Inferences and Dialogues in Ludics

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ABSTRACT – *We propose to use Ludics as a unified framework for the analysis of dialogue and as a reasoning system. Not only Ludics gives a denotational semantics for Linear Logic, but it uses interaction as a primitive notion. We first sketch a model for pragmatical and rhetorical aspects of dialogue after a brief review of the way the interactive aspect of dialogue may be represented in Ludics. Then we show how taking into account inferences that occur during a dialogue, with respect to a ISU-like model of dialogue. Through various examples we give an analysis of deductive inferences as well as processes making facts explicit that take place during knowledge updating.* [1]

1 Introduction

1.1 Inference and Dialogue: Interactive Viewpoint

Ludics [13] is a reconstruction of logic with *interaction* as a primitive notion, in the sense that the primary logical concepts are no more formulas and proofs but *cut-elimination* interpreted as an interaction between objects called *designs*. The sole purpose of a design is that it carries on the interaction: a design is nothing else but a set of potential paths where interaction may take place.

At first, let us recall that both the proof-theoretical notions of cut and of cut-elimination are closely related to the (more philosophical) notion of *inference*:
- A *cut* occurs between two proofs when one formula is the conclusion of the first proof, whereas the same formula is a hypothesis of the second one. Called modus ponens, it is the main ingredient of reasoning and it is the paradigm of deductive inference.
- Cut-elimination is one of the main results of proof theory: it states that it is possible (in Linear Logic as in Classical Logic) to *normalize* a proof, *i.e.* to transform a proof with cuts to an equivalent proof without cuts. As emphasized by Girard in [12], the cut-elimination process has also an inferential reading: it makes proofs *explicit*.

Secondly, let us observe that, in Ludics, we consider two modes of interaction: either we focus on the travel itself and we are interested in the possibility of the continuation, step after step, of a travel between two designs interacting each other, without paying attention to any context, or we focus on the transformation that this travel induces on some context or environment. In terms of Logic, the first case corresponds to a close situation: the interaction between proofs of dual formulas

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enables to decompose such formulas with respect to their structure [2], the second case corresponds to applying a modus ponens: from a proof of A and a proof of A 'implies' B, we get a proof of B. In terms of natural language and reasoning, the first case helps modelling interacting processes, *e.g. dialogues*, while the second one may be used for modelling *explicitation* processes, *e.g. calculus* or *deduction*.

1.2 Modelling Dialogues and Inferences in Ludics

Ludics has already been used in a series of papers for representing several aspects of Natural Language (*e.g.* [18, 19, 7]). They are all based on the idea that interaction is fundamental in Natural Language, dialogue being archetypical in that way. Remark that Ludics has also been used in the same spirit for formalizing web processes [6]. These modellings use the "primitive" objects of the interaction that Ludics set, upstream of a reconstruction of logic. In fact, Ludics is primarily *a theory of interaction* and its basic elements, *actions* and *paths*, are enough to grasp the rudimentary aspects of dialogical interactions. The formalisation of dialogue we obtain that way has to be related to the one obtained by means of Game Theory, see for example the study of dialogue done by Lascarides and Asher [16] or other works for the more specific case of argumentation dialogues [25, 21, 22, 20, 23]. However, note that several aspects of Ludics allow to go beyond what is done in Game Theory for modelling Natural Language: among them, an *action* has a more precise description than a *move* in Game Theory, and a specific action, the *daimon*, specifies the end of an interaction.

We must remark that the fact that Ludics is also a logical theory has not fully been taken into account in these previous works that consider Ludics for formalizing Natural Language. Let us shortly recall how Logic is retrieved from Ludics. The concrete basic steps of interaction (the actions) correspond to the basic steps of cut elimination. Hence a *path*, that is a sequence of actions used in an interaction, may correspond to the exploration of a formula through its main connective to its successive subformulas inside a proof. Finally, a design, that is a set of pairwise coherent paths, may correspond to a proof of a formula. However, a design abstracts from the notion of formula: a design has only loci, *i.e.* addresses, where interaction goes through. When a set of designs is given, the space where interaction with these designs may take place is also given. This space is defined by all the counter-designs of each design of this set. This gives an external viewpoint on designs, from which one may observe regularities. Formulas may then be retrieved as sets of designs closed relatively to these counter-designs. Indeed, exploring a set of designs reduces to exploring each design of this set. When it is closed, a set of designs describes all the ways to explore the object it represents, until its undecomposable elements. This has to be related to the concept of a formula, defined inductively by its connectives and subformulas that compose it until obtaining propositional variables or constants. Moreover, and this is an essential property of Ludics, some designs associated with a formula may denote proofs (of this formula): those designs that satisfy suitable properties, among which, precisely, the fact that the exploration can always continue until a suitable term. So, truthness of a formula requires the existence of some proof belonging to the set of designs associated to this formula.

