J. Fluid Mech. (2013), vol. 720, pp. 5–14. © Cambridge University Press 2013
doi:10.1017/jfm.2012.623

The competition between gravity and flow focusing in two-layered porous media

Herbert E. Huppert[1,2,]†, Jerome A. Neufeld[1,3,4] and Charlotte Strandkvist[5]

[1]Institute of Theoretical Geophysics, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, CMS Wilberforce Road, Cambridge CB3 0WA, UK

[2]Department of Mathematics, University of New South Wales, Sydney, New South Wales 2052, Australia

[3]BP Institute, University of Cambridge, Bullard Laboratories, Madingley Road, Cambridge CB3 0EZ, UK

[4]Department of Earth Sciences, University of Cambridge, Bullard Laboratories, Madingley Road, Cambridge CB3 0EZ, UK

[5]Centre for Mathematics and Physics in the Life Sciences and Experimental Biology, University College London, Gower Street, London WC1E 6BT, UK

(Received 13 July 2012; revised 6 December 2012; accepted 13 December 2012)

The gravitationally driven flow of a dense fluid within a two-layered porous media is examined experimentally and theoretically. We find that in systems with two horizontal layers of differing permeability a competition between gravity driven flow and flow focusing along high-permeability routes can lead to two distinct flow regimes. When the lower layer is more permeable than the upper layer, gravity acts along high-permeability pathways and the flow is enhanced in the lower layer. Alternatively, when the upper layer is more permeable than the lower layer, we find that for a sufficiently small input flux the flow is confined to the lower layer. However, above a critical flux fluid preferentially spreads horizontally within the upper layer before ultimately draining back down into the lower layer. This later regime, in which the fluid overrides the low-permeability lower layer, is important because it enhances the mixing of the two fluids. We show that the critical flux which separates these two regimes can be characterized by a simple power law. Finally, we briefly discuss the relevance of this work to the geological sequestration of carbon dioxide and other industrial and natural flows in porous media.

Key words: geophysical and geological flows, gravity currents, porous media

1. Introduction

The flow of fluids within porous media is of great importance in a host of environmental and industrial contexts. In many instances this flow is driven by the gradients in the hydrostatic pressure due to the density contrast between the injected fluid and the ambient, a flow termed a gravity current. In the environment these flows are manifest at the interface of fresh water and brine in underground aquifers and are important to many industrial problems, such as those involving

† Email address for correspondence: heh1@esc.cam.ac.uk

the flow of oil and carbon dioxide in geological reservoirs (Bear 1972; Phillips 2009). More recently, the rise in mean global surface temperature associated with the rise in atmospheric carbon dioxide (CO_2) concentrations has generated considerable interest in the injection of compressed CO_2 into large saline aquifers. This process, in which CO_2 is injected and spreads throughout the underground formation driven by the buoyancy contrast between the plume and the ambient fluid, is likely to be strongly influenced by the presence of permeability heterogeneities on all scales. These permeability heterogeneities are often vertically layered, corresponding to the geological history of the particular formation.

Buoyancy-driven flows, or gravity currents, within porous media have been studied by numerous authors and recently reviewed by Huppert (2006) and Phillips (2009). In cases in which an impermeable barrier directs the flow, they quickly relax to spreading with a self-similar form. When the depth of the gravity current is much less than the thickness of the permeable layer, solutions have been found in two dimensions (Bear 1972; Barenblatt 1996), where a current with constant input flux spreads as $t^{2/3}$, and in axisymmetric geometries (Lyle *et al.* 2005), where a current of constant input flux spreads as $t^{1/2}$ (although they also present results for arbitrary power law influx conditions). Anderson, McLaughlin & Miller (2003) investigated the propagation of a buoyant current in an unconfined, vertically layered porous medium. Their work showed that, for small variations in permeability, the similarity solution presented by Bear (1972) and Barenblatt (1996) describes the bulk flow well, with small variations of the underlying permeability structure resulting in correspondingly small perturbations to the current profile. This contrasts with the present work in which strong vertical variations in the permeability structure are considered. Such variations dramatically effect the flow field and result in a behaviour which is distinctly not self-similar.

