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Self-stabilization in self-organized Multihop Wireless Networks

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Thème 1 — Réseaux et systèmes
Projets ARES et Grand Large

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Abstract: In large scale multihop wireless networks, flat architectures are not scalable. In order to overcome this major drawback, clusterization is introduced to support self-organization and to enable hierarchical routing. When dealing with multihop wireless networks the robustness is a main issue due to the dynamicity of such networks. Several algorithms have been designed for the clusterization process. As far as we know, very few studies check the robustness feature of their clusterization protocols. Moreover, when it is the case, the evaluation is driven by simulations and never by a theoretical approach.

In this paper, we show that a clusterization algorithm, that seems to present good properties of robustness, is self-stabilizing. We propose several enhancements to reduce the stabilization time and to improve stability. The use of a Directed Acyclic Graph ensures that the self-stabilizing properties always hold regardless of the underlying topology. These extra criterion are tested by simulations.

Key-words: multihop wireless networks, clusterization, self-stabilization, scalability, density

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Auto-stabilisation dans les réseaux sans fil auto-organisés

Résumé : Dans les réseaux sans fil de grande dimension, les architectures de routage à plat ne passent pas à l'échelle. Pour pallier ce défaut majeur, des techniques de regroupement ont été introduites et permettent l’auto-organisation et le routage hiérarchique. Quand on considère des réseaux sans fil multisauts, la robustesse est également une propriété cruciale, en raison de la dynamicité du réseau. Plusieurs algorithmes ont été conçus pour permettre le regroupement des machines du réseau. À notre connaissance, très peu d’études tiennent compte du facteur robustesse. De plus, quand c’est le cas, l’évaluation se fait au moyen de simulations et non par une approche théorique.

Dans cet article, nous présentons un algorithme de regroupement auto-stabilisant qui semble présenter de bonnes propriétés de robustesse. Nous proposons plusieurs améliorations pour réduire le temps de stabilisation et améliorer la stabilité des regroupements. L’utilisation d’un graphe orienté acyclique assure que la propriété d’auto-stabilisation persiste indépendemment de la topologie sous-jacente. Les améliorations proposées sont testées par simulation.

Mots-clés : réseau sans fil multisauts, regroupement, auto-stabilisation, passage à l’échelle, densité
1 Introduction

Ad hoc networks or wireless sensor networks (wireless multihop networks) are composed of devices that communicate via wireless interfaces. They require no fixed infrastructure and no human intervention. Both are strongly based on self-organization and self-stabilization. Even if every mobile can move everywhere, and thus can disappear or appear in the network at any time, the network manages the changes in topology and provides the connectivity between any pair of terminals. If the current wireless cards allow the communication between mobiles that are in communication range, a routing protocol is required to provide the full connectivity of the network. As there are no dedicated devices in the network, all mobiles are potential routers. Such networks have become very popular due to their ease of use. Their applications range from the network extension when cabling is not possible or too expensive to spontaneous networks in case of natural disasters where the infrastructure has been totally destroyed by going through the monitoring and the collect of data with wireless sensor networks.

Due to the dynamics of such networks (devices mobility and/or instability of the wireless medium), routing protocols for fixed networks are not adapted. Ad hoc routing protocols proposed in the MANET working group at IETF\(^1\) are all flat routing protocols. It means that there is no hierarchy and all terminals have the same role. If flat protocols are quite effective on small and medium networks, they are not suitable on large scale networks due to bandwidth and processing overhead. Hierarchical routing seems to be more adapted to such large networks. It often relies on a specific partition of the network, called clusterization: the devices are gathered into clusters according to some criteria and specific routing protocols are used within and between the clusters. In addition to its scalable feature, such an organization presents numerous advantages such as synchronizing mobiles in a cluster or attributing new services zones. Several algorithms have been designed for the clusterization step. As far as we know, very few studies check the robustness feature of their clusterization protocols. Moreover, when it is the case, the evaluation is driven by simulations and never by a theoretical approach.

