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# Optimal Control Design for Robust Fuzzy Friction Compensation in a Robot Joint

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***Abstract*—This paper presents a methodology for the compensation of nonlinear friction in a robot joint structure based on a fuzzy local modeling technique. To enhance the tracking performance of the robot joint, a dynamic model is derived from the local physical properties of friction. The model is the basis of a precompensator taking into account the dynamics of the overall corrected system by means of a minor loop. The proposed structure does not claim to faithfully reproduce complex phenomena driven by friction. However, the linearity of the local models simplifies the design and implementation of the observer, and its estimation capabilities are improved by the nonlinear integral gain. The controller can then be robustly synthesized using linear matrix inequalities to cancel the effects of inexact friction compensation. Experimental tests conducted on a robot joint with a high level of friction demonstrate the effectiveness of the proposed fuzzy observer-based control strategy for tracking system trajectories when operating in zero-velocity regions and during motion reversals.**

***Index Terms*—Friction compensation, fuzzy modeling, fuzzy observers, linear matrix inequality (LMI), optimal $H_\infty$ control.**

## I. Introduction

IN CONTROL applications involving small displacement, low velocities, and motion reversal, friction modeling and compensation is of paramount importance. In particular, many physical phenomena such as stiction and presliding displacement can have a considerable influence on the system performance and stability; this can mainly result in stick-slip motions. In mechanical systems, nonlinearities are considered as a serious issue and have been the center of attention for many years. The large amount of research dealing with the problem has led to the development of various compensating strategies of nonlinear friction [1]–[3]. Some of the proposed approaches are based on reasonably accurate modeling of the nonlinearity, whereas others have considered the friction as part of the disturbances acting on the system [4]. In this case, a disturbance-rejection technique [5] or a nonlinear controller can be applied to improve the system performance [6]–[8]. In the first approach, friction is seen as a physical phenomenon characterized by microsliding displacements, varying breakaway force, and frictional lag. This has motivated the use of a dynamic model instead of the classical static friction–velocity map. Dynamic models have essentially been developed to give a better description of friction phenomena in mechanical systems characterized by the following physical observations:

1) presliding displacement: motion during stiction with contact deformation at zero velocity where friction is only a function of displacement;
2) frictional memory: effect observed in the form of hysteresis loops relating friction to input velocities.

Starting with the Dahl model [9], many dynamic models have been proposed: LuGre model [10], Leuven model, and many others [11]–[13]. In fact, these proposed dynamic models claim fidelity for the reproduction of friction behavior; however, the precision required in the context of friction compensation is associated with considerable identification effort due to the model complexity. Furthermore, the control algorithms based on these models are even more complicated at the design level and during implementation.

The idea is to represent local friction behavior by a dynamic linear model and then design a local friction observer for each model; the overall observer is constructed using the principle of parallel-distributed compensation resulting in a local-based friction compensator. Based on the general Stribeck [14], [15] curve with Dahl effects [9] and inspired from the dynamic nature of the bristle interpretation of friction phenomena [16], an equivalent dynamic model of nonlinear friction is designed to cancel the friction in the robot joint at low velocities. This model is used with a tracking controller that is primarily considered in the controlled robot joint.

The rest of this paper is organized as follows: Section II introduces a dynamic structure of friction in its general form in a simple model of a single robot joint. In Section III, the local modeling approach is then developed taking into account the identified friction behavior at different velocities. As a modeling control approach is based on dynamic fuzzy models, the friction parameters are locally identified for the model definition and used afterward for the design of a fuzzy observer of friction forces for compensation purposes. The overall control scheme is the sum of the nonlinear compensating term provided by the proposed observer that compensates the major part of friction and a robust $H_\infty$ controller design based on a linear matrix inequality (LMI) approach for disturbances and uncertain compensated term rejection in an outer loop. Finally, Section IV

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presents some experimental results to validate the control-system effectiveness in tracking different velocity ranges.

## II. Friction Dynamics and Robot Joint Modeling

Since the following development concerns a friction compensation task, we consider a single robot joint's dynamics, which can be described by

$$J\ddot{q} = \tau - F + \delta_0(t, q, \dot{q}, \ldots) \quad (1)$$

where $J$ is the inertia of the joint, $\tau$ is the control signal, $F$ represents the system's friction forces, $\delta_0$ is an unknown bounded function considered for the robust control design in Section III, which includes all disturbances and other nonlinear dynamics of the robot joint after cancellation, and $q$, $\dot{q}$, and $\ddot{q}$ are the position, velocity, and acceleration, respectively.