The effect of permeability variations has also been studied within the context of confined aquifers. When the current is confined between two relatively impermeable media, Huppert & Woods (1995) found the profile of the current, as seen in the frame of reference moving with the bulk flow velocity, spreads as $t^{1/2}$. For confined layers inclined to the horizontal Huppert & Woods (1995) were able to find similarity solutions for the spread of gravity currents through rocks with the permeability being a power of the vertical coordinate. More recently Cinar *et al.* (2006) experimentally investigated the injection and propagation of a multi-phase current within a confined bead pack with two distinct horizontal permeability layers. Using the width of the confined pack as the relevant length scale they characterized their experiments in terms of a Bond number (relating the importance of capillary forces to gravity) and a parameter characterizing the relative importance of vertical to horizontal flow.

In the present paper we consider the effects of a strong vertical contrast in permeability on the unconfined flow of a dense, miscible fluid. We begin in § 2 by presenting the geometry of the porous medium which motivates a scaling analysis for flow in a layered porous medium. In § 3 we experimentally consider the injection of a dense current into a vertically layered porous medium. The results of a suite of experiments, in which the injection rate, current properties and the permeabilities of the porous layers were varied are presented and compared to the theoretical scaling analysis. Finally, in § 5 we discuss the implications of these experiments for a host of environmental and geological processes.

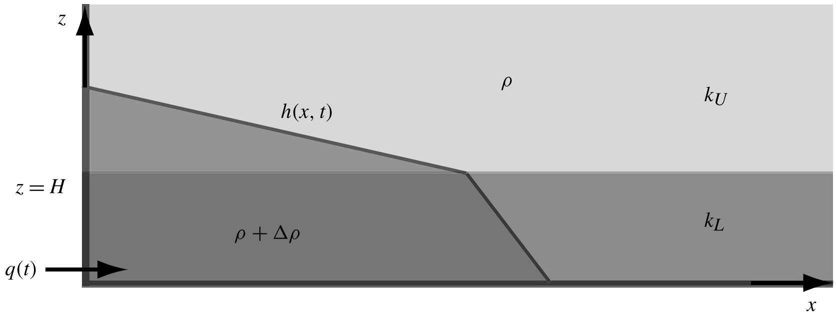


FIGURE 1. Schematic illustration of a current of density $\rho + \Delta\rho$ and height $h(x, t)$ intruding into a two-layered porous medium with interstitial fluid of density ρ and with permeability k_L between $0 < z < H$ and permeability k_U for $z > H$.

2. Geometry and scaling analysis

Consider the injection of fluid at a constant flux q and of density $\rho + \Delta\rho$ into a two-layered porous medium stratified with respect to permeability and saturated in fluid of density ρ as shown schematically in figure 1. We consider a finite lower layer of permeability k_L and depth H overlain by a semi-infinite layer of permeabililty k_U. Importantly we note that, with the Boussinesq approximation, the analysis applies both to the spreading of a dense fluid at the impermeable base of a two-layered porous medium as pictured in figure 1 and to the spreading of a buoyant fluid injected at the impermeable top of a two-layered porous medium, as might be the case for the injection of carbon dioxide.

The system can be modelled by considering flow within the two-layered porous medium governed by Darcy's law along with a statement of local conservation of mass as expressed by

$$\mu \boldsymbol{u} = -k(\boldsymbol{\nabla} p + \rho g \hat{\boldsymbol{z}}), \tag{2.1}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, \tag{2.2}$$

where the permeability k takes on the value k_L or k_U in the lower or upper layer respectively, $\boldsymbol{u}$ is the Darcy velocity, μ is the dynamic viscosity (assumed equal between ambient and injected fluids), p is the pressure and g is the acceleration due to gravity. Far from the source, fluid flow is driven primarily by the density difference between the injected fluid and the ambient. The pressure is mainly hydrostatic and the resultant horizontal flow is driven primarily by the horizontal gradients in the height of the fluid–fluid interface, $h(x, t)$. Therefore, in the limit of buoyancy-driven flow the fluid has characteristic buoyancy velocity $u_b = k_L g'/\nu$ where $g' \equiv g\Delta\rho/\rho$.

Previous studies of buoyancy-driven flow within homogeneous porous media by Huppert & Woods (1995), Barenblatt (1996) and Lyle *et al.* (2005) have modelled the evolution of the free surface of the buoyant current as self-similar due to the lack of a natural length scale. In the present paper the depth of the lower layer H provides a natural length scale (except at early times for a sufficiently small input flux) and hence no similarity solution can be found. Instead, the system is characterized by a competition between the effects of flow focusing through the high-permeability medium and the gravity-driven flow of a buoyant fluid.

