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Foster impedance data modeling via singly terminated *LC* ladder networks

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Abstract: In this work, a lossless model is developed for the given Foster impedance data. In the model, a 2-port short- or open-terminated LC ladder is used. After applying the proposed algorithm, a realizable driving-point reactance function that fits the given data is obtained. Next, this function is synthesized, resulting in the desired model. In the algorithm, there is no need to select a circuit topology for the model. Two examples are given to illustrate the utilization of the proposed modeling algorithm.

Key words: Foster impedance, LC ladder networks, lossless circuits, modeling, network synthesis

1. Introduction

It is necessary to obtain a model for the measured data from physical devices or systems in many engineering applications [1]. In engineering for communications, typical situations that require circuit models can be listed as follows: the characterization of circuits in terms of the minimum noise figure level or maximum power transfer capability [2], the synthesis of matching networks or microwave amplifiers [3], and the rapid simulation of analog and digital communication systems [4–7].

For example, a Foster function may be needed to improve the power transfer capability of a matching network in broadband applications [8,9]. In [10], a lossless unsymmetrical lattice network was used as a broadband matching network. Lossless circuits utilized in the arms of the lattice network are realized via reactance functions.

In the method proposed in [11], a Foster impedance function $X_f(\omega)$ on the $j\omega$-axis was described as:

$$X_f(\omega) = \sum_{r=1}^{n} \frac{k_r \omega}{p_r^2 - \omega^2} + k_\infty \omega - \frac{k_0}{\omega}. \tag{1}$$

The pole p_r has been selected by the modeler, and the residues (k_r, k_0, and k_∞) are computed by solving Eq. (1) for the given Foster data set. However, in the proposed approach, open- or short-terminated LC ladder networks are used. The resulting model topology is the natural consequence of the proposed algorithm.

In [12], again, open- or short-terminated LC ladder networks were utilized in the models, but a reflectance-based gradient method was proposed. This study, however, develops a reactance-based approach and the performance of the new proposed method is superior to the performance of the method proposed in [12].

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In the next section, lossless 1-port networks are summarized in line with [13] and then the scattering description of lossless 2-ports is provided. Finally, the proposed algorithm is presented and illustrated in terms of 2 examples.

2. Lossless 1-port networks

Consider the 1-port network seen in Figure 1, which contains only inductors and capacitors. Since no real power is dissipated in a purely reactive network, and its driving-point function is positive real, the poles and zeros of the driving-point reactance or susceptance must be imaginary or 0.

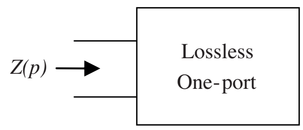

Figure 1. Lossless 1-port network.

Therefore, a driving-point reactance or susceptance function can be defined as an odd-ordered positive real function. Therefore, it is purely imaginary $\{$i.e. $Re\left[Z\left(j\omega\right)\right]=0\}$, and $Z\left(j\omega\right)$ can be written as:

$$Z\left(j\omega\right)=0+jX\left(j\omega\right). \tag{2}$$

The derivative of this function thus becomes:

$$\frac{dZ\left(j\omega\right)}{d\omega}=j\frac{dX\left(j\omega\right)}{d\omega}. \tag{3}$$

Since the derivative of $Z\left(j\omega\right)$ at the roots of $Z\left(j\omega\right)$ must be positive and real, a driving-point reactance has a derivative with a positive slope, and the poles and zeros must be mutually separated.

In the following, 2 conclusions can be written for a driving-point function of a pure reactance network as follows:

- The poles and zeros of $Z\left(j\omega\right)$ must be simple (or first-ordered), and they must occur on the $j\omega$-axis in conjugate pairs.
- The $j\omega$-axis poles and zeros must be mutually separated.

We can then write the driving-point impedance of the network seen in Figure 1 as:

$$Z(p)=\frac{N(p)}{D(p)}=\frac{A_1+pB_1}{A_2+pB_2}, \tag{4}$$

where N and D are the numerator and denominator polynomials, respectively, and the As and Bs are the even-ordered functions of p. For a reactance function, the highest- and lowest-order terms of $N(p)$ and $D(p)$ must differ in order by unity. If the orders of the numerator and denominator are equal, there would be resistance.

