SEISMIC-HAZARD ASSESSMENT: Conditional Probability

Supplies Needed

• calculator

PURPOSE

Previous exercises in this book have outlined methods for inferring the patterns and history of earthquake activity and faulting. This information is vital for assessing seismic hazard, but in its undigested form, it is not particularly useful to engineers, regional planners, or the general public. Earthquake-hazard is the bridge than connects relatively raw scientific data (fault patterns, slip rates, recurrence intervals, and ages of past earthquakes) with their practical applications. The purpose of this exercise is to illustrate some of the basic principles by which conditional probabilities are calculated.

INTRODUCTION

Conditional probability is defined as the likelihood that a given event – in this case an earthquake – will occur within a specified time period. This likelihood is based on information regarding past earthquakes in a given area and the basic assumption that future seismic activity will follow the pattern of activity observed in the past. Figure 1 is an example of a conditional-probability model for Southern California for the period, 1994-2024 (Working Group on California Earthquake Probabilities, 1995). The model gives the percent probability of a large earthquake during this 30-year period on each fault segment shown. Conditional probability is calculated only for those faults for which geologists have collected enough information to make an informed estimate of seismic hazards. This model predicts a 80-90% likelihood that an earthquake with magnitude equal to or greater than 7.0 will strike somewhere in Southern California before 2024, with the single greatest probability coming from the San Jacinto fault, just east of Los Angeles. This information, along with probabilities of maximum seismic shaking in different locations, is used by architects and engineers in designing structures within acceptable safety margins.

Conditional-probability predictions are only as good as the data used to create them. After the Loma Prieta earthquake struck the Santa Cruz mountains, just southeast of San Francisco, in 1989, some geologists called this a success for the conditional-probability

