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Minimal Super Technicolor

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ABSTRACT: We introduce novel extensions of the Standard Model featuring a supersymmetric technicolor sector (superttechnicolor). As the first minimal conformal superttechnicolor model we consider $\mathcal{N} = 4$ Super Yang-Mills which breaks to $\mathcal{N} = 1$ via the electroweak interactions. This is a well defined, economical and calculable extension of the SM involving the smallest number of fields. It constitutes an explicit example of a natural superconformal extension of the Standard Model featuring a well defined connection to string theory. It allows to interpolate, depending on how we break the underlying supersymmetry, between unparticle physics and Minimal Walking Technicolor. We consider also other $\mathcal{N} = 1$ extensions of the Minimal Walking Technicolor model. The new models allow all the standard model matter fields to acquire a mass.

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1. Introducing Minimal Super Technicolor

The Standard Model (SM) of particle interactions passes a large number of experimental tests. Yet we know that it cannot be the ultimate model of nature since it fails to explain the origin of matter-antimatter asymmetry and the abundance of cold dark matter. Several extensions of the SM have been proposed, and two stand out in the quest of a better theory: Supersymmetry and Technicolor.

The appeal of Supersymmetry resides in its higher level of space-time symmetries as well as in its often praised natural link to string theory. The most investigated route to introduce supersymmetry has been to supersymmetrize the SM and then invoke some

mechanism to break supersymmetry again, given that no sign of the superpartners has yet been observed in experiments. We do not know why the scale of supersymmetry breaking is higher than the electroweak one; this is simply an experimental input.

Technicolor declares the Higgs sector of the SM to be a low energy effective theory (see [1, 2] for up-to-date reviews and [3] for a review of the older models), in which the Higgs is not elementary but composite. The main appeal of technicolor is that we have already encountered this phenomenon in nature. Superconductivity is a time honored example while the relativistic version is ordinary Quantum Chromodynamics (QCD), both in the vacuum and at high quark matter density. Technicolor predicts the existence of a tower of massive states whose mass is of the order of the electroweak scale, although pseudo-Goldstone bosons can be lighter. This fact naturally explains why we have not detected technicolor yet. To give masses to the SM fermions one must, however, resort to another unknown sector.

In this article we would like to fuse the basic features of both models to construct novel explicit examples of superttechnicolor models possessing several interesting theoretical and phenomenological features. The superttechnicolor idea was put forward in [4], though the phenomenological viability of these early models seemed difficult to achieve. An important difference in our approach is that the underlying supersymmetric and technicolor theories, which can be resumed by decoupling, respectively, the technicolor fields or the superpartners by sending their masses to infinity, are both phenomenologically viable¹. Examples are the minimal supersymmetric standard model (MSSM) and/or the minimal walking technicolor (MWT) model [5, 6, 7, 8, 9]. The latter constitutes the backbone of the technicolor theory here.

The basic properties of the models we are about to introduce are:

- The models possess the highest degree of four-dimensional space-time symmetry compatible with experiments.
- The models can interpolate between already studied extensions of the SM at the TeV scale, such as unparticle physics [10, 11], MWT [5, 7] and traditional technicolor [12, 13] or supersymmetry (see [14] for a review). Hence the models can be used as a well defined laboratory to investigate different theoretical ideas.
- The technicolor models, even before being supersymmetrized, pass the precision electroweak tests.
- The models possess a clear and direct link to string theory in such a way that AdS/CFT techniques [15] are readily applicable to realistic extensions of the SM.

We start with the observation that the recently proposed minimal model of near conformal technicolor, i.e. MWT, is constituted by an $SU(2)$ gauge theory with four Weyl (two Dirac) fermions transforming according to the adjoint representation of the gauge group [5, 6, 7, 8, 9]. The $SU(2)_L \times SU(2)_R \subset SU(4)$ chiral symmetry of the model is then gauged

¹Though, in case we give up supersymmetry, we should introduce a kind of extended technicolor sector to generate the SM fermion masses.

under the electroweak interactions. The MWT model is interesting both theoretically [5, 16, 17, 18, 1, 2] and phenomenologically [19, 20, 21, 22]. It has also triggered a large lattice activity [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Another aspect that has not been emphasized enough is that, de facto, this model has the same number of degrees of freedom as $\mathcal{N} = 4$ Super Yang-Mills (4SYM) except for the absence of the associated three complex scalars. Once the missing scalars, transforming according to the adjoint representation of the $SU(2)$ gauge group are added to the theory, and taken to transform according to the two-index antisymmetric representation of the $SU(4)$ global symmetry, one recovers the 4SYM. This is the superttechnicolor model before embedding the electroweak symmetry. Note that the count of the number of degrees of freedom is a necessary but not sufficient condition for achieving 4SYM. One must also construct the superttechnicolor Lagrangian respecting $\mathcal{N} = 4$ symmetry. In this article we spell out the basic 4SYM Lagrangian in superfield notation and in terms of the physical component fields in Appendix A.

Besides symmetry arguments another equally relevant reason to look for a superttechnicolor extension of the SM is linked to the fact that the generation of the SM fermion masses is less involved than in models with total absence of scalars, although still natural. A recent analysis showing, at the effective Lagrangian level, how this can be achieved has appeared in [33]. This model makes use of fundamental scalars without the protection from supersymmetry and hence it belongs to the class of *unnatural* models.

We will hence construct a specific version of superttechnicolor, which we define as *Minimal $\mathcal{N} = 4$ Supersymmetric Technicolor* (M4ST). This model looks particularly appealing, in that it allows an $\mathcal{N} = 4$ superttechnicolor sector, which is broken down to $\mathcal{N} = 1$ supersymmetry only by weak and hypercharge interaction terms. It has a number of very interesting properties which we will elucidate below. Above all it allows for a direct link between a realistic model of nature and string theoretic model building and techniques. In fact one can explore several different regimes which will be only schematically described in the following sections. We summarize the M4ST Lagrangian written in terms of the component fields in Appendix B. In this way the model can readily be used for collider phenomenology and cosmological applications. This model could equally be termed *Minimal Superconformal Technicolor* (MCT) and hence MSCT=M4ST.

We also consider, in detail, other $\mathcal{N} = 1$ supersymmetric extensions of MWT for two specific choices of the hypercharge of the technifermion matter. We call this extension *Minimal Super Technicolor* (MST). This extension is less economical in the number of fields needed but has other interesting properties such as the natural presence of a gauge singlet superfield, which can be used to solve the μ problem of the MSSM, and a Higgs candidate already within the spectrum of MWT superpartners. The associated MST Lagrangian in terms of the component fields is summarized in Appendix C.

2. Minimal $\mathcal{N} = 4$ Super Technicolor (M4ST)

The earliest models of technicolor are known to have problems with the electroweak precision data. The simplest models of technicolor shown to pass the precision tests were put

forward recently [5, 6, 7]. In particular, as a starting point of our theory we will use the MWT [2] extension of the SM.

2.1 Minimal Walking Technicolor review

The gauge group is $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and the field content of the technicolor sector is constituted by four techni-fermions and one techni-gluon all in the adjoint representation of $SU(2)_{TC}$. The model features also a pair of Dirac leptons, whose left-handed components are assembled in a weak doublet, necessary to cancel the Witten anomaly [34] arising when gauging the new technifermions with respect to the weak interactions. Summarizing, the fermionic particle content of the MWT is given explicitly by

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \quad U_R^a, \quad D_R^a, \quad a = 1, 2, 3; \quad L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L, \quad N_R, \quad E_R. \quad (2.1)$$

The following generic hypercharge assignment is free from gauge anomalies:

$$\begin{aligned} Y(Q_L) &= \frac{y}{2}, & Y(U_R, D_R) &= \left(\frac{y+1}{2}, \frac{y-1}{2} \right), \\ Y(L_L) &= -3\frac{y}{2}, & Y(N_R, E_R) &= \left(\frac{-3y+1}{2}, \frac{-3y-1}{2} \right). \end{aligned} \quad (2.2)$$

The global symmetry of this technicolor theory, per se, is $SU(4)$ which breaks explicitly to $SU(2)_L \times U(1)_Y$ by the natural choice of the electroweak embedding [5, 7]. The vacuum choice is stable against the SM quantum corrections [35]. The latter leads also to splitting of the technibaryons allowing them to be natural candidates for cold dark matter of iTIMP type, i.e. isotriplets Technicolor Interacting Massive Particles [19, 36]. These dark matter candidates are of asymmetric type and require no fine-tuning of any of the parameters of the theory nor modification of the standard cosmological expansion model. The model is, however, sufficiently interesting to lead to several other kind of dark matter candidates [37, 38, 39, 40, 41, 42, 43, 44]. The first studies of collider phenomenology appeared in [45, 22, 21, 46, 20, 47] while the interesting topic of the finite temperature electroweak phase transition and its impact on the subsequent detection of gravitational waves have been investigated respectively in [48, 49] and [50]. These models can feature very light composite Higgs states as advocated in [6, 7, 8, 16] and supported by the recent analysis performed by Natale's group [51, 52, 53, 54, 55, 56].

An explicit construction of an extended technicolor type model addressing the problem of giving mass to the third generation of quarks and the new generation of leptons appeared in [9]. A less natural model introducing a novel scalar mimicking the effects of the extended technicolor interactions has also been introduced in [33, 57] following the pioneering work of Simmons [58], Kagan and Samuel [59], and Carone [60, 61]. More recently this type of models have been investigated also in [62, 63, 57]. Interesting related work can be also found in [64, 65].

Another interesting feature is that this model leads to a better unification of the SM couplings than the SM with an ordinary Higgs as shown in [66]. There we introduced

fermionic matter in the adjoint representation of the weak $SU(2)_L$ gauge group, and showed that, at the one loop level, a higher degree of unification of the SM couplings than in the MSSM can be achieved.

