N=4 Extended MSSM

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Minimal Supersymmetric Conformal Technicolor: The Perturbative Regime

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Abstract: We investigate the perturbative regime of the Minimal Supersymmetric Conformal Technicolor and show that it allows for a stable vacuum correctly breaking the electroweak symmetry. We find that the particle spectrum is richer than the MSSM one since it features several new particles stemming out from the new $\mathcal{N}=4$ sector of the theory. The parameter space of the new theory is reduced imposing naturality of the couplings and soft supersymmetry breaking masses, perturbativity of the model at the EW scale as well as phenomenological constraints. Our preliminary results on the spectrum of the theory suggest that the Tevatron and the LHC can rule out a significant portion of the parameter space of this model.
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1. Introduction

The earliest models of technicolor [1, 2] have problems with the electroweak (EW) precision data [3, 4, 5, 6]. Technicolor models must be extended in order to give mass to the standard model (SM) fermions [7, 8]. In these extensions one, typically, expects potentially large flavor changing neutral current (FCNC) processes. Using near conformal dynamics alleviates the FCNC problem [9, 10, 11]. For over a decade it was hoped that such a near conformal dynamics could strongly reduce the tension with precision data even if one had a large number of technidoublets gauged under the electroweak (EW) symmetry. However very recently it has been shown [12, 13, 14] that to be phenomenologically viable the near conformal models should contain the most minimal number of flavors gauged under the EW symmetry.

The simplest models of this type which are shown to pass the precision tests, or have the smallest deviation from the precision data, while still providing a (near) conformal behavior were put forward recently in [15, 16, 17, 18, 19, 20, 22, 21]. Among these, the Minimal Walking Technicolor (MWT) features the most economical particle content.

In MWT the gauge group is $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and the field content of the technicolor sector is constituted by two flavors of techni-fermions and one techni-gluon all in the adjoint representation of $SU(2)_{TC}$. The model features also a pair of Dirac leptons, whose left-handed components are assembled in a weak doublet, necessary to cancel the Witten anomaly [23] arising when gauging the new technifermions with respect to the weak interactions.

The model requires, however, still additional ingredients in order to give mass to the standard model (SM) fermions. For example, one may postulate the existence of an Extended Technicolor (ETC) sector, traditionally featuring new gauge interactions linking the SM fermions to the techniquarks, which can generate mass terms for the SM fermions (as well as for the techni-mesons and -baryons) via a new dynamical mechanism. Interesting developments recently appeared in the literature [24, 25, 26, 27, 28, 29]. Nonperturbative chiral gauge theories dynamics is expected to play a relevant role in models of ETC since it allows, at least in principle, the self breaking of the gauge symmetry. Recent progress on the phase diagrams of these theories has appeared in [30]. Another alternative is to reintroduce new bosons (*bosonic technicolor*) [31, 32, 33, 34, 35] able to give masses to the SM fermions using standard Yukawa interactions. Eventhough these models are phenomenologically viable, they suffer from a SM-like fine tuning and are therefore unnatural. Supersymmetric technicolor has been considered [36, 37] as a way to naturalize bosonic technicolor. Another possibility would be to imagine the new scalars also to be composite of some new strong dynamics.

In [38] we made the observation that the techni-fermions and techni-gluons of the Minimal Walking Technicolor fit perfectly in an $\mathcal{N} = 4$ supermultiplet, provided that we also include three scalar superpartners. In fact the $SU(4)$ global symmetry of MWT is nothing but the well known $SU(4)_R$ R symmetry of the $\mathcal{N} = 4$ Super Yang Mills (4SYM) theory. This is the global quantum symmetry that does not commute with the supersymmetry transformations.

Supersymmetrizing MWT in this way leads to an approximate $\mathcal{N} = 4$ supersymmetry of the technicolor sector that is broken only by EW gauge and Yukawa interactions. Due to approximate $\mathcal{N} = 4$ invariance the beta function of the technicolor gauge coupling is zero at one loop, i.e. the associated technicolor model is approximately conformal. We called this model Minimal Supersymmetric Conformal Technicolor (MSCT).

This model can also be viewed as the first extension of the SM featuring maximal supersymmetry in four dimensions when neglecting the EW gauging of the R-symmetry. MSCT constitutes an interesting theoretical as well as phenomenological model to explore since it naturally allows to investigate different regimes according to how strongly coupled the maximally supersymmetric Yang-Mills theory is taken to be. In this phenomenological work we analyze the situation in which such a theory is weakly coupled at the EW scale.

To determine the spectrum of the theory we first analyze the ground state and afterwards compute the masses. We will show that at tree level all the states are massive except for the lightest CP-even and -odd Higgses that will acquire mass at one loop.

We find that the physical spectrum is phenomenologically viable and can be investigated for possible discovery at the Large Hadron Collider (LHC) and the Tevatron.

We have also analyzed the running of the gauge and Yukawa couplings and discovered that the price to pay for having a heavier spectrum is to substantially shrink the perturbative region, in energy, of the model associated with the Yukawa couplings.

The paper is organized as follows: We recall the model details in Section 2 and in 3 we impose the minimization conditions on the scalar potential and derive the additional conditions for the stability of a non-trivial vacuum. In Section 4 we present the mass spectrum of MSCT in the perturbative regime (pMSCT), including the one-loop correction for the physical massless states, and finally in Section 5 we study the viability of pMSCT based on these results.

2. The Model

The fermionic particle content of the MWT is given explicitly as

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L , \qquad U_R^a , \quad D_R^a, \quad a = 1,2,3 \ ; \quad L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L , \qquad N_R \ , \ E_R \ , \tag{2.1}$$

where U and D are techni-fermions in the adjoint representation of $SU(2)_{TC}$, whose left-handed components form a doublet under $SU(2)_L$, and the chiral leptons required to cancel the Witten anomaly are denoted by N and E. The following generic hypercharge assignment is free from gauge anomalies:

$$\begin{aligned} Y(Q_L) =& \frac{y}{2} \ , & Y(U_R, D_R) &= \left(\frac{y+1}{2}, \frac{y-1}{2}\right) \ , \\ Y(L_L) =& -3\frac{y}{2} \ , & Y(N_R, E_R) &= \left(\frac{-3y+1}{2}, \frac{-3y-1}{2}\right) \ . \end{aligned} \tag{2.2}$$

The global symmetry of the technicolor theory, per se, is $SU(4)$ which breaks explicitly to $SU(2)_L \times U(1)_Y$ by the natural choice of the electroweak embedding [15, 17]. The vacuum choice is stable against the SM quantum corrections [39].

To build MSCT we set $y = 1$ in Eqs. (2.2) so that $\bar{D}_R^a$ is a singlet under EW symmetry and can play the role of the techni-gaugino.[1] We define the $\mathcal{N} = 4$ supermultiplet in terms $\mathcal{N} = 1$ superfields, whose scalar and fermionic components are expressed by:

$$\left(\tilde{U}_L, \ U_L\right) \in \Phi_1, \quad \left(\tilde{D}_L, \ D_L\right) \in \Phi_2, \quad \left(\tilde{\bar{U}}_R, \ \bar{U}_R\right) \in \Phi_3, \quad \left(G, \ \bar{D}_R\right) \in V, \tag{2.3}$$

where we used a tilde to label the scalar superpartner of each fermion. We indicated with Φ_i, $i = 1,2,3$ the three chiral superfields of 4SYM and with V the vector superfield. The superfields associated with the remaining MWT fermions N and E are given by:

$$\left(\tilde{N}_L, \ N_L\right) \in \Lambda_1, \quad \left(\tilde{E}_L, \ E_L\right) \in \Lambda_2, \quad \left(\tilde{\bar{N}}_R, \ \bar{N}_R\right) \in N, \quad \left(\tilde{\bar{E}}_R, \ \bar{E}_R\right) \in E. \tag{2.4}$$

The quantum numbers of the superfields in Eqs.(2.3,2.4) and of those labeled by H and H', which contain each a Higgs scalar weak doublet, are given in Table 1.

[1] In Section 5 we briefly comment on the alternative choice of $y = -1$.

Superfield	$SU(2)_{TC}$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\Phi_{1,2}$	Adj	1	$\Box$	1/2
Φ_3	Adj	1	1	-1
V	Adj	1	1	0
$\Lambda_{1,2}$	1	1	$\Box$	-3/2
N	1	1	1	1
E	1	1	1	2
H	1	1	$\Box$	1/2
H'	1	1	$\Box$	-1/2

Table 1: MSCT $\mathcal{N}=1$ superfields

The renormalizable lepton and baryon number[2] conserving superpotential for the MSCT is

$$P = P_{MSSM} + P_{TC}, \tag{2.5}$$

where P_{MSSM} is the minimal supersymmetric standard model (MSSM) superpotential, and

$$P_{TC} = -\frac{g_{TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c + y_U\epsilon_{ij}\Phi_i^a H_j \Phi_3^a + y_N\epsilon_{ij}\Lambda_i H_j N + y_E\epsilon_{ij}\Lambda_i H'_j E + y_R E\Phi_3^a\Phi_3^a. \tag{2.6}$$

In the last equation $\Phi_i^a = Q_i^a$, $i=1,2$, with a the technicolor index. Gauge invariance alone does not ensure the Yukawa coupling of the first term to be equal to g_{TC}, however, setting it to this value amounts to the $\mathcal{N}=4$ limit. We have also investigated in Appendix C the independent running of a more general Yukawa coupling and shown that it tends towards the g_{TC} value at low energies. This result further justifies our choice to set it equal to the technicolor gauge coupling itself.

