DEMOGRAPHIC RESEARCH

A peer-reviewed, open-access journal of population sciences

VOLUME 29, ARTICLE 41, PAGES 1127-1152
PUBLISHED 5 DECEMBER 2013
http://www.demographic-research.org/Volumes/Vol29/41/
DOI: 10.4054/DemRes.2013.29.41

Research Article

Cause-specific measures of life years lost

Per Kragh Andersen

Vladimir Canudas-Romo

Niels Keiding

This publication is part of the Special Collection on "Multistate Event History Analysis," organized by Guest Editors Frans Willekens and Hein Putter.

Table of Contents

1	Introduction	1128
2	A new definition of life years lost by cause of death	1131
3	Calculation of life years lost	1133
4	Comparison with other cause-specific measures of years lost	1136
5	An illustration: Danish and Russian national mortality data	1137
6	Discussion	1143
7	Acknowledgements	1144
	References	1145
	Appendices	1148

Demographic Research: Volume 29, Article 41
Research Article

Cause-specific measures of life years lost

Per Kragh Andersen [1]

Vladimir Canudas-Romo [2]

Niels Keiding [1]

Abstract

BACKGROUND
A new measure of the number of life years lost due to specific causes of death is introduced.

METHODS
This measure is based on the cumulative incidence of death, it does not require "independence" of causes, and it satisfies simple balance equations: "total number of life years lost = sum of cause-specific life years lost", and "total number of life years lost before age x + temporary life expectancy between birth and age $x = x$".

RESULTS
The measure is contrasted to alternatives suggested in the demographic literature and all methods are illustrated using Danish and Russian multiple decrement life-tables.

[1] Department of Biostatistics, University of Copenhagen, Ø. Farimagsgade 5 PB2099, DK-1014 Copenhagen, Denmark.
[2] Department of Biostatistics, University of Copenhagen, Ø. Farimagsgade 5 PB2099, DK-1014 Copenhagen, Denmark. Department of Population, Family and Reproductive Health. Johns Hopkins Bloomberg School of Public Health, USA. Max-Planck Odense Center. University of Southern Denmark.

1. Introduction

Measures of life lost to specific causes of death have been discussed since the famous debate on smallpox between Bernoulli (1766) and d'Alembert (1761), chronicled several times, perhaps in greatest detail by Karn (1931, 1933) and Dietz and Heesterbeek (2002). The dominant assumption has been that removal of a cause of death may be modeled by equating the corresponding death intensity to zero in the relevant multiple decrement model. The effect of removal of a cause of death can thus be measured by comparing life expectancy with and without that cause operating. This view is still prevalent in demography, e.g. in the authoritative textbook by Preston, Heuveline, and Guillot (2001: Box 4.2), who as an example calculated that for U.S. females in 1991:

> "... in the absence of neoplasms, life expectancy at birth would have been 82.46 years, a gain of 3.54 years relative to the life table with all causes present."

However, there have been grave doubts about the relevance of the so-called independent competing risks model. Formally, independent competing risks is defined as stochastic independence of latent cause-specific times to failure and it leads to a situation where the removal of one cause will leave the risk of dying from all other causes unchanged. Already Makeham (1874) concluded his attractive, concise exposition of the discussion that:

> "It will be observed that these solutions all proceed upon the assumption that the extermination of small pox does not affect the mortality arising from other causes. This must be proved before any reliance can be placed upon the conclusions arrived at. I give the investigation merely as an illustration of the comparative advantages of the different methods of solution."

As formulated by Karn (1931):

> "It is necessary to warn the reader that the theory of the life-table with a given disease eliminated as developed by Bernoulli, d'Alembert, Tremblay and Duvillard supposes that the mortality from the given disease is non-selective, i.e. that the population after removal of disease A is as susceptible to diseases B, C, D, etc. as it was before the elimination of that disease. This may possibly be true of certain diseases, but if a disease like phthisis or smallpox were eliminated the surviving population might be more subject to death from other diseases."

A careful demographic discussion of multiple causes of death was given by Manton and Stallard (1984), who chose the laudable path of starting with a chapter on death certificates, so that the reader could not possibly be ignorant of the tenuous nature of classification of causes of death. Manton and Stallard basically presented the classical 'cause-deletion' approach but made the independence assumption very explicit and suggested various elaborations of the underlying multi-state model to reduce the assumptions to various forms of *conditional* independence. Preston, Heuveline, and Guillot (2001: pp. 78-79) took the following problematic viewpoint:

> "Equation (4.4) says that the probability of remaining in the defined state between x and $x + n$ when many causes are operating ... is the product of each of the probabilities of remaining in that state if individual decrements were acting alone. (...) This multiplicative property pertains only when the outcomes ... are statistically independent: when one outcome does not depend on the others. Clearly, the assumption of independence ... entered at the point where we defined members of the set of decrements to be mutually exclusive and exhaustive. That is, the process of assignment of a cause to each particular decrement created a set of wholly separate and 'independent' entities. That these statistical entities are independent - admit no overlap or combinations or synergistic relations - does not mean that the underlying processes that they represent are independent. For example, it is very likely that an increase in the incidence of influenza in a population will raise death rates from certain cardiovascular diseases as well as from influenza. But whatever this synergistic relation among disease processes, the data will always come to the analyst in a set of cause-of-death assignments in which influenza and cardiovascular diseases are tidily separated; equation (4.4) will continue to hold."

In our view the independent competing risks model precisely assumes 'that the underlying processes that they represent are independent'. It is not enough that the risk categories 'come to the analyst as mutually exclusive and exhaustive'.

In his influential book, Chiang (1968, cf. 1961) summarized the current views of the time with the independent competing risks model as starting point, but, in addition to the 'net' probability of death from cause i in a hypothetical world where no other causes exist, he also described the 'crude' probability of death from cause i in this world where all other causes still operate. Cornfield (1957) preferred the terms 'pure' for net and 'mixed' for crude, while the Norwegian tradition (e.g., Sverdrup (1967), Hoem (1969), Andersen, Borgan, Gill, and Keiding (1993)) used 'partial' for net and 'influenced' for crude. The crude probability of death became the centre of attention in biostatistics and epidemiology, particularly through the influence of Prentice and Kalbfleisch in their important paper (Prentice et al. 1978) and book (Kalbfleisch and Prentice 1980), in which they seem (pp. 168-169) to have introduced the term *cumulative incidence*. A later influential paper is by Benichou and Gail (1990).

