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# THERMOELASTIC INVESTIGATIONS FOR FATIGUE LIFE ASSESSMENT

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## Abstract

An investigation is presented on the suitability and accuracy of a thermoelastic technique for the analysis of fatigue cracks. The stress intensity factor ranges $\Delta K_I$ and $\Delta K_{II}$ are determined from thermoelastic data recorded from around the tip of a sharp slot in a steel specimen under biaxial load, in order to assess the accuracy of the technique. $\Delta K_I$ and $\Delta K_{II}$ are determined to within 4 % and 9% of a theoretical prediction respectively. The results from a similar test on a fatigue crack under biaxial load are also presented. These show that thermoelastic stress analysis is a rapid and accurate way of analysing mixed-mode fatigue cracks. A discussion is given on the potential of thermoelastic stress analysis of propagating cracks.

## Introduction

In recent years thermoelastic stress analysis (TSA) has proved to be a useful tool for the study of fatigue cracks[1]. The stress intensity factor value obtained from thermoelastic analysis is equal to the range of the stress intensity factor, $\Delta K$, that occurs at the crack tip due to the applied cyclic load. This allows the actual crack driving force to be experimentally determined rather than being inferred from maximum and minimum stress intensity factors, which is the case with other experimental techniques. The direct determination of $\Delta K$ makes the technique ideal for use in fatigue life predictions, such as in a recent study of welded components[2,3].

Although accurate analyses have been performed for opening mode cracks and slots, only limited progress has been made for the determination of stress intensity factors for mixed-mode cracks using thermoelastic techniques[4,5,6,7,8]. Recently published data on the subject[5] show good agreement between theory and experiment for both $\Delta K_I$ and $\Delta K_{II}$ for central slots and cracks, and edge slots. However for mixed-mode edge cracks the differences between theory and experiment were up to 30%. No reasons could be given for the unusually large differences and plans for further investigation were indicated. The majority of the published data have been for cracks under predominantly mode I loading and the only data for predominantly mode II loading showed a difference between theory and experiment of up to 40%[4]. It was suggested that the theoretical solution may not be reliable at this extreme notch geometry for which there was only one numerical and no other experimental study in support. Although it is known that the majority of mixed-mode cracks found in engineering components eventually propagate as mode I cracks, in some cases such as turbine blades and rolling contact fatigue in rails, mixed-mode loading continues to dominate. Therefore this area of research is of importance and further investigation into the accurate application of the technique would be beneficial to industry.

All published TSA experiments in this area generate the mixed-mode conditions at a crack tip with the use of tensile loading of a plate containing a sharp slot or crack at an angle to the direction of loading. The cracks are grown under mode I loading and then cut out and loaded at an angle to the original direction of crack propagation[5]. For these types of mixed-mode experiments, a fatigue crack of a longer length, or different $K_{II}/K_I$ ratio, can only be tested using another specimen. It is considered that biaxial loading of a sample would give a more versatile experimental procedure and a better representation of the true strain field at the tip of a fatigue crack. Using only one biaxial specimen it is possible to record data for many different mixed-mode load cases by changing the applied loads. Such experiments using thermoelastic techniques have not been attempted previously. One reason may be the difficulty in growing a mixed-mode fatigue crack without the crack tip branching under biaxial loads. However Bold et al[9,10] have developed a load cycle which prevents such branching which enables true mixed-mode cracks of varying lengths to be analysed. The analysis of crack tip stress fields using thermoelasticity has been hampered also by the fact that, until recently, it has only been possible to use thermoelasticity to investigate crack tip parameters of stationary cracks, due to the limitations of single-point thermoelastic instruments such as SPATE (Stress Pattern Analysis by Thermal Emission)[*]. A scan of a high enough resolution for crack tip studies could take in excess of an hour to record, by which time the crack may have propagated. New instruments such as Deltatherm[†], which contain an array of detectors, enable thermoelastic data to be recorded in near real time.

A series of experiments has been performed using the latest technology to determine stress intensity factors at the tips of slots and cracks under biaxial fatigue loading using an algorithm which utilizes thermoelastic data[4]. The experiments were conducted in order to investigate, firstly, the accuracy achievable by the experimental technique under predominantly mode II loading conditions and secondly, the experimental analysis of true fatigue cracks grown under biaxial loading conditions.

## Experimental Method

### *Determination of $K_I$ and $K_{II}$ from Thermoelastic Data*

Thermoelasticity is based on the fact that under adiabatic and reversible conditions, a cyclically loaded structure experiences temperature variations that are proportional to the sum of the principal stresses[11]. These temperature variations may be measured using a sensitive infra-red detector, the signal from which, S, is related to the first stress invariant by the following equation:

$$\Delta(\sigma_1 + \sigma_2) = AS \tag{1}$$

where A is a calibration constant. An expression for this first stress invariant in the region of the

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[*] Manufactured by Image Automation, Ometron Division, Chislehurst, Kent,UK (production now discontinued)

[†] Manufactured by Stress Photonics Inc., Madison, Wisconsin, USA.

crack tip can be derived from stress field equations and used to determine the stress intensity factors. The stress intensity factor value obtained from thermoelastic analysis is equal to the range of the stress intensity factor, $\Delta K$, that occurs at the crack tip due to the applied cyclic load.

There are several different methods published which may be used to determine stress intensity factors from thermoelastic data and these have been reviewed by Tomlinson and Olden[1]. The approach which was used in the current experiments was developed by Tomlinson *et al*[4]. A Newton-Raphson iteration combined with a least squares approach is used to fit the equations describing the stress field around the crack tip, based on Mushkelishvili's approach, to the experimental data. This approach allows different applied stress fields to be described which may include non-uniform stress fields. The computerised analysis method requires the map of thermoelastic data to be interrogated at a number of points arranged in an array around the crack tip. The co-ordinates of these points and the thermoelastic signal are input into a computer program which calculates $\Delta K_I$ and $\Delta K_{II}$. The mean and variance of the least-squares fit of the solution to the data points are also calculated in order to give an indication of the accuracy of the results.

The co-ordinates of the data points are required relative to the position of the crack tip and therefore its location must be determined. If a slot is being analysed, the slot tip is relatively easy to identify on a map of thermoelastic data. However the tip of the fatigue crack is less easy to locate. In these experiments a new method[12] was utilised which uses both the phase and magnitude of the thermoelastic signal to locate the crack tip.

