

Syddansk Universitet

## Composite Goldstone Dark Matter: Experimental Predictions from the Lattice

Hietanen, Ari; Lewis, Randy; Pica, Claudio; Sannino, Francesco

*Published in:*  
Journal of High Energy Physics (Online)

*DOI:*  
[10.1007/JHEP12\(2014\)130](https://doi.org/10.1007/JHEP12(2014)130)

*Publication date:*  
2014

*Citation for published version (APA):*  
Hietanen, A., Lewis, R., Pica, C., & Sannino, F. (2014). Composite Goldstone Dark Matter: Experimental Predictions from the Lattice. *Journal of High Energy Physics (Online)*, 130. DOI: 10.1007/JHEP12(2014)130

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# Composite Goldstone Dark Matter: Experimental Predictions from the Lattice

Ari Hietanen<sup>♥</sup>, Randy Lewis<sup>♣</sup>, Claudio Pica<sup>♥</sup>, and Francesco Sannino<sup>♥</sup>

<sup>♥</sup> CP<sup>3</sup>-Origins & the Danish IAS, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark

<sup>♣</sup> Department of Physics and Astronomy, York University, Toronto, Ontario, M3J 1P3, Canada

## Abstract

We study, via first principles lattice simulations, the nonperturbative dynamics of  $SU(2)$  gauge theory with two fundamental Dirac flavors. The model can be used simultaneously as a template for composite Goldstone boson dark matter and for breaking the electroweak symmetry dynamically. We compute the form factor, allowing us to estimate the associated electromagnetic charge radius. Surprisingly we observe that the form factor obeys vector meson dominance even for the two color theory. We finally compare the model predictions with dark matter direct detection experiments. Our results are a foundation for quantitative new composite dynamics relevant for model building, and are of interest to current experiments.

Preprint: CP<sup>3</sup>-Origins-2013-30 DNRF90 & DIAS-2013-30

Unveiling the nature of dark matter (DM) constitutes a fundamental problem in physics. It plays an important role in large scale structure formation as well as the evolution of the Universe. DAMA [1], CoGeNT [2, 3], CRESST-II [4] and CDMS-Si [5] reported potential signals of weakly interacting massive particles while the remaining experiments report negative results [6–11] interpreted as upper bounds on interaction rates.

We explore the paradigm according to which DM is a composite pseudo-Goldstone boson (GB) [12]. Composite non-GB DM models appeared earlier [13]. The template is an  $SU(2)$  gauge theory with two fundamental fermion flavors termed  $U$  and  $D$  [14]. We view this theory as the kernel from which more elaborate models can grow. Our minimal template has the appeal to address simultaneously electroweak symmetry breaking and the origin of a naturally-light DM candidate [14]. The observed Higgs mass can arise due to top quark corrections to the mass of the lightest non-GB scalar in the theory [15]. The action has a global  $SU(4)$  symmetry, and the lattice simulations of [16] showed that it is dynamically broken to  $Sp(4)$ , thereby producing five GBs. By embedding the standard model (SM) within the  $SU(4)$  symmetry, one shows that three of these are eaten by the  $W^\pm$  and  $Z$  bosons; the remaining pair of GBs is taken to be the DM candidate and its antiparticle. Depending on the cross section for annihilation into SM fields one can have a symmetric (i.e. thermal relic density), asymmetric, or a mixed scenario [17]. An exact GB would be massless but the DM candidate can acquire a small mass from explicit symmetry breaking through new four-fermion interactions breaking the original  $SU(4)$  symmetry to  $SU_L(2) \times SU_R(2) \times U(1)$  while keeping the  $U$  and  $D$  fermions massless [14]. The interactions with ordinary matter relevant for direct detection experiments occur prevalently via the exchange of a Higgs or a photon. The photon interaction is due to the DM electric dipole moment that we wish to estimate, and in doing so we will show that vector meson dominance is at play even for the two color theory. Finally we will confront the theory predictions with direct DM detection measurements.