We may then enrich the framework of our modelling, using still Ludics, to grasp the part of dialogue that is relative to *inferences*. This is particularly interesting when we consider argumentation dialogue as the authors did in [8]. As remarked previously, this latter dimension of dialogues takes

[2] Obviously, since we cannot consider literally simultaneously a proof of A and a proof of $\neg A$, the notion of design relaxes the one of proof.

sense according to an environment or a context. We propose in this paper to integrate to our framework the Information State Update approach (ISU) developed by various authors [15, 1, 5, 16], according to which inferences are managed in a *context of dialogue*. Of course, the logical dimension of Ludics is fully relevant for considering correctness (or not) of deductive inferences.

1.3 Our contribution

As mentioned above, previous works, *e.g.* [18, 19, 7], were devoted to model the *surface* of dialogues, which concerns both the intra-locutor justification relation and the inter-locutors connection relation, *i.e.* relations that may exist between dialogical interventions. We assume that this interactive level is central in order to grasp and understand other dimensions of dialogues: semantics (as started in [17]), syntax and even lexical aspects. In this paper we are interested in the pragmatical and rhetorical dimensions. Our aim is to make more explicit the continuity between the interactive structure of dialogues and the eristic or dialectic role of interventions occurring in such dialogues. Our approach integrates two complementary aspects: one concerning pragmatical and rhetorical dimensions of dialogue, and another one concerning inferences that may take place inside an argumentation as well as the process of making facts explicit in informative dialogues. The content of this paper is organized as follows:
– in section 2, we sketch a model for pragmatical and rhetorical aspects of dialogue after a brief review of the modelling in Ludics of the interactive aspect of dialogues;
– in section 3, we show how taking into account inferences that occur during a dialogue, with respect to a ISU-like model of dialogue. Through various examples we give an analysis of deductive inferences as well as calculus/'explicitation' processes that take place during knowledge updating. Because of lack of space, we do not recall formal aspects of Ludics. However the reader may find complete presentations in [13, 3, 4].

2 Modelling Dialogues

The modelling of dialogues in Ludics may be considered as a refinement of what is done with Game Theory [16]. Roughly speaking, each locutor is a player in a game, the dialogical interventions are represented by *moves* and a dialogue itself is represented by means of an alternate sequence of moves, that is a *play*. Nevertheless, in Ludics, the duality, expressed by alternating positive and negative actions, does not correspond to a question/answer duality as in game semantics but to a production/reception duality in dialogue modelling. A positive action models the production of an utterance whereas a negative action is an expectation of utterances. A turn of speech is then twofold: producing an utterance to her interlocutor and expecting an utterance from her interlocutor. A dialogue defines then two sequences, called *paths*: each of them being the dialogue seen from the viewpoint of one locutor. Finally each path defines a design, hence the dialogue may be considered as the trace of the interaction between two designs, one for each locutor. We show in this section that the framework of Ludics, in particular the use of designs, is useful to give an account of various aspects of dialogues, namely presuppositions, divergence, argumentation, ...

The starting point of the formalisation is only based on the interactive feature of dialogues. This level is already enough to take into account some important elements of dialogues. We refine then this first approach in order to take care of other dimensions of dialogue. We focus on the pragmatical and rhetorical levels.

2.1 Interactive level

At this level, we only consider the interactive *surface* of dialogues: a model for the intra-locutor *justification relation* and for the inter-locutor *connection relation*. With respect to this interactive level, dialogues are only alternate sequences of interventions among which we may distinguish the one which initiates the exchange and the one which eventually ends the exchange. Interventions are only considered according to:
– their role in the flow of interventions: one intervention is anchored on a previous one and opens possible continuations of the dialogue (justification relation),
– the fact that they are produced by one locutor while they are in the same time received by the other locutor (connection relation).

An alternate sequence of interventions may then be represented by an alternate sequence of actions in Ludics, as illustrated in the following example:

EXAMPLE **1** *(C. Dickens, David Copperfield) The following dialogue is extracted from the novel "David Copperfield" of C. Dickens. The dialogue takes place between a coachman (***C***) and David (***D***), the coachman brings David to London:*

C	I_1 :	*You are going through, sir?*
D	I_2 :	*Yes, William. I am going to London.*
		I shall go down into Suffolk afterwards.
C	I_3 :	*Shooting, sir?*
D	I_4 :	*I don't know.*
C	I_5 :	*Birds is got wery shy, I'm told*
D	I_6 :	*So I understand*
C	I_7 :	*Is Suffolk your county, sir?*
D	I_8 :	*Yes, Suffolk's my county.*
C	I_9 :	*The dumplings is uncommon fine down there ...*

As a first approximation, an action $\kappa_1, \ldots, \kappa_9$ is associated to each utterance $I_1, \ldots, I_9$. The justification between actions is given Fig. 1, on right. The first action κ_1 is initial, indeed, with this intervention **C** initiates the dialogue. Actions κ_3, κ_7 and κ_9 are justified by the second intervention κ_2: in the three cases, these interventions refer to 'Suffolk', entity present in the second intervention that may introduce topics concerning hunting, native soil, gastronomy ... Each other intervention is justified by the action that precedes it immediately where the focus is given.