In order for the processing of subsequent data to be as efficient as possible it was first necessary to interface directly the data collection from the thermoelastic sensors to be utilised with the calculation algorithm of Tomlinson *et al*[4]. The calculation of the stress intensity factors requires data to be within the area which is dominated by the crack tip singularity, within the linear elastic stress field, and outside the area close to the crack tip where localised plasticity and triaxial stresses dominate, and heat conduction is possible. Heat is generated at the tip of a crack and therefore the adiabatic assumption is not valid very close to the crack tip[13]. It has been found also that data selected from the flanks of the crack are not well described by the stress field equations and must be masked[4]. The interface was therefore written in Visual Basic to enable both SPATE and DeltaTherm data to be input and then the operator selects areas to be masked where the data are invalid for the determination of the stress intensity factors. An array of approximately 100 data points are selected from the valid region of data around the crack tip and these are subsequently input directly to the algorithm to solve for $\Delta K_I$ and $\Delta K_{II}$. It was ensured that the data were collected from within the region of valid by plotting a graph of y versus $(1/S_{max}^2)$ [14], where $S_{max}$ is the maximum thermoelastic signal from a line at a distance y parallel to the crack, and only taking data from the linear region. Figure 1 shows how typical data points are collected on radial lines from the crack tip between an inner and an outer limit. An inner limit of 10ρ, where ρ is the notch radius, was used to mask the non-linear effects at the crack tip caused by plasticity and/or heat conduction. The outer limit was determined as a fraction of notch/crack length. A study on the location of the inner and outer limits of the data collection zones was carried out and it was found that an inner limit of radius = 1 mm could be used effectively for both predominantly mode I and mode II cases. The accurate location of the outer radius appeared to be significant for the predominantly Mode II cases. For short notches or cracks, the value of 0.4a, where a is the crack length, gave a good estimation for the outer limit of the singularity dominated zone. For relatively long cracks this value was reduced to approximately 0.2a.

The success of the interfacing procedure was tested by determining stress intensity factors from opening mode and predominantly mode I mixed-mode edge slots for which the algorithm is known to give accurate results, using SPATE data[16], and opening mode cracks using DeltaTherm data[2,3]. These studies showed that the interface and algorithm were robust and that the technique was highly reproducible and operator independent[3].

### *Specimen preparation and apparatus*

The specimens were made from 150M36 steel and were a cruciform shape with a central spark-eroded slot inclined at 45º, of width = 0.25 mm and length 2a = 6 mm, as in Figure 2. The slot was prepared by drilling a central hole with a diameter of 2 mm in the specimen and machining the slot from this point. The radius of the tip of the slot was 0.125 mm. The area of the specimen required for the determination of K from theory was calculated taking into account the stiffened edges of the specimen[17,18]. One side of the specimen was polished to enable any crack growth to be easily monitored using an optical microscope. The other side of the specimen was sprayed with a thin coat of matt black paint to increase emissivity and to obtain a uniform thermoelastic signal. A strain gauge rosette for calibration purposes was bonded to the polished side of the specimen in an area of near uniform stress. The load was applied using a 100 kN Denison Mayes Biaxial Testing Machine pictured in Figure 3. This machine has four actuators, two providing a vertical load and two providing a horizontal load. When all four actuators are pulling with equal force, specimen is loaded in equi-biaxial tension, which produces a mode I stress intensity on a starter crack at any angle. When one pair of actuators pull and the other pair push, a pure shear stress field is produced which applies a mode II stress intensity factor to a crack at 45º to the axes. The shape of the load waveforms and the response of the load cells were monitored using two oscilloscopes and the reference signal for the thermoelastic stress analysis was taken from one of the load cells.

### *Thermoelastic data collection*

The thermoelastic data for each series of experiments were recorded using the Deltatherm system pictured in Figure 3. For the initial tests investigating the accuracy of TSA for crack analysis the quality of the recorded data needed to be at an optimum. In order to improve the quality of the thermoelastic image the data may be collected over a longer time period. For the Deltatherm system, every increase in data collection time of a factor of four, doubles the signal to noise ratio of the data. An experiment was performed on one of the cruciform specimens which contained a fatigue crack of half length a = 15.15 mm. This was biaxially loaded to give an applied mode I stress intensity factor of 3.5 MPa√m. Thermoelastic data were recorded at integration times, that is the time over which the data are recorded, of 9, 18, 67, 131, and 195 seconds and the data are shown in Figure 4. It can be seen that for the very short integration time of 9 seconds the thermoelastic pattern which would be expected around a crack under Mode I loading is discernable, however the data are relatively noisy. As the integration time increases, the signal to noise ratio improves. The stress intensity factors for each data map were determined using the method outlined and the results are compared with a theoretical prediction as shown in Figure 5. These results show very little difference in the accuracy of the stress intensity factors determined for the decreasing noise levels, however the data recorded at the longest integration time appeared to give a more accurate answer and therefore this integration time was selected for the subsequent tests.

## Stress intensity factors from sharp slots

Although the ultimate aim was to investigate the use of thermoelasticity in the determination of fatigue cracks under biaxial load, it was decided that the accuracy of the proposed method should be tested initially using sharp slots. It was considered important to eliminate any factors which may introduce errors into the results and from previous work it had been found that fatigue cracks may exhibit crack closure and this had the effect of depressing the value of ΔK calculated[19]. So in order to prevent these effects masking the accuracy of the technique, data were first collected from around a sharp spark-eroded slot rather than a fatigue crack.

A sinusoidal load was applied to the cruciform specimen described, at a frequency of 8 Hz and a load ratio, R = 0 ($\sigma_{min} = 0$), along the two axes of the specimen in order to give the ratios of $\Delta K_{II}/\Delta K_I$ approximately equal to 0.5, 1, 1.5, 2, and 2.5. Then any effect of changing the load ratio was investigated by setting $\Delta K_{II}/\Delta K_I = 0.5$ and loading the specimen at R = 0.1, 0.2, 0.3, 0.5 and 0.7. At each load setting thermoelastic data were recorded around the slot tip for an integration time of 195 seconds using a DeltaTherm 1550 system and a typical map is shown in Figure 6. The thermoelastic signal was calibrated using the two orthogonal strain gauge rosettes shown in Figure 2, using a standard calibration method[4]. The signal was calibrated at regular intervals throughout the test programme since any change in ambient temperature can change the calibration constant. The stress intensity factor ranges, $\Delta K_I$ and $\Delta K_{II}$, were determined from each set of data using the method already described. The slot was further extended using a spark erosion technique to $2a = 12, 18, 24$ and 30 mm and the same procedure repeated at each of these slot lengths. The results for the variable $\Delta K_{II}/\Delta K_I$ tests are shown in Figure 7a) and those for the variable R ratio tests in Figure 7b) where the experimental values are compared to stress intensity factors determined from the theory developed by Bold *et al*[9,10]. The mode I stress intensity factor is represented by the closed symbols and the open symbols represent the mode II stress intensity factor.

