# Probabilistic Decision Making for Collision Avoidance Systems: Postponing Decisions

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***Abstract*— For collision avoidance systems to be accepted by human drivers, it is important to keep the rate of unnecessary interventions very low. This is challenging since the decision to intervene or not is based on incomplete and uncertain information. The contribution of this paper is a decision making strategy for collision avoidance systems which allows the system to occasionally postpone a decision in order to collect more information. The problem is formulated in the framework of statistical decision theory, and the core of the algorithm is to run a preposterior analysis to estimate the benefit of deciding with the additional information. A final decision is made by comparing this benefit with the cost of delaying the intervention. The proposed approach is evaluated in simulation at a two-way stop road intersection for stop sign violation scenarios. The results show that the ability to postpone decisions leads to a significant reduction of false alarms and does not impair the ability of the collision avoidance system to prevent accidents.**

## I. INTRODUCTION

Active safety systems are increasingly present in commercial vehicles, as part of a global effort to make roads safer. The purpose of such systems is to avoid or mitigate accidents through driver warnings or direct actions on the commands of the vehicles (braking, steering). As illustrated in Fig. 1, a typical Collision Avoidance (CA) system architecture is composed of *input* modules, *processing* modules, and *output* modules. A *situation assessment* module fuses the information obtained from different *sensors* and *databases* (e.g. digital map) and provides an estimate of the true state of the environment to a *risk assessment* module. The latter uses this information to compute the collision risk of the current situation. The role of the *decision making* module is to decide based on the collision risk whether or not to intervene (e.g. by warning the driver of an upcoming collision or by applying the brakes autonomously). The decision is forwarded to *actuators* or *human-machine interfaces* which perform the required actions. The three *processing* modules must take into account the uncertainties inherent to sensor measurements and model imperfections.

A major challenge for the *decision making* module is that it has to make decisions based on uncertain knowledge, and that the timing of interventions is critical:

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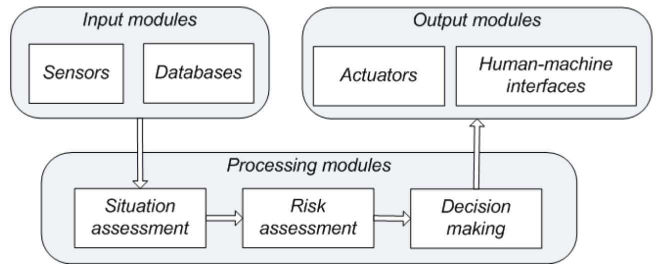


Fig. 1. Collision Avoidance system architecture.

- If an intervention is triggered at a time when the uncertainty about the occurrence of a collision is too large, there is a chance that it will end up being a false alarm. High false alarm rates are detrimental to the driver acceptance of CA systems and can lead to the user losing trust in the system [1].
- If the system waits until the last moment (certainty about the occurrence of a collision) to trigger an intervention, it might be too late to avoid the accident.

The decision to intervene or not relies on some metrics which quantify the criticality of the situation. A number of metrics have been proposed in the past, which are generally based on a measure of the "Time-To-X" (or TTX) where X corresponds to a relevant event in the near future. The most standard indicator is the Time-To-Collision (or TTC), which corresponds to the time remaining before a collision occurs. The decision to intervene can be based on a threshold on the TTC [2], [3], or on a comparison between the TTC and the time it would take for the vehicle to come to a full stop if emergency braking was applied [4]. As an alternative the authors of [5] designed a function which takes as an input the TTC and the speed of the ego vehicle and outputs whether the driver can avoid the collision by braking, or by steering, or if autonomous braking is needed. Similarly in [6] the entire space of combined steering, braking and accelerating maneuvers is considered when looking for collision-free trajectories. It is also possible to adapt the collision avoidance strategy based on the value of the TTC, e.g. when the TTC is still large it might be preferable to inform or warn the driver rather than to apply the brakes [7]. Another metric closely related to the TTC is the Time-To-React (or TTR), which corresponds to the time available for the driver to act before the collision becomes unavoidable. The idea is to simulate different driver actions (such as braking, accelerating, steering) and to identify the latest moment at which one of these maneuvers is able to avoid the collision [8]. Recently it has been suggested to incorporate a model of the driver's acceptance of the intervention in the decision making strategy [9]. The model assumes that an intervention is more likely to be accepted if the driver judges the situation to be critical, and the latter is estimated based on the driver's observations and predictions of the traffic situation.