One basic argument for focusing attention on the cumulative incidence is that it is an observable quantity in this world without counterfactual assumptions. Another is that it satisfies a simple balance equation, as seen below.

It should be mentioned that the cumulative incidence has been a well-integrated component also of demographic multiple decrement methodology for a long time. Thus the multiple decrement table in Box 4.1 of the above mentioned text by Preston, Heuveline, and Guillot (2001) contains a column ℓ_x^i describing how those who will ultimately die from cause i (neoplasms) will die out, the cumulative incidence being $1 - \ell_x^i/\ell_0^i$. Interestingly, Farr (1841) already had similar columns in his 'nosometrical table' describing the fate of a synthetic cohort of 'lunatics' according to time since admission into an asylum, the two competing events being death in the asylum and discharge from the asylum. Farr's calculation was in principle the same as that of today.

Demographers frequently apply methods for decomposing change in life expectancy over time and corresponding life years lost to account for the age and cause of death contribution. Methods to calculate such contributions have been developed by a United Nations (1982) report, Pollard (1982, 1988), Arriaga (1984), Pressat (1985) and Andreev (1982), see also Andreev et al. (2002), who focused on the difference in life expectancy between two periods of time. Keyfitz (1977, 1985) considered continuous change and derived a formula that relates the time-derivative of life expectancy to the entropy of life table survivorship, although not as a general method of decomposition. More recently, efforts have also been taken to calculate cause-decomposition considering continuous change (Vaupel and Canudas-Romo 2003, Beltran-Sanchez et al. 2008).

The purpose of the present paper is to propose a more descriptive concept of life years lost based only on the directly observable cumulative incidences and to compare this with some of the above-mentioned proposals in demography. A central property of the cumulative incidence is the simple balance equation

$$\text{Prob.(still alive)} + \text{Prob.(have died of cause 1)} + \cdots + \text{Prob.(have died of cause } k) = 1,$$

which holds when there are in total k mutually exclusive causes of death. From this a similar simple balance equation holds for the proposed measure of life years lost

$$\text{Years lost to cause } 1 + \cdots + \text{Years lost to cause } k = \text{Years lost to all causes.}$$

In Section 2 the new definition of life years lost due to specific causes is discussed following Andersen (2013), with computational details in Section 3 (and in the Appendix). Thus, Andersen (2013) introduced the quantity given in equation (3) below, discussed its mathematical properties in a biostatistical context, and showed how regression analysis targeting cause-specific years lost can be performed. The purpose of the present paper is to compare current methods from the demographic literature which are reviewed in Section 4 and all methods are illustrated in Section 5 using published life-tables from Denmark and Russia. A brief discussion is found in Section 6.

2. A new definition of life years lost by cause of death

We consider the standard multiple decrement life-table as in Preston, Heuveline, and Guillot (2001: Section 4.3) and denote the mortality rate at age a by $\mu(a)$. This is the sum of the cause-specific mortality rates $\mu^i(a)$ for causes $i = 1, \ldots, k$, which are assumed to be mutually exclusive and exhaustive:

$$\mu(a) = \sum_i \mu^i(a).$$

The probability at birth of surviving past age x is then

$${}_xp_0 = \exp(-\int_0^x \mu(a)da),$$

and the life expectancy at birth is

$${}_\omega e_0 = \int_0^\omega {}_ap_0 da,$$

where ω is the maximally attainable age. The *temporary life expectancy* between birth and age x is

$${}_xe_0 = \int_0^x {}_ap_0 da,$$

e.g., (Arriaga 1984). In biostatistics this quantity is known as the *restricted mean life time*, e.g., (Irwin 1949). The probability at birth of dying from cause i before age x, that is the cause i *cumulative incidence* as introduced in Section 1, is

$${}_xq_0^i = \int_0^x {}_ap_0 \mu^i(a)da$$

and it follows that the *balance equation*

$$ {}_xp_0 + \sum_i {}_xq_0^i = 1 \tag{1} $$

holds, since at age x every member of the population is either alive or has died from one of the k causes $i = 1, \ldots, k$. Integrating equation (1) from $x = 0$ to $x = a$ we get another balance equation,

$$ {}_ae_0 + \sum_i \int_0^a {}_xq_0^i dx = a, $$

saying that a, the maximum number of life years at birth before age a, can be decomposed into ${}_ae_0$, the temporary life expectancy between birth and age a, plus the sum

$$ {}_a\lambda_0 = \sum_i {}_a\lambda_0^i, \tag{2} $$

where

$$ {}_a\lambda_0^i = \int_0^a {}_xq_0^i dx. \tag{3} $$

In this respect ${}_a\lambda_0$ is the expected number of *life years lost before age* a and it was shown by Andersen (2013) that each term ${}_a\lambda_0^i$ defined in equation (3) can be interpreted as the expected number of life years lost before age a *due to cause* i.

Thus, in summary we have shown that the total number of years, a between birth and age a, can be written as the temporary life expectancy before age a plus the sum over causes of death, i, of the expected number of life years lost due to cause i before age a:

$$ a = {}_ae_0 + \sum_i {}_a\lambda_0^i. \tag{4} $$

Note that all quantities can be defined conditionally on survival untill age x. This is described in the Appendix. Also, note that there is an obvious age-decomposition of both the cumulative incidence, the temporary life expectancy, and the number of life years lost. This is because splitting the age interval from $a_0 = 0$ to $a_m = a$ into sub-intervals, say $[0, a_1), [a_1, a_2), \ldots, [a_{m-1}, a_m)$, we have

$$ {}_aq_0^i = \sum_{j=1}^m \int_{a_{j-1}}^{a_j} {}_xp_0 \mu^i(x) dx, \tag{5} $$

$$ {}_ae_0 = \sum_{j=1}^m \int_{a_{j-1}}^{a_j} {}_xp_0 dx, \tag{6} $$

and

$$ {}_a\lambda_0^i = \sum_{j=1}^{m} \int_{a_{j-1}}^{a_j} {}_xq_0^i dx. \tag{7}$$

This will be used when computing life-table based estimates for these quantities in Section 3.