The Lagrangian of the MSCT is

$$\mathcal{L} = \mathcal{L}_{MSSM} + \mathcal{L}_{TC} \ , \tag{2.7}$$

where the supertechnicolor Lagrangian $\mathcal{L}_{TC}$, by following the notation of Wess and Bagger [40], can be written in the form:

$$\mathcal{L}_{TC} = \frac{1}{2}\mathrm{Tr}\left(W^\alpha W_\alpha|_{\theta\theta} + \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}|_{\bar\theta\bar\theta}\right) + \Phi_f^\dagger \exp\left(2g_X V_X\right)\Phi_f|_{\theta\theta\bar\theta\bar\theta} + \left(P_{TC}|_{\theta\theta} + h.c.\right), \tag{2.8}$$

In the last equation

$$W_\alpha = -\frac{1}{4g}\bar{D}\bar{D}\exp\left(-2gV\right)D_\alpha \exp\left(2gV\right), \quad V = V^a T_A^a, \quad \left(T_A^a\right)^{bc} = -if^{abc}, \tag{2.9}$$

and

$$\Phi_f = Q, \Phi_3, \Lambda, N, E; \quad X = TC, L, Y \ . \tag{2.10}$$

[2] We assume all the superfields in Table 1 to have both lepton and baryon numbers equal to zero.

The product $g_X V_X$ is assumed to include the gauge charge of the superfield on which it acts. The charge is Y for $U(1)_Y$, and 1 (0) for a multiplet (singlet) of a generic group $SU(N)$. The technicolor vector superfield V_{TC} is identified with V defined in Eq.(2.3). The remaining vector superfields are those already defined in the MSSM [41] while the superpotential P_{TC} is given in Eq.(2.6). We have written the Lagrangian $\mathcal{L}_{TC}$ in terms of elementary fields in Appendix A. The full MSSM Lagrangian $\mathcal{L}_{MSSM}$ can be found in [41] and references therein.

3. Vacua and Stability Conditions

The gauge group breaking of pMSCT (excluding the color group) follows the pattern $SU(2)_{TC} \times SU(2)_L \times U(1)_Y \to U(1)_{EM} \times U(1)_{TC}$, were the first $U(1)$ on the right determines the conservation of the ordinary electromagnetic (EM) charge. Even though the breaking involves also the TC group, besides the EW one, we will still refer to it simply as EWSB.

Because of the symmetry of the Lagrangian, we are free to choose the vacuum of the techni-Higgs scalar to be aligned in the third direction of the $SU(2)_{TC}$ gauge space. We define the vacuum expectation values (vev)s of the scalar fields neutral under the residual symmetry by

$$\left\langle \tilde{D}_L^3 \right\rangle = \frac{v_{TC}}{\sqrt{2}}, \quad \left\langle \tilde{H}_0 \right\rangle = s_\beta \frac{v_H}{\sqrt{2}}, \quad \left\langle \tilde{H}'_0 \right\rangle = c_\beta \frac{v_H}{\sqrt{2}}, \tag{3.1}$$

where all the vevs are chosen to be real, $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$.[3] The vacuum expectation values of the remaining fields is chosen to be zero so that $U(1)_{EM}$ is conserved. The scalar potential is obtained from the D, F, and $soft$ terms of the Lagrangian given in Appendix A, and by the corresponding MSSM scalar potential. The potential is:

$$\begin{aligned} V_{in} = {} & M_Q^2 |\tilde{D}_L^3|^2 + \left(m_u^2 + |\mu|^2\right) |\tilde{H}_0|^2 + \left(m_d^2 + |\mu|^2\right) |\tilde{H}'_0|^2 - \left(b \tilde{H}_0 \tilde{H}'_0 + c.c.\right) \\ & + \frac{1}{8}\left(g_L^2 + g_Y^2\right) \left(|\tilde{D}_L^3|^2 - |\tilde{H}'_0|^2 + |\tilde{H}_0|^2\right)^2 . \end{aligned} \tag{3.2}$$

The terms depending on the phase of the different fields are the b term and its conjugate. As in the MSSM invariance under the $U(1)_Y$ symmetry together with the fact that $\tilde{H}$ and $\tilde{H}'$ have opposite hypercharges allow to redefine their vevs and b parameter to be real. The quartic terms in this potential cancel when $|\tilde{D}_L^3|^2 = |\tilde{H}'_0|^2 - |\tilde{H}_0|^2$. To make the potential bound from below we impose the Hessian of V_{in} to be semi-definite positive along this D flat plane, which gives the conditions:

$$\left(m_u^2 + |\mu|^2 - M_Q^2\right)\left(m_d^2 + |\mu|^2 + M_Q^2\right) \geqslant b^2, \; 2|\mu|^2 + m_u^2 + m_d^2 \geqslant 0 \; . \tag{3.3}$$

At the minimum V_{in} satisfies the equations

$$\partial_{\tilde{D}_L^3} V_{in}|_{\phi=<\phi>} = 0, \quad \partial_{\tilde{H}_0} V_{in}|_{\phi=<\phi>} = 0, \quad \partial_{\tilde{H}'_0} V_{in}|_{\phi=<\phi>} = 0 \; . \tag{3.4}$$

[3] Notice that, consistently with the rest of the paper, we indicated the scalar component of each Higgs weak doublet superfield with a tilde.

These equations can be solved with respect to the vevs and used to express the soft SUSY breaking parameters according to:

$$M_Q^2 = -\frac{1}{8}\left(g_L^2+g_Y^2\right)\left(v_{TC}^2-c_{2\beta}v_H^2\right), \tag{3.5}$$

$$m_u^2 = -\frac{1}{8}\left(g_L^2+g_Y^2\right)\left(v_{TC}^2-c_{2\beta}v_H^2\right)-|\mu|^2+b\,t_\beta^{-1}, \tag{3.6}$$

$$m_d^2 = \frac{1}{8}\left(g_L^2+g_Y^2\right)\left(v_{TC}^2-c_{2\beta}v_H^2\right)-|\mu|^2+b\,t_\beta, \tag{3.7}$$

where $t_\beta = \tan\beta$. We impose the trivial vacuum to be unstable, both on the $|\tilde{H}_0'|$, $|\tilde{H}_0|$ plane and the $|\tilde{D}_L^3|$ direction so that both $v_{TC},\ v_H > 0$, by requiring the corresponding Hessian of V_{in} evaluated at the origin of the moduli space to have (at least) one negative eigenvalue. This translates to the conditions

$$M_Q^2 < 0, \qquad \left(m_u^2+|\mu|^2\right)\left(m_d^2+|\mu|^2\right) < b^2. \tag{3.8}$$

Finally, requiring the potential to be stable at the vacuum point determines the extra conditions

$$m_{h_1^0}^2 > 0\,, \qquad m_{A_1}^2 > 0\,, \tag{3.9}$$

where $m_{h_1^0}$, m_{A_0} are defined in Eqs.(4.16,4.17,4.4). Without loss of generality one can choose $0<\beta<\pi/2$. After plugging Eqs.(3.5,3.6,3.7) in (3.3,3.8,3.9) all these conditions are satisfied for

$$0 < b < \frac{t_{2\beta}}{16}\left(g_L^2+g_Y^2\right)\left(v_{TC}^2-c_{2\beta}v_H^2\right), \quad c_{2\beta}v_H^2 < v_{TC}^2, \quad 0<\beta<\frac{\pi}{4}\,, \tag{3.10}$$

or

$$b > 0, \qquad \pi/4 \leqslant \beta < \pi/2\,. \tag{3.11}$$

We will investigate in the following the region of parameter space defined by the conditions (3.11) since, as it will become clear in in Section 5, this is the one which is phenomenologically appealing.

4. Mass Spectrum

The superpotential in Eq.(2.6) and the soft SUSY-breaking terms in Eq.(A.13) conserve both lepton L and baryon B numbers. From Eqs.(A.5,A.6) one can see that the terms generated by the superpotential respect the same conservation laws. This is true also for the remaining contributions to the Lagrangian given in Eqs.(A.2,A.3,A.4). Moreover, after EWSB the MSCT Lagrangian is still invariant under the residual $U(1)_{EM}\times U(1)_{TC}$.[4] We can therefore write the gauge boson, fermion, and scalar (squared) mass matrices in block diagonal form in the basis of EM- and TC-charges and L and B numbers. The mass matrices of all the SM fermions and their superpartners assume the same form, in terms of the Higgs vevs, as of those obtained in the MSSM and can be found for example in [41]. The EW gauginos, Higgs scalar doublets and their superpartners mix with the $\mathcal{N}=4$ technicolor sector. Finally the fields $N_L, \bar{N}_R$, and their scalar superpartners will not mix at tree level with other SM fields with EM charge $Q_{EM}=1$ (where we defined $Q_{EM}=T_L^3+Y$) by constuction.

[4] We can neglect $SU(3)_C$, since none of the fields presented here carries color charge.