The presence of a layer of permeability k_L and thickness H introduces two dimensionless parameters governing the evolution of this system. For flow within a

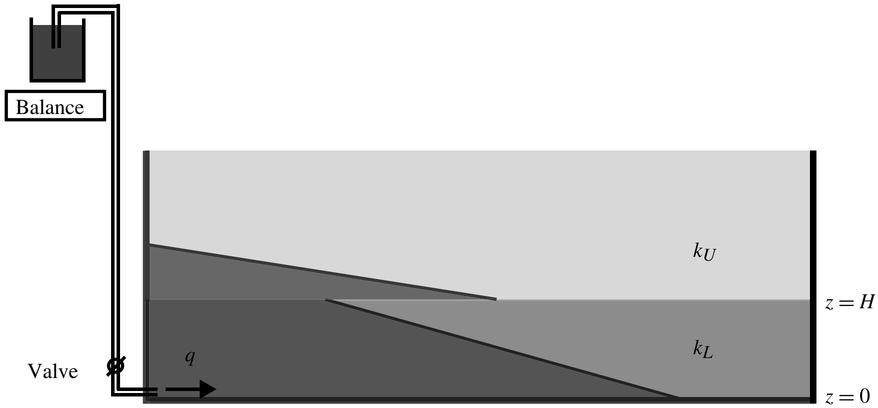


FIGURE 2. The experimental setup used to investigate gravity currents propagating through two-layered porous media. A constant flux was maintained by, and measured from, an elevated reservoir. The fluid then entered a porous medium composed of ballotini differing in size. The resultant currents were imaged using a digital camera.

porous media driven by gradients in the hydrostatic pressure, and hence depth, of the buoyant fluid we scale the depth of the fluid by H and time by H/u_b. We are left with two dimensionless groups. The ratio of upper and lower permeabilities

$$\Lambda = k_U/k_L, \tag{2.3}$$

and a non-dimensional measure of the input flux

$$Q \equiv \frac{q/H}{u_b} = \frac{q/H}{g'k_L/\nu}, \tag{2.4}$$

where q is the two-dimensional volumetric flux input. Embodied in this non-dimensional flux is the competition between radial flow driven by injection pressures near the source and horizontal propagation as characterized by the typical buoyancy velocity. We can therefore anticipate a change in the dominant balance between injection and buoyancy driven spreading for a critical value of the non-dimensional flux which can only depend on the ratio of permeabilities,

$$Q_C = f(\Lambda). \tag{2.5}$$

It is this relationship which we aim to experimentally investigate in the following section.

3. Experiments in two-layered systems

A total of 95 experiments were conducted in a cell of dimensions 120 cm × 20 cm and a gap width of 1 cm. The centimetre gap was then filled with two layers of glass ballotini, each layer containing a differing size, and therefore permeability (although not porosity). Three characteristic sizes of ballotini were used with mean diameters of 1, 2 and 3 mm. The depth of the lower layer was set at $H = 4$ or 6 cm to within an accuracy of the largest bead diameter. This two-layer porous medium was then saturated with fresh water, and the upper surface was left open to the atmosphere as depicted in figure 2.

Bead size (mm)	ϕ	Theoretical k (cm^2)	Experimental k (cm^2)
0.2	0.38	7.53×10^{-7}	3.17×10^{-7}
1.1	0.4	1.14×10^{-5}	1.20×10^{-5}
2.0	0.42	4.01×10^{-5}	4.89×10^{-5}
3.0	0.44	1.06×10^{-4}	1.36×10^{-4}

TABLE 1. Experimentally determined porosity ϕ and permeability k of a monodisperse porous medium composed of glass ballotini in a two-dimensional cell with a gap width of 1 cm as compared with the theoretical value given by the Kozeny–Carman relation neglecting any effects due to confinement by the sidewalls.

Each experiment was initiated by carefully releasing a constant flux of dense solution at the base of the narrow test cell. The density of the injected current was controlled by the addition of salt, and was visualized through the addition of blue dye. The input flux was maintained by a gravity drainage system and was measured to a high degree of accuracy by an electronic balance which recorded the mass of the reservoir as a function of time.