In the robot joint, friction dynamics can be expressed as

$$\begin{aligned} \dot{z} &= \eta(z, q, \dot{q}) \\ F &= N(z, q, \dot{q}) \end{aligned} \quad (2)$$

where $z$ represents an internal nonmeasurable state of friction [10]; $\eta$ and $N$ are nonlinear functions of $z$, $q$, and $\dot{q}$, which may also include hybrid dynamics usually needed for a more faithful reproduction of the friction physical behavior. It should be emphasized that this form represents a single-state dynamic model similar to many friction models like the Dahl and LuGre models. Furthermore, it is natural to see this model as a general form of these models, and most of the analyses related to the stability, passivity, and mathematical properties are directly applicable to this model. Therefore, in this paper, a friction compensator is proposed based on the fuzzy model structure, and an optimal controller design to guarantee the stability of a precompensated system is then reviewed.

The complex model structure is decomposed into a series of linear state-space time-invariant models. This will hold inside a set of velocities where the size of each set is decided according to how fast the dynamics of the identified input–output map is. For the friction model, this means that more models are required for a low-velocity region, i.e., the region where friction is known to be highly nonlinear.

Using local approximation techniques, (2) can be expressed in the form of a linear state-space model using a set of if–then rules, i.e.,

for rule $i = 1, \ldots, n$

$$\text{if} \quad \dot{q} \text{ is } \Omega_i \qquad \text{then} \qquad \begin{aligned} \dot{z} &= a_i z + b_i \dot{q} \\ F &= c_i z + d_i \dot{q} \end{aligned} \quad (3)$$

where $\Omega_i$ is the fuzzy set of velocities associated with the local dynamics; $a_i$, $b_i$, $c_i$, and $d_i$ are the parameters of the proposed model that are able to describe friction characteristics locally, and, consequently, they will be kept constant inside the equivalent set $\Omega_i$, which will be defined in Section III. Now, let $\mu_i(\dot{q})$ be the normalized membership function of the inferred fuzzy set $\Omega_i$, where $\Omega_{\text{all}} = \prod_{i=1}^{n} \Omega_i$ denotes the overall operating range of velocities of the considered system.

By applying a standard fuzzy inference method based on a singleton fuzzifier, product fuzzy inference, and a center average defuzzifier, the mechanism of estimation is an interpolation of all the identified local models along the operating range, i.e., (2) can be accurately reproduced by means of fuzzy dynamic models. However, it will strongly depend on the number of dynamic models used, the membership functions, and the identification method used [17]. However, the discontinuity occurring at zero velocity can be a big challenge, and switching functions are usually used as a solution to this problem [18], [19], i.e.,

$$\begin{aligned} \dot{z} &= \sum_{i=1}^{n} \mu_i(\dot{q}) a_i z + \sum_{i=1}^{n} \mu_i(\dot{q}) b_i \dot{q} \\ F &= \sum_{i=1}^{n} \mu_i(\dot{q}) c_i z + \sum_{i=1}^{n} \mu_i(\dot{q}) d_i \dot{q} \end{aligned} \quad (4)$$

where $\mu_i$ denotes the membership functions.

Fuzzy models are known to be universal function approximators [20], and this property gives (4) the ability to faithfully reproduce (2) by using some available tools for parameter identification and tuning, such as the adaptive neural fuzzy inference systems or genetic algorithms. However, the effort made to refine the model can be seriously compromised by the varying nature of friction. For this reason, local models are meant to reproduce the main feature of friction inside a certain set, and the simplicity of the chosen dynamics allows relatively easy design of the compensator.

Since the friction model structure has been established, we can define the parameters in (4) depending on the operating input velocity, namely, the stiction level, the presliding displacement, and the Stribeck effect. This will be detailed in Section III. Some effects such as varying breakaway force and frictional lag will not be taken into account in the design since we can avoid the complexity without affecting the performance of the designed compensator.

## III. Local-Based-Compensation Approach

The proposed friction compensation scheme is composed of two main control actions: 1) a nonlinear friction estimator generating a signal to be rejected for the elimination of friction-induced errors and 2) an optimal $H_\infty$ controller based on LMI design under the inexact friction compensation assumption.

Defining four parameters plus the size of each set $\Omega_i$ can be quite challenging, particularly if the model is expected to be reasonably accurate. Therefore, the method developed in this paper requires that the model represents the main features of friction inside each local set. The observer based on this model structure can then be refined using a set of gains to improve its convergence to realistic values. Since we can have prior knowledge about the main feature of friction and the velocity–friction torque map, it is possible to identify the model parameters in two successive steps: for the presliding displacement regime running at very low velocities and for higher velocities equivalent to the sliding regime.