4.1 Gauge Bosons

After EWSB has occurred some techni-gluons and EW gauge bosons acquire mass. The corresponding sector of the MSCT Lagrangian can be written as a function of the mass eigenstates as:

$$-\mathcal{L}_{g\text{-}mass} = g_{TC}^2 v_{TC}^2 G_\mu^+ G^{-\mu} + \frac{g_L^2}{2}\left(v_{TC}^2 + v_H^2\right) W_\mu^+ W^{-\mu} + \frac{g_L^2 + g_Y^2}{4}\left(v_{TC}^2 + v_H^2\right) Z_\mu Z^\mu \quad (4.1)$$

where

$$G_\mu^\pm = \frac{1}{\sqrt{2}}\left(G_\mu^1 \mp i\, G_\mu^2\right)\ ,\ W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\, W_\mu^2\right)\ ,\ Z_\mu = c_w W_\mu^3 - s_w B\ ,\ t_w = \frac{g_Y}{g_L}. \quad (4.2)$$

The $\pm$ exponent of the techni-gluon refers to the $U(1)_{TC}$ charge, while the $\pm$ exponent on the EW gauge bosons refer to the usual EM charge. The remaining, massless states are the techni-photon and the EW photon:

$$G_\mu = G_\mu^3\ , \qquad A_\mu = s_w W_\mu^3 + c_w B\ . \quad (4.3)$$

The phenomenological constraints on a new $U(1)$ massless gauge boson were studied in [42] and found to be phenomenologically viable. The tree-level masses of G, W and Z can be read off from Eq.(4.1):

$$m_G = g_{TC} v_{TC}\ , \qquad m_W = \frac{g_L}{2}\sqrt{v_{TC}^2 + v_H^2}\ , \qquad m_Z = \frac{m_W}{c_w}\ . \quad (4.4)$$

From these masses and the eigenstates in Eq.(4.2) it is immediate to evaluate the EW oblique parameters at tree level by using the formulas in [43]: we find $S = T = U = 0$ at tree level.

4.2 Fermions

The fermion mass terms are:

$$\begin{aligned}-\mathcal{L}_{f\text{-}mass} = {} & \frac{1}{2}\left(\chi^0\right)^T \mathcal{M}_n \chi^0 + \left(\chi_{tc}^+\right)^T \mathcal{M}_{tc}\chi_{tc}^- + \left(\chi^+\right)^T \mathcal{M}_c \chi^- \\ & + m_{tc\text{-}c}\left(\chi_{tc}^{++}\right)^T \chi_{tc}^{--} + \bar{m}_{tc\text{-}c}\left(\chi_{tc}^{+-}\right)^T \chi_{tc}^{-+} + m_{cc}\chi^{++}\chi^{--} + c.c.\ ,\end{aligned} \quad (4.5)$$

where[5]

$$\begin{aligned}\chi^0 &= \left(H_2, H_1', \tilde{W}_3, \tilde{B}, D_L^3, \bar{D}_R^3\right)\ , & \chi_{tc}^\pm &= \left(\frac{D_L^1 \mp i\, D_L^2}{\sqrt{2}}, \frac{\bar{D}_R^1 \mp i\, \bar{D}_R^2}{\sqrt{2}}\right), \\ \chi^+ &= \left(H_1, \frac{\tilde{W}_1 - i\,\tilde{W}_2}{\sqrt{2}}, U_L^3, \bar{N}_R\right)\ , & \chi^- &= \left(H_2', \frac{\tilde{W}_1 + i\,\tilde{W}_2}{\sqrt{2}}, \bar{U}_L^3, N_L\right), \\ \chi_{tc}^{\pm+} &= \frac{U_L^1 \mp i\, U_L^2}{\sqrt{2}},\quad \chi_{tc}^{\pm-} = \frac{\bar{U}_R^1 \mp i\, \bar{U}_R^2}{\sqrt{2}}\ , & \chi^{++} &= \bar{E}_R,\ \chi^{--} = E_L,\end{aligned} \quad (4.6)$$

[5] Notice that a tc subscript here and in the following indicates that the leftmost superscript $\pm$ refers to the techni-charge under $U(1)_{TC}$

and, at tree-level,

$$\mathcal{M}_n = \frac{1}{2}\begin{pmatrix} 0 & -2\mu & is_\beta g_L v_H & -is_\beta g_Y v_H & 0 & 0 \\ -2\mu & 0 & -ic_\beta g_L v_H & ic_\beta g_Y v_H & 0 & 0 \\ is_\beta g_L v_H & -ic_\beta g_L v_H & 2M_{\tilde{W}} & 0 & ig_L v_{TC} & 0 \\ -is_\beta g_Y v_H & ic_\beta g_Y v_H & 0 & 2M_{\tilde{B}} & -ig_Y v_{TC} & 0 \\ 0 & 0 & ig_L v_{TC} & -ig_Y v_{TC} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2M_D \end{pmatrix} , \tag{4.7}$$

$$\mathcal{M}_{tc} = \begin{pmatrix} 0 & ig_{TC} v_{TC} \\ -ig_{TC} v_{TC} & M_D \end{pmatrix} , \tag{4.8}$$

$$\mathcal{M}_c = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\mu & -is_\beta g_L v_H & -y_U v_{TC} & 0 \\ -ic_\beta g_L v_H & \sqrt{2}M_{\tilde{W}} & 0 & 0 \\ 0 & -ig_L v_{TC} & y_U s_\beta v_H & 0 \\ 0 & 0 & 0 & y_N s_\beta v_H \end{pmatrix} , \tag{4.9}$$

$$m_{tc\text{-}c} = -ig_{TC} v_{TC} + \frac{y_U s_\beta v_H}{\sqrt{2}} , \qquad m_{cc} = \frac{y_E c_\beta v_H}{\sqrt{2}} . \tag{4.10}$$

In the previous equations with the labels $n, tc, c, tc\text{-}c, cc$, we referred to, respectively, neutralinos, techni-neutralinos, charginos, techni-charginos, and doubly-charged chargino. Furthermore the barred fields indicate Hermitian conjugation while a tilde indicates the fermion superpartner of the corresponding gauge boson. $M_{\tilde{W}}$ and $M_{\tilde{B}}$ correspond to the wino and the bino soft masses, respectively. It is important for the phenomenological bounds on MSCT to notice that at tree-level, from the last equation,

$$m_t = \frac{y_t}{y_E} t_\beta m_{cc} , \tag{4.11}$$

where the subscript t here refers to the top quark.

The squared masses are obtained diagonalizing $\mathcal{M}_p \mathcal{M}_p^\dagger$, $p = n, tc, c$. We note that $\bar{D}_R^3$ has become the gaugino of the residual $U(1)_{TC}$ with mass M_D. For illustration we provide the explicit form of the techni-neutralino masses obtained diagonalizing the seesaw-like matrix in Eq.(4.8):

$$m_{tc^{0,1}} = \sqrt{\frac{M_D^2}{4} + g_{TC}^2 v_{TC}^2} \mp \frac{M_D}{2} . \tag{4.12}$$

4.3 Scalars

4.3.1 Tree-Level

The complete potential is given by

$$V = V_{TC} + V_{MSSM}, \qquad V_{TC} = -\mathcal{L}_D - \mathcal{L}_F - \mathcal{L}_{soft} - \left(\frac{1}{2} M_D \bar{D}_R^a \bar{D}_R^a + c.c.\right) , \tag{4.13}$$

where V_{MSSM} can be found in [41], while $\mathcal{L}_D, \mathcal{L}_F$, and $\mathcal{L}_{soft}$, are given in Appendix A. As for the SM fermions also the scalar superpartners do not mix, at the tree-level, with the

$\mathcal{N}=4$ techni-scalars or heavy scalar leptons. Therefore their mass spectrum assumes the same form as in the MSSM. The Higgs scalar fields, $\tilde{H}$ and $\tilde{H}'$, on the other hand, mix with the techni-scalars. The squared mass matrices of the CP-even and -odd EM neutral Higgs scalars are given by, respectively,

$$\mathcal{M}_h^2 = \frac{1}{4}\begin{pmatrix} \left(g_L^2+g_Y^2\right)s_\beta^2 v_H^2 + 4bt_\beta^{-1} & -c_\beta\left(g_L^2+g_Y^2\right)s_\beta v_H^2 - 4b & \left(g_L^2+g_Y^2\right)s_\beta v_H v_{TC} \\ -c_\beta\left(g_L^2+g_Y^2\right)s_\beta v_H^2 - 4b & c_\beta^2\left(g_L^2+g_Y^2\right)v_H^2 + 4bt_\beta & -c_\beta\left(g_L^2+g_Y^2\right)v_H v_{TC} \\ \left(g_L^2+g_Y^2\right)s_\beta v_H v_{TC} & -c_\beta\left(g_L^2+g_Y^2\right)v_H v_{TC} & \left(g_L^2+g_Y^2\right)v_{TC}^2 \end{pmatrix},$$

$$\left(\mathcal{M}_h^2\right)_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i^h\partial\phi_j^h}\right|_{\phi=\langle\phi\rangle}, \quad \phi^h = \Re\left(\tilde{H}_2, \tilde{H}_1', \tilde{D}_L^3\right), \tag{4.14}$$

and

$$\mathcal{M}_A^2 = \begin{pmatrix} bt_\beta^{-1} & b & 0 \\ b & bt_\beta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \left(\mathcal{M}_A^2\right)_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i^A\partial\phi_j^A}\right|_{\phi=\langle\phi\rangle}, \quad \phi^A = \Im\left(\tilde{H}_2, \tilde{H}_1', \tilde{D}_L^3\right) \tag{4.15}$$

From Eqs.(4.14,4.15) the squared masses of the CP-even and -odd Higgs scalars are

$$m_{h_0^0}^2 = m_{A_0}^2 = 0,\ m_{h_{1,2}^0}^2 = \frac{1}{2}\left(m_{A_1}^2 + m_Z^2 \mp \sqrt{\left(m_{A_1}^2 - m_Z^2\right)^2 + 4m_{A_1}^2 m_B^2 s_{2\beta}^2}\right),\ m_{A_1}^2 = \frac{2b}{s_{2\beta}}, \tag{4.16}$$

where we have defined the quantity:

$$m_B^2 = \frac{g_Y^2 + g_L^2}{4} v_H^2 \tag{4.17}$$

which does not correspond to any particle. In the limit $v_{TC}=0$, however, $m_B = m_Z$ and one recovers the MSSM results for the masses of the CP-even Higgs scalars.