Ludics allows for rebuilding the interaction that produces this alternate sequence of interventions as a trace. The alternate sequence of interactions $\kappa_1, \ldots, \kappa_9$ may be represented choosing the positive polarity for the interventions of **C**: in that way, the dialogical interaction is represented with the point of view of **C**. Reversing the polarities, one gets the point of view of **D**. The trace of interaction between the two designs is the dialogue. In left part of Fig. 1, the design on the left is the point of view of **C**, and the design on the right is the point of view of **D**, circled actions are the positive ones, black edges account for the justification relation, the flow of red arrows is the interaction between the two designs.

Viewing the dialogue as a trace makes explicit the fact that there are two points of view. Moreover it allows to observe the success or the failure of these two points of view, *i.e.* the fact that the dialogue may fail or end with a drop. Dialogues may badly end because of misunderstandings, disagreements, ... and it is necessary to be able to represent such situations. Ludics distinguishes

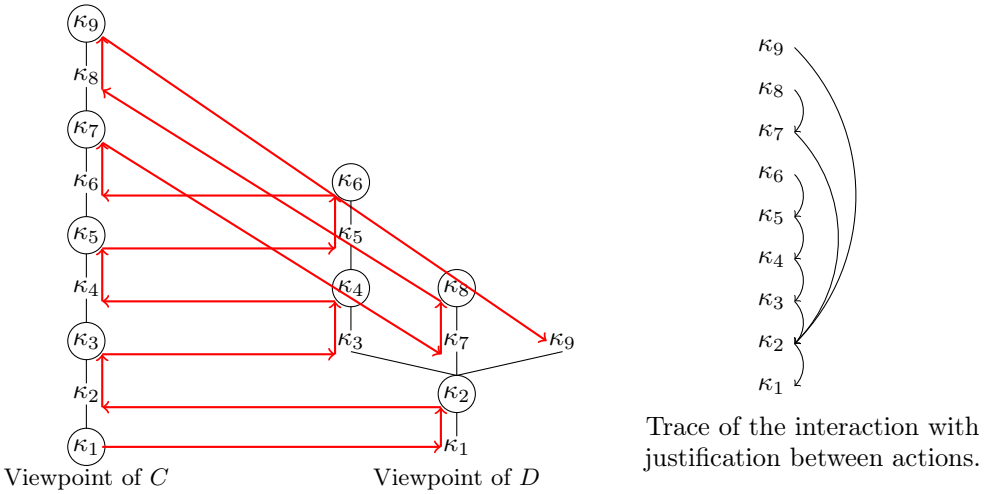


Figure 1: The interaction between two designs/viewpoints

two cases. An interaction is convergent when it ends with a daimon[3], it is divergent when a positive focus has no dual counterpart. These two cases allow for interpreting two standard final situations in dialogues: either the dialogue finishes well or there is a misunderstanding between the two locutors. The following example is given by M. Chemillier [2] to illustrate the difficulties that arise when one wants to isolate the logical part of a dialogue.

EXAMPLE **2** *A person* **P** *conducting a survey gives to a native the following informations:* All the Kpelle cultivate rice. Mister Smith does not cultivate rice*, the person* **P** *asks the following question to the native* **N***:*
- **P***: "Is Mister Smith a Kpelle ?" (κ_1)*
- **N***: "I do not know Mister Smith, I have never seen him." (κ_2)*

Here we consider only the question and its answer that we represent each by a single action (resp. κ_1 and κ_2). The person **P** expects a logically correct answer, *e.g. "no"*, we note κ_3. He plans also to receive an incorrect answer, *e.g. "yes"* or *"it may be the case"*, we note κ_4. Hence the design for the person **P** may be either $\textcircled{\kappa_1}$ followed by κ_3, or $\textcircled{\kappa_1}$ followed by κ_4. The interaction in Ludics is given Fig. 2: it is divergent as there is no negative action dual to κ_2 in what is expected in the design of **P**. The dialogue cannot continue.

2.2 Pragmatical and Rhetorical Levels

In order to take into account other dimensions of dialogues, the formalisation is refined. A first step is to replace the notion of action by the one of *dialogue act*. The notion of dialogue act is defined by Landragin [14] as "the minimal unit of communication in a dialogical context". A dialogue act is a communicational fact whose role is to fuel the dynamic and determine the shape of the dialogue. It may be explicit or implicit, verbal or not (*e.g.* an acknowledgment given as a gesture). It may appear

[3]A special positive action, which is not justified and which necessarily ends paths.

