XXIII...The Magnetopause Boundary

As the solar wind flows past the Earth, it applies pressure to the magnetic field of the earth, sweeping it back into a comet-like shape on the nighttime side of the Earth. The brunt of the solar wind pressure is exerted on the dayside field, compressing it. Only the restorative pressure of the magnetic field pushes against the solar wind, and a rough balance of these pressures occurs. Like a football scrimage line, this balance moves towards the earth when the solar wind pressure increases, and it moves outwards toward the sun as the solar wind pressure slackens. This scrimage line is called the magnetopause boundary.

Objective:

Students will explore in a quantitative way, how the distance to the 'bow shock' balance point depends on the solar wind pressure. They will explore the changes using actual satellite data, by both an algebraic and graphical process.

Teacher Note:

These activities are suited for advanced students of physics and math. Algebraic manipulation is required.

Mathematics:

There are three basic equations that define the solar wind pressure, the pressure from a magnetic field, and the magnetic field strength of the earth in space are defined as follows:

Equation 1
Solar Wind Pressure

$$P_w = \frac{1}{2} D v^2$$

Equation 2
Magnetic Field Pressure

$$P_m = \frac{1}{8\pi} F^2$$

Equation 3
Magnetic Field Force

$$F = 0.6 \frac{1}{R^3}$$

Equation 1 defines the 'ram' pressure produced by a wind with a density of D grams per cubic centimeter, moving at a speed of v measured in centimeters per second. Equation 2 defines the pressure exerted by a magnetic field which exerts a force of F measured in units of Gauss. Equation 3 defines the magnetic force, F, in terms of the distance from the source of the magnetism, R measured in multiples of the radius of Earth. At a distance of R=1.0 Earth radii, the surface magnetic field produces a force equal to 0.6 Gauss.

1) Because we want to define the distance, R, at which the magnetic and wind pressures balance, we need to substitute Equation 3 into Equation 2, and then set this equal to Equation 3. The basic idea is that Equation 1 equals Equation 2 when the magnetic and wind pressures are equal. This defines the condition at the front of the earths magnetic field as it feels the pressure of the wind. What we can deteermine by satellite observations is the value of the variabled D and V, so this means we know what Pw is numerically. What we need to do, then, is to solve for R, which will tell us at what distance the pressure balance occurs.

Substituting Equation 3 into Equation 2 gives us:

$$Pm = \frac{1}{8\pi}\left[0.6\frac{1}{R^3}\right]^2$$

2) The next step is to balance the two pressures by setting their equations equal to each other so that $\mathbf{P_w = P_m}$. If we now set Equation 1 equal to Equation 4 we get:

$$\frac{1}{2}DV^2 = \frac{1}{8\pi}\left[0.6\frac{1}{R^3}\right]^2$$

3) We now use a little algebra to simplify this formula, and solve for R:

$$\frac{1}{2}DV^2 = \frac{0.36}{8\pi \; R^6}$$

Square the quantities inside the brakets.

$$R^6 = \frac{0.72}{8\pi D V^2}$$

Multiply both sides by R^6 to remove the factor from the right-hand denominator, then divide both sides by $1/2DV^2$ to remove this factor from the left-hand side.

$$\boxed{R = \left[\frac{0.72}{8\pi D V^2}\right]^{\frac{1}{6}}}$$

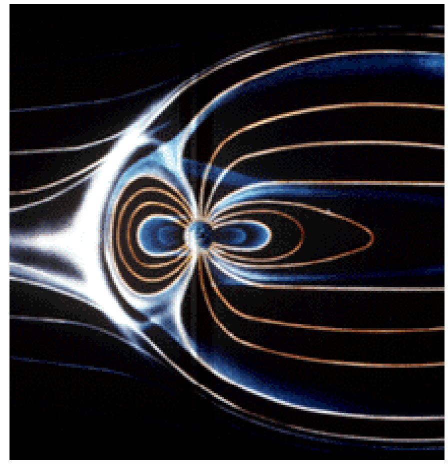
Raise both sides to the 1/6-th power, to obtain R on the left-hand side.