The dynamics of the MWT model is either near conformal or conformal as recent lattice simulations indicate [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

We want to investigate now the supersymmetrized version of this model. It will provide a natural ultraviolet completion of the model introduced in [33].

2.2 Upgrading MWT to $\mathcal{N} = 4$ Super Yang Mills

We start with the simple observation that the fermionic and gluonic spectrum fits perfectly in an $\mathcal{N} = 4$ supermultiplet, provided that we also include three scalar superpartners. In fact the $SU(4)$ global symmetry of MWT is nothing but the well known $SU(4)_R$ R symmetry of the 4SYM theory. This is the global quantum symmetry that does not commute with the supersymmetry transformations.

Having, at hand, already a great deal of the spectrum of 4SYM we explore the possibility of using this theory as a natural candidate for superttechnicolor. For the reader's convenience we have summarized the 4SYM Lagrangian in terms of the $\mathcal{N} = 1$ superfields, and in physical components in Appendix A. We refer to this appendix for the basic properties of the 4SYM theory, Lagrangian and notation.

We gauge part of the $SU(4)_R$ global symmetry of the superttechnicolor theory in order to couple the new supersymmetric sector to the weak and hypercharge interactions of the SM. We choose to do this in such a way that the model can still preserve $\mathcal{N} = 1$ supersymmetry. To this end one of the four Weyl technifermions is identified with the techni-gaugino and should be a singlet under the SM gauge group. The only possible candidates for this role are \bar{U}_R and \bar{D}_R , for $y = \mp 1$ respectively: we arbitrarily choose $y = 1$ and identify \bar{D}_R with the techni-gaugino. With this choice the charge assignments of the particles in eq.(2.1) under $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ are

$$\begin{aligned} Q_L &\sim \left(\mathbf{3}, \mathbf{1}, \mathbf{2}, \frac{1}{2} \right), \quad \bar{U}_R \sim (\mathbf{3}, \mathbf{1}, \mathbf{1}, -1), \quad \bar{D}_R \sim (\mathbf{3}, \mathbf{1}, \mathbf{1}, 0), \\ L_L &\sim \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{3}{2} \right), \quad \bar{N}_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}, 1), \quad \bar{E}_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}, 2). \end{aligned} \quad (2.3)$$

Based on these assignments we then define the scalar and fermion components of the $\mathcal{N} = 4$ superfields via

$$\left(\tilde{U}_L, U_L \right) \in \Phi_1, \quad \left(\tilde{D}_L, D_L \right) \in \Phi_2, \quad \left(\tilde{\bar{U}}_R, \bar{U}_R \right) \in \Phi_3, \quad (G, \bar{D}_R) \in V, \quad (2.4)$$

where we used a tilde to label the scalar superpartner of each fermion. We indicated with Φ_i , $i = 1, 2, 3$ the three chiral superfield of 4SYM and with V the vector superfield. Four more chiral superfields are necessary to fully supersymmetrize the MWT model, i.e.:

$$\left(\tilde{N}_L, N_L \right) \in \Lambda_1, \quad \left(\tilde{E}_L, E_L \right) \in \Lambda_2, \quad \left(\tilde{\bar{N}}_R, \bar{N}_R \right) \in N, \quad \left(\tilde{\bar{E}}_R, \bar{E}_R \right) \in E. \quad (2.5)$$

2.2.1 The Higgs Sector

In an ordinary technicolor model one assumes the techniquarks to condense due to the underlying technicolor dynamics. The SM Higgs can subsequently be identified as a state composed of the underlying techniquarks. This approach is not immediately applicable here since the superttechnicolor gauge theory, per se, has an exactly vanishing β function and hence the theory is conformal for any value of the technicolor gauge coupling. Of course, as it is well known, this theory represents still an extremely interesting nonperturbative model. If supersymmetry, or part of the supersymmetry, breaks one can still imagine a dynamical formation of the technifermion condensate.

Our goal, however, is to construct a calculable model able to interpolate between different scenarios and, in the first instance, preserving as much symmetry as possible. In this case, as one can see from the spectrum in eq.(2.3), that before invoking any dynamical mechanism, there is no candidate to play the role of the SM Higgs boson (a weak doublet with hypercharge $Y = \pm\frac{1}{2}$). We therefore introduce in the theory two Higgs doublet superfields with respective charge assignment

$$H \sim \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, \frac{1}{2} \right), \quad H' \sim \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{1}{2} \right), \quad (2.6)$$

where the presence of both $Y = \pm\frac{1}{2}$ superfields is needed to give mass by gauge invariant Yukawa terms to both the upper and lower components of the weak doublets of SM fermions. With this choice it is rather natural to take the MSSM to describe the supersymmetric extension of the Higgsless SM sector. All the MSSM fields are defined as singlets under $SU(2)_{TC}$. The resulting M4ST model is naturally anomaly-free, since both the MWT and the MSSM are such.

2.2.2 The M4ST Superpotential

The renormalizable superpotential for the M4ST, allowed by gauge invariance, and which we require additionally to conserve baryon and lepton number², and to be $\mathcal{N} = 4$ invariant in the limit of g_{TC} much greater than the other coupling constants, is

$$P = P_{MSSM} + P_{TC}, \quad (2.7)$$

where P_{MSSM} is the MSSM superpotential, and

$$P_{TC} = -\frac{g_{TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c + y_U\epsilon_{ij3}\Phi_i^a H_j \Phi_3^a + y_N\epsilon_{ij3}\Lambda_i H_j N + y_E\epsilon_{ij3}\Lambda_i H'_j E. \quad (2.8)$$

This superpotential describes an approximately conformal theory in the limit when g_{TC} is much greater than the gauge coupling constants g_Y , g_L , and Yukawa coupling constants y_U , y_N , y_E . Notice that the gauge invariance of the term proportional to g_{TC} in eq.(2.8) is guaranteed by the unbroken $SU(4)$ flavor symmetry and the requirement of gauge anomaly cancellation. There is no need to add further $\mathcal{N} = 1$ supersymmetry (SUSY) breaking terms

²We assume all the MWT particles to have baryon and lepton numbers equal to zero

to those of the MSSM, because the MWT particles are allowed to be mass degenerate with their superpartners since none of them has been yet observed.

Relaxing the requirements for the superpotential simply to that of gauge invariance, one would have to substitute g_{TC} with a generic Yukawa coupling constant y_{TC} , and to add the lepton number violating terms

$$P_{TC, \Delta L \neq 0} = y_{e,k} \epsilon_{ij3} \Lambda_i H_j e_k + y'_{N,k} \epsilon_{ij3} (l_k)_i H'_j N + y'_{E,k} \epsilon_{ij3} (l_k)_i \Lambda_j E, \quad (2.9)$$

where k here is a SM family index, e_1 is the chiral superfield having the SM left-handed positron as its fermion component, and l_1 is the weak doublet chiral superfield including the left-handed SM electron. Also, the MSSM part of the superpotential can be extended to include R-parity violating terms.

2.2.3 The M4ST Lagrangian and Spectrum

The Lagrangian of the M4ST is

$$\mathcal{L} = \mathcal{L}_{MSSM} + \mathcal{L}_{TC}, \quad (2.10)$$

where the supertechnicolor Lagrangian can be written in the form of eq.(A.2):

$$\mathcal{L}_{TC} = \frac{1}{2} \text{Tr} (W^\alpha W_\alpha |_{\theta\theta} + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} |_{\bar{\theta}\bar{\theta}}) + \Phi_f^\dagger \exp(2g_X V_X) \Phi_f |_{\theta\theta\bar{\theta}\bar{\theta}} + (P_{TC} |_{\theta\theta} + h.c.), \quad (2.11)$$

where

$$\Phi_f = Q, \Phi_3, \Lambda, N, E; \quad X = TC, C, L, Y, \quad (2.12)$$

with Q and Λ defined as the weak doublet superfields with components Φ_1, Φ_2 , and Λ_1, Λ_2 , respectively. The product $g_X V_X$ is assumed to include the gauge charge of the superfield on which it acts. The charge is Y for $U(1)_Y$, and 1 (0) for a multiplet (singlet) of a generic group $SU(N)$. The technicolor vector superfield V_{TC} is identified with V and its physical components are given in eq.(2.4). The remaining vector superfields are those already defined in the MSSM [14] while the superpotential P_{TC} is given in eq.(2.8). Finally, the first term on the right of eq.(2.11) and its Hermitian conjugate represent the kinetic Lagrangian terms of the self-interacting techni-gluon and techni-gaugino.

3. The M4ST Landscape

M4ST allows model builders to investigate in a well defined and computable way a large number of (perturbative and nonperturbative) inequivalent extensions of the (MS)SM. These inequivalent extensions are determined, partially, by the choice of the value of the coupling constant of the supertechnicolor sector near the electroweak scale as well as on the vacuum choice permitted by the flat directions and, finally, on the supersymmetry breaking pattern. It is not possible to exhaust in this work all the possibilities and, hence, we limit ourselves here to introduce the idea and the basic models of minimal super technicolor type. We, however, sketch some of the basic features of different limits one can take within the M4ST. Each specific model deserves to be studied on its own and some of these models will be investigated in more detail in future publications.

We identify two basic regimes. The perturbative one, in which the superttechnicolor coupling is sufficiently small allowing the new sector to be treated in perturbation theory and denote this model with pM4ST. We then introduce the case in which the superttechnicolor is strongly coupled and we will denote it as sM4ST.

3.1 Perturbative M4ST (pM4ST)

The simplest case to consider is the one in which the new sector is weakly coupled at the electroweak scale (pM4ST). In this case the spectrum of states, which can be observed at the electroweak scale, is constituted by the elementary fields introduced in (2.4) and (2.5). However, the detailed mass spectrum will depend on the structure of the SUSY breaking terms and on the corrections induced by the electroweak symmetry on the superttechnicolor sector.

