginning of the contraction phase. There is no universally accepted value for this quantity, since any bouncing cosmology model makes a different hypothesis on the initial state. Whatever the initial state of the universe, it must be built up so as to satisfy some minimum consistency requirements, in order to generate a viable cosmology. These requirements can help us picking a representative value for the initial energy density. For example, let us consider the solution of the flatness problem. If one supposes that curvature and energy density are comparable at the beginning of the contraction phase, the flatness problem is solved only if the contraction starts with $\rho_i < \rho_0$, where ρ_0 is the present energy density. By choosing the present value of the radiation density, $\rho_i = \rho_{\gamma,0} = 4.2 \times 10^{-14}$ J m^{-3}, the flatness problem is thus marginally solved. We will use this value for numerical estimates and discuss the dependence of the final results on this choice. As for the quantum gravity cut-off, we simply assume it to be exactly at the Planck scale $\rho_M = \rho_P$. Finally, the most important quantity is the transition scale ρ_c. We might want it to lie at energies at least higher than the nucleosynthesis, or even higher than the baryogenesis, in order not to spoil standard cosmological results. Then, we fix ρ_c at the energy density of a radiative fluid with the temperature $T_c = 1$ TeV, which corresponds to

$$\rho_c = 1.4 \times 10^{49} \left(\frac{T_c}{1 \text{ TeV}} \right)^4 \text{ J m}^{-3}. \tag{3.2}$$

Assuming that the fluid dominating the contraction is pure radiation, with $\alpha = 1/3$, the anisotropy growth from the initial state to the bounce is

$$\frac{\sigma_M^2/\rho_M}{\sigma_i^2/\rho_i} \simeq 5.4 \times 10^{-34} \left(\frac{T_c}{1 \text{ TeV}} \right)^6 \left(\frac{\rho_{\gamma,0}}{\rho_i} \right)^{1/2}. \tag{3.3}$$

In this setup the initial shear is suppressed by 34 orders of magnitude with respect to the dominant component energy density in the course of the pre-bounce phase. For comparison, if we assume no transition to a quadratic regime, the anisotropy grows by 63 orders of magnitude in a radiation-dominated contraction from $\rho_{\gamma,0}$ to ρ_P.

With the values just used for this example, we can easily calculate that the suppression would perfectly balance the growth if we chose $T_c = 2.5 \times 10^5$ TeV. The same would happen if we kept $T_c = 1$ TeV and set the initial energy density at $\rho_i = 3 \times 10^{-67} \rho_{\gamma,0}$.

Eq. (3.3) is for a radiation-dominated contraction, which is the easiest case to realize, because all particles tend to become relativistic as the temperature increases. However, a dust-dominated contraction would be particularly interesting from the point of view of cosmological perturbations. In fact, it is well-known that the constant mode of the curvature perturbation would develop a scale-invariant spectrum [19] and that this would be easily transferred to the constant mode of the post-bounce without invoking any exotic mechanism at the bounce [20]. Unfortunately, a contraction dominated by non-relativistic particles would sooner or later become relativistic and thus replaced by a radiation-dominated one. Non-relativistic particles can be mimicked by a scalar field with a positive exponential potential, but the tracking solution is a repeller in a contracting universe [21]. It is interesting to note that Eq. (2.1) with $\alpha = 0$ can be obtained by a K-essence model with the

Lagrangian

$$\mathcal{L} = \rho_c \left[C \left(\frac{\chi}{\rho_c} \right)^{1/4} \pm 1 \right]^2, \tag{3.4}$$

where C is a dimensionless parameter and $\chi = (\dot{\phi})^2/2$.

Now, assuming that a stable dust-like contraction exists, we want to test the efficiency of the quadratic correction in solving the anisotropy problem with $\alpha = 0$. Keeping $\rho_M = \rho_P$, $\rho_i = \rho_{\gamma,0}$, $T_c = 1$ TeV, and setting $\alpha = 0$, Eq. (3.1) becomes

$$\frac{\sigma_M^2/\rho_M}{\sigma_i^2/\rho_i} \simeq 10^{-2} \left(\frac{T_c}{1 \text{ TeV}} \right)^8 \left(\frac{\rho_{\gamma,0}}{\rho_i} \right). \tag{3.5}$$

Still we have a small suppression of the anisotropies, which is a considerable step beyond the huge growth of anisotropies in a pure dust contraction with $P \simeq 0$. This result can be improved by lowering T_c. Otherwise, one has to assume that the initial level of anisotropies is sufficiently low from the beginning. Keeping anisotropies under control is essential if one wants to generate a viable primordial spectrum of perturbations by a dust-like contraction. It is easy to show that once the perturbation mode is outside the horizon the spectrum of the curvature perturbation is not affected by the presence of non-linearities in the equation of state. Therefore, the anisotropies must be largely negligible at least until the horizon exit of the modes of interest for CMB observations and the formation of large-scale structures. Since the horizon exit of these modes occurs during the low energy linear phase, anisotropies in the initial state must be very low in any case.

4. Conclusions

In this work we have clearly demonstrated that the inclusion of non-linear terms in the EoS of the matter source driving the contraction of a bouncing cosmology model has a huge benefic effect on the suppression of the anisotropies. We have explicitly considered a quadratic correction, but it is easy to imagine (and we have explicitly checked) that more general non-linear terms would yield a completely analogous result.

It can be objected that if the quadratic term in the EoS is included in the source driving the post-bounce expansion as well, then it would act in the opposite way, by temporarily enhancing anisotropies. However, in general, one does not expect the nature of the dominant source to be the same before and after the bounce. Then, if there is any post-bounce non-linear regime, it is sufficient that the transition to the standard linear regime occurs at a higher temperature than the pre-bounce transition. In such a way, the post-bounce would be protected from the return of anisotropies.

Using this mechanism, it is possible to recover sources with EoS coefficient $w < 1$ as possible candidates for driving a pre-bounce contraction. In fact, the approach to the bounce is no longer menaced by the establishment of chaotic BKL oscillations. Models of bouncing cosmologies can then safely use e.g. standard radiative sources for the pre-bounce contraction without worrying about anisotropies. Furthermore, this mechanism is particularly interesting for any tentative revival of dust-like sources as possible candidates

for driving pre-bounce contractions. By introducing non-linear corrections, one of the most severe shortcomings preventing the development of bouncing cosmological models can be finally overcome.

Acknowledgments

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