**The lattice method and results:** In the continuum, the Lagrangian for our template is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{U}(i\gamma^\mu D_\mu - m_U)U + \bar{D}(i\gamma^\mu D_\mu - m_D)D. \quad (1)$$

On the lattice we use the Wilson plaquette action with Wilson fermions. Mesons will couple to local operators of the form  $\mathcal{O}_{\bar{X}Y}^{(\Gamma)}(x) = \bar{X}(x)\Gamma Y(x)$ , and  $\sqrt{2}\mathcal{O}_{\bar{U}U\pm\bar{D}D}^{(\Gamma)}(x) = \mathcal{O}_{\bar{U}U}^{(\Gamma)}(x) \pm \mathcal{O}_{\bar{D}D}^{(\Gamma)}(x)$ , where  $\Gamma$  denotes any product of Dirac matrices and  $X, Y$  are either  $U, D$  or  $D, U$ . Baryons couple to

$$\mathcal{O}_{XY}^{(\Gamma)}(x) = X^T(x)(-i\sigma^2)CY(x), \quad (2)$$

$$\mathcal{O}_{UU\pm DD}^{(\Gamma)}(x) = \frac{1}{\sqrt{2}}\left(\mathcal{O}_{UU}^{(\Gamma)}(x) \pm \mathcal{O}_{DD}^{(\Gamma)}(x)\right), \quad (3)$$

where the Pauli structure  $-i\sigma^2$  acts on color indices while the charge conjugation operator  $C$  acts on Dirac indices. A photon can couple to a local vector operator such as  $\mathcal{O}_{\bar{U}U\pm\bar{D}D}^{(\gamma_\mu)}$  which is a conserved current in the continuum limit but not in the lattice theory. It is advantageous to work directly with the lattice conserved currents,

$$V_\mu^X(x) = \frac{1}{2}\bar{X}(x + \hat{\mu})(1 + \gamma_\mu)U_\mu^\dagger(x)X(x) - \frac{1}{2}\bar{X}(x)(1 - \gamma_\mu)U_\mu(x)X(x + \hat{\mu}), \quad (4)$$

where  $U_\mu(x) \in SU(2)$  is the gauge link matrix. With Eq. (4) we can produce the electromagnetic current,

$$V_\mu(x) = \frac{1}{2}V_\mu^U(x) - \frac{1}{2}V_\mu^D(x). \quad (5)$$

A three-point correlation function that probes the elastic form factor of the DM candidate can be fitted in two ways (see, e.g., [18]). One is the simultaneous fit method:

$$\frac{C_{UD}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f)}{|Z_\Pi|^2} = \frac{e^{-(t_f-t)E_\Pi(\vec{p}_f)} e^{-(t-t_i)E_\Pi(\vec{p}_i)}}{2E_\Pi(\vec{p}_f) 2E_\Pi(\vec{p}_i)} F_\Pi(p_i + p_f)_\mu,$$

$$C_{UD}^{(2)}(t_i, t_f, \vec{p}) = |Z_\Pi|^2 \frac{e^{-(t_f-t_i)E_\Pi(\vec{p})}}{2E_\Pi(\vec{p})} + \sum_{\text{excited } n} |Z_n|^2 \frac{e^{-(t_f-t_i)E_n(\vec{p})}}{2E_n(\vec{p})}, \quad (6)$$

where we have used the standard definition of the form factor,  $\langle \Pi(\vec{p}_f) | V_\mu(0) | \Pi(\vec{p}_i) \rangle = F_\Pi(Q^2)(p_i + p_f)_\mu$ , and  $Q^2 = (\vec{p}_f - \vec{p}_i)^2 - (E_\Pi(\vec{p}_f) - E_\Pi(\vec{p}_i))^2$ . For any chosen lattice momentum, the fit parameters are the energies  $E_\Pi$  and  $E_n$ , the coefficients  $|Z_\Pi|^2$  and  $|Z_n|^2$ , and the form factor  $F_\Pi(Q^2)$ . Besides the method of Eq. (6), the form factor can also be obtained from a ratio method, valid for  $t_i \ll t \ll t_f$ :

$$F_\Pi(Q^2) = \frac{C_{UD}^{(3)}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) C_{UD}^{(2)}(t_i, t, \vec{p}_f)}{C_{UD}^{(2)}(t_i, t, \vec{p}_i) C_{UD}^{(2)}(t_i, t_f, \vec{p}_f)} \frac{2E_\Pi(\vec{p}_f)}{E_\Pi(\vec{p}_i) + E_\Pi(\vec{p}_f)}. \quad (7)$$

