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Variance Risk Premiums on the S&P 500, Nasdaq 100, Euro Stoxx 50, FTSE 100, SMI, DAX and the United States Oil Fund

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Abstract

In this thesis I investigate the variance risk premiums, defined as the difference between the markets implied variance and subsequent realised variance, in equity index and oil futures markets. I describe how the square of listed volatility indices approximates the markets risk neutral expectancy of future 30-day variance and quantify the variance risk premium over a period of around 11 years for the equity indices and for a period of 4 years in the oil futures market. The variance risk premiums are found to be strongly negative and statistically significant in the equity indices. The results in the oil futures market are mixed, but found to be strongly negative and statistically significant considering the much used log variance risk premium measure. In addition I am able to rationalise how the market prices variance risk by employing an extension of the classical Sharpe ratio that accounts for investor aversion of higher moments in the return distributions. Furthermore I find that the classical capital asset pricing model predicts the correct sign of the variance risk premiums, but fails to explain their magnitude. Since none has described the statistical properties of the variance risk premiums on European equity indices in a similar manner to mine my thesis may serve as a fruitful addition to the existing literature of the variance risk premiums.

Acknowledgements: I would like to thank professor Sjur Westgaard for his encouragement and help. Many thanks are also owed to PHD candidate Steinar Veka for always being online or readily available on the phone to help me through obstacles and discuss results. Finally I would like to thank my fellow students Torgeir Kråkenes and Morten Nitteberg for valuable comments and thoughts. The responsibility for any errors or misconceives expressed in this thesis rests on my shoulders alone.
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1 Introduction

By using the square of listed volatility indices as an approximation for the fair variance swap rate the aim of this thesis is to do an empirical investigation of the variance risk premium in the S&P 500 and Nasdaq 100 equity indices in the U.S and the FTSE 100, SMI, DAX and Stoxx 50 equity indices in Europe. In addition, I will use the same methodology to investigate the variance risk premium in the U.S crude oil market. More specifically, I will quantify the average variance risk premium in defined periods for all of these markets and look at the risk return relationship from investing in variance swaps. Finally I will investigate if the observed risk premiums are explained by the classical capital asset pricing model (CAPM). Essentially my aim is to do a statistical analysis of how the market prices variance risk in these markets which is important to effectively manage risk and investments, price and hedge derivative securities and to understand the behaviour of asset prices in general.

Trading of volatility contracts started in the beginning of the 1990s and since then volatility has emerged as an asset class in its own right. However, it was not until the end of the 1990s, when researchers developed acceptable ways of valuing volatility contracts, that trading began on a larger scale. Today investors are able to hedge and speculate on variance through a number of financial contracts such as futures and options on volatility indices, variance swaps and volatility swaps. The demand for volatility is likely to stem from speculation, portfolio insurance and diversification. A number of businesses like pension funds, insurance companies and other financial institutions are implicitly short volatility and use variation based contracts to hedge against this exposure. On the other hand, more risk willing institutions like hedge funds and investment banks have been willing to take the short side of this contracts hoping for superior returns. The variance swap is the most commonly traded volatility contract, a estimate made by Risk magazine in 2007 placed the daily volume in the over-the-counter (OTC) variance swaps market on major equity indices to be USD 5M at risk per percentage point change in volatility. A long position in this contract pays off the difference between realised variance over a given period and an agreed upon fixed variance rate. Being long a variance swap provides a pure bet on future realised variance. The fixed rate that sets the initial value of this contract equal to zero is named the fair variance rate and represents the markets expected future variance over the life of the swap. The variance risk premium is then the difference between the ex post realised variance and the fair variance swap rate. A positive variance risk premium implies that ex post realised variance were higher than the market expected ex ante over some period and the holder of a variance swap receives a positive payoff.

Variance and volatility contracts are traded over-the-counter (OTC) and the very best is probably to gather historic data from large dealers in these markets and use actual traded
swap rates to investigate the variance risk premium. Other researchers have synthesized the fair variance swap rates from a large set of historic option prices. Even if these two approaches probably are more theoretically appealing I will use the square of VIX, VXN, VFTSE, VDAX, VStoxx, VSMI and OXV volatility indices as an approximation for the 30-day fair variance swap rate. The methodology used by exchanges to set the price of an volatility index is based on the same replication arguments that is used by traders in the OTC markets and quotations of these indices are easily obtained from the CBOE homepage and the Thomson Reuters Ecowin database.

Earlier research of the variance risk premium has primarily focused on US stocks and equity indices and mainly on the S&P 500 index. Newly constructed volatility indices on European equity indices enable me to extend the knowledge of the statistical description of equity index variance risk premium. To the best of my knowledge, none has investigated the variance risk premium on European equity indices in a similar manner to mine. It is therefore interesting to compare my results with the results reported in earlier research on this topic. I include oil market variance in my thesis in order to facilitate an extension of the study of Trolle and Schwartz [2009] to the very recent years where we have seen extraordinary swings in the price of crude oil. Much similar to both Carr and Wu [2007] and Trolle and Schwartz [2009] I aim to figure out the sign and magnitude of the variance risk premium, find the risk adjusted returns from selling short dated variance swaps, check if the variance risk premiums are constant over time and check to which extent the variance risk premiums are explained by the capital asset pricing model. Having this knowledge and understanding of how the market prices variance risk is important for any investor as he is faced with both the uncertainty of future return, captured by the return variance, and the uncertainty of that return variance itself (variance risk).

The thesis proceeds in the following way: section 2 discusses different reasons for the existence of a variance risk premium in the light of well established financial theory and presents important research papers related to the study of variance risk, section 3 presents the data, section 4 discusses alternative ways of replicating variance and presents the methodology used to estimate the variance risk premium, section 5 presents and discuses the results, section 6 concludes and finally there is an appendix where figures and tables of the results are presented.
2 Literature review

In this section I provide a short overview of the literature related to the variance risk premium. First I present some theories of why a variance risk premium may exist. Second, I present some of the main papers leading up to the development of volatility as an asset class and finally, I present some previous empirical findings of the variance risk premium.

2.1 Why would there be a variance risk premium?

The variance risk premium, defined as a consistent and non-negligible difference between the markets price or expectancy of future variance and subsequent realised variance, may either be explained as a result of consistent market inefficiencies, economic fundamentals or investors risk aversion.

Black [1976] and Christie [1982] where among the first to document the now well known negative relationship between stock returns and volatility. The explanation they put forth is the "leverage effect hypothesis". Negative stock returns increases leverage which makes the stock (equity) riskier and results in higher volatility. Another explanation to the asymmetric relationship between returns and volatility is the "volatility feedback effect" put forth by amongst others Campbell and Hentschel [1992]. According to the feedback effect anticipations of higher volatility raises the required return on equity and as a consequence stock prices drop. The Constant Elasticity of Volatility (CEV) model by Cox [1996] is one example of a model that takes these effects into account to model asset prices. Because of these characteristics being long volatility is likely to act as an insurance against negative returns, a long volatility position gives a positive payout when the investor needs it the most and it is reasonable to assume that he is willing to pay a premium for this insurance effect. Following this economic argument variance risk is priced as a consequence of its negative relationship with index returns.

As Carr and Wu [2007] explains, in the stochastic volatility model of e.g Heston [1993] variance varies unconditionally of the underlying. And if this is a good representation variance variation represents an independent risk source that might ask for a risk premium on its own in addition to the premium on return variance.

Another reason for the existence of a negative variance risk premium may be found in behavioural finance theory. As first demonstrated by Kahneman and Tversky [1979] people have a tendency to strongly prefer avoiding losses to acquiring gains. As a consequence investors will be less willing to be short variance than long since possible losses are theoretically unlimited. In line with the prospect theory it is logical to assume that investors
may be willing to pay a premium to insure themselves against large losses and also against volatile markets since people seem to dislike highly uncertain states.

2.2 Some historical developments

Brenner and Galai [1989] points out that volatility has exhibited substantial instability and that at the time of writing there were no effective tools available to hedge against changes in volatility. They were among the first to advocate for the construction of a volatility index that should be used as the underlying asset for futures and options enabling investors to hedge against changing volatility. They propose several ways of constructing this index, among them a combination of historical and implied volatilities.

Fleming et al. [1993] explains how the original volatility index (VIX) is constructed as an average of S&P 100 at the money option implied volatilities. Concurrently the Chicago Board Options Exchange (CBOE) introduces VIX.

Dupire [1993] and Neuberger [1994] independently showed that, under the assumption of a continuous price process, a forward contract that pays \(\ln(\frac{S_T}{S_0})\) could be used to hedge and speculate on variance. Building on this idea Carr and Madan [1998] and Demeterfi et al. [1999] came up with a way of replicating and valuing a variance swap. Independently they showed that the fair value of total variance is given by the value of an infinite strip of European options.

Following these findings, in 2003 CBOE redefined the construction of the VIX volatility index see CBOE/VIX-WhitePaper [2009]. The new VIX uses options on the S&P 500 index and its computation now involves a discrete approximation to the theoretical results in Carr and Madan [1998] and Demeterfi et al. [1999]. For a good description of the major differences between the new and the old VIX see Carr and Wu [2006]. CBOE now lists VIX futures and options. In addition to VIX, CBOE calculates several other volatility indices on equity indices, currencies and commodities.

Based on the same methodology volatility indices on main European equity indices was launched around 2005-2006. See Alexander [2008b].

2.3 Previous empirical findings

Coval and Shumway [2001] estimates the variance risk premium on the S&P 500 and the S&P 100 from the returns of zero-beta, at-the-money straddle positions. They find a significantly negative variance risk premium and additionally conclude that factors other than market risk must be priced in option returns.
Bakshi [2003] find evidence of a negative volatility risk premium on the S&P 500 by examining the statistical properties of delta-hedged option portfolios.

Bondarenko [2004] synthesizes variance swap rates from market prices of traded options on the S&P 500 and find a negative discrete return of -26.66% from holding long positions in monthly variance swaps. He also argues that a considerable portion of hedge funds historical returns comes from selling variance risk.