The phenomenology is extremely rich with several novel weakly coupled particles, such as the new techni-up and techni-down, and their respective superpartners, which can emerge at the LHC. The superpartners will be very similar to ordinary squarks but will carry technicolor instead of color. All the weak processes involving the production of squarks at colliders should be re-investigated to take into account the presence of these new states.

Here we stress, instead, a specific feature of the spectrum associated to the massless, neutral, and weakly interacting techni-fermion, namely D_L . The introduction of the mass term via an explicit Yukawa coupling to a Higgs scalar would break SUSY non-softly and hence render the model unnatural. The techni-fermion D_L , because of its weak charge, can be produced in particle-antiparticle pairs by the Z boson decay. The phenomenology of a massless D_L , because of its coupling to the Z boson, might be difficult to reconcile with the experimental data.

The chiral superfield kinetic term on the right hand side of eq.(2.11) generates the Yukawa coupling $\sqrt{2}g_{TC}\epsilon^{abc}\tilde{D}_L^{a\dagger}D_L^b\bar{D}_R^c$ which, in case the *techni-Higgs* \tilde{D}_L develops a vacuum expectation value, would make D massive by breaking $SU(2)_{TC} \times SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$. In this analysis we have assumed that superttechnicolor symmetry is unbroken at the electroweak scale (up to the effects induced by SUSY breaking in the MSSM sector). A more promising way to tackle this problem is to consider, instead, a dynamical symmetry breaking by the nonperturbative technicolor dynamics. This is typically what happens in any (nonsupersymmetric) technicolor model.

We can also imagine to give a mass to D , relaxing possibly tight phenomenological constraints on the pM4ST, through non-renormalizable interaction terms. From these considerations it appears clear that a thorough phenomenological study of the pM4ST is needed.

3.2 Strong M4ST, (sM4ST), AdS/CFT and Unparticle or the Holographic Super Technicolor

If we assume the superttechnicolor dynamics to be strongly coupled at the electroweak scale, then we must use non-perturbative methods to investigate the effects of the new sector on the MSSM dynamics and vice versa. For example, we can no longer use the single particle

state interpretation in terms of the underlying degrees of freedom of the superttechnicolor model but rather must use an *unparticle* language given that the superttechnicolor model is exactly conformal, before coupling it to the MSSM. The model resembles the one proposed in [67] in which, besides a technicolor sector, one has also coupled a natural unparticle composite sector. If no SUSY breaking terms are added directly to the 4SYM sector then conformality will be broken only via weak and hypercharge interactions.

An important further point is that one can use the machinery of the AdS/CFT correspondence to make reliable computations in the nonperturbative sector, considering the effects of the electroweak interactions as small perturbations.

3.3 Natural 4th Super Family

The M4ST, as its predecessor, the MWT, predicts the natural occurrence of a fourth family of leptons around the electroweak energy scale, put forward first in [7, 8]. The physics of these fourth family of leptons has been studied in [68, 69]. In [68] we focussed especially on detailed collider physics phenomenology while taking into account cosmological limits and providing a detailed discussion of the mixing with the other generations. Precision data and collider phenomenology were investigated in [69]. We note that MWT technicolor can be considered as the precursor of the renewed interest in a fourth family at the LHC given that, from the weak interactions point of view, the model has a fourth family of both (techni)quarks and leptons, and historically appeared before the suggestions of [70] and [71]. Besides, the electroweak precision data comparison is also very similar to the ones we investigated within MWT³. In [7, 8] we also showed that there is no problem with precision data. From the electroweak point of view there is little difference between the MWT and a fourth-family extended SM at the electroweak scale.

Since the M4ST is a supersymmetrized version of the MWT the former now features a novel and natural super 4th family of leptons, besides the techniquarks, awaiting to be discovered at colliders, albeit with more exotic electric charges. The new electron will be doubly charged and will have a number of interesting signatures at colliders.

We have introduced a very minimal supersymmetric extension of MWT and shown that one can use 4SYM as a direct extension of the SM of particle interactions. We have briefly mentioned several possibilities which we will explore in the near future. This is, however, not the only way we can supersymmetrize MWT.

4. Minimal $\mathcal{N} = 1$ Super Technicolor (MST)

A more straightforward supersymmetrization of the MWT can be obtained simply adding a superpartner for each particle in eq.(2.1) and for the techni-gluon G . The resulting model, which we call minimal super technicolor (MST), is anomaly free for the hypercharge assignment of eq.(2.2). This is so since the techni-gaugino, the only new Weyl fermion among the techni-superpartners, is a singlet under $SU(3)_C \times SU(2)_L \times U(1)_Y$, and transforms according to the real representation of $SU(2)_{TC}$, which also guarantees that there is no topological anomaly [34].

³This is so since in the MWT the composite Higgs can be very light.

Requiring the gauge anomalies (including the gravitational one) to cancel leaves two hypercharges undetermined ⁴. We assume, however, for MST still the hypercharge assignment displayed in eq.(2.2), since it does not require the introduction of additional Higgs weak doublets.

We define the chiral superfields (left-handed transforming):

$$\left(\tilde{U}_L, U_L\right) \in \Phi_1, \quad \left(\tilde{D}_L, D_L\right) \in \Phi_2, \quad \left(\tilde{\bar{U}}_R, \bar{U}_R\right) \in U, \quad \left(\tilde{\bar{D}}_R, \bar{D}_R\right) \in D, \quad (4.1)$$

all transforming according to the adjoint representation to the $SU(2)_{TC}$ gauge group and the gauge superfield

$$(G, \lambda) \in V. \quad (4.2)$$

The techni-singlet superfields are defined in eq.(2.5).

The cancellation of the Witten anomaly in the theory requires possible additional chiral superfields to come in pairs, while generating fermion masses, respecting supersymmetry, requires the Higgs doublets to be at least two. We choose to introduce in the theory the two Higgs superfields whose charges are defined in eq.(2.6). Although for some value of y this is not the minimal choice, it does allow us to obtain a phenomenologically viable model for any value of y .

The first coefficient of the beta function of the MST is $\beta_0 = 3N - 4N = -N = -2$, where in the last equality we used the fact that $N = 2$, i.e. the gauge group is $SU(2)_{TC}$. The first term is the contribution of the vector superfield and the second term counts the number of chiral superfields in the adjoint representation of the gauge group times the quadratic casimir for the adjoint representation of $SU(N)$ (which is N with our normalization of the generators). Since this coefficient is negative, we can use perturbation theory for the MST to analyze the spectrum and physical processes at collider experiments and for cosmology. This can be considered a virtue, eventhough this analysis does not take into account the effects of SUSY breaking. If the breaking occurs at very high energies, with respect to the electroweak scale, it will affect the running of the coupling constant. For example, if all the superpartners are decoupled then one recovers the dynamics of MWT which is the one of an asymptotically free gauge theory with a possible infrared fixed point around the electroweak scale.

4.1 The MST Superpotential for $y = 1$

The superpotential of the theory is dictated by the value of the hypercharge parameter y in eq.(2.2), since the gauge invariance of a generic term in the superpotential, an analytical function of the chiral superfields, depends on the hypercharge assignment. We find the models obtained for $y = \pm 1, \pm \frac{1}{3}$, particularly appealing, since for these values of y it is possible to write mass and Yukawa terms involving only the superfields in eq.(2.5) and eq.(4.1), which we refer to simply as techni-superfields. For $y = 0$ one can write mass

⁴We note that that of eq.(2.2) is not the only possible anomaly free charge assignment. Given that in these models there is no need for an ETC sector, we can avoid the constraint $Y(U_L) + Y(\bar{U}_R) = -\frac{1}{2} = Y(H')$ (or equivalently $Y(D_L) + Y(\bar{D}_R) = \frac{1}{2} = Y(H)$), since the role of the techni-fermion condensates is played by two fundamental Higgs scalars, which generate the mass of the SM fermions.

mixing terms for U and D , and for E and N , respectively. For any y Yukawa coupling terms involving either one of the two Higgses are allowed, and so all the fermions in the theory can acquire mass⁵.

In sec.(2) we already studied the minimal model for $y = 1$: by substituting Φ_3 with U , g_{TC} with y_{TC} in eq.(2.8), and by adding the terms $m_D D^a D^a$, $y_D \epsilon_{ij} \Phi_i^a H_j' D^a$ to eq.(2.8) and $y_D' \epsilon_{ij} \Phi_i^a l_j D^a$ to eq.(2.9) (where $a = 1, 2, 3$ is the $SU(2)_{TC}$ gauge index and we suppressed, as we do for the rest of the paper, the family index k) one obtains the full superpotential of the $y = 1$ MST, given by eq.(2.7). D now naturally acquires a mass term, in contrast to the pM4ST case. The corresponding $y = 1$ MST phenomenology can more easily be put in agreement with experimental data than in the pM4ST case.

The superpotential for $y = -1$ is similar to the $y = 1$ case and hence will not be studied further here.

4.2 The MST Superpotential for $y = \pm 1/3$

For $y = \pm \frac{1}{3}$ the MST presents both a gauge singlet and a Higgs candidate (with corresponding hypercharge $\pm \frac{1}{2}$): the gauge singlet can be used to solve the MSSM μ problem in an NMSSM fashion, while the Higgs candidate can be used in principle to reduce further the particle content of the theory. Indeed for $y = -\frac{1}{3}$ we can identify the Higgs superfields H with Λ and H' with l (a generic MSSM leptonic doublet superfield). With this choice we are allowed to discard the extra Higgs superfields, since the theory has neither topological nor gauge anomalies. Such a theory, though, would suffer from some naturalness and phenomenological problems: with reference to H' replaced by a SM lepton it would be difficult to accommodate the relatively large mass splitting between the ordinary neutrino and its scalar superpartner which now is identified with one the higgses. Another phenomenological obstruction would be that all the Yukawa terms, giving mass to the lower components of the weak doublets, violate lepton number conservation. These operators are strongly constrained by experiments. We will therefore study the model with $y = -\frac{1}{3}$ which includes both the H and H' : the more economical model can be retrieved by sending the masses of these two extra Higgs doublets to infinity.