3. Calculation of life years lost

The temporary life expectancy and the number of life years lost can be estimated based on data collected in a multiple decrement life-table. To do this, consider a single age interval, say from x to $x + n$. The data needed for the calculations include:

- ℓ_x, the fraction of survivors at age x, (so that we assume $\ell_0 = 1$). This estimates ${}_xp_0$.
- ${}_nL_x$, the average number of person-years lived between ages x and $x + n$. This estimates the contribution $\int_x^{x+n} {}_ap_0 da$ to the temporary life expectancy from the interval, cf. equation (6).
- ${}_nd_x^i$, the fraction of deaths from cause i and the total fraction of deaths ${}_nd_x = \sum_i {}_nd_x^i$ between ages x and $x + n$. Thus, ${}_nd_x^i$ estimates the contribution $\int_x^{x+n} {}_ap_0\mu_i(a)da$ to the cumulative incidence from the interval, cf. equation (5).

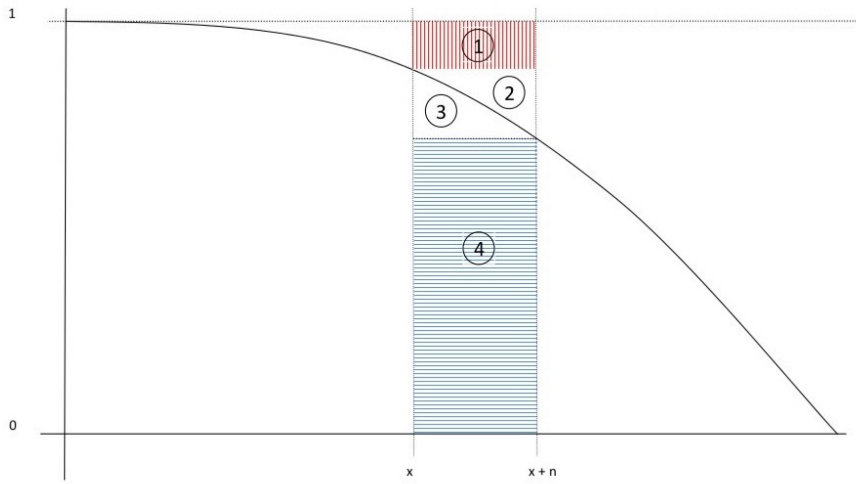
These values may have been calculated from raw counts of age- and cause-specific deaths and age-specific person-years at risk as described by Preston, Heuveline, and Guillot (2001: Ch. 3).

The average number of years, ${}_nL_x$, lived between ages x and $x+n$ can be represented as the area under the survival curve; see Figure 1. Likewise, the average number of years lost in the age interval from x to $x+n$, say ${}_n\daleth_x$, is the area above the curve and, obviously,

$$ {}_nL_x + {}_n\daleth_x = n. \tag{8}$$

Here, ${}_n\daleth_x$ estimates the contribution $\sum_i \int_x^{x+n} {}_aq_0^i da$ to the years lost from the age interval from x to $x + n$, cf. equation (7).

Figure 1: Life table survival curve, ℓ_x, and number of life years lost between ages x and $x + n$,



The person-years lived can be divided into contributions from those remaining alive during the interval (area 4 in Figure 1) and those who die (area 3), leading to

$$ {}_nL_x = n\ell_{x+n} + {}_nA_x \cdot {}_nd_x \tag{9} $$

where ${}_nA_x$ is the average number of person-years lived in the interval by those dying in it (e.g., Preston, Heuveline, and Guillot (2001: Ch. 3), where it was denoted ${}_na_x$). Similarly, the years lost are composed of contributions from those who had died prior to age x (area 1) and from those who die in the interval (area 2):

$$ {}_n\daleth_x = n(1 - \ell_x) + (n - {}_nA_x)_nd_x. $$

To subdivide the lost years into contributions from each cause, the latter expression is modified, as follows:

$$ {}_n\daleth_x^i = n(1 - \ell_x) \cdot {}_xf_0^i + (n - {}_nA_x)_nd_x \cdot {}_nR_x^i $$

where ${}_xf_0^i$ is the life-table based estimate of the probability at birth of dying from cause i before age x (that is, the cumulative incidence) and ${}_nR_x^i = \frac{{}_nd_x^i}{{}_nd_x}$. If the age interval from

0 to x is divided into sub-intervals $[a_{j-1}, a_j)$, as described in Section 2, we have

$$ {}_x f_0^i = \sum_j {}_{n_j} d_{a_{j-1}}^i , $$

where $n_j = a_j - a_{j-1}$ is the width of interval $j = 1, \ldots, m$. Not all published life-tables include the quantities ${}_n A_x$, however, equation (9) may be solved for ${}_n A_x$ and substituted into the expression for ${}_n \daleth_x^i$ leading to

$$ {}_n \daleth_x^i = n(1 - \ell_x) \cdot {}_x f_0^i + (n \cdot \ell_x - {}_n L_x)_n R_x^i . $$

Note that both the quantities ${}_a \lambda_0^i$ and their estimates based on the life-table data

1) are additive: ${}_a \lambda_0 = \sum_i {}_a \lambda_0^i$ and ${}_n \daleth_x = \sum_i {}_n \daleth_x^i$, and thereby do a "book keeping" of where the years lost go in the population,

2) together with the temporary life expectancy add to the total number of life years: ${}_a e_0 + {}_a \lambda_0 = a$ and ${}_n L_x + {}_n \daleth_x = n$, cf. equations (4) and (8).

Finally, the years lost contribution by age and cause of death can also be used to compare life expectancies between two populations. Life expectancy can be expressed as a function of the total number of years and number of years lost, for example for country c_1:

$$ {}_x e_0^{c_1} = x - {}_x \lambda_0^{c_1} $$

and then the difference between the life expectancies of two populations can be obtained by looking at their years lost as

$$ {}_x e_0^{c_1} - {}_x e_0^{c_2} = {}_x \lambda_0^{c_2} - {}_x \lambda_0^{c_1} . $$

We can now calculate both the age- and cause-contribution to the number of years lost by calculating those terms in each of the populations and then subtracting them. A similar strategy can be applied if the interest is to calculate the change over time in the number of years lost in one population. In that case c_1 and c_2 will correspond to the first and second time respectively.

In the Appendix we show how the calculations can, alternatively, be based on a "modified life-table" with constant mortality rates in each age interval.