Bollerslev et al. [2011] uses high frequency data of VIX and S&P 500 from 1990-2004 and find a average 30-day volatility risk premium of -7.40%.

Carr and Wu [2007] synthesize the 30-day fair variance swap rate using historic option prices on five US equity indices and 35 individual stocks. They find strongly negative variance risk premiums on the indices and predominantly negative, but mixed results on the individual stocks. They define the log return of a variance swap as the log variance risk premium and report annualised Sharpe ratios of 0 to 0.98 from selling variance swaps considering this log return measure. Additionally they make an attempt to explain the negative variance premiums in the classical asset pricing model and find that the negative correlation between index returns and volatility generates strongly negative beta values, but concludes that this beta can only explain a small portion of the observed negative variance risk premiums. Furthermore, they conclude that also the Fama-French three factor model are incapable of explaining the magnitude of the negative variance risk premiums.

Trolle and Schwartz [2009] examines the variance risk premium on natural gas, crude oil and on the S&P 500 using a approach similar to that of Carr and Wu [2007]. They find negative variance risk premiums in all markets considered and report annualised Sharpe ratios from shorting variance of 0.35 for natural gas, 0.59 for crude oil and 1.02 for S&P 500. Additionally they conclude that the absolute magnitude of the variance risk premium is time-varying and correlated with the level of the variance swap rate. Furthermore, they also find that it is difficult to explain the level and variation in energy variance risk premiums with systematic factors.
3 Data

In this chapter I present the data I have used to quantify the variance risk premium on the different indices I examine. I present the length of my time series and the source of my data. The symbols of the underlying indices and their respective volatility indices I apply throughout my text along with the exchange that list them are collectively presented in table 3.1.

3.1 Data presentation

S&P 500: The S&P 500 is a market capitalization weighted index of 500 large-cap U.S public companies and is considered as the core index for U.S equities. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin, ticker: SPX.

VIX: Before the Chicago Board Options Exchange (CBOE) redesigned the VIX index it was calculated as the average of eight near at the money Black-Scholes implied volatilities for S&P 100 option prices. The methodology used to calculate the VIX index was changed in 2003. The new VIX measures the market's expectation of 30-day volatility on the S&P 500 and is based on a model independent approach taking only market prices of options into account, explained in section 4. Upon renewal of the index CBOE backdated the new VIX to 1990. CBOE still lists the old VIX under the ticker VXO. Daily closing quotes from January 2000 to May 2011 of (new) VIX are downloaded from Thomson Reuters Ecowin.

Nasdaq 100: The Nasdaq 100 is a semi market capitalization weighted index of the 100 largest non-financial companies listed on the Nasdaq stock market exchange. Companies of all nationalities may be included in the index. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

VXN: Measures market's 30-day expected volatility on the Nasdaq 100 index by applying the VIX methodology, explained in section 4, to market prices of Nadaq 100 options. The index was renewed at the same time as the VIX (2003). Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

Euro Stoxx 50: Is a market capitalization weighted index of 50 large-cap European public companies designed by STOXX Ltd. The index is widely followed by media and investors as an indicator of European equity performance. Stocks from Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom may be
included in the index. Exchange traded funds that tracks the performance of the index is available to all investors from several trading platforms and exchanges. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**VStoxx:** Measures market’s 30-day expected volatility on the Euro Stoxx 50 index by applying the VIX methodology, explained in section [4] to market prices of Euro Stoxx 50 index options. Note that Stoxx Ltd. constructs volatility indices of several maturities from 30 days to 360 days. For my purpose I have only examined the near term (30-day) expected volatility. The index was launched in 2006, the early years of my data is based on back-calculation. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**FTSE 100:** Is a market capitalization weighted index of the 100 largest UK companies listed on the London Stock Exchange. Recognised as the # 1 UK equity performance indicator. Daily closing prices from January 2000 to May 2011 are downloaded from Thompson Reuters Ecowin.

**VFTSE:** Measures market’s 30-day expected volatility on the FTSE 100 index by applying the VIX methodology, explained in section [4] to market prices of FTSE 100 index options. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**SMI:** The Swiss Market Index (SMI) is a market capitalization weighted index of the 20 largest companies by market-cap listed on the SIX Swiss stock exchange. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**VSMI:** Measures market’s 30-day expected volatility on the SMI index by applying the VIX methodology, explained in section [4] to market prices of SMI index options. The index was launched in 2005 and the early years of my time series rely on back-calculation. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**DAX:** The Deutscher Aktien Index (DAX) is a market capitalization weighted index of the 30 largest companies by market-cap listed on the Frankfurt stock exchange. Daily closing prices from January 2000 to May 2011 are downloaded from Thomson Reuters Ecowin.

**VDAX:** Measures market’s 30-day expected volatility on the DAX index by applying the VIX methodology, explained in section [4] to market prices of DAX index options. As with the VIX index there exists a old and a new VDAX, VDAX-New was launched in 2005. The Old VDAX was constructed from Black and Scholes implied volatilities similar to the old VIX. Daily closing prices from January 2000 to May 2011 are downloaded from
USO: The United States Oil Fund LP (USO) is an exchange traded fund that tracks the changes in WTI light, sweet crude oil future prices. The ETF is listed on NYSE Arca under the ticker USO. Daily closing prices from May 2007 to May 2011 are downloaded from the CBOE homepage.

OXV: The CBOE Oil ETF volatility index (OXV) measures the market’s expectation of 30-day volatility of crude oil prices by applying the VIX methodology, explained in section 4, to market prices of USO options. The index was launched in mid 2008. Daily closing prices from May 2007 to May 2011 are downloaded from the CBOE homepage.

Table 3.1: Table of data symbols, length of time series and sources. All data are daily. The symbols presented here are used consistently throughout the text

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Exchange</th>
<th>Period</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>Equity Index</td>
<td>NYSE</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>Nasdaq 100</td>
<td>Equity Index</td>
<td>Nasdaq</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>Euro Stoxx 50</td>
<td>Equity Index</td>
<td>Eurex</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>Equity Index</td>
<td>London SE</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>SMI</td>
<td>Equity Index</td>
<td>Swiss SE</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>DAX</td>
<td>Equity Index</td>
<td>Frankfurt SE</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>USO</td>
<td>Oil (ETF)</td>
<td>NYSE Arca</td>
<td>2007:5 - 2011:5</td>
</tr>
<tr>
<td>VXN</td>
<td>Volatility Nasdaq 100</td>
<td>CBOE</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>VStoxx</td>
<td>Volatility Euro Stoxx 50</td>
<td>Eurex</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>VFTSE</td>
<td>Volatility FTSE 100</td>
<td>Eurex</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>VSMI</td>
<td>Volatility SMI</td>
<td>Eurex</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>VDAX</td>
<td>Volatility DAX</td>
<td>Eurex</td>
<td>2000:1 - 2011:5</td>
</tr>
<tr>
<td>OXV</td>
<td>Volatility USO</td>
<td>CBOE</td>
<td>2007:5 - 2011:5</td>
</tr>
</tbody>
</table>

3.2 Data quality

Data quality as such is considered a non-issue as all my data is obtained directly from trusted sources. Most of my time series are downloaded from Thomson Reuters Ecowin while two of my time series is downloaded from the CBOE homepage see table 3.1. The quantification of the different measures of the variance risk premium is done in Microsoft Excel while the illustrations, regressions and statistical analysis is done in Oxmetrics. For the equity time series I have intentionally chosen them to start at the same date so that they overlap to make my results internally comparable. The length of these time series stretch over a period of approximately eleven and a half years or 2956 trading days. Even though most of the volatility indices on equities was first launched or renewed to
their current methodology around 2005, the exchanges responsible for their construction back-calculated them according to the new methodology in order to serve the academic and investor community with historical data. For the VIX and VXN indices this is done as long back as to 1990. For the European volatility indices this back-calculation is done back to January 2000, thus restricting my time series since I want them to overlap. The only ingredient in the construction of the volatility indices is market prices of options. Therefore, the historical record of these indices only relies on a liquid options market and a reliable dataset of historical option prices. As far as I have been able to figure out there is no reason to suspect that there was a lack of liquidity in the index options for the markets I examine. It is also fair to assume that the team responsible for the construction had access to reliable records of these option prices. No mention of any such possible error sources are given in the papers describing their methodology, furthermore I suspect that considerable resources was invested into the construction and testing procedure of these indices before they where launched to the market. The volatility indices themselves are just theoretical products and are not directly tradeable, there are however futures and options (cash settled) which are tradable. The CBOE Crude Oil Volatility Index (OXV) was first introduced in July 2008 and is only back-calculated to May 2007. The short length of this time series may be a problem.
4 Methodology

In this section I will explain the intuition behind the replication strategy of a variance swap, explain how the square of listed volatility indices can be used as an approximation for the 30-day variance swap rate, present the measures of the variance risk premium and explain the risk adjusted performance measures I apply.