The massless eigenstates h_0^0, π_Z (the longitudinal degree of freedom of the Z boson), and A_0, are expressed by

$$h_0^0 = N_h\left(s_\beta v_{TC}, c_\beta v_{TC}, c_{2\beta} v_H\right)\cdot\phi^h, \qquad N_h^{-2} = v_{TC}^2 + c_{2\beta}^2 v_H^2, \tag{4.18}$$

$$\pi_Z = N_Z\left(s_\beta v_H, -c_\beta v_H, v_{TC}\right)\cdot\phi^A, \qquad N_Z^{-2} = v_{TC}^2 + v_H^2, \tag{4.19}$$

$$A_0 = N_A\left(s_\beta v_{TC}, -c_\beta v_{TC}, -v_H\right)\cdot\phi^A, \qquad N_A^{-2} = v_{TC}^2 + v_H^2, \tag{4.20}$$

with $\phi^{h,A}$ defined respectively in Eqs. (4.14,4.15). The masslessness of h_0^0 and A_0 will not survive at the one-loop level.

The remaining scalar squared mass matrices are given in Appendix B. By using these results and those given in Eqs.(4.4,4.7,4.14,4.15), and taking into account the multiplicities of each mass matrix, we can calculate the supertrace of the tree level squared mass matrices, defined by

$$S\mathrm{Tr}\mathcal{M}^2 = \sum_j (-1)^{2j}(2j+1)\,\mathrm{Tr}\mathcal{M}_j^2, \qquad j = 0, \frac{1}{2}, 1, \tag{4.21}$$

where $\mathcal{M}_j$ are the complete squared mass matrices of scalars, femions, and gauge bosons. We obtain:

$$S\mathrm{Tr}\mathcal{M}^2 = 2\left(-M_{\tilde{B}}^2 - 3M_{\tilde{W}}^2 - 3M_D^2 + 2m_d^2 + 2m_u^2 + 2M_L^2 + M_N^2 + M_E^2 + 6M_Q^2 + 3M_U^2\right) , \tag{4.22}$$

where the numerical factors in front of the SUSY breaking squared mass parameters reflect the degrees of freedom of the corresponding fields. The equation above shows that the SUSY invariant contributions to the squared mass matrices cancel out, as they should.

4.3.2 One-Loop

We calculate the one-loop contributions to the CP-even and -odd neutral (both under $U(1)_{EM}$ and $U(1)_{TC}$) scalars. We expect the lightest eigenstates, h_0^0 and A_0, that are accidentally massless at tree level, to receive non-zero contributions to their masses from the one-loop effective potential. The one loop potential is [44]:

$$\Delta V_1 = \frac{1}{64\pi^2} S\mathrm{Tr}\left[\mathcal{M}^4(\phi)\left(\ln\frac{\mathcal{M}^2(\phi)}{\mu_r^2} - \frac{3}{2}\right) + 2\mathcal{M}^2(\phi)\,\mu_r^2\right] , \tag{4.23}$$

where $\mathcal{M}^2(\phi)$ are field-dependent mass matrices not evaluated at their vevs, defined by:

$$\left(\mathcal{M}^2(\phi)\right)_{ij} = \frac{\partial^2 V}{\partial\phi_i\partial\phi_j} , \tag{4.24}$$

and μ_r is the renormalization scale. The last term in Eq.(4.23) renormalizes the one-loop contributions to the scalar masses to zero when $\mu_r^2 = \mathcal{M}^2(\langle\phi\rangle)$.[6] The last term in Eq.(4.23) gives a very small contribution to ΔV_1 since only the SUSY breaking terms (generally small to avoid a large fine tuning) do not cancel in the supertrace, and therefore we neglect it. To minimize the correction from higher order contributions to V, we take μ_r equal to the mass of the heaviest particle among the eigenstates presented in Sections 4.1, 4.2, and the last subsection.

The one-loop mass matrix correction, $\Delta\mathcal{M}_a^2$, for any real field a with n components can be extracted from ΔV_1 by numerically evaluating the derivatives of the mass eigenvalues with respect to the fields evaluated on the vevs [45], where

$$(\Delta\mathcal{M}_a^2)_{ij} = \left.\frac{\partial^2\Delta V_1(a)}{\partial a_i\partial a_j}\right|_{a=\langle a\rangle} + \Delta M_{ij}^2 , \tag{4.25}$$

$$\left.\frac{\partial^2\Delta V_1(a)}{\partial a_i\partial a_j}\right|_{a=\langle a\rangle} = \sum_k \frac{1}{32\pi^2}\frac{\partial m_k^2}{\partial a_i}\frac{\partial m_k^2}{\partial a_j}\ln\left.\frac{m_k^2}{\mu_r^2}\right|_{a=\langle a\rangle} + \sum_k \frac{1}{32\pi^2}m_k^2\frac{\partial^2 m_k^2}{\partial a_i\partial a_j}\left.\left(\ln\frac{m_k^2}{\mu_r^2} - 1\right)\right|_{a=\langle a\rangle} ,$$

$$\begin{aligned}\Delta M_{ij}^2 &= -\frac{\delta_{ij}}{\phi_i^h}\left.\frac{\partial\Delta V_1(\phi^h)}{\partial\phi_i^h}\right|_{\phi^h=\langle\phi^h\rangle} \\ &= -\sum_k \frac{1}{32\pi^2}m_k^2\frac{\delta_{ij}}{\phi_i^h}\frac{\partial m_k^2}{\partial\phi_i^h}\left.\left(\ln\frac{m_k^2}{\mu_r^2} - 1\right)\right|_{\phi^h=\langle\phi^h\rangle} .\end{aligned} \tag{4.26}$$

[6] In case there is more than one field, one should use different scales μ_r for each contribution to the supertrace to get an exactly vanishing one-loop correction to the mass.

The second term in Eq.(4.25) takes into account the shift in the minimization conditions (see [45]), and m_k^2 is the set of mass eigenvalues of the field dependent mass matrix $\mathcal{M}^2(\phi)$. Notice that ΔM_{ij}^2 has to be included in the expression of $(\Delta\mathcal{M}_a^2)_{ij}$ only when a_i are the CP-even or -odd Higgses, since ΔM_{ij}^2 gives the shift of the soft mass parameters of the scalar fields that develop a non-zero vev. The Goldstone bosons do not contribute to $\Delta\mathcal{M}_a^2$.

In this first estimate we compute $\Delta\mathcal{M}^2$ for the neutral Higgses neglecting the contributions from top-stop mass splitting. We consider the fields given in Table 1, plus the W and B bosons and their superpartners. In this way the supertrace receives contributions only from the soft mass terms. We therefore consider our results for the one-loop masses of the CP-even and -odd Higgses an estimate of the values that can be obtained when taking into account the full MSCT spectrum.

It is seen that except for the ordinary EM neutral Goldstone boson which can be interpreted as the longitudinal component of the Z boson no other neutral scalar is massless. The mass of the lightest physical states, h_0^0 and A_0, has a strong dependence on the size of the Yukawa couplings in the superpotential, Eq.(2.6). A random scan of the parameter space, with the constraint that the SUSY breaking scale, given in Eqs.(A.13,3.2), is around the TeV region and with $\pi/4 < \beta < \pi/2$, gives:[7]:

$$\begin{aligned} m_{h_0^0} &= 10.6 \pm 5.5 \text{ GeV}\,, \qquad g_{TC} = y_U = y_N = y_E = y_R = 1\,, \\ m_{h_0^0} &= 125 \pm 54 \text{ GeV}\,, \qquad g_{TC} = y_U = y_N = y_E = y_R = \pi\,. \end{aligned} \tag{4.27}$$

From the scan we read off the central value for the mass of the lightest Higgs and the associated standard deviation. The latter represents the spread in the distributions of the values of the parameters. We have also tried to reach a larger value of the masses by optimizing the search around the maximum value of the initial sample of parameters and obtain in this case $m_{h_0^0}^{max} = 30.5$ GeV and $m_{h_0^0}^{max} = 276$ GeV for the same choice of Yukawas above.

The mass of A_0 for the parameter values that maximize $m_{h_0^0}$ is $m_{A_0} = 8\,(27)$ GeV for $g_{TC} = ... = 1\,(\pi)$. It is interesting to notice also that $m_{A_0} = 0$ for $a_{TC} = 0$: consequentially in the following we take the soft parameter a_{TC} to be rather large (though still within the TeV region). In the following section we impose the experimental bounds on the mass spectrum to determine its phenomenological viability and use the renormalization group equations to determine the perturbative range of our results.

5. Phenomenological Viability

The lower bounds on the mass of the lightest neutralino and chargino are [47]:

$$m_{\chi_0^0} > 46 \text{ GeV}\,,\ m_{\chi_0^\pm} > 94 \text{ GeV}\,. \tag{5.1}$$

These limits refer to the MSSM, but are rather general, since they are extracted mostly from the Z decay to neutralino-antineutralino pair the former, and from photo-production

[7] We have checked that the potential is actually at a minimum and have considered two sample values of the Yukawa couplings.

of a chargino-antichargino pair at LEPII the latter. We can therefore assume these limits to hold also for the MSCT. Because of their generality and independence from the coupling strength (as long as it is not negligible), we use the lower bound on the chargino mass also for the mass of the doubly-charged chargino E.

The presence of the term proportional to y_R in the superpotential, Eq.(2.6) allows it to decay into singly charged ordinary particles. Therefore it escapes cosmological constraints on charged stable particles. The techni-gaugino $\bar{D}_R^3$ is an EW singlet fermion and therefore is a right-handed neutrino, which can be very light. Because of this, and the fact that the mass of the lightest techineutralino, Eq.(4.12), is a monotonically decreasing function of the mass of $\bar{D}_R^3$, we assume M_D to be small with respect to the $g_{TC}v_{TC}$ energy scale.