The real part of this function can be expressed as:

$$Re\left\{Z(p)\right\}_{p=j\omega}=\frac{A_1A_2+\omega^2B_1B_2}{A_2^2+\omega^2B_2^2}. \tag{5}$$

For a realizable network, $Re\{Z(j\omega)\}$ must be positive and real; therefore, it follows from Eq. (5) that $A_1A_2 + \omega^2 B_1 B_2 \geq 0$, and equality ($A_1A_2 + \omega^2 B_1 B_2 = 0$) yields reactance networks. Hence, there are only 2 possibilities for the real part of the function to be 0: either A_1 and B_2 must be 0, or A_2 and B_1 must be 0 in Eq. (5). Under these conditions, the driving-point impedance will have 1 of the following forms:

$$Z(p) = \frac{N(p)}{D(p)} = \frac{pB_1}{A_2}, \tag{6a}$$

$$Z(p) = \frac{N(p)}{D(p)} = \frac{A_1}{pB_2}. \tag{6b}$$

3. Scattering description of lossless 2-ports

The scattering matrix for a lumped-element lossless 2-port (Figure 2) can be written as [14]:

$$S(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \frac{1}{g(p)} \begin{bmatrix} h(p) & \mu f(-p) \\ f(p) & -\mu h(-p) \end{bmatrix}, \tag{7}$$

where $g(p)$, $h(p)$, and $f(p)$ are real polynomials in complex frequency $p = \sigma + j\omega$, μ is a constant, and $g(p)$ is a strictly Hurwitz polynomial. The 3 polynomials $g(p)$, $h(p)$, and $f(p)$ are related by the Feldtkeller equation as $g(p)g(-p) = h(p)h(-p) + f(p)f(-p)$.

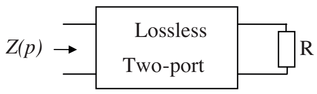

Figure 2. Lossless 2-port terminated by resistance $R = 0$ or $R = \infty$.

The polynomial $f(p)$ is either even or odd if the 2-port is reciprocal. In this case, if $f(p)$ is even, $\mu = +1$, and if $f(p)$ is odd, $\mu = -1$. Therefore, for a lossless reciprocal 2-port:

$$\mu = \frac{f(-p)}{f(p)} = \pm 1, \tag{8}$$

and the Feldtkeller equation can be written as:

$$g(p)g(-p) = h(p)h(-p) + \mu f(p)^2. \tag{9}$$

We can then write the input impedance of the network seen in Figure 2 as:

$$Z(p) = \frac{N(p)}{D(p)} = \frac{1 + S_{11}(p)}{1 - S_{11}(p)}. \tag{10}$$

For a short- or open-terminated network $|S_{11}(p)| = 1$ or, equivalently, $S_{11}(p)S_{11}(-p) = 1$. Therefore, from Eq. (7), it can be stated that:

$$S_{11}(p)S_{11}(-p) = \frac{h(p)}{g(p)}\frac{h(-p)}{g(-p)} = 1. \tag{11}$$

If $h(p)$ is equal to $g(p)$, $S_{11} = 1$ at all frequencies (trivial solution). Otherwise, it can be concluded that:

$$h(p) = \pm g(-p) = \gamma\, g(-p), \tag{12}$$

and

$$S_{11}(p) = \pm\frac{g(-p)}{g(p)} = \gamma\frac{g(-p)}{g(p)}. \tag{13}$$

At $p = 0$, $S_{11}(p)$ can be either $\gamma = +1$ or $\gamma = -1$, corresponding to an open termination or to a short termination, respectively.

Using the Feldtkeller equation and Eq. (12), it can be said that:

$$\begin{aligned} g(p)g(-p) &= h(p)h(-p) + f(p)f(-p) \\ &= \gamma\, g(-p)\cdot\gamma\, g(p) + f(p)\, f(-p) \\ &= g(p)\, g(-p) + f(p)\, f(-p). \end{aligned} \tag{14}$$

Hence, $f(p) = 0$, if the lossless 2-port is terminated in a short or an open connection.

If the driving point impedance is written using Eqs. (10) and (13), then the following expression can be obtained:

$$Z(p) = \frac{1+S_{11}(p)}{1-S_{11}(p)} = \frac{1+\gamma\frac{g(-p)}{g(p)}}{1-\gamma\frac{g(-p)}{g(p)}} = \frac{g(p)+\gamma\, g(-p)}{g(p)-\gamma\, g(-p)}. \tag{15}$$

If $\gamma = -1$, $Z(p) = \frac{g_o(p)}{g_e(p)}$, which is equal to Eq. (6a), and if $\gamma = +1$, then $Z(p) = \frac{g_e(p)}{g_o(p)}$, which is equal to Eq. (6b), where the subscripts "e" and "o" refer to the even and odd parts, respectively.