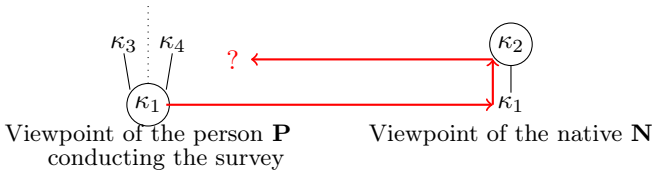


Figure 2: A divergent dialogue about Kpelle.

as one or more propositions, but also as part of a proposition (word, adverb, ...). It expresses an entitlement or a decision of the speaker, and also its acknowledgment by the addressee. In some sense, it is quite close to a speech act. However a speech act may correspond to several dialogue acts as shown in following examples. Dialogue acts are indeed more elementary than speech acts. They can be seen as the basic blocks from which one builds interpretation for dialogical interventions or even utterances. Formally, a dialogue act may be defined using the complete definition of action in Ludics: a polarity, an address and a finite set of new addresses which may be used as adresses of following actions. A dialogue act is such an action together with the expression that reveals the dialogue act in the intervention. Such an expression may be a proposition, a word (*e.g.* a single adverb, a noun), a prosodic feature, a non verbal sign (a nod, a shake, a slap, ...). In trivial cases, an intervention is a unique dialogue act. Otherwise a turn of speech has to be decomposed into a sequence of dialogue acts, hence may give rise to a complex design. Note that the representation of an utterance in terms of dialogue acts is dependent of the context of the dialogue, and in particular of past interventions that occurred.

Definition 1 (Dialogue Act) *A* dialogue act κ *is:*

- *either a proper dialogue act, that is to say a tuple* (ϵ, ξ, I, e) *where*
  - *the finite sequence of integers* ξ *is the* focus *of* κ*: the location of the act with respect to the dialogical interaction one considers,*
  - I *is the* ramification *of* κ*: the openings created by the dialogue act on which new dialogue acts may be produced,*
  - e *is the expression of the dialogue act, that is to say the language or communicational fact by means of which the dialogue act manifests itself,*
  - *the polarity* ϵ *of the act may be positive* $(+)$ *or negative* $(-)$*. The act is positive for the locutor that produces it except when this act is a constraint from its interlocutor (see below presupposition case). These acts have a dual polarity when received by the interlocutor.*
- *or a particular positive dialogue act, still called* daimon *anot noted* $(\dagger, e)$*, that registers the end of an interaction that went well. In that case, the expression* e *of the dialogue act may often be empty.*

Our modelling of dialogues in Ludics, together with the notion of dialogue act, allows for analysing different pragmatical aspects as well as rhetorical features. These two dimensions appear as being closely linked: both concern primitively dialogues, both are central in argumentation

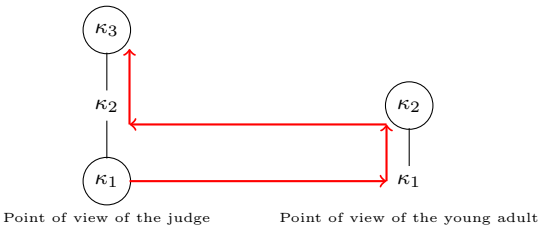
dialogues, both are at the frontier of linguistics and philosophy and both are closely related to Logic. For example, the pragmatical notion of *speech acts* is systematically used for describing argumentation dialogues by means of an almost exhaustive list of *speech acts of argumentation*. The notion of fallacies seems being at first relevant for rhetorical issue but the *presuppositions* are one of the current study objects of Pragmatics. Without trying to radically separate these two dimensions, the sole interactive level of dialogues, enriched by the notion of dialogue act enables to characterize different fallacies as well as speech acts of argumentation, according to their behaviour with respect to interaction (see also [18, 8]). We illustrate below the modelling of fallacies in Ludics on an example of presupposition.

2.2.1 Presupposition in Dialogues

A presupposition is an implicit assertion concerning the world, whose validity is accepted in the dialogue.

EXAMPLE **3** *Let us consider this well known example due to Aristotle; a judge asks a young delinquent this question: "Have you stopped beating your father?".*

Answering this question by yes or no supposes that the answer of the implicit question is yes:
– Q_1*: "Did you beat your father?"*
– R*: "Yes."*
– Q_2*: "Have you stopped beating him?"*
This may be modelled by the following interaction between two designs:



where the dialogue acts κ_1, κ_2 and κ_3 refer respectively to interventions : "Did you beat your father?"; "Yes"; "Have you stopped beating him?".

The question asked by the judge, "Have you stopped beating your father?", is a sequence of three dialogue acts: $(\kappa_1, Q_1)(\kappa_2, R)(\kappa_3, Q_2)$. If the young adult continues the dialogue, he implicitly assumes that the two (partial) designs do not diverge. He accepts to justify the intervention he will have on the negative dialogue act dual to (κ_3, Q_2), this dialogue act being anchored on the positive dialogue act dual to (κ_2, R): he assumes he would have answered yes to the implicit question.