Other useful limits on the parameters are obtained by using the fact that the smallest eigenvalue of a semi-positive definite square matrix is smaller or equal to any eigenvalue of the principal submatrices. From the absolute square of the neutralino mass matrix, Eq.(4.7), we get

$$\begin{aligned} M_{\tilde{B}}^2 &> (46\ \text{GeV})^2 - \frac{g_Y^2}{4}\left(v_H^2 + v_{TC}^2\right) = (13.5\ \text{GeV})^2\ ,\ \mu > 46\ \text{GeV}\,, \\ v_{\text{TC}} &> 2\frac{46\ \text{GeV}}{\sqrt{g_L^2+g_Y^2}} = 124\ \text{GeV}\,,\ v_H < 213\ \text{GeV}\,, \end{aligned} \tag{5.2}$$

where we used, from Eq.(4.4),

$$\sqrt{v_H^2 + v_{TC}^2} = 246\ \text{GeV}. \tag{5.3}$$

From the absolute square of the chargino mass matrix and the doubly-charged chargino mass, Eqs.(4.9,4.10), we get

$$M_{\tilde{W}}^2 > (94\ \text{GeV})^2 - \frac{1}{2}c_\beta^2 g_L^2 v_H^2 = (63.5\ \text{GeV})^2\ ,\ \frac{y_E c_\beta v_H}{\sqrt{2}} > 94\ \text{GeV}\,. \tag{5.4}$$

From Eq.(4.11), with $m_t = 173$ GeV, and the bounds (5.2,5.4), it follows that

$$y_t > \frac{173}{213}\sqrt{\frac{1}{\frac{1}{2} - \frac{94^2}{y_E^2 213^2}}}\,. \tag{5.5}$$

This last bound is plotted in Figure 1, where the shaded area shows the values of y_t and y_E excluded by the experiment: it is evident from the plot in Figure 1 that either y_t or y_E is constrained to be larger than about 1.3.[8]

One of our goals is to determine if the perturbative MSCT mass spectrum is phenomenologically viable. The other is to determine the range of energy where the model remains perturbative. We anticipate that the Yukawa couplings are the ones driving the model towards a strongly coupled regime.

[8] Had we chosen the hypercharge parameter y=-1 rather than 1, the constraints in Eqs.(5.2,5.4,5.5) would be the same with y_E and y_N interchanged. although a more detailed study would be necessary, we expect that the choice $y = -1$ produces the same general results and conclusions that we present in this paper for y=1.

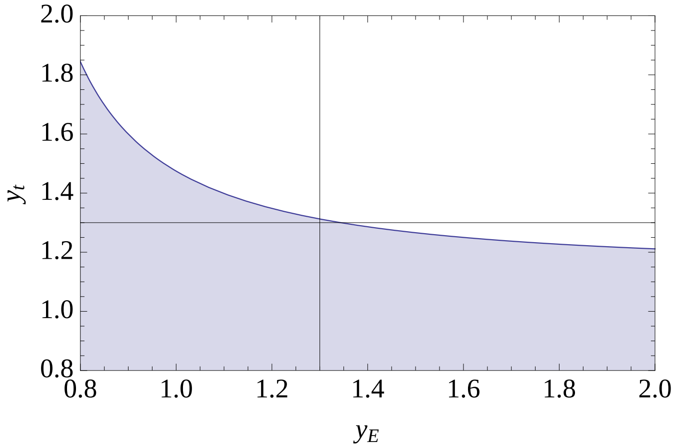

Figure 1: Shaded area shows experimentally excluded values of the Yukawa couplings y_t and y_E.

By using the couplings renormalization group equations (RGE) given in Appendix C we find that a phenomenologically reasonable compromise for the values of y_t and y_E allowing perturbativity at the energy scale of a few TeVs, for the elementary processes occurring at the LHC, is respectively, 1.65 and 2.2. The value of the lightest chargino mass is controlled mainly by y_U, which we take equal to y_t, while $y_{TC} = y_N = 1.1$[9], since they are less constrained to be large. In Figure 2 are plotted $y_{TC}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale M: the couplings are normalized for $M = m_Z$ to $y_t = y_U = 1.65$, $y_E = 2.2$, $y_N = y_{TC} = 1.1$. Having shown that the Yukawa couplings enter the nonperturbative region very early in energy scale we cannot discuss perturbative unification of the couplings in this model since we should take into account the nonperturbative effects of this sector on the running of the gauge couplings.

By maximizing the minimum eigenvalue of $\mathcal{M}_n \cdot \mathcal{M}_n^\dagger$ on the parameter space allowed by the bounds (5.2,5.4,5.5) with $y_t = 1.65$ and $y_E = 2.2$, we obtain

$$m_{\chi_0^0}^{max} = 46 \text{ GeV}\,, \; m_{\chi_0^\pm} = 60 \text{ GeV}, \tag{5.6}$$

where $m_{\chi_0^\pm}$ is calculated at the point in parameter space that maximizes $m_{\chi_0^0}$. With these chosen values of the Yukawa couplings at the EW scale, a Landau pole for the Yukawas arises around 2.3 TeV and the chargino mass is light compared to the experimental bounds Eqs.(5.2,5.4). A scan of the allowed parameter space, with the same values of the Yukawa couplings shows that it is not possible to satisfy both constraints in Eq.(5.1) simultaneously. However, by further lowering the Landau pole scale we can achieve a phenomenologically viable spectrum:

$$\begin{aligned} &m_{\chi_0^0}^{max} = 47 \text{ GeV}\,, \; m_{\chi_0^\pm}^{max} = 96 \text{ GeV}\,, \; m_{h_0^0} = 95 \text{ GeV}\,, \; m_{A_0} = 32 \text{ GeV}\,, \\ &M_{pole} = 400 \text{ GeV}\,. \end{aligned} \tag{5.7}$$

[9]In calculating the RGE we assumed a generic coupling y_{TC} in place of g_{TC} in Eq.(2.6). Amusingly y_{TC} quickly converges , when running towards the infrared, to the assumed value of g_{TC} in agreement with the findings in [46]. Therefore it is a good approximation to assume the technicolor sector to be $\mathcal{N} = 4$ SUSY by taking $y_{TC} = g_{TC}$ at low energies, no matter what the value of y_{TC} is at higher energies.

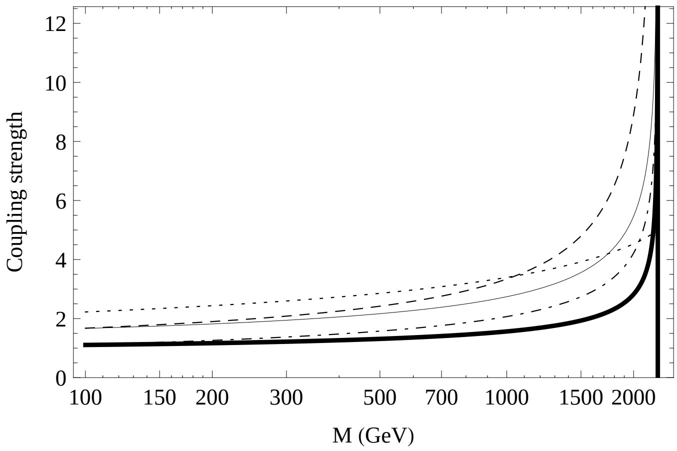

Figure 2: Plot of $y_{TC}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale M: the couplings are normalized for $M = m_Z$ to $y_t = y_U = 1.65$, $y_E = 2.2$, $y_N = y_{TC} = 1.2$.

The results correspond to having chosen at the EW scale the values $y_N = 1.8$, $y_t = g_{TC} = y_U = 2.3$, $y_E = 2.4$.[10]

6. Conclusions and Outlook

We have investigated the perturbative regime of MSCT and shown that it allows for a stable vacuum correctly breaking the EW symmetry, and also that the particle spectrum is richer than the MSSM one. This occurs since the model features several new particles stemming out from the $\mathcal{N} = 4$ sector of the theory.

The MSCT, in the perturbative regime, satisfies the current experimental constraints for the mass spectrum of the model which in turn requires Yukawa couplings larger than the ones in the (MS)SM. By running the renormalization group equation we observed that the Yukawa sector becomes nonperturbative close to the TeV scale.

We have also initiated a preliminary study of the parameter space of the model and reduced it by imposing naturality of the couplings and masses, one loop vacuum stability, perturbativity of the model at the EW scale as well as phenomenological constraints.

Our preliminary results on the spectrum of the MSCT model indicate that the Tevatron and the LHC can rule out a significant portion of the parameter space of this model. Part of the particle spectrum is very similar to the one of the MSSM, however, MSCT also features several new light states, with respect to the EW scale, such as doubly charged particles. Therefore an interesting experimental signature would be the discovery of a doubly charged particle together with a very light chargino and/or neutralino. Finally, since the Yukawa couplings are larger than the SM ones we expect 100% increase of several production cross sections such as the Higgs scalar (h_0^0) one via the gluon-gluon fusion process.

[10] Viable masses for χ_0^0 and $\chi_0^\pm$ can be obtained for a Landau pole energy arising at as high an energy as 1 TeV.

Another characteristic of the model relevant for collider experiments is that due to the presence of one extra Higgs-type particle sector, coming from the supertechnicolor sector, the spectrum features scalars and pseudoscalars lighter than in the MSSM case. These states will also yield interesting signatures at collider experiments. We plan to explore, in the future, in detail the processes relevant at colliders experiments, as well as the dark matter phenomenology which will be substantially different than in the MSSM.