4. Comparison with other cause-specific measures of years lost

In this section we show two alternative methods which are related to the cause-specific measure of years lost introduced in Section 2. The first is the life expectancy decomposition suggested by Beltran-Sanchez et al. (2008) and the second is the average number of life-years lost as a result of death presented by Vaupel and Canudas-Romo (2003). To introduce the former, note that we can always write

$$_xp_0 = {}_xp_0^{1*} \cdots {}_xp_0^{k*},$$

where

$$_xp_0^{i*} = \exp(-\int_0^x \mu^i(a)da),$$

e.g., Preston, Heuveline, and Guillot (2001: Section 4.3). If causes are "independent" then, as mentioned in Section 1, $_xp_0^{-i*} = \prod_{i' \neq i} {}_xp_0^{i'*}$ is the x-year survival probability at birth when cause i is eliminated and $_xe_0^{-i*} = \int_0^x {}_ap_0^{-i*}da$ the corresponding temporary life expectancy. Another definition of cause-specific years lost is then

$$_x\lambda_0^{i*} = {}_xe_0^{-i*} - {}_xe_0, \tag{10}$$

(Beltran-Sanchez et al. 2008).

This measure is 1) not additive 2) requires (as mentioned in Section 1) "independence" of causes, i.e., it refers to a hypothetical population where cause i is no longer operating and where (by independence) the mortality rates from causes $i' \neq i$ are still given by $\mu^{i'}(a)$. Note that the question of what would happen if certain causes were eliminated is not addressed via $_a\lambda_0^i$ for which all computations are performed in "this world" where all causes operate.

A final definition of years lost is

$$_x\lambda_0^{\dagger} = \int_0^x {}_{(x-a)}e_a \cdot {}_ap_0\mu(a)da$$

(see Vaupel and Canudas-Romo (2003) who called it $e^{\dagger}$). This is the average temporary life expectancy between age at death and age x and in the language of Gardner and Sanborn (1990) it is a measure of "years of potential life lost". It leads to a third measure of cause-specific years lost

$$_x\lambda_0^{i\dagger} = \frac{\int_0^x {}_{(x-a)}e_a \cdot {}_ap_0\mu^i(a)da}{_xq_0^i}$$

where the average temporary life expectancy at death is only taken among those who die from cause i before age x. This measure

1) is not additive
2) does not require "independence".

However, redefining ${}_x\lambda_0^{i\dagger}$ without division by ${}_xq_0^i$, that is, as

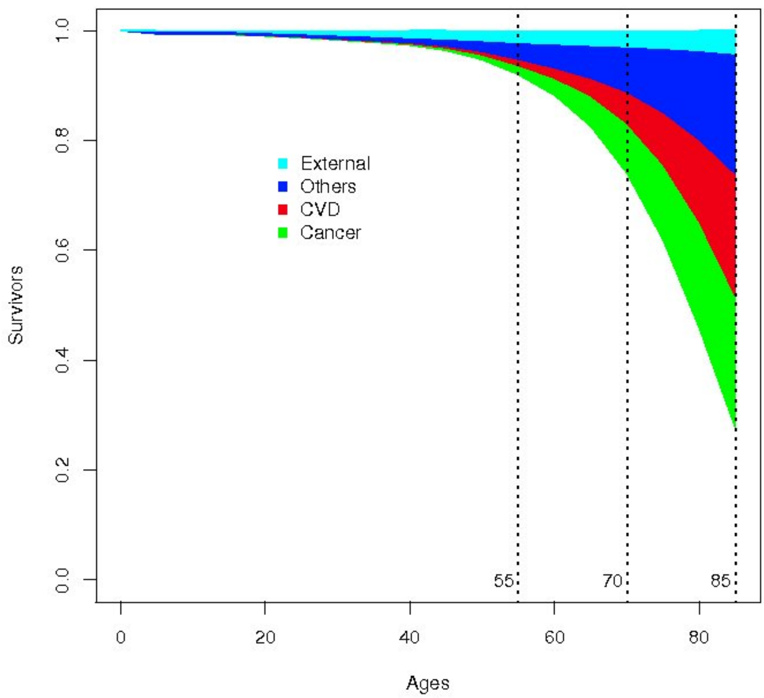
$$ {}_x\lambda_0^{i\dagger} = \int_0^x {}_{(x-a)}e_a \cdot {}_ap_0\mu^i(a)da \tag{11} $$

additivity is obtained though the years lost do not add to $x - {}_xe_0$. It can be shown that if we further replace ${}_{(x-a)}e_a$ by $x - a$ in equation (11) (which Vaupel and Canudas-Romo (2003) did not do) we get a measure of "premature (to x) years of potential life lost" (Gardner and Sanborn 1990), which is identical to ${}_a\lambda_0^i$ defined in equation (3).

5. An illustration: Danish and Russian national mortality data

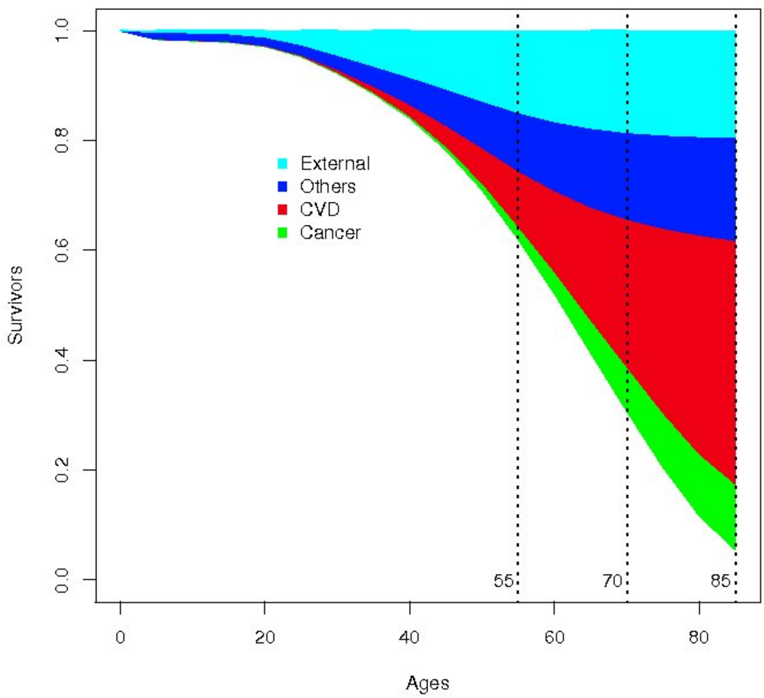
Life-tables and cause of death data from 2005 from Denmark and Russia were extracted from the Human Mortality Data base (HMD) and from the World Health Organisation (WHO). Based on these example data we will illustrate the different measures of cause-specific numbers of life years lost before different ages. Tables 1-2 show the results and Figures 2-3 show the survival curves including the cumulative incidences (${}_xq_0^i$) on which the suggested calculations of years lost rely. Four competing causes of death are considered: cancer, cardio-vascular diseases (CVD), external causes (accidents, suicides, homicides), and other causes.