3.1. *Experimental determination of porosity and permeability*

The permeability of a porous medium composed solely of monodisperse spheres can be characterized by the familiar Kozeny–Carman relationship (Bear 1972; Acton, Huppert & Worster 2001),

$$k = \frac{d^2}{180}\frac{\phi^3}{(1-\phi)^2}, \tag{3.1}$$

where k is the permeability, d is the diameter of the spheres and ϕ is the porosity of the medium. In our narrow, two-dimensional cell we anticipate regions of anomalously high permeability near the sidewalls associated with a decrease in the packing fraction. Therefore, we experimentally verified the effective permeability of the two-dimensional cell by measuring the propagation of fixed flux gravity currents in a uniform porous medium for the four bead sizes listed in table 1. The front position x_N of a two-dimensional gravity current as a function of time t is well know and, for fixed flux q, is given by

$$x_N(t) = \eta_N\,(q u_b)^{1/3}\,t^{2/3}, \tag{3.2}$$

where $\eta_N = 1.4819$ and $u_b = kg'/\phi\nu$ (Huppert & Woods 1995). The porosity of the medium ϕ was determined by filling the void spaces with a known volume of water.

4. Experimental results

We carried out a suite of 95 experiments over which the input flux, depth of the lower layer, permeabilities and permeability ratios were varied to examine quantitatively the role of permeability structure on the propagation of two-dimensional gravity currents. This extensive parameter search can be grouped according to five differing permeability ratios $\Lambda = 2.1,\ 4.0,\ 8.5,\ 19.4$ and 77.4. For each permeability ratio the input flux q and the reduced gravity g' were varied and the profile of the current was examined visually. We classified each flow as either overriding or non-overriding depending on whether the profile across the interface was discontinuous

(*a*)
(*b*)
(*c*)

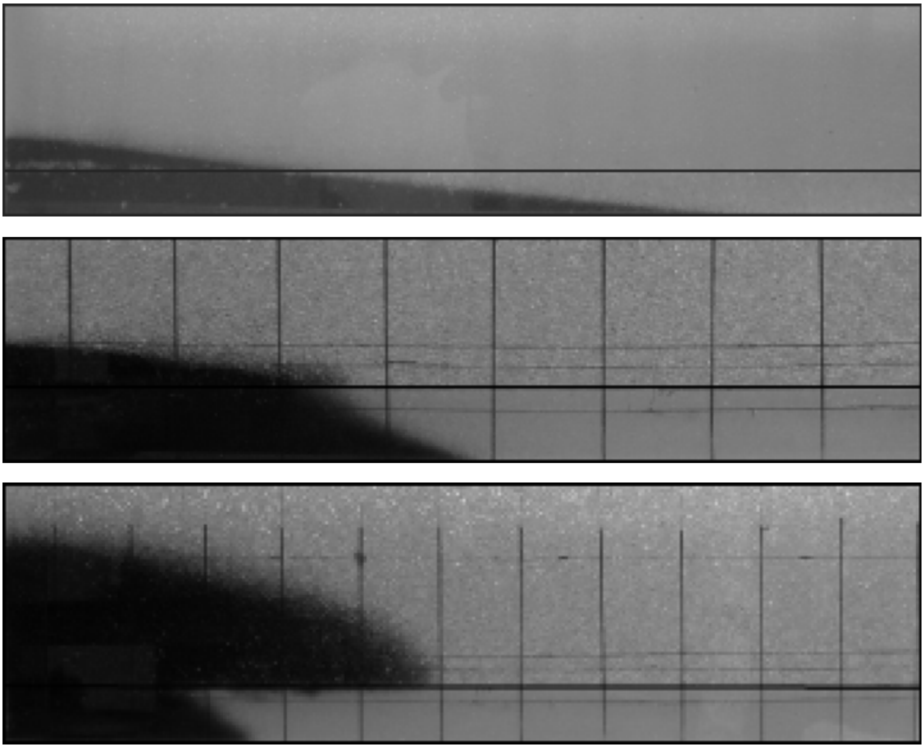


FIGURE 3. The profiles of three representative experiments in two-layer porous media are shown. In (*a*) the lower layer is more permeable than the upper layer ($k_L > k_U$) and thus flow is focused along the base of the Hele–Shaw cell. In (*b,c*) the upper layer is more permeable than the lower ($k_U > k_L$). For the relatively low flux shown in (*b*) the current does not override, while above a critical flux, shown in (*c*) the current follows the high-permeability path and overrides the lower layer.

or continuous, respectively. Examples of these experimental profiles are shown in figure 3(*a*–*c*). In all cases dense fluid is input at the bottom left of the diagram. In figure 3(*a*) the lower layer is more permeable than the upper layer ($k_U < k_L$) and so gravity acts in the direction of the high permeability. Flow is therefore focused along the bottom boundary of the two-dimensional cell and the length of the current in the lower layer is enhanced over what it would be if the medium was uniform with the same permeability as the lower layer.