4.1 Capturing variation

4.1.1 Volatility strategies

A simple option is a bet on both the direction of the underlying and on volatility, if you are net long options you also have a long position in volatility. The value of your portfolio increases with market volatility, if you are net short options the opposite is true. It is possible to construct portfolio of options that is, in principle, not sensitive to the direction of change of the underlying asset. One such strategy is known as an at the money (ATM) straddle which is a option portfolio constructed of simultaneous long positions in identical ATM call and put options on the same underlying. At inception,

![Profit diagram of at the money straddle](image)

Figure 4.1: Profit diagram of at the money straddle, in principle not sensitive to the direction of change of the underlying asset

while the straddle is ATM, it is not sensitive to the change of the underlying. However, as soon as the underlying moves, the straddle will move out of the money and become a directional trade. Option prices are also sensitive to changes in the risk free interest rate and to changes in dividends.
4.1.2 Capturing volatility by delta-hedging

Under the assumption of zero interest rates and no dividends a single option that is delta hedged continuously will only change in value if realised volatility changes from the implied volatility at which the option was bought. Remember that the classical Black-Scholes-Merton (BSM) assumptions involve constant volatility. At first, one would think that delta hedging a single option continuously is a good way to trade volatility. As we will see, it is not. From the BSM formula we know that vega, an options sensitivity to small changes in the volatility of the underlying, is given by (4.1). When a single option is continuously delta hedged the profit or loss from this portfolio only depends on changes in volatility (assuming zero interest rates and no dividends), the problem is that the magnitude of this volatility exposure is dependent upon the options moneyness. The exposure to changes in volatility peaks when the option is at the money and falls of rapidly when the option moves in or out of the money, see figure 4.1.2.\(^1\)

\[
\text{Volatility vega} = Se^{(b-r)T} n(d_1)\sqrt{T}
\]  

(4.1)

\[d_1 = \frac{\ln S - \ln K + (r - b + 0.5\sigma^2)T}{\sigma\sqrt{T}}\]

\[n(d_1)\]

Figure 4.2: The options sensitivity to changes in the volatility of the underlying is clearly largest when the option is at the money.

The difference between realised and implied volatility will count much more when the option is at the money. Another limitation of capturing volatility by delta-hedging a

\(^1\)Note that S is the price of the underlying, b is the cost of carry and r is the risk free interest rate. N represents the cumulative distribution function of the standard normal distribution and \(d_1 = \frac{\ln S - \ln K + (r - b + 0.5\sigma^2)T}{\sigma\sqrt{T}}\) where K is the strike price, \(\sigma\) is the standard deviation of the returns and T represents the time to maturity.
single option is that it requires very frequent and costly delta hedging.

4.1.3 Replicating the log contract

A pure trade on volatility using option portfolios or by delta hedging a single option is practically impossible and very costly. It will be labour intensive and imperfect. The easy way to trade variation is to use volatility swaps or variance swaps. Similar to other derivative contracts the theoretical prices of these products are found by the price of the portfolio that replicates their payoff. While we will see that there exists a model independent and robust replication for variance swaps this is not the case for volatility swaps. Replicating volatility swaps requires a arbitrage free stochastic volatility model to determine the evolution of the volatility of volatility see Demeterfi et al. [1999] and Gatheral [2006]. For this reason, using variance swaps is the most common and easiest way (at least from the market makers perspective) to trade variation. It is also the reason why the construction of volatility indices like the VIX is based on replicating variance not volatility.

The payoff to a variance swap is totally independent of the level of the underlying, having issued or bought a variance swap you are only exposed to changes in variance of the underlying asset. From the BSM formula we know that the variance (volatility squared) vega, an options sensitivity to small changes in the variance of the underlying, is given by

\[
Variance\ vega = S^{(b-r)T}n(d_1)\frac{\sqrt{T}}{2\sigma}
\]  

(4.2)

From (4.2) the sensitivity of variance vega to changes in the underlying is obviously different from zero. What we need to replicate the payoff to a variance swap is a portfolio of options that gives us a portfolio of variance vega that is independent of the level of the underlying. Neuberger [1994] showed that the log contract whose payoff at expiration is \(\log\left(\frac{S_T}{F}\right)\) can be used to trade realised variance. Based on the idea of replicating this contract Demeterfi et al. [1999] showed that a portfolio of options weighted inversely proportional to its strike price becomes totally independent of the underlying inside the strike range as long as the option strikes is evenly spaced and the underlying follows an diffusion, that is it evolves continuously without jumps.

In figure 4.1.3 we get some sense to how this works as we see that the resulting portfolio of options has a variance vega that is different from zero and independent of the underlying. The intuition is understood by comparing the variance exposure of the equally weighted portfolio of options (blue-dashed line) with the portfolio weighted inversely proportional to the strike (red-solid line). From (4.2) and the graphs to the left in figure 4.1.3 each
Figure 4.3: Illustration of variance exposure to a portfolio of options weighted inversely proportional to strike price

option of higher strike brings a variance exposure proportional to its strike, therefore you will need a diminishing proportion (inversely proportional to square of strike) of options of higher strikes in the portfolio to get a constant exposure to variance. This is the intuition behind the fact that in the theoretical case of an infinite strip of European options it is possible to construct a portfolio of options that is purely dependent on variance for all levels of the underlying.
4.2 Variance swap payoff

At maturity, the payoff to a long variance swap is equal to the difference between the realized variance over the life of the contract and the fair variance swap rate that is constant and set at initiation of the contract.

\[ [RV_{t,T} - K_t^2] \cdot N \]  

(4.3)

Where \( RV_{t,T} \) is the \textit{ex post} realised variance, \( K_t \) is the variance swap strike and \( N \) is the variance notional.\footnote{The notional for a variance swap can be expressed either as variance notional or vega notional. Variance notional will represent the \( N \) p/l for each variance point difference between \( RV_{t,T} \) and \( K_t^2 \). Vega notional represents the average p/l for a 1\% change in volatility.}

Note that the variance strike is \textit{quoted} in terms of annualized volatility, \( K_t \), the payout to a variance swap is based on the difference between annualized realised variance and the level of variance implied by the strike (the strike squared) over the life of the swap.\footnote{Since this may be confusing at first read I will re-explain it: If you go to a dealer of variance swaps and ask him for a quote you will get \( K_t \), simply the square root of the actual fair variance swap rate, the payment you receive is based on \( K^2 \) the fair variance swap rate. This is just a matter of market convention.}

Figure 4.4: The long side of a variance swap contract pays fixed and receives floating while the short side of the contract receives fixed and pays floating

4.3 Calculating the variance swap rate

This section is based on McDonald [2005] and Demeterfi et al. [1999]. My intention is to provide the reader with an easy to understand presentation of how the fair variance swap rate is calculated.

The fair variance swap rate, \( K_t^2 \), is defined as the value that makes the net present value of the contract equal to zero. It is the solution to:

\[ E^Q_t [RV_{0,T}] - K_t^2 = 0 \]  

(4.4)

Where \( Q \) indicates the risk-neutral measure and \( t \) denotes time of initiation of the variance.
swap. No-arbitrage implies that the variance swap rate equals the risk neutral expectation of future realised variance at time $t$.

$$K_t^2 = E_t^Q [RV_{0,T}]$$ (4.5)

Neuberger [1994] showed that it is possible to trade realised variance with a contract that pays

$$\ln \left( \frac{S_T}{S_0} \right)$$ (4.6)

at expiration ($T$). This contract is known as the log contract and is not traded in the market. Assuming that the underlyer follows the diffusion:

$$\frac{dS_t}{S_t} = (\alpha - \delta)dt + \sigma(S_t, X_t, t)dZ$$ (4.7)

we use Ito’s lemma to find the process of the log of the underlyer:

$$d[\ln(S_t)] = \left[ \frac{\partial \ln(S_t)}{\partial t} + \frac{\partial \ln(S_t)}{\partial S} \mu_S + \frac{1}{2} \frac{\partial^2 \ln(S_t)}{\partial S^2} \sigma^2 S^2 \right] dt + \frac{\partial \ln(S_t)}{\partial S} \sigma S dZ$$

$$= \left( (\alpha - \delta) - \frac{1}{2} \sigma^2 \right) dt + \sigma dZ$$

Integrate from 0 to $T$ to obtain:

$$\int_0^T d[\ln(S_t)] dt = \int_0^T (\alpha - \delta) dt - \frac{1}{2} \int_0^T \sigma^2 dt + \int_0^T \sigma dZ$$

$$\ln \left( \frac{S_T}{S_0} \right) = (\alpha - \delta) T - \frac{1}{2} \int_0^T \sigma^2 dt + \int_0^T \sigma dZ$$

$$\frac{1}{T} \int_0^T \sigma^2 dt = 2(\alpha - \delta) - \frac{2}{T} \ln \left( \frac{S_T}{S_0} \right) + \frac{2}{T} \int_0^T \sigma dZ$$

Assuming zero dividends ($\delta$) and a constant risk free rate ($r$) we obtain the following:

4 Note that $\frac{\partial \ln(S_t)}{\partial t} = 0$, $\frac{\partial \ln(S_t)}{\partial S} = \frac{1}{S}$ and that $\frac{\partial^2 \ln(S_t)}{\partial S^2} = -\frac{1}{S^2}$.

5 Note that $\int_0^T d[\ln(S_t)] dt = [\ln(S_t)]_0^T = \ln(S_T) - \ln(S_0) = \ln \left( \frac{S_T}{S_0} \right)$.
expectation under the risk neutral measure:

\[ E^Q \left[ \frac{1}{T} \int_0^T \sigma^2 \, dt \right] = 2r - \frac{2}{T} E^Q \left[ \ln \left( \frac{S_T}{S_0} \right) \right] \] (4.8)

We see that the r.h.s is equal to realised variance from inception (0) to expiration (T). To ease the understanding we set the risk free rate equal to zero and rearrange to get

\[ E^Q \left[ \int_0^T \sigma^2 \, dt \right] = -2 E^Q \left[ \ln \left( \frac{S_T}{S_0} \right) \right] \] (4.9)

From (4.9) we explicitly see that when the underlying follows the diffusion (4.7) we can estimate the fair value of total variance by replicating the payoff to a short position in the log contract (4.6). Combining (4.5) and (4.9) we get

\[ K^2_t = E^Q \left[ \int_0^T \sigma^2 \, dt \right] = -2 E^Q \left[ \ln \left( \frac{S_T}{S_0} \right) \right] \] (4.10)