Figure 2: Life table distribution of survivors and death by cause of death, males from Denmark 2005



Source: HMD and WHO.
Note: Green: cancer, red: CVD, light blue: external, dark blue: others.

Figure 3: Life table distribution of survivors and death by cause of death, males from Russia 2005



Source: HMD and WHO.
Note: Green: cancer, red: CVD, light blue: external, dark blue: others.

Table 1: Three different measures of years lost, Danish males 2005

			Years lost ${}_x\lambda_0^i$, equation (3)				
Age, x	${}_xe_0$	${}_x\lambda_0$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	53.83	1.17	0.11	0.06	0.16	0.84	1.17
70	66.55	3.45	0.54	0.32	0.56	2.03	3.45
85	74.54	10.46	2.00	1.49	1.81	5.16	10.46
			Cause-eliminated ${}_x\lambda_0^{i*}$, equation (10)				
Age, x	${}_xe_0$	${}_x\lambda_0$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	53.83	1.17	0.19	0.16	0.45	0.63	1.42
70	66.55	3.45	0.84	0.56	0.79	1.36	3.55
85	74.54	10.46	2.75	2.01	1.04	2.91	8.70
			${}_x\lambda_0^{i\dagger}$, equation (11)				
Age, x	$x - {}_x\lambda_0^\dagger$	${}_x\lambda_0^\dagger$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	50.93	4.07	0.92	0.56	1.00	1.58	4.07
70	61.90	8.10	2.56	1.71	1.13	2.70	8.10
85	72.68	12.32	3.68	3.32	1.26	4.06	12.32

Source: HMD and WHO.

From Tables 1-2 it follows that, no matter which measure of years lost is used, there are large differences between the two countries. Thus, for any age x, the total number of years lost before x is considerably larger in Russia. Also, patterns in differences between causes are captured by all methods: Cancer is a major cause of years lost only in Denmark, whereas external causes and CVD dominate in Russia.

Looking at the finer details, however, we do consider it advantageous that cause-specific measures of years lost add up to the corresponding total, as it is the case for both the suggested measure ${}_x\lambda_0^i$ and for ${}_x\lambda_0^{i\dagger}$ but not for the cause-eliminated number ${}_x\lambda_0^{i*}$. Further, the suggested ${}_x\lambda_0^i$ has a simple and useful relationship with the curves in Figures 2-3. Thus, the cause-specific years lost are simply the colored areas above the survival curve up to the given threshold age x.

Table 2: Three different measures of years lost, Russian males 2005

			Years lost $_x\lambda_0^i$, equation (3)				
Age, x	$_xe_0$	$_x\lambda_0$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	49.11	5.89	0.21	0.62	2.00	3.06	5.89
70	56.11	13.89	0.69	2.01	4.43	6.75	13.89
85	58.62	26.38	1.43	4.38	8.13	12.45	26.38
			Cause-eliminated $_x\lambda_0^{i*}$, equation (10)				
Age, x	$_xe_0$	$_x\lambda_0$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	49.11	5.89	0.24	0.97	2.58	1.81	5.60
70	56.11	13.89	0.77	3.20	4.29	3.15	11.39
85	58.62	26.38	1.40	6.89	5.09	4.01	17.39
			$_x\lambda_0^{i\dagger}$, equation (11)				
Age, x	$x - {_x\lambda_0^\dagger}$	$_x\lambda_0^\dagger$	**Cancer**	**CVD**	**External**	**Other**	**Sum**
55	42.97	12.03	0.81	3.06	4.66	3.49	12.03
70	54.49	15.51	1.47	5.11	4.95	3.99	15.51
85	68.29	16.71	1.62	5.98	4.98	4.14	16.71

Source: HMD and WHO.

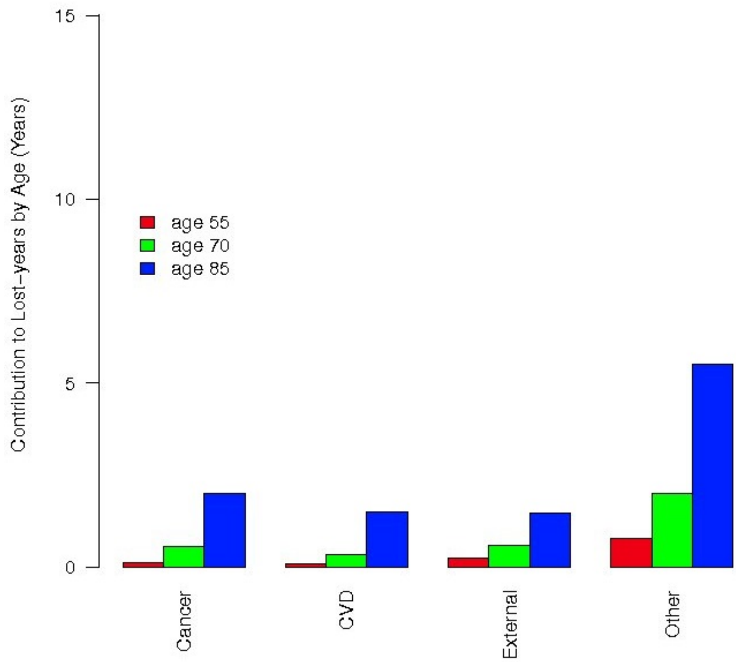
The method, as explained in Section 4, lends itself to be used for other decompositions, e.g. by country and by age. Thus, in Table 3 the difference in temporary life expectancy between Denmark and Russia has been decomposed by age and cause and it can be seen, for example, that a main component in the 10.44 year difference in temporal life expectancy at age 70 comes from external causes (3.88 years). On the other hand, by age 85 more years lost are due to cancer in Denmark than in Russia. This might be due to the early deaths in Russia from other causes of death, a fact that highlights the relevance of competing causes of death. The age-specific contributions used for calculating the decomposition in Table 3 are displayed graphically in Figures 4-5.