In this paper we introduce the possibility for the Collision Avoidance system to postpone the decision to intervene. Our objective is to implement the fact that in some situations the new observations obtained by waiting will reduce the uncertainty about the occurrence of a collision, therefore the decision will be more reliable if it is made later using this additional information. The important question is whether the potential gain brought by the additional information outweighs the cost of waiting. In order to answer this question, our decision making approach runs a *preposterior analysis* to determine the expected value and cost of the additional information. In Section II some background on statistical decision making is presented. Section III introduces the proposed approach for a decision making strategy which allows the CA system to postpone the decision to intervene, and Section IV describes a possible implementation. The approach was evaluated in simulation for stop sign violation scenarios, and the results are provided in Section V.

## II. Background: Preposterior analysis for decision making

### A. Statistical decision theory

Statistical decision theory is concerned with helping a decision maker select the best alternative to a problem in the presence of uncertain knowledge [10], [11]. A decision making problem is defined by the following basic elements:

- The *alternatives* to choose from, represented by the random variable $A = \{a \in \mathcal{A}\}$.
- The *state of nature* (or *state of the world*), about which uncertain knowledge is available, represented by the random variable $X = \{x \in \mathcal{X}\}$.
- The *cost function* $c(x, a)$, defined for each combination of alternatives and states of nature $(x, a) \in \mathcal{X} \times \mathcal{A}$.
- A *decision criterion*, used to select an alternative based on statistical knowledge about the state of nature and the cost function. An example decision criterion is to select the alternative $a^* \in \mathcal{A}$ which minimizes the expected cost. When $X$ is discrete, $a^*$ is defined as:

$$a^* = \arg\min_{a \in A} \sum_x c(x, a) \times P(x|y) \quad (1)$$

where $y$ represents the information from which a probability distribution on $X$ is inferred.

### B. Deciding with additional information

Moreover the decision maker is sometimes faced with an additional choice: whether or not to collect additional information before making the decision. The use of additional information might reduce the uncertainty about the state of nature and therefore help select a better alternative. However access to additional information usually has a cost, whether it is monetary (cost of running a survey) or time-related (cost of postponing the decision for the purpose of data collection). The question to be solved is whether the cost of additional information outweighs the potential gain that more information would bring. This analysis is sometimes called *preposterior analysis* because it attempts to estimate what improvement would be brought by a data sample before seeing the actual data sample.

The value of the additional information can be quantified by means of the Expected Value of Sample Information (EVSI). It corresponds to the additional expected payoff possible through knowledge of the additional information and is computed in three steps as follows:

***Step 1:*** Compute the expected cost of the optimal decision without using the additional information:

$$EC = \min_{a \in \mathcal{A}} \sum_x c(x, a) \times P(x|y) \quad (2)$$

***Step 2:*** Compute the expected cost of the optimal decision using the additional information, by integrating over all the predicted possible outcomes of the information sample and using the posterior probabilities for the states of nature:

$$\widehat{EC} = \int_{\widehat{y}} P(\widehat{y}) \times [\min_{a \in \mathcal{A}} \sum_x c(x, a) \times P(x|y, \widehat{y})] \, d\widehat{y} \quad (3)$$

where the random variable $\widehat{Y} = \{\widehat{y} \in \widehat{\mathcal{Y}}\}$ represents the predicted additional information sample.