4. Proposed modeling algorithm

Inputs:

- $Z(j\omega_i) = jX_f(\omega_i); i = 1, 2, \ldots, N$: Given Foster impedance data.
- $\omega_i = 2\pi\, f_i, i = 1, 2, \ldots, N$: Measurement frequencies.
- $r_k = j\beta_k; k = 1, 2, \ldots, n$: The initial imaginary poles and zeros of the driving-point impedance of the model ($Z_{in}(p)$). Since the poles and zeros are mutually separated, if the smallest ($j\beta_1$) is a pole, a zero must be placed at $\omega = 0$, and if the smallest is a zero, a pole must be placed at $\omega = 0$. Hence, the number of reactive elements in the Foster model is $nc = n + 1$.
- γ : Termination type, $\gamma = -1$ for short termination and $\gamma = +1$ for open termination.
- δ : The stopping criteria of the sum of the square errors.

Computational steps:

Step 1: If the given impedance and measurement frequencies are not normalized, select a frequency normalization number (f_n) and an impedance normalization number (R_n), and normalize them as follows: $Z_n(j\omega) = \frac{Z(j\omega)}{R_n}$ and $\omega_n = \frac{f}{f_n}$.

Step 2: Decide if the smallest initial root is a pole or zero, then calculate the conjugates of the imaginary poles and zeros such as $r_k^* = -j\beta_k$, and by using the pole or zero at $\omega = 0$, form the polynomials $N(p)$ and $D(p)$.

Step 3: Calculate the input impedance $Z_{in}(j\omega) = \frac{N(j\omega)}{D(j\omega)}$ at the frequency points.

Step 4: Calculate the error via $\varepsilon(j\omega) = Z_n(j\omega) - Z_{in}(j\omega)$ and $\delta_c = \sum |\varepsilon(j\omega)|^2$.

Step 5: If $\delta_c \leq \delta$, synthesize the input impedance function $Z_{in}(p) = \frac{N(p)}{D(p)}$. Now it is necessary to denormalize the element values as $L = \frac{L^{(n)} R_n}{2\pi f_n}$ and $C = \frac{C^{(n)}}{2\pi f_n R_n}$, where $L^{(n)}$ and $C^{(n)}$ are the normalized inductor and capacitor values, respectively. If $\delta_c > \delta$, go to the next step, otherwise, stop.

Step 6: Change the initial imaginary poles and zeros via any constrained optimization routine, and return to Step 2.

5. Examples

5.1. Example 1

To be able to compare the performance of the newly proposed method and the method in [12], the example given in [12] is solved here again.

First, an impedance is formed by an inductor in series with the parallel combination of a capacitor and a resistor, with the normalized values $L^{(n)} = 1$, $C^{(n)} = 2$, and $R^{(n)} = 1$. Next, the imaginary part of this impedance is used to obtain the Foster impedance data. The calculated Foster impedance data are listed in Table 1.

Table 1. Foster impedance data given in [12].

ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
X_f	0.1010	0.2077	0.3248	0.4552	0.6000	0.7588	0.9302	1.1122	1.3028	1.5000

In this step, open termination ($\gamma = +1$) and 4 elements ($n_c = 4$) were selected for the model. The initial imaginary poles and zeros of the driving-point impedance of the model ($Z_{in}(p)$) were selected as $r_1 = j2$, $r_1 = j4$, and $r_1 = j6$ in an ad hoc manner. The first root ($r_1 = j2$) was selected as a zero, so a pole was placed at $\omega = 0$. Applying the proposed algorithm, the driving-point impedance of the model is determined as:

$$Z_{in}(p) = \frac{N(p)}{D(p)},$$

where

$$N(p) = p^4 + 14.9151p^2 + 0.0197$$

and

$$D(p) = -3.8618p^3 - 12.8557p.$$

The synthesis of this impedance function results in the equivalent circuit presented in Figure 3.

In Figure 4, the Foster impedance curves of the data in Table 1, which were obtained from the newly proposed method and given in [12], are shown. It is clear that the performance of the proposed method marks an improvement.

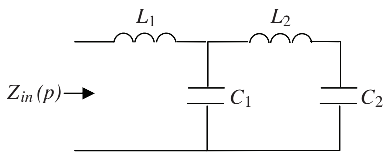

Figure 3. Obtained model for the Foster impedance data given in Table 1. $L_1^{(n)} = 0.2589, L_2^{(n)} = 0.9017, C_1^{(n)} = 0.3333$, and $C_2^{(n)} = 652.2403$ (normalized values).