In this article, we apply self-stabilization principles over a clusterization protocol proposed in [16] and which presents good properties of robustness. With a theoretical approach, which can be applied to several clusterization schemes, we show that, under some assumptions, the algorithm is self-stabilizing. We also improve the robustness by adding extra-advanced features and we show that the resulting algorithm is still self-stabilizing. These properties are further validated by simulations. A state-of-the-art on clustering in multihop wireless networks is given in Section 2. The description of the chosen clusterization algorithm is done in Section 3. In Section 4, we provide a formal analysis, showing that this algorithm is self-stabilizing. We also discuss some improvements for robustness in this section. The properties of the proposed algorithm and the different improvements are evaluated by simulations in Section 5.

\(^1\)http://www.ietf.org/html.charters/manet-charter.html
2 State-of-the-art

Flat routing protocols (like the classical reactive or proactive protocols) are not really suitable for large wireless multihop networks. Indeed, such routing protocols become ineffective on a large scale because of bandwidth (flooding of control messages) and processing (routing table computation) overhead. One solution to solve this scalability problem is to introduce a hierarchical routing by gathering geographically close nodes into clusters [14, 17, 10]. Several techniques for clusters formation and cluster-heads selection have been proposed. All solutions aim to identify subsets of nodes within the network and to bind each of them to a unique leader. Some solutions try to gather nodes into homogeneous clusters by using either an identity criteria (e.g., the lowest identity [12, 3]) or a fixed connectivity criteria (for instance maximum degree [5], 1-hop clusters [2, 6, 13] and k-hops clusters [7]) or a connectivity and identity criteria (max-min d-cluster [1]). To maintain the clusters structure, most of the solutions try to keep a fixed cluster diameter [9, 5, 1], a fixed cluster radius [15] or a constant number of nodes in the clusters [18].

These solutions are not adapted to large multihop wireless networks. First, a small modification in the network topology (due to the mobility of one node for instance) often implies new computations to build the new clusters and to elect the cluster-heads. Moreover, building and maintaining clusters with a constant feature (like diameter or the number of nodes) may generate a significant number of useless clusters. For instance, why separating a set of nodes that can communicate with each other just because it can not fit into one cluster since the constant feature is not respected? In [16], a density criteria is proposed. This metric allows to limit the exchanged traffic generated while clusters are re-built and the nodes’ tables updated. Therefore it presents good properties of robustness. In this paper, we show that this algorithm is self-stabilizing and its robustness can be enhanced with some extra rules.

3 Density driven clustering algorithm

The model. The system is composed of a set \( V \) of nodes and each node has a unique identifier. Each node \( p \) can communicate with a subset \( N_p \subseteq V \) of nodes determined by the range of the radio signal; \( N_p \) is called the neighborhood of node \( p \). Note that \( p \) does not belong to \( N_p \) (\( p \notin N_p \)). We assume that communication capability is bidirectional: \( q \in N_p \) iff \( p \in N_q \). Define \( N^1_p = N_p \) and for \( i > 1 \), \( N^i_p = N^{i-1}_p \cup \{ r \mid (\exists q \in N^{i-1}_p, r \in N_q) \} \) (let’s call \( N^i_p \) the \( i \)-neighborhood of \( p \)). We assume that the distribution of nodes is sparse: there is some known constant \( \delta \) such that for any node \( p \), \( |N_p| \leq \delta \). Note that a control on density can be done by adjusting their communication range and/or powering off nodes in areas that are too dense.