### *Discussion of Results*

From Figure 7(a) the experimental data for both $\Delta K_I$ and $\Delta K_{II}$ compare well with the theoretical estimations. For the mode I stress intensity factors in Figure 7(a)i) all the data appear to be within 10% of theory with an average difference of 4.5 %. The mode II data in Figure 7(a)ii) show slightly more scatter than the mode I data, with the average error as 6.5 %. All the results appear more accurate for the longer slots, however for the shortest slot the area in which valid data could be collected was relatively small, due to the small linear elastic zone and this was thought to be the reason for the less favourable comparison with theory. The larger discrepancy for the shortest slot at the low $K_{II}/K_I$ ratio may be accounted for in the mode II results, by the fact that the value of $K_{II}$ was small.. From the results for increasing R ratio (Figure 7(b)), the average differences between experiment and theory were 4.1% and 8.8% for $\Delta K_I$ and $\Delta K_{II}$ respectively. Again the 3 mm slot showed more discrepancy between experiment and theory, especially for the mode II values of the stress intensity factor.

Overall, this series of tests showed that accurate stress intensity factors may be calculated from thermoelastic data under both mode I and mode II dominant, mixed-mode loading conditions. The current work used the same algorithm to determine the stress intensity factors as the previous work[4] on predominantly mode II edge slots. In these previous experiments, the stress intensity factors determined by experiment showed marked differences when compared with numerical solutions. Comparison of current and previous experimental accuracy, suggests that these previous errors could be due to the limitations in data collection of the SPATE 8000 system when compared with the higher resolution Deltatherm system and the improved accuracy of locating the position of the crack tip.

## Determination of stress intensity factors from mixed-mode fatigue cracks

A series of tests were performed to determine the stress intensity factors from fatigue cracks subject to mixed-mode, biaxial loading using thermoelasticity. It is believed that no data of this type have been published previously. A slot of length 2a = 6 mm was spark eroded in to a specimen of the same design as in Figure 2. In order to initiate crack growth in the direction of the slot, a pre-crack was grown from this starter notch using an equi-biaxial (Mode I) load, to a length of 2a = 10 mm. Then a mixed-mode load was applied. In order to prevent branching of the propagating crack, a successive load cycle which was developed by Bold *et al*[9] was utilised. Bold had found that no combination of tensile mean stress and cyclic shear stress could be found that produced more than about one millimetre of crack growth before branch crack growth occurred, when these loads were applied simultaneously. In his successive cycle, a mode I load is applied and removed before the fully reverse mode II cycle is applied, as shown in Figure 8. The combination of mode I and mode II loading produces coplanar crack growth, that is growth which is both perpendicular to the maximum tensile stress of the initial tensile part of the cycle and in the direction of the maximum shear part of the overall cycle. An attempt was made to record thermoelastic data under this load cycle, however it was found that the reference signal required for TSA was crucial to successful data collection. The infrared data recorded by the detector include background radiation as well as infrared emissions due to the thermoelastic effect. In order to filter infrared background noise and extract the meaningful data, the raw infrared detector signal is correlated to a reference signal which corresponds to the varying load. This may be taken from, for example, a load cell, a function generator or a strain gauge bonded to the test piece. The infrared signal and its corresponding reference signal are compared using an electronic signal-processing device, and the detector signal is refined to give a meaningful measure of temperature variation due to the load cycle. The most common reference signal used in thermoelastic tests is the sine wave, from which it is straightforward to calculate the change in the sum of the principal stresses. The reference signal used here was taken from the X-axis load cell (see Figure 8). After consultation with the manufacturers of the DeltaTherm system, Stress Photonics Inc., it was concluded that the DeltaVision software could not recognise this reference signal for correlation with the infrared signal. The output from strain gauge bonded to the test piece may be used as a reference signal, however it needs to be in an area of high strain in order to give a reliable clean signal of sufficient magnitude to correlate with the thermoelastic signal[11]. The authors have found strain gauge reference signals to be quite noisy and if a low signal is amplified then the noise is also amplified. Therefore it has been found to be preferable to use an area of high strain. In this case the strain gauge would have to be bonded immediately ahead of the crack to obtain a high signal and also an accurate description of the load cycle experienced by the crack. But since the crack is propagating then the strain gauge would block the crack growth. The solution to this problem was to grow the crack under successive loading and then record thermoelastic data under a reduced sine load range, e.g. in Figure 8, ensuring that the crack did not grow nor branch during the data collection. This solution meant that the proposed investigation of the ability of the method to monitor propagating mixed-mode cracks could not be achieved at this stage, however further research into possible solutions to the problem is being performed.

It had been intended to prevent crack closure whilst growing the cracks, but this had not been possible since, for the successive load cycle where the opening mode R = 0, closure is difficult to prevent[9,10]. However when recording data at higher R ratios any effect of crack closure on the values of the stress intensity factors is minimised since the crack is being loaded in the fully open part of the cycle. Therefore thermoelastic data were recorded under a sinusoidal load of load ratio, R = 0.7 at the levels shown in Figure 8 at a frequency of 8 Hz, to give an applied $\Delta K_{II}/\Delta K_I = 0.45$. The stress intensity factors were determined using the same procedure as outlined previously and results (Figure 9) were obtained for fatigue cracks of length 2a = 12, 18, 24 and 30 mm. For R = 0.7 an average difference between experiment and theoretical predictions of 4.3% and 5% was obtained for $\Delta K_I$ and $\Delta K_{II}$ respectively which is comparable to the accuracy obtained for the specimens containing slots and also comparable to the results obtained by Dulieu-Barton et al for central cracks[5] which exhibited differences of up to 6%. Since the results from the slot and crack experiments are of comparable accuracy this indicates that the assumption that there was no effect from crack closure was correct and that thermoelastic data can be used effectively for the analysis of fatigue cracks. A further paper on the effects of crack closure is in preparation.