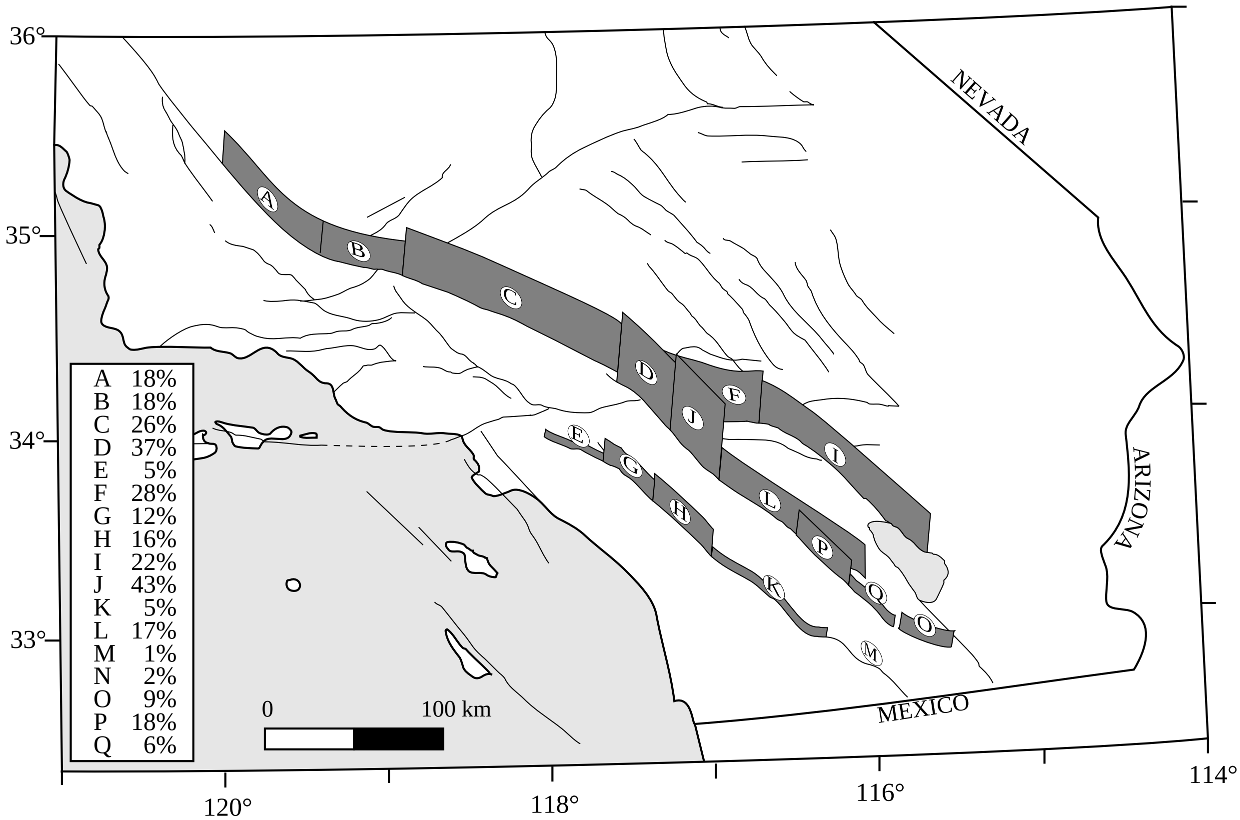

Figure 1. Probabilities of fault rupture for the period 1994 to 2024. Width of the shaded bars indicate percent probability for each fault segment. (After Working Group on California Earthquake Probabilities, 1995)

approach. A probability model published in 1988 had assigned this area a 30% chance of a major earthquake in the subsequent 30 years, the second highest value of any segment of the San Andreas fault. At the same time, however, the same model had called this particular probability "equivocal," assigning it the lowest rating on its reliability scale (an E, on a reliability scale from A to E). Many geologists consider such estimates of reliability to be at least as important as the conditional probability itself.

EARTHQUAKE RECURRENCE

Conditional probability is based on models of how and when earthquakes recur. Previous exercises in this book used various types of geologic data to estimate earthquake *recurrence intervals,* the average time between earthquakes on a given fault. In this exercise, we need to examine this concept a bit more closely. Our understanding of earthquake recurrence is fundamentally based on the *elastic-rebound model,* which states that earthquakes occur when elastic strain along a fault exceeds the strength of the rock. Earthquakes release the strain built up during the preceding years.

Figure 2 illustrates three different examples of earthquake recurrence, based on t different interpretations of the elastic-rebound model. The first one, the *characteristic-earthquake model,* assumes that a given fault segment is characterized by earthquakes with approximately the same magnitudes and amount of slip. Given a constant long-term strain rate, these characteristic earthquakes would occur at approximately equal intervals. In the elastic-rebound model, characteristic earthquakes would occur only where two strict requirements are met:

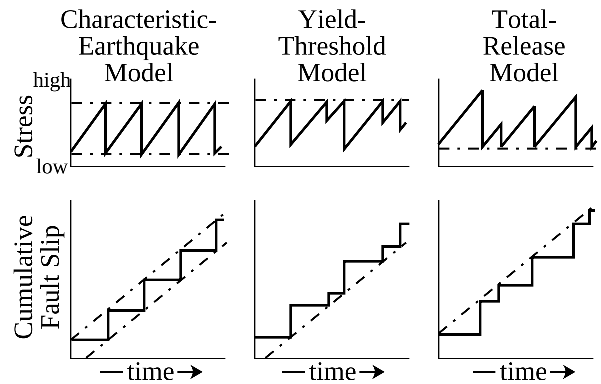

Figure 2. Models of earthquake recurrence. (After Shimazaki and Nakata, 1980)

A) the fault has a constant, predictable strain threshold, and earthquakes occur when strain exceeds that threshold

B) earthquakes on the fault release all accumulated strain.

In the *yield-threshold model,* Requirement A is met, but not Requirement B. In the *total-release model,* Requirement B is met, but not Requirement A. In both of these models, earthquakes recur periodically, but with unequal recurrence intervals.

In both the *yield-threshold* and the *total-release models,* recurrence intervals are not constant, but like exam grades in a large university class, the intervals may follow a predictable distribution (Figure 3). Various statistical distributions can be used, but this exercise assumes a normal (or *Gaussian)* distribution. Remember that a normal distribution can be described by a mean (μ) and a standard deviation (σ); about 68% of all values fall in the range between $\mu-\sigma$ and $\mu+\sigma$, and 95% fall between $\mu-(2\sigma)$ and $\mu+(2\sigma)$. A statistical probability table (Table 1) can be used to find the probability that the next interval between earthquakes will exceed a <u>predicted</u> duration of time (T).