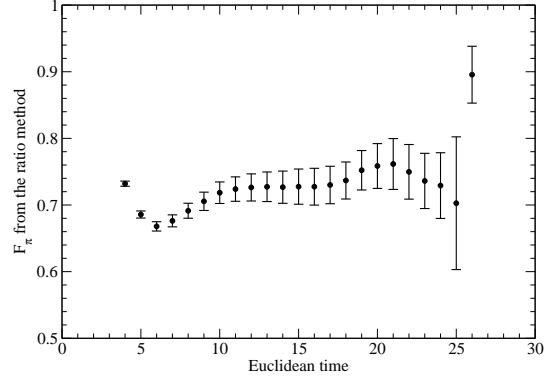
This equation is very convenient because all  $Z_n$  have canceled away, and the ratio  $E_\Pi(\vec{p}_i)/E_\Pi(\vec{p}_f)$  is easy to obtain from the lattice two-point functions. All that remains is to fit the ratio to a constant for each value of  $Q^2$ . A feature of Eq. (7) is that the only two-point function that extends all the way from  $t_i$  to  $t_f$  has momentum  $\vec{p}_f$ . We maximize numerical precision by always choosing  $\vec{p}_f = \vec{0}$ .

The  $U$  and  $D$  fermions in our action have electroweak charges that are constrained by anomaly cancellation: they form a left-handed weak doublet, right-handed weak singlets, and have electric charges  $Q_U = +1/2$  and  $Q_D = -1/2$ . Neither fermion carries QCD color. The DM GB has a valence structure  $UD$ . Because it is symmetric under  $U \leftrightarrow D$ , the DM candidate has no electroweak elastic form factors if the theory has exact isospin symmetry. Isospin breaking is expected to occur in nature given that it is already present for the ordinary quarks, and is welcome in the present context since it serves to further diminish, or eliminate, tensions with precision data. The ultimate origin of isospin breaking might be four-fermion interactions arising from higher-energy physics [14], but we can mimic it here by simply using two different explicit masses for the  $U$  and  $D$  fermions. Our lattice study therefore follows standard lattice QCD methods. See [18, 19] and references therein.

In the large  $N_c$  limit the form factor can be written as a sum over vector meson poles [20]. In practice those sums are dominated by the lightest vector mesons. Perhaps surprisingly, this large  $N_c$  result has long been known to work rather well for certain QCD observables despite the seemingly small value of  $N_c = 3$ . A QCD example that exactly parallels our  $m_U \neq m_D$  effects is the neutral kaon, which has a nonzero form factor arising from  $m_d \neq m_s$ . The experimental determination of the neutral kaon charge radius [21] is dominated by the difference between  $\rho^0$  and  $\phi$  meson exchanges,

$$F_{K^0}(Q^2) \approx -\frac{1}{3} \left( \frac{m_\rho^2}{m_\rho^2 + Q^2} \right) + \frac{1}{3} \left( \frac{m_\phi^2}{m_\phi^2 + Q^2} \right), \quad (8)$$

with  $\langle r^2 \rangle_{K^0} = -6 \frac{dF_{K^0}}{dQ^2} \Big|_{Q^2=0}$ . If vector meson dominance were also applicable to our  $N_c = 2$  case, then lattice determinations of the vector meson masses would provide



**FIG. 1:** Ratio definition of the GB form factor, Eq. (7), for momentum  $(p_x, p_y, p_z) = (1, 1, 0) \times 2\pi/L$  in the ensemble having  $(\beta, m_0) = (2.2, -0.72)$ .

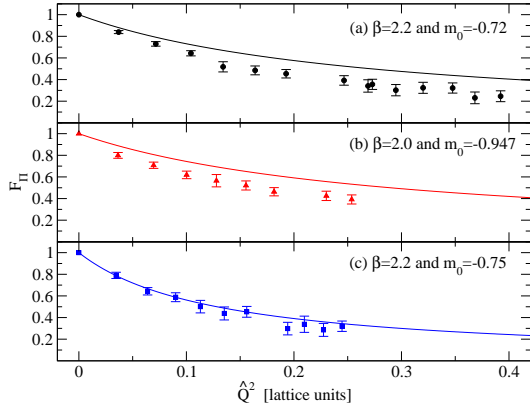
estimates of all GB form factors. The following discussion presents a lattice simulation of the GB form factor in the  $m_U = m_D$  limit and shows that the large  $N_c$  result is indeed reflected in our  $N_c = 2$  theory.