2.2.2 Argumentation Dialogues

Argumentative dialogues show some peculiarities. In particular speech acts that compose them are limited in number: assertions, arguments, denial, concessions, ... In addition, a notion of winnings appears, which seems specific to argumentative dialogues. In a dialogue whose aim is to exchange information, to share knowledge or a feeling, the question of whether one of the speakers wins

is quite irrelevant. But to know who is *right* after a controversy is essential. An argumentative dialogue is distinguished from (common) dialogues in the sense that two arguments are opposed, each party having his thesis to defend or advance. The dialogue is then mainly an exchange of arguments and counter-arguments with the sole purpose that the dialogue ends when a thesis is considered *winning*.

In order to take care of argumentative dialogues, we give below an account of speech acts of argumentation by means of dialogue acts. It is necessary to precise what is a "gain". One of the locutors becomes a "loser" in the following situation: he abandons the dispute conceding the last arguments of his addressee (who becomes the winner): the winner has the last word. We can precise the situations when a daimon is used: either the set of addresses where a dialogue act may take place is empty, hence the daimon is the only possible dialogue act and the opponent had really "the last word", or there exist still locations where actions may take place and the locutor prefers to quit the dialogue. Note that this notion of winner is linked to a specific dialogue: it may be the case that the current loser may be a winner with another opponent. We must mention that the notion of "winning" design exists in Ludics: a design is winning if, in particular, it does not contain the daimon. Hence every interaction with such a design is either divergent or ends with a daimon in the opposite design: such a design wins every time.

The speech acts of argumentation:

- **To claim a thesis** (to affirm, to assert). This corresponds to posing a proposition on which the interlocutor may continue the dialogue by negating it or by conceding it or by asking more explanations. Such a proposition may constitute the initial thesis of a dialogue, it may also be an additional element for an on-going discussion. In our Ludics framework, this is represented by a proper and positive dialogue act $(+, \xi, I, e)$, that may be initial or justified by one of the previous dialogue acts that introduces one of the subjects of the dialogue.

- **To argue**. This corresponds to posing propositions that serve as argumentative premises and in the same time the sound structuring between these premises and the conclusion, *i.e.* the fact on which the locutor argues, that should be a location open to discussion. For example, the locutor may affirm a thesis A and justify it with B and $B \Rightarrow A$, hence propositions that are posed are actions corresponding to the two elements B and $B \Rightarrow A$. This speech act is represented by a unique positive proper dialogue act $(+, \xi, I, e)$. Contrarily to the previous item, the ramification I, hence the locations opened by this act, cannot be a singleton: it contains at least all the premises and the relation between the premises and the conclusion.

- **To negate** (to refuse). We consider this speech act as a kind of thesis, a claim of the locutor. However such a claim cannot be an initial act of the dialogue: it is a refusal of a previous claim[4]. In a general setting, in such a turn of speech, the locutor may negate and pose a counter-argumentation by giving new facts. It is represented as a proper positive dialogue act $(+, \xi, I, e)$, necessarily justified by a previous act that introduces the proposition that is negated.

- **To ask, to request some justification**. With respect to the structure of a dialogue, there is no difference with the previous item except the linguistic form it has.

[4]Note that an initial claim may also be a negation. There is a difference between beginning a dialogue with, *e.g.*, "Paul did not come yesterday" and saying "You are wrong. Paul did not come." as a reply to "Paul came to the party yesterday."

- **To concede**. This consists in accepting one assertion that the interlocutor claimed. Hence the dialogue will not continue on this element. As such, a concession is only a part of a turn of speech: either the dialogue ends with this (last) concession and the locutor abandons, or the locutor continues its intervention on another element still under discussion. A sequence of two dialogue acts is associated with the concession: $(+, \xi, \{0\}, e_0), (-, \xi.0, \emptyset, e_1)$ where
  - the first one, positive, enables to express what is conceded, its focus is the one created by the affirmation that one concedes and its ramification is a singleton.
  - The second dialogue act is a negative one, it is focused on $\xi.0$: the locus that the immediate previous act created and it has an empty ramification. The effect is that the conceded affirmation disappears.

- **To abandon**. This act ends an argumentation dialogue in a convergent way. The locutor does not mention any misunderstanding but recognizes he has nothing more to add. The positive dialogue act $(\dagger, e)$ fits exactly to this situation. Its use finishes the interaction.

In order to illustrate the effect of an intervention during an argumentation dialogue, we give below an example due to Prakken [23] and that we analyzed more exhaustively in [9].