In figure 3(*b,c*) the upper layer is more permeable than the lower layer ($k_U > k_L$) leading to a competition between gravity and flow focusing along the high-permeability zone. Thus, for small values of the non-dimensional input flux (shown in figure 3*b*) flow is predominantly along the bottom boundary; the current does not override. However, above a critical flux Q_C, flow is focused along the high-permeability layer leading to an overriding current as shown in figure 3(*c*).

The division between overriding and non-overriding currents is most clearly shown in figures 4 and 5. The former figure presents the results as separate functions of H, g' and $g'H$, as suggested by (2.4). A straight line with the same slope, 0.36, can be drawn through all of the results. In figure 5 we show how, for permeability ratio $\Lambda = 8.5$, the ultimate behaviour of the current depends on the dimensional

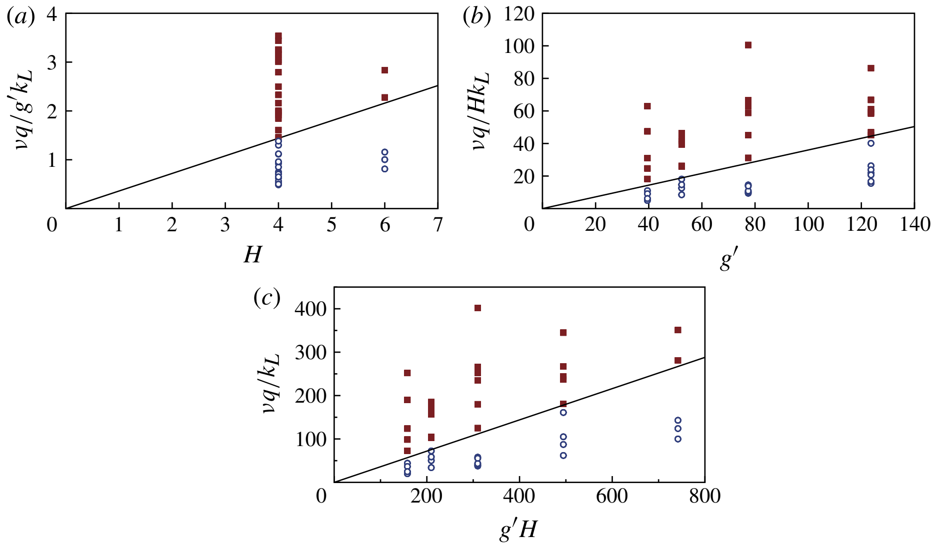


FIGURE 4. (Colour online) Collapse of the data for three different combinations of parameters. A ■ indicates an overriding current, while a ○ indicates a non-overriding current. In each case the line is drawn with slope 0.36 delineating the two regimes.

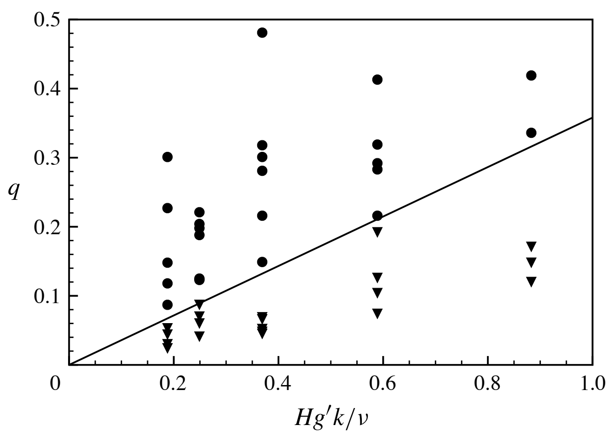


FIGURE 5. Regime diagram separating overriding from non-overriding currents. The dimensional flux q (cm^2 s^{-1}) is plotted as a function of the grouping $Hg'k_L/\nu$ (cm^2 s^{-1}).

flux q, the height H of the lower layer, and the characteristic velocity in the porous medium $g'k_L/\nu$ thereby completely confirming the validity of the dimensionless grouping

$$Q = \frac{q}{Hg'k_L/\nu}. \tag{4.1}$$

Finally, the results of the 95 experiments are summarized in figure 6. In total five different permeability ratios were investigated ($\Lambda = 2.1, 4.0, 8.5, 19.4, 77.4$) in the manner shown in figure 5. Each set of experiments at fixed Λ provide a critical