The density metric criteria. The notion of density, firstly introduced in [16], characterizes the relative importance of a node in the network and in its 1-neighborhood. The
underlying idea is that if some nodes move in \( N_p \), changes will affect the microscopic view of node \( p \) (for instance its degree \(|N_p|\) will change) but its macroscopic view will not drastically change since globally the network does not change and its \( N_p \) globally remains the same. The density will smooth changes down in \( N_p \) by considering the ratio between the number of links and the number of nodes in the 1-neighborhood. The definition is the following:

**Definition 1** The density of a node \( p \in V \) is

\[
d_p = \frac{|e = (v, w) \in E \text{ s.t. } w \in \{p\} \cup N_p \text{ and } v \in N_p|}{|N_p|}
\]

**Cluster-head selection and cluster formation.** Due to space limitation, we describe the heuristic process informally. The algorithm and its analysis are more detailed in [16]. Each node locally computes its density value and regularly broadcasts it to all its 1-neighbors. Each node is thus able to compare its density value to its 1-neighbors’ and decide by itself whether it joins one of them (the one with the highest density value) or it wins in its 1-neighborhood and elects itself as a cluster-head. If there are some joint winners, the smallest identity is used to decide between them. In this way, two neighbors can not be both cluster-heads. If node \( p \) has joined node \( w \), we will say that \( w \) is node \( p \)'s parent (and denote it \( w = \mathcal{F}(p) \)). A node’s parent can also have joined another node and so on. The cluster-head will be the node which has elected itself as its own cluster-head. A cluster can then extend itself until it reaches a cluster frontier.

**Features.** This metric has been studied in [16] with both simulations and a stochastic analysis. It outlined that the number of cluster-heads computed with this metric is bounded and decreases when the nodes intensity increases. This is an advantage because if many nodes are in the communication range of each other, there is no need to separate them into different clusters as they can hear each other. Moreover, this heuristic has revealed to be more stable towards nodes mobility than other metrics, like the degree and the max-min metrics [16].

**Example.** To illustrate this heuristic, let’s take the following example given in Fig 1. In its 1-neighborhood topology, node \( a \) has two 1-neighbors (\( N_a = \{d, i\} \)) and two links (\( \{(a, d), (a, i)\} \)) and node \( b \) has four 1-neighbors (\( N_b = \{c, d, h, i\} \)) and five links (\( \{(b, c), (b, d), (b, h), (b, i), (h, i)\} \)). Table 1 shows the final result.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
<th>( f )</th>
<th>( h )</th>
<th>( i )</th>
<th>( j )</th>
</tr>
</thead>
<tbody>
<tr>
<td># Neighbors</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td># Links</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>1-density</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.5</td>
<td>1.5</td>
<td>1.25</td>
<td>1.5</td>
<td>1.25</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 1: Results of our heuristics on the illustrative example.

In our example, node \( c \) joins its neighbor node \( b \) which density is the highest (\( \mathcal{F}(c) = b \)). Yet, the node with the highest density in node \( b \)'s neighborhood is \( h \). Thus, \( \mathcal{F}(b) = h \). As
node $h$ has the highest density in its own neighborhood, it becomes its own cluster-head: $H(h) = h$ and thus: $H(c) = H(b) = H(h) = h$. To sum up, node $c$ joins $b$ which joins $h$ and all three of them belong to the cluster which cluster-head is $h$. Moreover, we have $d_j = d_f$. We thus use the ID to decide between both nodes. Let’s assume that node $j$ has the smallest ID: $F(f) = j$ and $F(j) = j$ so $H(f) = H(j) = j$. Finally, we obtain two clusters organized around two cluster-heads: node $h$ and node $j$. (See the right side on Figure 1).

4 Self-stabilization

In this section we study the self-stabilizing properties of the density-driven clustering algorithm presented in [16]. We follow the same assumptions and principles as the ones given in [11].

**Hypothesis.** We assume that the implementation of CSMA/CA satisfies the following point: there exists a constant $\tau > 0$ such that the probability of a frame transmission without collision is at least $\tau$ (this corresponds to typical assumptions for multi-access channels [4]; the independence of $\tau$ for different frame transmissions indicates that we assume a memoryless probability distribution in a Markov model).