### *A. Local Approach Applied to Dynamic Friction Modeling*

In the zero-velocity zone and during microsliding motions, the frictional force in (4) can be expressed by the Dahl-effect

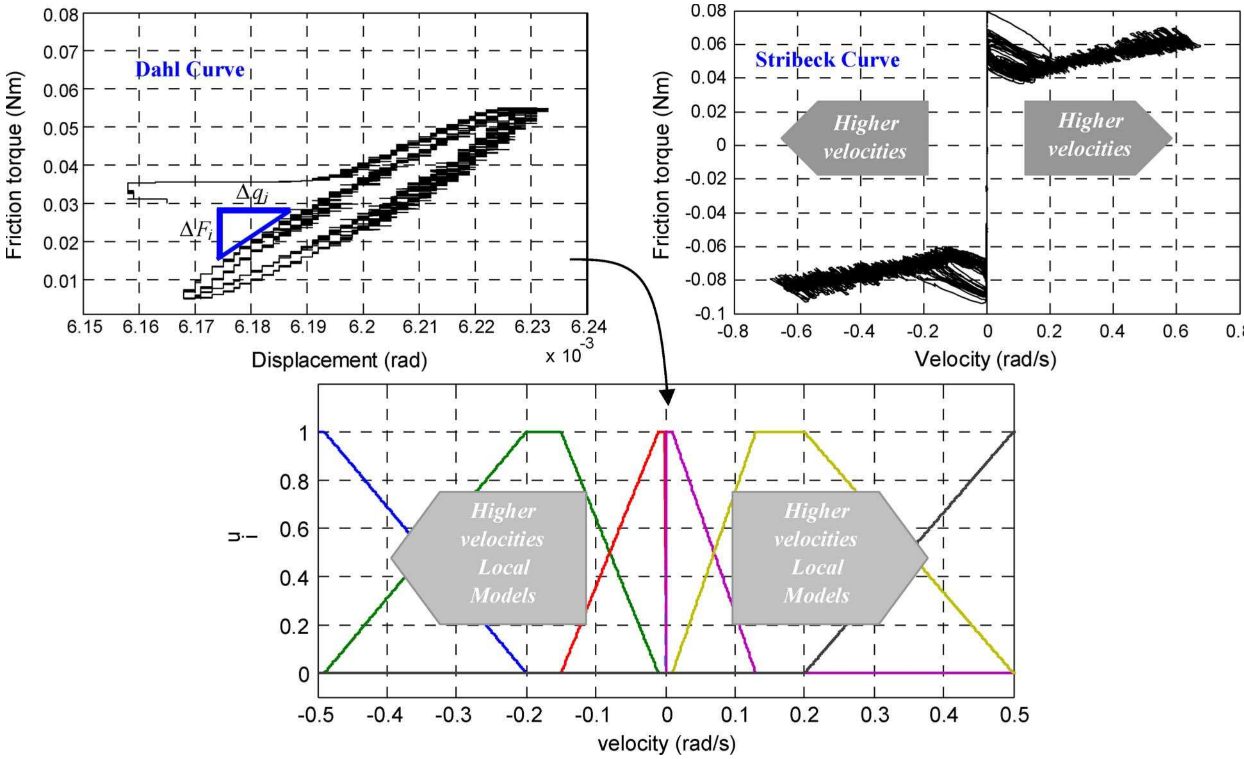


Fig. 1. Basic idea in modeling friction phenomena. At zero velocity, friction is basically related to position and becomes exclusively dependent on the velocity in the sliding regime; the membership functions allow a mixed regime and soft switching between dynamics and ensure good representation of friction forces in the robot joint. (Top right) Stribeck curve characterizing friction–velocity relationship; it might be clearly asymmetric in reality. (Top left) Dahl curve illustrating microdisplacements regime ($\sigma_0 = \Delta F_i/\Delta q_i$) in the robot joint: experimental curve.

formula, where friction is a function of displacement, and the dynamics due to velocity are not taken into account. This can be written as

$$F = \sigma_0 z \approx \sigma_0 q. \tag{5}$$

Therefore, $z \approx q$, which allows us to determine $c_i = \sigma_0 = \Delta F/\Delta q$ from the Dahl curve shown in Fig. 1. Similarly, the internal state of friction $z$ is consequently equal to the displacement, and the friction model dynamics can be completed as follows:

$$\dot{z} \approx \dot{q}. \tag{6}$$

Comparing (4) and (6) gives the parameter $b_i = 1$ that holds for presliding regime. Note that (5) and (6) are a special case of (4).

