Time-dependent current-voltage relation in electron guns
Byung Mook Weon and Jung Ho Je

Citation: Journal of Applied Physics 97, 036101 (2005); doi: 10.1063/1.1846938
View online: http://dx.doi.org/10.1063/1.1846938
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/97/3?ver=pdfcov
Published by the AIP Publishing

Articles you may be interested in
Measurement of back-bombardment temperature rise in microwave thermionic electron guns

Time-dependent current-voltage characteristics of Al/p-CdTe/Pt x-ray detectors

Temperature dependence of current-voltage characteristics in highly doped Ag/ p -GaN/In Schottky diodes

Errata: Theory and Design of Thermionic Electron Beam Guns

Progress Toward a Gigawatt-Class Annular Beam Klystron with a Thermionic Electron Gun
AIP Conf. Proc. 625, 159 (2002); 10.1063/1.1498194
The Child–Langmuir law shows that the space-charge limited current is proportional to the voltage raised to 3/2 power.\(^1\)\(^\text{-}^3\) This relationship between the current and the voltage is valid for all cathode–anode geometries and we can write \(I = pV^{3/2}\), where the constant \(p\), called the perveance, only depends on the form of the electrodes (in the nonrelativistic limit).\(^3\)\(^\text{-}^4\) The classical scaling of the limiting current to the 3/2 power of gap voltage is widely used in the fields of high-current emission diodes, vacuum microelectronics, high-power microwave sources, accelerator physics, and sheath physics.\(^3\)\(^\text{-}^5\) However, in the quantum regime (nanometer scale), the classical scaling (3/2 power) is no longer valid.\(^6\)

In electron devices which use free electrons, the cathode is one of the most important parts.\(^3\) When thermionic emission is applied, the cathode material must have low work function and high melting point to avoid significant evaporation at working temperature.\(^2\) The two main types of thermionic cathodes are oxide and dispenser (or impregnated) cathodes.\(^3\)\(^\text{-}^6\) The oxide cathodes of Ba, Sr, and Ca on Ni base metal are widely used, because of their low work function (\(\sim 1\) eV) and high current density (\(\sim 5\) A/cm\(^2\) under dc load) at low working temperatures (900–1100 K).\(^3\)\(^\text{-}^6\) Recently, we developed our understanding of the complex activation and degradation processes of oxide cathodes. An observation of Ba enrichment on real cathode surface using \textit{in situ} synchrotron x-ray absorption spectroscopy revealed that the activation process is a complex process,\(^7\) and the heterogeneity (simultaneous, multiple mechanisms) is responsible for the stretched exponential decay of the current on life.\(^8\)

We note that the voltage decay over time seems to be universal in typical thermionic cathodes.\(^6\) The main reasons for the voltage decay are the sintering shrinkage and/or the evaporation of emission materials.\(^6\) These phenomena may induce the infinitesimal but significant change of the electron gun geometry in nature. In this study, we focus on a general situation where the current and the voltage in electron guns decay over time as in oxide cathodes. The voltage decay suggests a variation of the perveance and the power in the Child–Langmuir law during operation over a long period of time in the electron guns using oxide cathodes. We derive a general expression for the time-dependent current–voltage relation as \(I(t) = p V(t)^{\delta(t)}\), where \(\delta(t)\) is a time-dependent exponent and the perveance, \(p(t)\), is a function of \(\delta(t)\). The exponent \(\delta(t)\) indicates the deviation of the classical Child–Langmuir relation \((I = p V^{3/2})\). This deviation is attributed to the gradual change of the electron gun geometry over time. \(\copyright\) 2005 American Institute of Physics.

\[\text{(DO1: 10.1063/1.1846938)}\]
\[ \delta(t) = A t^B, \]  

which is determined by the two constants, \( A \) and \( B \), which are again estimated as \( A = \exp(\alpha_1 - \alpha_2) \) and \( B = (\beta_1 - \beta_2) \), from Eqs. (1) and (2). In particular, \( \delta(t) \) is primarily determined by the constant \( B \). \( \delta(t) \) increases or decreases over time as \( B \) is positive or negative, respectively.

On the basis of Eq. (3), we can derive a general formula for the time-dependent perveance and the time-dependent Child–Langmuir relation. Equation (3) can be transformed as follows:

\[ \frac{I(0)}{V(0)^{\delta(0)}} = \frac{I(t)}{V(t)^{\delta(t)}}. \]  

Comparing with the perveance, \( p(0) = I(0)/V(0)^{3/2} \), in the Child–Langmuir relation, which is time independent, Eq. (5) can be considered as time-dependent perveance

\[ p(t) = \frac{I(0)}{V(0)^{\delta(t)}}. \]  

Then, we note that the perveance \( p(t) \) is a function of \( \delta(t) \). From Eqs. (5) and (6), we finally derive the following general equation of time-dependent Child–Langmuir relation:

\[ I(t) = p(t)V(t)^{\delta(t)}. \]  

This relation indicates that the perveance \( p(t) \) and the power \( \delta(t) \) would be time dependent, when \( I(t) \) and \( V(t) \) are time dependent. Most importantly, the time dependence of the Child–Langmuir relation is determined by the time dependence of \( \delta(t) \). The well-known Child–Langmuir relation of the classical scaling (3/2 power) is one special case when \( \delta(t) \) becomes 3/2. Once \( \delta(t) \) can be evaluated, then we will be able to expect the time dependence of the perveance, which is very important to control the beam drive conditions in oxide cathodes during operation.