The simulations performed for this project use some of the lattice parameters from [16] that are closest to the chiral limit, but now with a larger lattice volume (namely  $L^4 = 32^4$ , which removes all finite volume effects) and more configurations. A complete analysis of 500 configurations at  $(\beta, m_0) = (2.2, -0.72)$  provides a first result for the form factor. To consider discretization effects an analysis of 300 configurations at  $(\beta, m_0) = (2.0, -0.947)$  is performed. To study chiral extrapolation effects, an analysis of 300 configurations at  $(\beta, m_0) = (2.2, -0.75)$  is performed. All ensembles were created with the HiRep code [22] for fully-dynamical plaquette-action  $SU(2)$  gauge theory with two flavors ( $U$  and  $D$ ) of mass-degenerate Wilson fermions.

We choose the outgoing GB to be at rest in our form factor computations, so momentum flows from the incoming GB to the photon coupling. All momentum directions are averaged for each configuration. We use Dirichlet boundary conditions in the time direction for fermions. The GB creation operator is placed at the fifth time step from the lattice's left edge ( $t_i = 4$ ) and the annihilation operator is placed at the fifth from the right ( $t_f = 27$ ).

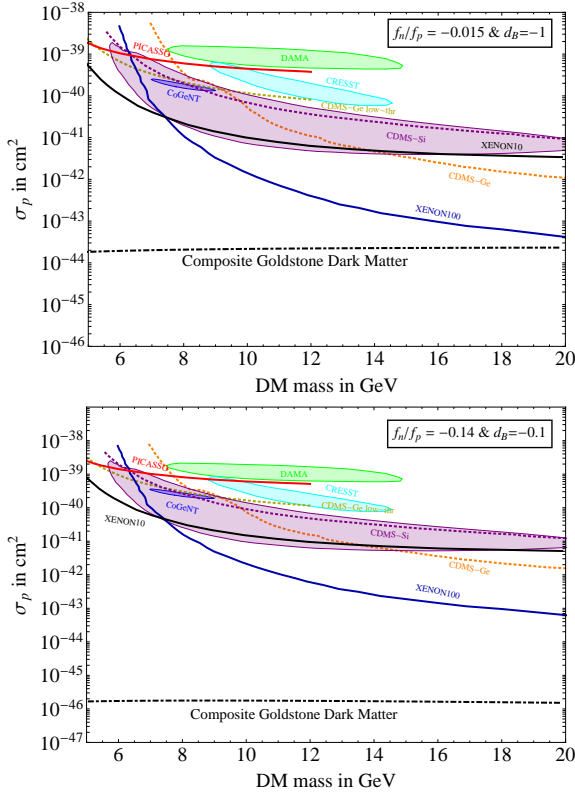
As an example, Fig. 1 shows the raw form factor data for the right-hand side of Eq. (7) with one particular momentum in the  $(\beta, m_0) = (2.2, -0.72)$  ensemble. There is a broad range of Euclidean times between  $t_i$  and  $t_f$  where the ratio is indeed constant, allowing the form factor to be read from the plot.

Numerical results for the form factors are compared to vector meson dominance in Fig. 2. The form factor consistently has the shape of a simple meson pole but only panel (c), with our lightest fermion, obeys vector meson dominance. Panels (a) and (b) have a heavier fermion and virtually identical vector mesons on our finer and coarser lattices, respectively. Because the form



**FIG. 2:** Lattice results for the GB form factor, with statistical errors. The curves are the predictions from a vector meson pole,  $(1 + Q^2/m_\rho^2)$ , with the vector mass taken from the corresponding lattice simulation.

factors in (a) and (b) are consistent with each other, there is no indication of discretization errors. A direct statistical comparison in Fig. 2 gives  $\chi^2/\text{d.o.f.} =$  (a) 11, (b) 12 and (c) 0.94. We conclude that the lightest vector meson dominates the form factor for sufficiently light fermions.