***Step 3:*** Subtract the two expected costs to obtain the expected gain of using additional information:

$$EVSI = EC - \widehat{EC} \quad (4)$$

The decision about whether or not to use additional information to make the decision is then made by comparing the EVSI with the cost of the additional information.

## III. Proposed approach: Preposterior analysis for Collision Avoidance systems

We propose to apply the framework described above to Collision Avoidance (CA) systems. The contribution of this paper is to propose a strategy for the decision making module (see Fig. 1) which gives the possibility to postpone the decision making in cases where the following two conditions are fulfilled:

*Condition 1:* It is estimated that the additional observations obtained by waiting until time $t + 1$ would reduce the uncertainty about the occurrence of a collision in the future and therefore lead to a better decision.

*Condition 2:* It is estimated that the collision will still be avoidable by the CA system if it intervenes at time $t + 1$ instead of time $t$.

The expected benefit of the proposed strategy is a reduction of the rate of false alarms (thanks to *Condition 1*), while reaching the same collision avoidance rate as strategies which do not give the possibility to postpone the decision (thanks to *Condition 2*).

### A. Formulation of the decision problem

We formulate the problem as a "statistical decision making with additional information" problem [10], [11] with the following elements:

- The *state of nature* is defined as the occurrence of a collision involving the ego vehicle at some point in the time period $[t,\ t+T]$, where $t$ is the current time and $T$ is the time horizon considered by the CA system:
$$\mathcal{X} = \{collision,\ \neg collision\} \tag{5}$$
In the context of CA systems, the probability distribution on the *state of nature* is provided by a *risk assessment* module as illustrated in Fig. 1. The time horizon $T$ is generally set as a compromise between computation time and risk to miss a conflict [7].
- The *alternatives* to choose from are for the CA system to intervene or not:
$$\mathcal{A} = \{intervene,\ \neg intervene\} \tag{6}$$
- The *cost function* is defined such that both unnecessary interventions of the CA system and failures to intervene are penalized:
$$c(x, intervene) = \begin{cases} 0 & if\ x = collision \\ c_1 & if\ x = \neg collision \end{cases} \tag{7}$$
$$c(x, \neg intervene) = \begin{cases} c_2 & if\ x = collision \\ 0 & if\ x = \neg collision \end{cases} \tag{8}$$
- The *decision criterion* implements the two conditions stated in the introduction of this section. It is based on the comparison between the value and the cost of additional information. The former is represented by the Expected Value of Sample Information (EVSI). The latter is represented by the Expected Cost of Waiting (ECW), a metric which quantifies the effect of waiting on the ability of the CA system to avoid the collision.
$$\begin{array}{l} IF\ (EVSI > 0)\ AND\ (ECW = 0) \\ \qquad postpone\ decision\ until\ t+1 \\ ELSE \\ \qquad make\ decision\ using\ Eq.\ 1 \end{array} \tag{9}$$

The computation of EVSI and ECW are detailed below.

### B. Value of additional information:

The Expected Value of Sample Information (EVSI) is computed as in Eq. 4 by subtracting the expected costs of deciding with and without additional information.

The expected cost $EC$ of deciding without additional information is computed as the expected cost of making a decision using the observations obtained from the sensors until the current time $t$:
$$EC = \min_{a \in \mathcal{A}} \sum_x c(x, a) \times P(x|z_{0:t}) \tag{10}$$
with $z_t$ the observations provided by the sensors at time $t$.

The expected cost $\widehat{EC}$ of deciding with additional information is computed as the expected cost of making a decision using the observations obtained from the sensors until the current time $t$ and the observations that would be obtained if we waited until time $t+1$:
$$\widehat{EC} = \int_{z_{t+1}} P(z_{t+1}) \times [\min_{a \in \mathcal{A}} \sum_x c(x, a) \times P(x|z_{0:t+1})]\, dz_{t+1} \tag{11}$$
with $Z_{t+1} = \{z_{t+1} \in \mathcal{Z}\}$ a random variable representing the observations provided by the sensors at time $t+1$.

