## The Algebra of Earth Science



## Introduction

Many of the measured parameters in Earth science are changing with time. Are these changes linear or not? If they are linear (with a good
 correlation), then we can predict future changes with some confidence.

## Increasing Carbon Dioxide

The Carbon Dioxide ( $\mathrm{CO}_{2}$ ) in the atmosphere is relatively small (less than 1 part in a 1000), but it is an important "greenhouse gas" that traps infrared light from the Earth, making the Earth warm up. Is the amount increasing linearly (each year adds the same amount)?

Take the data of $\mathrm{CO}_{2}$ versus time and plot it on normal $x-y$ graph paper (or use a graphing calculator). Does the resulting plot look linear? If so, draw a line through the data (you can use a stick of spaghetti to find the best line that covers the most data points). Since the line is a very good fit, you can estimate what it will do at a later time by extending the line. What year should the amount of $\mathrm{CO}_{2}$ exceed 370 parts per million?

Now look at the third column. This data represents the global average temperature measurement, as a difference from the average of the years 1961-1990, measured in degrees Celsius. Plot this data as a function of time, on x-y graph paper or a graphing calculator (shown here as squares). Is this trend linear? Is the fit a good fit (are all the points very close to a line, or is there more scatter? (Can you use a spaghetti noodle to cover all the points, or do you need a fettuccini noodle?)

## Data sources:

$\mathrm{CO}_{2}$ : (from WRI)
Data from Siple station and Law Dome ice cores, CDIAC Temperature: (from GCMD)
Jones, P.D., D.E. Parker, T.J. Osborn, and K.R. Briffa. 2000. Global and hemispheric temperature anomalies land and marine instrumental records. In Trends: A Compendium of Data on Global Change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

| Year | CO2 (ppm) (Parts per million) | Global Temperature Anomaly <br> (iC from 1961-1990) |
| :---: | :---: | :---: |
| 1965 | 320 | -0.16 |
| 1966 | 321 | -0.07 |
| 1967 | 322 | -0.08 |
| 1968 | 323 | -0.1 |
| 1969 | 324 | 0.03 |
| 1970 | 326 | -0.03 |
| 1971 | 326 | -0.19 |
| 1972 | 327 | -0.04 |
| 1973 | 330 | 0.09 |
| 1974 | 330 | -0.18 |
| 1975 | 331 | -0.12 |
| 1976 | 332 | -0.22 |
| 1977 | 334 | 0.06 |
| 1978 | 335 | -0.03 |
| 1979 | 337 | 0.07 |
| 1980 | 339 | 0.11 |
| 1981 | 340 | 0.13 |
| 1982 | 341 | 0.06 |
| 1983 | 343 | 0.25 |
| 1984 | 344 | 0.03 |
| 1985 | 346 | 0.01 |
| 1986 | 347 | 0.1 |
| 1987 | 349 | 0.25 |
| 1988 | 351 | 0.25 |
| 1989 | 353 | 0.19 |
| 1990 | 354 | 0.34 |
| 1991 | 355 | 0.29 |
| 1992 | 356 | 0.14 |
| 1993 | 357 | 0.19 |
| 1994 | 359 | 0.26 |
| 1995 | 361 | 0.38 |
| 1996 | 363 | 0.22 |
| 1997 | 364 | 0.43 |
| 1998 | 367 | 0.59 |
| 1999 | 369 | 0.33 |


(teachers - blank out the solution plots before handing this out to students)
If you plot both the second column and the third column data on a single plot, the data from the third column will all be very low on the graph. In cases like this, we often use a different y-axis scale for one set of data than the other (shown above).

## Are the effects correlated?

Two effects are correlated if changes in one are proportional to changes in the other. We explore correlations by plotting one variable versus the other one, rather than each one versus time. Note that there is more scatter on this plot than in the $\mathrm{CO}_{2}$ versus time plot. The correlation coefficient, $r$, is 0.75 which is good but not as good as .99 (the $\mathrm{CO}_{2}$ versus time plot). A value of 1 is a perfect correlation and allows accurate predictions. Lower correlations show trends but cannot be used to make good predictions.