For simplicity and knowing that the sum of membership functions $\mu_i(\dot{q})$ at any point of the operating domain is equal to 1, parameters $b_i$ and $c_i$ can be kept constant for all operating points without losing the capability of the proposed structure to describe the friction behavior. Basically, the main features of nonlinear friction are captured by internal state $z$ characterizing the stiff nature of friction, which is combined, in the proposed structure, with a component having a damping effect on the system.

At relatively higher velocities, friction is more velocity dependent, and, for the steady-state regime, two domains can be distinguished. 1) At relatively low velocities, the nonlinear part is characterized by mixed dynamics and a negative damping term due to the Stribeck velocity; 2) at higher velocities, the linear part is characterized only by the viscous friction as a positive damping term, as shown in Fig. 1.

The steady-state characteristics of the proposed structure of (4) may then be found. By letting $dq/dt = 0$ and taking into account the parameters previously identified using (5) and (6), we can write

$$F_{\text{ss}} = \left( -\sum_{i=1}^{n} \mu_i(\dot{q}) \frac{\sigma_0}{a_i} + \sum_{i=1}^{n} \mu_i(\dot{q}) d_i \right) \dot{q}. \tag{7}$$

The rest of the parameters can be deduced from (7) by comparison using the identified level of friction. The steady-state friction can be represented by a static map between friction and velocity; it takes the so-called Stribeck curve form, which is experimentally identified at constant velocities. Thus, $d_i = d_0$, which represents the damping term associated with the viscous friction at relatively high velocities, and $a_i = \alpha_i$ will be varying with the velocity and takes the value calculated from $F_i$, which represents the friction level at the velocity $\dot{q}_i$, i.e.,

$$\alpha_i = -\frac{\sigma_0}{F_i} \dot{q}_i. \tag{8}$$

$\alpha_i$ can be defined in a bounded domain described by the following inequality: $-(\sigma_0/F_S) \geq (\alpha_i/|\dot{q}_i|) \geq -(\sigma_0/F_C)$ with respect to all operating points except for $\dot{q} = 0$ (rad/s) and $F_i = 0$ $(\text{N} \cdot \text{m})$, where $F_C$ and $F_S$ represent, in this case, the levels of the Coulomb and static frictions, respectively.

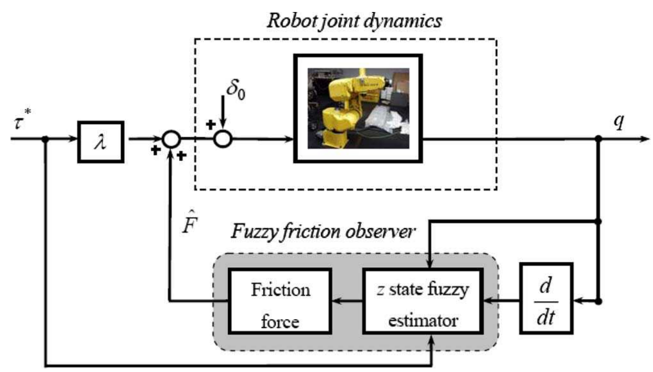


Fig. 2. Friction compensation in robot joint based on the local design: minor loop control.

The final form describing the internal state dynamics and the output of the proposed model structure can be written after substitution of all the identified parameters as

$$\dot{z} = \sum_{i=1}^{n} \mu_i(\dot{q})\alpha_i z + \dot{q}$$
$$F = \sigma_0 z + d_0 \dot{q}. \tag{9}$$

Equation (8) is important since it represents the bounds that encompass the nonlinear behavior of friction, which has a direct influence on the stability of the overall proposed control system. The validation of this model can be done via a simple comparison of its parameters with other existing models, so that the mathematical properties such us linearization, passivity, and stability can be shown. Some comparative simulations can be found in [21]. Further development for the generalization of the model and the validation is currently being performed.

### B. Observer-Based Friction Compensator Design

The proposed friction compensator is derived from a reformulation of the friction dynamics in (9). A rejection of disturbances caused by inexact friction estimation is achieved by the compensating gains acting as a local integral action [22]. These gains are chosen within a predefined domain, and their values will be fixed during the experiments to reach the best performance. The outer closed-loop system will satisfy the robust stability condition under the following assumptions: 1) inexact compensation resulting from uncertain estimation, namely, $\Delta\sigma_0$ and $\Delta d_0$; 2) varying parameters resulting from the fuzzy modeling $\alpha_i$; 3) controller-design parameters in the precompensation loop, such as $l_i$, $\kappa_i$, and $\kappa_i'$; and 4) existence of disturbances $\delta_0$. Fig. 2 shows the proposed friction compensation control scheme applied to the robot joint. Fig. 3 shows the local representation of the frequency response of the system with friction before and after introducing the precompensator, which demonstrates a clear improvement on the local dynamics and allows the robust design of the control law considering uncertainties in the compensation.