### C. Cost of additional information

The Expected Cost of Waiting (ECW) is computed as the difference between the probability that the CA system will be able to avoid the potential collision if it intervenes now and if it intervenes at time $t+1$.

### D. Summary

The proposed decision making strategy relies on the computation of several terms:

- In order to compute the Expected Value of Sample Information (EVSI) it is necessary to calculate the collision probability $P(x|z_{0:t})$, the collision probability $P(x|z_{0:t+1})$, and the probability of a future observation $P(z_{t+1})$.
- In order to compute the Expected Cost of Waiting (ECW) it is necessary to estimate whether the CA system will be able to avoid the potential collision if it intervenes now and if it intervenes at time $t+1$.

## IV. Implementation

There are numerous examples in the literature of algorithms which can compute the terms listed above. In this section we describe one possible implementation which builds on our previous work on risk assessment [12].

### A. Probabilistic motion model

In this previous work the joint motion of vehicles in a traffic scene is modeled by a Dynamic Bayesian Network (DBN) using four categories of variables:

- $I_t^n$ represents the maneuver being performed by vehicle $n$ at time $t$ (e.g. keep lane, change lanes). We call it $I$ as in "Intention", since the maneuver performed by a vehicle reflects the *intended maneuver* of the driver.
- $E_t^n$ represents the maneuver that vehicle $n$ is expected to perform at time $t$ according to the traffic laws (e.g. keep lane, change lanes). We call it $E$ as in "Expectation", since it represents the *expected maneuver*.
- $\Phi_t^n$ represents the *physical state* of vehicle $n$ at time $t$ (e.g. position, speed).
- $Z_t^n$ represents the *measurements* available about vehicle $n$ at time $t$. They often correspond to a noisy version of a subset of the *physical state* variables.

$I_t^n$, $E_t^n$, and $\Phi_t^n$ are hidden variables, while $Z_t^n$ is observable. For more clarity in the equations, in the remaining of this paper factored stated will be used to represent the conjunction of variables for the $N$ vehicles in the scene, e.g. $Z_t \triangleq (Z_t^1 ... Z_t^N)$.

The proposed joint distribution of the DBN over all the vehicles is as follows [12]:

$$P(E_{0:T} I_{0:T} \Phi_{0:T} Z_{0:T}) = P(E_0 I_0 \Phi_0 Z_0) \times \prod_{t=1}^{T} \times \prod_{n=1}^{N} [P(E_t^n | I_{t-1} \Phi_{t-1}) \times P(I_t^n | I_{t-1}^n E_t^n) \times P(\Phi_t^n | \Phi_{t-1}^n I_t^n) \times P(Z_t^n | \Phi_t^n)] \quad (12)$$

which corresponds to a classic Markov state-space model linking $I_t^n$, $\Phi_t^n$, and $Z_t^n$, augmented by the *expected maneuver* $E_t^n$ which is derived from the previous situational context $(I_{t-1}\Phi_{t-1})$ and has an influence on the intended maneuver $I_t^n$. For the interested reader more details about this model can be found in the previously published papers describing this DBN [12], [13].

### B. Bayesian inference for risk estimation

Inference on variables in the DBN described above is performed using a particle filter, which means that at each timestep the probability density function of the hidden variables $I_t$, $E_t$, and $\Phi_t$ is approximated by a set of weighted samples called particles. The set of $N_{particles}$ particles at time $t$ is denoted:

$$\{H_{i,t}, w_{i,t}\}_{i=1:N_{particles}} \quad (13)$$

with $H_{i,t} = (I_t E_t \Phi_t)$ the state of particle $i$ at time $t$ and $w_{i,t}$ the weight of particle $i$ at time $t$.