To summarize, we now know that the fair swap rate for a variance swap represents the risk neutral expectation of realised variance from inception to expiration of the contract. And we know that realised variance can be captured by replicating the expected payoff to a short position in a log contract. Carr and Madan [1998] and Demeterfi et al. [1999] showed that it is possible to replicate the payoff of the log contract by a portfolio of options. Introducing a new arbitrary parameter \( S_* \) so that for every \( K \) (strike) less than \( S_* \) a put option is used and for every \( K \) higher than \( S_* \) a call option is used Demeterfi et al. [1999] decomposes the log payoff in the following way

\[ - \ln \frac{S_T}{S_*} = - \frac{S_T - S_*}{S_*} \]

\[ + \int_0^{S_*} \frac{1}{K^2} \text{Max}(K - S_T, 0) \, dK \]

\[ + \int_{S_*}^{\infty} \frac{1}{K^2} \text{Max}(S_T - K) \, dK \]

6Remembering that \( dZ(t) \) is \( N(0, \sigma) \) distributed so that the expectation of (4.7) is \( r \, dt \) under the risk neutral measure
We add and subtract \( \ln(S_0) \) on the l.h.s and take expectations to obtain

\[
-E^Q \ln \left( \frac{S_T}{S_0} \right) + \ln \frac{S^*}{S_0} = -\frac{F_{0,T}(S) - S^*}{S^*} + e^{rT} \left[ \int_0^{S^*} \frac{1}{K^2} P(K) \, dK \right] + e^{rT} \left[ \int_{S^*}^{\infty} \frac{1}{K^2} C(K) \, dK \right]
\]

Take this expression and substitute for \(-E^Q \ln \left( \frac{S_T}{S_0} \right)\) in 4.8 to obtain:

\[
E^Q \left[ \int_0^T \sigma^2 dt \right] = \frac{2}{T} \left[ rT - \ln \left( \frac{S_T}{S_0} \right) - \frac{F_{0,T}(S) - S^*}{S^*} \right] + 2e^{rT} \left[ \int_0^{S^*} \frac{1}{K^2} P(K) \, dK + \int_{S^*}^{\infty} \frac{1}{K^2} C(K) \, dK \right]
\]

If we set \( S^* \) equal to the forward price \( F_{0,T} \) we are left with:

\[
K_i^2 = E^Q \left[ \int_0^T \sigma^2 dt \right] = \frac{2e^{rT}}{T} \left[ \int_0^{F_{0,T}} \frac{1}{K^2} P(K) \, dK + \int_{F_{0,T}}^{\infty} \frac{1}{K^2} C(K) \, dK \right]
\] (4.11)

Assuming (i) arbitrage free markets, (ii) that an infinite strip of European call and put options with maturity \( T \) is traded in the market and (iii) that the underlying follows a diffusion the formula in 4.11 gives us the expected future realised variance in a completely model independent way. With model independent I mean that the formula calculates the expected future realised variance directly from observed market prices of options instead of backing out implied volatilities from an option pricing model.

4.3.1 Using the square of listed volatility indices to approximate 30-day variance rate

This section is based on Gatheral [2006] and will show that, based on their specific methodology, the square of listed volatility indices is a very good approximation to the constant 30-day fair variance swap rate. Please understand VIX as an abbreviation for Volatility Index, the vol. indices are all based on the same methodology so the arguments presented here is equally valid for all of the volatility indices.

From the description of the VIX family of indices CBOE/VIX-WhitePaper [2009] the
volatility indices are defined:\[8\]

\[
VIX^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} Q_i(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]
\] (4.12)

where \(F\) is the forward price and \(Q_i\) is the price of the out of the money option with strike \(K_i\). Therefore \(Q_i\) is a call option when \(K_i\) is above the forward price and a put option when \(K_i\) is below the forward price. \(K_0\) is defined to be the first strike below the forward price \(F\).

From [Demeterfi et al. 1999] the fair value of total variance is given by the value of an infinite strip of European options:

\[
K_{i,T}^2 = E \left[ \int_t^T \sigma^2_S \, dt \right] = 2 \left\{ \int_{-\infty}^0 dk p(k) + \int_0^\infty dk c(k) \right\}
\] (4.13)

\[
\frac{VIX^2 T}{2} = \int_0^F \frac{dK}{K^2} P(K) + \int_F^\infty \frac{dK}{K^2} C(K)
\]

\[
= \int_0^{K_0} \frac{dK}{K^2} P(K) + \int_{K_0}^\infty \frac{dK}{K^2} C(K) + \int_F^{K_0} \frac{dK}{K^2} (P(K) - C(K))
\]

\[
\approx \int_0^\infty \frac{dK}{K^2} Q(K) + \frac{1}{K_0^2} \int_{K_0}^F dK (K - F)
\]

\[
= \int_0^\infty \frac{dK}{K^2} Q(K) + \frac{1}{K_0^2} \left[ \frac{(K - F)^2}{2} \right]_F^{K_0}
\]

\[
= \int_0^\infty \frac{dK}{K^2} Q(K) - \frac{1}{K_0^2} \frac{(K_0 - F)^2}{2}
\]

discretization of this last expression yields

---

\[8\] The VIX, VXN, OXV, VSMI, VDAX, VFTSE and VSTOXX indices are all part of a consistent family of indices based on the VIX methodology described in [CBOE/VIX-WhitePaper 2009]. See for ex. VSMI-Factsheet or VStoxx-Factsheet and Alexander 2008b.
or with non-zero interest rates and the split between put and call options we have

\[
\frac{VIX^2 T}{2} = \sum_i \frac{\Delta K_i}{K_i^2} Q_i(K_i) - \frac{1}{2} \left[ \frac{K_0^2 - 2FK_0 + F^2}{K_0^2} \right]
\]

\[
VIX^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} Q_i(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2
\]

In this way we see that the square of listed volatility indices represents a discrete approximation to the theoretical results presented in section 4.3. The last term is there to correct for the fact that there may be no option with a strike equal to the current forward price. A constant maturity of 30-days are obtained by interpolating between the near-term options with less than 30 days to maturity and the next term options with more than 30 days to maturity, see CBOE/VIX-WhitePaper [2009] for details.

### 4.4 Calculating the realised variance

Consistent with the construction of the volatility indices, see CBOE/VIX-WhitePaper [2009], corresponding to each 30-day variance swap rate I compute the annualised 30-day realised variance as:

\[
RV_{t,T} = \frac{365}{30} \sum_{t=1}^{30} \left( \log \frac{P_t}{P_{t-1}} \right)^2
\]  

(4.14)

Where \( P_t \) is the daily closing price of the index. Assuming mean returns are zero, as is standard in variance swap contracts Alexander [2008b], it is not necessary to demean the average return.

### 4.5 Measuring the variance risk premium

I define three return measures used for variance swaps which have also been used most extensively in earlier research on the variance risk premia. These return measures are also the empirical measures of the variance risk premium. Both measures 4.15 and 4.16 is treated as excess returns over the risk free rate.  

\[9\] I have used the LIBOR of 3-month maturity obtained from Thompson Reuters Ecowin as an approximation for the risk free rate. Demeaning the risk free rate is done in order to ensure that we are in fact considering the variance risk premium similar to the way one would consider the risk premium on any
4.5.1 Discrete return

Since $VIX^2$ is a good approximation of variance swap rate for a 30-day contract, the return of a long variance swap with a maturity of 30 days in discrete terms can be approximated in the following way\(^{10}\)

\[
\text{Return variance swap} = \frac{(RV_{t,T} - VIX^2_t)}{VIX^2_t}
\]

4.5.2 Logarithmic return

Similarly the continuously compounded return from a long position in a 30-day variance swap is approximated in the following way

\[
\text{Return variance swap} = \log \left( \frac{RV_{t,T}}{VIX^2_t} \right)
\]

In line with amongst others Carr and Wu [2007], Trolle and Schwartz [2009], I define this as the log variance risk premium. The sample mean of \(4.15\) will be the average variance risk premium in discrete terms while the sample mean of \(4.16\) will be the average variance risk premium in logarithmic terms. When comparing these two estimates of the variance risk premium one should bear in mind that the logarithmic return always will be lower than the discrete return. As a consequence the logarithmic transformation may lead to an underestimation of the variance risk premium as mentioned by Bondarenko [2007]. According to Carr and Wu [2007] the main benefit of the transformation is that the distribution of the logarithmic returns will be closer to normality. This discussion will be treated in further detail in section 5 and outlined with histograms of both distributions in appendix C.

4.5.3 Payoff measure

The third measure of the variance risk premium is the payoff to a variance swap with a notional of $N=100$. The risk premium in payoff terms

\[
RP_{\text{payoff}} = \left[ RV_{t,T} - K^2_t \right] \times 100
\]

The sample mean of this measure is an estimate of the average variance risk premium in other risky investment\(^{10}\)

\(^{10}\)As in the text above VIX is to be interpreted as an abbreviation for volatility index. The return measures are calculated for every volatility index considered not just the VIX.
monetary units. Concurrently it is a measure of the absolute magnitude of the variance risk premium in percentage terms.

4.6 Risk adjusted performance measures

This section explains the risk adjusted performance measures I examine on the strategy of shorting 30-day variance swaps. A risk adjusted performance measure tries to answer the following question "should we include an asset in our portfolio, assuming that future returns will have the same distribution as in a historical sample". Since returns are random and investors are risk averse, this means that a risk adjusted performance measure will typically not rank the strategy with the highest historical return first. In short, most risk adjusted performance measures compares returns to the risk of a strategy, returns could be viewed as income while risk is considered to be a cost. Or more formally, investors are assumed to have mean-variance preferences such that they maximize expected utility under a utility function that is increasing with the mean or return and decreasing with variance or risk. And naturally, it is also assumed that the history of the investment strategy is a good predictor of future return and risk. I will not attempt to solve the full mean-variance portfolio problem, instead I calculate risk adjusted performance measures to gauge the profitability of shorting 30-day variance swaps. They facilitate the opportunity to compare the strategy of shorting 30-day variance swaps to any other investment strategy and to compare my results to those found in previous studies. Additionally, the calculation of the risk adjusted performance measures of this strategy serve as a starting point for the discussion of how the market prices variance risk.