The applied control ensuring the quadratic stability of the system given by (1) with friction modeled in (9) yields the following dynamics:

$$\text{if} \quad \dot{q} = \dot{q}_i$$
$$\text{then} \quad \dot{\hat{z}} = \alpha_i \hat{z} + \kappa_i q + \dot{q} - l_i . \tau^* \tag{10}$$
$$\hat{F} = \lambda\tau^* + \kappa_i' q + \sigma_0 \hat{z} + d_0 \dot{q}. \tag{11}$$

In (11), $\lambda > 0$ is a fixed positive gain of the feedback controller that can be defined at the robust $H_\infty$ design stage; $\kappa_i$ and $\kappa_i'$ are small positive gains added to the dynamics of the local model to satisfy the quadratic stability criteria and $H_\infty$ control performance for the resulting polytopic uncertain form described by (1), (10), and (11). The precompensated dynamic model is characterized by bounded disturbances and uncertainty boxes that can be classified into two types: 1) parameters related to modeling uncertainties and mismatch in friction compensation such as $\Delta\sigma_0$ and $\Delta d_0$; they can be varying locally or set to a value that represents the worst mismatch situation for all the operating domain; and 2) design parameters such as $l_i$, $\kappa_i$, and $\kappa_i'$ that will be defined locally and that can be decided later in the experiments after the calculation of $H_\infty$ controller gains using the LMI approach. $\kappa_i$ and $\kappa_i'$ are set to a small value around zero velocity and then set to zero for the remaining operating velocities; they are necessary to find a solution to the set of LMIs into the overall domain. The optimal control problem is then formulated as follows: seeking a single quadratic Lyapunov function that enforces the design objectives for all plants in the predefined polytope and, in other terms, finding a stabilizing state feedback control $\tau^*$ that minimizes the closed-loop rms gain of the plant from $\xi_\infty = q$ to $\delta_0$. This problem can be transformed into an LMI problem, and the rms gain is guaranteed not to exceed some prescribed performance value $\gamma$ if there exists a positive matrix $P_\infty$ that satisfies the inequalities [23], shown at the bottom of the page, where all parameters for the robust design are given as follows:

$$A_i = \begin{bmatrix} \alpha_i & \kappa_i & 1 \\ 0 & 0 & 1 \\ \Delta\sigma_0 & \kappa_i' & \Delta d_0 \end{bmatrix}$$
$$B_{1i} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad B_{2i} = \begin{bmatrix} l_i \\ 0 \\ \lambda \end{bmatrix}$$
$$C_1' = [0 \quad 1 \quad 0]$$
$$D_1' = 0 \qquad D_2' = [0 \quad 0 \quad 0].$$

The estimation mechanism in (10) uses the dynamics of (9) added to an error-compensating term modulated by a local

$$\begin{bmatrix} (A_i + B_{2i}K)P_\infty + P_\infty(A_i + B_{2i}K)^T & B_{i1} & P_\infty\left(C_1' + D_2'K\right)^T \\ B_{1i}^T & -I & D_1'^T \\ \left(C_1' + D_2'K\right)P_\infty & D_1' & -\gamma^2 I \end{bmatrix} < 0, \qquad P_\infty > 0$$

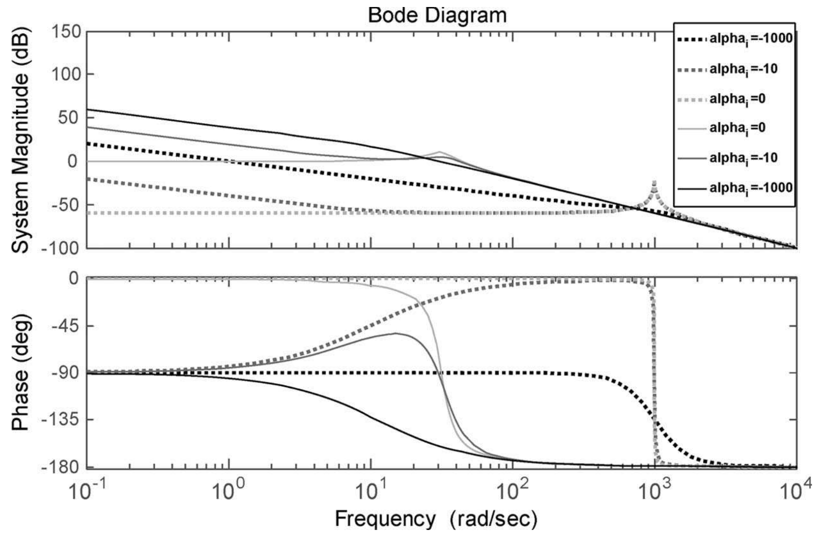


Fig. 3. Frequency response of the considered system (dotted line $\tau/q$) before and (solid line $\tau^*/q$) after precompensation; three local models are shown for different ranges of velocities (at presliding regime and for higher velocities).