Table 3: Decomposition of the male life expectancy gap between Denmark and Russia in 2005

Age, x	${}_xe_0$ Denmark	${}_xe_0$ Russia	Difference	Cause-contribution to difference Cancer	CVD	External	Other	Sum
55	53.83	49.11	4.72	0.11	0.56	1.83	2.22	4.72
70	66.55	56.11	10.44	0.15	1.69	3.88	4.72	10.44
85	74.54	58.62	15.92	-0.58	2.89	6.32	7.29	15.92

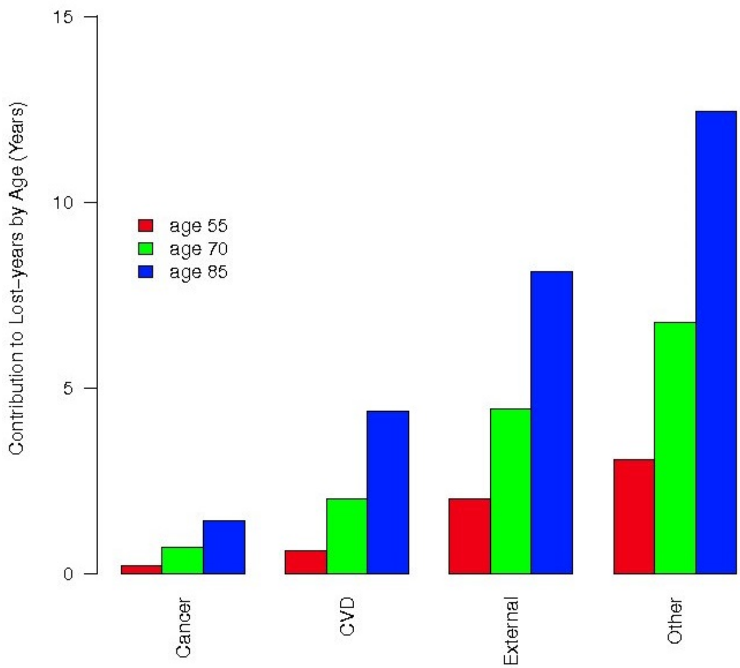
Source: HMD and WHO.

Figure 4: Contribution to years lost by age for males from Denmark 2005



Source: HMD and WHO.

Figure 5: Contribution to years lost by age for males from Russia 2005



Source: HMD and WHO.

6. Discussion

Cause-specific decomposition of mortality is an old theme in demography and public health, as hinted in Section 1. Much of that literature involves counterfactual assumptions about what would happen if some cause were eliminated. More recently, there has been considerable interest in using years lost to particular causes of death as a measure of disease burden, both at the global (e.g., Lopez et al. (2006) and the local (e.g., Aragon et al. (2008)) levels. In these applications it is common to attach various weighting schemes to the lost years, thereby generating concepts such as disability-adjusted life years; see Gardner and Sanborn (1990) for an early concise and critical survey. Furthermore, the actual mortality experience is often related to some standard mortality regime.

All proposals, including ours, make the same assumption, namely that there exists an unequivocal classification of causes of death. This issue is not always discussed in depth in methodological expositions, although we refer to Manton and Stallard (1984) for their exemplary discussion related to the USA. As an example from the applied literature we may mention the detailed description by Lopez et al. (2006) of the formidable problems in obtaining comparable cause-of-death data for world-wide comparisons.

Our proposal in this paper is to focus on the basic, directly observable data and to develop a concept of life years lost from the classical concept of crude probability of cause-specific death (going back at least to Farr (1841)) or cumulative cause-specific incidence of death, as the concept is now usually termed in biostatistics and epidemiology. In contrast to many other proposals, this measure is additive in the sense that it satisfies the obvious balancing equation that the sum of years lost to a set of mutually exclusive and exhaustive causes equals the total years lost, and as seen in Figs. 2-3 this may be attractively illustrated in simple survival graphs. Also, the measure lends itself to further decompositions, e.g. according to age or according to country, as illustrated using life-tables from Denmark and Russia in Section 5.

7. Acknowledgements

The second author's work was supported by grant funding from the European Research Council - ERC Starting Grant 240795.

References

Andersen, P. (2013). Decomposition of number of years lost according to causes of death. *Statistics in Medicine*. doi:10.1002/sim.5903.

Andersen, P., Borgan, Ø., Gill, R., and Keiding, N. (1993). *Statistical Models Based on Counting Processes*. New York: Springer.

Andreev, E. (1982). Metod komponent v analize prodoljitelnosty zjizni. [The method of components in the analysis of length of life]. *Vestnik Statistiki* 9: 42–47.

Andreev, E., Shkolnikov, V., and Begun, A. (2002). Algorithm for decomposition of differences between aggregate demographic measures and its application to life expectancies, Gini coefficients, health expectancies, parity-progression ratios and total fertility rates. *Demographic Research* 7(14): 499–522. doi:10.4054/DemRes.2002.7.14.

Aragon, T., Lichtensztajn, D., Katcher, B., Reiter, R., and Katz, M. (2008). Calculating expected years of life lost for assessing local ethnic disparities in causes of premature death. *BMC Public Health* 8(16). doi:10.1186/1471-2458-8-116.

Arriaga, E. (1984). Measuring and explaining the change in life expectancies. *Demography* 21(1): 83–96. doi:10.2307/2061029.

Beltrán-Sánchez, H., Preston, S., and Canudas-Romo, V. (2008). An integrated approach to cause-of-death analysis: cause-deleted life tables and decompositions of life expectancy. *Demographic Research* 19(35): 1323–1350. doi:10.4054/DemRes.2008.19.35.

Benichou, J. and Gail, M. (1990). Estimates of absolute cause-specific risk in cohort studies. *Biometrics* 46(3): 813–826. doi:10.2307/2532098.

Bernoulli, D. (1766). Essai d'une nouvelle analyse de la mortalité, causée par la petite vérole, et des avantages de l'inoculation pour le prévenir. *Histoire avec les Mémoires, Académie Royal des Sciences Paris 1760:* 1–45.

Chiang, C. (1961). A stochastic study of the life table and its applications. III. the follow-up study with the consideration of competing risks. *Biometrics* 17(1): 57–78. doi:10.2307/2527496.

Chiang, C. (1968). *Introduction to Stochastic Processes in Biostatistics*. New York: Wiley.

Cornfield, J. (1957). The estimation of the probability of developing a disease in the presence of competing risks. *American Journal of Public Health* 47(5): 601–607. doi:10.2105/AJPH.47.5.601.

d'Alembert, J. (1761). Sur l'application du calcul des probabilities à l'inoculation de la petite vérole. In: d'Alembert, J. (ed.). *Opuscules Mathematiques 2*. 26.

Dietz, K. and Heesterbeek, J. (2002). Daniel Bernoulli's epidemiological model revisited. *Mathematical Biosciences* 180(1–2): 1–21. doi:10.1016/S0025-5564(02)00122-0.