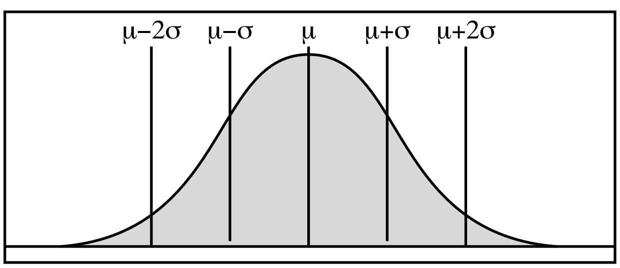

Figure 3. A normal distribution.

Table 1 is simply a list of 300 probabilities, one for each of 300 *normalized values* (N(T) for normalized time intervals). A normalized value is simply a value that is scaled for the mean and standard deviation of that distribution:

$$N(T) = \frac{T - \mu}{\sigma} \qquad (1)$$

For example, $N(\mu+2\sigma) = 2.00$. In order to use Table 1, calculate N(T) to two decimal places, and then find the integer and the first decimal (e.g., 2.0 for N(T)=2.00) on the vertical axis of the table and the second decimal (_._0 for N(T)=2.00) on the horizontal axis. Finally, find the value within the table at the intersection of those two axes (0.023 for N(T)=2.00). Remember that this value is the probability from (0.0 to 0.5) that the actual earthquake recurrence interval will exceed T, the predicted interval.

Table 1. Probabilities that an actual value will exceed a predicted value, based on a normal distribution. Find the first two digits of N(T) on the vertical axis and the last digit on the horizontal axis. For N(x)<0 (values of T less than μ), *subtract* the indicated probability from 1.000.

N(T)	_._0	_._1	_._2	_._3	_._4	_._5	_._6	_._7	_._8	_._9
0.0_	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1_	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.2_	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.3_	0.382	0.378	0.375	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4_	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.5_	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6_	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7_	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.8_	0.212	0.209	0.206	0.203	0.201	0.198	0.195	0.192	0.189	0.187
0.9_	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.0_	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.1_	0.136	0.134	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2_	0.115	0.113	0.111	0.109	0.108	0.106	0.104	0.102	0.100	0.099
1.3_	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.4_	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5_	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6_	0.055	0.054	0.053	0.052	0.051	0.050	0.049	0.048	0.047	0.046
1.7_	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.8_	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.9_	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.0_	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.1_	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2_	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3_	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.4_	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5_	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6_	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7_	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8_	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9_	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001

Example 1:

Find the probability that a fault will rupture in the next 25 years if the fault has ruptured in 1739, 1785, 1832, 1850, 1913, and 1979.

These six earthquakes define five inter-earthquake intervals, 46, 47, 18, 63, and 66 years long. The mean of these intervals is:

$$\mu = \frac{46 + 47 + 18 + 63 + 66}{5} = 48 \text{ years}$$

The standard deviation of these intervals is:

$$\sigma = \frac{|46-48| + |47-48| + |18-48| + |63-48| + |66-48|}{5}$$

$$\sigma = \frac{2 + 1 + 30 + 15 + 18}{5} = 13.2 \text{ years}$$

Thus the recurrence-interval distribution for this fault is 48±13.2 years. If you are doing this exercise in 2006, you know that the current recurrence interval (since 1979) is at least

27 years. The question here is: what is the likelihood that the fault will rupture in the next 25 years (between 2006 and 2031)? The probability that this recurrence interval will fall between 27 and 52 years is:

$$P[27\text{-}52] = (P[27] - P[52]) \div P[27], \tag{2}$$

which simply says that the probability of the recurrence interval falling in the range, 27 to 52 years, equals (the probability of the interval exceeding 27 years minus the probability that it will exceed 52 years) divided by the 27-year probability. You find P(27) and P(52) by finding the normalized values (Equation 1) and then using Table 1:

$$N(52) = \frac{T - \mu}{\sigma} = \frac{52 - 48}{13.2} = 0.30$$

$$N(27) = \frac{T - \mu}{\sigma} = \frac{27 - 48}{13.2} = -1.59$$

The probability – P(52) – on Table 1 that corresponds to N(T)=0.30 is 0.382. Finding P(27) is only slightly more complicated, because N(27) is negative. As Table 1 instructs you, find the value in the table that corresponds to +N(x), and then subtract that value from 1.000:

$$\text{for } N(T)=-1.59,\ P(T) = 1.000 - 0.056 = 0.944$$

Using the values of P(27) and P(52) and Equation 2:

$$P[27\text{-}52] = (P(27) - P(52)) \div P(27) = 0.944 - 0.382 = 0.447$$

This means that there is a 44.7% likelihood that a major earthquake will occur on this fault in the 25-year period between 2006 and 2031.