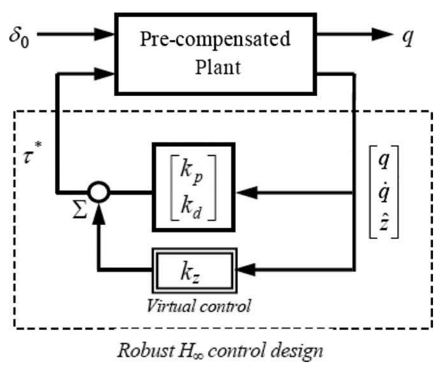


Fig. 4. LMI-based robust controller: outer loop design.

gain $l_i$ and local-feedback terms. The local gains can be derived from linear design techniques to ensure the stable behavior of the inner loop representing the precompensated system with friction separately. Then, the stability of the overall controlled system is taken into account by solving the LMI, and the existence of a Lyapunov quadratic matrix $P_\infty$ leads to the following overall controller expression: $\tau = \lambda\tau^* + \hat{F}$, where $\lambda$ is a positive gain of the controller that will be set to 0.5 and

$$\tau^* = KX = k_p q + k_d \dot{q} + k_z \hat{z}$$
$$\dot{\hat{z}} = \sum_{i=1}^{n} \mu_i(\dot{q})\alpha_i \hat{z} + \sum_{i=1}^{n} \mu_i(\dot{q})\kappa_i q + \dot{q} - \sum_{i=1}^{n} \mu_i(\dot{q}) l_i . \tau^* \tag{12}$$

where $K$ represents the calculated state feedback vector of the optimal controller; $k_p$, $k_d$, and $k_z$ are the position, velocity, and friction state gains, respectively [24]. The third part of the control element $\tau^*$ in (12) is termed "virtual control" and can be seen as an additive compensation term of friction and a stabilizing part of the control at the same time.

The term "virtual control" is used to describe the fact that the state $z$ is nonmeasurable and has been introduced to describe friction. The experimental results have shown that this can bring a slight improvement in terms of disturbance rejection, although

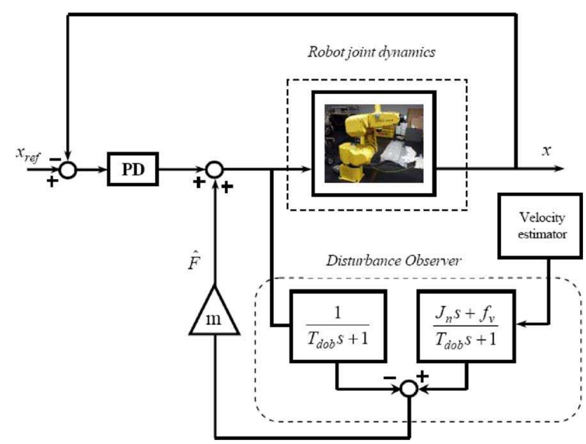


Fig. 5. Comparative control methods. $m = 0$: PD control, $m = 1$: DOB.

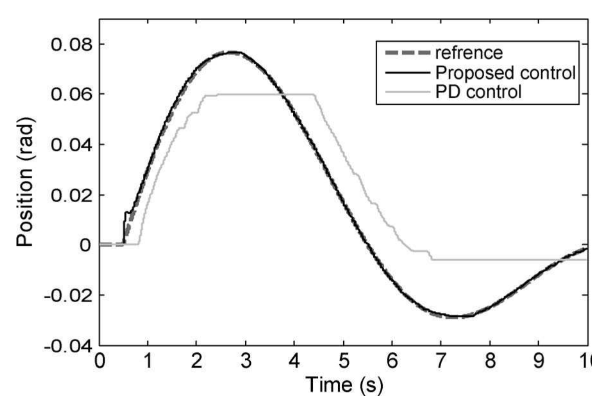


Fig. 6. Position tracking performances before and after compensation.