**Notation.** We describe algorithms using the notation of guarded assignment statements: $G \rightarrow S$ represents a guarded assignment, where $G$ is a predicate of the local variables of a node, and $S$ is an assignment to local variables of the node. If predicate $G$ (called the *guard*) holds, then assignment $S$ is executed, otherwise $S$ is skipped. Some guards can be event predicates that hold upon the event of receiving a message. We assume that all such guarded assignments execute atomically when a message is received. At any system state, where a given guard $G$ holds, we say that $G$ is *enabled* at that state. The $\parallel$ operator is the nondeterministic composition of guarded assignments; ($\parallel q : q \in M_p : G_q \rightarrow S_q$) is a closed-form expression of $G_{q_1} \rightarrow S_{q_1} \parallel G_{q_2} \rightarrow S_{q_2} \parallel \cdots \parallel G_{q_k} \rightarrow S_{q_k}$, where $M_p = \{q_1, q_2, \ldots, q_k\}$.

**Execution Semantics.** The life of computing at every node consists of the infinite repetition of evaluating its guarded actions. We assume that every action is executed within a constant time finding a guard and executing its corresponding assignment or skipping it when the guard is *false*. Generally, we suppose that when a node executes its program,
all statements with true guards are executed within a constant time (done, for example, in round-robin order).

**Shared Variable Propagation.** Some variables of nodes are designated as shared variables. Following the scheme presented in [11], nodes periodically transmit the values of their shared variables, based on a timed discipline. Beyond periodic retransmission, an assignment to a shared variable causes peremptory transmission: if a statement \( G \rightarrow S \) assigns a shared variable, then we suppose that there is a transformation of the statement into a computation that slows execution down so that it does not exceed some desired rate, and also provides randomization to avoid collision in messages that carry the shared variable values. One technique for implementing such \( G \rightarrow S \) is presented in [11]. In the remaining of the section, we assume that nodes use the scheme of [11] to learn \( N_p \) and \( N_p^2 \).

### 4.1 Constant Height DAG Construction

In the chosen clustering algorithm, as in every clustering algorithm using the node identification as last decision, the worst case is met when every nodes has the same deciding value, i.e. in our case the density value, and when nodes’ identifiers are unique in the network and badly distributed. In such a case, the algorithm builds only one cluster which may have a diameter as big as the diameter of the network. This may cause scalability problems because the stabilization time is likely to depend on this diameter. Moreover, it is obvious that building such a cluster is useless as we could have used the network without clusters instead. To overcome this drawback, it can be useful to give nodes smaller names (also named colors), from a constant space of names, in a way which ensures that names are locally unique. A DAG (Directed Acyclic Graph) can be constructed by using these identifiers and by orienting edges between neighbors from the higher identifier to the lower one.

Our constant height DAG construction is based on the randomized technique described in [11], but uses a much smaller name-space \( |\gamma| \) equals \( \delta^6 \) or even \( \delta \) is sufficient in our case). Let \( \text{Id}_p \) be a shared variable that belongs to the domain \( \gamma \); variable \( \text{Id}_p \) is the name of node \( p \). Another variable is used to collect the names of neighboring nodes: \( \text{Cids}_p = \{ \text{random}(\gamma \setminus \text{Cids}_p) \text{otherwise} \}

The algorithm for constant height DAG construction is the following:

\[
N1: \quad \text{true} \rightarrow \text{Id}_p := \text{newId}(\text{Id}_p)
\]

**Theorem 1** Algorithm N1 self-stabilizes with probability 1 in an expected constant time to a DAG which height is at most \( |\gamma| + 1 \).
The proof of this theorem is similar to the one in [11]. There are two competing motivations for tuning the parameter $\gamma$. On one hand, a large value of $|\gamma|$ decreases the expected convergence time of $N_1$. On the other hand, a small value of $|\gamma|$ decreases the DAG's height, and thus the expected convergence time of subsequent algorithms.