1) Ground-rupturing earthquakes have occurred on the Parkfield segment of the San Andreas fault in 1857, 1881, 1901, 1922, 1934, and 1966. What is the probability that another earthquake will **not** have occurred at Parkfield between 1966 and the present date?

Seismic-Hazard Assessment

The Parkfield segment of the San Andreas fault is noteworthy because it has ruptured so regularly in the past that, in 1985, the U.S. Geological Survey made a formal prediction that there was a 90% probability that the Parkfield segment would generate a magnitude 5.5-6.0 earthquake by 1993. The fault has remained embarrassingly quiet ever since. In fact, this author hesitates to even mention Parkfield for fear that the fault segment will rupture the day after this book goes to press. The Earth is complex, and there are a number of reasons why a fault might deviate from the recurrence predicted by the past earthquakes, including:

- Earthquake clustering – Some faults are characterized by periods of closely-spaced earthquakes, followed by periods of inactivity.
- Fault-segment triggering – Different segments of the same fault are not truly independent. An earthquake on one segment often triggers rupture on adjacent segments.
- Inadequate period of record – All earthquake-recurrence models are studies in small numbers. Statisticians prefer distributions that consist of hundreds of values, but seismologists rarely have data for more than a handful of past recurrence intervals.

THE WASATCH FAULT ZONE

The 343 km-long Wasatch fault zone marks the eastern boundary of the Basin and Range province, separating the region from the Colorado Plateau and the Middle Rocky Mountains to the east. The Wasatch fault zone underlies a populated corridor that is home to 80% of the inhabitants of Utah. The fault zone is subdivided into ten distinct segments (Figure 4), including six more-active central segments and four less-active segments on the margin of the fault zone. All of the segments except the Brigham City segment have ruptured in the last 1500 years, but none in historical time.

Extensive trenching has been done at sites along the Wasatch fault zone to characterize the history of earthquakes in the recent geologic past. Exercise 9 in this book outlines how fault trenches are used in studies of paleoseismology, and Figure 5 illustrates one trench cut across the Wasatch fault. The right-hand side of Figure 4 summarizes the available information on the number and timing of earthquakes on the six central fault segments during the past 6000 years. In the questions on the following pages, you will use the fault-trench information (the best published data currently available) to estimate conditional probabilities for this fault zone.

2) Summarizing the age information in Figure 4, trenches along the Wasatch fault zone record earthquake recurrence intervals of:

   1200, 525, 1725, 1050, 3975, 2100, 2625, 2250, 3675, and 900 years

   Find the mean and standard deviation of this recurrence-interval population.