We in turn discuss the experimental results. We investigated the degradation behaviors of the commercial electron guns with oxide cathodes for cathode ray tubes (CRTs) made by LG.Philips Displays. Particularly, we measured the maximum emission current and the applied cutoff voltage from the 24 oxide cathodes in electron guns, while they operated at a temperature of 1100 K in the space-charge limit regime over a long period of time (~9000 h). The initial values of the perveance of the electron guns were measured as 4.7 \( \mu \)perv (10^{-6} \( A/V^{3/2} \)) on average. We found that both the current and the voltage in the electron guns with oxide cathodes indeed followed the stretched exponential degradation behavior. Combining the mathematical formulas for the current and the voltage leads to the derivation of a simple, general equation for the time-dependent current–voltage relation.

Figure 1 shows the stretched exponential degradation behaviors of the maximum emission current (open circles) and the applied cutoff voltage (open squares), respectively. The straightness of the plot in Fig. 1 verifies the validity of the stretched exponential decay of the current (open circles) or the voltage (open squares), as already described in Eq. (1) or Eq. (2). The data were measured between 1000 and 9000 h. (The data below 1000 h were eliminated because there was little degradation of the emission current.) The correlation between the current decay data (open circles) and Eq. (1) is very high \( r = \frac{0.9595, n = 240, p < 0.0001}{\text{The intercept } (\alpha_1) \text{ and the slope } (\beta_1) \text{ are estimated as } “-10.091 83 \pm 0.174 37” \text{ and } “1.109 35 \pm 0.021 11,” \text{ respectively. We are able to see a high correlation between the voltage data (open squares) and Eq. (2) } (r = 0.9258, n = 240, p < 0.0001). The intercept (\alpha_2) \text{ and slope } (\beta_2) \text{ are estimated as } “-9.88876 \pm 0.203 45” \text{ and } “0.930 72 \pm 0.024 63,” \text{ respectively.}}

The exponent \( \delta(t) \) can be easily estimated using Eq. (4) and the values of \( \alpha_1, \alpha_2, \beta_1, \) and \( \beta_2 \) estimated in Fig. 1, as illustrated in Fig. 2. We are able to see that the \( \delta(t) \) increases over time. This indicates that the perveance decreases with time according to Eq. (6). This result implies that there would be a significant deviation from the classical Child–Langmuir relation, as can be expected from Eq. (7). Interestingly, the extrapolation of \( \delta(t) \) below 1000 h (dashed line) demonstrates that \( \delta(t) \) approaches 3/2 at \( t=0 \). This result shows that the current–voltage relation would follow the

\[ \delta(t) = A t^B. \]
classical Child–Langmuir relation in the initial stage of operation.

Now, we discuss the time-dependent perveance and the time-dependent Child–Langmuir relation. Usually the perveance is believed to depend on the gun geometry.5 Our findings represent that the perveance and the Child–Langmuir relation are time dependent because of the stretched exponential time dependences of the current and the voltage. This clearly indicates that the geometry gradually changes over a long period of time. Therefore, we believe that the time-dependent deviation of the Child–Langmuir relation from the classical scaling is probably due to the change of the geometry over time.9 Another possible factor is the increasing contribution of the temperature-limited current to the total current.10 This result implies that the stability of the geometry during operation is very important for stable performance with high perveance3,4,9,11 and high reliability,10 which are required in a number of applications, where charge or current density is a key parameter.

On the other hand, we consider special cases of time-independent deviation from the classical Child–Langmuir relation, where $\delta(t)$ is a constant, but not necessarily 3/2, in Eq. (4). When both the current and the voltage follow the simple exponential decay (in the case of $\beta_1=\beta_2=1$) or when the slope of the stretched exponential decay of the current is identical with that of the voltage (in the case of $\beta_1=\beta_2 \neq 1$), the time-independent deviation from the Child–Langmuir relation can occur. However, we believe that those special situations of $\beta_1=\beta_2=1$ or $\beta_1=\beta_2 \neq 1$ would occur very seldom, compared to the general situations of $\beta_1 \neq \beta_2$ in nature.

In conclusion, we derived general equations of the time-dependent perveance and the time-dependent Child–Langmuir relation in electron guns especially using oxide cathodes, based on the stretched exponential behaviors of the emission current and the applied voltage. From the experimental results for 9000 h, we found that $I(t)$ and $V(t)$ follow the stretched exponential decay. The general expression for the time-dependent current–voltage relation was derived as $I(t)=p(t)V(t)^{\delta(t)}$, where $\delta(t)=At^d$ and $p(t)$ is a function of $\delta(t)$. Especially, the time dependence of $\delta(t)$ and as a result the time dependence of the perveance $p(t)$ indicate the degree of the time-dependent deviation of the Child–Langmuir relation from the classical scaling (3/2 power). This is attributed to the gradual change of the electron gun geometry over time. We believe that the findings in this study would be universal in electron guns of various applications as well as in oxide cathodes.

2J. Langmuir, Phys. Rev. 2, 450 (1913); 21, 419 (1923).