Farr, W. (1841). Report on the mortality of lunatics. *Journal of the Statistical Society of London* 4(1): 17–33. doi:10.2307/2338035.

Gardner, J. and Sanborn, J. (1990). Years of potential life lost (YPLL) – What does it measure? *Epidemiology* 1(4): 322–329. doi:10.1097/00001648-199007000-00012.

Hoem, J. (1969). Purged and partial Markov chains. *Skandinavisk Aktuarietidskrift* 52: 147–155.

Human Mortality Data base (HMD) (2013). Human mortality data base. [electronic resource]. University of California, Berkeley, USA and Max Planck Institute for Demographic Research, Germany. www.mortality.org or www.humanmortality.de.

Irwin, J. (1949). The standard error of an estimate of expectation of life, with special reference to expectation of tumourless life in experiments with mice. *Journal of Hygiene* 47(2): 188–189. doi:10.1017/S0022172400014443.

Kalbfleisch, J. and Prentice, R. (1980). *The Statistical Analysis of Failure Time Data*. New York: Wiley.

Karn, M. (1931). An inquiry into various death-rates and the comparative influence of certain diseases on the duration of life. *Annals of Eugenics* 4(3–4): 279–302. doi:10.1111/j.1469-1809.1931.tb02080.x.

Karn, M. (1933). A further study of methods of constructing life tables when certain causes of death are eliminated. *Biometrika* 25(1–2): 91–101. doi:10.2307/2332264.

Keyfitz, N. (1977). What difference does it make if cancer were eradicated? An examination of the Taeuber paradox. *Demography* 14(4): 411–418. doi:10.2307/2060587.

Keyfitz, N. (1985). *Applied Mathematical Demography. 2nd ed.* New York: Springer.

Lopez, A., Mathers, C., Ezzati, M., Jamison, D., and Murray, C. (2006). *Global Burden of Disease and Risk Factors*. Washington, DC: World Bank Institute.

Makeham, W. (1874). On an application of the theory of the composition of decremental forces. *Journal of the Institute of Actuaries* 18: 317–322.

Manton, K. and Stallard, E. (1984). *Recent Trends in Mortality Analysis*. Orlando: Academic Press.

Pollard, J. (1982). The expectation of life and its relationship to mortality. *Journal of the Institute of Actuaries* 109(2): 225–240. doi:10.1017/S0020268100036258.

Pollard, J. (1988). On the decomposition of changes in expectation of life and differentials in life expectancy. *Demography* 25(2): 265–276. doi:10.2307/2061293.

Prentice, R., Kalbfleisch, J., Peterson, A., Flournoy, N., Farewell, V., and Breslow, N. (1978). The analysis of failure time data in the presence of competing risks. *Biometrics* 34(4): 541–554.

Pressat, R. (1985). Contribution des écarts de mortalité par âge à la différence des vies moyennes. *Population* 40(4–5): 765–770. doi:10.2307/1532986.

Preston, S., Heuveline, P., and Guillot, M. (2001). *Demography*. Oxford: Blackwell.

Sverdrup, E. (1967). Statistiske metoder ved dødelighetsundersøkelser. Statistical Memoirs, Institute of Mathematics, University of Oslo.

United Nations (1982). *Levels and Trends of Mortality Since 1950, Study 74*. New York: United Nations; Department of International Economic and Social Affairs.

Vaupel, J. and Canudas-Romo, V. (2003). Decomposing change in life expectancy: A bouquet of formulas in honor of Nathan Keyfitz's 90th birthday. *Demography* 40(2): 201–216. doi:10.1353/dem.2003.0018.

World Health Organisation (2013). World mortality database. [electronic resource]. www3.who.int/whosis/mort/.

Appendix

First, note that all quantities presented in Section 2 can be defined conditionally on survival untill age x. Thus, the probability of surviving past age $x+n$ among those who survived until age x is

$$ {}_np_x = \exp(-\int_x^{x+n} \mu(a)da), $$

the temporary life expectancy between ages x and $x+n$ is

$$ {}_ne_x = \int_x^{x+n} {}_ap_x da, $$

and, among those still alive at age x, the probability of dying from cause i before age $x+n$ is

$$ {}_nq_x^i = \int_x^{x+n} {}_ap_x \mu^i(a)da. $$

We still have the balance equation

$$ {}_np_x + \sum_i {}_nq_x^i = 1. $$

Furthermore,

$$ {}_n\lambda_x^i = \int_x^{x+n} {}_aq_x^i da $$

is the expected number of life years lost before age $x+n$ due to cause i among those alive at age x and it follows that

$$ {}_ne_x + \sum_i {}_n\lambda_x^i = n. $$

Second, as an alternative to the life-table estimates presented in Section 3, all quantities may be estimated from age- and cause-specific mortality rates. To this end, the interval $[0,\omega)$ is split into disjoint age intervals $[a_{k-1}, a_k), k = 1, \ldots, K$ with $a_0 = 0, a_K = \omega$ and we assume that the mortality rates $\mu(a)$ are *piecewise constant*, i.e.

$$ \mu(a) = \mu_k \text{ when } a \in [a_{k-1}, a_k), k = 1, \ldots, K. $$

In this case, the survival function ${}_ap_0 = \exp(-\int_0^a \mu(x)dx)$ evaluated at age a_k is:

$$ {}_{a_k}p_0 = \exp(-\sum_{j=1}^{k} \mu_j(a_j - a_{j-1})), k = 1, \ldots, K, $$

and for $a \in [a_k, a_{k+1})$

$$ {}_{a}p_0 = {}_{a_k}p_0 \exp(-\mu_{k+1}(a - a_k)). $$

Further, we have

$$ {}_{(a_k - a_m)}p_{a_m} = \exp(-\sum_{j=m+1}^{k} \mu_j (a_j - a_{j-1})), k = 1, \ldots, K, k > m, $$

and, more generally, for $a \in [a_{j-1}, a_j)$ and $x \in [a_{m-1}, a_m)$

$$ {}_{(x-a)}p_a = \exp(-\mu_j(a_j - a)) \cdot {}_{(a_{m-1} - a_j)}p_{a_j} \cdot \exp(-\mu_m(x - a_{m-1})). $$