4) We are now ready to substitute some typical values for the solar wind density (D) and speed (V) to see what the average distance is to this equilibrium point in space. The equations have been simplified in their form by selecting the quantities D and V in terms of grams per cubic centimeter and centimeters per second. The equation will then provide a value for R in terms of the distance in units of Earth's radius. Example, for R = 2.5, this means that the distance is 2.5 times the Earth's radius or 2.5 x 6378 kilometers = 15,945 kilometers.

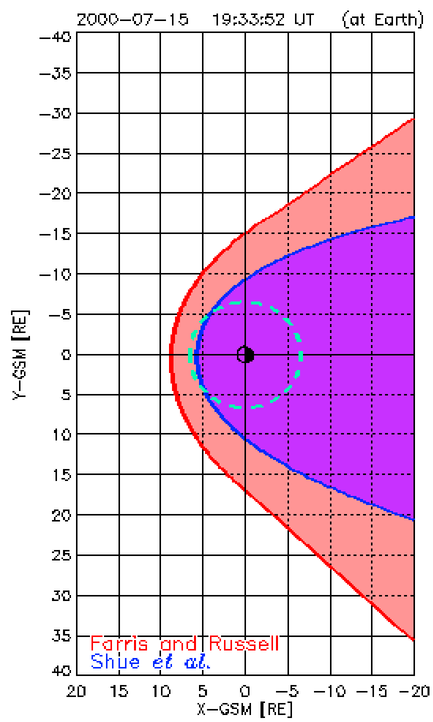
The typical speed of the solar wind is 450 kilometers/second or **V= 4.5 x 10^7 centimeters/sec**. The typical density of the wind is about 8 hydrogen atoms per cubic centimeter. Since 1 hydrogen atoms has a mass of 1.6 x 10^{-24} grams, the density equals 8 x 1.6 x 10^{-24} gm/cc or **D = 1.28 x 10^{-23} gm/cc**. If we substitute these values for D and V into the equation, we get:

$$\boxed{R = (1105451)^{1/6} = 10.2 \text{ Earth radii.}}$$

5) Plot on a scale model the following features looking down on the earth from above its North Pole:



1) The disk of the earth.
2) A selected direction to the Sun
3) The position of the day-night terminator on the Earth.
4) The circular orbit representing the positions of the geosynchronous satellites at a distance of 6.4 Earth radii from the center of the Earth.
5) The distance of the magnetopause calculated from Step 5. Note, it is located along the line connecting the center of the Earth and the Sun at Local Noon on the day-side of the Earth.



Assume that the shape of the magnetosphere is roughly a parabolic shape (called a *paraboloid* in 3-dimensions) with a focus at the Earth's center, a vertex at the magnetopause distance, and fanning out towards the night-time side of the Earth. Draw this as a dotted line on your scale model.

6) Visit the NASA, ACE satellite data archive at, and note the values for the solar wind speed and its density. Calculate D and V as in Step 4, and re-calculate the magnetopause distance for the solar wind conditions today. Plot a parabola with the vertex distance of the Bow Shock.

7) During the July 14, 200 'Bastille Day' storm, the parameters for the solar wind were measured to be V = 1700 kilometers/sec and 95 atoms/cc. Calculate the magnetopause distance for this severe storm as in Step 6.

Questions:

1) Where was the magnetopause compared to the orbit of the geosynchronous communication satellites?
2) Which quantity has the strongest impact on changes in the magnetopause location, the density of the solar wind, or the speed of the wind?
3) If a solar wind monitoring satellite is located 1.5 million kilometers closer to the Sun than Earth's orbit, how much warning would you get if the wind speed were 1,500 km/sec or only 500 km/sec?

To see a real-time calculation of the magnetopause position based on NASA, ACE data, visit **http://pixie.spasci.com/DynMod/**