4.6.1 The Sharpe ratio

As first developed by Sharpe [1966] the Sharpe ratio is perhaps the best known risk adjusted performance measure. It is defined as the excess return on an asset over the risk free rate divided by the standard deviation of the asset returns distribution. Assuming investors have mean-variance preferences they will choose the strategy with the highest Sharpe ratio if given the choice between strategy A and B (not allowing for a mix of them).

\[ SR = \frac{RP_{\log, \text{discrete}}}{\sigma_{RP}^{\text{NW}}} \sqrt{\frac{365}{30}} \]  
(4.18)

Where \( RP \) is the average variance risk premium in either log or discrete return terms, as defined in 4.16 and 4.15, and \( \sigma_{RP}^{\text{NW}} \) is standard deviations estimated with the approach of
Newey and West [1987] with a maximum lag-length equal 30-days, the maturity of each
swap in calendar days in order to adjust for the autocorrelation caused by the overlap in
observations. The multiplication with $\sqrt{\frac{365}{30}}$ is done to obtain annualised Sharpe ratios.

4.6.2 Adjusting the Sharpe ratio for higher moments

When ranking different investment performances the standard measure of the Sharpe ratio
is limited to returns that are normally distributed. It is intuitive to think that investors
are averse not only to a high volatility but also to negative skewness and excess kurtosis.
Skewness, known as the third moment of a distribution, describes the asymmetry in a
return distribution. Investors would be averse to negative skewness in the returns since a
negatively skewed distribution occurs when there is more or less frequent observations of
extreme negative outcomes without the same observations of extreme positive outcomes
to "offset" the negative ones. Kurtosis known as the fourth moment, measures the peaked-
ness of a distribution. Normally distributed returns have a kurtosis coefficient of 3, excess
kurtosis is kurtosis in excess of 3 and indicates a distribution that has a higher peak than
the normal. Investors would be averse to excess kurtosis since a higher probability mass
just around the mean indicates that more of the variance is due to infrequent extreme
observations. This means that extremely low and high returns has a higher probability of
occurrence than that dictated by a normal distribution with the same variance. For this
reasons I will calculate the Sharpe ratio adjusted for the third and forth moment in case
I find significant evidence of skewness or excess kurtosis.

The Sharpe ratio adjusted for higher moments as described in Alexander [2008a] p. 260
is calculated in the following way

$$\text{ASR}_2 = \text{SR} + \left( \frac{\mu_3}{6} \right) \text{SR}^2 - \left( \frac{\mu_4}{24} \right) \text{SR}^3$$

(4.19)

where SR is the standard Sharpe ratio, $\mu_3$ is the estimated skewness and $\mu_4$ is the excess
kurtosis.

11 This is the exact procedure applied by both Trolle and Schwartz [2009] and Carr and Wu [2007]. The
Newey West consistent standard deviations is found using the statistics software program Stata.
5 Empirical Results

5.1 Properties of variation

Appendix B displays the time series of realised volatility and the respective volatility indices for all the indices I examine. The displays of the equity indices clearly shows how equity volatility reacts to corporate and geopolitical events. Across the board we see how the volatility indices rapidly increases and peaks at very high levels after the September 2011 terrorist attacks, the WorldCom bankruptcy in July 2002 and the bankruptcy of Lehman Brothers in September 2008. The time-series on Nasdaq volatility displays a very high volatility in general from around March 2000 when the index peaked until the end of 2002 when it bottomed. For the United States Oil fund ETF (USO), where I only have data from May 2007 till May 2011, we also see the extreme levels of volatility in oil futures prices leading up to and after the culmination of the financial crisis.

5.2 Variance risk premium

I calculate the three measures of the 30-day variance risk premium for S&P 500, Nasdaq 100, Stoxx 50, SMI, FTSE, DAX equity indices based in the period from January 2000 till May 2011 a total of 2956 trading days. In addition, I calculate the same three measures on the variance risk premium for the United States Oil Fund (USO) in the period from May 2007 till May 2011 a total of 1014 trading days.

Tables D.1, D.2 and D.3 summarizes the descriptive statistics on the 30-day log, discrete and payoff measures of the variance risk premium respectively. Reported t-statistics is the hetroscedastic and autocorrelation robust (HACSE) t-statistics obtained in Oxmetrics under the null-hypothesis of zero risk premium. The time series of the risk premiums have been tested for hetroscedasticity and autocorrelation using the Box Pierce test and rejected their respective null-hypothesis for all markets on all three measures of the variance risk premium. For the equity indices it is clear from the tables that the variance risk premium is significantly negative on all three measures. The average log variance risk premium (table E.1) is most negative for the Nasdaq 100 (-58.1 %) followed by S&P 500 (-48.6%), FTSE 100(-48,2%), Euro Stoxx 50 (-44,6%), SMI (-41,8%) and DAX (-35.3%). The mean variance payoff is also most negative for the Nasdaq 100 (-5.07) followed by Euro Stoxx 50 (-1.65), FTSE 100 (-1.26), DAX (-1.11), S&P 500 (-1.09) and SMI (-0.91). The reason why the rank of the indices change when you compare these two measures is that the distribution of the returns are very different between the indices. The mean log variance risk premium in the oil market is the most negative of all assets with a mean log risk premium of -63.1%. Considering the payoff measure it is the second most negative asset
with a mean payoff of -3.42. The discrete variance risk premiums are also highly negative and significant in all markets with the exception of the USO index. For the USO index I find a relatively small and insignificant positive variance risk premium of 5.5% in discrete terms. The reason for this is that the asymmetric return properties are much more visible and the sample mean of the discrete measure is being pulled up by observations of extreme positive returns. We also see from table D.2 that the same effect makes the sample mean of the discrete variance risk premium less negative than the logarithmic variance risk premium. In short the distribution of the discrete measure is very positively skewed. The figures in appendix A displays the time series of the log variance risk premium. The negative mean as well as the high variation and extreme peaks in both positive and negative direction are well described in these figures. Take note of the September 2011 terrorist attacks, the WorldCom bankruptcy in July 2002 and the bankruptcy of Lehman Brothers in September 2008. These events all causes observations of extreme positive returns from longing variance swaps in logarithmic terms. In the tables of the descriptive statistics we understand that this is even more true in discrete terms.

5.3 Return distributions

In appendix C the histogram of the variance risk premium are presented both in terms of the log and discrete variance risk premium measure. Remembering that these two measures of the variance risk premium also serves as measures of the return from investing in variance swaps these histograms displays the return distributions from long positions in 30-day variance swaps. It is clear from visual inspection of any of these histograms that neither the distribution of the log variance risk premium or the discrete variance risk premium is normally distributed. In tables D.1 and D.2 the descriptive statistics of both measures are presented respectively. The distributions is positively skewed and highly peaked for all assets except USO which exhibits a negative skew in log variance risk premium terms. I test for normality using the Jarque-Berra test and the results reported in tables D.1 and D.2 show that none of the distributions are normally distributed at any significance level. The distributions of the log variance risk premium is much closer to normal, but are nonetheless rejected by the Jarque-Berra test. As a part of the Jarque-Berra test I also separately test if the skewness and excess kurtosis coefficients are different from zero. The test fails to reject the null of zero excess kurtosis at the 5% level of the log variance risk premium measure of the USO index. In all other markets the null hypotheses of zero skew and excess kurtosis are rejected at any significance level.
5.4 Sharpe ratios from shorting 30-day variance swaps

In tables D.4 and D.5 annualised Sharpe ratios from shorting variance swaps considering the log and discrete return measures are reported respectively. The reported number in the row "Sharpe ratio" is the measure described in 4.6.1 while the row "Adjusted Sharpe ratio" refers to the Sharpe adjusted for higher moments described in 4.6.2. As I have explained earlier the standard measure of the Sharpe ratio is limited to returns that are normally distributed. Since I have found that my returns are far from normal, considering any measure, the attractiveness of selling variance on these indices should be ranked according to the Sharpe ratio that adjusts for higher moments. From table D.4 we see that it has been most attractive to sell variance on the Nasdaq index and least attractive to sell variance on the DAX index with adjusted Sharpe ratios of 0.762 and 0.497 respectively. In table D.5 when discrete returns from shorting variance swaps is considered, the asymmetry of the returns become way more pronounced which we can see from the larger difference between the standard Sharpe ratio measure and the adjusted Sharpe. On this measure we see that selling variance on the Nasdaq index is the least attractive of the equity indices with an adjusted Sharpe ratio of 0.0146. Here the most attractive strategy have been to sell variance on the FTSE 100 index with an adjusted Sharpe ratio of 0.216. To put these numbers into perspective Carr and Wu [2007] report unadjusted Sharpe ratios in log return terms of 0.98 and 0.55 from selling variance on the S&P 500 and Nasdaq 100 from January 1996 to February 2003. Trolle and Schwartz [2009] find a Sharpe ratio of 0.59 from selling variance swaps on oil. In the oil market I find a Sharpe ratio of 0.49 in log return terms. To put these numbers into further perspective a large investment portfolio may typically be assumed to have an expected excess return in the range of 3 to 6% p.a over some fairly long investment horizon, say 5-10 years, the expected standard deviation may be in the range of 15 to 30 % p.a. This gives annualised Sharpe ratios between 0.1 and 0.4. Clearly, in the context of Sharpe ratios selling equity index and oil variance seems to be a very attractive investment strategy. However, one should be careful when comparing investment strategies based on Sharpe ratio alone, particularly when the return distributions are asymmetric like they typically are for derivative securities. I have tried to account for this in calculating the adjusted Sharpe ratio measure that accounts for skewness and excess kurtosis. In log return terms, where the asymmetry is less visible, selling variance still appears to be a superior investment strategy. This is less true when returns are considered in discrete terms where the asymmetric payoff is really pronounced. In discrete terms and adjusting for skewness and excess kurtosis the Sharpe ratios are more comparable to those obtained from holding a large diversified investment portfolio. By construction, log returns will always be lower than its discrete counterpart. In addition, the Sharpe ratios are suppressed by the much more visible asymmetry of discrete returns. To me, it makes a lot more sense...
to measure the attractiveness of selling variance in terms of discrete returns. I really do not understand why previous research focuses so heavily on the log return measure, one claim is that its return distribution conforms more to the normal. Indeed it does, and as a consequence one may be led to believe that superior investment performances may be obtained by selling variance swaps. To me the log transformation underestimates the variance risk premium and in a way hides the true distribution of the returns. I find Sharpe ratios in the range of -0.04 to 0.216 considering discrete returns and adjusting for the asymmetric return distributions I find from selling variance on equity indices and oil futures. Assuming that the returns of a large investment portfolio are much closer to being normally distributed this is comparable to Sharpe ratios one might typically expect from holding such a portfolio over a fairly long period. Given its asymmetric nature and assuming that this asymmetry is priced in the market the market seems to charge a premium of variance risk that is less extreme than previous research suggest. It is in line with the risk premiums charged by investors in other more well known markets and taken this as a fair price of risk the highly asymmetric nature of variance risk helps to rationalise the way variance risk is priced by the market.