It follows that the temporary life expectancy between ages a_m and a_k (with $a_m < a_k$) is

$$ {}_{(a_k - a_m)}e_{a_m} = \int_{a_m}^{a_k} {}_{(a-a_m)}p_{a_m} da = \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} {}_{(a-a_m)}p_{a_m} da $$

$$ = \sum_{j=m+1}^{k} {}_{(a_{j-1}-a_m)}p_{a_m} \int_{a_{j-1}}^{a_j} \exp(-\mu_j(a - a_{j-1}))da $$

$$ = \sum_{j=m+1}^{k} {}_{(a_{j-1}-a_m)}p_{a_m} \frac{1}{\mu_j}(1 - \exp(-\mu_j(a_j - a_{j-1}))), $$

and, for $a \in [a_{j-1}, a_j)$

$$ {}_{(a_k - a)}e_a = \int_{a}^{a_j} {}_{(x-a)}p_a dx + \int_{a_j}^{a_k} {}_{(x-a)}p_a dx $$

$$ = \frac{1}{\mu_j}(1 - \exp(-\mu_j(a_j - a))) + \exp(-\mu_j(a_j - a)) {}_{(a_k - a_j)}e_{a_j}. $$

The conditional probability of death from cause i before age a_k, given survival untill age a_m is

$$ {}_{(a_k - a_m)}q^i_{a_m} = \int_{a_m}^{a_k} {}_{(a-a_m)}p_{a_m} \mu^i(a) da = \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} {}_{(a-a_m)}p_{a_m} \mu^i(a) da $$

$$ = \sum_{j=m+1}^{k} {}_{(a_{j-1}-a_m)}p_{a_m} \int_{a_{j-1}}^{a_j} \exp(-\mu_j(a - a_{j-1}))\mu^i_j da $$

$$= \sum_{j=m+1}^{k} {}_{(a_{j-1}-a_m)}p_{a_m} \frac{\mu_j^i}{\mu_j}(1 - \exp(-\mu_j(a_j - a_{j-1})))$$

and, thereby, for $a \in [a_k, a_{k+1})$:

$${}_{(a-a_m)}q_{a_m}^i = {}_{(a_k-a_m)}q_{a_m}^i + \int_{a_k}^{a} {}_{(x-a_m)}p_{a_m}\mu_{k+1}^i dx$$

$$= {}_{(a_k-a_m)}q_{a_m}^i + {}_{(a_k-a_m)}p_{a_m} \frac{\mu_{k+1}^i}{\mu_{k+1}}(1 - \exp(-\mu_{k+1}(a - a_k))).$$

It follows that the number of years lost due to cause i between ages a_m and a_k is:

$${}_{(a_k-a_m)}\lambda_{a_m}^i = \int_{a_m}^{a_k} {}_{(a-a_m)}q_{a_m}^i da = \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} {}_{(a-a_m)}q_{a_m}^i da$$

$$= \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} \left({}_{(a_{j-1}-a_m)}q_{a_m}^i + {}_{(a_{j-1}-a_m)}p_{a_m} \frac{\mu_j^i}{\mu_j}(1 - \exp(-\mu_j(a - a_{j-1})))\right)da$$

$$= \sum_{j=m+1}^{k} (a_j - a_{j-1}){}_{(a_{j-1}-a_m)}q_{a_m}^i$$

$$+{}_{(a_{j-1}-a_m)}p_{a_m} \frac{\mu_j^i}{\mu_j}((a_j - a_{j-1}) - \frac{1}{\mu_j}(1 - \exp(-\mu_j(a_j - a_{j-1})))).$$

Note that (as it should be!)

$$\sum_i {}_{(a_k-a_m)}\lambda_{a_m}^i + {}_{(a_k-a_m)}p_{a_m} = a_k - a_m.$$

Finally, we look at

$${}_{(a_k-a_m)}\lambda_{a_m}^{\dagger} = \int_{a_m}^{a_k} {}_{(a_k-a)}e_a \cdot {}_{(a-a_m)}p_{a_m}\mu(a)da$$

or the corresponding cause-specific measure

$${}_{(a_k-a_m)}\lambda_{a_m}^{i\dagger} = \int_{a_m}^{a_k} {}_{(a_k-a)}e_a \cdot {}_{(a-a_m)}p_a\mu^i(a)da.$$

We have, for the all-cause quantity

$$ {}_{(a_k-a_m)}\lambda^{\dagger}_{a_m} = \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} {}_{(a_k-a)}e_a \cdot {}_{(a-a_m)}p_{a_m} \mu_j da $$

$$ = \sum_{j=m+1}^{k} \int_{a_{j-1}}^{a_j} \big(\frac{1}{\mu_j}(1-\exp(-\mu_j(a_j-a))) + \exp(-\mu_j(a_j-a))_{(a_k-a_j)}e_{a_j}\big) $$
$$ \times \exp(-\mu_j(a-a_{j-1}))_{(a_{j-1}-a_m)}p_{a_m}\mu_j da $$

$$ = \sum_{j=m+1}^{k} \big[{}_{(a_{j-1}-a_m)}p_{a_m}\big(\frac{1}{\mu_j}(1-\exp(-\mu_j(a_j-a_{j-1}))) - (a_j-a_{j-1})\exp(-\mu_j(a_j-a_{j-1}))\big) $$
$$ +\mu_j(a_j-a_{j-1})\exp(-\mu_j(a_j-a_{j-1}))_{(a_k-a_j)}e_{a_j} \cdot {}_{(a_{j-1}-a_m)}p_{a_m}\big] $$
$$ = \sum_{j=m+1}^{k} {}_{(a_{j-1}-a_m)}p_{a_m} $$
$$ \times\big(\frac{1}{\mu_j}(1-\exp(-\mu_j(a_j-a_{j-1}))) + (a_j-a_{j-1})\exp(-\mu_j(a_j-a_{j-1}))(\mu_j \cdot {}_{(a_k-a_j)}e_{a_j}-1)\big). $$

For the cause-specific quantity ${}_{(a_k-a_m)}\lambda^{i\dagger}_{a_m}$ each term should be multiplied by μ^i_j/μ_j.

If, in the expression for ${}_{(a_k-a_m)}\lambda^{i\dagger}_{a_m}$, we replace ${}_{(a_k-a)}e_a$ by $a_k - a$, i.e.

$$ {}_{(a_k-a_m)}\lambda^{i\dagger\prime}_{a_m} = \int_{a_m}^{a_k} (a_k-a) \cdot {}_{(a-a_m)}p_{a_m}\mu^i(a) da $$

we simply (by partial integration) get ${}_{(a_k-a_m)}\lambda^{i}_{a_m}$.