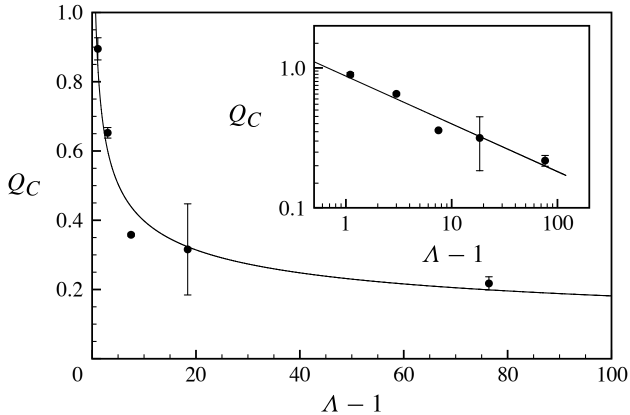


FIGURE 6. Phase diagram separating overriding from non-overriding currents. Results of all 95 experiments are plotted in terms of the non-dimensional flux Q_C as a function of the permeability difference $\Lambda - 1$.

non-dimensional flux Q_C which exhibits a power law dependance on the permeability ratio of the form

$$Q_C = c\,(\Lambda - 1)^{-n}, \tag{4.2}$$

where our fit of the experimental data provides the constants $c = 0.93 \pm 0.05$ and $n = 0.34 \pm 0.04$.

Perhaps most importantly, for fluxes above the critical value the currents are found to spread preferentially in the upper, higher-permeability layer. This leads to a situation in which dense fluid overlays light fluid, an unstable stratification. As exemplified by the images in figure 7 the currents become unstable to a Rayleigh–Taylor instability which promotes mixing between the two fluids. Indeed, although we are constrained by the finite length of the tank, it might be conjectured that the overriding state may ultimately be transient. However, this is not consistent with our observations which show that, for length scales accessible by our experiments, the velocity of the upper current always exceeds the lower.

5. Discussion and conclusions

We have shown that even a two-layer porous medium can lead to quite different and surprising styles of propagation of a gravity current of relatively heavy fluid. The differences are due to the competition between gravity, which tends to make the heavy fluid flow along the confining bottom boundary, and flow focusing, which attracts moving fluid to regions of higher permeability. When the lower layer is of larger permeability than that above it, gravity and flow focusing act in consort. The transport of the gravity current in the lower layer is enhanced over that if the porous medium was all at the same permeability of the lower layer. This occurs because much of the current then propagates in an almost confined lower layer (totally confined if the permeability of the upper layer was zero).

In the reverse, and in some sense more interesting case, when the lower layer is less permeable than the upper layer, the two effects of gravity and flow focusing act in

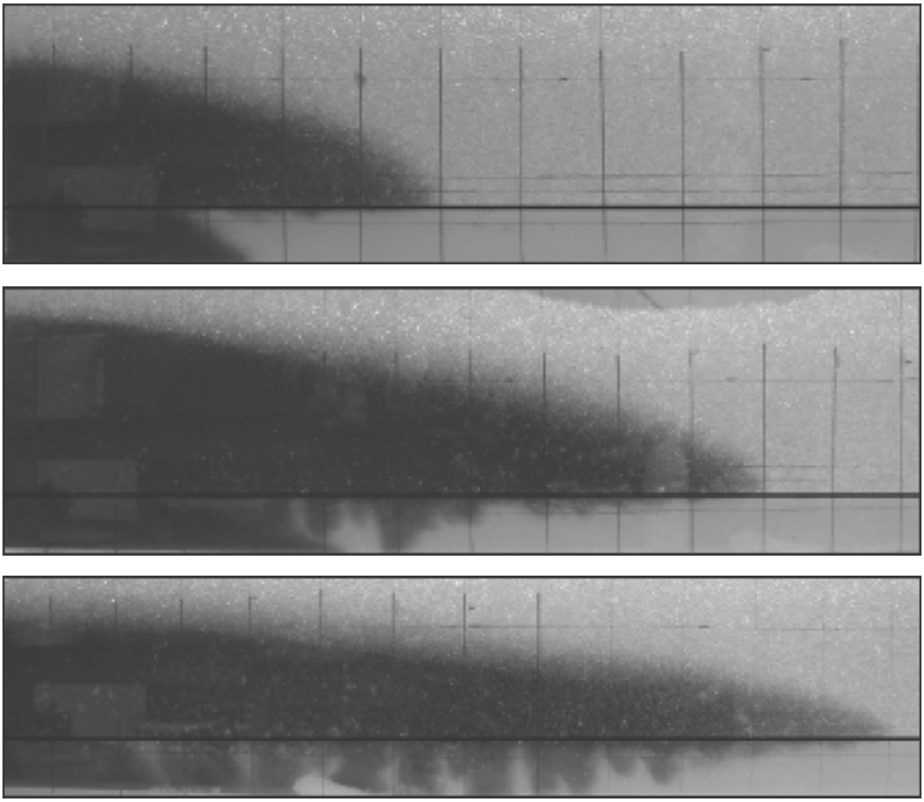
(*a*)
(*b*)
(*c*)