The risk estimation algorithm proposed in [12] exploits the fact that 90% of road accidents are caused by driver error [14]. The probability of a collision is computed as the probability that the intention of drivers differ from what is expected of them, i.e. $P(\exists n \in N : I_t^n \neq E_t^n | z_{0:t})$. Using the particle filter, this inference can be performed by summing up the weights of the current particles which verify the condition $(\exists n \in N : \ I_t^n \neq E_t^n)$:

$$P([X = collision] | z_{0:t}) = \sum_{i:\ (\exists n \in N:\ I_t^n \neq E_t^n)} w_{i,t} \quad (14)$$

### C. Value of additional information

In this section we describe how the probabilistic motion model described above can be used to compute the terms $P(x|z_{0:t+1})$ and $P(z_{t+1})$ which are needed to compute the EVSI (see Section III-D).

The probability of future observations $P(z_{t+1})$ can be calculated in two steps. The first one is to run the prediction step in the particle filter to obtain a probability distribution on $\Phi_{t+1}$. The second step is to use the sensor model $P(Z|\Phi)$ to compute the probability of an observation $z_{t+1}$.

Following this, the collision probability at time $t+1$ can be computed in two steps. The first step is to execute the update step in the particle filter with observations $z_{t+1}$. The second step is to sum up the weights of the particles which verify the condition $(\exists n \in N : \ I_{t+1}^n \neq E_{t+1}^n)$:

$$P([X = collision] | z_{0:t+1}) = \sum_{i:\ (\exists n \in N:\ I_{t+1}^n \neq E_{t+1}^n)} w_{i,t+1} \quad (15)$$

### D. Cost of additional information

In this section we describe how the probabilistic motion model described above can be used to estimate the ability of the CA system to avoid a collision, in order to compute the ECW (see Section III-D).

First of all we define the Time-To-Collision (TTC), and the Time-To-Stop (TTS). The TTC can be computed as the time that is left until a collision occurs if both vehicles involved in the collision continue on the same course and at the same speed [15]. The TTS corresponds to the time needed by a vehicle to reach a full stop after the CA system intervenes. If we consider a CA system where the intervention consists in applying the brakes autonomously, the TTS can be computed as follows [16]:

$$TTS_t = \frac{s_t}{\delta} + T_{machine} \quad (16)$$

with $s_t$ the speed of the ego vehicle at time $t$, $\delta = 7$ m/s² the deceleration applied by the CA system, and $T_{machine} = 0.4$ s the average braking system response time [16]. If instead we consider a CA system where the intervention consists in warning the driver, the response time of the driver has to be taken into account in the computation of the TTS [16].

The probability that the potential collision can be avoided if the CA system intervenes now can be computed by summing the weights of the current particles which verify the condition $(TTC_t > TTS_t)$. Similarly, the probability that the potential collision can be avoided if the CA system intervenes at time $t+1$ can be computed using the particles predicted for time $t+1$ instead of the current particles. As a result the ECW can be computed as:

$$ECW = \sum_{i:\ (TTC_t > TTS_t)} w_{i,t} - \sum_{i:\ (TTC_{t+1} > TTS_{t+1})} w_{i,t+1} \quad (17)$$

## V. Evaluation

The implementation described in Section IV was run on a dual core 2.26 GHz processor PC, with 400 particles in the filter and with new observations $z_t$ made available every 200 ms. In its current non-optimized state the code runs at 1.5Hz when run on one core only, however the particle filter code is highly parallelizable and it is expected that it would run approximately two times faster if it was run on both cores, and four times faster on a four cores computer.

### A. Scenarios

Tests were run in simulation for collision scenarios and no-collision scenarios at a two-way stop road intersection. The PreScan simulator [17] was used to generate trajectories belonging to four different scenarios. All of the scenarios involve an "Ego Vehicle" (EV) driving on the main road towards the intersection and an "Other Vehicle" (OV) approaching the intersection from a secondary road and performing various maneuvers, as illustrated in Fig. 2. Scenarios 1, 2, and 3 are collision scenarios where the EV and the OV collide after the OV violated the stop sign.