CO2 and Temperature


Other data which are related to global warming may not be measured over as long a time history, but still are useful to examine for trends. One is the retreat of glaciers (measured in meters from a time in the past, in this case 1985).

Take these two data sets and plot them versus the year of measurement individually, as in the first plot of the last page.

Which data set has a better linear fit versus time?

Why do you suppose that is true?

| Year | Nose Position of <br> Rainbow Glacier <br> (relative to 1985) | Annual Galveston <br> Average Sea Level |
| :---: | :---: | :---: |
| 1985 | 0 | 1521 |
| 1986 | -11 | 1535 |
| 1987 | -22 | 1497 |
| 1988 | -33 | 1483 |
| 1989 | -44 | 1519 |
| 1990 | -55 | 1581 |
| 1991 | -60 | 1611 |
| 1992 | -75 | 1578 |
| 1993 | -96 | 1564 |
| 1994 | -116 | 1575 |
| 1995 | -137 | 1599 |
| 1996 | -161 | 1530 |
| 1997 | -181 | 1577 |
| 1998 | -201 | 1620 |
| 1999 | -241 | 1589 |
| 2000 | -246 | 1556 |

Now plot the second column data against the third column data, much as in the lower plot of the previous page. Are the two data correlated?

Now, be careful - just because two data sets are correlated does not mean that one causes the other - they could both be caused by a third effect. In this case global warming makes the glacier retreat. The melting glaciers do cause the sea level to rise, but also the sea level rises because the water is warming up and expanding.

| Year | World Population <br> (millions) |
| :---: | :---: |
| 0 | 170 |
| 200 | 190 |
| 400 | 190 |
| 500 | 190 |
| 600 | 200 |
| 700 | 207 |
| 800 | 220 |
| 900 | 226 |
| 1000 | 254 |
| 1100 | 301 |
| 1200 | 360 |
| 1300 | 360 |
| 1400 | 350 |
| 1500 | 425 |
| 1600 | 545 |
| 1700 | 600 |
| 1750 | 629 |
| 1800 | 813 |
| 1850 | 1128 |
| 1900 | 1550 |
| 1950 | 2521 |
| 1990 | 5266 |
| 2000 | 6055 |
|  |  |

## Non-linear Trends

Not all trends in Earth science are linear trends. Some trends are quadratic, or even exponential (the rate of increase is increasing with time). The World's population growth is a non-linear data set. Let's look at the data (from the U.S. census and the World Resources International web sites) (next page).

Plot this data versus time.

If you only look at the first thousand years, the trend in population is very nearly linear. Disease, famines and plagues caused decreases in the total world population - for example the Bubonic plague of the 1200's. However, if you continue the plot past 1500 , it is clear that the population is growing much faster than the linear trend of the first century.

Plotting the $y$-axis on a logarithmic scale helps make the plot a little more linear. A logarithmic axis has each step being a factor of ten in value, not a simple addition. Trends which are linear when plotted using a logarithmic axis, are called exponentials. An exponential has each year being a constant multiple of the previous year.

The world's population is rising even faster than an exponential - the slope of the curve on the logarithmic plot near 2000 is much larger than the slopes prior to that. This is because the birth rate alone causes an exponential growth. In addition, advance in medicine, immunization and nutrition helps people to live longer than before, adding to the numbers of humans - thus the term "population explosion". It is not surprising that overpopulation is the root cause of many, if not most, of Earth's environmental problems. A sustainable world is one in which the numbers of people are in balance with

 the world's resources to supply them.

For the best sources of numeric Earth data:

World Resources Institute:
NASA Global Change Master Directory.
ESIP Federation Home Page
http://earthtrends.wri.org/index.cfm http://gcmd.nasa.gov/md/index.html http://www.esipfed.org