further experiments and analysis are needed. Furthermore, it should be noted that this scheme relies on the worst case design using local models of friction with uncertain compensation and external disturbances. In our case, the quadratic stability of the precompensated system is checked for the varying parameters resulting either from the friction compensation mismatch or the design choice. This allows us to tune and choose the observer

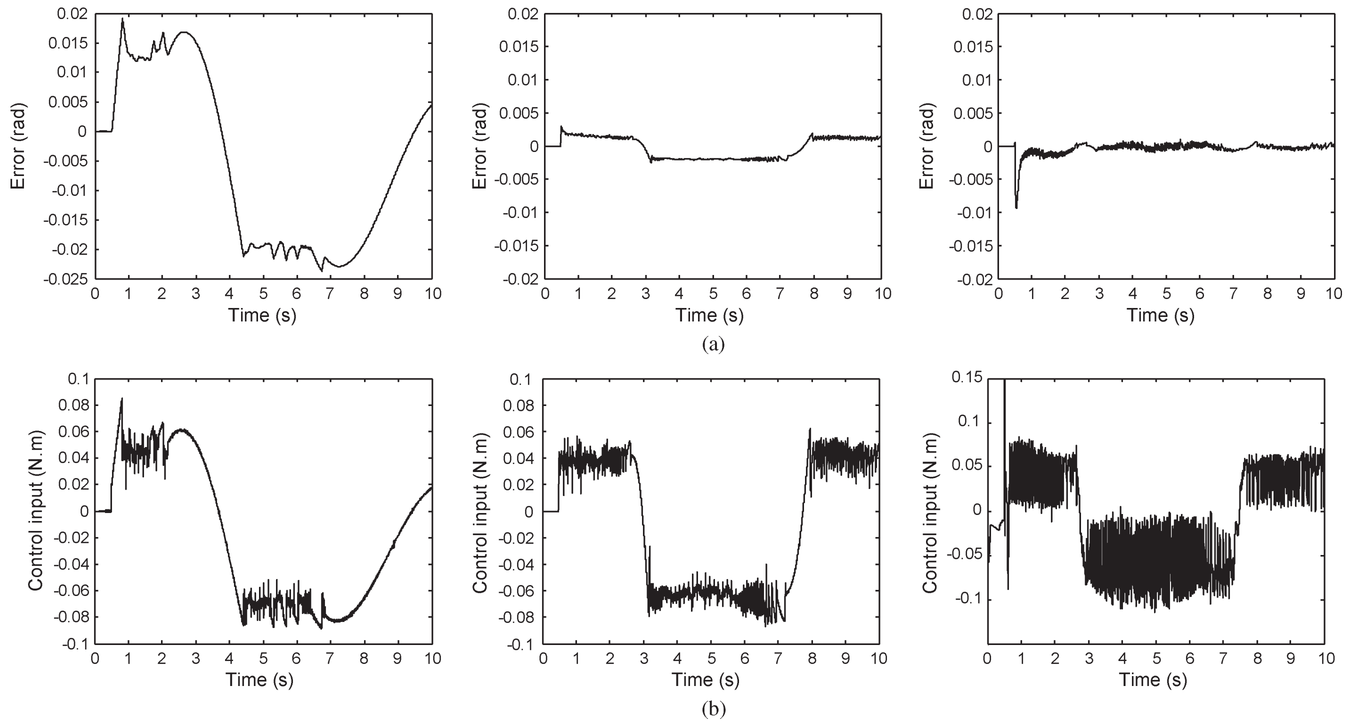


Fig. 7. Experimental results showing tracking performances. (Left) Under PD control. (Middle) DOB. (Right) Proposed method.

local gains $l_i$ that ensure the best tracking performance without compromising the stability of the overall system.

For the velocity range of $[-0.5, 0.5]$ rad/s, seven local models are used to reproduce the behavior of the nonlinear shape of the Stribeck curve that characterizes the dynamic friction inside the slow-motion regime set, including the reversal velocity region. By applying a standard fuzzy inference method, i.e., using a singleton fuzzifier, product fuzzy inference, and a center average defuzzifier, the mechanism of estimation will work as an interpolator of all the relevant linear estimators [25]. The control action combines a direct friction compensation ensured by the fuzzy observer and the action of an optimal tracking controller shown in Fig. 4.