## Determination of stress intensity factors from propagating cracks

Although it was not possible to record meaningful thermoelastic data from propagating mixed-mode cracks due to the absence of a suitable reference signal, it is possible to record data from a propagating mode I crack. The DeltaTherm system has a 256 x 320 array of infrared detectors, which provide the means to observe near real time fatigue crack growth. The only limitation to real time data collection is the amount of noise in the image which can be due to the nature of the surface of the material and the movement of the component during cyclic loading. As discussed earlier, this noise can be minimised by integrating the image over a number of cycles, however if the crack is propagating then the number of cycles is limited to the number over which the stress pattern cannot be observed to change. In the biaxial tests already described, a longer integration period of over 3 minutes was selected to ensure very high quality data, however in many cases this length of integration would not be necessary.

An experiment was carried out on a repair welded specimen similar to those described in references 2 and 3. Single edge notched tensile plate specimens of geometry shown in Figure 10 were manufactured from a section cut from a multipass submerged arc butt-welded ferritic steel plate (dimensions: 40 mm thick, 3 m wide and 12 m long) where the weld had been repaired. A spark eroded notch of 4 mm in length was introduced, as shown, and one side of the plate was polished to enable the crack growth to be easily monitored using an optical microscope. The other side of the panel was sprayed with matt black paint to increase emissivity and to obtain a uniform thermoelastic signal. A uniform cyclic tensile load was applied along the longitudinal axis of the panel at a frequency of 16 Hz. The specimen was loaded with a range, $\Delta P$ of 43.6 kN, with an R-ratio of 0.1. Thermoelastic data were recorded at intervals as the crack grew through the weld using a DeltaTherm 1550 system. Each data map was integrated over 30 seconds and the data are shown for a sample of four crack lengths in Figure 11. It may be observed that the maximum rate at which data were recorded was 0.00057 mm/cycle. If the SPATE system had been used, a scan of a similar resolution would over an hour to record, during which time the crack would grow by 28.8 mm at this rate. During the recording of each of these images the crack grew by an average of less than 0.1 mm. The data were relatively noisy, due to the nature of the welded material and the movement of the panel during cyclic loading but as was shown in the previous study[2], by smoothing the data prior to any quantitative analysis using a 3x3 mean filter, reliable stress intensity factors may be determined. The mean filter is contained in the Deltatherm software and works by moving a "kernal" over the entire image one pixel at a time. The 3x3 kernal is used to find the weighted average of the area and replace the central pixel with that value. The corner, side, and centre pixels are weighted 1/16, 1/8, and 1/4 respectively. Even with the level of noise present, results from these welded plates have been shown to be reproducible and independent of operator[3]. Data collection and processing can be achieved in less than 5 minutes, which is over an order of magnitude faster than the SPATE system could achieve. Therefore this study indicates the potential of the thermoelastic technique for studying crack tip parameters of propagating cracks in near-real time.

## Conclusions

It has been shown that accurate stress intensity factors may be calculated from thermoelastic data under both mode I and mode II dominant, mixed-mode loading conditions.

Investigations have taken place into growth of mixed-mode cracks and recording data as the fatigue crack grows. Data can be recorded under a sine load, however the issue of recording thermoelastic data under the successive loading cycle is still under investigation. Thermoelastic data recorded in near real time from a propagating mode I fatigue crack have been presented indicating the potential of the technique for fatigue crack analysis.

## Acknowledgements

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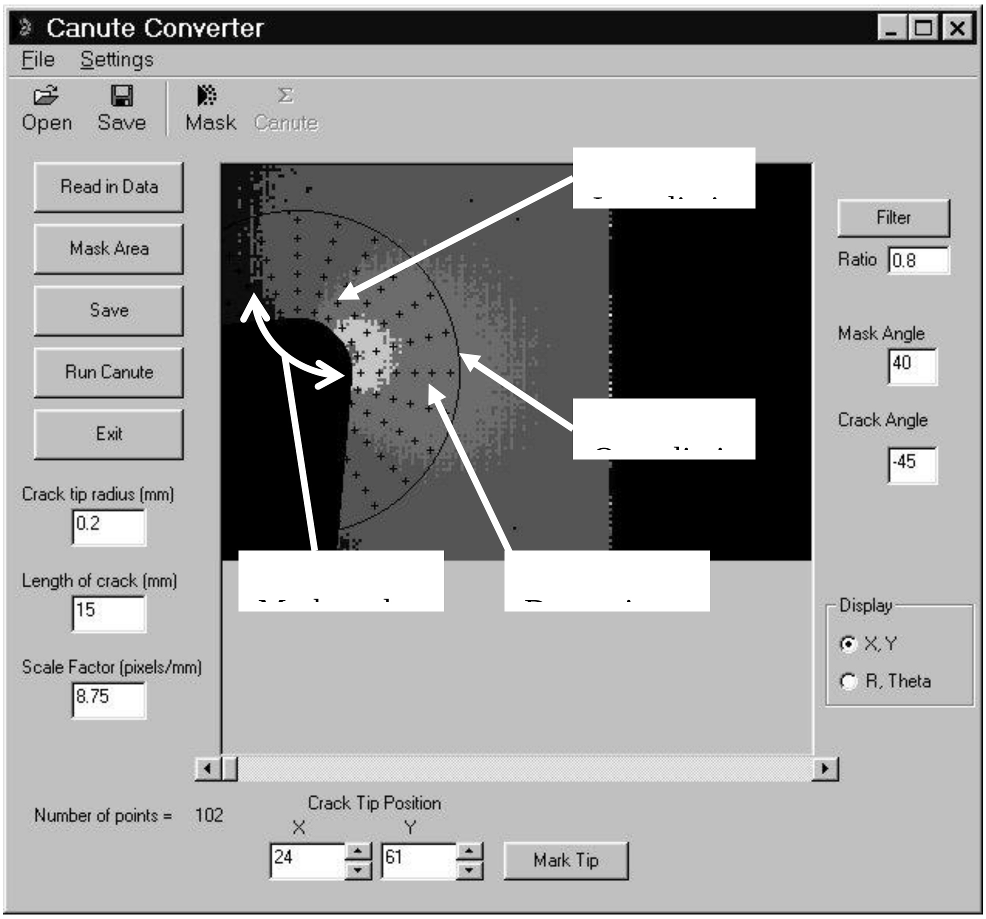


Crack tip radius (mm) 0.2

Length of crack (mm) 15

Scale Factor (pixels/mm) 8.75

Ratio 0.8

Mask Angle 40

Crack Angle -45

Display
● X, Y
○ R, Theta

Number of points = 102

Crack Tip Position
X 24
Y 61

**Figure 1** Showing the how a set of valid data points were selected using the Visual Basic interface. Typical data are collected on radial lines from the crack tip between an inner and an outer limit.