EXAMPLE **4** *A juridical controversy between a plaintiff P and a defendant D starts as follows:*

I_1	*Plaintiff:*	*I claim that defendant owes me 500 euro.*
I_2	*Defendant:*	*I dispute plaintiff's claim.*
I_3	*Plaintiff:*	*Defendant owes me 500 euro by r_1 since we conclude a valid sales contract, I delivered but defendant did not pay.*
I_4	*Defendant:*	*I concede that plaintiff delivered and I did not pay, but I dispute that we have valid contract.*

To start, we focus on the argumentative dialogue. According to our modelling, the two first interventions I_1 and I_2 create each only a new locus, respectively L_1 and L_2. On the contrary, the intervention L_3 is represented by a dialogue act which creates four new loci. Indeed the utterance I_3 '*Defendant owes me 500 euro by r_1 since we conclude a valid sales contract, I delivered but defendant did not pay*' makes explicit four argumentative elements to support his assertion '*Defendant owes me 500 euro'*. These argumentative elements are: a law article r_1; the fact that both parties concluded a valid contract; the fact that plaintiff delivered his service and at last, the fact that defendant did not pay 500 euros. By means of this dialogue act, we indicate that the dialogue may continue using loci $L_{3_1}, \ldots, L_{3_4}$ respectively associated with these four elements.

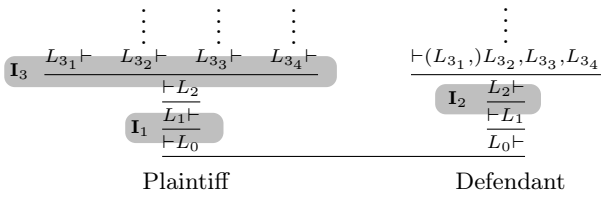


Figure 3: A juridical example: Interventions I_1 to I_3

Intervention I_4 is more complex, it is not given into account by a unique dialogue act but by a sequence of several ones. Indeed, in intervention I_4, defendant concedes two plaintiff's elements:

'*plaintiff delivered*' and '*I did not pay*'. These two concessions are successively represented, each of them being a positive dialogue act followed by a negative one with an empty ramification, at first above L_{3_4} then above L_{3_3}. At last, intervention I_4 ends by negating the factual proposition '*The parties concluded a valid sales contract*', this is represented by a positive dialogue act focused on L_{3_2} and with a singleton as ramification. The argumentative dialogue until this fourth intervention is represented Fig. 4.

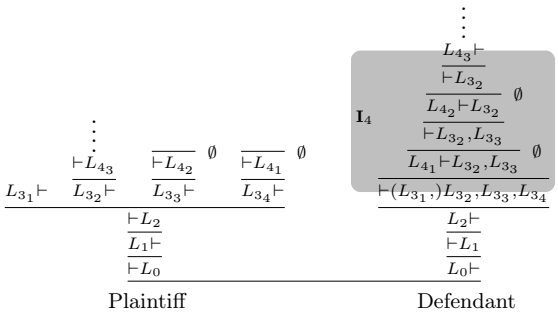


Figure 4: A juridical example: Intervention I_4

2.2.3 Other Speech Acts

Providing an information or asking a wh-question may also be represented. In such cases, dialogue acts must be combined with operations on a knowledge base.

- When a locutor gives an information i, she not only produces a dialogue act $(+, L_i, \{0\}, v)$, by means of which she expresses the value v of the information, but also she *relocates* this information from her knowledge base to the locus of the dialogue.

- When a locutor asks a wh-question, she not only formulates her question but she is also ready to receive and register an answer. Even if the speech act starts with a first positive dialogue act $(+, \xi, \{0\}, qwh)$, where qwh is the expression of the question, it is not reduced to the dialogical expression. The speech act continues with other *internal calculi*. The previous dialogue act is followed by all the possibilities to receive an answer and then to register it by means of a *fax*.

We develop in the next section a way to compute inferences[5] by means of internal calculi performed by locutors.

3 Inferences in Dialogues

In dialogues, interactions occur not only at the *surface* level, *i.e.* the distribution of turn of speech, but also between the contents of interventions or with knowledge. We focus in this section on

[5]Making facts explicit as well as making deductive inferences.

inference modelling. We use such a term to refer to calculi that make explicit elements that are exchanged during a dialogue.

For that purpose, we have to give an account for the *state of knowledge* (dialogue occurring for exchanging information) or the *state of commitments* (argumentation dialogue) for each locutor and which both constitute the context of the dialogue. We give an account for these contextual elements by means of two *context (of dialogue) states*: one for each locutor. These context states are increased during the dialogue by elements arising from the dialogue. Then a dialogue between two protagonists S and A is represented similarly to, say, [10, 11] or [20], that is by a sequence of dialogue states. The specificity here is the definition of the dialogue state: it is a triple of data, the context state of S, the context state of A, and the net of two designs associated with the current dialogue.