5.5 Time-variation in variance risk premium

As in [Carr and Wu 2007] and [Trolle and Schwartz 2009] I test for time-variation in the variance risk premium by running the following two regressions:

\[
\log RV(t, T) = a + b \log K^2(t, T) + \epsilon
\]  

(5.1)

and

\[
RV(t, T) = a + bK^2(t, T) + \epsilon
\]  

(5.2)

The regressions are estimated by OLS in Oxmetrics, the results are reported in table E.8. As they explain in the above mentioned papers a constant variance risk premium in either payoff or log terms would imply that the slope coefficient (b) in both of these models is one. Furthermore, absence of variance risk premium in either payoff or log terms would imply that the constant term (a) in these regressions are zero. The tested hypothesis are \(a=0\) and \(b=1\). Reported t-statistics and p-values are computed from the heteroscedastic and autocorrelation robust standard errors reported in Oxmetrics after having tested for the presence of both using the Box Pierce test. I find that the slope coefficients (b) of (5.2) are significantly less than one in all markets except for the S&P 500. When the slope
coefficient of (5.2) is less than one it means that the absolute magnitude of the variance risk premium is non-constant or time-varying and negatively correlated with the level of the variance swap rate. In this case the absolute magnitude of the variance risk premium tends to become more negative when the variance swap rate increases. The results from (5.1) where we regress the log of the realised variance against the log of the variance swap indicates a non time-varying risk premium in log terms. The slope coefficient (b) of (5.1) are much closer to one and significantly different from one only in the case of DAX and S&P 500. This indicates that the variance risk premium in log terms are much closer to being a constant time-series than the absolute magnitude of the risk premium and that there is a much weaker correlation between the log of realised variance and the log of the variance swap rate.

5.6 CAPM regressions

Under the classical asset pricing model (CAPM) investors only prices market risk since all other risk sources are assumed to be diversified away by holding a large set of uncorrelated assets. In this setting the expected excess return of an asset can be fully explained by the beta of an asset on the market portfolio as \( R_i^e = \beta_i R_M^e \). Where beta is defined as \( \beta = \frac{Cov(R_i^e, R_M^e)}{\sigma_M^2} \). Presumably buyers of variance swaps are willing to accept a negative expected return since variance swap returns are negatively correlated to index returns. Being long a variance swap should then bring beneficial diversification effects outweighing its negative expected return, if this is the case estimated betas must become very negative. Table E.5 lists the historical correlations between excess index returns and log variance swap returns. The correlations are all negative with the correlation between S&P 500 excess return and log variance swap return on the S&P as the most negative with a correlation coefficient of -0.182. I make an attempt to investigate if CAPM are able to explain the negative expected return of being long variance swaps by running the regressions:

\[
\log \left( \frac{RV_i(t,T)}{K_{i,T}} \right) = \alpha_i + \beta_i R_M^e(t,T) + \epsilon_i
\]

I have used the respective underlying indices as market proxies in each CAPM regressions. This is not consistent with CAPM theory which states that the market portfolio is the value-weighted portfolio of all assets. Nevertheless, running these regressions give an indication as to whether or not the negative expected return on variance swaps may be explained by the negative correlation between variance swap and index returns. As can be seen in table E.7, I find negative and highly significant intercept and slope coefficients for the regression in equation (5.3) in all markets. The alpha coefficients are almost identical to the unconditional means of the log variance risk premium reported in table 27.
For some markets they are more negative, in others they are less negative. In the CAPM setting this indicates that variance risk is overpriced by the market. It also tells us that the negative Beta coefficients cannot fully account for the negative variance risk premiums, even though it predicts the correct sign of the risk premium. In the CAPM setting the abnormal returns made from selling variance risk should quickly attract heavy selling to bring its price down to equilibrium. This does not seem to happen and CAPM fails to explain the magnitude of the variance risk premium. The results I find are in large very similar to those found by Carr and Wu [2007]. They conclude that either there is a large inefficiency in the market for variance or else the majority of the variance risk is generated by an independent risk factor that the market prices heavily. For CAPM to hold it relies on some strict assumptions among them that returns are normally distributed. Above I concluded that none of the return distributions of the log variance risk premium are normally distributed. This may well be one of the main reasons to why CAPM fails so heavily in explaining the variance risk premium.

\[12\]

In addition, they also try to explain the variance risk premium with the Fama-French three factor model and end up concluding much in the same way.
6 Conclusion

In this thesis I use the notion of a variance swap, a derivative security that pays off the difference between market implied variance and subsequent realised variance, to analyse and quantify the statistical properties of the variance risk premium in equity and oil markets. I describe how the square of listed volatility indices is a good approximation of the fair variance swap rate and use the square of listed volatility indices as an approximation of the 30-day fair variance swap rate. Since the fair variance swap rate represents the markets risk neutral expectation of future realised variance I calculate the variance risk premium as the difference between the fair swap rate and subsequent realised variance. In all I calculate three different measures of the variance risk premium; the log variance risk premium which is the sample average return of a long variance swap in logarithmic terms, the discrete variance risk premium which is the sample average discrete return of a long variance swap and the variance risk premium in payoff terms which is the sample average in monetary units of a long variance swap with a notional of 100. I find significantly negative 30-day variance risk premiums on the S&P 500, Nasdaq 100, Euro Stoxx 50, FTSE 100, SMI and DAX equity indices considering any of these three measures. I also apply the same methodology to the United States Oil Exchange Traded Fund (ETF) and find significantly negative 30-day log and payoff variance risk premium but a insignificant positive discrete variance risk premium.

I also find that the historical distribution of the variance risk premiums are closer to normal in logarithmic terms than in discrete terms but that none of them can be said to be normally distributed based on a Jarque Berra test of normality. In discrete terms the variance risk premiums are extremely asymmetric and highly peaked. Given that the variance risk premiums are negative it is possible to make positive returns by short selling variance swaps. Similar to other studies on the variance risk premium I investigate if the historical returns of this strategy is in proportion to its risk by calculating annualised Sharpe ratios. Since I find that the return distribution of this strategy is highly peaked and negatively skewed I extend the classical measure of the Sharpe ratio to adjust for higher moments, assuming that investors are averse not only to a high variation in the returns. In discrete return terms and adjusting for higher moments the strategy of short selling variance swaps seems less superior than previous studies suggests. Adjusting for higher moments I find annualised discrete return Sharpe ratios of 0.015, 0.13, 0.16, 0.18, 0.21, 0.22 from shorting 30-day variance swaps on the Nasdaq 100, SMI, DAX, S&P 500, Euro Stoxx 50 and FTSE 100 equity indices respectively. Contrary to what other studies suggests the risk adjusted performance I find from selling short dated equity index variance swaps seems to be more in line with the risk of the strategy and not much different from what one might expect from holding a well diversified investment portfolio. The actual
profitability of this strategy is of course also dependent upon several other important factors not considered in this thesis such as bid-ask spreads and other transaction costs. Assuming that investors are averse to highly variate, peaked and negatively skewed return distributions my results rationalize the way variance risk is priced by the market.

To better understand the dynamic properties of the variance risk premiums I check for time variation by regressing the log of realised variance on the log of the variance swap rate and by regressing the realised variance on the variance swap rate. I find that the variance risk premium in log return terms are more of a constant time-series than the absolute magnitude of the variance risk premium. The absolute magnitude of the variance risk premiums are time varying and negatively correlated with the variance swap rate. This suggests that the market re-evaluates the risk of variance itself and that observations of high variance risk premiums in absolute terms are often followed by observations of negative risk premiums.

When I investigate whether the classical asset pricing model (CAPM) are able to explain the observed negative log variance risk premium I find that the negative correlation between index returns and the log variance risk premiums generate highly negative and significant Beta coefficients. Nevertheless these negative Beta coefficients fails to explain the magnitude of the log variance risk premiums and I find alpha coefficients that are almost identical to the unconditional mean of the log variance risk premium. CAPM assumes returns to be normally distributed and similar to what my analysis of the Sharpe ratios suggest I suspect that this is one of the main reasons to why CAPM fails so much in explaining the size of the log variance risk premiums. I argue that it is logical to assume that investors are averse to both highly peaked and negatively skewed return distributions, and that this risk is priced in the variance swap market. Since CAPM fails to account for this it underestimates the size of the variance risk premiums derived from the return of variance swaps. Since I have focused on making my study comparable to previous studies one shortfall of my study is that I do not check if CAPM are able to explain the discrete variance risk premiums.