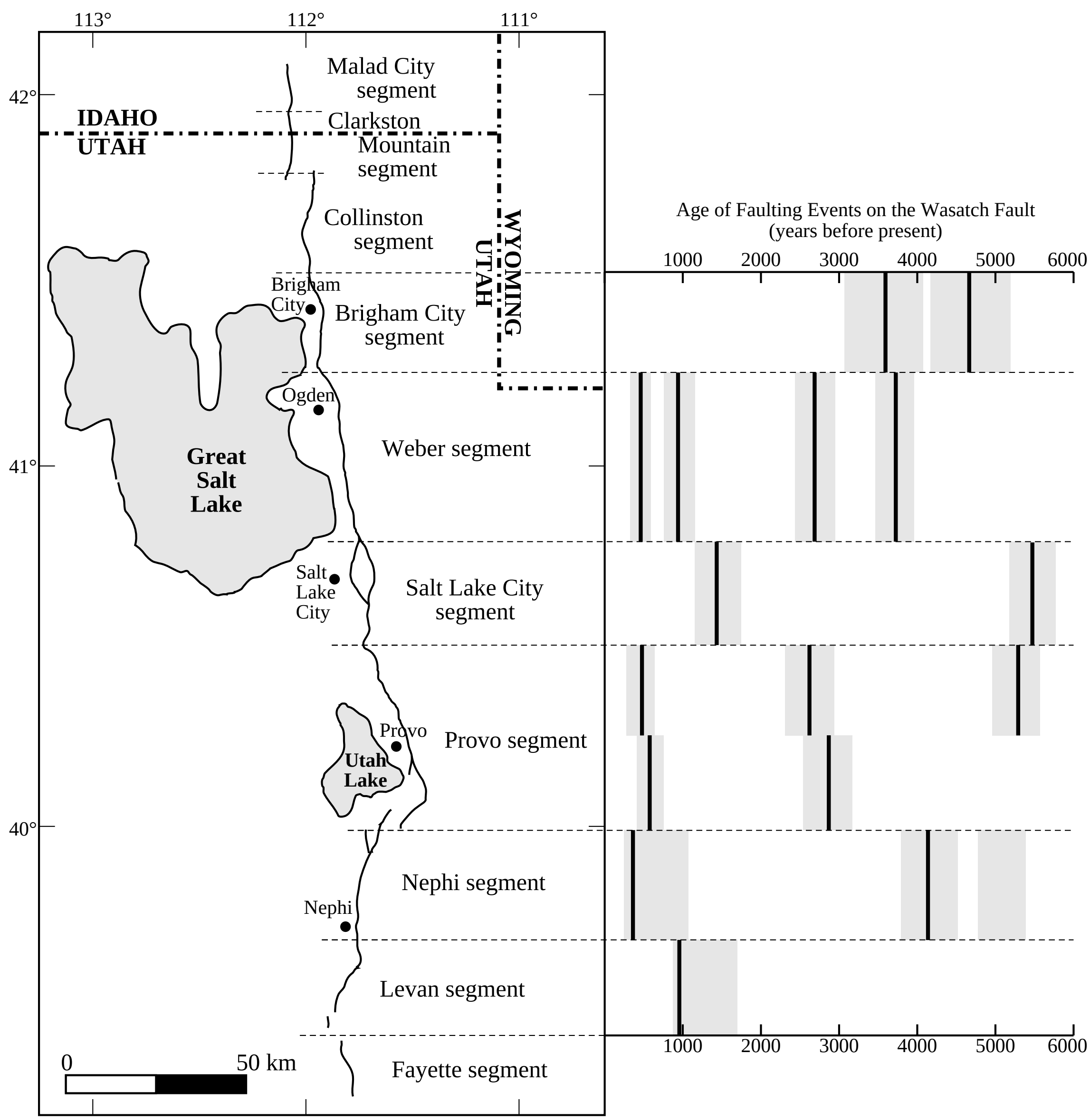

Figure 4. Location map of the Wasatch fault zone and late Holocene earthquake history from fault trenches. (After Gori and Hays, 1991)