Since the MST with $y = -\frac{1}{3}$ can be obtained easily from the $y = \frac{1}{3}$ MST, we start from this simpler case and then extend it to obtain the $y = -\frac{1}{3}$ model.

The hypercharge assignment for $y = \frac{1}{3}$ corresponds to that of a SM family (assuming that that includes also a right-handed neutrino):

$$\begin{aligned} Y(Q_L) &= \frac{1}{6}, & Y(\bar{U}_R, \bar{D}_R) &= \left(-\frac{2}{3}, \frac{1}{3}\right), \\ Y(L_L) &= -\frac{1}{2}, & Y(\bar{N}_R, \bar{E}_R) &= (0, 1). \end{aligned} \quad (4.3)$$

Following the notation of eqs.(2.7,2.8,2.9) we write the extension of the MSSM superpo-

⁵The techni-gaugino clearly requires a SUSY breaking term, rather than a superpotential mass term.

tential as

$$\begin{aligned}
P_{TC} = & s_N N + \frac{1}{2} m_N N N + m_\Lambda \epsilon_{ij} \Lambda_i H_j + y_N N^3 + y_U \epsilon_{ij} \Phi_i^a H_j U^a + y_D \epsilon_{ij} \Phi_i^a H_j' D^a \\
& + y'_N \epsilon_{ij} \Lambda_i H_j N + y_E \epsilon_{ij} \Lambda_i H_j' E + y'_D \epsilon_{ij} \Phi_i^a \Lambda_j D^a + y_H \epsilon_{ij} H_i H_j' N + y_e \epsilon_{ij} l_i \Lambda_j e,
\end{aligned} \tag{4.4}$$

to which we can add the lepton number violating terms⁶

$$P_{TC, \Delta L \neq 0} = y'_E \epsilon_{ij} \Lambda_i l_j E + y''_D \epsilon_{ij} \Phi_i^a l_j D^a + y_n \epsilon_{ij} l_i H_j N + y'_e \epsilon_{ij} l_i H_j' E + y''_e \epsilon_{ij} \Lambda_i H_j' e. \tag{4.5}$$

It is interesting to notice that the term proportional to m_N in eq.(4.4) and that proportional to y_n in eq.(4.5) generate the Lagrangian terms required to give mass to the neutrino in a natural way (that is allowing for y_n to be of the same order as the other Yukawa coupling constants) by seesaw mechanism.

The hypercharge assignment for $y = -\frac{1}{3}$ corresponds to minus that of a SM family (still assuming that that includes a right-handed neutrino):

$$\begin{aligned}
Y(Q_L) = -\frac{1}{6}, \quad Y(\bar{U}_R, \bar{D}_R) = \left(-\frac{1}{3}, \frac{2}{3}\right), \\
Y(L_L) = \frac{1}{2}, \quad Y(\bar{N}_R, \bar{E}_R) = (-1, 0).
\end{aligned} \tag{4.6}$$

We switch the weak singlet superfields U and D , and N and E , of eq.(4.1) to match the label with the (absolute) value of the EM charge:

$$U \rightleftharpoons D, \quad N \rightleftharpoons E. \tag{4.7}$$

Applying these substitutions to eq.(4.4), and replacing the Higgs superfields H and H' respectively with Λ and l , as required by gauge invariance, we find:

$$\begin{aligned}
P_{TC} = & s_N N + \frac{1}{2} m_N N N + m_\Lambda \epsilon_{ij} \Lambda_i H_j' + y_N N^3 + y_U \epsilon_{ij} \Phi_i^a H_j' U^a + y_D \epsilon_{ij} \Phi_i^a H_j D^a \\
& + y'_N \epsilon_{ij} \Lambda_i H_j' N + y_E \epsilon_{ij} \Lambda_i H_j E + y'_D \epsilon_{ij} \Phi_i^a \Lambda_j D^a + y_H \epsilon_{ij} H_i H_j' N.
\end{aligned} \tag{4.8}$$

Notice that the correspondent of the last term in eq.(4.4) does not appear in eq.(4.8) because, by symmetries, this term vanishes identically, i.e. $y_e \epsilon_{ij} l_i l_j e = 0$.⁷

The lepton number violating terms include also mass-mixing terms obtained coupling techni-singlet and MSSM leptonic superfields with opposite hypercharges. These are in addition to a number of Yukawa terms. These arise as a direct consequence of l and Λ having the same charge assignments as H' and H . We can now add to P_{MSSM} also the superpotential

$$\begin{aligned}
P_{TC, \Delta L \neq 0} = & m'_\Lambda \epsilon_{ij} \Lambda_i l_j + m_e E e + y'_U \epsilon_{ij} \Phi_i^a l_j U^a + y''_N \epsilon_{ij} \Lambda_i l_j N + y_n \epsilon_{ij} l_i H_j N \\
& + y'_n \epsilon_{ij} l_i \Lambda_j N + y_e E e N,
\end{aligned} \tag{4.9}$$

where the terms proportional to y_n and y'_n allow, together with that proportional to m_N in eq.(4.8), to solve the neutrino mass naturalness problem.

⁶We consider all the techni-superfields to have baryon and lepton number equal to zero.

⁷Here we neglect generation mixing terms, that otherwise would give a non-zero contribution to the previous operator.

4.3 Minimal Susy Breaking Terms

We neglect for now the most general expression for the SUSY breaking Lagrangian, and write only those terms involving exclusively the gauge singlet N and the Higgses H and H' . The corresponding potential is:

$$V_{soft} = \left[a_H \tilde{N}_R \left(\tilde{H}_1 \tilde{H}'_2 - \tilde{H}_2 \tilde{H}'_1 \right) + a_N \tilde{N}_R^3 + b_N \tilde{N}_R^2 + c_N \tilde{N}_R + h.c. \right] + M_N^2 \tilde{N}_R \tilde{N}_R. \quad (4.10)$$

It would be interesting to investigate the possibility of a phenomenologically viable solution of both the MSSM μ and neutrino mass problems within the frame of the $y = \frac{1}{3}$ MST. It would be furthermore interesting to determine the level of fine tuning required by such a possible solution, and what is the size of this tuning relative to that of the MSSM, which is of the order of 1%. The corresponding SUSY-breaking potential for $y = -\frac{1}{3}$ MST is the same as of eq.(4.10).

5. Conclusions

We have presented novel extensions of the SM featuring an $\mathcal{N} = 4$ or $\mathcal{N} = 1$ supertechnicolor sector. These models are minimal and direct supersymmetric generalizations of the MWT model. We started from the observation that the MWT model has the same degrees of freedom of 4SYM except for the absence of the six real scalars. Following this trail we added the six scalars and constructed an extension of the SM naturally featuring a supersymmetrized version of MWT which was 4SYM. We used as basic model, before adding the new supertechnicolor model and supersymmetry breaking interactions, the MSSM, so that we could give mass to all the SM particles. In the MSSM we then embedded the 4SYM in such a way that the extended supersymmetry, of the supertechnicolor sector, is broken to $\mathcal{N} = 1$ only via weak and hypercharge gauge interactions. Since the original MWT model contains also a natural 4th family of leptons, needed to cure the topological Witten anomaly, we introduced in the theory also a 4th family of lepton superfields. We then constructed the superpotential for the full theory and provided the Lagrangian in terms of superfields as well as the corresponding physical components. The resulting model was termed in short M4ST. Depending on the way supersymmetry breaks, the value of the technicolor coupling constant around the electroweak scale, and the value assumed by several other natural couplings one is allowed to investigate several vastly different physical scenarios. We have suggested several possible models ranging from ordinary technicolor to unparticle models as well as completely perturbative extensions. We recall that the new sector coupled to the MSSM is, per se, conformal and hence it can be seen as a well defined model of unparticle (when the technicolor coupling constant is sufficiently strong that the single particle interpretation is no longer viable). The advantage is that even when the supertechnicolor coupling constant is taken to be large, one can use AdS/CFT methods to determine a number of features ranging from the computation of the unparticle spectrum to thermodynamical properties which will be investigated in the future. Besides, the model can benefit from, and provide further motivation for, lattice studies of supersymmetry (see [72, 73, 74] for recent interesting lattice investigations). The M4ST model can now be used

to predict interesting signals for collider phenomenology, as well as a model for cosmological applications, for investigating a closer connection with string theory, and finally, one can make use of the AdS/CFT methods to investigate explicit physical phenomena for beyond standard model physics at the TeV scale.

For completeness we have also considered the case in which the MWT superttechnicolor extension is directly an $\mathcal{N} = 1$ gauge theory, the MST. Here more fields than in the case of the M4ST are needed. We constructed the superpotential for several choices of the technifields hypercharge. The models feature also a fourth generation of super leptons. Because the beta function of the MST per se is positive (before supersymmetry breaking), one can investigate the phenomenology of MST in the perturbative regime, which can be seen as an advantage over models in which the theory is asymptotically free. Of course, if one chooses to decouple the superpartners at a very high energy one will recover the MWT theory, which is strongly coupled at the electroweak scale.

Any other model of superttechnicolor can be constructed in a similar way, by basically merging a technicolor theory with its supersymmetric counterpart (in our case the MWT and the MSSM, respectively). The dynamics at the electroweak scale will, however, depend on the type of gauge interactions and supermatter representation with respect to the technicolor interactions of the specific model. To this scope the reader can find useful the knowledge of the $SU(N)$ supersymmetric phase diagram for matter in different representations [17].