Since the negative relationship between index returns and and variance alone seems incapable of explaining the variance risk premiums, future research should focus on how investors prices asymmetric return distributions and how this may further rationalise how the market prices variance risk. Additionally one must test even more if the variance risk premiums may be explained by systematic fundamental factors. Another interesting topic is to figure out if variation based derivatives should be included in long term investment portfolios. Furthermore it could also be interesting to investigate how variation based contracts could or should be used to manage risk and investments.
References


A Figures of Time series of variance variance swap returns

A.1 Time series of S&P 500 variance swap returns

Figure A.1: Time series of $\log\left( \frac{RV_{t,T}}{VIX_t^2} \right)$, the log excess return on a 30-day variance swap on the S&P 500 index. Calculated from daily data on VIX and S&P 500 2000:1 - 2011:5
A.2 Time series of Nasdaq 100 variance swap returns

Figure A.2: Time series of $\log\left(\frac{RV_{t,T}}{VXN_t^2}\right)$, the log excess return on a 30-day variance swap on the Nasdaq 100 index. Calculated from daily data on VXN and Nasdaq 100 2000:1 - 2011:5
A.3 Time series of Stoxx 50 variance swap returns

Figure A.3: Time series of $\log \left( \frac{RV_{t,T}}{V_{Stoxx_t^2}} \right)$, the log excess return on a 30-day variance swap on the Stoxx 50 index. Calculated from daily data on VStoxx and Stoxx 50 2000:1 - 2011:5
A.4 Time series of FTSE 100 variance swap returns

Figure A.4: Time series of $\log \left( \frac{RV_{t,T}}{VFTSE_t^2} \right)$, the log excess return on a 30-day variance swap on the FTSE 100 index. Calculated from daily data on VFTSE and FTSE 100 2000:1 - 2011:5
A.5 Time series of SMI index variance swap returns

Figure A.5: Time series of $\log\left(\frac{RV_{t,T}}{VSMI^2_t}\right)$, the log excess return on a 30-day variance swap on the VSMI index. Calculated from daily data on SMI and VSMI 50 2000:1 - 2011:5
A.6 Time series of DAX variance swap returns

Figure A.6: Time series of $\log\left(\frac{RV_{t,T}}{VDAX_{t}^{2}}\right)$, the log excess return on a 30-day variance swap on the DAX index. Calculated from daily data on VDAX and DAX 50 2000:1 - 2011:5
A.7 Time series of USO variance swap returns

Figure A.7: Time series of $\log \left( \frac{RV_{t,T}}{O XV_{t}^{2}} \right)$, the log excess return on a 30-day variance swap on the USO index. Calculated from daily data on OXV and USO 2007:5 - 2011:5
B Time series of volatility indices and realised volatility

B.1 Time series of VIX and realised volatility for S&P 500

Figure B.1: VIX (red solid) and subsequent realised volatility (blue dashed). Notice the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VIX^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.
B.2 Time series of VXN and realised volatility for Nasdaq 100

Figure B.2: VXN (red solid) and subsequent realised volatility (blue dashed). The Nasdaq peaked in March 2000 and the uncertainty that followed the dot-com burst is well described in this time series. Notice also here the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VXN^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.
B.3 Time series of VStoxx and realised volatility for Stoxx50

Figure B.3: VStoxx (red solid) and subsequent realised volatility (blue dashed). Notice the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VStoxx^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.
B.4 Time series of VFTSE and realised volatility on FTSE 100

Figure B.4: VFTSE (red solid) and subsequent realised volatility (blue dashed). Notice the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VFTSE^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section [4] this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.
### B.5 Time series of VSMI and realised volatility on SMI

![Figure B.5: VSMI (red solid) and subsequent realised volatility (blue dashed). Notice the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VSMI^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.](image-url)
B.6 Time series of VDAX and realised volatility on DAX

Figure B.6: VDAX (red solid) and subsequent realised volatility (blue dashed). Notice the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $VIX^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative.
B.7 Time series of OXV and realised volatility on USO 2007:5 - 2011:5

Figure B.7: OXV (red solid) and subsequent realised volatility (blue dashed). Notice, also for oil, the unprecedented levels of implied and realised volatility sparked by the collapse of Lehman Brothers in late 2008. Note that throughout my thesis $OXV^2$ is taken as a proxy for the fair variance rate so that using my methodology, that is thoroughly explained in section 4, this figure is a time series of expected future 30-day volatility in annualized terms (red solid line) and subsequent realised 30-day volatility. When, as is most often the case, the red solid line lies above the blue dashed line the return from a long variance swap is negative and the variance risk premium is negative by any measure.
C Histograms of discrete and logarithmic returns on long 30-day variance swaps

C.0.1 Histogram of returns on 30-day variance swaps S&P 500

Figure C.1: Histogram of returns on variance swaps S&P 500, based on daily data 2000:1 - 2011:5.

Top: Histogram of $\log\left(\frac{RV_{t,T}}{VIX_t^2}\right)$. Bottom: Histogram of $\frac{(RV_{t,T} - VIX_t^2)}{VIX_t^2}$. The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal.
C.0.2 Histogram of returns on 30-day variance swaps Nasdaq 100

Figure C.2: Histogram of returns on variance swaps Nasdaq 100, based on daily data 2000:1 - 2011:5.

Top: Histogram of \( \log \left( \frac{RV_{t,T}}{VXN_T^2} \right) \). Bottom: Histogram of \( \frac{(RV_{t,T} - VXN_T^2)}{VXN_T^2} \). The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal distribution.
C.0.3 Histogram of returns on 30-day variance swaps Stoxx 50

Figure C.3: Histogram of returns on variance swaps Stoxx 50, based on daily data 2000:1 - 2011:5.
Top: Histogram of $\log \left( \frac{RV_{t,T}}{V_{Stoxx_t}} \right)$. Bottom: Histogram of $\frac{(RV_{t,T} - V_{Stoxx_t^2})}{V_{Stoxx_t^2}}$. The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal.
C.0.4 Histogram of returns on 30-day variance swaps FTSE 100

Figure C.4: Histogram of returns on variance swaps FTSE 100, based on daily data 2000:1 - 2011:5.

Top: Histogram of \( \log \left( \frac{R_{V_{t,T}}}{\sqrt{V_{FTSE_t}^2}} \right) \). Bottom: Histogram of \( \frac{(R_{V_{t,T}} - V_{FTSE_t}^2)}{V_{FTSE_t}^2} \). The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal.
Figure C.5: Histogram of returns on variance swaps SMI, based on daily data 2000:1 - 2011:5.

Top: Histogram of $\log\left(\frac{RV_{t,T}}{VSMI_t^2}\right)$. Bottom: Histogram of $\frac{(RV_{t,T} - VSMI_t^2)}{VSMI_t^2}$. The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal.
C.0.6 Histogram of returns on 30-day variance swaps DAX

Figure C.6: Histogram of returns on variance swaps DAX, based on daily data 2000:1 - 2011:5.

Top: Histogram of $\log\left(\frac{RV_{l,T}}{VDAX_t^2}\right)$. Bottom: Histogram of $\frac{(RV_{l,T} - VDAX_t^2)}{VDAX_t^2}$. The returns are highly negative on average, but positively skewed (observations of extreme positive returns). The returns are excess returns over the risk free rate and in annualized terms. The distribution of log variance risk premium (top) is closer to the normal.
C.0.7 Histogram of returns on 30-day variance swaps USO

Figure C.7: Histogram of returns on variance swaps based on daily data of USO and OXV 2007:5 - 2011:5.

Top: Histogram of $\log\left(\frac{RV_{t,T}}{OXV_t^2}\right)$. Bottom: Histogram of $\frac{(RV_{t,T} - OXV_t^2)}{OXV_t^2}$. 
# D Empirical Results and Analysis tables

Table D.1: Descriptive statistics 30-day log variance risk premium in annualized terms. T-Statistic HACSE refers to the hetroscedastic and autocorrelation consistent t-test performed on the mean variance risk premium ($H_0=$Mean variance risk premium is zero).

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>Nasdaq</th>
<th>Stoxx 50</th>
<th>FTSE 100</th>
<th>SMI</th>
<th>DAX</th>
<th>USO</th>
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<td>-0.212</td>
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</table>

Table D.2: Descriptive statistics 30-day discrete variance risk premium in annualized terms. T-Statistic HACSE refers to the hetroscedastic and autocorrelation consistent t-test performed on the mean variance risk premium ($H_0=$Mean variance risk premium is zero).

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
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<th>SMI</th>
<th>DAX</th>
<th>USO</th>
</tr>
</thead>
<tbody>
<tr>
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<td>183.76</td>
<td>213.53</td>
<td>225.2</td>
<td>96.18</td>
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Table D.3: Descriptive statistics variance swap payoff. The estimated payoff is \( (RV_{t,T} - K_{t,T}) \times 100 \). So the mean here for each index represents the average payoff on a 30-day variance swap in the sample period. T-statistics is the HACSE robust t-statistics reported by oxmetrics under the null of a mean = 0.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
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<th>SMI</th>
<th>DAX</th>
<th>USO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>-45.786</td>
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<td>55.545</td>
<td>51.385</td>
<td>50.7</td>
<td>49.173</td>
<td>47.682</td>
<td>50.89</td>
<td>34.806</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>5.41</td>
<td>9.109</td>
<td>5.889</td>
<td>5.117</td>
<td>5.421</td>
<td>5.67</td>
<td>8.106</td>
</tr>
<tr>
<td>Skewness</td>
<td>5.429</td>
<td>-0.136</td>
<td>3.097</td>
<td>4.107</td>
<td>3.012</td>
<td>3.346</td>
<td>0.555</td>
</tr>
<tr>
<td>t-test, p-value, HACSE</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Jarque-Berra test, p-values</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Skewness, t-test, t-statistic</td>
<td>120.57</td>
<td>3.0222</td>
<td>68.785</td>
<td>91.212</td>
<td>66.898</td>
<td>74.308</td>
<td>8.7</td>
</tr>
<tr>
<td>Ex. Kurtosis t- test, t- statistic</td>
<td>545.42</td>
<td>93.043</td>
<td>266.33</td>
<td>401.05</td>
<td>320.75</td>
<td>281.77</td>
<td>25.489</td>
</tr>
</tbody>
</table>