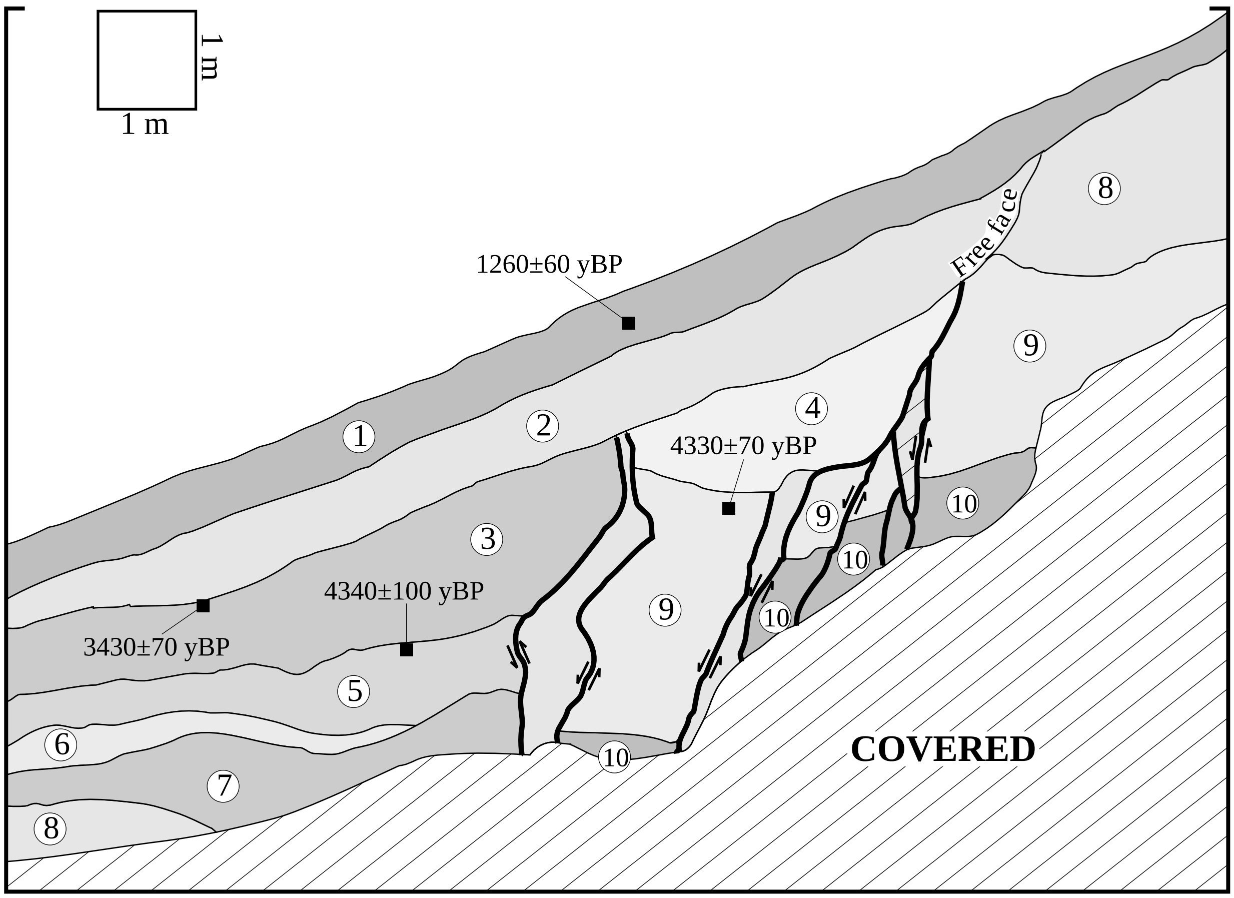

Figure 5. Part of trench BC-1 on the Brigham City segment of the Wasatch fault zone illustrating an earthquake that ruptured the ground surface just after 3430 years BP. (After Gori and Hays, 1991)

3) All of the Wasatch fault segments have ruptured in the last 1500 years except the Brigham City segment. The last ground-rupturing earthquake there occurred approximately 3430 years ago. Find the probability that a major earthquake will occur on the Brigham City segment during the next 25 years.

4) The Salt Lake City segment of the Wasatch fault zone last ruptured about 1500 years ago. Find the probability that a major earthquake will occur on this segment during the next 25 years.

5) Explain why the conditional probability for the Brigham City segment is so different from that of the Salt Lake City probability.

CONDITIONAL PROBABILITY AND GROUND SHAKING

Recurrence-interval information helps define the probability of future earthquakes, but it says nothing about their magnitudes of their effects. That information must come from other sources, such as fault slip rates, past dimensions of rupture, past earthquake magnitudes, and site-specific conditions. Combined with recurrence probabilities, this additional information allows us to calculate probabilities of different levels of seismic ground shaking. Figure 6 is a regional map of ground-shaking hazard across the U.S., based on recurrence-interval and ground-shaking probabilities. Estimates of ground-shaking risk are vital to architects, engineers, and planners in earthquake-prone areas.

Like recurrence intervals, ground-shaking estimates for a specific site (usually expressed as acceleration) can be described by a mean value and associated standard deviation (a normal distribution). Such estimates must be firmly based on shaking intensities during past earthquakes or other data. The probability of seismic shaking exceeding a given value of acceleration (A) in a pre-specified duration of time (T) is expressed as:

$$P(A,T) = P(A) * P(T). \qquad (3)$$

Seismic-Hazard Assessment

P(A,T) is the probability that *both:* 1) an earthquake will occur in the pre-specified time, and 2) the ground shaking will exceed an acceleration of A during that earthquake.

Example 2:

A site is characterized by seismic shaking of 0.5±0.3 g (50%±30% of the acceleration of gravity) as a result of rupture on a nearby fault. If there is a 12% chance of the fault rupturing in the next 50 years, what is the probability that this site will experience seismic acceleration greater than 0.7 g in that 50-year period?