A. $\mathcal{N} = 4$ Super Yang Mills: Notation and Lagrangian

The $\mathcal{N} = 4$ supersymmetric Lagrangian for an $SU(N)$ gauge theory can be written in terms of three $\mathcal{N} = 1$ chiral superfields Φ_i , $i = 1, 2, 3$ and one $\mathcal{N} = 1$ vector superfield V , all in the adjoint representation of $SU(N)$. The superpotential for this Lagrangian reads (see [75] and references therein)

$$P = -\frac{g}{3\sqrt{2}}\epsilon_{ijk}f^{abc}\Phi_i^a\Phi_j^b\Phi_k^c, \quad j, k = 1, 2, 3; a, b, c = 1, \dots, N^2 - 1; \quad (\text{A.1})$$

where g is the gauge coupling constant, and f^{abc} the structure constant. This superpotential is invariant under $SU(3)$ transformations over the flavor index. The full Lagrangian is indeed invariant under $SU(4)$ transformations because the $\mathcal{N} = 4$ supersymmetry algebra is invariant under the same transformations of the supercharges.

Following the notation of Wess and Bagger [76] we write

$$\mathcal{L} = \frac{1}{2}\text{Tr} (W^\alpha W_\alpha|_{\theta\theta} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}) + \Phi_i^\dagger \exp(2gV) \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + (P|_{\theta\theta} + h.c.) \quad (\text{A.2})$$

where

$$W_\alpha = -\frac{1}{4g}\bar{D}\bar{D}\exp(-2gV)D_\alpha\exp(2gV), \quad V = V^a T_A^a, \quad (T_A^a)^{bc} = -if^{abc}, \quad (\text{A.3})$$

and with Φ_i having gauge components Φ_i^a . In terms of the component fields eq.(A.2) can be expressed as

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F^{\mu\nu a}F_{\mu\nu}^a - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - D^\mu\phi_i^{a\dagger}D_\mu\phi_i^a - i\bar{\psi}_i^a\bar{\sigma}^\mu D_\mu\psi_i^a \\ & + \sqrt{2}gf^{abc}\left(\phi_i^{a\dagger}\psi_i^b\lambda^c + \bar{\lambda}^c\bar{\psi}_i^b\phi_i^a\right) + \frac{g}{\sqrt{2}}\epsilon_{ijk}f^{abc}\left(\phi_i^a\psi_j^b\psi_k^c + \bar{\psi}_k^c\bar{\psi}_j^b\phi_i^{a\dagger}\right) \\ & + \frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e \end{aligned} \quad (\text{A.4})$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c, \quad D_\mu\xi^a = \partial\xi^a - gf^{abc}A_\mu^b\xi^c, \quad \xi = \lambda, \psi_i, \phi_i. \quad (\text{A.5})$$

Here λ is the gaugino, while ψ_i and ϕ_i are respectively the fermionic and scalar component of Φ_i . To make explicit the $SU(4)$ R-symmetry of the Lagrangian the following change of variables provides useful:

$$\varphi_{rs}^a = -\varphi_{sr}^a, \quad \varphi_{i4}^a = \frac{1}{2}\phi_i^a, \quad \varphi_{ij}^a = \frac{1}{2}\epsilon_{ijk}\phi_k^{a\dagger}, \quad \eta_i^a = \psi_i^a, \quad \eta_4^a = \lambda^a; \quad r, s = 1, \dots, 4. \quad (\text{A.6})$$

The symmetry of the Lagrangian can be made manifest by rewriting eq.(A.4) as

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F^{\mu\nu a}F_{\mu\nu}^a - \text{Tr}D^\mu\varphi^{a\dagger}D_\mu\varphi^a - i\bar{\eta}_r^a\bar{\sigma}^\mu D_\mu\eta_r^a \\ & - \sqrt{2}gf^{abc}\left(\varphi_{rs}^{a\dagger}\eta_r^b\eta_s^c + \bar{\eta}_r^c\bar{\eta}_s^b\varphi_{rs}^a\right) \\ & + \frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\text{Tr}\varphi^{b\dagger}\varphi^c\text{Tr}\varphi^{d\dagger}\varphi^e. \end{aligned} \quad (\text{A.7})$$

Under $SU(4)$ φ^a transforms as a **6**, η^a as a **4**, and A_μ^a as a **1**, leaving the Lagrangian in eq.(A.7) unchanged.

B. M4ST Lagrangian in Components

The Lagrangian of a supersymmetric theory can, in general, be defined by

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{g-Yuk} + \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_{P-Yuk} + \mathcal{L}_{soft}, \quad (\text{B.1})$$

where the labels refer to the kinetic terms, the Yukawa ones given by gauge and superpotential interactions, the D and F scalar interaction terms, and the soft SUSY breaking ones. All these terms can be expressed in function of the physical fields of the theory with the help of the following equations:

$$\mathcal{L}_{kin} = -\frac{1}{4}F_j^{\mu\nu a}F_{j\mu\nu}^a - i\bar{\lambda}_j^a\bar{\sigma}^\mu D_\mu\lambda_j^a - D^\mu\phi_i^{a\dagger}D_\mu\phi_i^a - i\bar{\psi}_i^a\bar{\sigma}^\mu D_\mu\psi_i^a, \quad (\text{B.2})$$

$$\mathcal{L}_{g-Yuk} = \sum_j i\sqrt{2}g_j \left(\phi_i^\dagger T_j^a \psi_i \lambda_j^a - \bar{\lambda}_j^a \bar{\psi}_i T_j^a \phi_i \right), \quad (\text{B.3})$$

$$\mathcal{L}_D = -\frac{1}{2} \sum_j g_j^2 \left(\phi_i^\dagger T_j^a \phi_i \right)^2, \quad (\text{B.4})$$

$$\mathcal{L}_F = - \left| \frac{\partial P}{\partial \phi_i^a} \right|^2, \quad (\text{B.5})$$

$$\mathcal{L}_{P-Yuk} = -\frac{1}{2} \left[\frac{\partial^2 P}{\partial \phi_i^a \partial \phi_l^b} \psi_i^a \psi_l^b + h.c. \right], \quad (\text{B.6})$$

where i, l run over all the scalar field labels, while j runs over all the gauge group labels, and a, b are the corresponding gauge group indices. Furthermore, we normalize the generators in the usual way, by taking the index $T(F) = \frac{1}{2}$, where

$$\text{Tr}T_R^a T_R^b = T(R)\delta^{ab},$$

with R here referring to the representation (F =fundamental). The SUSY breaking soft terms, moreover, are obtained by re-writing the superpotential in function of the scalar fields alone, and by adding to it its Hermitian conjugate and the mass terms for the gauginos and the scalar fields.

Using these equations it is rather straightforward to write the Lagrangian of the M4ST defined in sec.(2). We refer to [14] and references therein for the explicit form of \mathcal{L}_{MSSM} in terms of the physical fields of the MSSM, and focus here only on \mathcal{L}_{TC} . The kinetic terms are trivial and therefore we do not write them here. The gauge Yukawa terms are given by

$$\begin{aligned} \mathcal{L}_{g-Yuk} = & \sqrt{2}g_{TC} \left(\tilde{U}_L^b U_L^c \tilde{D}_R^a - D_R^a \tilde{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b D_L^c \tilde{D}_R^a - D_R^a \tilde{D}_L^b \tilde{D}_L^c + \tilde{U}_R^b \tilde{U}_R^c \tilde{D}_R^a - D_R^a U_R^b \tilde{U}_R^c \right) \epsilon^{abc} \\ & + i\frac{g_L}{\sqrt{2}} \left(\tilde{Q}_L^i Q_L^j \tilde{W}^k - \tilde{W}^k \tilde{Q}_L^i \tilde{Q}_L^j + \tilde{L}_L^i L_L^j \tilde{W}^k - \tilde{W}^k \tilde{L}_L^i \tilde{L}_L^j \right) \sigma_{ij}^k \\ & + i\sqrt{2}g_Y \sum_p Y_p \left(\tilde{\psi}_p \psi_p \tilde{B} - \tilde{B} \tilde{\psi}_p \tilde{\psi}_p \right), \quad \psi_p = U_L^a, D_L^a, \bar{U}_R^a, N_L, E_L, \bar{N}_R, \bar{E}_R, \end{aligned} \quad (\text{B.7})$$

where \tilde{W}^k and \tilde{B} are respectively the wino and the bino, σ^k the Pauli matrices, $i, j = 1, 2$; $k, a, b, c = 1, 2, 3$; and the hypercharge Y_p is given for each field ψ_p in eqs.(2.3).

The D terms are given by

$$\mathcal{L}_D = -\frac{1}{2} \left(g_{TC}^2 D_{TC}^a D_{TC}^a + g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y \right) + \frac{1}{2} \left(g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y \right)_{MSSM}, \quad (\text{B.8})$$

where

$$\begin{aligned} D_{TC}^a &= -i\epsilon^{abc} \left(\tilde{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b \tilde{D}_L^c + \tilde{U}_R^b \tilde{U}_R^c \right), \quad D_L^k = \frac{\sigma_{ij}^k}{2} \left(\tilde{Q}_L^i \tilde{Q}_L^j + \tilde{L}_L^i \tilde{L}_L^j \right) + D_{L,MSSM}^k \\ D_Y &= \sum_p Y_p \tilde{\psi}_p \tilde{\psi}_p + D_{Y,MSSM}. \end{aligned} \quad (\text{B.9})$$