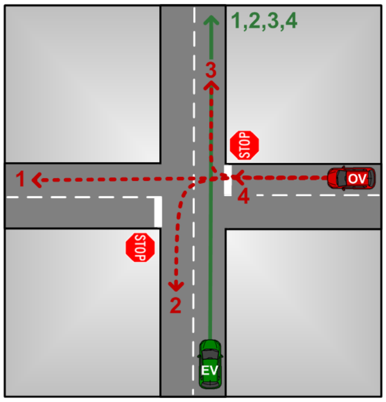


Fig. 2. The four simulated scenarios. For each scenario the maneuver of the "Ego Vehicle" (EV) is shown in plain green and the maneuver of the "Other Vehicle" (OV) is shown in dotted red.

Scenario 4 is a no-collision scenario where the OV stops at the stop line and yields to the EV. A total of 250 collision instances and 300 no-collision instances were simulated, by varying the speed profiles of the two vehicles.

At each timestep the EV has access to information $z_t$ about the position, orientation, and speed of itself and the OV. In the real world this information could for example be obtained via vehicle-to-vehicle communication [13].

### B. Decision making strategies

We consider a CA system which can apply the brakes on the EV to try to avoid collisions, and we compare the performances of two decision making strategies.

The *baseline* strategy follows the classic approach described in Section II-A which is to select the alternative $a^* \in \mathcal{A}$ which minimizes the expected cost, without considering the potential value of additional information to make a decision. It is interesting to note that this strategy is equivalent to making the CA system intervene whenever the collision probability (see Eq. 14) exceeds a predefined threshold $\lambda$ with $\lambda = \frac{c_1}{c_1+c_2}$ [18]. This strategy was used in our previous work [12], and a precision / recall analysis led us to set the threshold to $\lambda = 0.3$. Here we use this previous result and set the costs such that $c_1 = \frac{\lambda}{1-\lambda} \times c_2$ with $\lambda = 0.3$.

The *proposed* strategy corresponds to the algorithm described in Section IV, and uses the same costs $c_1$ and $c_2$ as the *baseline* strategy. The difference is that the *proposed* strategy can postpone the decision if it estimates that waiting would bring useful additional information and still leave enough time for the CA system to avoid the collision.

### C. Performance metrics

The performances of the two strategies are compared based on three metrics:

- The rate of missed interventions: $\frac{NM}{NC}$, with $NM$ the number of collision instances where the CA system never intervened before the collision occurred and $NC$ the number of collision instances.

| | *Baseline* approach | *Proposed* approach |
|---|---|---|
| Missed interventions | 0.0% | 0.0% |
| Avoided collisions | 81.2% | 81.2% |
| False alarms | 6.5% | 3.9% |

Fig. 3. Performances of the *proposed* approach and the *baseline* approach.

- The rate of avoided collisions: $\frac{NA}{NC}$, with $NA$ the number of collision instances where the CA system intervened and successfully avoided the collision and $NC$ the number of collision instances.
- The rate of false alarms: $\frac{NF}{NN}$ with $NF$ the number of no-collision instances where the CA system intervened and $NN$ the number of no-collision instances.

### D. Results

The performances of the *proposed* approach and the *baseline* approach are shown in Fig. 3, and commented below.

***Missed interventions and avoided collisions:*** As expected the rate of missed interventions and avoided collisions is identical for the two approaches, since the *proposed* strategy postpones a decision only if the collision is still avoidable at time $t+1$. The non-avoided collisions (18.8% of collision instances) correspond to instances where the CA system intervened but emergency braking was not enough to avoid the collision. Typically, this happens when the OV slows down as if to stop when approaching the intersection and then accelerates at the last moment instead of stopping.

***False alarms:*** In our dataset the possibility to delay decisions leads to a 40% reduction of false alarms. If the driver acceptance was studied for CA systems using the *proposed* and the *baseline* decision making strategies, we expect that this difference in the false alarms rate would have a strong impact.