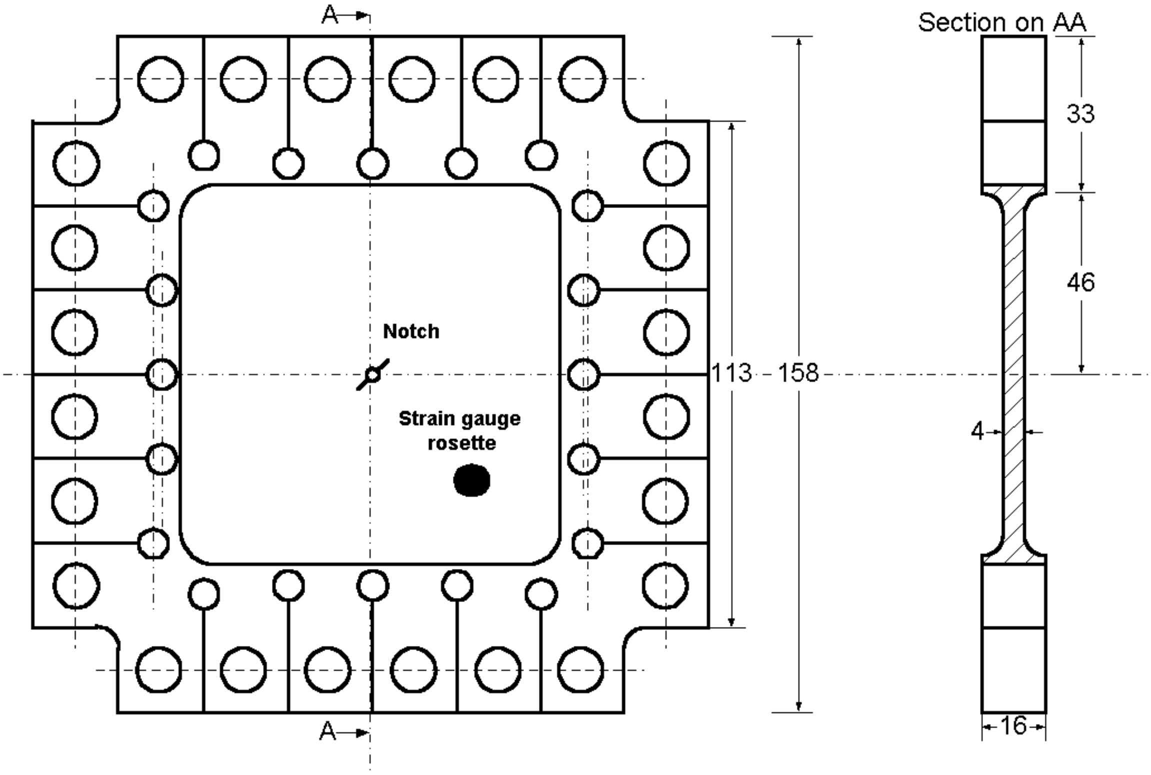


**Figure 2** The cruciform specimen used for the thermoelastic tests, showing the position of the strain gauge rosette and the spark eroded notch