FIGURE 7. The long time evolution of an overriding current shows the importance ultimate importance of Rayleigh–Taylor fingering leading to enhanced mixing.

opposition. In this case for an input flux

$$q < q_c = 0.9(g' k_L H/\nu)\,(k_U/k_L - 1)^{-1/3} \tag{5.1}$$

(where the variables have been defined in the text), the effects of gravity dominate. For $q > q_c$ the increased resistance to flow in the higher permeability upper layer dominates and the relatively dense current initially overrides the less dense fluid in the lower layer. This gravitationally unstable situation is then subject to Rayleigh–Taylor instability, which enhances the mixing between the injected fluid and the ambient fluid in the interstitials of the porous medium. This is possibly the most important consequence of the two-layer system.

Flow in porous media has a wide range of industrial and natural applications, including groundwater flow, the spreading of contaminated spills, the flow of compressed carbon dioxide in a 'super-critical' state through a storage reservoir during sequestration, the process of enhanced oil recovery and the seepage of sea water through the ground to contaminate fresh aquifers intended possibly to supply drinking water for humans or nutrient-laden water for plants. Typically in these situations the porous medium is heterogeneous with a permeability which can be a very strong function of depth. Our experiments and physical reasoning indicate that even in the simple situation of a two-layer permeability there can be a quite different flow field dependent on the permeability constraints. The extra mixing generated by the heterogeneous permeabilities can be significant. Also, because in this case the fluid makes contact with a larger volume of porous medium, effects such as dissolution, chemical reaction and residual trapping will be enhanced.

Acknowledgements

We thank E. Owen and M. Hallworth who helped in this study. During the course of the project H.E.H. was in receipt of a Royal Society Wolfson Merit Award and J.A.N. was supported by fellowships from the Lloyds Tercentenary Trust, the Leverhulme Trust and a Royal Society University Research Fellowship.

REFERENCES

ACTON, J. M., HUPPERT, H. E. & WORSTER, M. G. 2001 Two-dimensional viscous gravity currents flower over a deep porous medium. *J. Fluid Mech.* **440**, 359–380.

ANDERSON, D. M., MCLAUGHLIN, R. M. & MILLER, C. T. 2003 The averaging of gravity currents in porous media. *Phys. Fluids* **10**, 2810–2829.

BARENBLATT, G. I. 1996 *Scaling, Self-similarity, and Intermediate Asymptotics*. Cambridge University Press.

BEAR, J. 1972 *Dynamics of Fluids in Porous Media*. Dover.

CINAR, Y., JESSEN, K., BERENBLYUM, R., JUANES, R. & ORR, F. M. Jr. 2006 An experimental and numerical investigation of crossflow effects in two-phase displacements. *Soc. Petrol. Engng J.* **90568**, 216–226.

HUPPERT, H. E. 2006 Gravity currents: a personal perspective. *J. Fluid Mech.* **554**, 299–322.

HUPPERT, H. E. & WOODS, A. W. 1995 Gravity-driven flows in porous layers. *J. Fluid Mech.* **292**, 55–69.

LYLE, S., HUPPERT, H. E., HALLWORTH, M., BICKLE, M. & CHADWICK, A. 2005 Axisymmetric gravity currents in a porous medium. *J. Fluid Mech.* **543**, 293–302.

PHILLIPS, O. M. 2009 *Geological Fluid Dynamics: Sub-surface Flow and Reactions*. Cambridge University Press.