Since our model features, at the EW scale, a new $\mathcal{N} = 4$ supertechnicolor sector, collider experiments have the possibility to explore directly string theory. This is so since the new scalars coming from this sector can be directly identified with the *extra* six space coordinates of ten dimensional supergravity. This link is even more clear when considering the present supertechnicolor sector in the nonperturbative regime which can be investigated using AdS/CFT techniques and will be investigated elsewhere.

Acknowledgments

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A. MSCT Lagrangian

The Lagrangian of a supersymmetric theory can, in general, be defined by

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{g-Yuk} + \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_{P-Yuk} + \mathcal{L}_{soft}, \tag{A.1}$$

where the labels refer to the kinetic terms, the Yukawa ones given by gauge and superpotential interactions, the D and F scalar interaction terms, and the soft SUSY breaking ones. All these terms can be expressed in function of the elementary fields of the theory with the help of the following equations:

$$\begin{aligned}
\mathcal{L}_{kin} &= -\frac{1}{4} F_j^{\mu\nu a} F^a_{j\mu\nu} - i\bar{\lambda}^a_j \bar{\sigma}^\mu D_\mu \lambda^a_j - D^\mu \phi_i^{a\dagger} D_\mu \phi^a_i - i\bar{\chi}^a_i \bar{\sigma}^\mu D_\mu \chi^a_i \ , &\text{(A.2)}\\
\mathcal{L}_{g-Yuk} &= \sum_j i\sqrt{2} g_j \left(\phi_i^\dagger T^a_j \chi_i \lambda^a_j - \bar{\lambda}^a_j \bar{\chi}_i T^a_j \phi_i \right) \ , &\text{(A.3)}\\
\mathcal{L}_D &= -\frac{1}{2} \sum_j g_j^2 \left(\phi_i^\dagger T^a_j \phi_i \right)^2 \ , &\text{(A.4)}\\
\mathcal{L}_F &= -\left| \frac{\partial P}{\partial \phi^a_i} \right|^2 \ , &\text{(A.5)}\\
\mathcal{L}_{P-Yuk} &= -\frac{1}{2} \left[\frac{\partial^2 P}{\partial \phi^a_i \partial \phi^b_l} \chi^a_i \chi^b_l + h.c. \right], &\text{(A.6)}
\end{aligned}$$

where i, l run over all the scalar field labels, while j runs over all the gauge group labels, and a, b are the corresponding gauge group indices. Furthermore, we normalize the generators in the usual way, by taking the index $T(F) = \frac{1}{2}$, where

$$\mathrm{Tr} T^a_R T^b_R = T(R) \delta^{ab},$$

with R here referring to the representation (F=fundamental). The SUSY breaking soft terms, moreover, are obtained by re-writing the superpotential in function of the scalar fields alone, and by adding to it its Hermitian conjugate and the mass terms for the gauginos and the scalar fields.

We refer to [41] and references therein for the explicit form of $\mathcal{L}_{MSSM}$ in terms of the elementary fields of the MSSM, and focus here only on $\mathcal{L}_{TC}$. The kinetic terms are trivial and therefore we do not write them here. The gauge Yukawa terms are given by

$$\begin{aligned}
\mathcal{L}_{g-Yuk} = & \sqrt{2} g_{TC} \left(\bar{\tilde{U}}^b_L U^c_L \bar{D}^a_R - D^a_R \bar{U}^b_L \tilde{U}^c_L + \bar{\tilde{D}}^b_L D^c_L \bar{D}^a_R - D^a_R \bar{D}^b_L \tilde{D}^c_L + \tilde{U}^b_R \bar{U}^c_R \bar{D}^a_R - D^a_R U^b_R \bar{\tilde{U}}^c_R \right) \epsilon^{abc} \\
& + i \frac{g_L}{\sqrt{2}} \left(\bar{\tilde{Q}}^i_L Q^j_L \tilde{W}^k - \bar{\tilde{W}}^k \bar{Q}^i_L \tilde{Q}^j_L + \bar{\tilde{L}}^i_L L^j_L \tilde{W}^k - \bar{\tilde{W}}^k \bar{L}^i_L \tilde{L}^j_L \right) \sigma^k_{ij} \\
& + i\sqrt{2} g_Y \sum_p Y_p \left(\bar{\tilde{\chi}}_p \chi_p \tilde{B} - \bar{\tilde{B}} \bar{\chi}_p \tilde{\chi}_p \right), \ \chi_p = U^a_L, D^a_L, \bar{U}^a_R, N_L, E_L, \bar{N}_R, \bar{E}_R \ ,
\end{aligned} \tag{A.7}$$

where $\tilde{W}^k$ and $\tilde{B}$ are respectively the wino and the bino, σ^k the Pauli matrices, $i, j = 1, 2$; $k, a, b, c = 1, 2, 3$; and the hypercharge Y_p is given for each field χ_p in Table 1.

The D terms are given by

$$\mathcal{L}_D = -\frac{1}{2}\left(g_{TC}^2 D_{TC}^a D_{TC}^a + g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y\right) + \frac{1}{2}\left(g_L^2 D_L^k D_L^k + g_Y^2 D_Y D_Y\right)_{MSSM}, \tag{A.8}$$

where

$$\begin{aligned} D_{TC}^a &= -i\epsilon^{abc}\left(\bar{\tilde{U}}_L^b \tilde{U}_L^c + \bar{\tilde{D}}_L^b \tilde{D}_L^c + \tilde{U}_R^b \bar{\tilde{U}}_R^c\right), \ D_L^k = \frac{\sigma_{ij}^k}{2}\left(\bar{\tilde{Q}}_L^{i\,a}\tilde{Q}_L^{j\,a} + \bar{\tilde{L}}_L^i \tilde{L}_L^j\right) + D_{L,MSSM}^k \\ D_Y &= \sum_p Y_p \bar{\tilde{\chi}}_p \tilde{\chi}_p + D_{Y,MSSM}. \end{aligned} \tag{A.9}$$

In these equations the $D_{L,MSSM}^k$ and $D_{Y,MSSM}$ auxiliary fields are assumed to be expressed in function of the MSSM elementary fields [41]. The rest of the scalar interaction terms[11] is given by

$$\begin{aligned} \mathcal{L}_F &= -g_{TC}^2\Big[\left(\tilde{U}_L^b\bar{\tilde{U}}_L^b + \tilde{D}_L^b\bar{\tilde{D}}_L^b + \bar{\tilde{U}}_R^b\tilde{U}_R^b\right)^2 - \left(\tilde{U}_L^b\bar{\tilde{U}}_L^c + \tilde{D}_L^b\bar{\tilde{D}}_L^c + \bar{\tilde{U}}_R^b\tilde{U}_R^c\right)\left(\bar{\tilde{U}}_L^b\tilde{U}_L^c + \bar{\tilde{D}}_L^b\tilde{D}_L^c\right. \\ &+ \left.\tilde{U}_R^b\bar{\tilde{U}}_R^c\right)\Big] - y_U^2\Big[\left(\tilde{H}_1\tilde{D}_L^a - \tilde{H}_2\tilde{U}_L^a\right)\left(\bar{\tilde{H}}_1\bar{\tilde{D}}_L^a - \bar{\tilde{H}}_2\bar{\tilde{U}}_L^a\right) + \tilde{U}_R^a\bar{\tilde{U}}_R^a\left(\tilde{H}_1\bar{\tilde{H}}_1 + \tilde{H}_2\bar{\tilde{H}}_2\right) \\ &+ \tilde{U}_R^a\bar{\tilde{U}}_R^b\left(\bar{\tilde{U}}_L^a\tilde{U}_L^b + \bar{\tilde{D}}_L^a\tilde{D}_L^b\right)\Big] - y_N^2\Big[\left(\bar{\tilde{N}}_L\bar{\tilde{H}}_2 - \bar{\tilde{E}}_L\bar{\tilde{H}}_1\right)\left(\tilde{N}_L\tilde{H}_2 - \tilde{E}_L\tilde{H}_1\right) \\ &+ \tilde{N}_R\bar{\tilde{N}}_R\left(\tilde{H}_1\bar{\tilde{H}}_1 + \tilde{H}_2\bar{\tilde{H}}_2 + \tilde{N}_L\bar{\tilde{N}}_L + \tilde{E}_L\bar{\tilde{E}}_L\right)\Big] - y_E^2\Big[\left(\bar{\tilde{N}}_L\bar{\tilde{H}}_2' - \bar{\tilde{E}}_L\bar{\tilde{H}}_1'\right)\left(\tilde{N}_L\tilde{H}_2' - \tilde{E}_L\tilde{H}_1'\right) \\ &+ \tilde{E}_R\bar{\tilde{E}}_R\left(\tilde{H}_1'\bar{\tilde{H}}_1' + \tilde{H}_2'\bar{\tilde{H}}_2' + \tilde{N}_L\bar{\tilde{N}}_L + \tilde{E}_L\bar{\tilde{E}}_L\right)\Big] - y_R^2\left(\tilde{U}_R^a\tilde{U}_R^a\bar{\tilde{U}}_R^b\bar{\tilde{U}}_R^b + 4\bar{\tilde{U}}_R^a\tilde{U}_R^a\bar{\tilde{E}}_R\tilde{E}_R\right) \\ &+ \Big\{\sqrt{2}y_U g_{TC}\epsilon^{abc}\Big[\tilde{U}_L^b\tilde{D}_L^c\left(\bar{\tilde{H}}_1\bar{\tilde{D}}_L^a - \bar{\tilde{H}}_2\bar{\tilde{U}}_L^a\right) + \tilde{U}_R^a\bar{\tilde{U}}_R^b\left(\tilde{U}_L^c\bar{\tilde{H}}_1 + \tilde{D}_L^c\bar{\tilde{H}}_2\right)\Big] \\ &- y_U y_N\tilde{U}_R^a\bar{\tilde{N}}_R\left(\bar{\tilde{U}}_L^a\tilde{N}_L + \bar{\tilde{D}}_L^a\tilde{E}_L\right) - y_N y_E\,\tilde{N}_R\bar{\tilde{E}}_R\left(\bar{\tilde{H}}_1\tilde{H}_1' + \bar{\tilde{H}}_2\tilde{H}_2'\right) \\ &+ y_R\bar{\tilde{U}}_R^a\Big[2\sqrt{2}g_{TC}\epsilon^{abc}\bar{\tilde{U}}_L^b\bar{\tilde{D}}_L^c\bar{\tilde{E}}_R + 2y_U\bar{\tilde{E}}_R\left(\bar{\tilde{D}}_L^a\bar{\tilde{H}}_1 - \bar{\tilde{U}}_L^a\bar{\tilde{H}}_2\right) + y_E\bar{\tilde{U}}_R^a\left(\bar{\tilde{E}}_L\bar{\tilde{H}}_1' - \bar{\tilde{N}}_L\bar{\tilde{H}}_2'\right)\Big] \\ &+ h.c.\Big\} + \mathcal{L}_{mix}, \end{aligned} \tag{A.10}$$