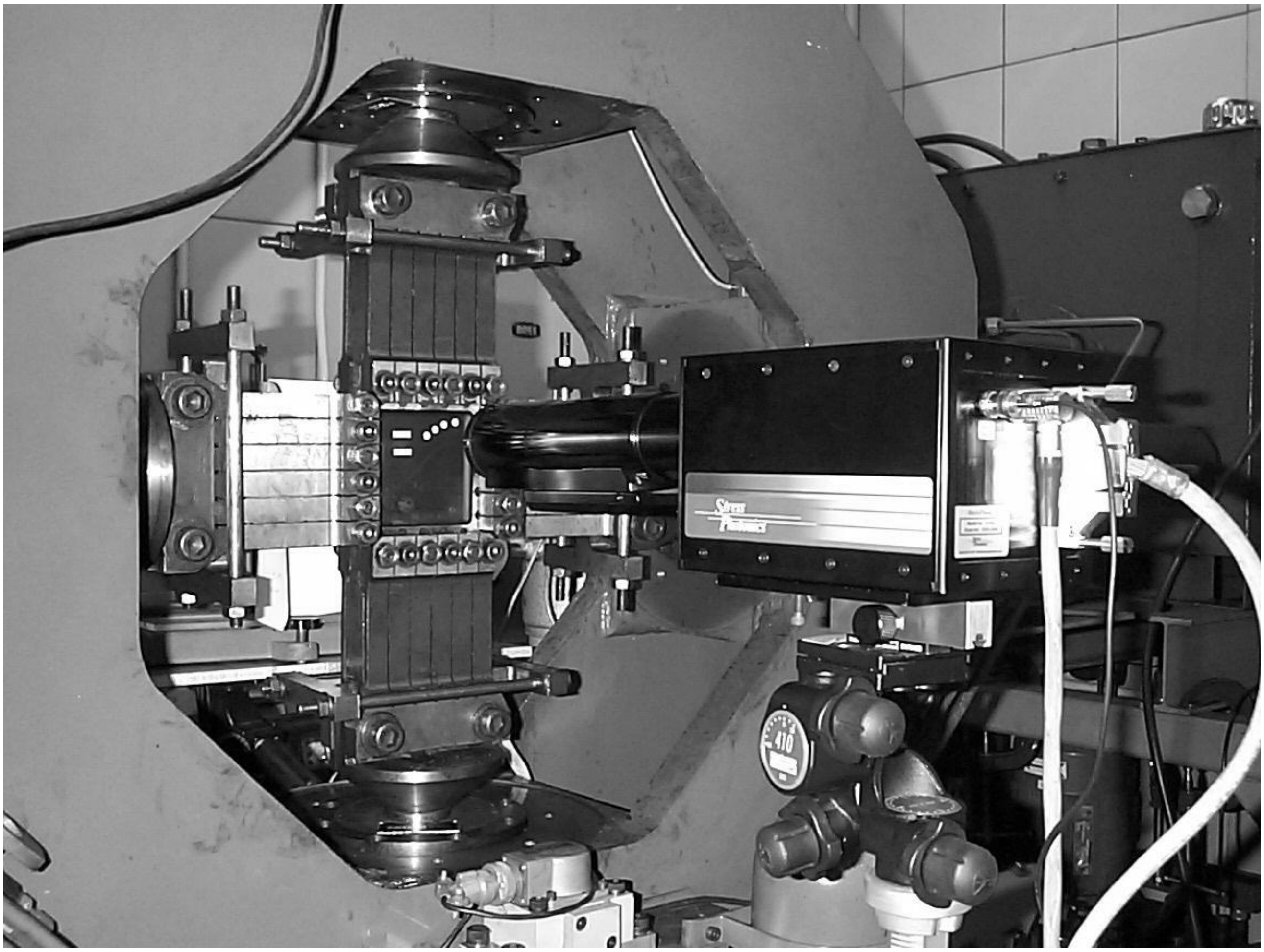
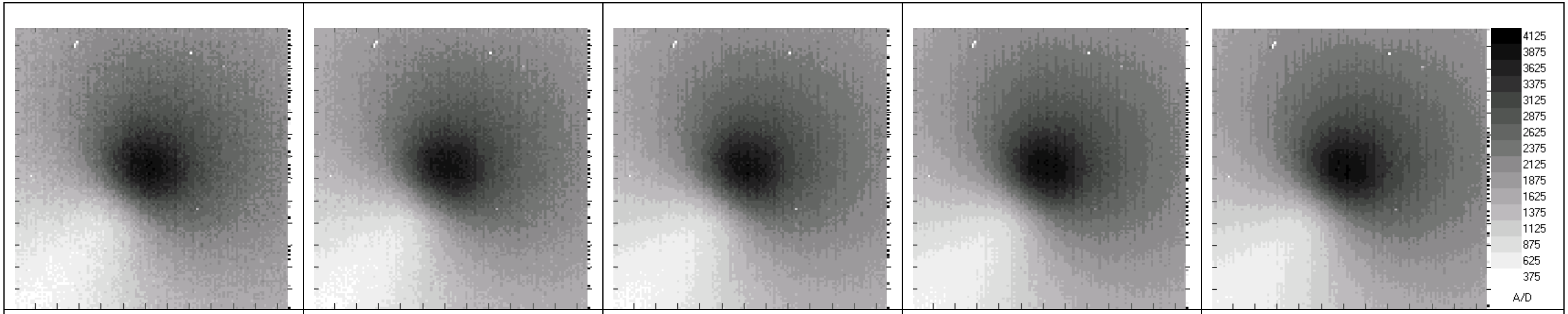


**Figure 3** The cruciform specimen loaded in the Denison Mayes Biaxial test machine and the Deltatherm 1550 camera head.



a) 9 seconds b) 18 seconds c) 67 seconds d) 131 seconds e) 195 seconds

**Figure 4** Thermoelastic data around a crack under mode I loading with increasing integration times

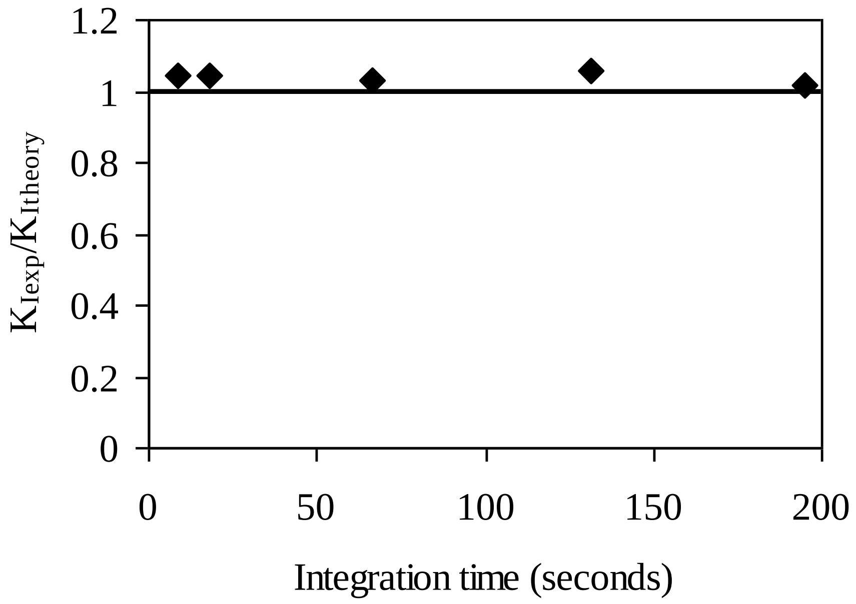


**Figure 5** The normalised stress intensity factors determined from the thermoelastic data shown in Figure 3 which were recorded at increasing integration times