3.1 Information Exchanges

By means of the notion of *context state*, we may complete the account of speech acts performed during a dialogue with exchange of information. The dialogue acts that we pointed in section 2.2.3 are only the publicly useful parts of more complex elements contained in (private) context states.

- **To ask a wh-question**. In such a speech act, the locutor not only formulates her question but she is ready to receive and register an answer. The full speech act (see Fig. 5) may be taken into account by a design which contains not only the dialogue act corresponding to the formulation of the question but also the *inference tools* to *compute* the reception and the registration of possible answers. The base of such a design contains two positive loci: one for anchoring the dialogue act and one for registering the answer. It is built as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash L_{k_0}}\,\emptyset}{L_{k_0=data} \vdash L_{k_0}}\,\emptyset}{\vdash L_{val=k_0}, L_{k_0}}}{L_{val} \vdash L_{k_0}}}{\vdash L_{k_0}, L_i} \quad \ldots \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash L_{k_n}}\,\emptyset}{L_{k_n=data} \vdash L_{k_n}}\,\emptyset}{\vdash L_{val=k_n}, L_{k_n}}}{L_{val} \vdash L_{k_n}}}{\vdash L_{k_n}, L_i}}{L_{wh} \vdash L_i}}{\vdash L_0, L_i}\,\kappa_1$$

Figure 5: Speech act for a wh-question

The speech act starts with a first positive dialogue act $\kappa_1 = (+, L_0, \{wh\}, qwh)$, where qwh is the expression of the question, something like *What is the value of i ?*. It is followed by all the anticipation of possible answers, *i.e.* several negative actions, namely the $(-, L_{wh}, \{k\})$'s for each value k of the data[6]. Then the design contains several actions which enable to register on another locus L_i the value v of the data expected. Indeed when a sequence of dialogue act $(+, L_k, \{0\}, on - data - i)(L_{val}, \{0\}, \epsilon)(+, L_{val=v}, \emptyset, val = v)$ is given by the other locutor, the questioner perfoms internally the interaction between this answer-design and his question-design and the result is exactly the design corresponding to the information *the value of the data i is equal to v*, that is the following design: $(+, L_i, \{0\}, thedatais)(-, L_{wh}, \epsilon)(+, L_v, \emptyset, equal - to - v)$.

[6]Infinite number of answers may be taken into account nicely using Terui's c-designs [24]

3.2 Argumentative Disputes

In case of controversies, we model the commitment states of each locutor; that is the part of context states consisting in the statements which are publicly assumed by each locutor (often as propositions). Indeed, one of the issue of an argumentation dialogue is to evaluate the correctness of such commitments and to eventually update them.

The commitment state is composed by commitment elements (C.E.). Here, for simplicity reasons, we only consider them occurring as *factual propositions* and *inferential propositions*. Moreover we give a minimal account of a factual proposition 'F': a C.E. reduced to a unique design based on $\vdash L_F$. The minimal account of an inferential proposition 'F implies G' is a C.E. also reduced to a unique design based on $L_F \vdash L_G$[7].

The management of commitment states, their updating and their use in the dialogue in progress make use of the logical aspect of Ludics. We give below an account of the negation of propositions.

- **Negation.** We distinguish between a positive factual proposition: *something is the case* and a negative factual proposition: *something is not the case*. The C.E. associated with the negative factual proposition is the design which expresses the logical negation of the positive one. More precisely the following designs are respectively associated with the factual propositions 'F *is the case*' and 'F *is not the case*', when F is an *atomic* factual proposition (L_F, resp. L_{notF}, is the location of F, resp. *not* F):

$$\frac{}{\vdash L_F}\ \emptyset \qquad\qquad \frac{\dfrac{\dfrac{}{\vdash}\ \dagger}{L_F \vdash}\ \emptyset}{\vdash L_{notF}}$$

  Factual propositions that are logically decomposable may also be represented as C.E. In such cases designs start with actions corresponding to the adequate logical decomposition.

  As an example, the following designs are respectively associated with the factual propositions: '*the conjunction of* F *and* G *is the case*' (located at L_{FandG}) and '*the conjunction of* F *and* G *is not the case*' (located at $L_{not(FandG)}$):

$$\frac{\dfrac{\dfrac{}{\vdash L_F}\ \emptyset}{L_{notF} \vdash} \quad \dfrac{\dfrac{}{\vdash L_G}\ \emptyset}{L_{notG} \vdash}}{\vdash L_{FandG}} \qquad \frac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash}\ \dagger}{L_F \vdash}\ \emptyset}{\vdash L_{notF}}}{L_G \vdash L_{notF}}\ \emptyset}{\dfrac{\vdash L_{notF}, L_{notG}}{\dfrac{L_{FandG} \vdash}{\vdash L_{not(FandG)}}}} \qquad \frac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash}\ \dagger}{L_G \vdash}\ \emptyset}{\vdash L_{notG}}}{L_F \vdash L_{notG}}\ \emptyset}{\dfrac{\vdash L_{notF}, L_{notG}}{\dfrac{L_{FandG} \vdash}{\vdash L_{not(FandG)}}}}$$