***Decisions:*** We further analyze the results obtained for the *proposed* approach by looking at the reasons behind the decisions made by the system in different situations. We define 3 cases:

1) The system postpones the decision, i.e. $(EVSI > 0)$ and $(ECW = 0)$.
2) The system estimates that it would be too dangerous to postpone the decision, i.e. $(ECW > 0)$.
3) The system estimates that the additional information obtained by postponing the decision would not help make a better decision, i.e. $(EVSI = 0)$.

For the 250 collision instances, the percentage of instances belonging to each of these 3 cases is displayed in Fig. 4. When the time-to-collision is larger than 5 s, approximately 10% of the decisions are postponed by the system. The reason why a large majority of the decisions are not postponed is that the system considers that the additional information would not be useful. Indeed when the vehicles are far away from the intersection it is difficult to predict whether the drivers intend to stop, and waiting 200 ms will not bring information which will help discriminate between violating and

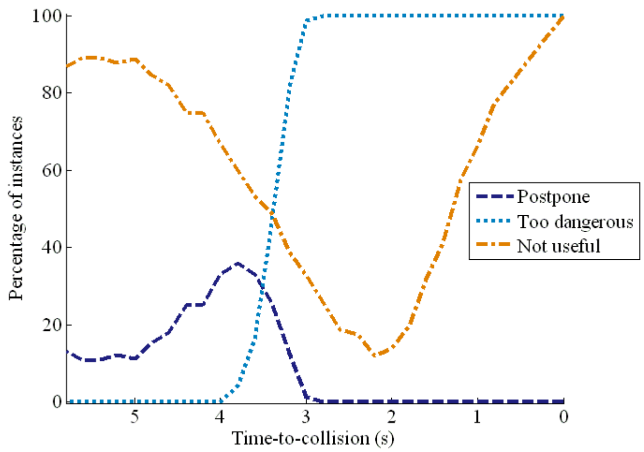


Fig. 4. Decisions made by the *proposed* approach as a function of the time remaining before the collision.

compliant behaviors. When the time-to-collision becomes closer to 4 s the percentage of postponed decisions increases to reach 36%. This rise coincides with a decrease of the "Not useful" cases, since the motion of a vehicle contains more hints about the drivers' intention to stop as the vehicles get closer to the intersection. As the time-to-collision becomes closer to 3 s we observe a steep rise of the "Too dangerous" curve, as it becomes more and more difficult to avoid the collision. As a consequence of this rise, the percentage of postponed decisions quickly declines even if using additional information becomes more and more useful. For a time-to-collision between 2 s and 3 s no decision gets postponed since it would be too dangerous, while the system estimates that waiting would provide useful additional information. Finally when the time-to-collision is below 2 s postponing the decision becomes less and less useful, since the intentions of the drivers to stop or not at the intersection are already obvious. Postponing the decision would also be dangerous, and the system never does so.

For the 300 no-collision instances, we found that every false alarm was generated in situations where $(EVSI > 0)$ and $(ECW > 0)$. This means that the system estimates that the additional information which would be obtained by waiting would help make a better decision, but the decision does not get postponed because it would be too dangerous.

## VI. Conclusions and Future Work

This paper presented a decision making strategy for Collision Avoidance systems which can postpone the decision to intervene in order to collect additional information. The core idea is that in some situations the information which would be obtained by waiting would reduce the uncertainty about the occurrence of a collision, and therefore help make a better decision. The algorithm was tested in simulation at a two-way stop intersection for collision scenarios and no-collision scenarios involving two vehicles. A comparative evaluation with a decision making strategy which does not allow postponing decisions showed that our approach generates fewer false alarms and avoids as many collisions.

The algorithm presented in this paper considers that decisions can be postponed as long as the collision is still avoidable. For driver acceptance and safety reasons, in future work we wish to take into account the comfort of the driver in the decision making process. In particular the algorithm will be modified so that the cost of postponing a decision is larger if it implies a stronger deceleration. We also plan to show the generality of the approach by applying it to other scenarios (e.g. obstacle avoidance on the highway) and with other state-of-the-art risk assessment strategies.

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