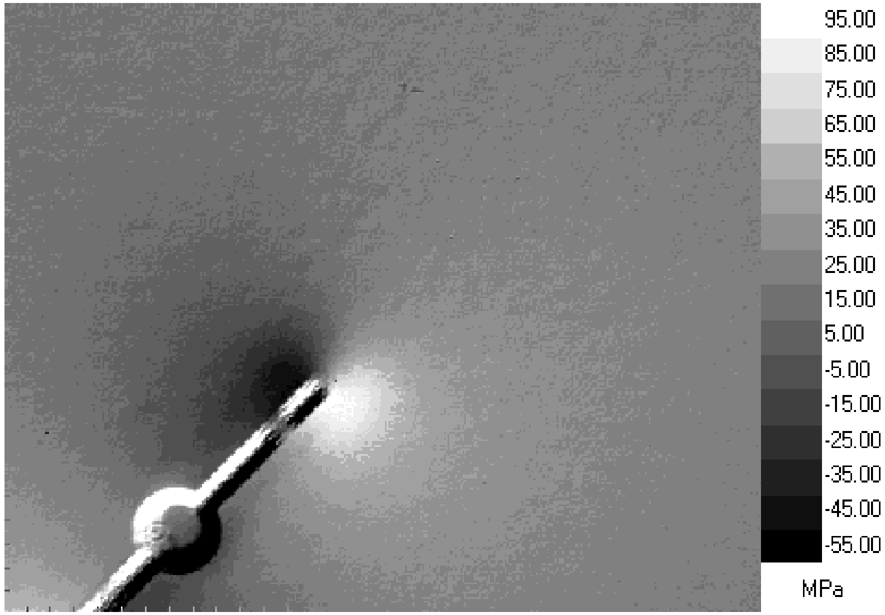
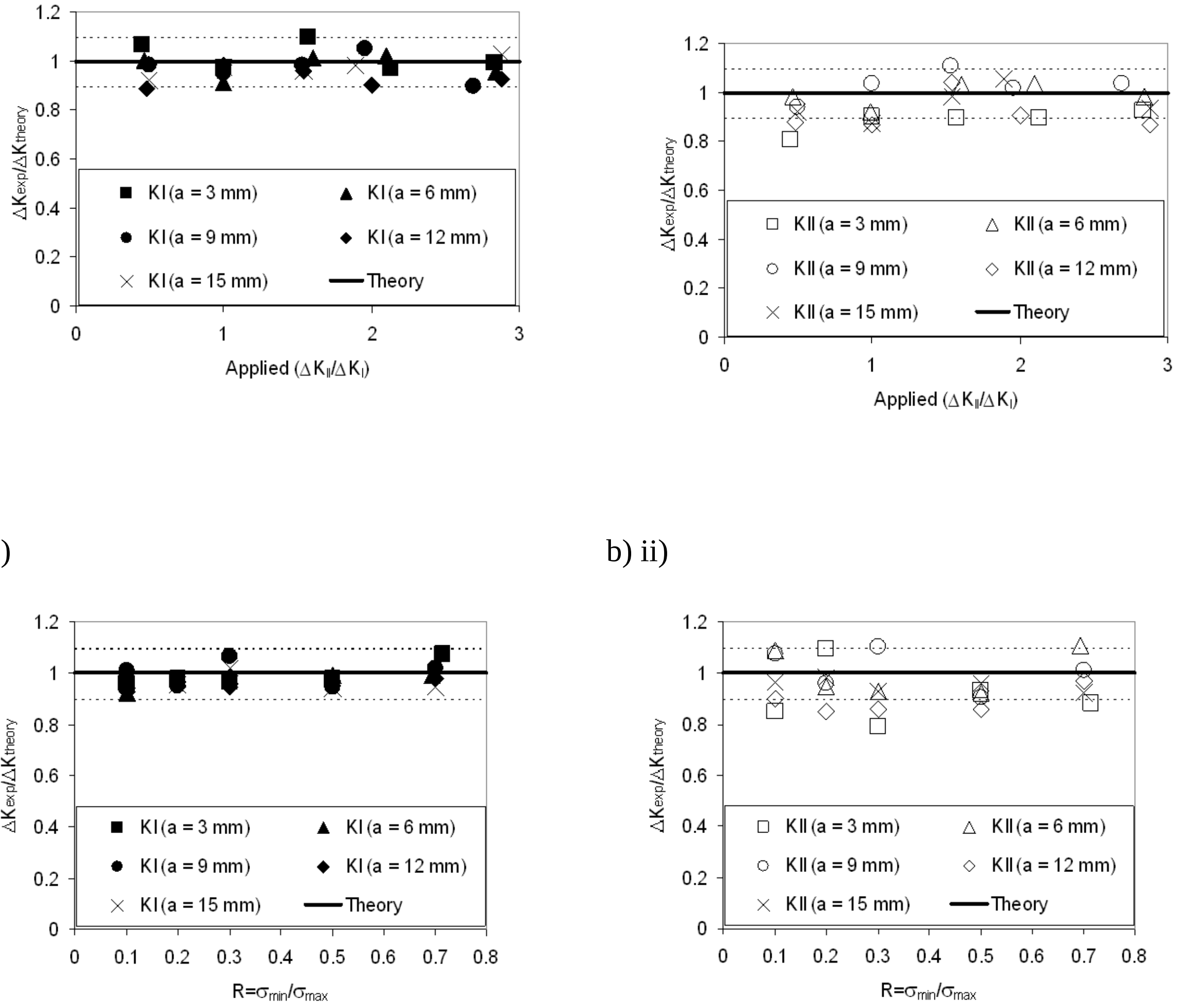


**Figure 6** Showing the thermoelastic signal around a machined slot of length, 2a = 12 mm. The applied $\Delta K_{II}/\Delta K_{I} = 2$ and R = 0.



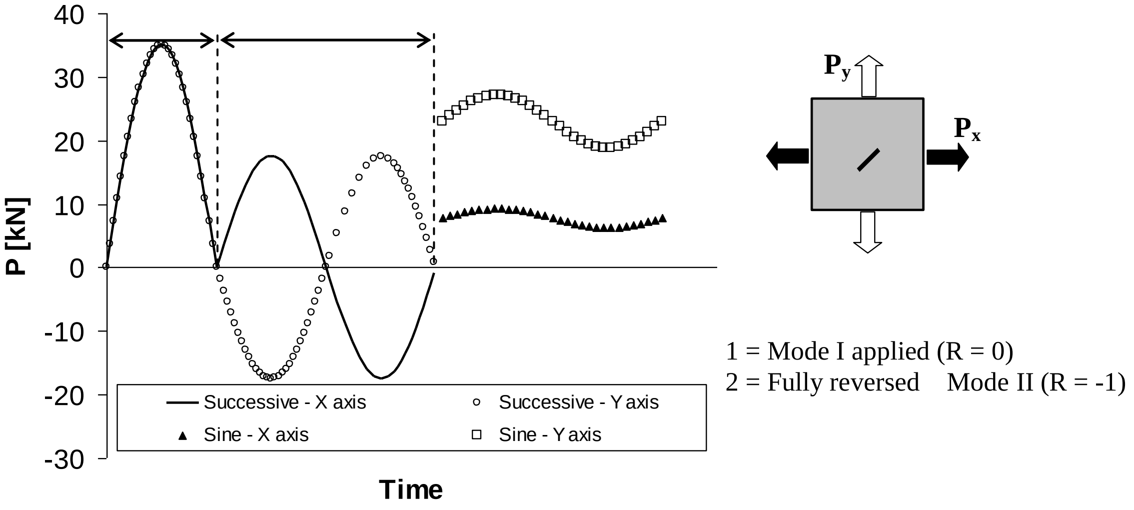
a) i)

a) ii)

b) i)

b) ii)