  Remarks :

[7] Notice that we do not suppose that a factual proposition is necessarily a *logical formula*. In Ludics the notion of logical formulas is replaced by the more general notion of behaviour, *i.e.* a closed set of designs with same base. Here we follow a still more general setting by putting in a commitment state some sets of designs not necessarily closed. Even if we do not completely exploit this in this paper, it should give the opportunity of a fine-grained account of knowledges. Indeed some knowledges are assumed as fully explicit propositions while others may be only partially assumed: one or several logical propositions may be associated with a given utterance without their full (until atomic ones) decompositions being known.

1. Loci are indexed by their intuitive interpretation. These addresses have not to be confused with the logical notation of propositions.
2. For a strictly commutative interpretation of the conjunction, we need both the two designs (in the right in previous figure) in the C.E. associated with *the conjunction of F and G is not the case.*

Let us illustrate how such an argumention dialogue modelling works on the example presented in section 2.2.2: In the juridic dialogue, intervention I_3 results from a logical calculus. Let us detail how we interpret this intervention with respect to the commitment state of plaintiff P. Plaintiff gives several informations: *there is a valid contract between P and D*, *P delivered*, and P applies the law r_1, the implicit content of r_1 is *'If there is a valid sales contract between a vendor and an emptor and if the vendor delivers then the emptor has to pay'*. The following C.E. are added to the commitment state of P by this third intervention:

- The design associated with the law r_1 is based on $L_{Cont-and-Del} \vdash L_{to.pay}$. This is a rather elaborated design:

$$\mathfrak{D}_{r_1} = \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash L_{to.pay}}\,\emptyset}{L_{Del} \vdash L_{to.pay}}\,\emptyset}{\vdash L_{notDel}, L_{to.pay}}}{L_{Cont} \vdash L_{notDel}, L_{to.pay}}\,\emptyset}{\vdash L_{notCont}, L_{notDel}, L_{to.pay}}}{L_{Cont-and-Del} \vdash L_{to.pay}}$$

- The designs respectively associated with the assertions '*there is a valid contract between P and D*' and '*P delivered*' are the following:

$$\mathfrak{D}_{cont} = \dfrac{}{\vdash L_{Cont}}\,\emptyset \qquad \mathfrak{D}_{del} = \dfrac{}{\vdash L_{Del}}\,\emptyset$$

- There is also in the commitment state of P the following design associated with the fact that defendant did not pay (interpretation of intervention I_1):

$$\mathfrak{D}_{not.paid} = \dfrac{\dfrac{\dfrac{}{\vdash}\,\dagger}{L_{to.pay} \vdash}\,\emptyset}{\vdash L_{notPay}}$$

Loci $L_{3_1}, \ldots, L_{3_4}$ respectively associated with the argumentative elements in intervention I_3 are delocations of loci on which the four designs $\mathfrak{D}_{r_1}$, $\mathfrak{D}_{cont}$, $\mathfrak{D}_{del}$ and $\mathfrak{D}_{not.paid}$ are anchored[8]. A computation using these designs yields a contradiction: indeed, the result of the interaction between designs $\mathfrak{D}_{cont}$, $\mathfrak{D}_{del}$ and $\mathfrak{D}_{r_1}$ (by using additional logical steps) is the following design:

$$\mathfrak{D}_{to.pay} = \dfrac{}{\vdash L_{to.pay}}\,\emptyset$$

Then the interaction between this latter design and $\mathfrak{D}_{not.paid}$ results in a logical contradiction.

The following of the dialogue will consist in giving arguments until the contradiction may be solved.

[8]We may notice by this way the continuity from internal computations to the interactive level of a dialogue.

4 Conclusion

Ludics is a theory of Logic however its objects are more general than formulas and proofs. This possibility to move around the logical frontier is interesting for studying argumentation as done in [8]. In particular, we are not obliged neither to associate a *logical formula* to each dialogical intervention nor to grasp the streams of arguments by means of already logically correct inferences. Since we may separate the dialogical dimension, according to which the streams of arguments take place, and the inferential dimension, we may grasp one of the main issue of controversies: go towards the elucidation of a statement which was not a priori given and take into account correctness/incorrectness of asserted deductive inferences without requiring non monotonic logics. Finally, such a framework enables to model cross-effects between the various levels of dialogue analysis as one kind of objects, *i.e.* designs, may represent dialogues viewed as interacting processes as well as *explicitation* processes, or in other words as interaction is at the core of Ludics and gives an account for cut as well as cut-elimination. Our future research will focus on improving the relation between surface vs pragmatical and rhetorical levels, and on extending this model to other levels of dialogue analysis.

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