4.2 Density-driven Clusters Construction

Each node $p$ maintains two shared variables, denoted by $d_p$ and $H(p)$. $d_p$ denotes the density of node $p$ given in Definition 1 and $H(p)$ denotes the cluster-head chosen by $p$. We define $\prec$ as a binary total order such that $p \prec q$ if and only if $d_p < d_q$ or $(d_p = d_q) \land (Id_q < Id_p)$. Let $\max_{\prec}$ denote the maximum function associated to this total order. When a node $p$ computes the result of $\prec$ or $\max_{\prec}$, it uses the cached values of its neighborhood (assuming $\boxtimes Id_p = Id_p$ and $\boxtimes d_p = d_p$).

We now define the $\text{clusterHead}$ choice function:

$$\text{clusterHead} = \begin{cases} 
Id_p & \text{if } \forall q \in N_p, q \prec p \\
H(\max_{\prec}\{q \in N_p\}) & \text{otherwise}
\end{cases}$$

The cluster-head algorithm runs as follows:

\begin{align*}
R1: & \quad \text{true } \rightarrow d_p := \text{density} \\
R2: & \quad \text{true } \rightarrow H(p) := \text{clusterHead}
\end{align*}

Lemma 1 Each node $p$ has a correct density value $d_p$ within an expected constant time.

Proof. After an expected constant time, each node $p$ has a correct view of its neighborhood at distance two. Then, after $R1$ is executed, the density $d_p$ of $p$ is correct. ■

Lemma 2 Each node $p$ has a correct cluster-head value $H(p)$ within an expected constant time.

Proof. Assume that all nodes have correct density values (this is true after an expected constant time by Lemma 1). After the shared variable $d_p$ has been communicated without collision to all nodes in $N_p$ (this occurs in an expected constant time), each node has a correct cache value of all density values in its neighborhood. We now consider the DAG induced by the $\prec$ relation (thereafter denoted by $\text{DAG}_{\prec}$). In an expected constant time, the roots of $\text{DAG}_{\prec}$ have a correct cluster-head value (that is their own identifier). Now assume that all nodes up to distance $n$ from the roots of the $\text{DAG}_{\prec}$ have a correct cluster-head value. When $R2$ is executed at nodes at distance $n + 1$ from the roots of $\text{DAG}_{\prec}$, those nodes get a correct cluster-head value (because the cluster-head is deterministically determined (i) by the density and local topology – which are fixed – and (ii) by the cluster-head values of nodes at distance up to $n$ from the roots of the $\text{DAG}_{\prec}$). By induction, the time needed for stabilization is proportional to the height of the $\text{DAG}_{\prec}$.
We now prove that the height of the DAG is bounded by a constant value. Node identifiers are bounded by a constant $\gamma$. The number of edges in the neighborhood at distance one is bounded by $\delta^2$, the number of neighbors at distance one is bounded by $\delta$, so the number of possible values for the density function is at most $\delta^3$. Overall, the name-space of values in the DAG is $\gamma \delta^3$, which is bounded by a constant. As a result, the height of the DAG is also bounded by a constant.

The algorithm stabilizes in an expected time proportional to the height of the DAG, and the height of the DAG is constant, so the expected time for stabilization is also constant.

4.3 Improving Stability

We improve the stability of the algorithm by adding some selection criterion. First, when two nodes compete for being cluster-heads (they have the same density value), the winner will be, first, the one which was cluster-head before (if it exists), and then the one with the lowest DAG Id (as defined in Section 4). This scheme adds stability into clusters organization by limiting clusters reconstruction. Cluster-heads remain cluster-heads as long as possible. It is a good property since the only cluster-heads’ role is to give an identity to the clusters. This refinement preserves the structure of our stabilization proof, since it is equivalent to define the total order relation $\prec$ as $p \prec q$ if and only if $(d_p < d_q)$ or $(d_p = d_q) \land (H(q) = Id_q) \land (H(p) \neq Id_p)$ or $(d_p = d_q) \land (H(p) \neq Id_p) \land (H(q) \neq Id_q) \land (Id_q < Id_p)$. In addition, the height of the new DAG is similar to the height of the previous one.