## IV. Experimental Results and Evaluation

Experiments were performed on a joint of a FANUC robot to evaluate the proposed control strategy. The experimental setup consists of a 700-MHz PC running the operating system RT-Linux, connected by an optical cable to a digital servo adapter that provides signal interfacing between the PC and a servo-amplifier module. The control algorithm is implemented in C language. The gains of the observer were tuned in during the experiments after defining all the model and controller parameters. Since the current work deals with friction compensation, only one isolated joint will be used in the experiments, and the results can be extended to other joints. We should also note that the extension to other joints can be fruitful for relatively slow motions since other nonlinear dynamics are velocity dependent and can be seen as minor disturbances; otherwise, they should be compensated beforehand. The control algorithm, as implemented, depends on the velocity, which is, by the way, estimated using the signal of a position encoder and can have a direct influence on the quality of the control signal. A good estimation of the velocity by differentiation-low-pass filtering of the signal acquired from the encoder is then used for better signal quality.

To evaluate the proposed control designed for friction compensation, experiments were performed on a robot joint system for a trajectory tracking task, with different velocity ranges. Comparisons with other control methods, namely, proportional derivative (PD) control and disturbance observer (DOB)-based control, are reported. Fig. 5 shows the results obtained with PD control for $m = 0$ and DOB control for $m \neq 0$. We used linear techniques to determine the parameters of the comparative control. Basically, these methods use only the linear parts of the considered robot joint dynamics, so that the pole placement used for PD design or the inverse model to form the DOB filters was calculated using the nominal parameters of joint inertia $J_n$ and viscous friction $f_v$. Note that only the linear part of the system consisting of inertia and viscous friction as a damping factor is used for the control design in Fig. 4 [26]. The reference trajectory $q_{\text{ref}} = (1 - 0.1(t - 0.5)) \sin(2\pi f(t - 0.5))$ is shown in Fig. 6. Therefore, the robot joint will be operated in the low-velocity region with $f = 0.1$ Hz and will be performing many velocity reversals during the experiment.

Since the robot joint comprises a considerable friction component, PD control has serious limitations and shows residual-tracking errors that cannot be eliminated even with high PD gains.

Around zero velocity, it is clear that the tracking performance of the robot joint is severely affected by friction, as shown in Fig. 7. The fuzzy observer with a gain scheduling property is proposed as an efficient way to compensate friction errors without using highly excessive control input for the local operating range.

TABLE I
ROOT MEAN-SQUARE TRACKING ERRORS

| PD Control | DOB control | LMI based (proposed) |
|---|---|---|
| 0.0125 rad | 0.0013 rad | 0.0006 rad |

TABLE II
CONTROLLER PARAMETERS

| | |
|---|---|
| $K_P$ (N.m/rad) | 3.6 |
| $K_D$ (N.m.s/rad) | 0.21 |
| $k_p$(N.m/rad), $k_d$, (N.m.s/rad) $k_z$ (N.m/rad) | 1.8, 0.14, 1574.9 |
| $\lambda$ | 0.5 |
| $\kappa_i$ , $\kappa'_i$ | 0.001, 0.02 |
| $\dot{q}_i$ (rad/s) | -0.5,-0.1,-0.01,-0.001,0,0.001,0.01,0.1,0.5 |
| $l_i$ | 0,-0.5,-1.5,-4.5,-1.5,0.5,0 |

Fig. 7 shows a clear reduction in the friction-induced error. This can be explained by the fact that a good estimation of friction by the fuzzy observer and the disturbance rejection leads to robust performances. There is a large residual error due to friction at zero and low velocity; this error can be minimized by the use of a disturbance observer, but the performances reached by the DOB remain limited due to the highly nonlinear nature of friction in the low-velocity regime for the chosen reference trajectory. After robust friction compensation, the tracking error is bounded and minimized to a value less than 0.002 rad, and the robot joint responds more smoothly during velocity reversal.

The tracking performances can be measured by the calculated recursive mean-square error that is reported for all cases in Table I.

By using compensating gains in the low-velocity region, the observer was able to give better results in friction estimation and reduction of the tracking error. On the other hand, the $H_\infty$ controller has been designed to handle a bounded compensation mismatch since the friction phenomenon itself is inherently variable and very difficult to model with accuracy.

## V. CONCLUSION

A dynamic friction structure based on a local modeling approach has been proposed for the compensation of friction in motion-control systems. Motivated by the dynamic nature of friction, the estimation mechanism uses local properties and adds a component to the control signal to cancel friction effects at low velocities. The proposed control scheme relies on local identified parameters and a relatively simpler design technique than other model-based friction compensation methods. On the other hand, the robust control design via the LMI approach ensures robustness and performance under some severe assumptions like uncertain friction compensation and fuzzy varying gains for the observer. The number of tuning parameters (see Table II) is related also to the number of models and, therefore, can increase the complexity of the design. This can be the basis for further developments and investigations, and a robust adaptive control can be proposed.

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