**Figure 7** Graphs of the normalised stress intensity factor for the machined slots versus (a) increasing applied $\Delta K_{II}/\Delta K_I$ and (b) increasing R ratio



1 = Mode I applied (R = 0)
2 = Fully reversed Mode II (R = -1)

**Figure 8** The loads applied to each axis of the load machine for the successive load cycle for crack growth under biaxial load, where the applied successive $\Delta K_{II}/\Delta K_{I} = 1$; and an example of the sinusoidal load cycle used for thermoelastic data collection, where $\Delta K_{II}/\Delta K_{I} = 0.45$ and $R = 0.7$

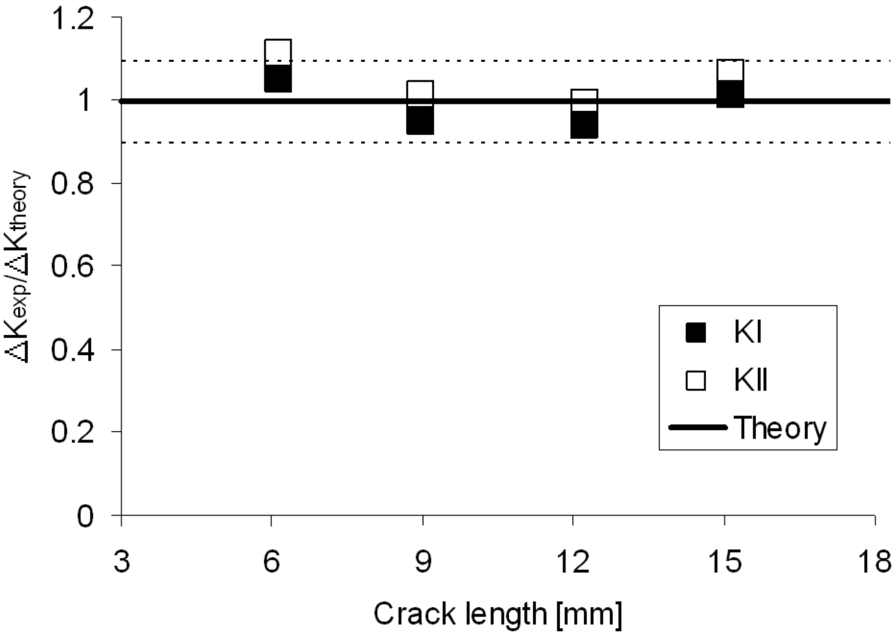


**Figure 9** The stress intensity factors $\Delta K_I$ and $\Delta K_{II}$ with increasing crack length. $\Delta K_{II}/\Delta K_I = 0.45$ and $R = 0.7$

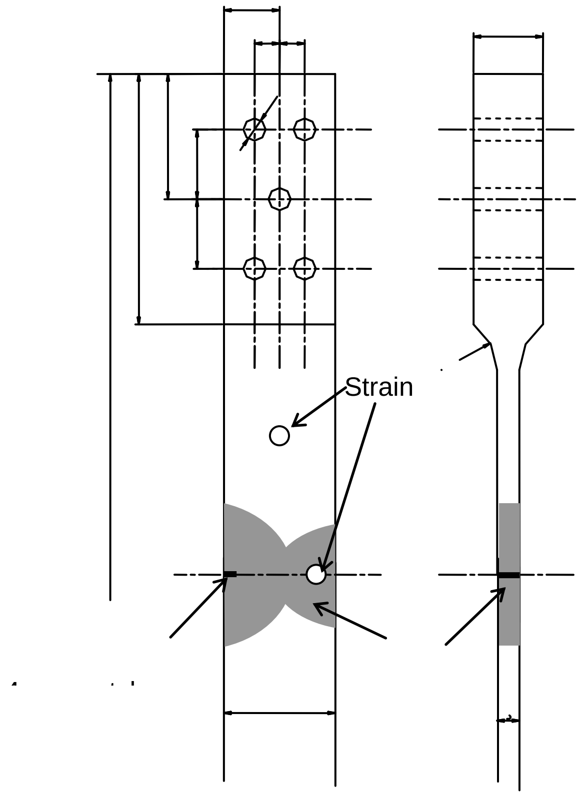
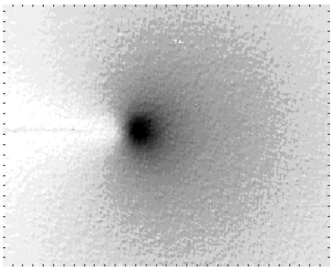
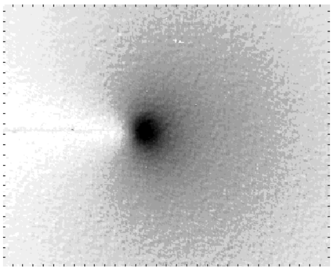


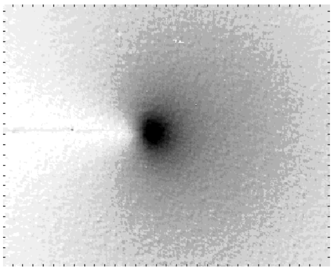
**Figure 10** Repair welded specimen used for thermoelastic tests on propagating cracks, showing the area of the weld, the position of the strain gauges for calibration purposes, and the spark eroded notch



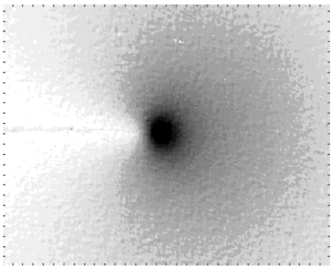
a = 15.08 mm; N = 525570 cycles; da/dN = 0.00015 mm/cycle



a = 15.78 mm; N = 527420 cycles; da/dN = 0.00038 mm/cycle



a = 16.01 mm; N = 529810 cycles; da/dN = 0.00010 mm/cycle



a = 17.18 mm; N = 531880 cycles; da/dN = 0.00057 mm/cycle

**Figure 11** Thermoelastic data recorded from a welded steel specimen at a range of growth rates (scale = 15.2 pixels/mm)

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