**FIG. 3:** Direct detection experimental constraints as detailed in the text. The composite GB DM cross section is shown for two values of  $d_B$ .

**Dark form factor:** The charge radius of a scalar cou-

ples to the photon as follows [23]

$$\mathcal{L}_\gamma = ie \frac{d_B}{\Lambda^2} \phi^* \overleftrightarrow{\partial}_\mu \phi \partial_\nu F^{\mu\nu}. \quad (9)$$

In our case we have a specific expression for the coefficient,

$$\frac{d_B}{\Lambda^2} = \frac{1}{Q^2} \left[ \frac{1}{2} \frac{m_{\rho_u}^2}{m_{\rho_u}^2 + Q^2} - \frac{1}{2} \frac{m_{\rho_D}^2}{m_{\rho_D}^2 + Q^2} \right] = \frac{m_{\rho_u}^2 - m_{\rho_D}^2}{2m_{\rho_u}^2 m_{\rho_D}^2} + \text{h.o.},$$

which, for small isospin breaking ( $m_{\rho_u} \approx m_{\rho_D} \equiv m_\rho$ ), corresponds to  $\Lambda = m_\rho$ , and  $d_B = (m_{\rho_u} - m_{\rho_D})/m_\rho$ . The numerical value of  $m_\rho$  is obtained by extrapolating our lattice data for  $m_\rho$  and  $f_\pi/Z_A$ , shown in Figs. 7 and 6 of [16] respectively, to the chiral limit. Using the finer lattice spacing ( $\beta = 2.2$ ) a quadratic extrapolation gives  $m_\rho = 0.30$  in lattice units. Similarly, fitting with a ratio of linear functions gives  $f_\pi/Z_A = 0.029$ . We conclude that  $m_\rho = \frac{0.30}{0.029} (246 \text{ GeV}) = 2.5 \pm 0.2 \text{ TeV}$ , where the error bar is the difference between the coarse and fine lattices.

From [23], the cross section for a DM particle  $\phi$  scattering from a proton through photon exchange is

$$\sigma_p^\gamma = \frac{\mu^2}{4\pi} \left( \frac{8\pi\alpha d_B}{\Lambda^2} \right)^2 \quad (10)$$

where  $\mu = m_\phi m_p / (m_\phi + m_p) < m_p$  and  $m_p$  the proton mass. The only remaining unknown is  $|d_B|$  which is clearly less than unity. We therefore have the first lattice-determined upper bound on the cross section for a model of composite GB DM,

$$\sigma_p^\gamma < 2.3 \times 10^{-44} \text{ cm}^2. \quad (11)$$

Besides the photon interactions we expect also a composite Higgs exchange [14, 17, 23, 24]. The relevant terms in the Lagrangian connecting our DM candidate to the composite Higgs are  $\frac{d_1}{\Lambda} h \partial_\mu \phi^* \partial^\mu \phi + \frac{d_2}{\Lambda} m_\phi^2 h \phi^* \phi$ . A complete classification for all the possible operators can be found in [25]. We cannot yet determine via first principles simulations these form factors, however from the pseudo-GB nature of the DM field  $\phi$ , we expect  $d_1$  and  $d_2$  to be order unity and  $\Lambda \approx m_\rho$ .

Making the further minimal assumption that the composite Higgs state couples to the SM fermions with a strength proportional to their masses, as for the ordinary Higgs, the zero momentum transfer cross section of  $\phi$  scattering off a nucleus with  $Z$  protons and  $A - Z$  neutrons is [17, 23]  $\sigma_A = \frac{\mu_A^2}{4\pi} |Z f_p + (A - Z) f_n|^2$ , where  $f_n = d_H f \frac{m_p}{m_H^2 m_\phi}$ ,  $f_p = f_n - \frac{8\pi\alpha d_B}{\Lambda^2}$ ,  $\mu_A$  is the  $\phi$ -nucleus reduced mass,  $f \sim 0.3$  parametrizes the Higgs to nucleon coupling, and we have defined  $d_H = -\frac{d_1 + d_2}{v_{EW} \Lambda} m_\phi^2$  [24].