The first step in this problem is to find P(A), which is the probability of A (0.7 g) being exceeded during any one earthquake. In order to do this, first calculate N(A):

$$N(A) = \frac{0.7\text{ g} - \mu}{\sigma} = \frac{0.7\text{ g} - 0.5\text{ g}}{0.3\text{ g}} = 0.67$$

using Table 1, P(A) = 0.251 = 25.1%

Now combine this information with the probability of recurrence (P(50)=12%):

$$P(A,T) = P(A) * P(T) = 0.251 * 0.120 = 0.030$$

This means that there is a 3.0% likelihood of an earthquake occurring on this fault and causing ground acceleration greater than 0.7 g at the site in question.

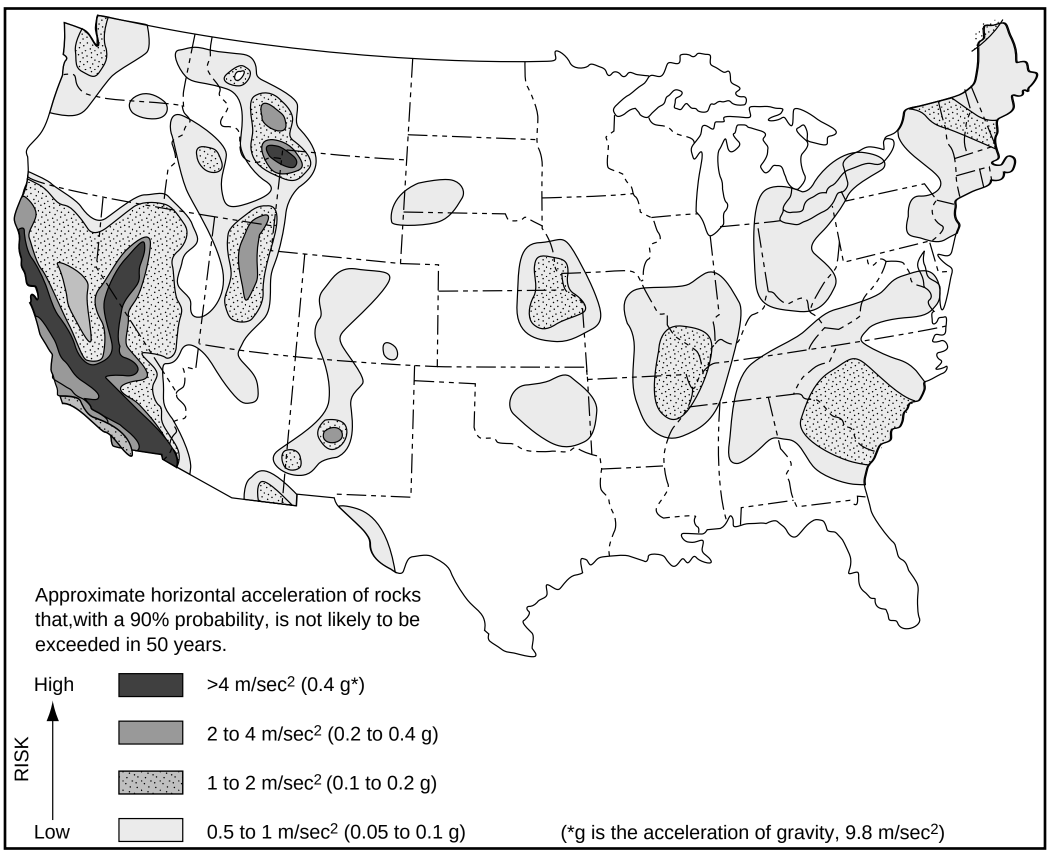

Figure 6. Ground-shaking hazards in the U.S. (From Algermissen and Perkins, 1976)

6) A new construction project is being planned for a site 10 km from the Salt Lake City segment of the Wasatch fault zone. That site is characterized by seismic ground acceleration of 0.4±0.2 g (for a M=7.0 earthquake; Joyner and Boore, 1988). Find the probability that the site will experience accelerations greater than 0.7 g during the next 100 years.

7) If the same site in Question 6 were instead 10 km from the Brigham City segment, what would be the probability of exceeding 0.7 g in a 100-year period?

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