In these equations the $D_{L,MSSM}^k$ and $D_{Y,MSSM}$ auxiliary fields are assumed to be expressed in function of the MSSM physical fields [14]. The rest of the scalar interaction terms⁸ is given by

$$\begin{aligned} \mathcal{L}_F &= -g_{TC}^2 \left[\left(\tilde{U}_L^b \tilde{U}_L^b + \tilde{D}_L^b \tilde{D}_L^b + \tilde{U}_R^b \tilde{U}_R^b \right)^2 - \left(\tilde{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b \tilde{D}_L^c + \tilde{U}_R^b \tilde{U}_R^c \right) \left(\tilde{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b \tilde{D}_L^c \right. \right. \\ &\quad \left. \left. + \tilde{U}_R^b \tilde{U}_R^c \right) \right] - y_U^2 \left[\left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) \left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) + \tilde{U}_R^a \tilde{U}_R^a \left(\tilde{H}_1 \tilde{H}_1 + \tilde{H}_2 \tilde{H}_2 \right) \right. \\ &\quad \left. + \tilde{U}_R^a \tilde{U}_R^b \left(\tilde{U}_L^a \tilde{U}_L^b + \tilde{D}_L^a \tilde{D}_L^b \right) \right] - y_N^2 \left[\left(\tilde{N}_L \tilde{H}_2 - \tilde{E}_L \tilde{H}_1 \right) \left(\tilde{N}_L \tilde{H}_2 - \tilde{E}_L \tilde{H}_1 \right) \right. \\ &\quad \left. + \tilde{N}_R \tilde{N}_R \left(\tilde{H}_1 \tilde{H}_1 + \tilde{H}_2 \tilde{H}_2 + \tilde{N}_L \tilde{N}_L + \tilde{E}_L \tilde{E}_L \right) \right] - y_E^2 \left[\left(\tilde{N}_L \tilde{H}_2' - \tilde{E}_L \tilde{H}_1' \right) \left(\tilde{N}_L \tilde{H}_2' - \tilde{E}_L \tilde{H}_1' \right) \right. \\ &\quad \left. + \tilde{E}_R \tilde{E}_R \left(\tilde{H}_1' \tilde{H}_1' + \tilde{H}_2' \tilde{H}_2' + \tilde{N}_L \tilde{N}_L + \tilde{E}_L \tilde{E}_L \right) \right] + \left\{ \sqrt{2} y_U g_{TC} \epsilon^{abc} \left[\tilde{U}_L^b \tilde{D}_L^c \left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) \right. \right. \\ &\quad \left. \left. + \tilde{U}_R^b \tilde{U}_L^c \tilde{H}_1 \tilde{U}_R^a + \tilde{U}_R^b \tilde{D}_L^c \tilde{H}_2 \tilde{U}_R^a \right] - y_U y_N \tilde{U}_R^a \tilde{N}_R \left(\tilde{U}_L^a \tilde{N}_L + \tilde{D}_L^a \tilde{E}_L \right) \right. \\ &\quad \left. - y_N y_E \tilde{N}_R \tilde{E}_R \left(\tilde{H}_1 \tilde{H}_1' + \tilde{H}_2 \tilde{H}_2' \right) + h.c. \right\} + \mathcal{L}_{mix}, \end{aligned} \quad (\text{B.10})$$

with \mathcal{L}_{mix} defined in function of the F auxiliary fields associated with the MSSM two Higgs super-doublets:

$$\begin{aligned} \mathcal{L}_{mix} &= - \sum_{\phi_p} \left(F_{\phi_p, TC} F_{\phi_p, MSSM}^\dagger + h.c. \right), \quad \phi_p = H_1', H_2', H_1, H_1, \quad F_{H_1', TC} = -y_E \tilde{E}_L \tilde{E}_R, \\ F_{H_2', TC} &= y_E \tilde{N}_L \tilde{E}_R, \quad F_{H_1, TC} = -y_U \tilde{D}_L^a \tilde{U}_R^a - y_N \tilde{E}_L \tilde{N}_R, \quad F_{H_2, TC} = y_U \tilde{U}_L^a \tilde{U}_R^a + y_N \tilde{N}_L \tilde{N}_R. \end{aligned} \quad (\text{B.11})$$

The corresponding MSSM auxiliary fields F can be found in [14] and references therein. Also, in the eqs.(B.10,B.11) we used \tilde{H} and \tilde{H}' to indicate the scalar Higgs doublets, for consistency with the rest of the notation where the tilde identifies the scalar component of a chiral superfield or the fermionic component of a vector superfield. The remaining

⁸We neglect here and in the following the lepton-number violating terms given by the superpotential in eq.(2.9), and consider the constants in the superpotential to be real in first approximation to avoid the contribution of CP violating terms.

Yukawa interaction terms are determined by the superpotential, and can be expressed as

$$\begin{aligned}
\mathcal{L}_{P-Yuk} = & \sqrt{2}g_{TC}\epsilon^{abc} \left(U_L^a D_L^b \tilde{U}_R^c + U_L^a \tilde{D}_L^b \bar{U}_R^c + \tilde{U}_L^a D_L^b \bar{U}_R^c \right) + y_U \left[(H_1 D_L^a - H_2 U_L^a) \tilde{U}_R^a \right. \\
& + \left(\tilde{H}_1 D_L^a - \tilde{H}_2 U_L^a \right) \bar{U}_R^a + \left(H_1 \tilde{D}_L^a - H_2 \tilde{U}_L^a \right) \bar{U}_R^a \left. \right] + y_N \left[(H_1 E_L - H_2 N_L) \tilde{N}_R \right. \\
& + \left(H_1 \tilde{E}_L - H_2 \tilde{N}_L \right) \bar{N}_R + \left(\tilde{H}_1 E_L - \tilde{H}_2 N_L \right) \bar{N}_R \left. \right] + y_E \left[(H_1' E_L - H_2' N_L) \tilde{\tilde{E}}_R \right. \\
& + \left(H_1' \tilde{E}_L - H_2' \tilde{N}_L \right) \bar{\tilde{E}}_R + \left(\tilde{H}_1' E_L - \tilde{H}_2' N_L \right) \bar{\tilde{E}}_R \left. \right] + h.c.. \tag{B.12}
\end{aligned}$$

The soft SUSY breaking terms, finally, can be written straightforwardly starting from the superpotential in eq.(2.8), to which we add the techni-gaugino and scalar mass terms as well:

$$\begin{aligned}
\mathcal{L}_{soft} = & - \left[a_{TC}\epsilon^{abc} \tilde{U}_L^a \tilde{D}_L^b \tilde{U}_R^c + a_U \left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) \tilde{U}_R^a + a_N \left(\tilde{H}_1 \tilde{E}_L - \tilde{H}_2 \tilde{N}_L \right) \tilde{N}_R \right. \\
& + a_E \left(\tilde{H}_1' \tilde{E}_L - \tilde{H}_2' \tilde{N}_L \right) \tilde{\tilde{E}}_R + \frac{1}{2} M_D \bar{D}_R^a \bar{D}_R^a + h.c. \left. \right] - M_Q^2 \tilde{Q}_L^a \tilde{Q}_L^a - M_U^2 \tilde{U}_R^a \tilde{U}_R^a \\
& - M_L^2 \tilde{L}_L \tilde{L}_L - M_N^2 \tilde{N}_R \tilde{N}_R - M_E^2 \tilde{\tilde{E}}_R \tilde{\tilde{E}}_R. \tag{B.13}
\end{aligned}$$

C. MST Lagrangian in Components

In this appendix we write the $y = \frac{1}{3}$ MST Lagrangian, determined by the superpotential in eqs.(2.7,4.4), in terms of its physical components. The full Lagrangian can be derived using eqs.(B.1,B.2,B.3,B.4,B.5,B.6), as we did for the M4ST Lagrangian in the previous appendix. The MST Lagrangian's kinetic terms (eq.(B.2)) are trivial and therefore we do not write them here. The gauge Yukawa terms, independent of the superpotential, are given by

$$\begin{aligned}
\mathcal{L}_{g-Yuk} = & \sqrt{2}g_{TC} \left(\tilde{U}_L^b U_L^c \lambda^a - \bar{\lambda}^a \bar{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b D_L^c \lambda^a - \bar{\lambda}^a \bar{D}_L^b \tilde{D}_L^c + \tilde{U}_R^b \bar{U}_R^c \lambda^a - \bar{\lambda}^a U_R^b \tilde{U}_R^c + \tilde{D}_R^b \bar{D}_R^c \lambda^a \right. \\
& - \left. \bar{\lambda}^a D_R^b \tilde{D}_R^c \right) \epsilon^{abc} + i \frac{g_L}{\sqrt{2}} \left(\tilde{Q}_L^i Q_L^j \tilde{W}^k - \tilde{W}^k \tilde{Q}_L^i \tilde{Q}_L^j + \tilde{L}_L^i L_L^j \tilde{W}^k - \tilde{W}^k \tilde{L}_L^i \tilde{L}_L^j \right) \sigma_{ij}^k \\
& + i\sqrt{2}g_Y \sum_p Y_p \left(\tilde{\psi}_p \psi_p \tilde{B} - \tilde{B} \bar{\psi}_p \bar{\psi}_p \right), \quad \psi_p = U_L^a, D_L^a, \bar{U}_R^a, \bar{D}_R^a, N_L, E_L, \bar{N}_R, \bar{E}_R, \tag{C.1}
\end{aligned}$$

where $i, j = 1, 2$; $k, a, b, c = 1, 2, 3$; and the hypercharge Y_p is given for each field ψ_p in eqs.(4.3).