The event rate formulae and derivation for generic couplings  $f_n$  and  $f_p$  can be found in [24]. In the upper and lower panel of Fig. 3 we show the favored regions and exclusion contours in the  $(m_\phi, \sigma_p)$  plane for (upper

panel)  $f_n/f_p = -0.015$  corresponding to  $d_B = -1$  and  $d_1+d_2 = 1$  and (lower panel)  $f_n/f_p = -0.14$  corresponding to  $d_B = -0.1$  and again  $d_1 + d_2 = 1$ . The green contour is the  $3\sigma$  favored region by DAMA/LIBRA [26] assuming no channeling [27] and that the signal arises entirely from Na scattering; the blue region is the 90% CL favored region by CoGeNT; the light blue region corresponds to CRESST [4] results; the dashed orange line corresponds to CDMS-Ge [9] and the magenta one to CDMS-Ge-lowthr for masses below 10 GeV [9]; the bound [11] and the allowed region in purple corresponds to CDMS-Si [5]; the red, black and blue lines are respectively the exclusion plots from the PICASSO [28], Xenon10 [6] and Xenon100 [8] experiments.

We find that the theoretical composite GB DM cross sections (the black dot-dashed curves) are below the exclusion limits set by the most stringent experiments but can be accessed by future experiments for sufficiently large weak isospin breaking. The maximal size of the cross section with ordinary matter, at low energies, is set by having explicitly shown, via lattice simulations, that

in this theory the relevant form factors are saturated by a single vector meson exchange whose mass is in the 2.5 TeV energy range.

One can envision natural models with larger cross sections. These would require smaller values of the vector masses which can be obtained, for example, by rendering the theory near conformal by either adding new matter gauged under the composite dynamics and singlet with respect to SM interactions [14, 29], and/or changing the matter representation or the composite gauge group [30, 31]. Lattice investigations of non-GB composite DM were performed in [32]. Other electroweak embeddings, where the Higgs is a pseudo-GB, are possible [33].

This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada and the Danish National Research Foundation DNRF:90 grant. Computing facilities were provided by the Danish Centre for Scientific Computing and Canada's Shared Hierarchical Academic Research Computing Network (SHARCNET: <http://www.sharcnet.ca>). We thank E. Del Nobile for updates on the latest experimental results.