with $\mathcal{L}_{mix}$ defined in function of the F auxiliary fields associated with the MSSM two Higgs super-doublets:

$$\begin{aligned} \mathcal{L}_{mix} &= -\sum_{\phi_p}\left(F_{\phi_p,TC}F^{\dagger}_{\phi_p,MSSM} + h.c.\right), \ \phi_p = H_1', H_2', H_1, H_1, \ F_{H_1',TC} = -y_E\tilde{E}_L\bar{\tilde{E}}_R, \\ F_{H_2',TC} &= y_E\tilde{N}_L\bar{\tilde{E}}_R, \ F_{H_1,TC} = -y_U\tilde{D}_L^a\bar{\tilde{U}}_R^a - y_N\tilde{E}_L\bar{\tilde{N}}_R, \ F_{H_2,TC} = y_U\tilde{U}_L^a\bar{\tilde{U}}_R^a + y_N\tilde{N}_L\bar{\tilde{N}}_R. \end{aligned} \tag{A.11}$$

The corresponding MSSM auxiliary fields F can be found in [41] and references therein. Also, in the Eqs.(A.10,A.11) we used $\tilde{H}$ and $\tilde{H}'$ to indicate the scalar Higgs doublets, for consistency with the rest of the notation where the tilde identifies the scalar component of a chiral superfield or the fermionic component of a vector superfield. The remaining

[11]We consider the constants in the superpotential to be real to avoid the contribution of CP violating terms.

Yukawa interaction terms are determined by the superpotential, and can be expressed as

$$\begin{aligned}
\mathcal{L}_{P-Yuk} &= \sqrt{2} g_{TC} \epsilon^{abc} \left(U_L^a D_L^b \tilde{\bar{U}}_R^c + U_L^a \tilde{D}_L^b \bar{U}_R^c + \tilde{U}_L^a D_L^b \bar{U}_R^c \right) + y_U \left[(H_1 D_L^a - H_2 U_L^a) \tilde{\bar{U}}_R^a \right. \\
&+ \left(\tilde{H}_1 D_L^a - \tilde{H}_2 U_L^a \right) \bar{U}_R^a + \left(H_1 \tilde{D}_L^a - H_2 \tilde{U}_L^a \right) \bar{U}_R^a \Big] + y_N \Big[(H_1 E_L - H_2 N_L) \tilde{\bar{N}}_R \\
&+ \left(H_1 \tilde{E}_L - H_2 \tilde{N}_L \right) \bar{N}_R + \left(\tilde{H}_1 E_L - \tilde{H}_2 N_L \right) \bar{N}_R \Big] + y_E \Big[(H_1' E_L - H_2' N_L) \tilde{\bar{E}}_R \\
&+ \left(H_1' \tilde{E}_L - H_2' \tilde{N}_L \right) \bar{E}_R + \left(\tilde{H}_1' E_L - \tilde{H}_2' N_L \right) \bar{E}_R \Big] - y_R \bar{U}_R^a \left(\bar{U}_R^a \tilde{\bar{E}}_R + \tilde{\bar{U}}_R^a \bar{E}_R \right) \\
&+ h.c.. \qquad \text{(A.12)}
\end{aligned}$$

The soft SUSY breaking terms, finally, can be written straightforwardly starting from the superpotential in Eq.(2.6), to which we add the techni-gaugino and scalar mass terms as well:

$$\begin{aligned}
\mathcal{L}_{soft} &= - \Big[a_{TC} \epsilon^{abc} \tilde{U}_L^a \tilde{D}_L^b \tilde{\bar{U}}_R^c + a_U \left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) \tilde{\bar{U}}_R^a + a_N \left(\tilde{H}_1 \tilde{E}_L - \tilde{H}_2 \tilde{N}_L \right) \tilde{\bar{N}}_R \\
&+ a_E \left(\tilde{H}_1' \tilde{E}_L - \tilde{H}_2' \tilde{N}_L \right) \tilde{\bar{E}}_R + a_R \tilde{\bar{U}}_R^a \tilde{\bar{U}}_R^a \tilde{\bar{E}}_R + \frac{1}{2} M_D \bar{D}_R^a \bar{D}_R^a + c.c. \Big] - M_Q^2 \tilde{\bar{Q}}_L^a \tilde{Q}_L^a \\
&- M_U^2 \tilde{\bar{U}}_R^a \tilde{U}_R^a - M_L^2 \tilde{\bar{L}}_L \tilde{L}_L - M_N^2 \tilde{\bar{N}}_R \tilde{N}_R - M_E^2 \tilde{\bar{E}}_R \tilde{E}_R. \qquad \text{(A.13)}
\end{aligned}$$

B. Scalar Squared Mass Matrices

The techni-Higgs squared mass matrix is

$$\begin{aligned}
\mathcal{M}_{tc\text{-}h}^2 &= \frac{1}{2} \begin{pmatrix} g_{TC}^2 v_{TC}^2 & -g_{TC}^2 v_{TC}^2 \\ -g_{TC}^2 v_{TC}^2 & g_{TC}^2 v_{TC}^2 \end{pmatrix} , \ (\mathcal{M}_{tc\text{-}h}^2)_{ij} = \left. \frac{\partial^2 V}{\partial \phi_i^{tc\text{-}h} \partial \phi_j^{tc\text{-}h}} \right|_{\phi = \langle \phi \rangle} , \\
\phi^{tc\text{-}h} &= \Re \left(\frac{\tilde{D}_L^1 - i \tilde{D}_L^2}{\sqrt{2}}, \frac{\tilde{D}_L^1 + i \tilde{D}_L^2}{\sqrt{2}} \right) , m_{h^{TC}} = g_{TC} v_{TC} \,. \qquad \text{(B.1)}
\end{aligned}$$

The massless eigenstate in the last matrix is the longitudinal degree of freedom of the techni-photon G in Eq.(4.3):

$$\pi_{TC} = \frac{1}{\sqrt{2}} (1, 1) \cdot \phi^{tc\text{-}h} \,. \qquad \text{(B.2)}$$

The charged-Higgs squared mass matrix is

$$\mathcal{M}_{h^\pm}^2 = \begin{pmatrix} \mathcal{M}_{hc}^2 & 0 \\ 0 & \mathcal{M}_{hl}^2 \end{pmatrix} , \qquad \text{(B.3)}$$

$$
\begin{aligned}
\left(\mathcal{M}^2_{hc}\right)_{11} &= \frac{1}{4}\left(4b\mathrm{ct}_\beta + c^2_\beta g^2_L v^2_H - v^2_{TC}\left(g^2_L - 2y^2_U\right)\right),\ \left(\mathcal{M}^2_{hc}\right)_{12} = b + \frac{1}{4}c_\beta g^2_L v^2_H s_\beta \\
\left(\mathcal{M}^2_{hc}\right)_{13} &= \frac{1}{4}v_H s_\beta v_{TC}\left(g^2_L - 2y^2_U\right),\ \left(\mathcal{M}^2_{hc}\right)_{14} = -\frac{a_U v_{TC}}{\sqrt{2}}, \\
\left(\mathcal{M}^2_{hc}\right)_{22} &= bt_\beta + \frac{1}{4}g^2_L\left(v^2_H s^2_\beta + v^2_{TC}\right),\ \left(\mathcal{M}^2_{hc}\right)_{23} = \frac{1}{4}c_\beta g^2_L v_H v_{TC}, \\
\left(\mathcal{M}^2_{hc}\right)_{24} &= -\frac{\mu v_{TC} y_U}{\sqrt{2}},\ \left(\mathcal{M}^2_{hc}\right)_{33} = \frac{1}{4}v^2_H\left(c_{2\beta}g^2_L + 2s^2_\beta y^2_U\right), \\
\left(\mathcal{M}^2_{hc}\right)_{34} &= \frac{1}{\sqrt{2}}v_H\left(a_U s_\beta - \mu c_\beta y_U\right), \\
\left(\mathcal{M}^2_{hc}\right)_{44} &= \frac{1}{4}\left(g^2_Y\left(c_{2\beta}v^2_H - v^2_{TC}\right) + 2y^2_U\left(v^2_H s^2_\beta + v^2_{TC}\right) + 4M^2_U\right), \\
\left(\mathcal{M}^2_{hl}\right)_{11} &= M^2_L + \frac{1}{2}s^2_\beta v^2_H y^2_N + \frac{1}{8}\left(g^2_L + 3g^2_Y\right)\left(c_{2\beta}v^2_H - v^2_{\mathrm{TC}}\right), \\
\left(\mathcal{M}^2_{hl}\right)_{12} &= \frac{1}{\sqrt{2}}v_H\left(a_N s_\beta - \mu c_\beta y_N\right), \\
\left(\mathcal{M}^2_{hl}\right)_{22} &= M^2_N + \frac{1}{4}g^2_Y v^2_{\mathrm{TC}} + \frac{1}{4}v^2_H\left(y^2_N - c_{2\beta}\left(g^2_Y + y^2_N\right)\right),
\end{aligned} \tag{B.4}
$$

$$
\left(\mathcal{M}^2_{h^\pm}\right)_{ij} = \left.\frac{\partial^2 V}{\partial\phi^{h^\pm}_i\partial\phi^{h^\pm}_j}\right|_{\phi=\langle\phi\rangle},\quad \phi^{h^\pm} = \Re\left(\tilde{H}_1, \tilde{H}'_2, \tilde{U}^3_L, \tilde{\bar{U}}^3_R, \tilde{N}_L, \tilde{\bar{N}}_R,\right). \tag{B.5}
$$