The D terms are given by

$$\mathcal{L}_D = -\frac{1}{2} \left(g_{TC}^2 D_{TC}^a D_{TC}^a + g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y \right) + \frac{1}{2} \left(g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y \right)_{MSSM}, \tag{C.2}$$

where

$$\begin{aligned}
D_{TC}^a = & -i\epsilon^{abc} \left(\tilde{U}_L^b \tilde{U}_L^c + \tilde{D}_L^b \tilde{D}_L^c + \tilde{U}_R^b \tilde{U}_R^c + \tilde{D}_R^b \tilde{D}_R^c \right), \\
D_L^k = & \frac{1}{2} \sigma_{ij}^k \left(\tilde{Q}_L^i \tilde{Q}_L^j + \tilde{L}_L^i \tilde{L}_L^j \right) + D_{L,MSSM}^k, \quad D_Y = \sum_p Y_p \tilde{\psi}_p \tilde{\psi}_p + D_{Y,MSSM}. \tag{C.3}
\end{aligned}$$

In these equations the $D_{L,MSSM}^k$ and $D_{Y,MSSM}$ auxiliary fields are assumed to be expressed in function of the MSSM physical fields [14]. The rest of the scalar interaction terms⁹ is given in terms of the F auxiliary fields by

$$\mathcal{L}_F = - \sum_{\phi_p} F_{\phi_p} F_{\phi_p}^\dagger, \quad \phi_p = U_L, D_L, \bar{U}_R, \bar{D}_R, N_L, E_L, \bar{N}_R, \bar{E}_R, H_1, H_2, H'_1, H'_2, \nu_L, e_L, \bar{e}_R, \quad (\text{C.4})$$

where

$$\begin{aligned} F_{U_L} &= y_U \tilde{H}_2 \tilde{U}_R^a + y_D \tilde{H}'_2 \tilde{D}_R^a + y'_D \tilde{E}_L \tilde{D}_R^a, \\ F_{D_L} &= -y_U \tilde{H}_1 \tilde{U}_R^a - y_D \tilde{H}'_1 \tilde{D}_R^a - y'_D \tilde{N}_L \tilde{D}_R^a, \\ F_{\bar{U}_R} &= y_U \left(\tilde{U}_L^a \tilde{H}_2 - \tilde{D}_L^a \tilde{H}_1 \right), \\ F_{\bar{D}_R} &= y_D \left(\tilde{U}_L^a \tilde{H}'_2 - \tilde{D}_L^a \tilde{H}'_1 \right) + y'_D \left(\tilde{U}_L^a \tilde{E}_L - \tilde{D}_L^a \tilde{N}_L \right), \\ F_{N_L} &= m_\Lambda \tilde{H}_2 + y'_N \tilde{H}_2 \tilde{N}_R + y_E \tilde{H}'_2 \tilde{E}_R - y'_D \tilde{D}_L \tilde{D}_R - y_e \tilde{e}_L \tilde{e}_R, \\ F_{E_L} &= -m_\Lambda \tilde{H}_1 - y'_N \tilde{H}_1 \tilde{N}_R - y_E \tilde{H}'_1 \tilde{E}_R + y'_D \tilde{U}_L \tilde{D}_R + y_e \tilde{\nu}_L \tilde{e}_R, \\ F_{\bar{N}_R} &= s_N + m_N \tilde{N}_R + 3y_N \tilde{N}_R^2 + y'_N \left(\tilde{N}_L \tilde{H}_2 - \tilde{E}_L \tilde{H}_1 \right) + y_H \left(\tilde{H}_1 \tilde{H}'_2 - \tilde{H}_2 \tilde{H}'_1 \right), \\ F_{\bar{E}_R} &= y_E \left(\tilde{N}_L \tilde{H}'_2 - \tilde{E}_L \tilde{H}'_1 \right), \end{aligned} \quad (\text{C.5})$$

and for the MSSM physical fields, referring to [14] and references therein for the MSSM auxiliary F fields,

$$\begin{aligned} F_{H_1} &= -(m_\Lambda \tilde{E}_L + y_U \tilde{D}_L^a \tilde{U}_R^a + y'_N \tilde{E}_L \tilde{N}_R - y_H \tilde{H}'_2 \tilde{N}_R) + F_{H_1, MSSM}, \\ F_{H_2} &= m_\Lambda \tilde{N}_L + y_U \tilde{U}_L^a \tilde{U}_R^a + y'_N \tilde{N}_L \tilde{N}_R - y_H \tilde{H}'_1 \tilde{N}_R + F_{H_2, MSSM}, \\ F_{H'_1} &= -(y_D \tilde{D}_L^a \tilde{D}_R^a + y_E \tilde{E}_L \tilde{E}_R + y_H \tilde{H}_2 \tilde{N}_R) + F_{H'_1, MSSM}, \\ F_{H'_2} &= y_D \tilde{U}_L^a \tilde{D}_R^a + y_E \tilde{N}_L \tilde{E}_R + y_H \tilde{H}_1 \tilde{N}_R + F_{H'_2, MSSM}, \\ F_{\nu_L} &= y_e \tilde{E}_L \tilde{e}_R + F_{\nu_L, MSSM}, \\ F_{e_L} &= -y_e \tilde{N}_L \tilde{e}_R + F_{e_L, MSSM}, \\ F_{\bar{e}_R} &= y_e \left(\tilde{\nu}_L \tilde{E}_L - \tilde{e}_L \tilde{N}_L \right) + F_{\bar{e}_R, MSSM}. \end{aligned} \quad (\text{C.6})$$

The remaining Yukawa interaction terms are determined by the superpotential, and can

⁹We neglect here and in the following the lepton-number violating terms given by the superpotential in eq.(4.4) and assume the constants in eq.(4.4) to be real to avoid CP violating contributions.

be expressed as

$$\begin{aligned}
\mathcal{L}_{P-Yuk} = & -\frac{1}{2}m_N\bar{N}_R\bar{N}_R - m_\Lambda(N_L H_2 - E_L H_1) - 3y_N\tilde{N}_R\bar{N}_R\bar{N}_R - y_U\left[(U_L^a H_2 - D_L^a H_1)\tilde{U}_R^a\right. \\
& + \left.(U_L^a \tilde{H}_2 - D_L^a \tilde{H}_1\right)\bar{U}_R^a + \left.(\tilde{U}_L^a H_2 - \tilde{D}_L^a H_1)\bar{U}_R^a\right] - y_D\left[(U_L^a H_2' - D_L^a H_1')\tilde{D}_R^a\right. \\
& + \left.(U_L^a \tilde{H}_2' - D_L^a \tilde{H}_1')\bar{D}_R^a + \left.(\tilde{U}_L^a H_2' - \tilde{D}_L^a H_1')\bar{D}_R^a\right] - y'_N\left[(N_L H_2 - E_L H_1)\tilde{N}_R\right. \\
& + \left.(N_L \tilde{H}_2 - E_L \tilde{H}_1)\bar{N}_R + \left.(\tilde{N}_L H_2 - \tilde{E}_L H_1)\bar{N}_R\right] - y_E\left[(N_L H_2' - E_L H_1')\tilde{E}_R\right. \\
& + \left.(N_L \tilde{H}_2' - E_L \tilde{H}_1')\bar{E}_R + \left.(\tilde{N}_L H_2' - \tilde{E}_L H_1')\bar{E}_R\right] - y'_D\left[(U_L^a E_L - D_L^a N_L)\tilde{D}_R^a\right. \\
& + \left.(U_L^a \tilde{E}_L - D_L^a \tilde{N}_L)\bar{D}_R^a + \left.(\tilde{U}_L^a E_L - \tilde{D}_L^a N_L)\bar{D}_R^a\right] - y_H\left[(H_1 H_2' - H_2 H_1')\tilde{N}_R\right. \\
& + \left.(H_1 \tilde{H}_2' - H_2 \tilde{H}_1')\bar{N}_R + \left.(\tilde{H}_1 H_2' - \tilde{H}_2 H_1')\bar{N}_R\right] - y_e[(\nu_L E_L - e_L N_L)\tilde{e}_R \\
& + \left.(\nu_L \tilde{E}_L - e_L \tilde{N}_L)\bar{e}_R + (\tilde{\nu}_L E_L - \tilde{e}_L N_L)\bar{e}_R\right] + h.c.. \tag{C.7}
\end{aligned}$$

The soft SUSY breaking terms, finally, can be written straightforwardly starting from the superpotential in eq.(4.4), to which we add the techni-gaugino and scalar mass terms as well:

$$\begin{aligned}
\mathcal{L}_{soft} = & -\left[a_N\tilde{N}_R^3 + b_N\tilde{N}_R^2 + c_N\tilde{N}_R + b_\Lambda\left(\tilde{N}_L\tilde{H}_2 - \tilde{E}_L\tilde{H}_1\right) + a_H\left(\tilde{H}_1\tilde{H}_2' - \tilde{H}_2\tilde{H}_1'\right)\tilde{N}_R\right. \\
& + a_U\left(\tilde{U}_L^a\tilde{H}_2 - \tilde{D}_L^a\tilde{H}_1\right)\tilde{U}_R^a + a_D\left(\tilde{U}_L^a\tilde{H}_2' - \tilde{D}_L^a\tilde{H}_1'\right)\tilde{D}_R^a + a'_N\left(\tilde{N}_L\tilde{H}_2 - \tilde{E}_L\tilde{H}_1\right)\tilde{N}_R \\
& + a_E\left(\tilde{N}_L\tilde{H}_2' - \tilde{E}_L\tilde{H}_1'\right)\tilde{E}_R + a'_D\left(\tilde{U}_L^a\tilde{E}_L - \tilde{D}_L^a\tilde{N}_L\right)\tilde{D}_R^a + a_e\left(\tilde{\nu}_L\tilde{E}_L - \tilde{e}_L\tilde{N}_L\right)\tilde{e}_R + \frac{1}{2}M_\lambda\bar{\lambda}^a\bar{\lambda}^a \\
& \left. + h.c.\right] - M_Q^2\tilde{Q}_L^a\tilde{Q}_L^a - M_U^2\tilde{U}_R^a\tilde{U}_R^a - M_D^2\tilde{D}_R^a\tilde{D}_R^a - M_L^2\tilde{L}_L\tilde{L}_L - M_N^2\tilde{N}_R\tilde{N}_R - M_E^2\tilde{E}_R\tilde{E}_R. \tag{C.8}
\end{aligned}$$

References

- [1] F. Sannino, arXiv:0911.0931 [hep-ph].
- [2] F. Sannino, arXiv:0804.0182 [hep-ph].
- [3] C. T. Hill and E. H. Simmons, Phys. Rept. **381**, 235 (2003) [Erratum-ibid. **390**, 553 (2004)]
- [4] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B **189**, 575 (1981).
- [5] F. Sannino and K. Tuominen, Phys. Rev. D **71**, 051901 (2005) [arXiv:hep-ph/0405209].
- [6] D. K. Hong, S. D. H. Hsu and F. Sannino, Phys. Lett. B **597**, 89 (2004) [arXiv:hep-ph/0406200].
- [7] D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D **72**, 055001 (2005) [arXiv:hep-ph/0505059].
- [8] D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D **73**, 037701 (2006) [arXiv:hep-ph/0510217].
- [9] N. Evans and F. Sannino, arXiv:hep-ph/0512080.
- [10] H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007) [arXiv:hep-ph/0703260].
- [11] H. Georgi, Phys. Lett. B **650**, 275 (2007) [arXiv:0704.2457 [hep-ph]].