Second, if a node $p$ is a 1-neighbor of two different cluster-heads $u$ and $v$ (which are not directly linked), it will initiate a fusion between $u$ and $v$'s clusters: if $p$ has chosen $v$ as cluster-head, that means that $u \prec v$ and that $v$ will remain a cluster-head unlike $u$. This ensures that (i) a cluster-head is not too off-centered in its own cluster, (ii) a cluster has at least a diameter of two, and (iii) that two cluster-heads are distant of at least three hops. Again, this refinement preserves our stabilization proof, since it is sufficient to use the alternative $\text{clusterHead}$ function:

$$\text{clusterHead} = \begin{cases} 
    \text{Id}_p & \text{if } (\forall q \in N_p, q \prec p) \\
    \text{Id}_q & \text{if } (\forall q \in N^2_p, \text{s.t. } H(q) = \text{Id}_q \implies q \prec p) \\
    H(\max_{\prec} \{q \in N_p\}) & \text{otherwise}
\end{cases}$$

The condition for being a cluster-head thus becomes “I am locally maximal (in the sense of $\prec$) and any cluster-head in my 2-neighborhood is smaller than me (and they should not remain cluster-heads)”. The remaining of the algorithm (and thus proof) is the same.
5 Simulations

As mentioned in Section 4, we suppose that there exists a constant $\tau > 0$ such that the probability of a frame transmission without collision is at least $\tau$. Yet, we can suppose that in a bounded time $\Delta(\tau)$, each node is able to locally broadcast one frame and then receive all packets sent by its 1-neighbors. Such a $\Delta(\tau)$ time unit is called a step, during which each node can receive each packet of all its 1-neighbors. After one step, each node can discover its 1-neighbors. After two steps, each node can compute its 2-neighbors and then its density. After only three steps, each node knows its parent. Then, the number of steps required to discover its cluster-head identity directly depends on the distance from the node to its cluster-head and is bounded by the depth of the tree. Table 2 summarizes the information a node is able to compute at the end of constant number of steps.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>What a node learns from its neighbors</th>
<th>What it can then compute</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-neighbors</td>
<td>neighborhood table</td>
</tr>
<tr>
<td>Step 2</td>
<td>1-neighbors + 2-neighbors</td>
<td>its density</td>
</tr>
<tr>
<td>Step 3</td>
<td>1-neighbors + 2-neighbors + its neighbors’ density</td>
<td>its father</td>
</tr>
</tbody>
</table>

Table 2: Sum up of exchanged information

We performed simulations in order to evaluate the performance of the proposed heuristic and estimate the importance of the introduction of the DAG. Nodes are randomly deployed using a Poisson process with different intensity levels $\lambda$ in a $1 \times 1$ square with various transmission ranges $R$ varying from 0.05 to 0.1. Each given statistic is the average over 1000 simulations. All these results are fully described in the extended version.

To build the DAG, each node randomly chooses a DAG Id between 0 and $\delta^2$ where $\delta$ is the maximum node’s degree in the graph as defined in section 3. For this, each node randomly chooses a DAG Id and then compares it to its neighbors’ ones. If DAG Ids are the same, the node with the smallest "normal" Id chooses another DAG Id and so on until every node has a different DAG Id than the ones of its 1-neighbors. For simulations on a grid and on a random geometry topology with $\lambda$ equal to 1000, the number of steps required to build the DAG does not take a lot of time since it only requires two steps on average, whatever $R$ is. Therefore, building the DAG is not costly. Table 3 summarizes the mean number of steps used to build the DAG for different transmission range values and a Poisson process intensity equal to 1000 in a random geometry topology and within a grid.