Table D.4: Annualised Sharpe ratios from shorting variance swaps considering the log return measure. The measure Sharpe ratio refers to the standard Sharpe ratio described in sec. 4.6.1. Adjusted Sharpe ratios are the Sharpe ratios adjusted for higher moments as described in sec. 4.6.2.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>Nasdaq</th>
<th>Stoxx 50</th>
<th>FTSE 100</th>
<th>SMI</th>
<th>DAX</th>
<th>USO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>48.60%</td>
<td>58.10%</td>
<td>44.60%</td>
<td>48.20%</td>
<td>41.80%</td>
<td>35.30%</td>
<td>63.10%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>235.4%</td>
<td>205.6%</td>
<td>226.9%</td>
<td>233.2%</td>
<td>257.0%</td>
<td>224.6%</td>
<td>311.5%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.672</td>
<td>-0.798</td>
<td>-0.95</td>
<td>-1.096</td>
<td>-0.997</td>
<td>-0.791</td>
<td>0.212</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>1.647</td>
<td>2.377</td>
<td>1.892</td>
<td>1.759</td>
<td>1.702</td>
<td>1.673</td>
<td>0.5317</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.72017</td>
<td>0.98589</td>
<td>0.685484</td>
<td>0.721092</td>
<td>0.567291</td>
<td>0.548321</td>
<td>0.706559</td>
</tr>
<tr>
<td>Adjusted Sharpe ratio</td>
<td>0.63645</td>
<td>0.761709</td>
<td>0.585693</td>
<td>0.598629</td>
<td>0.500868</td>
<td>0.497193</td>
<td>0.716384</td>
</tr>
</tbody>
</table>

Table D.5: Risk adjusted performance from shorting variance swaps (discrete) Annualised Sharpe ratios from shorting variance swaps considering the discrete return measure. The measure Sharpe ratio refers to the standard Sharpe ratio described in sec. 4.6.1. Adjusted Sharpe ratios are the Sharpe ratios adjusted for higher moments as described in sec. 4.6.2.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>Nasdaq</th>
<th>Stoxx 50</th>
<th>FTSE 100</th>
<th>SMI</th>
<th>DAX</th>
<th>USO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>24.90%</td>
<td>34.30%</td>
<td>21.70%</td>
<td>23.20%</td>
<td>14.70%</td>
<td>15.30%</td>
<td>-5.50%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>279.1%</td>
<td>219.2%</td>
<td>277.4%</td>
<td>289.8%</td>
<td>361.7%</td>
<td>280.8%</td>
<td>344.2%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.311137</td>
<td>0.545885</td>
<td>0.272847</td>
<td>0.279255</td>
<td>0.141764</td>
<td>0.190074</td>
<td>-0.05574</td>
</tr>
<tr>
<td>Adjusted Sharpe ratio</td>
<td>0.183861</td>
<td>0.014645</td>
<td>0.208501</td>
<td>0.216271</td>
<td>0.126205</td>
<td>0.160357</td>
<td>-0.05737</td>
</tr>
</tbody>
</table>

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Table D.6: Historical correlations between excess return on variance swaps and excess index returns. Excess return on variance swaps (denoted VS) are along the top line. Excess index return are found in rows 2:7 in the first column.

<table>
<thead>
<tr>
<th></th>
<th>VS S&amp;P 500</th>
<th>VS Nasdaq</th>
<th>VS Stoxx 50</th>
<th>VS SMI</th>
<th>VS FTSE 100</th>
<th>VS DAX</th>
<th>VS USO</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>-0.182</td>
<td>-0.153</td>
<td>-0.179</td>
<td>-0.172</td>
<td>-0.173</td>
<td>-0.169</td>
<td>-0.115</td>
</tr>
<tr>
<td>Nasdaq</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stoxx 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTSE 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table D.7: Correlations between log variance swap returns on the different indices. Highly correlated across the board. Higher correlations between indices on the same continent.

<table>
<thead>
<tr>
<th></th>
<th>VS S&amp;P 500</th>
<th>VS Nasdaq</th>
<th>VS Stoxx 50</th>
<th>VS SMI</th>
<th>VS FTSE</th>
<th>VS DAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>VS S&amp;P 500</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS Nasdaq</td>
<td>0.833</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS Stoxx 50</td>
<td>0.737</td>
<td>0.593</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS SMI</td>
<td>0.666</td>
<td>0.505</td>
<td>0.804</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS FTSE</td>
<td>0.724</td>
<td>0.614</td>
<td>0.861</td>
<td>0.808</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>VS DAX</td>
<td>0.702</td>
<td>0.534</td>
<td>0.926</td>
<td>0.769</td>
<td>0.786</td>
<td>1</td>
</tr>
</tbody>
</table>

Table D.8: CAPM regression results. The Log excess return from investing in variance swaps (Log variance risk premium) is regressed on the excess return from investing in their underlying index. T-statistics and p-values are computed from heteroscedastic and autocorrelation (HAC) robust standard errors reported in Oxmetrics. Regressions are estimated by OLS in Oxmetrics.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>Nasdaq</th>
<th>Stoxx 50</th>
<th>SMI</th>
<th>FTSE</th>
<th>DAX</th>
<th>USO</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha coefficient</td>
<td>-0.478</td>
<td>-0.581</td>
<td>-0.461</td>
<td>-0.423</td>
<td>-0.474</td>
<td>-0.338</td>
<td>-0.641</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.003</td>
</tr>
<tr>
<td>$R^2$</td>
<td>23.9%</td>
<td>14.0%</td>
<td>28.9%</td>
<td>24.1%</td>
<td>27.9%</td>
<td>26.9%</td>
<td>6.23%</td>
</tr>
</tbody>
</table>
Table D.9: Summary of regression results time-variation in variance risk premium. $K(t,T)$ is the variance swap rate and $RV(t,T)$ is the realised variance estimated as described in section [4]. T-statistics and p-values under the null hypotheses of $a=0$ and $b=1$ are computed from HAC standard errors reported in Oxmetrics. Regressions are estimated by OLS in Oxmetrics.

<table>
<thead>
<tr>
<th></th>
<th>( \log RV(t,T) = a + b \log K(t,T) + \epsilon )</th>
<th>$R^2$</th>
<th>( RV(t,T) = a + bK(t,T) + \epsilon )</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>\begin{align*} a &amp;= -1.005, \quad b = 1.087, \quad R^2 = 0.679 \ t-stat, HACSE &amp;= -5.201, \quad b = 2.695, \quad p-values, HACSE = 0.0000, \quad b = 0.0035 \end{align*}</td>
<td>-0.003</td>
<td>\begin{align*} a &amp;= 0.951, \quad b = 0.5976 \quad R^2 = 0.539 \ t-stat, HACSE &amp;= 2.695, \quad b = -0.211, \quad p-values, HACSE = 0.0035 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>Nasdaq</td>
<td>\begin{align*} a &amp;= -0.277, \quad b = 0.956, \quad R^2 = 0.765 \ t-stat, HACSE &amp;= -1.635, \quad b = -0.574, \quad p-values, HACSE = 0.1022, \quad b = 0.5660 \end{align*}</td>
<td>0.0263</td>
<td>\begin{align*} a &amp;= 0.611, \quad b = 0.693, \quad R^2 = 0.579 \ t-stat, HACSE &amp;= -0.574, \quad b = -9.796, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>Stoxx 50</td>
<td>\begin{align*} a &amp;= -0.791, \quad b = 1.055, \quad R^2 = 0.666 \ t-stat, HACSE &amp;= -4.207, \quad b = 1.895, \quad p-values, HACSE = 0.0000, \quad b = 0.0583 \end{align*}</td>
<td>0.0221</td>
<td>\begin{align*} a &amp;= 0.766, \quad b = 0.693, \quad R^2 = 0.503 \ t-stat, HACSE &amp;= 0.766, \quad b = -5.223, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>FTSE 100</td>
<td>\begin{align*} a &amp;= -0.220, \quad b = 1.035, \quad R^2 = 0.6604 \ t-stat, HACSE &amp;= -3.723, \quad b = 1.139, \quad p-values, HACSE = 0.0000, \quad b = 0.2548 \end{align*}</td>
<td>0.0288</td>
<td>\begin{align*} a &amp;= 0.726, \quad b = 0.789, \quad R^2 = 0.420 \ t-stat, HACSE &amp;= 0.766, \quad b = -4.215, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>SMI</td>
<td>\begin{align*} a &amp;= -0.212, \quad b = 0.967, \quad R^2 = 0.575 \ t-stat, HACSE &amp;= -1.043, \quad b = -0.908, \quad p-values, HACSE = 0.0002, \quad b = 0.2548 \end{align*}</td>
<td>0.0892</td>
<td>\begin{align*} a &amp;= 0.644, \quad b = 0.346, \quad R^2 = 0.330 \ t-stat, HACSE &amp;= -9.140, \quad b = -0.140, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>DAX</td>
<td>\begin{align*} a &amp;= -0.704, \quad b = 1.057, \quad R^2 = 0.670 \ t-stat, HACSE &amp;= -3.811, \quad b = 2.002, \quad p-values, HACSE = 0.0000, \quad b = 0.0454 \end{align*}</td>
<td>0.0131</td>
<td>\begin{align*} a &amp;= 0.845, \quad b = 0.427, \quad R^2 = 0.565 \ t-stat, HACSE &amp;= -3.187, \quad b = -0.187, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
<tr>
<td>USO</td>
<td>\begin{align*} a &amp;= -0.430, \quad b = 1.017, \quad R^2 = 0.667 \ t-stat, HACSE &amp;= -1.309, \quad b = 0.4009, \quad p-values, HACSE = 0.1907, \quad b = 0.6885 \end{align*}</td>
<td>-0.203</td>
<td>\begin{align*} a &amp;= 0.880, \quad b = 0.1833, \quad R^2 = 0.781 \ t-stat, HACSE &amp;= -2.569, \quad b = 0.0103, \quad p-values, HACSE = 0.0000 \end{align*}</td>
<td></td>
</tr>
</tbody>
</table>