- 
- [1] R. Bernabei *et al.* [DAMA and LIBRA Collaborations], *Eur. Phys. J. C* **67**, 39 (2010) [arXiv:1002.1028 [astro-ph.GA]].
- [2] C. E. Aalseth *et al.* [CoGeNT Collaboration], *Phys. Rev. Lett.* **106**, 131301 (2011) [arXiv:1002.4703 [astro-ph.CO]].
- [3] C. E. Aalseth, P. S. Barbeau, J. Colaresi, J. I. Collar, J. Diaz Leon, J. E. Fast, N. Fields and T. W. Hossbach *et al.*, *Phys. Rev. Lett.* **107**, 141301 (2011) [arXiv:1106.0650 [astro-ph.CO]].
- [4] G. Angloher, M. Bauer, I. Bavykina, A. Bento, C. Bucci, C. Ciemiak, G. Deuter and F. von Feilitzsch *et al.*, *Eur. Phys. J. C* **72**, 1971 (2012) [arXiv:1109.0702 [astro-ph.CO]].
- [5] R. Agnese *et al.* [CDMS Collaboration], [arXiv:1304.4279 [hep-ex]].
- [6] J. Angle *et al.* [XENON10 Collaboration], *Phys. Rev. Lett.* **107**, 051301 (2011) [arXiv:1104.3088 [astro-ph.CO]].
- [7] E. Aprile *et al.* [XENON100 Collaboration], *Phys. Rev. Lett.* **107**, 131302 (2011) [arXiv:1104.2549 [astro-ph.CO]].
- [8] E. Aprile *et al.* [XENON100 Collaboration], *Phys. Rev. Lett.* **109**, 181301 (2012) [arXiv:1207.5988 [astro-ph.CO]].
- [9] Z. Ahmed *et al.* [CDMS-II Collaboration], *Phys. Rev. Lett.* **106**, 131302 (2011) [arXiv:1011.2482 [astro-ph.CO]].
- [10] Z. Ahmed *et al.* [CDMS Collaboration], arXiv:1203.1309 [astro-ph.CO].
- [11] R. Agnese *et al.* [CDMS Collaboration], [arXiv:1304.3706 [astro-ph.CO]].
- [12] S. B. Gudnason, C. Kouvaris and F. Sannino, *Phys. Rev. D* **73**, 115003 (2006) [hep-ph/0603014]; *idem*, *Phys. Rev. D* **74**, 095008 (2006) [hep-ph/0608055].
- [13] S. Nussinov, *Phys. Lett. B* **165**, 55 (1985). S. M. Barr, R. S. Chivukula and E. Farhi, *Phys. Lett. B* **241**, 387 (1990).
- [14] T. A. Ryttov and F. Sannino, *Phys. Rev. D* **78**, 115010 (2008) [arXiv:0809.0713 [hep-ph]].
- [15] R. Foadi, M. T. Frandsen and F. Sannino, *Phys. Rev. D* **87**, 095001 (2013) [arXiv:1211.1083 [hep-ph]].
- [16] R. Lewis, C. Pica and F. Sannino, *Phys. Rev. D* **85**, 014504 (2012) [arXiv:1109.3513 [hep-ph]].
- [17] A. Belyaev, M. T. Frandsen, S. Sarkar and F. Sannino, *Phys. Rev. D* **83**, 015007 (2011) [arXiv:1007.4839 [hep-ph]].
- [18] F. D. R. Bonnet *et al.* [Lattice Hadron Physics Collaboration], *Phys. Rev. D* **72**, 054506 (2005) [hep-lat/0411028].
- [19] B. B. Brandt, A. Juttner and H. Wittig, arXiv:1306.2916 [hep-lat].
- [20] P. Masjuan, E. Ruiz Arriola and W. Broniowski, *Phys. Rev. D* **87**, 014005 (2013) [arXiv:1210.0760 [hep-ph]].
- [21] J. Beringer *et al.* [Particle Data Group Collaboration], *Phys. Rev. D* **86**, 010001 (2012).
- [22] L. Del Debbio, A. Patella and C. Pica, *Phys. Rev. D* **81**, 094503 (2010) [arXiv:0805.2058 [hep-lat]].
- [23] R. Foadi, M. T. Frandsen and F. Sannino, *Phys. Rev. D* **80**, 037702 (2009) [arXiv:0812.3406 [hep-ph]].
- [24] E. Del Nobile, C. Kouvaris and F. Sannino, *Phys. Rev. D* **84**, 027301 (2011) [arXiv:1105.5431 [hep-ph]].
- [25] E. Del Nobile and F. Sannino, *Int. J. Mod. Phys. A* **27**, 1250065 (2012) [arXiv:1102.3116 [hep-ph]].
- [26] C. Savage, G. Gelmini, P. Gondolo and K. Freese, *Phys. Rev. D* **83**, 055002 (2011) [arXiv:1006.0972 [astro-ph.CO]].
- [27] N. Bozorgnia, G. B. Gelmini and P. Gondolo, *JCAP* **1011**, 019 (2010) [arXiv:1006.3110 [astro-ph.CO]].
- [28] S. Archambault *et al.* [PICASSO Collaboration], *Phys. Lett. B* **711**, 153 (2012) [arXiv:1202.1240 [hep-ex]].
- [29] D. D. Dietrich and F. Sannino, *Phys. Rev. D* **75**, 085018 (2007) [hep-ph/0611341].
- [30] M. T. Frandsen and F. Sannino, *Phys. Rev. D* **81**, 097704 (2010) [arXiv:0911.1570 [hep-ph]].
- [31] A. Hietanen, C. Pica, F. Sannino and U. I. Sonderegard, *Phys. Rev. D* **87**, no. 3, 034508 (2013) [arXiv:1211.5021 [hep-lat]].
- [32] T. Appelquist, R. C. Brower, M. I. Buchoff, M. Cheng, S. D. Cohen, G. T. Fleming, J. Kiskis and M. F. Lin *et al.*, *Phys. Rev. D* **88**, 014502 (2013) [arXiv:1301.1693 [hep-ph]].
- [33] D. B. Kaplan and H. Georgi, *Phys. Lett. B* **136**, 183 (1984).