<table>
<thead>
<tr>
<th>$R$</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid</td>
<td>2.20</td>
<td>2.17</td>
<td>2.06</td>
<td>2.01</td>
<td>2.01</td>
<td>2.0</td>
</tr>
<tr>
<td>Random geometry</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>1.9</td>
<td>2.0</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 3: Number of steps needed to build the DAG over two different 1000 nodes distributions
We measure the following criteria: number of cluster-heads per surface unit, clusterization tree length (also in order to evaluate time of stabilization) and cluster-head eccentricity. Table 4 shows these criteria for \( \lambda = 1000 \) and different values of \( R \). We note \( e(\mathcal{H}(u)/\mathcal{C}) = \max_{v \in \mathcal{C}(u)} (d(\mathcal{H}(u), v)) \) in number of hops, the eccentricity of the cluster-head of node \( u \) inside its cluster, where \( d(u, v) \) is the minimum number of hops to reach \( v \) from \( u \). Whatever the transmission radius is (and so the degree), we can note that the mean cluster-head eccentricity and tree length do not vary too much. This confirms our assumption that the transmission of the cluster-head identity can be expected within a constant and low time. At last, let’s note that in the cases where nodes and node’s Id are homogeneously and randomly distributed, the use of the DAG does not bring much help. This is due to the fact that in such a nodes distribution, a node uses very rarely the Id to choose his parent because density values are well-distributed and scarcely equal.

We now consider a scenario where nodes are distributed over a grid with Ids increasing from left to right and from the bottom to the top. All interior nodes will have the same value density and the only criteria to select a cluster-head is the Id. As the nodes’ Ids are not well distributed, all nodes will finally join the same head. Table 5 shows the obtained results in this case. One can note that the DAG construction is very useful in such a case as it allows to drastically reduce the number of steps needed before stabilization. Figure 2 shows an example of clusters organization obtained for a radius equal to \( R = 0.05 \).

<table>
<thead>
<tr>
<th>( R = 0.05 )</th>
<th>( R = 0.08 )</th>
<th>( R = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>With DAG</td>
<td>No DAG</td>
<td>With DAG</td>
</tr>
<tr>
<td># clusters</td>
<td>61.0</td>
<td>61.4</td>
</tr>
<tr>
<td>( e(\mathcal{H}(u)/\mathcal{C}(u)) )</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>average tree length</td>
<td>2.7</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 4: Clusters features on a random geometric graph

<table>
<thead>
<tr>
<th>( R = 0.05 )</th>
<th>( R = 0.08 )</th>
<th>( R = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>With DAG</td>
<td>No DAG</td>
<td>With DAG</td>
</tr>
<tr>
<td># clusters</td>
<td>52.8</td>
<td>1.0</td>
</tr>
<tr>
<td>( e(\mathcal{H}(u)/\mathcal{C}(u)) )</td>
<td>3.4</td>
<td>29.1</td>
</tr>
<tr>
<td>average tree length</td>
<td>3.7</td>
<td>83.4</td>
</tr>
</tbody>
</table>

Table 5: Clusters characteristics on a grid

We have also tested the extra criterion given in Section 4.3 to improve the stability. Due to space limitation, we will not present all the results. We performed simulations where nodes move randomly at a randomly chosen speed during 15 minutes. We computed the percentage of cluster-heads which remained cluster-heads after each 2 seconds. For a node mobility between 0 to 1.6m/s (for pedestrians) the percentage of cluster-heads reelection is
about 82% with our improvement rules and 78% without. For a mobility from 0 to 10 m/s (for cars) this percentage is 31% with the new rules against 25% without. Thus, our improvements are useful in terms of cluster-head stability.

6 Conclusion

In this paper, we have addressed several issues concerning the self-stabilization on the clusterization process in multihop wireless network. We have proved that the clusterization algorithm based on the density criteria, defined in [16] is self-stabilizing. We have proposed different enhancements to reduce the stabilization time and to improve stability of the cluster-heads. Note that our contribution regarding the self-stabilization could be applied to several clusterization metrics as for instance the node’s degree ([3]).

In the future, several possible extensions of this work are open to investigation. It could be interesting to derive sharp bounds on the stabilization as a function of the mobility, e.g., speed of the nodes, mobility model, frequency of links failure, etc. Based on these bounds, we also plan to study hierarchical self-stabilization algorithms. Finally, we also
want to consider energy constraints in the stabilization algorithm and we are investigating energy-efficient organization algorithms.

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