- [12] S. Weinberg, Phys. Rev. D **19**, 1277 (1979).
- [13] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
- [14] S. P. Martin, arXiv:hep-ph/9709356.
- [15] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [16] D. D. Dietrich and F. Sannino, Phys. Rev. D **75**, 085018 (2007) [arXiv:hep-ph/0611341].
- [17] T. A. Ryttov and F. Sannino, Phys. Rev. D **76**, 105004 (2007) [arXiv:0707.3166 [hep-th]].
- [18] T. A. Ryttov and F. Sannino, Phys. Rev. D **78**, 065001 (2008) [arXiv:0711.3745 [hep-th]].
- [19] M. T. Frandsen and F. Sannino, arXiv:0911.1570 [hep-ph].
- [20] A. Belyaev, R. Foadi, M. T. Frandsen, M. Jarvinen, F. Sannino and A. Pukhov, Phys. Rev. D **79**, 035006 (2009) [arXiv:0809.0793 [hep-ph]].
- [21] R. Foadi, M. T. Frandsen and F. Sannino, Phys. Rev. D **77**, 097702 (2008) [arXiv:0712.1948 [hep-ph]].
- [22] R. Foadi, M. T. Frandsen, T. A. Ryttov and F. Sannino, Phys. Rev. D **76**, 055005 (2007) [arXiv:0706.1696 [hep-ph]].
- [23] S. Catterall and F. Sannino, Phys. Rev. D **76**, 034504 (2007) [arXiv:0705.1664 [hep-lat]].
- [24] L. Del Debbio, M. T. Frandsen, H. Panagopoulos and F. Sannino, JHEP **0806**, 007 (2008) [arXiv:0802.0891 [hep-lat]].
- [25] S. Catterall, J. Giedt, F. Sannino and J. Schneible, JHEP **0811**, 009 (2008) [arXiv:0807.0792 [hep-lat]].
- [26] A. J. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, JHEP **0905**, 025 (2009) [arXiv:0812.1467 [hep-lat]].
- [27] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, Phys. Rev. D **80**, 074507 (2009) [arXiv:0907.3896 [hep-lat]].
- [28] A. J. Hietanen, K. Rummukainen and K. Tuominen, Phys. Rev. D **80**, 094504 (2009) [arXiv:0904.0864 [hep-lat]].
- [29] S. Catterall, J. Giedt, F. Sannino and J. Schneible, arXiv:0910.4387 [hep-lat].
- [30] A. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, Nucl. Phys. A **820**, 191C (2009).
- [31] F. Bursa, L. Del Debbio, L. Keegan, C. Pica and T. Pickup, arXiv:0910.2562 [hep-ph].
- [32] F. Bursa, L. Del Debbio, L. Keegan, C. Pica and T. Pickup, arXiv:0910.4535 [hep-ph].
- [33] M. Antola, M. Heikinheimo, F. Sannino and K. Tuominen, arXiv:0910.3681 [hep-ph].
- [34] E. Witten, Phys. Lett. B **117**, 324 (1982).
- [35] D. D. Dietrich and M. Jarvinen, Phys. Rev. D **79**, 057903 (2009) [arXiv:0901.3528 [hep-ph]].
- [36] R. Foadi, M. T. Frandsen and F. Sannino, Phys. Rev. D **80**, 037702 (2009) [arXiv:0812.3406 [hep-ph]].
- [37] S. B. Gudnason, C. Kouvaris and F. Sannino, Phys. Rev. D **73**, 115003 (2006) [arXiv:hep-ph/0603014].

- [38] S. B. Gudnason, C. Kouvaris and F. Sannino, Phys. Rev. D **74**, 095008 (2006) [arXiv:hep-ph/0608055].
- [39] K. Kainulainen, K. Tuominen and J. Virkajarvi, Phys. Rev. D **75**, 085003 (2007) [arXiv:hep-ph/0612247].
- [40] C. Kouvaris, Phys. Rev. D **77**, 023006 (2008) [arXiv:0708.2362 [astro-ph]].
- [41] M. Y. Khlopov and C. Kouvaris, Phys. Rev. D **77**, 065002 (2008) [arXiv:0710.2189 [astro-ph]].
- [42] M. Y. Khlopov and C. Kouvaris, Phys. Rev. D **78**, 065040 (2008) [arXiv:0806.1191 [astro-ph]].
- [43] C. Kouvaris, Phys. Rev. D **78**, 075024 (2008) [arXiv:0807.3124 [hep-ph]].
- [44] K. Kainulainen, K. Tuominen and J. Virkajarvi, arXiv:0912.2295 [astro-ph.CO].
- [45] A. R. Zerwekh, Eur. Phys. J. C **46**, 791 (2006) [arXiv:hep-ph/0512261].
- [46] R. Foadi and F. Sannino, Phys. Rev. D **78**, 037701 (2008) [arXiv:0801.0663 [hep-ph]].
- [47] R. Foadi, M. Jarvinen and F. Sannino, Phys. Rev. D **79**, 035010 (2009) [arXiv:0811.3719 [hep-ph]].
- [48] J. M. Cline, M. Jarvinen and F. Sannino, Phys. Rev. D **78**, 075027 (2008) [arXiv:0808.1512 [hep-ph]].
- [49] M. Jarvinen, T. A. Rytto and F. Sannino, Phys. Rev. D **79**, 095008 (2009) [arXiv:0903.3115 [hep-ph]].
- [50] M. Jarvinen, C. Kouvaris and F. Sannino, arXiv:0911.4096 [hep-ph].
- [51] A. Doff and A. A. Natale, arXiv:0912.1003 [hep-ph].
- [52] A. Doff, A. A. Natale and P. S. Rodrigues da Silva, Phys. Rev. D **80**, 055005 (2009) [arXiv:0905.2981 [hep-ph]].
- [53] A. Doff and A. A. Natale, Phys. Lett. B **677**, 301 (2009) [arXiv:0902.2379 [hep-ph]].
- [54] A. Doff and A. A. Natale, Int. J. Mod. Phys. E **16**, 2979 (2007).
- [55] A. Doff, A. A. Natale and P. S. Rodrigues da Silva, Phys. Rev. D **77**, 075012 (2008) [arXiv:0802.1898 [hep-ph]].
- [56] A. Doff and A. A. Natale, Phys. Lett. B **641**, 198 (2006) [arXiv:hep-ph/0510201].
- [57] A. R. Zerwekh, arXiv:0907.4690 [hep-ph].
- [58] E. H. Simmons, Nucl. Phys. B **312**, 253 (1989).
- [59] A. Kagan and S. Samuel, Phys. Lett. B **270**, 37 (1991).
- [60] C. D. Carone and E. H. Simmons, Nucl. Phys. B **397**, 591 (1993) [arXiv:hep-ph/9207273].
- [61] C. D. Carone, E. H. Simmons and Y. Su, Phys. Lett. B **344**, 287 (1995) [arXiv:hep-ph/9410242].
- [62] V. Hemmige and E. H. Simmons, Phys. Lett. B **518**, 72 (2001) [arXiv:hep-ph/0107117].
- [63] C. D. Carone, J. Erlich and J. A. Tan, Phys. Rev. D **75**, 075005 (2007) [arXiv:hep-ph/0612242].
- [64] R. S. Chivukula, A. G. Cohen and K. D. Lane, Nucl. Phys. B **343**, 554 (1990).

- [65] R. Sekhar Chivukula, N. D. Christensen, B. Coleppa and E. H. Simmons, Phys. Rev. D **80**, 035011 (2009) [arXiv:0906.5567 [hep-ph]].
- [66] S. B. Gudnason, T. A. Rytto and F. Sannino, Phys. Rev. D **76**, 015005 (2007) [arXiv:hep-ph/0612230].
- [67] F. Sannino and R. Zwicky, Phys. Rev. D **79**, 015016 (2009) [arXiv:0810.2686 [hep-ph]].
- [68] M. T. Frandsen, I. Masina and F. Sannino, arXiv:0905.1331 [hep-ph].
- [69] O. Antipin, M. Heikinheimo and K. Tuominen, JHEP **0910**, 018 (2009) [arXiv:0905.0622 [hep-ph]].
- [70] B. Holdom, JHEP **0608**, 076 (2006) [arXiv:hep-ph/0606146].
- [71] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007) [arXiv:0706.3718 [hep-ph]].
- [72] S. Catterall, JHEP **0506**, 027 (2005) [arXiv:hep-lat/0503036].
- [73] J. W. Elliott, J. Giedt and G. D. Moore, Phys. Rev. D **78**, 081701 (2008) [arXiv:0806.0013 [hep-lat]].
- [74] J. Giedt, R. Brower, S. Catterall, G. T. Fleming and P. Vranas, Phys. Rev. D **79**, 025015 (2009) [arXiv:0810.5746 [hep-lat]].
- [75] N. Dorey and S. P. Kumar, JHEP **0002**, 006 (2000) [arXiv:hep-th/0001103].
- [76] J. Wess and J. Bagger, *Princeton, USA: Univ. Pr. (1992) 259 p*