The massless eigenstate in the Hermitian matrix $\mathcal{M}^2_{hc}$, Eq(B.4), is the longitudinal degree of freedom of the W gauge boson:

$$
\pi_W = N_W\left(s_\beta v_H, -c_\beta v_H, v_{TC}\right)\cdot\phi^{h^\pm},\ N^{-2}_W = v^2_{TC} + v^2_H. \tag{B.6}
$$

The remaining eigenvalues of $\mathcal{M}^2_{hc}$ and those of $\mathcal{M}^2_{hl}$ are all non-zero: they have rather lengthy and not particularly instructive expressions, and therefore we do not write them here.

The techni-charged Higgs squared mass matrix is

$$
\mathcal{M}^2_{tc\text{-}h^\pm} = \begin{pmatrix} \mathcal{M}^2_d & -\mathcal{M}^2_o \\ \mathcal{M}^2_o & \mathcal{M}^2_d \end{pmatrix}, \tag{B.7}
$$

$$
\begin{aligned}
\left(\mathcal{M}^2_d\right)_{11} &= \frac{1}{4}c_{2\beta}g^2_L v^2_H + \frac{1}{2}s^2_\beta y^2_U v^2_H - \frac{1}{4}\left(g^2_L - 4g^2_{TC}\right)v^2_{TC},\ \left(\mathcal{M}^2_d\right)_{12} = \frac{1}{\sqrt{2}}v_H\left(a_U s_\beta - \mu c_\beta y_U\right) \\
\left(\mathcal{M}^2_d\right)_{22} &= M^2_U + \frac{1}{4}\left(4g^2_{TC} - g^2_Y\right)v^2_{TC} + \frac{1}{4}v^2_H y^2_U + \frac{1}{4}c_{2\beta}v^2_H\left(g^2_Y - y^2_U\right), \\
\left(\mathcal{M}^2_o\right)_{ij} &= \frac{1}{\sqrt{2}}a_{TC}v_{VC}\epsilon_{ij},\ \left(\mathcal{M}^2_{tc\text{-}h^\pm}\right)_{ij} = \left.\frac{\partial^2 V}{\partial\phi^{tc\text{-}h^\pm}_i\partial\phi^{tc\text{-}h^\pm}_j}\right|_{\phi=\langle\phi\rangle}, \\
\phi^{tc\text{-}h^\pm} &= \Re\left(\frac{\tilde{U}^1_L - i\tilde{U}^2_L}{\sqrt{2}}, \frac{\tilde{\bar{U}}^1_R + i\tilde{\bar{U}}^2_R}{\sqrt{2}}\right)\bigcup\Im\left(\frac{\tilde{U}^1_L - i\tilde{U}^2_L}{\sqrt{2}}, \frac{\tilde{\bar{U}}^1_R + i\tilde{\bar{U}}^2_R}{\sqrt{2}}\right).
\end{aligned} \tag{B.8}
$$

The doubly charged-Higgs squared mass matrix is

$$\begin{aligned}
\left(\mathcal{M}^2_{h^{2\pm}}\right)_{11} &= M_L^2 + \frac{1}{2}c_\beta^2 v_H^2 y_E^2 - \frac{1}{8}\left(g_L^2 - 3g_Y^2\right)\left(c_{2\beta}v_H^2 - v_{\rm TC}^2\right), \qquad \text{(B.9)}\\
\left(\mathcal{M}^2_{h^{2\pm}}\right)_{12} &= \frac{1}{\sqrt{2}}v_H\left(\mu s_\beta y_E - a_E c_\beta\right), \left(\mathcal{M}^2_{h^{2\pm}}\right)_{22} = \frac{1}{2}\left(v_{\rm TC}^2 - \frac{1}{2}c_{2\beta}v_H^2\right)g_Y^2 + M_E^2 + \frac{1}{2}c_\beta^2 v_H^2 y_E^2
\end{aligned}$$

$$\left(\mathcal{M}^2_{h^{2\pm}}\right)_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i^{h^{2\pm}}\partial\phi_j^{h^{2\pm}}}\right|_{\phi=\langle\phi\rangle}, \;\; \phi^{h^{2\pm}} = \Re\left(\tilde{E}_L, \tilde{\bar{E}}_R,\right). \qquad \text{(B.10)}$$

The eigenvalues of $\mathcal{M}^2_{h^{2\pm}}$ and $\mathcal{M}^2_{tc\text{-}h^\pm}$ are all non-zero: they have rather lengthy and not particularly instructive expressions, and therefore we do not write them here.

C. Renormalization Group Equations

We now compute the running of the Yukawa couplings. The effects of the top Yukawa coupling are included but the other MSSM Yukawa couplings are neglected. We explicitly consider an $SU(2)$ technicolor gauge group and the hypercharge assignment for the new fields as given in the Table 1. However we denote the number of technicolors by N and the hypercharges of the non MSSM fields by $Y(\Phi)$, $Y(3)$, $Y(\Lambda)$, $Y(N)$, and $Y(E)$ (where the first two hypercharges refer to the weak doublet with components $\Phi_{1,2}$ and to Φ_3) to better identify the source of the different numerical factors in the expressions below:[12]

$$\begin{aligned}
\beta_{y_{TC}} &= y_{TC}\left(2\gamma_\Phi + \gamma_3\right)\\
\beta_{y_U} &= y_U\left(\gamma_\Phi + \gamma_3 + \gamma_U\right)\\
\beta_{y_N} &= y_N\left(\gamma_\Lambda + \gamma_U + \gamma_N\right)\\
\beta_{y_E} &= y_E\left(\gamma_\Lambda + \gamma_D + \gamma_E\right)\\
\beta_{y_t} &= y_t\left(\gamma_U + \gamma_t + \gamma_q\right)
\end{aligned}$$

[12] We neglect in this context the coupling y_R since it does not give any contribution to the masses at tree level and can be taken to be negligibly small.

with

$$
\begin{aligned}
\gamma_\Phi &= \frac{1}{16\pi^2}\left(2N(y_{TC}^2 - g_{TC}^2) + y_U^2 - \frac{3}{2}g_L^2 - 2\frac{3}{5}Y(\Phi)^2 g_Y^2\right) \\
\gamma_3 &= \frac{1}{16\pi^2}\left(2N(y_{TC}^2 - g_{TC}^2) + 2y_U^2 - 2\frac{3}{5}Y(3)^2 g_Y^2\right) \\
\gamma_U &= \frac{1}{16\pi^2}\left((N^2-1)y_U^2 + 3y_t^2 + y_N^2 - \frac{3}{2}g_L^2 - \frac{3}{10}g_Y^2\right) \\
\gamma_D &= \frac{1}{16\pi^2}\left(y_E^2 - \frac{3}{2}g_L^2 - \frac{3}{10}g_Y^2\right) \\
\gamma_\Lambda &= \frac{1}{16\pi^2}\left(y_N^2 + y_E^2 - \frac{3}{2}g_L^2 - 2\frac{3}{5}Y(\Lambda)^2 g_Y^2\right) \\
\gamma_N &= \frac{1}{16\pi^2}\left(2y_N^2 - 2\frac{3}{5}Y(N)^2 g_Y^2\right) \\
\gamma_E &= \frac{1}{16\pi^2}\left(2y_E^2 - 2\frac{3}{5}Y(E)^2 g_Y^2\right) \\
\gamma_t &= \frac{1}{16\pi^2}\left(2y_t^2 - \frac{8}{15}g_Y^2 - \frac{8}{3}g_3^2\right) \\
\gamma_q &= \frac{1}{16\pi^2}\left(y_t^2 - \frac{3}{2}g_L^2 - \frac{1}{30}g_Y^2 - \frac{8}{3}g_3^2\right)
\end{aligned}
$$

The one loop running of the gauge couplings is given by:

$$\beta_{g_a} = \frac{1}{16\pi^2} g_a^3 \left[\Sigma_i I_a(i) - 3C_a(G)\right]$$

where $I_a(i)$ is the Dynkin index of the superfield i (1/2 for each fundamental of SU(N), $3Y(i)/5$ for U$(1)_Y$, where $Y(i)$ gives the hypercharge of the superfield i) and obtain:

$$\beta_{g_Y} = \frac{15}{16\pi^2} g_Y^3 \ , \quad \beta_{g_L} = \frac{3}{16\pi^2} g_L^3 \ , \quad \beta_{g_3} = \frac{-3}{16\pi^2} g_3^3 \ .$$

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