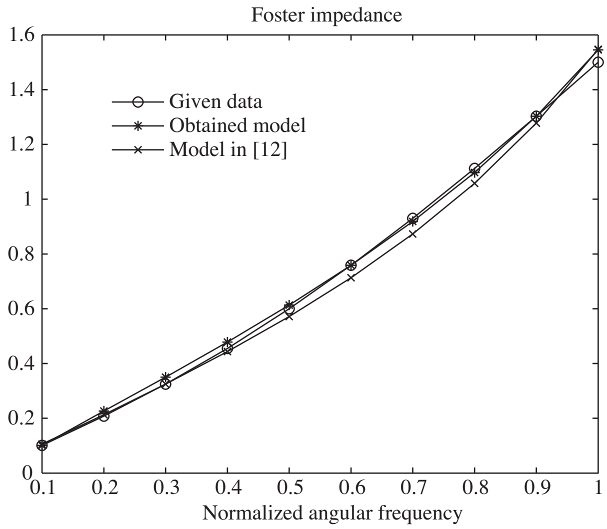

Figure 4. Comparison of the Foster impedance curves for the given data, the new model, and that in [12].

5.2. Example 2

In this example, a data set measured from a dipole antenna is used and an LC ladder model for the antenna is obtained. The normalized measurement frequencies and impedance values are given in Table 2. For frequency normalization, $f_n = 275$ MHz was used, and $R_n = 51864\Omega$ was used for the impedance normalization.

Table 2. Foster impedance data for dipole antenna.

ω	0.1091	0.2364	0.3273	0.4545	0.5455	0.6727	0.7636	0.8909	0.9818	1.0000
X_f	–0.0655	–0.0191	–0.0063	0.0062	0.0145	0.0302	0.0502	0.1459	0.8666	1.0000

In this example, open termination ($\gamma = +1$) and 4 elements ($n_c = 4$) were selected for the model. The initial imaginary roots were selected as $r_1 = j1$, $r_1 = j2$, and $r_1 = j3$. The first root ($r_1 = j1$) was selected as a zero, and so a pole was placed at $\omega = 0$. Applying the proposed algorithm, the driving-point impedance function of the model is determined as:

$$Z_{in}(p) = \frac{N(p)}{D(p)},$$

where

$$N(p) = 1.3059p^4 + 6.6568p^2 + 1$$

and

$$D(p) = -135.6013p^3 - 135.6506p.$$

The synthesis of this impedance function results in the equivalent circuit shown in Figure 5. The normalized element values are $L_1^{(n)} = 0.00963$, $L_2^{(n)} = 0.03208$, $C_1^{(n)} = 135.6506$, and $C_2^{(n)} = 31.16722$. After denormalization, the following actual element values are obtained:

$L_1 = \frac{L_1^{(n)} \cdot R_n}{2\pi f_n} = 289.05$ nH, $L_2 = \frac{L_2^{(n)} \cdot R_n}{2\pi f_n} = 963.91$ nH,

$C_1 = \frac{C_1^{(n)}}{2\pi f_n R_n} = 1.5137$ pF, $C_2 = \frac{C_2^{(n)}}{2\pi f_n R_n} = 0.3478$ pF.

A comparison of the original and reconstructed impedance curves is illustrated in Figure 6.

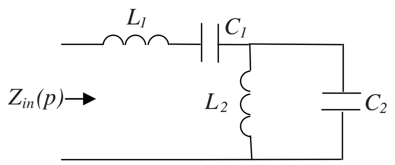

Figure 5. Obtained model for the dipole antenna.

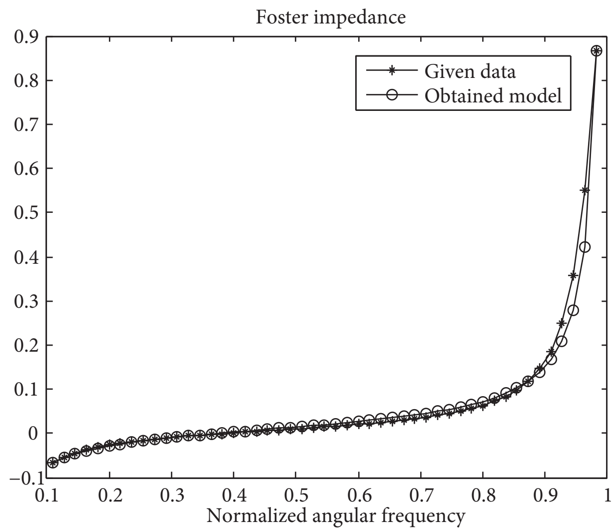

Figure 6. Foster impedance curves.

6. Conclusion

In this study, a new algorithm was introduced to model the measured or computed Foster impedance data. In the models, short- or open-terminated LC ladder networks were used. Two examples illustrated the implementation of the modeling method and served as a proof-of-principle. The modeling method is quite simple and straightforward, and it is an important tool for many applications like broadband matching and device modeling.

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